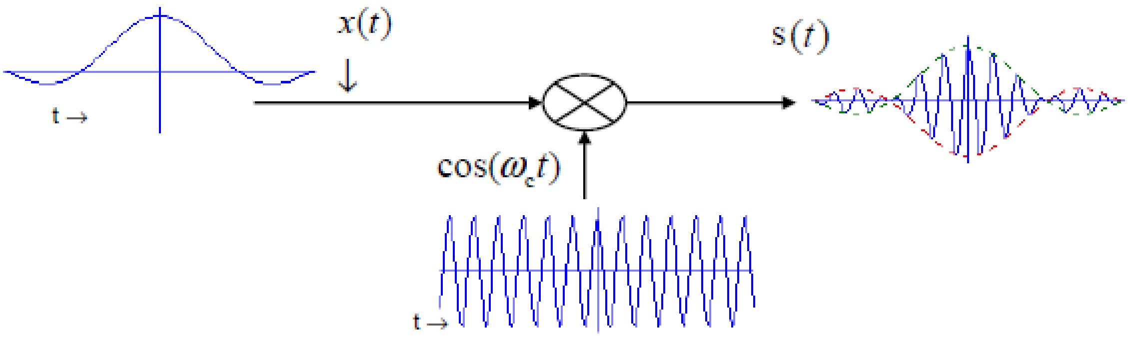
Amplitude Modulation (AM)

- Amplitude modulation is when the amplitude of a high frequency carrier signal is changed in accordance with the intensity of a message signal.

- In amplitude modulation, only the amplitude of the carrier wave is changed, but the frequency of the modulating (message) signal remains the same.

- Amplitude modulation is done by an electronic circuit called a modulator.



- A carrier signal is described as:

$$c(t) = V_c \cos(\omega_c t) \qquad (1.72)$$

where V_c is the amplitude of carrier.

- To amplitude modulate the carrier its amplitude is changed in accordance with the level of the message signal, which is described as:

$$m(t) = V_m \cos(\omega_m t) \qquad (1.73)$$

where V_m is the amplitude of message signal.

- The amplitude of the AM signal is given by:

$$\text{Amplitude of AM signal } = V_c + V_m \cos(\omega_m t) \qquad (1.74)$$

- The AM signal is given as:

$$s(t)_{AM} = \text{ Amplitude of AM signal } \times \cos(\omega_c t)$$

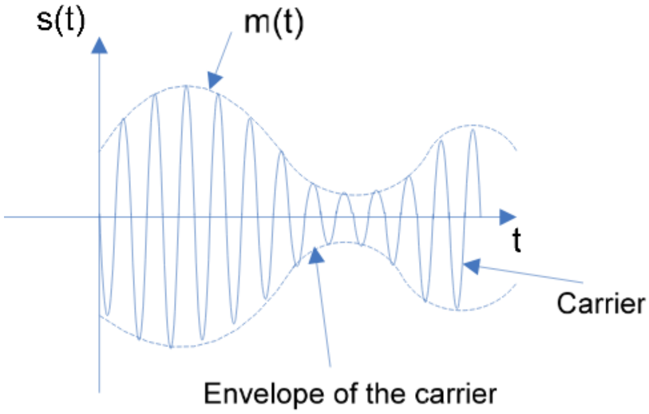
$$s(t)_{AM} = \left[V_c + m(t)\right]\cos(\omega_c t)$$

$$s(t)_{AM} = \left[V_c + V_m \cos(\omega_m t)\right]\cos(\omega_c t)$$

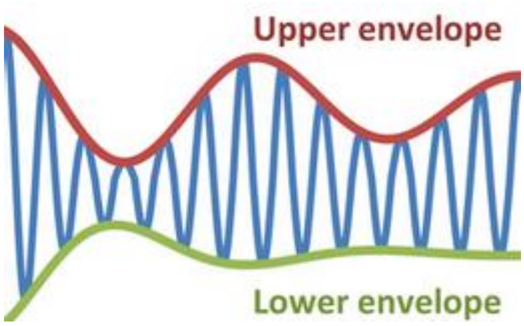
$$s(t)_{AM} = V_c \cos(\omega_c t) + V_m \cos(\omega_c t)\cos(\omega_m t) \qquad (1.75)$$

- Using the identity: $\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$, the above equation becomes:

$$s(t)_{AM} = V_c \cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c + \omega_m)t) + \frac{V_m}{2}\cos((\omega_c - \omega_m)t) \qquad (1.76)$$



- An envelope of an oscillating signal is a smooth curve outlining its extremes.



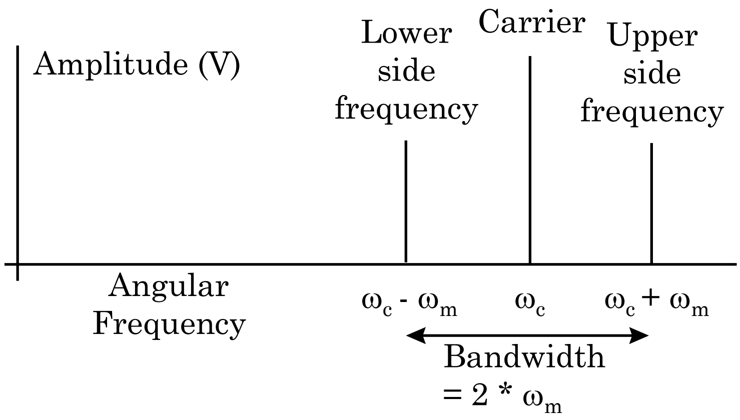
This is a signal made up of 3 components, i.e.

1. The carrier signal at frequency, ω_c.
2. The upper side frequency, $\omega_c + \omega_m$.
3. The lower side frequency, $\omega_c - \omega_m$

- The bandwidth is given as:

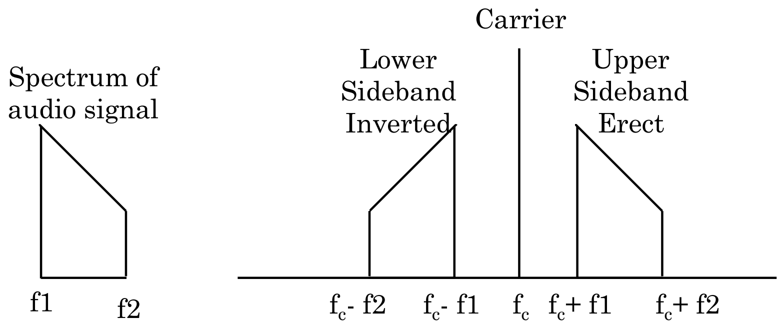
$$BW = (\omega_c + \omega_m) - (\omega_c - \omega_m) = 2\omega_m \text{ rad/s} \qquad (1.77)$$

- The spectrum of the 3 signals is shown below:



- If the message signal is made up of a range of frequencies from f_1 to f_2 (as is normally the case) rather than a single frequency the output signal will be a band of frequencies, contained in

- the upper side band (USB), erect and
- the lower side band (LSB), inverted.



<u>N. B.:</u> the bandwidth of the AM signal is twice the frequency of the highest modulating signal frequency.

<u>Modulation Factor/ Modulation Index</u>, m

- An important consideration in amplitude modulation is to describe the depth of modulation, i.e. the extent to which the amplitude of carrier signal is changed by the message signal.

- This is described by a factor called modulation factor or modulation index, m .

$$\text{Modulation factor or modulation index, } m = \frac{V_m}{V_c} \qquad (1.78)$$

- Using the modulation index, the AM signal can be expressed as:

$$s(t)_{AM} = V_c \cos(\omega_c t) + \frac{mV_c}{2}\cos((\omega_c + \omega_m)t) + \frac{mV_c}{2}\cos((\omega_c - \omega_m)t) \qquad (1.79)$$

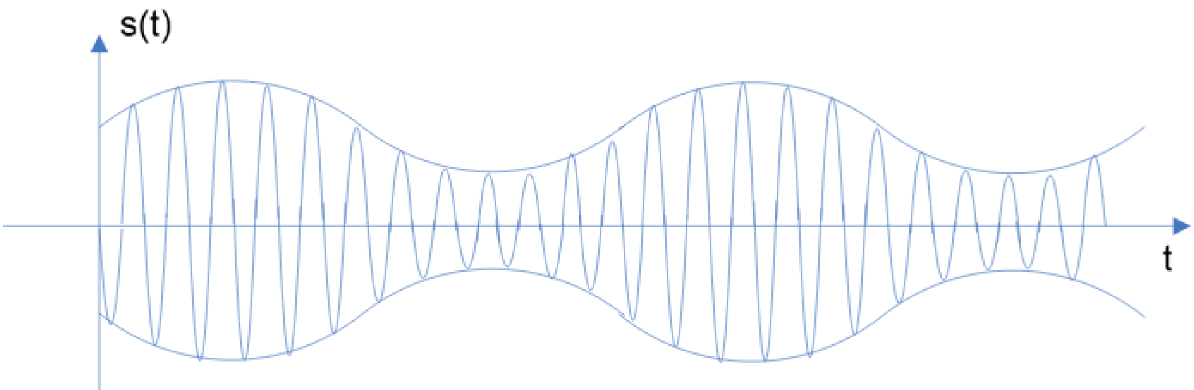
- If $m = 0.5$, the carrier amplitude varies by 50% above and below its unmodulated level.

- For $m = 1$ it varies by 100%.

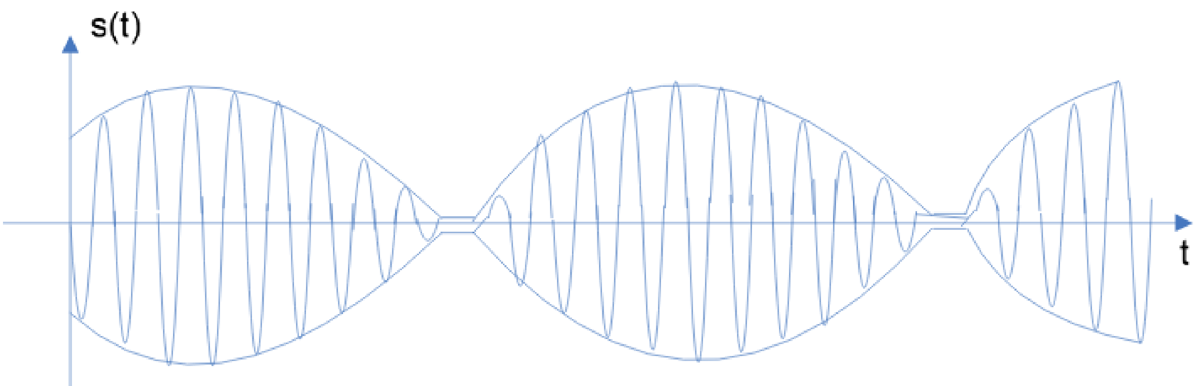
- Modulation depth greater than 100% is generally to be avoided as it creates distortion. This is known as over modulation.

- The greater the degree of modulation, i.e. modulation factor, the stronger and clearer will be the audio signal.

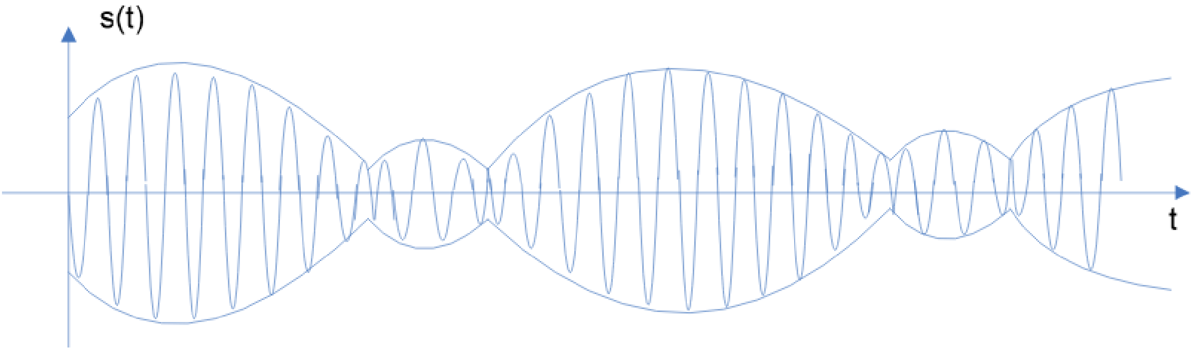
For modulation index less than 1, i.e. under modulated



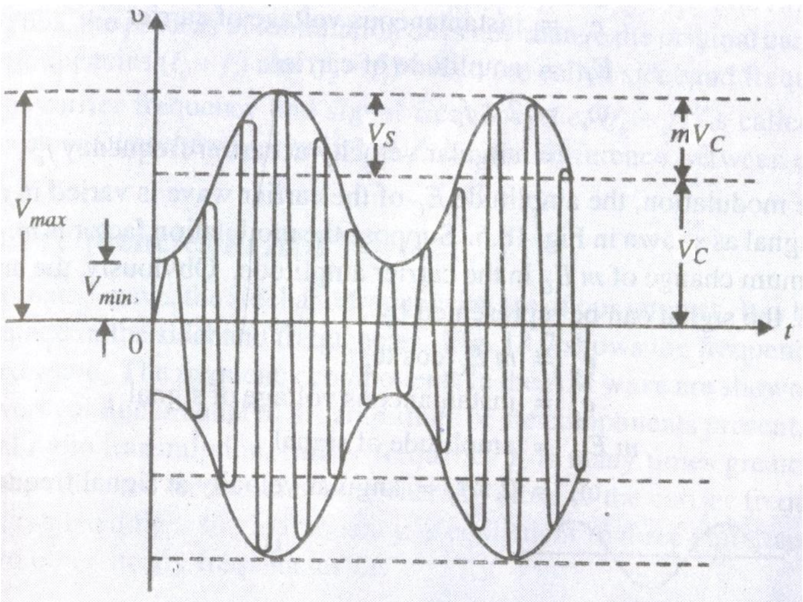
For modulation index = 1



For modulation index greater than 1, i.e. over modulated



- If the maximum and minimum amplitudes of the amplitude modulated signal are $V_{\max}$ and $V_{\min}$ respectively, then the modulation index can be expressed as:



$$m = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} \qquad (1.80)$$

<u>Power in AM Wave</u>

- In practice, the AM wave is a voltage or current signal.

- For a function, $A\cos(\omega t)$, its RMS value is:

$$RMS\left[A\cos(\omega t)\right] = \sqrt{\frac{1}{T}\int_0^T \left[A\cos(\omega t)\right]^2 dt} = A\sqrt{\frac{1}{T}\int_0^T \cos^2(\omega t)\, dt} = A\sqrt{\frac{1}{T}\int_0^T \frac{1+\cos(2\omega t)}{2} dt}$$

$$RMS\left[A\cos(\omega t)\right] = A\sqrt{\frac{1}{T}\int_0^T \frac{1}{2}dt + \underbrace{\frac{1}{2T}\int_0^T \cos(2\omega t)\,dt}_{0}} = A\sqrt{\frac{T}{2T}} = \frac{A}{\sqrt{2}} \qquad (1.81)$$

- The power of $A\cos(\omega t)$ can then be expressed as:

$$\text{Power}\left[A\cos(\omega t)\right] = \left(RMS\left[A\cos(\omega t)\right]\right)^2 = \left(\frac{A}{\sqrt{2}}\right)^2 \qquad (1.82)$$

- The AM signal can be expressed as:

$$s(t)_{AM} = V_c\cos(\omega_c t) + \frac{mV_c}{2}\cos\left((\omega_c + \omega_m)t\right) + \frac{mV_c}{2}\cos\left((\omega_c - \omega_m)t\right)$$

- Based on the above, the power of the carrier signal can be expressed as:

$$\text{Power of carrier, } P_C = \left(\frac{V_c}{\sqrt{2}}\right)^2 \qquad (1.83)$$

- The power of the upper sideband can be expressed as:

$$\text{Power of upper sideband, } P_U = \left(\frac{mV_c}{2\sqrt{2}}\right)^2 \qquad (1.84)$$

- The power of the lower sideband can be expressed as:

$$\text{Power of lower sideband, } P_L = \left(\frac{mV_c}{2\sqrt{2}}\right)^2 \qquad (1.85)$$

- Therefore, the total power in the AM signal is given as:

$$P_T = P_C + P_U + P_L$$

$$P_T = \left(\frac{V_c}{\sqrt{2}}\right)^2 + \left(\frac{mV_c}{2\sqrt{2}}\right)^2 + \left(\frac{mV_c}{2\sqrt{2}}\right)^2$$

$$P_T = \frac{V_c^2}{2} + \frac{m^2V_c^2}{8} + \frac{m^2V_c^2}{8}$$

$$P_T = \frac{V_c^2}{2} + \frac{V_m^2}{8} + \frac{V_m^2}{8} \qquad (1.86)$$

- The efficiency of an AM signal is given as:

$$\eta_{AM} = \frac{\text{Total sideband power}}{\text{Total power in AM signal}} = \frac{P_U + P_L}{P_T}$$

$$\eta_{AM} = \frac{\frac{m^2V_c^2}{8} + \frac{m^2V_c^2}{8}}{\frac{V_c^2}{2} + \frac{m^2V_c^2}{8} + \frac{m^2V_c^2}{8}} = \frac{m^2}{2+m^2} \qquad (1.87)$$

- Assume that the AM signal is dissipated in a load resistor, R.

- The power in the carrier signal is given as:

$$P_c = \frac{V_c^2}{2R} \qquad (1.88)$$

- The power in each of the sidebands is given as:

$$P_S = \frac{m^2V_c^2}{8R} \qquad (1.89)$$

- The total power is:

$$P_T = P_c + 2P_s = P_c\left(1 + \frac{m^2}{2}\right) \qquad (1.90)$$

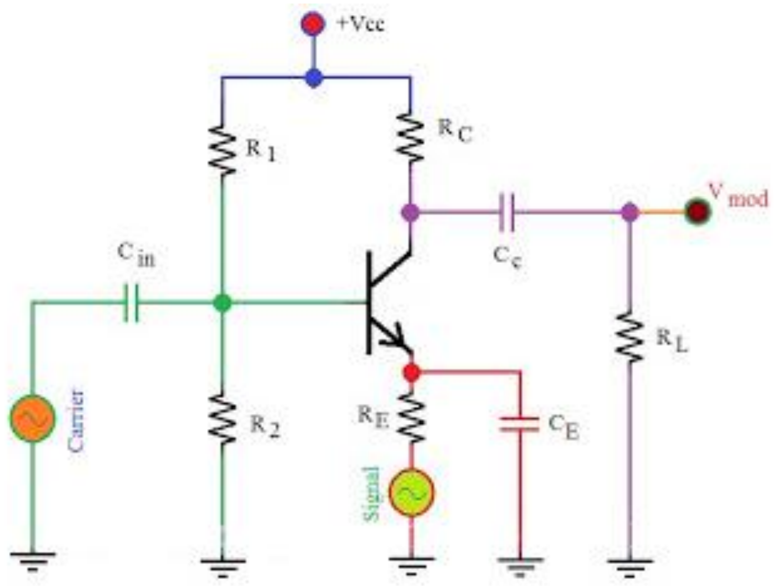
- The fraction of the power in the carrier is:

$$\frac{P_c}{P_T} = \frac{1}{1+\frac{m^2}{2}} \qquad (1.91)$$

- All along the discussion has been focused on Double-Sideband Large-Carrier (DSB-LC) modulation.

Simple AM Modulator

- The figure below shows a simple AM modulator.



- It is essentially a CE amplifier with a voltage gain of A_v.
- The carrier signal is the input to the amplifier.
- The modulating signal is applied in the emitter resistance circuit.
- The amplifier circuit amplifies the carrier by a factor A_v, so that the output is $v_c A_v$.
- Since the modulating signal is a part of the biasing circuit, it produces low frequency variations in the emitter circuit. This in turn causes variations in A_v.
- The result is that amplitude of the carrier varies in accordance with the strength of the modulating signal.
- Consequently, amplitude modulated output is obtained across R_L.

Double-Sideband Suppressed-Carrier (DSB-SC) and Single-Sideband (SSB) Amplitude Modulation

- Because most of the output power from an amplitude modulator is contained in the carrier there will be major power savings if the carrier can be suppressed.
- The carrier amplitude and frequency do not change and so it does not contain any signal information.

- Thus, the suppression of the carrier will not cause any of the information in the signal to be lost.

- Each sideband is the image of the other and one of them may be suppressed without the loss of any information.

- All of the information is conveyed through the use of a single sideband with no carrier.

- When the carrier and both side bands are transmitted it is called full amplitude modulation.

- If only the carrier is suppressed, we have double-sideband suppressed-carrier (DSB-SC) modulation.

- If the carrier and one side band are suppressed, we have single-sideband (SSB) or single-sideband suppressed-carrier (SSB-SC) modulation.

<u>Summary</u>

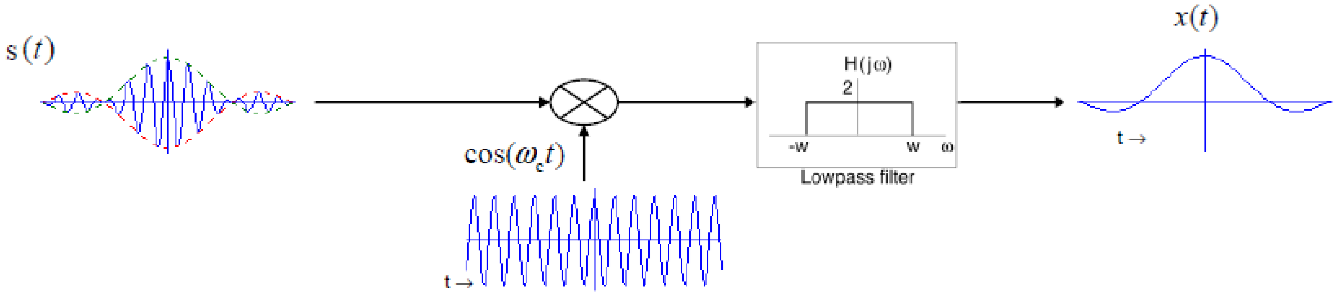
Amplitude Modulation Type	Modulated signal	Efficiency	Bandwidth
Double-Sideband Large-Carrier (DSB-LC) Modulation	$s(t)_{AM} = V_c \cos(\omega_c t) + \frac{V_m}{2}\cos((\omega_c + \omega_m)t) + \frac{V_m}{2}\cos((\omega_c - \omega_m)t)$	$\eta_{AM} = \frac{\text{Total sideband power}}{\text{Total power in AM signal}} = \frac{P_U + P_L}{P_C + P_U + P_L}$	$BW_{AM} = 2\omega_m$
Double Sideband Suppressed Carrier (DSB-SC) Modulation	$s(t)_{DSB-SC} = \frac{V_m}{2}\cos((\omega_c + \omega_m)t) + \frac{V_m}{2}\cos((\omega_c - \omega_m)t)$	$\eta_{DSB-SC} = \frac{\text{Total sideband power}}{\text{Total power in AM signal}} = \frac{P_U + P_L}{P_U + P_L}$	$BW_{DSB-SC} = 2\omega_m$
Single-Sideband (SSB) or Single-Sideband Suppressed-Carrier (*SSB*-SC) modulation	$s(t)_{SSB} = \frac{V_m}{2}\cos((\omega_c \pm \omega_m)t)$	$\eta_{AM} = \frac{\text{Total sideband power}}{\text{Total power in AM signal}} = \frac{P_U}{P_U} \text{ or } \frac{P_L}{P_L}$	$BW_{SSB} = \omega_m$

Limitations of Amplitude Modulation

- Noisy reception
- Low efficiency
- Small operating range
- Lack of audio quality

AM Demodulation

- Demodulation is the process of recovering the message signal from the modulated signal.



- After mixing, the output of the mixer is:

$$\hat{s}(t)_{AM} = \left[V_c + m(t)\right]\cos(\omega_c t) \times \cos(\omega_c t) = \left[V_c + m(t)\right]\cos^2(\omega_c t)$$

$$\hat{s}(t)_{AM} = \left[V_c + m(t)\right]\left(\frac{1+\cos(2\omega_c t)}{2}\right) = \frac{V_c + m(t)}{2} + \frac{V_c + m(t)}{2}\cos(\omega_c t) \qquad (1.92)$$

- After passing $\hat{s}(t)_{AM}$ through the lowpass filter we obtain:

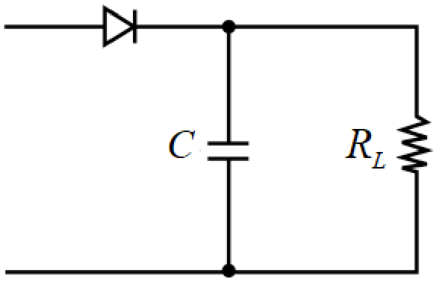
$$\frac{V_c + m(t)}{2} \qquad (1.93)$$

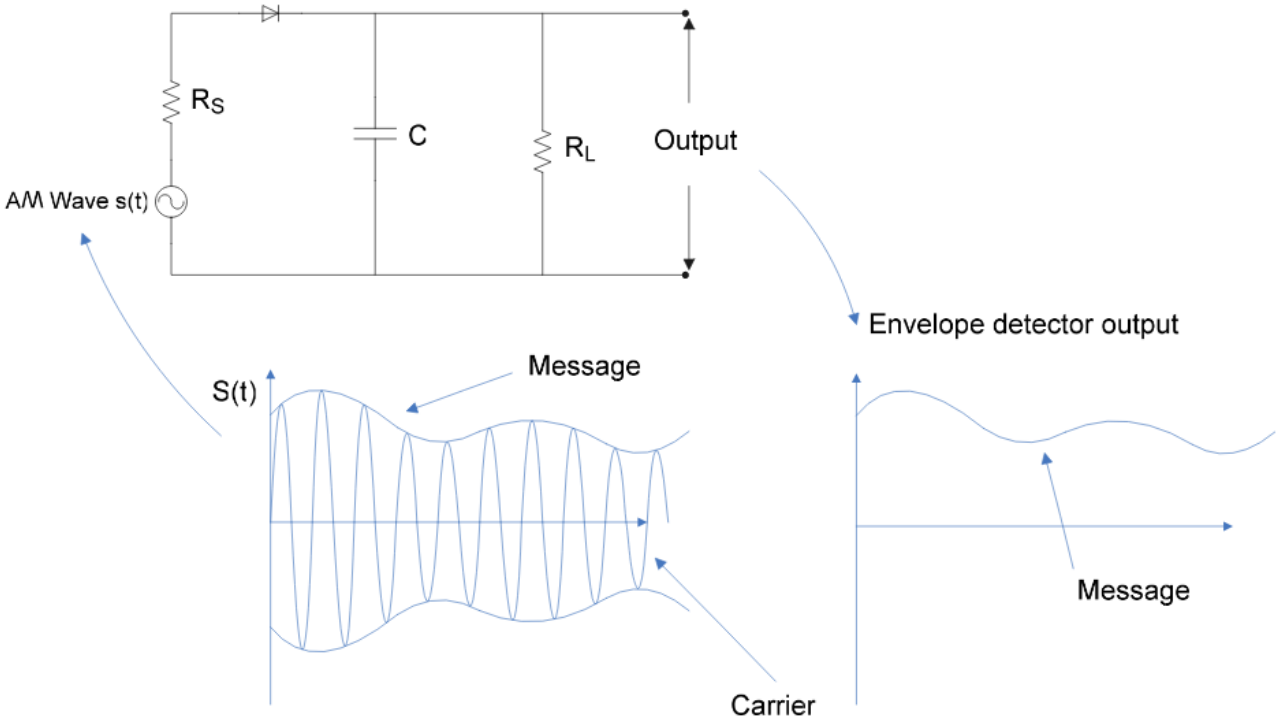
- After passing through a capacitor to block the DC component we obtain:

$$\frac{m(t)}{2} \qquad (1.94)$$

- This signal is then amplified using an appropriate amplifier to recover the original signal.

- When full wave transmission (DSB-LC) is used the most commonly used AM demodulator is the diode detector also known as an envelope detector.





The operation of the above circuit is as follows:

- On the positive half-cycle of the input signal, the diode is forward biased and the capacitor, C charges up rapidly to the peak value of the input signal.
- When the input signal falls below this value, the diode becomes reverse biased and the capacitor C discharges slowly through the load resistor R_L.
- This discharging process continues until the next positive half cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

<u>Limitations of the Diode Detector Demodulator</u>

- If the time constant $R_L C$ in the envelope detector is too long relative to the period of the highest frequency modulating signal it will not be able to follow the peaks and troughs of the envelope giving rise to diagonal clipping. However, to minimise ripple we want to make $R_L C$ as large as possible. In practice we should therefore choose a value:

$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{f_m} \qquad (1.95)$$

- If $R_L C$ is too short then there will be excessive RF ripple and the output power will be reduced.

- Because the diode is a non-linear device there will be some distortion in the demodulated signal.

- In general, $R_L C$ must be a lot longer than the period of the carrier and a lot shorter than the period of the modulating signal.

- R_L must be a lot larger than the forward resistance of the diode to maintain detection.

Exercises

1. The maximum peak-to-peak voltage of an AM wave is $16mV$ and the minimum peak-to-peak voltage is $4mV$. Calculate the modulation factor.

2. A carrier of $100V$ and $1200kHz$ is modulated by a $50V$, $1000Hz$ sine wave. Find the modulation factor.

3. A $2500kHz$ carrier is modulated by audio signal with frequency span of $50-15000Hz$. What are the frequencies of lower and upper sidebands? What bandwidth of RF amplifier is required to handle the output?

4. An AM wave is represented by the expression, $v = 5\left[1+0.6\cos(6280t)\right]\sin\left(211\times10^4 t\right) volts$

   i) What are the amplitudes of the carrier and message signal?
   ii) What frequency components are contained in the modulated wave and what is the amplitude of each?

5. A sinusoidal carrier voltage of frequency $1MHz$ and amplitude $100 volts$ is amplitude modulated by a sinusoidal voltage of frequency $5kHz$ producing 50% modulation. Calculate the frequency and amplitude of lower and upper sideband terms.

6. A carrier wave of 500 watts is subjected to 100% amplitude modulation. Find:
   i) Power in sidebands
   ii) Power of modulated wave