



# Vehicles Dynamics, Planning and Control of Robotic Cars

Final Report

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# Assignment 1

## Tire Model Exercises - 1

### 1.1 Exercise 1 – Pure Longitudinal Slip

Q. Using the Pacejka Magic Formula, plot the longitudinal tire force  $F_{x0}$  obtained in pure longitudinal slip conditions, as a function of slip  $\kappa \in [-1, 1]$ . Which comments are you able to make about the obtained graph?

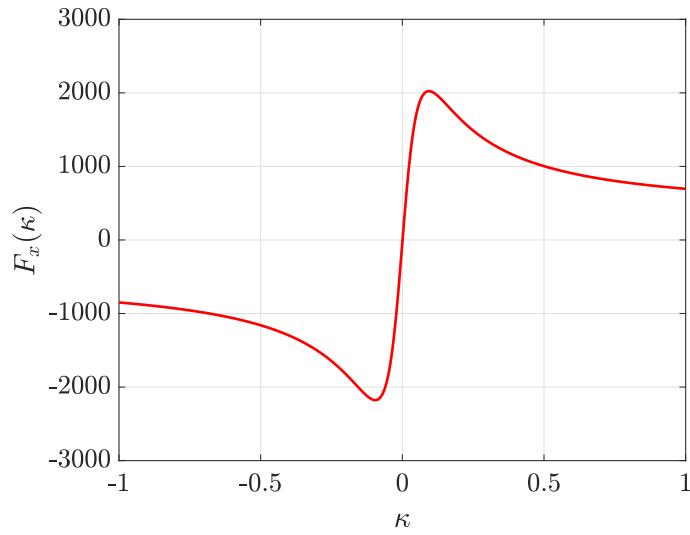


Figure 1.1:  $F_{x0}$  as a function of slip

When  $\kappa$  (longitudinal slip) increases,  $F_x$  grows until it reaches a saturation limit. Furthermore, the slip zone grows, and the adherence area decreases. That means, the total tire force  $F_x$  keeps growing until it reaches a peak and a saturation limit as shown in the graph below.

$\kappa$  (longitudinal slip) is positive when the tire is accelerating, negative during braking, and reaches -1 when the wheel locks.

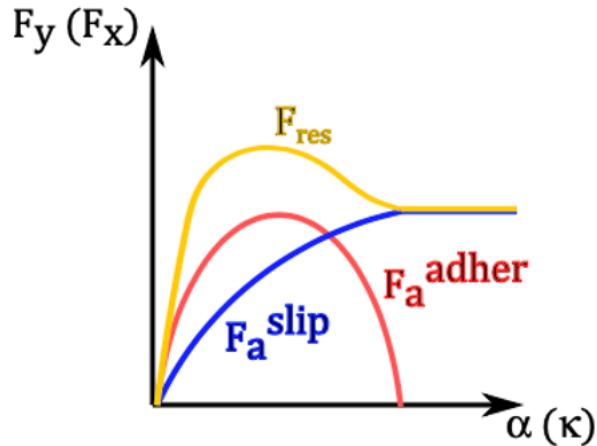


Figure 1.2: Slip and adherence forces

**Q.** If you were supposed to design a traction control system for maximizing vehicle longitudinal acceleration, which would be the target value of longitudinal slip  $\kappa$  that you would try to achieve?

Acceleration is directly proportional to the force [ $F=ma$ ] if the mass is constant. If I want to maximize the vehicle longitudinal acceleration, I would need to maximize the longitudinal force [ $F_x$ ].  $F_x$  is highest at the saturation limit, which in this example happens at slip  $\kappa = 0.094$ , and yields an  $F_x$  of 2022.99 N.

**Q.** Assuming that wheel rotational speed is  $\omega = 70$  rad/s, tire effective rolling radius is  $R_e = 0.2$  m, while the longitudinal component of tire contact point speed  $v_{Cx} = 13$  m/s, compute the longitudinal slip  $\kappa$ . In these conditions, is the wheel accelerating, braking or is it in pure rolling? Compute also the corresponding longitudinal tire force  $F_{x0}$

Using Matlab, the calculated longitudinal slip  $\kappa = 0.0769$  and the calculated longitudinal force  $F_x$  is = 1990.65 N. the longitudinal slip is positive ( $>0$ ), which means that the wheel is accelerating.

**Q.** Compute the cornering stiffness  $Cf\kappa$ , that is the derivative for  $\kappa = 0$  of the  $F_{x0}$ . Up to which value of  $\kappa$  is the linear approximation of Pacejka curve acceptable?

The cornering Stiffness  $C_f\kappa$  is equal to the derivative of the longitudinal force with respect to the longitudinal slip when the longitudinal slip is equal to zero. This means its equal to the slope at the origin ( $x=y=0$ ) or equal to BCD function as shown in Figure 1.3. The calculated cornering stiffness is  $C_f\kappa = 47909.4$

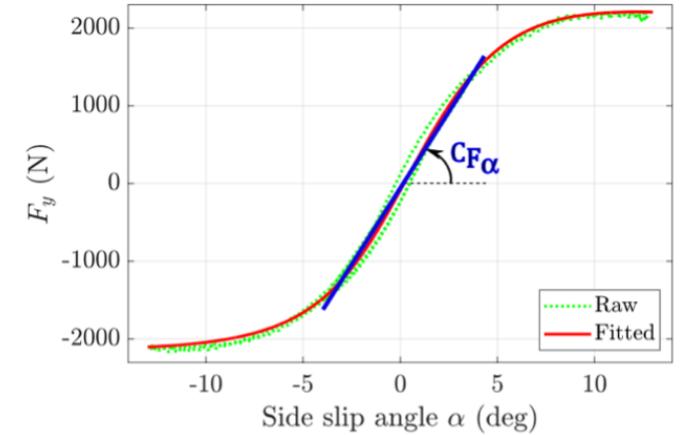


Figure 1.3: cornering stiffness as a linear approximation

The linear approximation allows to neglect the complex Pacejka Magic Formula, but it is valid only for small  $\kappa$ . At  $\kappa = 0.02$ , the percent difference is already at 10% as shown in Figure 1.4.

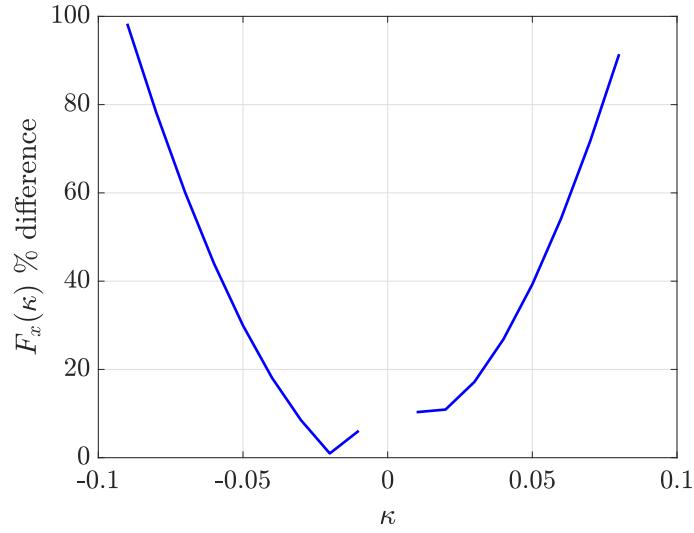


Figure 1.4: % difference in  $\kappa$  using linear approximation vs Pacejka formula

## 1.2 Exercise 2 - Combined Slip

**Q.** Assume that the tire contact point velocity components along the tire x and y axes are  $v_{Cx} = 15 \text{ m/s}$  and  $v_{Cy} = -1.3 \text{ m/s}$ , respectively. Calculate the side slip angle  $\alpha$ . Moreover, compute the combined tire force  $F_x$  using this value of  $\alpha$ , for a longitudinal slip  $\kappa = 0.08$ .

Alpha can be calculated using the practical slip approach:

$$\text{side slip angle } \alpha = -\arctan\left(\frac{v_{sy}}{v_{Cx}}\right) = -\arctan\left(\frac{v_{Cy}}{v_{Cx}}\right) \quad (1.1)$$

Using equation 1.2 to calculate  $G_{xa}$  (weighing function). Once calculated,  $F_{x0}$  can now be multiplied by  $G_{xa}$  (weighing function) to get the combined tire force  $F_x$ .

$$G_{xa} = -D_{xa} \cos(C_{xa} \arctan(B_{xa}(\alpha + S_{Hxa}))) \quad (1.2)$$

$$F_{x0} = D_x \sin(C_x \arctan(B_x \kappa_x - E_x(B_x \kappa_x - \arctan(B_x \kappa_x)))) \quad (1.3)$$

$$F_x = G_{xa} F_{x0} \quad (1.4)$$

Using Matlab to calculate the side slip angle  $\alpha$  and combined tire force:

calculated side slip alpha = 0.086451  
 calculated weighing function = 0.724417  
 calculated combined force  $F_x$  = 1450.424623

**Q.** Plot the combined longitudinal tire force  $F_x$  as a function of  $\kappa \in [-1, 1]$ , for the following levels of side slip angle  $\alpha = \{0, 2, 4, 6, 8\}$  degrees. Which comments can you make about the 5 curves obtained in this way? Finally, plot the weighting function  $G_{xa}$  as a function of  $\kappa \in [-1, 1]$  for each of the previously defined values of  $\alpha$ , and briefly comment also these 5 curves.

Figure 1.5 shows plots obtained for the combined longitudinal force  $F_x$  for each side slip angle as a function of the longitudinal slip  $\kappa$ . The maximum combined longitudinal force  $F_x$  keeps decreasing with higher side slip  $\alpha$ .

Figure 1.6 shows the plots obtained for the weighing function  $G_{xa}$  for each side slip angle as a function of the longitudinal slip  $\kappa$ . Higher side slip  $\alpha$  decreases the weighing function, which

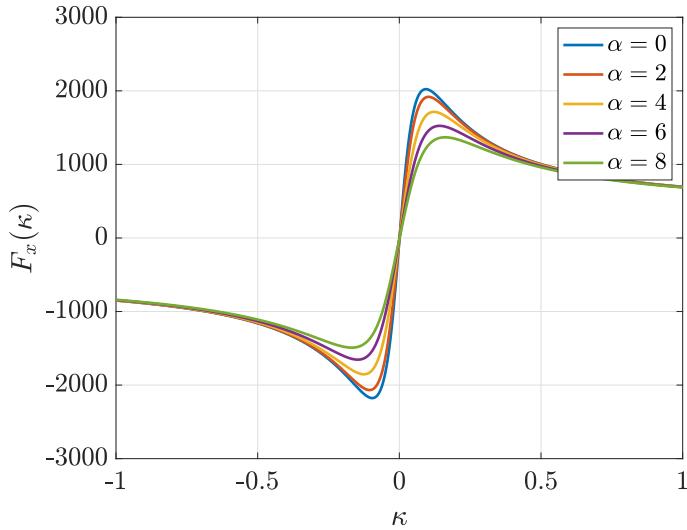


Figure 1.5: combined longitudinal force  $F_x$  as a function of  $\kappa$

in effect decreases the combined longitudinal force  $F_x$ . The effect of the weighing function is quite more potent around  $\kappa = 0$ , and that effect decreases the further away we are from it.

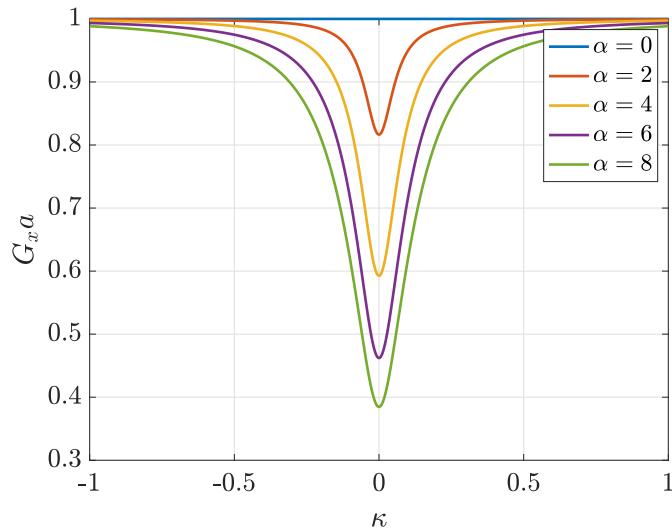


Figure 1.6: Weighing function  $G_{xa}$  as a function of  $\kappa$

# Assignment 2

## Tire Model Exercises - 2

### 2.1 Exercise 1 – Understanding Tire Data

Q. Plot the raw data in different graphs, specifically focusing on  $\kappa$ ,  $\alpha$ ,  $\gamma$ ,  $F_z$  and pressure  $P$ . Comment on what you see. What is, according to you, the main target of these tests?

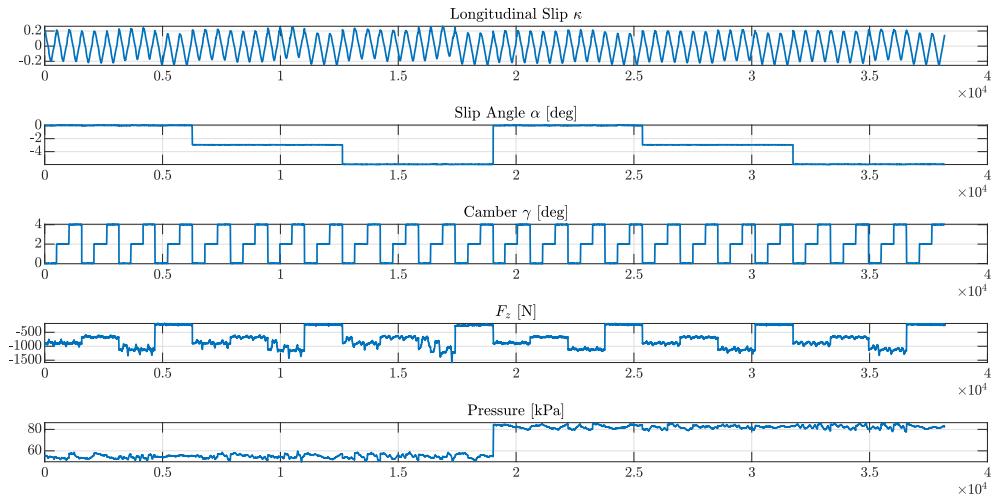


Figure 2.1: raw data

From the five plotted variables in Figure 2.1, it appears that the longitudinal slip, slip angle, camber, vertical force, and pressure are all showing repeat patterns. That means that those five variables are controlled to measure and test their effect on the longitudinal force  $F_x$ . Note that the longitudinal slip  $\kappa$  for each test spanned from 0.2 to -0.2 and then back again to 0.2.

The data extracted for the following questions only took the first half of each separate test. Furthermore, only data that had  $P = 83$  kPa were used, because the data seemed noisier with  $P = 55$  kPa, especially with  $F_z$  data.

**Q. Focus on the data with  $\alpha = 0$  and  $\gamma = 0$ , and plot the curves  $F_x$  vs  $\kappa$  for each of the 4 vertical loads  $F_z$  used in the experiments. Plot the 4 curves on the same graph, with different colors. Comment on what you see.**

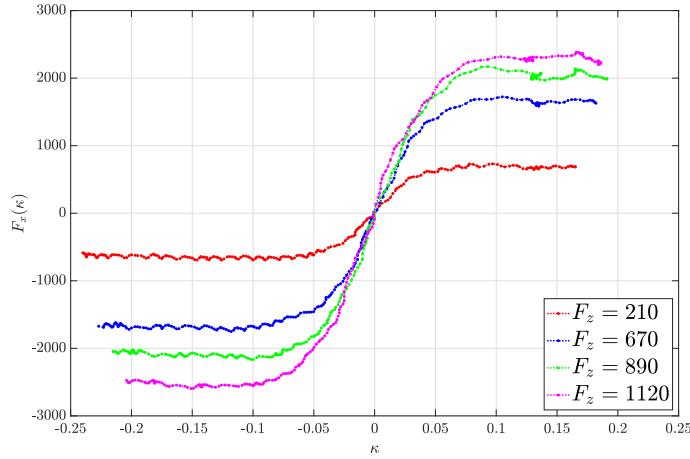


Figure 2.2:  $\kappa$  vs  $F_x$  based on vertical force

Figure 2.2 suggests that the Longitudinal Force  $F_x$  increases with the vertical force  $F_z$ . There is some dependency on the vertical force. If you double the vertical force, it does not mean the longitudinal force will double. In some parts there is a linear dependency, but in others it is not the case.

**Q. Focus on the data with  $\gamma = 0$  and  $F_z = 150$  lbf  $\approx 670$ N, and plot the curves  $F_x$  vs  $\kappa$  for each of the 3 side slip angles  $\alpha$  used in the experiments. Plot the 3 curves on the same graph, with different colors. Comment on what you see.**

The Longitudinal Force  $F_x$  in Figure 2.3 shows an inverse relationship with the side slip angle.

## 2.2 Exercise 2 - Fitting Tire Data

**Q. First consider the data with  $F_z = F_{z0} = 890$ N,  $\gamma = 0$  and  $\alpha = 0$ , and fit the coefficients  $X1 = \{p_{Cx1}, p_{dx1}, p_{Ex1}, p_{Ex4}, p_{Kx1}, p_{Hx1}, p_{Vx1}\}$ . Plot the fitted curve  $F_x$**

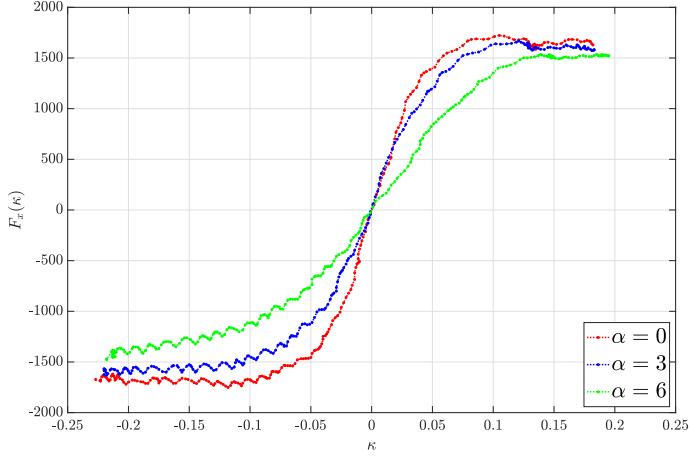


Figure 2.3:  $\kappa$  vs  $F_x$  at vertical force = 670 based on slip angle

vs  $\kappa$  that you obtained in these nominal conditions, together with the raw data.

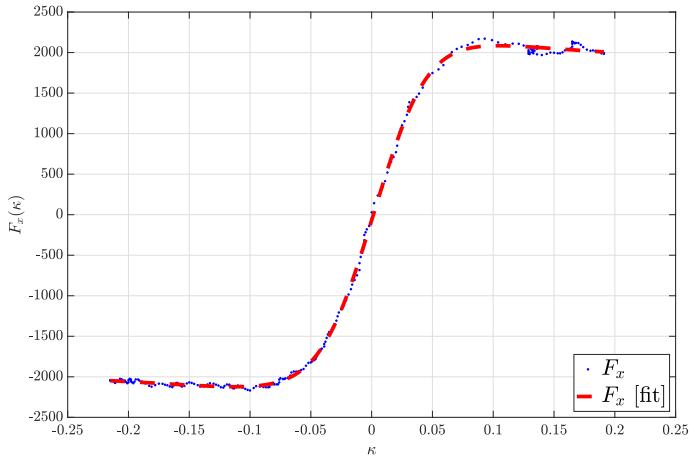


Figure 2.4: Fitted  $F_{x0}$  after optimizing first 7 parameters compared to test data vs  $\kappa$

In Figure 2.4, the data was filtered based on the following criteria:  $F_{z0} = 890$ ,  $\alpha = 0$ ,  $\gamma = 0$ . The first 7 coefficients were optimized using the *fmincon* function in Matlab. I started with a random coefficients initial guess vector. The initial guess vector has proven to be quite important as running the code multiple times showed a curve that did not fit the data at all, which means the optimizer was stuck at a local minima. However, most of the time the initial guess gave a very good fit that converged to the correct global minima. Later on I optimized the initial guesses to give consistent good fitting results based on trial and error.

**Q.** Now consider the data with the 4 different values of  $F_z$ , but still  $\gamma = 0$  (and  $\alpha = 0$ ). This enables the fitting of the parameters:  $\mathbf{X2} = \{p_{dx2}, p_{Ex2}, p_{Ex3}, p_{Hx2}, p_{Kx2}, p_{Kx3}, p_{Vx2}\}$ . Plot the fitted and raw curves  $F_{x0}$  vs  $\kappa$  for the 4 values of  $F_z$  and comment the

results.

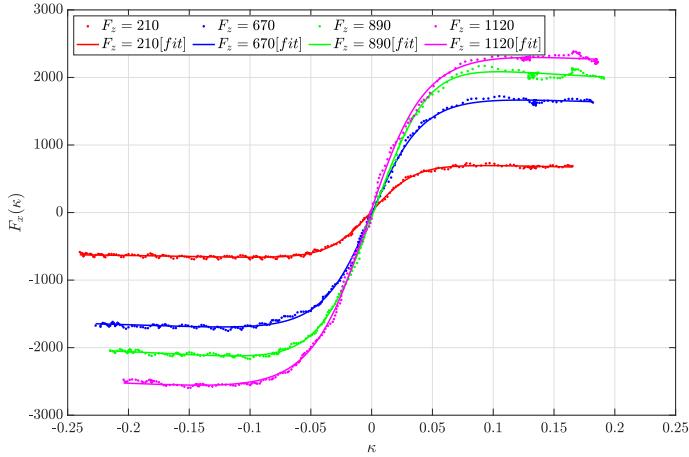


Figure 2.5: Fitted  $F_x$  vs test data after second fitting for each  $F_z$

The real  $F_x$  data for each separate  $F_z$  are plotted against the output of the second fitting in Figure 2.5. The fitted data appear to match the test data quite well. This means the fitting results are adequate to approximate the longitudinal force  $F_x$  for vertical forces  $F_z$  that are different from the originally tested one.

**Q.** Now consider the data with the 3 different values of  $\gamma$ , but with  $F_z = F_{z0}$  (and  $\alpha = 0$ ). Plot the fitted and raw curves  $F_{x0}$  vs  $\kappa$  for the 3 values of  $\gamma$  and comment the results.

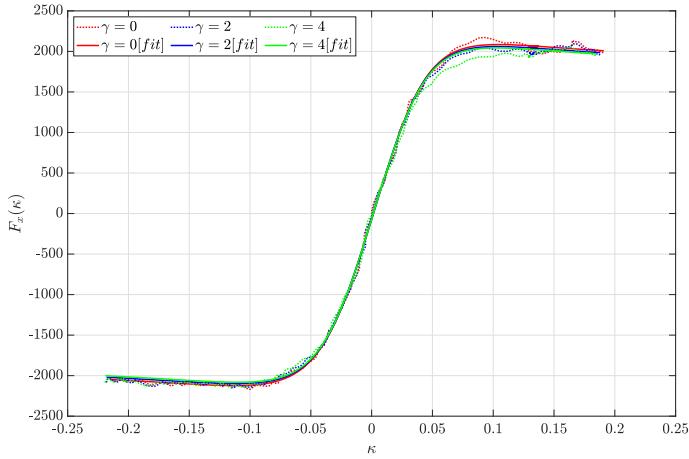


Figure 2.6: Fitted  $F_x$  vs test data after third fitting for each  $\gamma$

Figure 2.6 shows the results of the third fitting. The data from  $\gamma = 0$  fits properly as shown previously in Figure 2.4. However, the fitting on the other  $\gamma$  values did not match completely. Trying different initial values for  $p_{dx2}$  did not yield any better results.

# Assignment 3

## Vehicle Data Analysis Exercises

### 3.1 Exercise 1 – Understanding Vehicle Data

Q. Plot lateral and longitudinal velocity

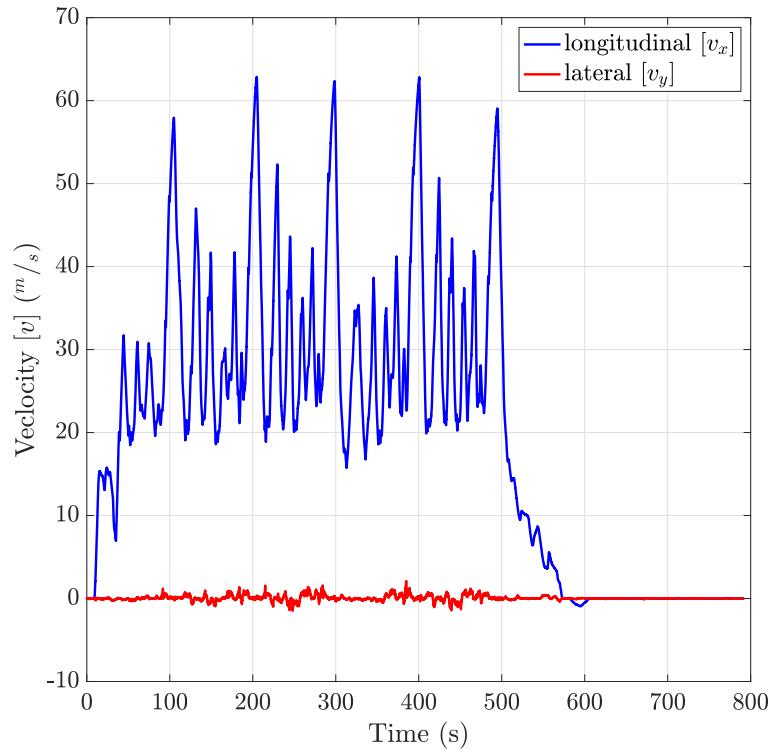


Figure 3.1: lateral and longitudinal velocity vs time

From Figure 3.1, the magnitude of the longitudinal velocity is higher than the lateral velocity.

This matches what was expected as the vehicle mainly moves longitudinally and only laterally while slipping.

**Q. Evaluate the longitudinal speed using the Hall-effect wheel speed sensors and compare the data with the INS data**

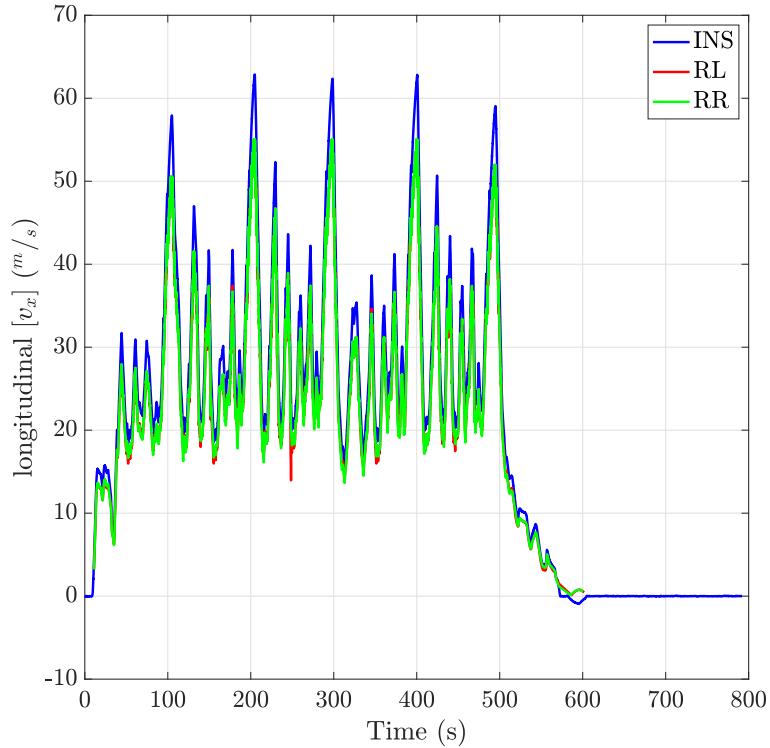


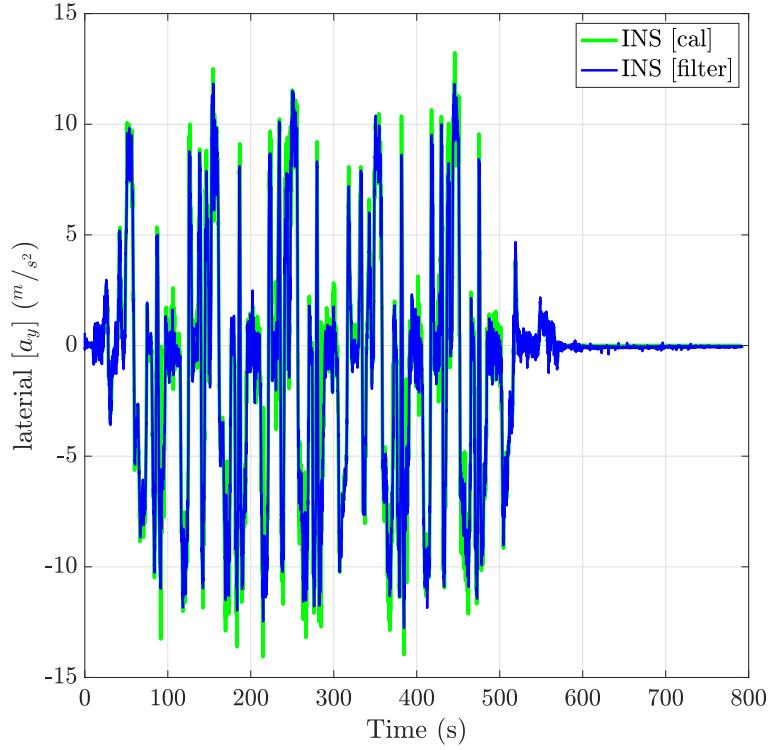
Figure 3.2: INS vs RL/RR hall effect sensors  $v_x$

The longitudinal speed for the vehicle was calculated using the hall effect sensor data. Only the rear wheels [left and right] were used for the calculation. That is because they do not have a torque applied on them and they are “free rolling”. Every change in voltage [tick] means a full revolution of the tire. The time difference between 2 ticks was calculated [ $\bar{T}$ ] and used in equation 3.1 to estimate the longitudinal speed [Where c is the wheel circumference]:

$$v_{wheel} = \frac{c}{\bar{T}} \quad (3.1)$$

The INS velocity is sometimes bigger than the hall effect calculated speeds, especially during braking [partial wheel lock]. There is no obvious difference between them (Figure 3.2).

**Q. Evaluate the lateral acceleration using the relation with the yaw-rate and the longitudinal speed.**



*Figure 3.3: filtered INS vs calculated  $a_y$*

The lateral acceleration was calculated by multiplying the yaw rate by the longitudinal velocity, and is shown in Figure 3.3.

**Q. Comparing the longitudinal acceleration measured by INS with the one obtained by derivation of the longitudinal speed measured from the Hall sensors.**

The derived acceleration [shown in Figure 3.4] is quite noisier than the one provided by INS. The derived acceleration was filtered using a moving mean. This resulted in a far better matching acceleration compared to the INS data.

**Q. Evaluate the side slip angle.**

In Figure 3.5, the side slip angle was calculated using the longitudinal and lateral speed. derived acceleration is quite a lot noisier than the one provided by the INS.

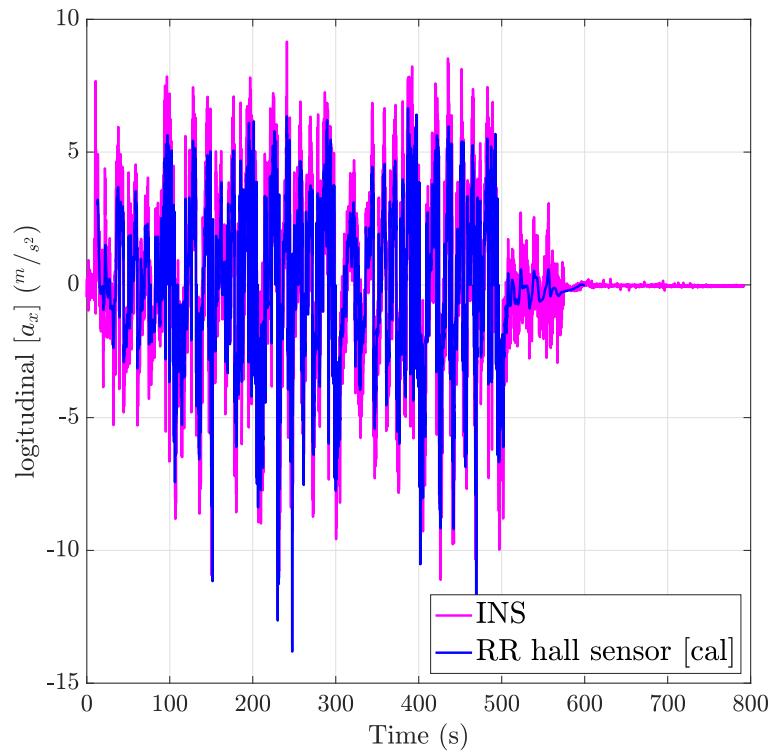


Figure 3.4: INS vs RR wheel hall effect derived  $a_x$

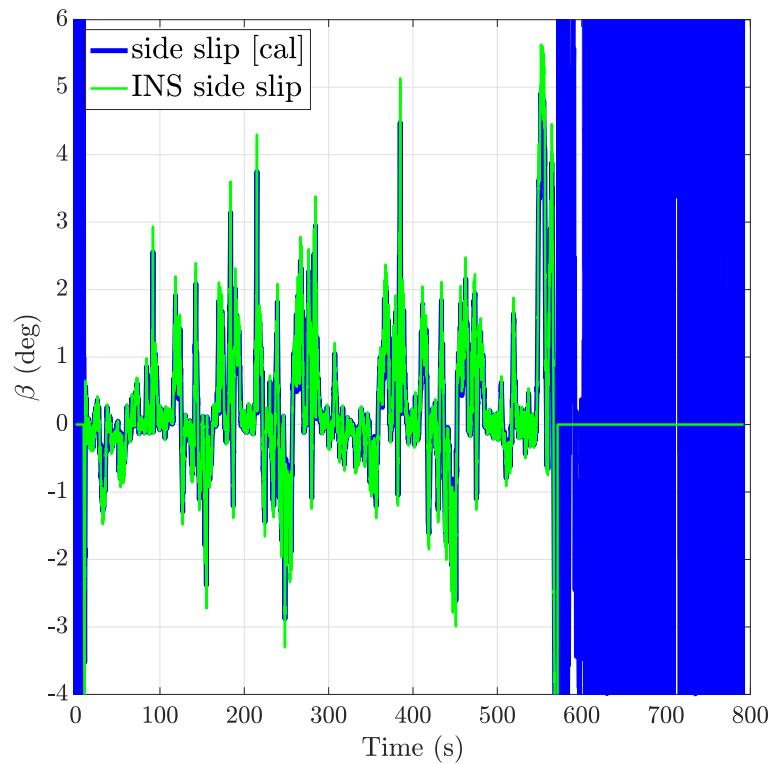


Figure 3.5: INS vs calculated side slip angle

# Assignment 4

## Vehicle Model Exercises

### 4.1 Exercise 1 - Vehicle Model Implementation

Q. For each maneuver, plot and comment the main results that you obtain, particularly focusing on tire forces and moments ( $\{F_x, F_y, F_z, M_z\}$ ) and tire slips ( $\{\kappa, \alpha\}$ ).

1. initial conditions:  $u_0 = 30$  km/h  
simulation timing:  $T_s = 0.001$  s,  $T_f = 20$  s  
requested pedal: req\_pedal = 1  
requested steering wheel angle: req\_steer = 0 deg.

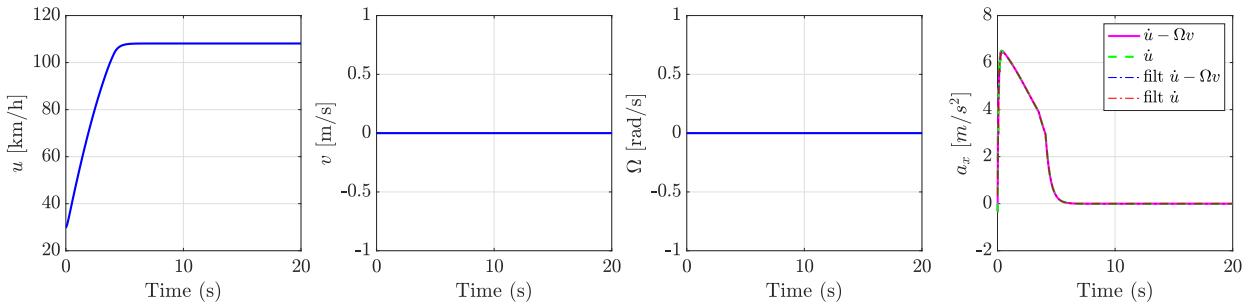


Figure 4.1: vehicle motion graphs [maneuver #1]

Figure 4.1 shows the motion graphs of the first maneuver. The vehicle starts from 30 km/h, with no steering input and full throttle. The velocity increased until it reached full speed of 108 km/h within 5 seconds. All upcoming graphs are influenced by the acceleration profile.

## Assignment 4 – Vehicle Model Exercises

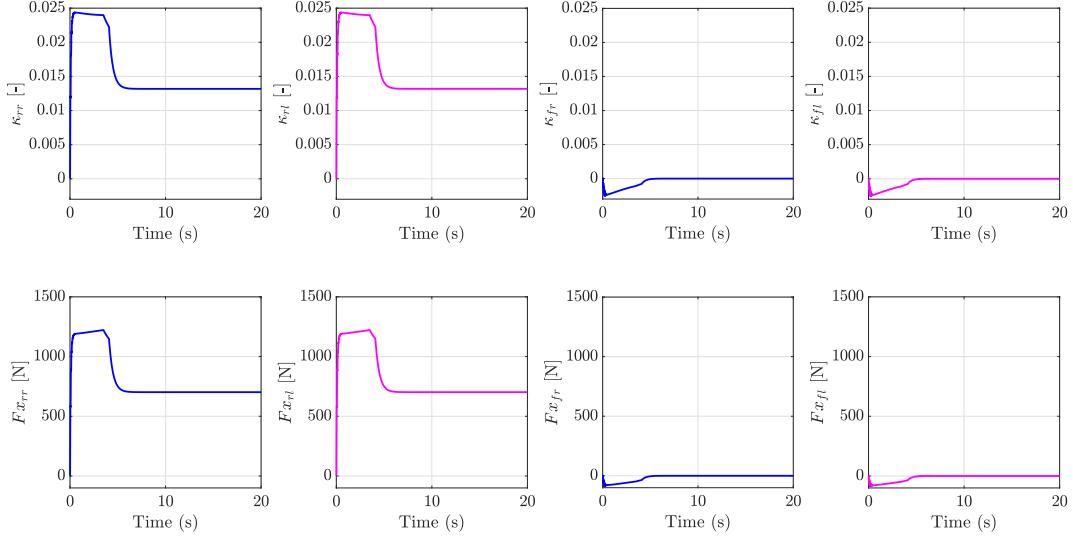


Figure 4.2: longitudinal slip  $\kappa$  and longitudinal forces  $F_x$  [maneuver #1]

The graphs in Figure 4.2 shows no difference between both right tires [and similarly the left tires] as the vehicle was moving straight without steering input. However, both rear tires show much higher  $\kappa$  and  $F_y$ . This was expected as the vehicle is rear-wheel drive and torque applied from the motors would increase both the slip and force experienced by the rear tires.

During the vehicle's acceleration to maximum speed [between 0 and 5 seconds] all tires showed higher slip and force. Once maximum speed is reached [acceleration is zero], the slips and forces decrease across all tires. The front tires go to zero, while the rears do not, as the motors must still keep applying [decreased] torque to keep the vehicle at speed.

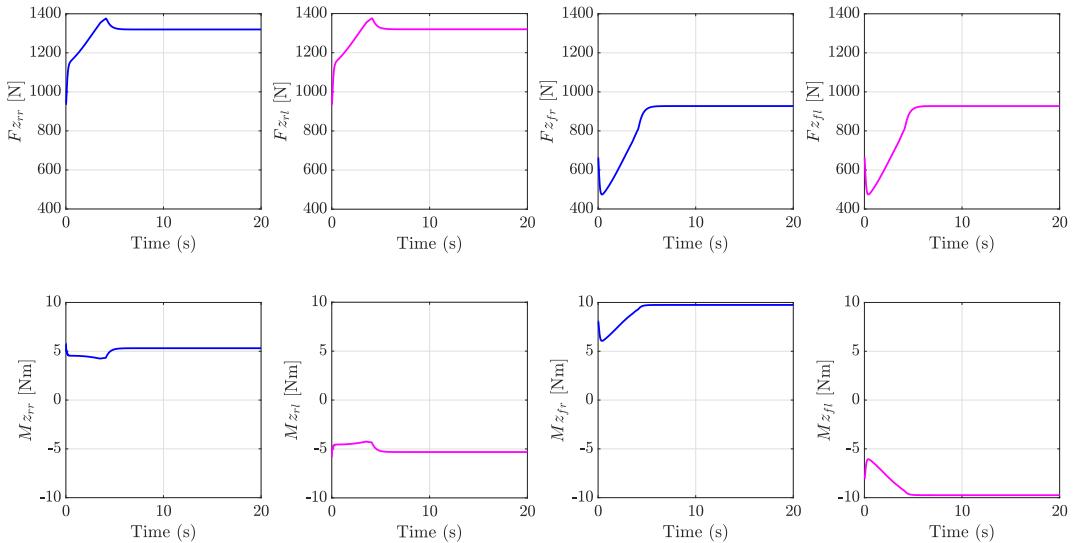


Figure 4.3: Vertical forces  $F_z$  and self aligning torque  $M_z$  [maneuver #1]

## Assignment 4 – Vehicle Model Exercises

The aerodynamics load  $F_A$  increases as speed increases. As shown in Figure 4.3, the vertical force  $F_z$  increases across all tires because it relies on  $F_A$ . They later reach steady state once the car stops accelerating. the rear tires show a peak in  $F_z$  due to the sharp drop in acceleration  $a_y$  towards nearing maximum speed.

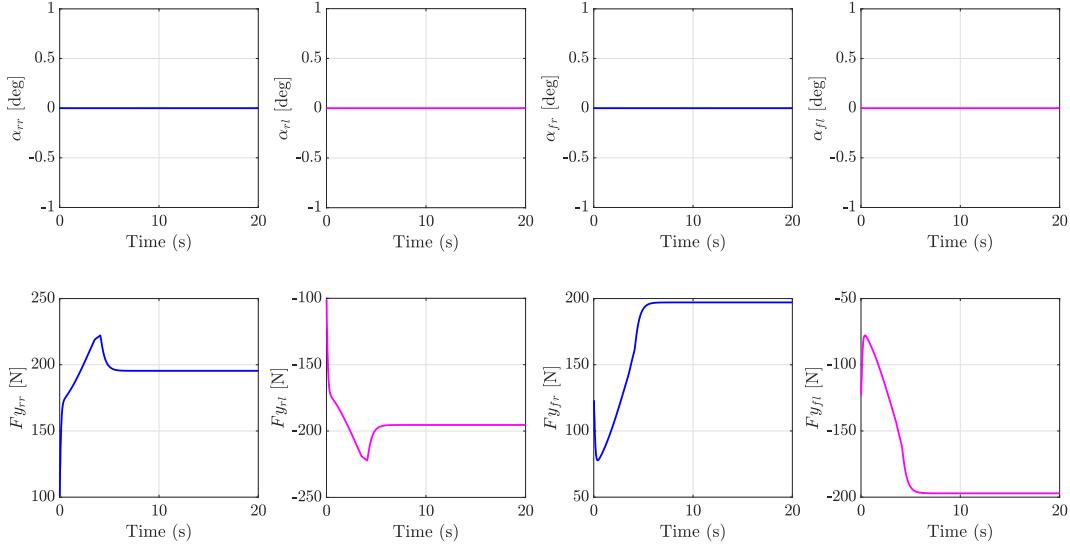


Figure 4.4: side slip angle  $\alpha$  and lateral forces  $F_y$  [maneuver #1]

As previously mentioned, the vehicle has no steering input, which is why the side slip angle  $\alpha$  for all tires are zero [Figure 4.4]. Furthermore, the magnitude of the lateral forces on the right tires are equal [and similarly the left tires]. The right tires' lateral forces are opposite in direction compared to the left tires and equal in magnitude, thus making them cancel out keeping the vehicle moving straight.  $F_y$  is smaller than  $F_x$  for all the tires. The self aligning torque  $M_z$  profiles previously shown in Figure 4.3 resemble the lateral forces profiles for each tire as they are trying to counter their effect on the steering.

During acceleration, the lateral forces  $F_y$  across all tires increase because the vertical forces  $F_z$  increased. The lateral forces relies on the vertical forces, which is why both the  $F_y$  and  $F_z$  profiles match for front and rear tires.

## Assignment 4 – Vehicle Model Exercises

2. initial conditions:  $u_0 = 100 \text{ km/h}$   
 simulation timing:  $T_s = 0.001 \text{ s}$ ,  $T_f = 1.5 \text{ s}$   
 requested pedal: req\_pedal = -1  
 requested steering wheel angle: req\_steer = 0 deg.

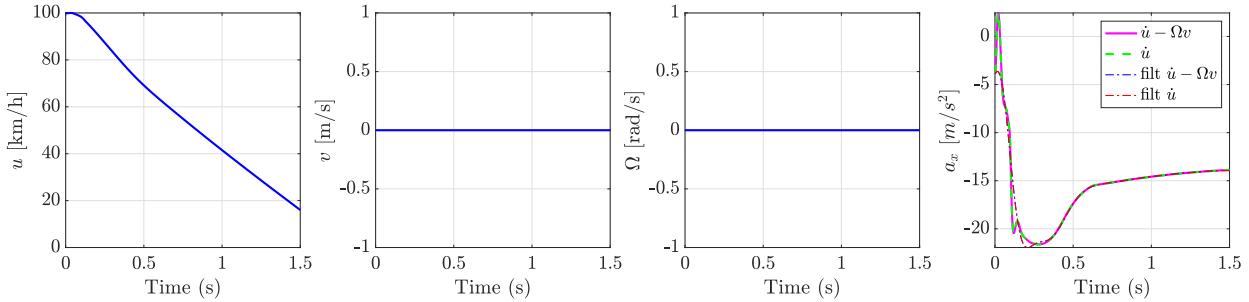


Figure 4.5: vehicle motion graphs [maneuver #2]

Figure 4.5 shows the motion graphs of the second maneuver. The vehicle starts from 100 km/h, with no steering input and applying full brakes. The velocity dropped to 16 km/h within the specified 1.5 seconds. The deceleration shows a decrease after 0.27 seconds as the rear tires start slipping and going into a full wheel lock as shown in Figure 4.6.

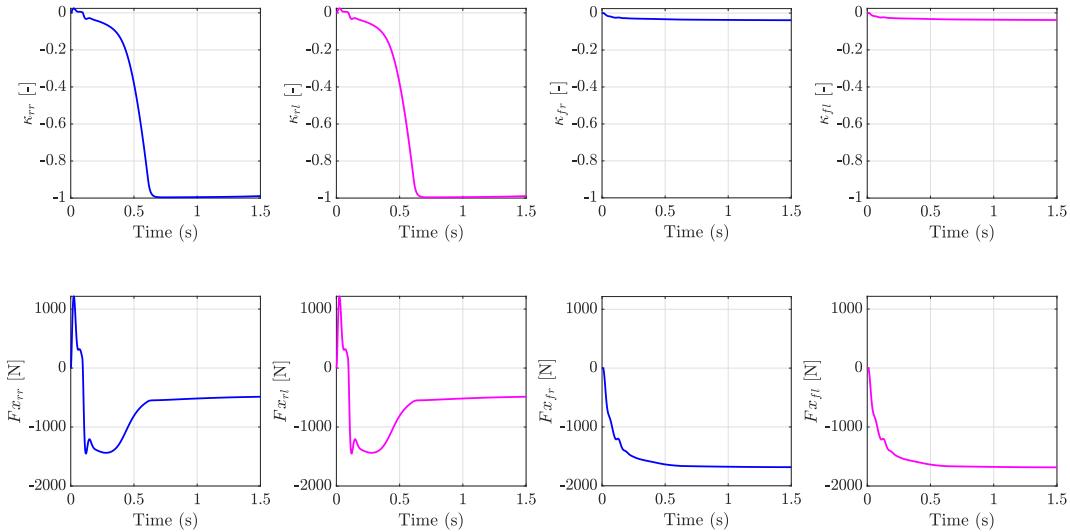


Figure 4.6: longitudinal slip  $\kappa$  and longitudinal forces  $F_x$  [maneuver #2]

This maneuver also doesn't have steering input, which is why the graphs in Figure 4.6 show no difference between both right and left tires. The longitudinal forces  $F_x$  for all tires increase in the opposite direction because the vehicle is decelerating. The longitudinal slip  $\kappa$  for the rear tires go to -1, which means both tires went into wheel lock.  $F_x$  decreases once wheel lock starts to occur causing lower efficiency in braking and the deceleration rate decreases.

## Assignment 4 – Vehicle Model Exercises

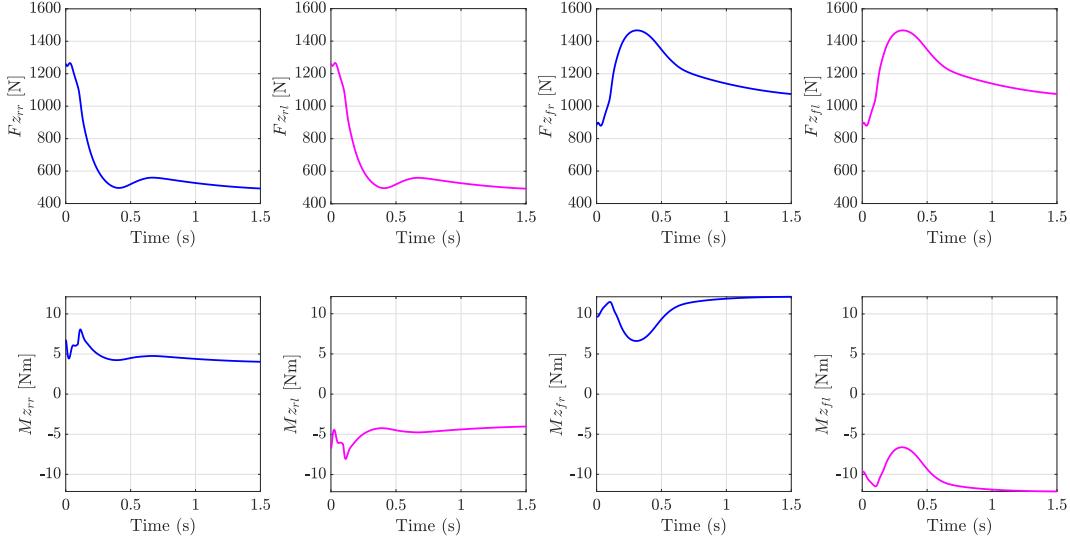


Figure 4.7: Vertical forces  $F_z$  and self aligning torque  $M_z$  [maneuver #2]

Figure 4.7 shows the vertical force  $F_z$  has decreased in the rear tires while increasing in the front. This is attributed to the longitudinal load transfer to the front due to the braking. The  $F_z$  forces on all tires showed a peak around 0.27 seconds due to the rear wheels locking.

No steering input means no side slip angle  $\alpha$  for all tires [Figure 4.8]. And similar to maneuver #1, the right tires' lateral forces are opposite in direction compared to the left tires and equal in magnitude. The self aligning torque  $M_z$  profiles in Figure 4.7 for each tire are trying to counter the  $F_y$ 's effect on the steering.  $F_y$  showed the same trend as  $F_z$ , increasing in the front while decreasing in the rear tires.

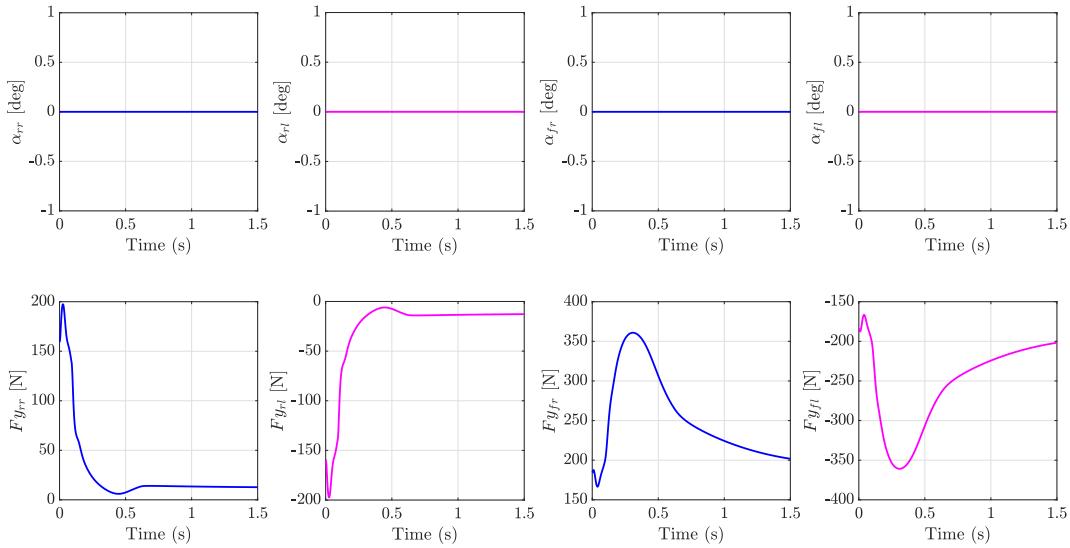


Figure 4.8: side slip angle  $\alpha$  and lateral forces  $F_y$  [maneuver #2]

3. initial conditions:  $u_0 = 50 \text{ km/h}$   
simulation timing:  $T_s = 0.001 \text{ s}$ ,  $T_f = 1.5 \text{ s}$   
requested pedal: req\_pedal = 0.5  
requested steering wheel angle: req\_steer = 20 deg.

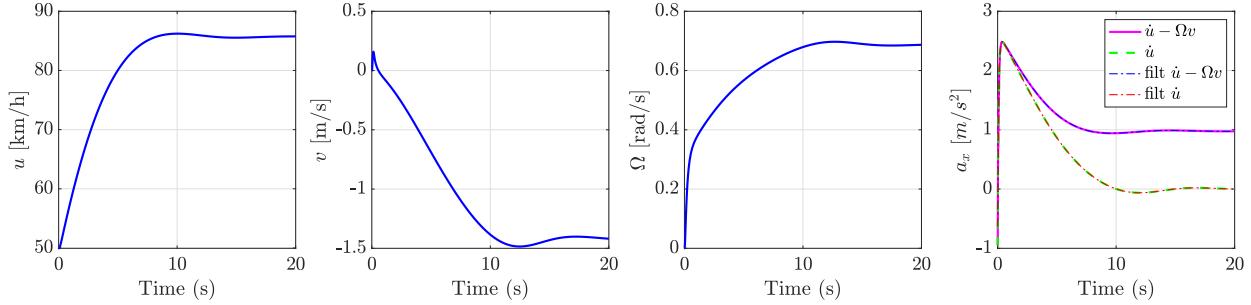


Figure 4.9: vehicle motion graphs [maneuver #3]

Figure 4.9 shows the motion graphs of the third maneuver. The longitudinal speed  $u$  keeps increasing until it reaches full saturation for half throttle. The lateral speed  $v$  shows the same profile as  $u$ .  $v$  is negative indicating the vehicle is turning left. The yaw rate  $\Omega$  follows the same pattern, increasing during acceleration, and constant once acceleration is zero.

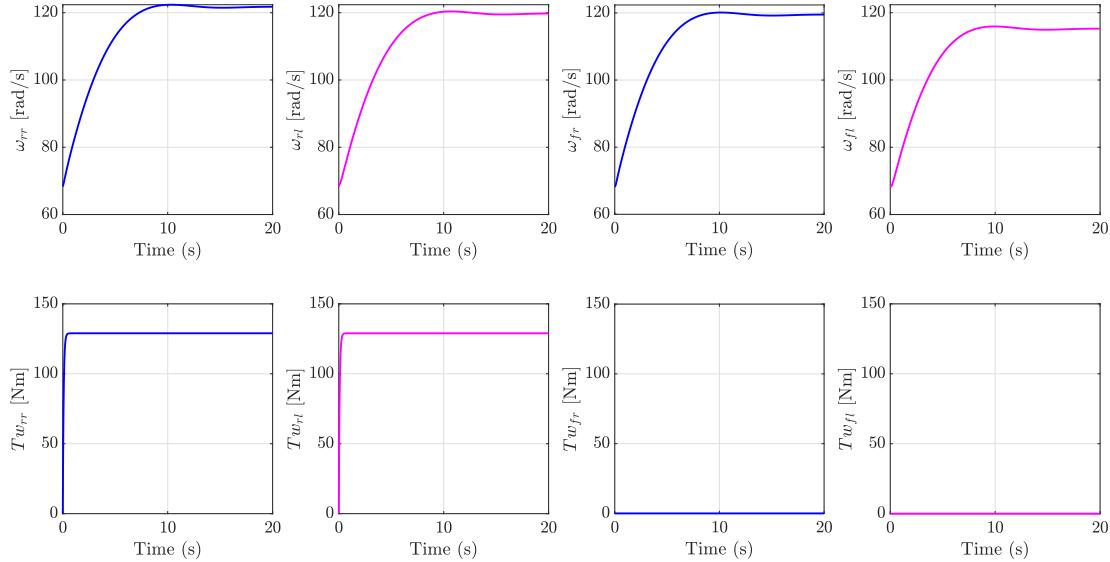


Figure 4.10: Wheel rates  $\omega$  and Torques  $T_w$  [maneuver #3]

The angular velocity  $\omega$  are different for different for each wheel as the outer [right] tires would need to rotate faster than the inner [left] tires [Figure 4.10]. The torque experienced by the rear tires are equal to each other, while the front ones are zero.

## Assignment 4 – Vehicle Model Exercises

The longitudinal slip  $\kappa$  relies on  $\omega$  and can be approximately calculated using Equation 4.1:

$$\kappa_{ij} \simeq \frac{\omega_{ij} R_{ij} - u}{u} \quad i \in \{f, r\} \quad j \in \{r, l\} \quad (4.1)$$

Figure 4.11 shows  $\kappa$  and  $F_x$ . As there was no torque on the front tires,  $\kappa$  and  $F_x$  on the front tires were small.  $\kappa$  for the rear tires are different because they are rotating at different speeds. However, both rear tires experienced the same  $F_x$ .

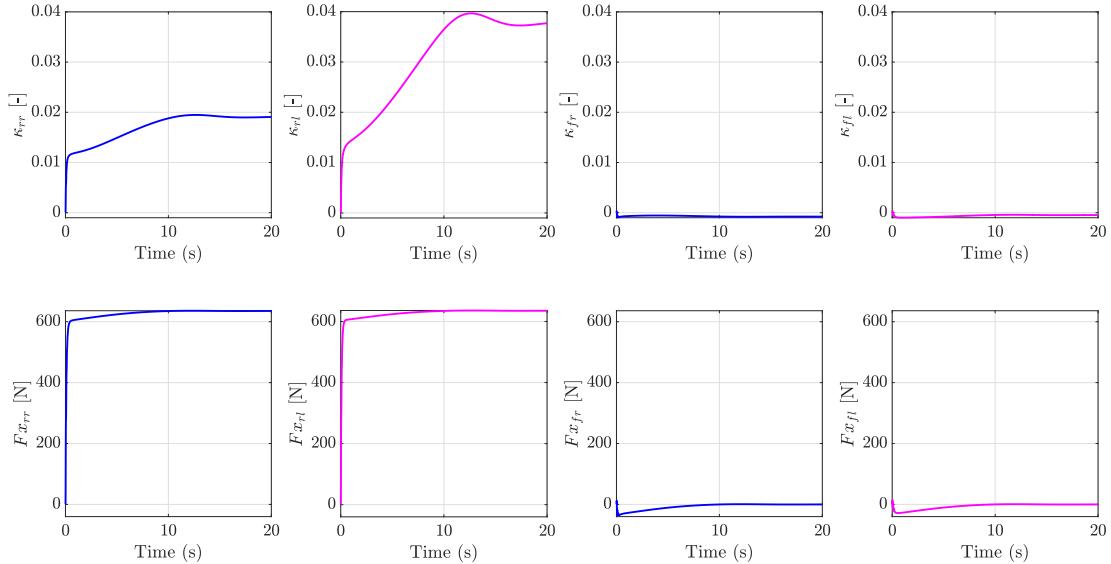


Figure 4.11: longitudinal slip  $\kappa$  and longitudinal forces  $F_x$  [maneuver #3]

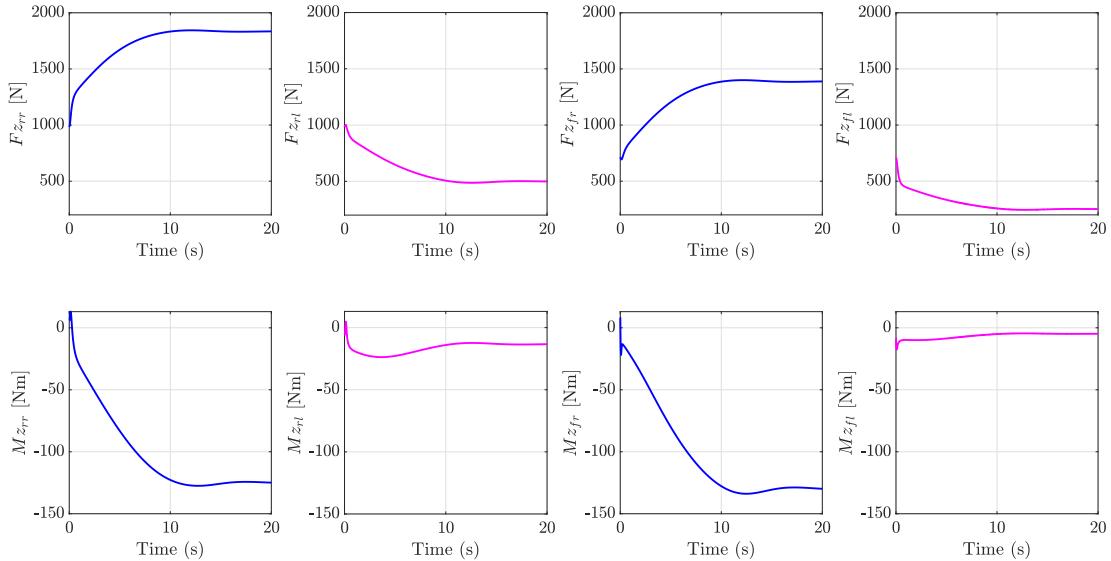


Figure 4.12: Vertical forces  $F_z$  and self aligning torque  $M_z$  [maneuver #3]

## Assignment 4 – Vehicle Model Exercises

Figure 4.12 shows the vertical forces  $F_z$  for the right [outer] tires increasing then becoming constant when acceleration becomes zero. The left [inner] tires experienced the opposite because of the lateral load transfer to the right side of the vehicle due to the left turn.

The lateral side slip  $\alpha$  for all tires are calculated using Equation 4.2:

$$\alpha_{ij} = -\arctan\left(\frac{vc_{yij}}{vc_{xij}}\right) \quad i \in \{f, r\} \quad j \in \{r, l\} \quad (4.2)$$

$\alpha$  appears to be consistent across the four tires as shown in Figure 4.13. However, the rear tires are slightly higher due to the torque applied by the motors.

The right side tires have higher lateral forces  $F_y$  compared to the right. This is because  $F_y$  relies on  $F_z$ , as the lateral load transferred to the right side during the left turn maneuver. Both  $\alpha$  and  $F_y$  become constant once the acceleration is equal to zero. The self aligning torque  $M_z$  profiles in Figure 4.12 for each tire are trying to counter the  $F_y$ 's effect on the steering.

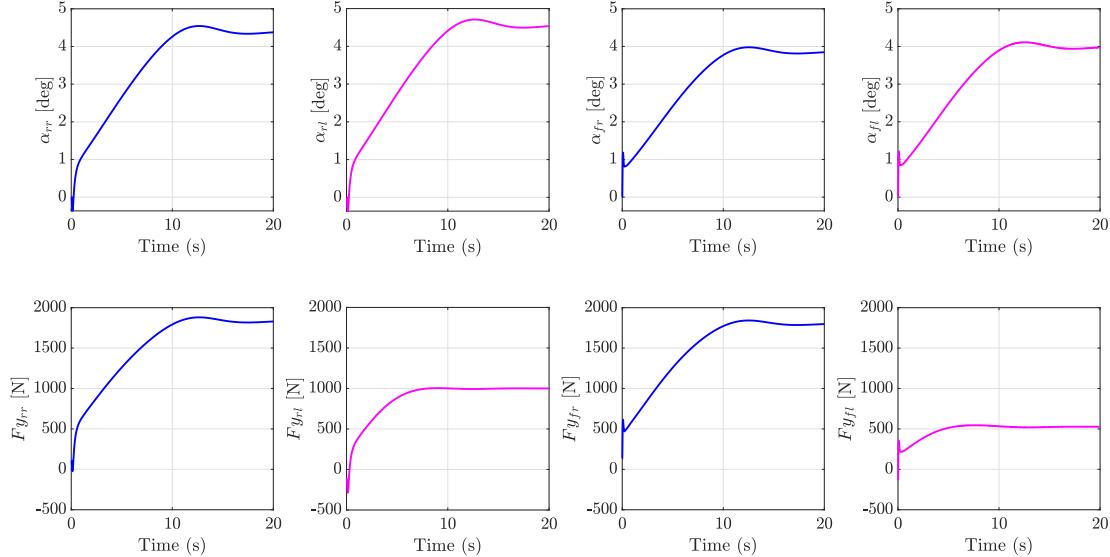


Figure 4.13: side slip angle  $\alpha$  and lateral forces  $F_y$  [maneuver #3]

# Assignment 5

## Handling Identification

5.1 Exercise 1 - Sine Steer Maneuvers

5.2 Exercise 2 - Constant Steer Maneuvers