Linear Programming (PROL)

Duration: 1h30.
No documents are allowed.
Only ***non-programmable*** *pocket calculators are allowed.*

We're considering the following list of linear programs respectively given the names (A), (B), (C), (D) and (E), (F).

$$(A)\begin{cases}\text{maximize} & x_1+x_2+x_3\\ \text{subject to}\\ & 2x_1+3x_2+\ x_3\le 1\\ & x_1-4x_2+3x_3\ge 2\\ \text{with} & x_1,\,x_2\ge 0.\end{cases}\qquad (B)\begin{cases}\text{minimize} & -x_1-x_2-x_3+x_4\\ \text{subject to}\\ & 2x_1+3x_2+\ x_3-\ x_4+\le 1\\ & -x_1+4x_2-3x_3+3x_4\ \le -2\\ \text{with} & x_1,\,x_2,\,x_3,\,x_4\ge 0.\end{cases}$$

$$(C)\begin{cases}\text{minimize} & -5x_1-6x_2-9x_3-8x_4\\ \text{subject to}\\ & x_1+2x_2+3x_3+\ x_4\le 5\\ & -x_1-\ x_2-2x_3-3x_4\ge -3\\ \text{with} & x_1,\,x_2,\,x_3,\,x_4\ge 0.\end{cases}\qquad (D)\begin{cases}\text{maximize} & 5x_1+6x_2+9x_3+8x_4\\ \text{subject to}\\ & x_1+2x_2+3x_3+\ x_4\le 5\\ & x_1+\ x_2+2x_3+3x_4\le 3\\ \text{with} & x_1,\,x_2,\,x_3,\,x_4\ge 0.\end{cases}$$

$$(E)\begin{cases}\text{maximize} & x_1-2x_2-x_3\\ \text{subject to}\\ & 2x_1+\ x_2-x_3\le -1\\ & x_1-2x_2+x_3\le -2\\ \text{with} & x_1,\,x_2,\,x_3\ge 0.\end{cases}\qquad (F)\begin{cases}\text{maximize} & 5x_1+6x_2+9x_3+8x_4\\ \text{subject to}\\ & x_1+2x_2+3x_3+\ x_4\ \le 5\\ & x_1+\ x_2+2x_3+3x_4+x_5=3\\ \text{with} & x_1,\,x_2,\,x_3,\,x_4,\,x_5\ge 0.\end{cases}$$

1 Equivalence and Duality

Question 1-1. Which previous programs are equivalent to each others? Sort them into groups of equivalent classes.

Question 1-2. Give standard form of (F). What is the slack form of (E)?

Question 1-3. What are the dual programs (C) and (D)?

Question 1-4. Which programs (D), (E) and (F) have basic feasible solution? Give a feasible solution of (A).

2 Simplex Algorithm

Question 2-5. Solve linear program (D).

Question 2-6. Out of previous solution of (D) give a solution of its dual.

Question 2-7. Transform program (E) into an equivalent linear program having feasible basic solution.