Linear Programming

Complemetary Exam

Duration: 1h30.
No documents are allowed.
*Only **non-programmable** pocket calculators are allowed.*

We're considering the following list of linear programs respectively given the names (A), (B), (C), (D), (E) and (F).

$$(A) \begin{cases} \text{maximize} & 3x_1 + 2x_2 + 3x_3 \\ \text{subject to} & \\ & -x_1 - 2x_2 - 3x_3 \le 5 \\ & 2x_1 + x_2 + x_3 \le 3 \\ & -x_1 - x_2 + x_3 \le 1 \\ \text{with} & x_1,\ x_2,\ x_3 \ge 0. \end{cases}$$

$$(B) \begin{cases} \text{minimize} & x_1 + x_2 - x_3 \\ \text{subject to} & \\ & -x_1 - 2x_2 + 3x_3 \le 1 \\ & -x_1 - x_2 + 2x_3 \ge 2 \\ \text{with} & x_1,\ x_2,\ x_3 \ge 0. \end{cases}$$

$$(C) \begin{cases} \text{maximize} & 3x_1 + 2x_2 + 3x_3 \\ \text{subject to} & \\ & -x_1 - 2x_2 - 3x_3 \le 5 \\ & 2x_1 + x_2 + x_3 \le 3 \\ & -x_1 - x_2 + x_3 \le 1 \\ \text{with} & x_1,\ x_2 \ge 0. \end{cases}$$

$$(D) \begin{cases} \text{maximize} & -x_1 - x_2 + x_3 \\ \text{subject to} & \\ & -x_1 + 2x_2 - 3x_3 + x_4 = 1 \\ & x_1 + x_2 - 2x_3 \le -2 \\ \text{with} & x_1,\ x_2,\ x_3,\ x_4 \ge 0. \end{cases}$$

$$(E) \begin{cases} \text{maximize} & -x_1 - x_2 + x_3 \\ \text{subject to} & \\ & -x_1 - 2x_2 + 3x_3 \le 1 \\ & -x_1 - x_2 + 2x_3 \ge 2 \\ \text{with} & x_1,\ x_2,\ x_3 \ge 0. \end{cases}$$

$$(F) \begin{cases} \text{maximize} & 6x_1 + 4x_2 + 6x_3 \\ \text{subject to} & \\ & -x_1 - 2x_2 - 3x_3 \le 5 \\ & 2x_1 + x_2 + x_3 \le 3 \\ & x_1 + x_2 - x_3 \ge -1 \\ \text{with} & x_1,\ x_2 \ge 0. \end{cases}$$

1 Equivalence and Duality

Question 1-1. Which previous programs are equivalent to each others? Sort them into groups of equivalent classes.   3 P.

Question 1-2. Give standard form of (F). What is the slack form of (B)?   2 P.

Question 1-3. What are the dual programs of (A) and (E)?   2 P.

Question 1-4. Guess feasible solutions of (A) and (D).

2 P.

2 Simplex Algorithm

Question 2-5. Solve linear program (A) using the simplex algorithm.

4 P.

Question 2-6. Out of previous solution of (A) give an optimal point of its dual. What is its optimal objective value?

2 P.

Question 2-7. Solve dual of linear program (E).

5 P.