

Personal solutions to GR book by Carroll chapter

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Baset

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1.

Let the initial velocity of the particle with respect to the observer S' be $u'_0 \equiv (u_0^{x'}, u_0^{y'})$. For simplicity, we assume that the collision happens at $t' = 0$. Moreover, we assume observers S and S' are aligned along the x -axis and that when $t' = 0$ then $t = 0$. Let at $t' = 0$ the line that describes the mirror be $-x' = y'$. The line that describes the particle is $\frac{x'}{u_0^{x'}} = \frac{y'}{u_0^{y'}}$. For now, $u_0^{x'}, u_0^{y'} \neq 0$. Now, we will find the angle between these two lines:

$$\tan \theta' = \left| \frac{\tan \frac{\pi}{4} - \frac{u_0^{y'}}{u_0^{x'}}}{1 + \tan \frac{\pi}{4} \frac{u_0^{y'}}{u_0^{x'}}} \right| = \left| \frac{1 - \frac{u_0^{y'}}{u_0^{x'}}}{1 + \frac{u_0^{y'}}{u_0^{x'}}} \right| \quad (1)$$

We let the $u_0^{y'} = u_0^{x'} \tan \alpha' = u_0^x \tan \alpha'$. Then, we have that:

$$\tan \theta' = \left| \frac{1 - \tan \alpha'}{1 + \tan \alpha'} \right| \quad (2)$$

But in the frame S , we have that the $u_0^y = \frac{u_0^{y'} + v}{1 + u_0^{y'} v}$. Also, the mirror's coordinates in S are:

$$y = \gamma(y' + vt') = \gamma y' |_{t'=0} \quad (3)$$

Therefore, w.r.t. to S at $t = 0$ the mirror is $y = -\frac{x}{\gamma}$. The particle's coordinates in S are $y = \frac{u_0^{y'} + v}{(1 + u_0^{y'} v) u_0^x} x$. Therefore, the angle between the particle and the mirror is:

$$\tan \theta = \left| \frac{\gamma - \frac{u_0^{y'} + v}{(1 + u_0^{y'} v) u_0^x}}{1 + \gamma \frac{u_0^{y'} + v}{(1 + u_0^{y'} v) u_0^x}} \right| \implies \tan \theta = \left| \frac{\gamma - \frac{\tan \alpha' + v/u_0^x}{1 + \tan \alpha' u_0^x v}}{1 + \gamma \frac{\tan \alpha' + v/u_0^x}{1 + \tan \alpha' u_0^x v}} \right| \quad (4)$$