**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: - B. 0.2676 { P(X>50) = 1 - P(X<50) = 0.26 }

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: -

1. False.

P (X>44) = 0.1586 = 15.86% = 63 employees approx. (1% = 4 employees)

P(38<X<44) = 0.3413 = 34.13% = 136 employees approx.

Hence more employees at the processing center are older than 44 than between 38 and 44 is false.

1. Ture.

P (X<30) = 0.0912 = 9.12% = 36 employees approx.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: - 2 *X*1 = 2 x N (μ, σ2) = N (2 μ, 4 σ2)

*X*1 + *X*2 = N (μ, σ2) + N (μ, σ2) = N (2 μ, 2 σ2)

It means that Var 2 *X*1 has 2 times more variance value than *X*1 + *X*2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: - D. 48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: -

1. Total Profit = Profit1 + Profit2 = N (5, 32) + N (7, 42) = N (12, 52)

Total Profit in rupee = N (12, 52) \*45 = N (540, 225)

Rupee range containing 95% probability = (99.00, 980.99)

Hence rupee range containing 95% probability is 99 million and 980.99 million.

1. Z = (X-μ)/σ (wherein from z table, 5th percentiles = -1.645)

X = μ + Z σ = 540 + (-1.645 \* 225) = 169.875

Hence the 5th percentile of profit (in Rupees) for the company = 169.875 million

1. Probability of Division 1 making a loss P(X<0) = 0.0477 = 4.77%

Probability of Division 2 making a loss P(X<0) = 0.0400 = 4.00%

Hence Division 2 has a larger probability of making loss in a given year.