Vehicles Dynamics, Planning and Control of Robotic Cars

Final Report

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1 Tire Model Exercises - 1

Exercise 1 – pure longitudinal slip

Q. Using the Pacejka Magic Formula, plot the longitudinal tire force F_{x0} obtained in pure longitudinal slip conditions, as a function of slip $\kappa \in$ [-1, 1]. Which comments are you able to make about the obtained graph?

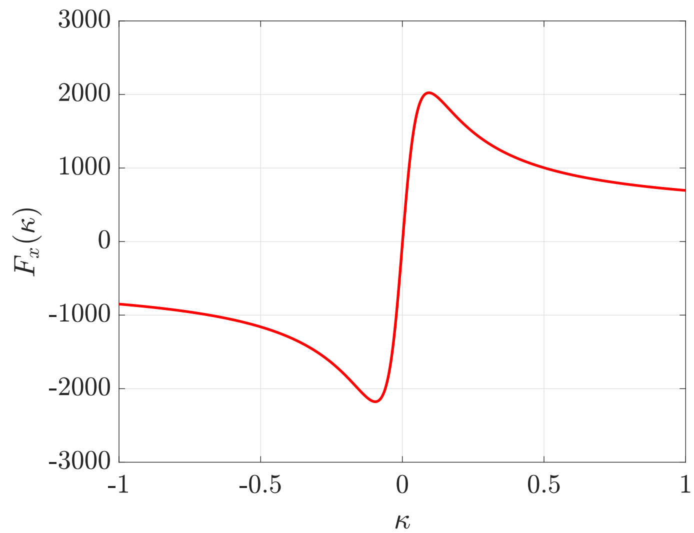

Figure 1: F_{x0} as a function of slip

When κ (longitudinal slip) increases, F_x grows until it reaches a saturation limit. Furthermore, the slip zone grows, and the adherence area decreases. That means, the total tire force F_x keeps growing until it reaches a peak and a saturation limit as shown in the graph below. κ (longitudinal slip) is positive when the tire is accelerating, negative during braking, and reaches -1 when the wheel locks.

Q. If you were supposed to design a traction control system for maximizing vehicle longitudinal acceleration, which would be the target value of longitudinal slip κ that you would try to achieve?

Acceleration is directly proportional to the force [F=ma] if the mass is constant. If I want to maximize the vehicle longitudinal acceleration, I would need to maximize the longitudinal force [F_x]. F_x is highest at the saturation limit, which in this example happens at slip κ = 0.094, and yields an F_x of 2022.99 N.

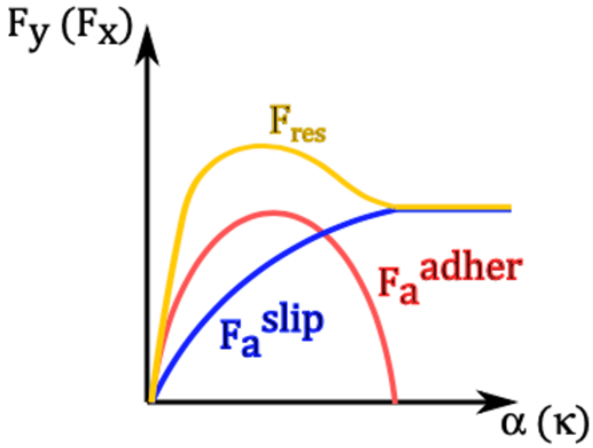

Figure 2: Slip and adherence forces

Q. Assuming that wheel rotational speed is $\omega = 70$ rad/s, tire effective rolling radius is $R_e = 0.2$ m, while the longitudinal component of tire contact point speed $v_{Cx} = 13$ m/s, compute the longitudinal slip κ. In these conditions, is the wheel accelerating, braking or is it in pure rolling? Compute also the corresponding longitudinal tire force F_{x0}

Using Matlab, the calculated longitudinal slip $\kappa = 0.0769$ and the calculated longitudinal force F_x is = 1990.65 N. the longitudinal slip is positive (>0), which means that the wheel is accelerating.

Q. Compute the cornering stiffness Cfκ, that is the derivative for $\kappa = 0$ of the F_{x0}. Up to which value of κ is the linear approximation of Pacejka curve acceptable?

The cornering Stiffness Cfκ is equal to the derivative of the longitudinal force with respect to the longitudinal slip when the longitudinal slip is equal to zero. This means its equal to the slope at the origin (x=y=0) or equal to BCD function as shown in Figure 3. The calculated cornering stiffness is Cfκ = 47909.4

The linear approximation allows to neglect the complex Pacejka Magic Formula, but it is valid only for small κ. At $\kappa = 0.02$, the percent difference is already at 10% as shown in Figure 4.

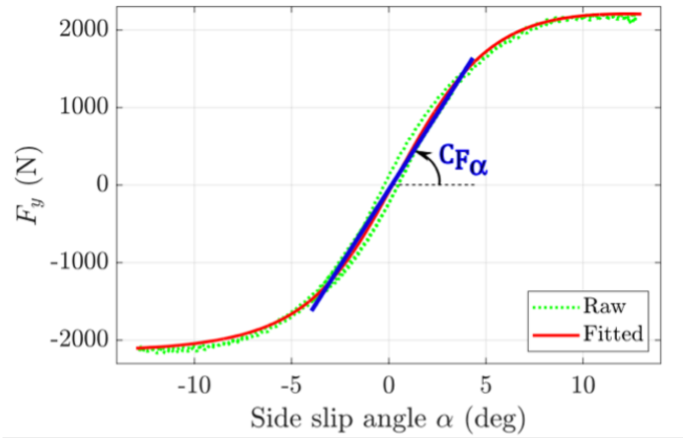

Figure 3: cornering stiffness as a linear approximation

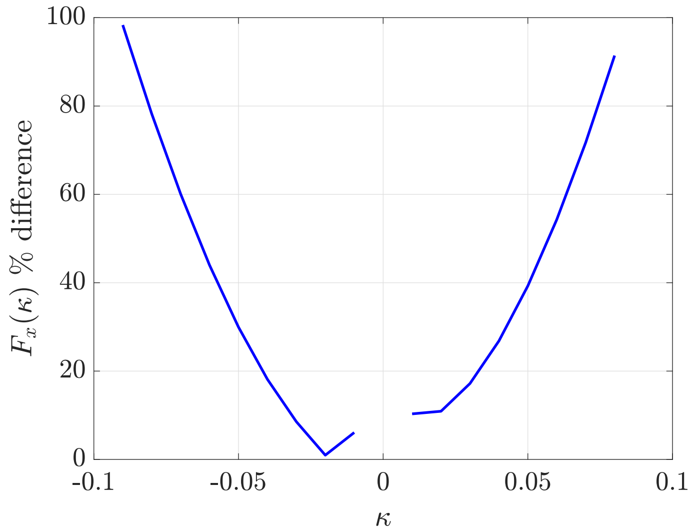

Figure 4: % difference in κ using linear approximation vs Pacejka formula

Exercise 2 - combined slip

Q. Assume that the tire contact point velocity components along the tire x and y axes are υ_{Cx} = 15 m/s and υ_{Cy} = -1.3 m/s, respectively. Calculate the side slip angle α. Moreover, compute the combined tire force F_x using this value of α, for a longitudinal slip κ = 0.08.

Alpha can be calculated using the practical slip approach:

$$\text{side slip angle } \alpha = -\arctan\left(\frac{\upsilon_{sy}}{\upsilon_{Cx}}\right) = -\arctan\left(\frac{\upsilon_{Cy}}{\upsilon_{Cx}}\right) \tag{1}$$

Using equation 2 to calculate G_{xa} (weighing function). Once calculated, F_{x0} can now be multiplied by G_{xa} (weighing function) to get the combined tire force F_x.

$$G_{xa} = -D_{xa}\cos(C_{xa}\arctan(B_{xa}(\alpha + S_{Hxa})) \tag{2}$$

$$F_{x0} = D_x \sin(C_x \arctan(B_x\kappa_x - E_x(B_x\kappa_x - \arctan(B_x\kappa_x))) \tag{3}$$

$$F_x = G_{xa}F_{x0} \tag{4}$$

Using Matlab to calculate the side slip angle α and combined tire force:

calculated side slip alpha = 0.086451
calculated weighing function = 0.724417
calculated combined force F_x = 1450.424623

Q. Plot the combined longitudinal tire force F_x as a function of $\kappa \in$ [-1, 1], for the following levels of side slip angle $\alpha = \{0, 2, 4, 6, 8\}$ degrees. Which comments can you make about the 5 curves obtained in this way? Finally, plot the weighting function G_{xa} as a function of $\kappa \in$ [-1, 1] for each of the previously defined values of α, and briefly comment also these 5 curves.

Figure 5 shows plots obtained for the combined longitudinal force F_x for each side slip angle as a function of the longitudinal slip κ. The maximum combined longitudinal force F_x keeps decreasing with higher side slip α.

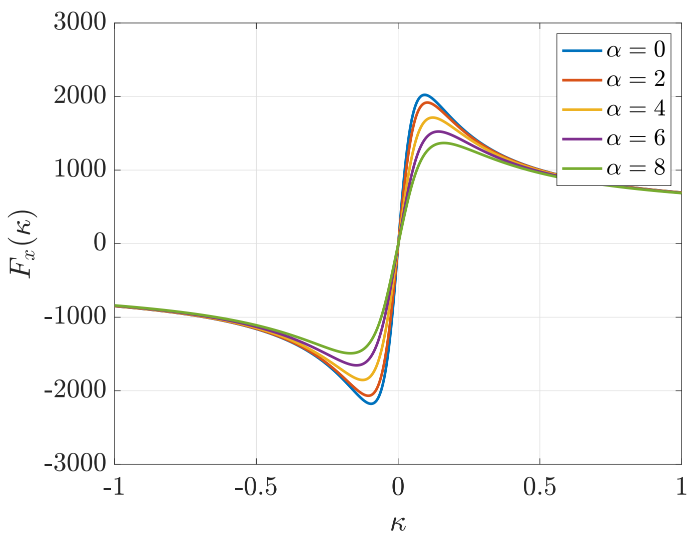

Figure 5: combined longitudinal force F_x as a function of κ

Figure 6 shows the plots obtained for the weighing function G_{xa} for each side slip angle as a function of the longitudinal slip κ. Higher side slip α decreases the weighing function, which in effect decreases the combined longitudinal force F_x. The effect of the weighing function is quite more potent around $\kappa = 0$, and that effect decreases the further away we are from it.

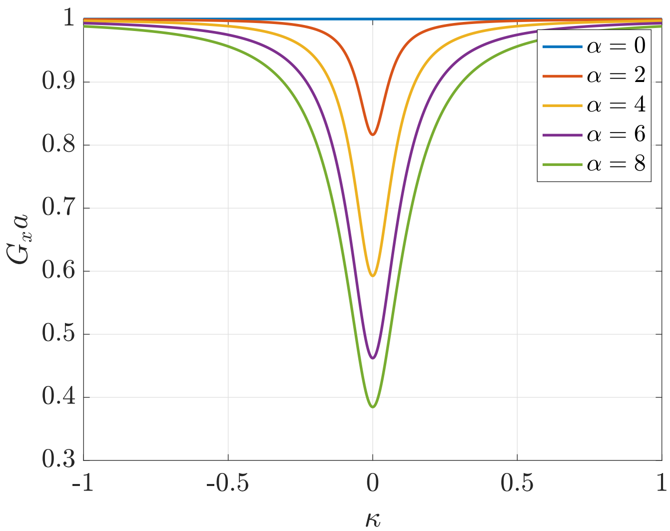

Figure 6: Weighing function G_{xa} as a function of κ

2 Tire Model Exercises - 2

Exercise 1 – Understanding tire data

Q. Plot the raw data in different graphs, specifically focusing on κ, α, γ, F_z and pressure P. Comment on what you see. What is, according to you, the main target of these tests?

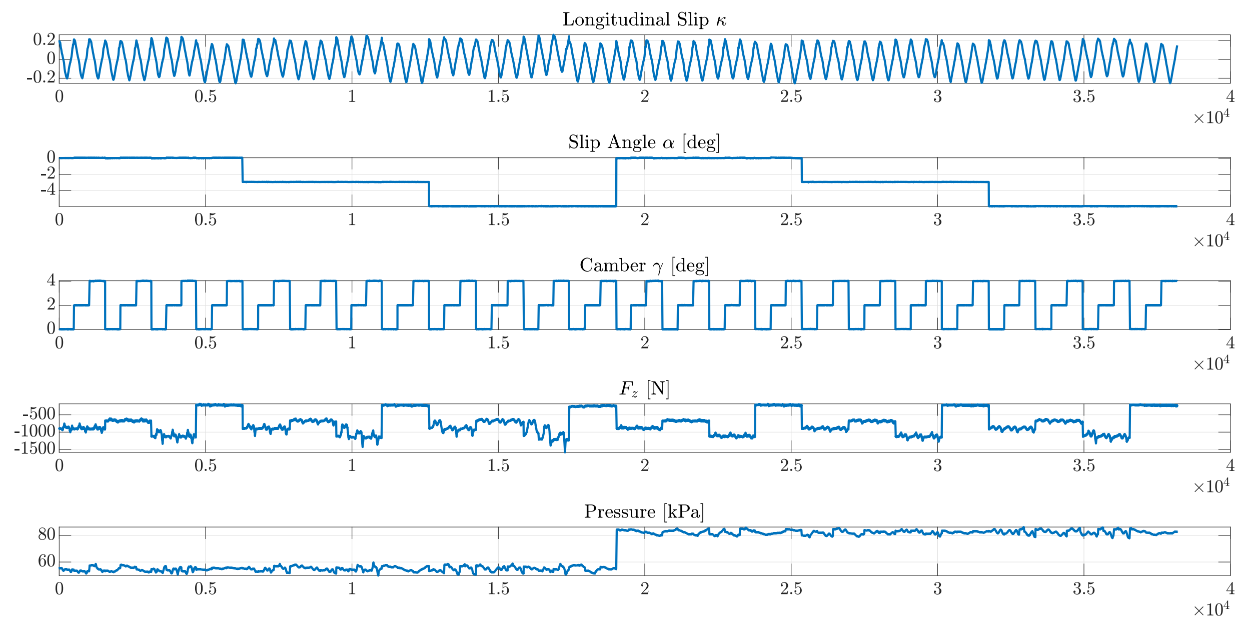

Figure 7: raw data

From the five plotted variables in Figure 7, it appears that the longitudinal slip, slip angle, camber, vertical force, and pressure are all showing repeat patterns. That means that those five variables are controlled to measure and test their effect on the longitudinal force F_x. Note that the longitudinal slip κ for each test spanned from 0.2 to -0.2 and then back again to 0.2. The data extracted for the following questions only took the first half of each separate test. Furthermore, only data that had P = 83 kPa were used, because the data seemed noisier with P = 55 kPa, especially with F_z data.

Q. Focus on the data with α =0 and γ =0, and plot the curves F_x vs κ for each of the 4 vertical loads F_z used in the experiments. Plot the 4 curves on the same graph, with different colors. Comment on what you see.

Figure 8 suggests that the Longitudinal Force F_x increases with the vertical force F_z. There is some dependency on the vertical force. If you double the vertical force, it does not mean

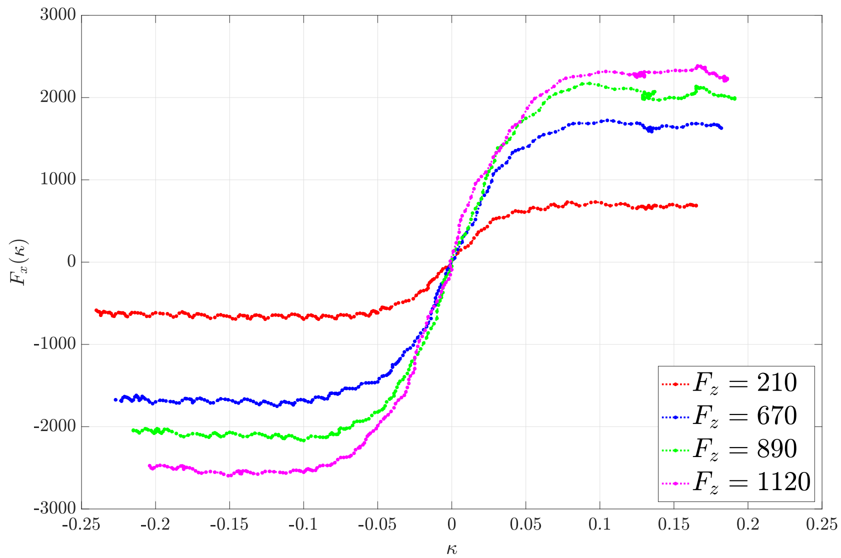

Figure 8: κ vs F_x based on vertical force

the longitudinal force will double. In some parts there is a linear dependency, but in others it is not the case.

Q. Focus on the data with γ =0 and F_z = 150 lbf ≈ 670N, and plot the curves F_x vs κ for each of the 3 side slip angles α used in the experiments. Plot the 3 curves on the same graph, with different colors. Comment on what you see.

The Longitudinal Force F_x in Figure 9 shows an inverse relationship with the side slip angle.

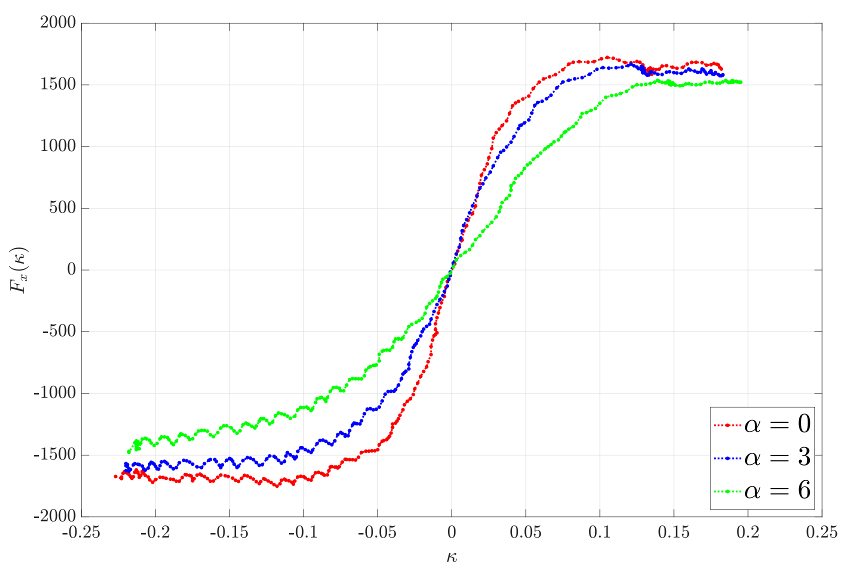

Figure 9: κ vs F_x at vertical force = 670 based on slip angle

Exercise 2 - Fitting tire data

Q. First consider the data with $F_z = F_{z0}$ =890N, γ =0 and α =0, and fit the coefficients X1 = $\{p_{Cx1}, p_{dx1}, p_{Ex1}, p_{Ex4}, p_{Kx1}, p_{Hx1}, p_{Vx1}\}$. Plot the fitted curve F_x vs κ that you obtained in these nominal conditions, together with the raw data.

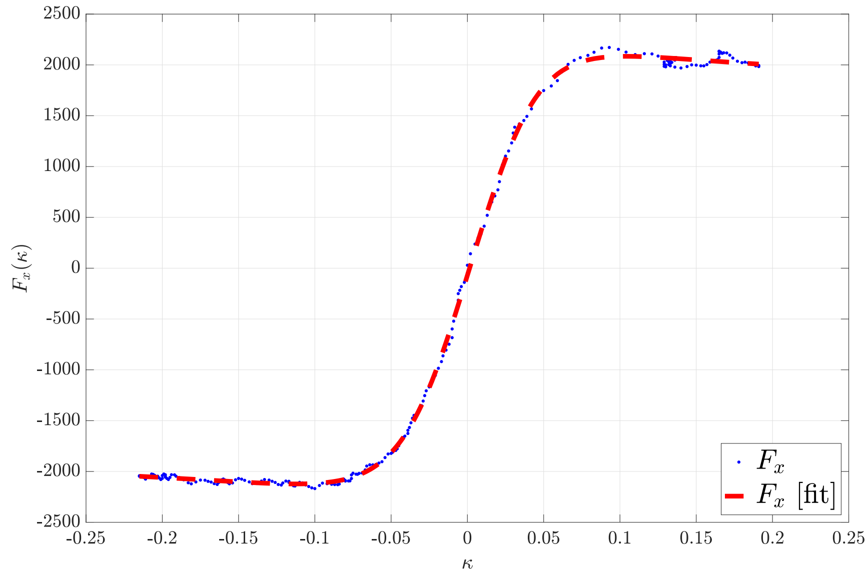

Figure 10: Fitted F_{x0} after optimizing first 7 parameters compared to test data vs κ

In Figure 10, the data was filtered based on the following criteria: $F_{z0} = 890$, $\alpha = 0$, $\gamma = 0$. The first 7 coefficients were optimized using the $fmincon$ function in Matlab. I started with a random coefficients initial guess vector. The initial guess vector has proven to be quite important as running the code multiple times showed a curve that did not fit the data at all, which means the optimizer was stuck at a local minima. However, most of the time the initial guess gave a very good fit that converged to the correct global minima. Later on I optimized the initial guesses to give consistent good fitting results based on trial and error.

Q. Now consider the data with the 4 different values of F_z, but still $\gamma = 0$ (and $\alpha = 0$). This enables the fitting of the parameters: X2 =$\{p_{dx2}, p_{Ex2}, p_{Ex3}, p_{Hx2}, p_{Kx2}, p_{Kx3}, p_{Vx2}\}$. Plot the fitted and raw curves F_{x0} vs κ for the 4 values of F_z and comment the results.

The real F_x data for each separate F_z are plotted against the output of the second fitting in Figure 11. The fitted data appear to match the test data quite well. This means the fitting results are adequate to approximate the longitudinal force F_x for vertical forces F_z that are different from the originally tested one.

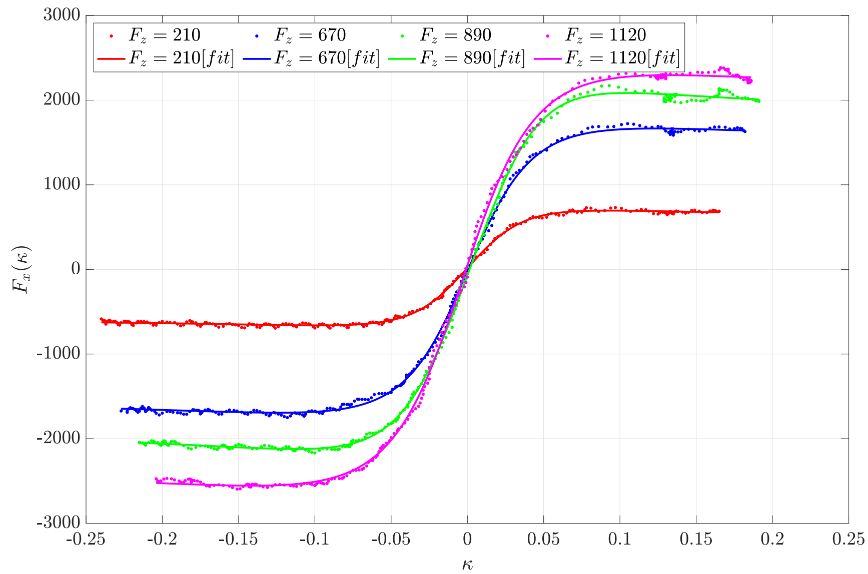

Figure 11: Fitted F_x vs test data after second fitting for each F_z

Q. Now consider the data with the 3 different values of γ, but with $F_z = F_{z0}$ (and $\alpha = 0$). Plot the fitted and raw curves F_{x0} vs κ for the 3 values of γ and comment the results.

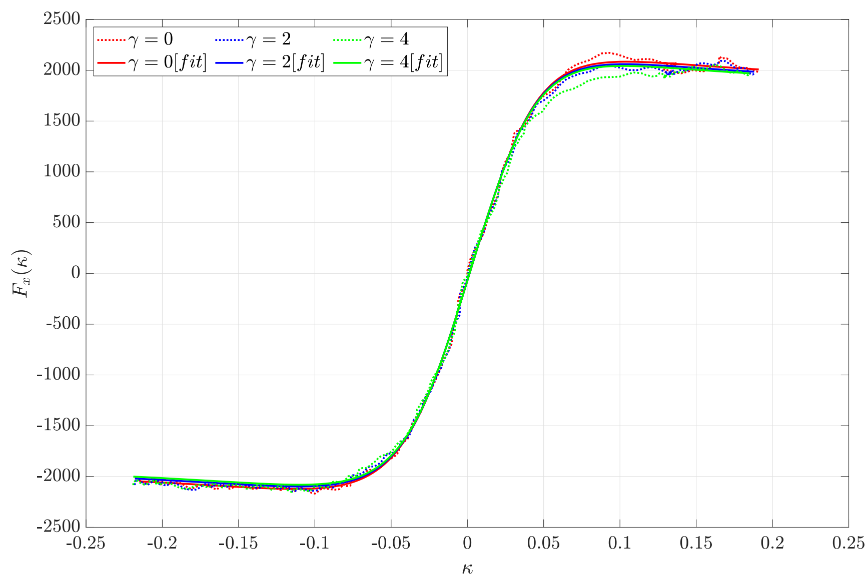

Figure 12: Fitted F_x vs test data after third fitting for each γ

Figure 12 shows the results of the third fitting. The data from $\gamma = 0$ fits properly as shown previously in Figure 10. However, the fitting on the other γ values did not match completely. Trying different initial values for p_{dx2} did not yield any better results.

3 Vehicle Data Analysis Exercises

Exercise 1 – Understanding vehicle data

Q. Plot lateral and longitudinal velocity

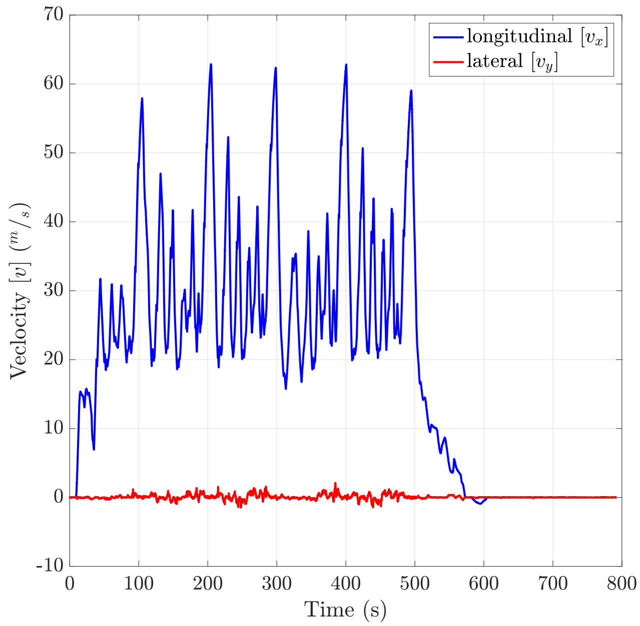

Figure 13: lateral and longitudinal velocity vs time

From Figure 13, the magnitude of the longitudinal velocity is higher than the lateral velocity. This matches what was expected as the vehicle mainly moves longitudinally and only laterally while slipping.

Q. Evaluate the longitudinal speed using the Hall-effect wheel speed sensors and compare the data with the INS data

The longitudinal speed for the vehicle was calculated using the hall effect sensor data. Only the rear wheels [left and right] were used for the calculation. That is because they do not have a torque applied on them and they are "free rolling". Every change in voltage [tick] means a full revolution of the tire. The time difference between 2 ticks was calculated [$\overline{T}$] and used in equation 5 to estimate the longitudinal speed [Where c is the wheel circumference]:

$$v_{wheel} = \frac{c}{\overline{T}} \tag{5}$$

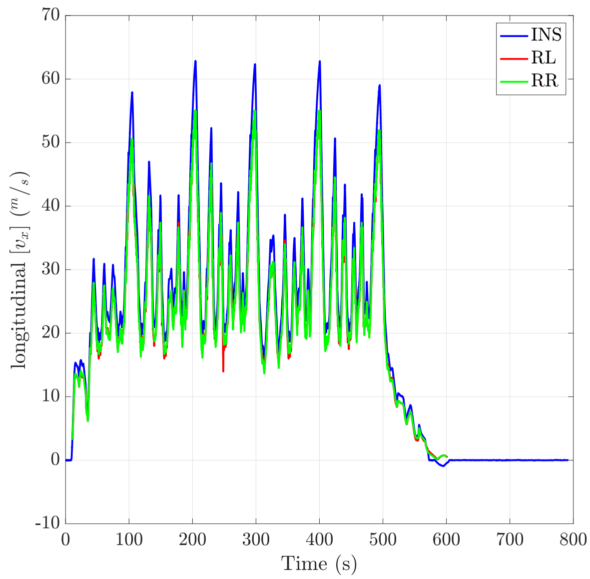

Figure 14: INS vs RL/RR hall effect sensors v_x

The INS velocity is sometimes bigger than the hall effect calculated speeds, especially during braking [partial wheel lock]. There is no obvious difference between them (Figure 14).

Q. Evaluate the lateral acceleration using the relation with the yaw-rate and the longitudinal speed.

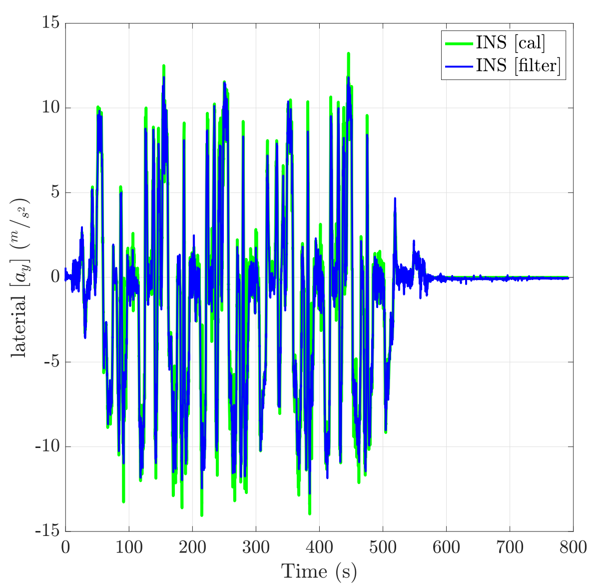

Figure 15: filtered INS vs calculated a_y

The lateral acceleration was calculated by multiplying the yaw rate by the longitudinal velocity, and is shown in Figure 15.

Q. Comparing the longitudinal acceleration measured by INS with the one obtained by derivation of the longitudinal speed measured from the Hall sensors.

The derived acceleration [shown in Figure 16] is quite noisier than the one provided by INS. The derived acceleration was filtered using a moving mean. This resulted in a far better matching acceleration compared to the INS data.

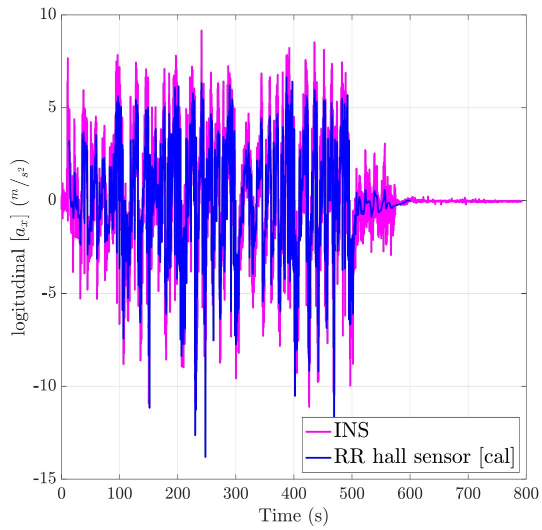

Figure 16: INS vs RR wheel hall effect derived a_x

Q. Evaluate the side slip angle.

In Figure 17, the side slip angle was calculated using the longitudinal and lateral speed. derived acceleration is quite a lot noisier than the one provided by the INS.

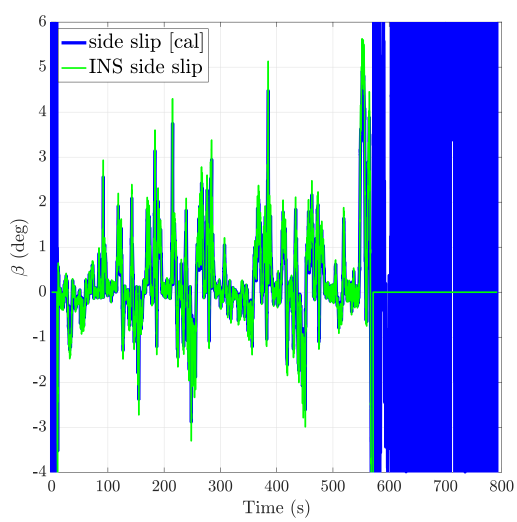

Figure 17: INS vs calculated side slip angle

4 Vehicle Model Exercises

Exercise 1 – Vehicle model implementation

Q. For each maneuver, plot and comment the main results that you obtain, particularly focusing on tire forces and moments ({F_x, F_y, F_z, M_z}) and tire slips ({κ, α}).

1. initial conditions: u_0 = 30 km/h
   simulation timing: T_s = 0.001 s, T_f = 20 s
   requested pedal: req_pedal = 1
   requested steering wheel angle: req_steer = 0 deg.

Answer to 1

2. initial conditions: u_0 = 100 km/h
   simulation timing: T_s = 0.001 s, T_f = 1.5 s
   requested pedal: req_pedal = -1
   requested steering wheel angle: req_steer = 0 deg.

Answer to 2

3. initial conditions: u_0 = 50 km/h
   simulation timing: T_s = 0.001 s, T_f = 1.5 s
   requested pedal: req_pedal = 0.5
   requested steering wheel angle: req_steer = 20 deg.

Answer to 3