Chaotic Catastrophes and Jungian Dynamics: A Search for Metaphors

Like others before him, Ken Wilber describes humanity's insatiable and often ungraceful search for wholeness as "all joys yearn[ing] for infinity."[1] If we agree, then we in the West are faced with overcoming a paradigm that has dismissed, pathologized, and obscured that infinity and its attendant chaos behind a veneer of order and predictability at the expense of our personal and collective wholeness. The Greeks famously had a hard time accepting the idea of *apeiron*, which denotes "infinity" and "boundlessness." A major feature of Greek philosophy therefore sought to "save the appearances" – to coarse grain the chaotic edges of everyday experience by using mathematics as a metaphorical tool for explaining away chaos as a mere illusion.[2] This legacy of this Axial age leitmotif was then amplified in modernity by the metaphysical and scientific contributions of René Descartes and Sir Isaac Newton, which galvanized the superiority of certainty in part by hypostatizing the mathematics that Plato and his compatriots had applied only metaphorically. Several factors conspired in the twentieth to deconstruct this worldview, including the unfolding revelations of quantum mechanics wherein the predictability of the Newtonian/Cartesian project gives way to indeterminacy and probability. In retrospect, these early twentieth century advances in physics may have been permissive causes of a resurgence in romantic conceptions of wholeness, interdependence, participatory empirical inquiry, and complexity. This phenomenon has opened the aperture of scientific inquiry as much as it has illuminated the role of chaos in our personal lives.

The integration of *apeiron* into Western epistemology moves glacially, however, for the same reason it does so for us personally: it demands a relaxing of the ego and a relinquishing of

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[1] Wilber, Ken. 1981. *Up from Eden*. Wheaton, Illinois: Quest Books. 16.

[2] Jones, Roger S. 1992. *Physics for the Rest of Us : Ten Basic Ideas of Twentieth-Century Physics That Everyone Should Know-- and How They Have Shaped Our Culture and Consciousness*. Chicago: Contemporary Books. 102-103.

control over the environment that strikes us as a return to a pre-historic, magical naïveté. Unfortunately, the imbalance between order and chaos has generated problems for the Western psyche, leading it, for example, to project its unacknowledged chaos onto the others and the external world.[3] This paper will examine one dynamic in particular—the modern (mostly Western) "faith crisis"—to illustrate the synergetic forces of chaos and order, each of which are critical to the tension needed to dialectically move consciousness to more expansive stages. It will attempt to model faith crises as a "catastrophe" or "bifurcation" point on a person's psycho-spiritual developmental trajectory, and as such, the chaotic dynamics of the logistic map and strange attractors may yield strategies for integrating the experience and anticipating future development. A review of relevant aspects of chaos theory will lay the groundwork for the suggestion that personal and collective transformations in consciousness require chaos.

Chaos

We are familiar with the idea of "chaos" as a frenetic feeling about the disorderliness of our everyday experience, but in the discipline of chaos theory it refers to an extreme sensitivity of dynamical systems to its initial conditions, sometimes illustrated using the well-known "Butterfly Effect." The former can be more accurately described as "entropic chaos," in which objects follows the arrow of time to a maximum point of entropy and irreversible disintegration. The latter, however, attempts to explain behavior in an interdependent (i.e., complex) system where lower entropy patterns of order can appear and disappear at random. One important lesson from chaos theory is that simple functions can achieve indescribable dynamics when their outputs are reapplied as inputs *ad infinitum* in a process called "iteration." Chaos is a study of the

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[3] Van Eenwyk, John R. 1997. *Archetypes & Strange Attractors : The Chaotic World of Symbols*. Studies in Jungian Psychology by Jungian Analysts, 75. Toronto: Inner City Books. 157.

complexity that arises when the dependencies among a system's elements becomes important and its processes iterate. In such a system, whose whole is greater than the sum of its parts, removing an element degrades the overall system behavior disproportionately more than the individual contribution of that element.[4] In the mathematics of chaos theory, the goal is not a perfectly descriptive model of reality; the focus shifts from quantitative precision to qualitative pattern mapping.[5] Theorists working in this domain frequently resort to sophisticated visualization techniques that represent system variables in an abstract plane called "phase space," enabling observers to watch systems evolve as variable values change. Two representations in phase space are particularly relevant to this paper: one-dimensional bifurcation diagrams using the logistic map, and multi-dimensional "strange attractors."

When some functions are iterated, they oscillate among more than one output. Their trajectories in phase space are said to "bifurcate" regularly, first two, then four, sixteen, thirty-two. Almost without exception, in fact, quadratic functions experience period doubling behavior until it eventually returns chaotic outputs. This phenomenon can be shown using the logistic map, which models growth under environmental constraint:

$$x_{n+1} = \lambda\, x_n(1-x_n)$$

In the logistic map, x is a number between zero and one that represents the ratio of an existing value (e.g., a population) to the maximum possible value. That value can change based on some growth rate λ. As with population growth, however, behavior modeled with the logistic function is constrained by environmental factors, such as food availability. The parenthetical $(1-x_n)$ models those constraints, and this part of the equation is responsible for its non-linearity when

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[4] Miller, John H, and Scott E Page. 2007. *Complex Adaptive Systems : An Introduction to Computational Models of Social Life. Princeton Studies in Complexity*. Princeton, N.J.: Princeton University Press. 9.

[5] Capra, Fritjof. 1996. *The Web of Life : A New Synthesis of Mind and Matter*. London: HarperCollins.135-136.

iterated. Without the constraining factors, the iterated map would not yield chaotic output that ultimately leads to surprising stability at elevated levels of λ. Figures 1 and 2 below are produced with open-source Python libraries[6] and show the distinct outcomes when the equation is iterated 1 and 100 times respectively.

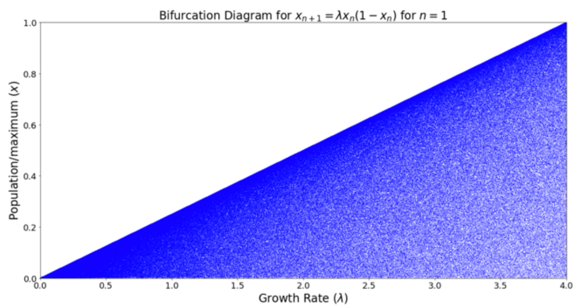

Figure 1

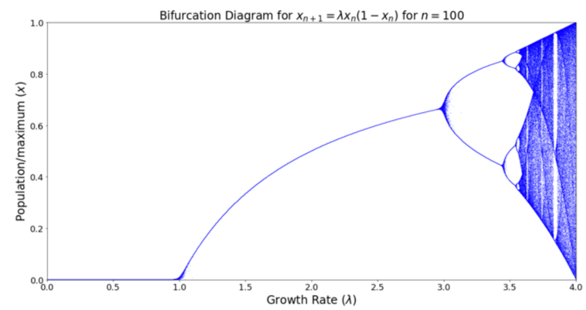

Figure 2

Run only a single time for all values of λ (figure 1), the relationship between the population and growth rate appears linear. Of course, this method does not account for feeding the new population in as the input for recalculation. When that iterative approach is taken, the graph yields a spectacular acceleration from periodic behavior to chaos or aperiodicity. In figure 2, we see that with growth rates (λ) of less than 1, the constraints of the environment wipe out the population. Between a λ of 1 and 3, the function outputs a stable, near deterministic population. As shown in figure 2, a λ of 1.5 is sufficient to maintain the population somewhere around 30 percent of the maximum (i.e., $x = 0.2$). When the λ parameter is pushed to 3, a critical point in the system's evolution appears as a bifurcation.

This parameter "pushing" increases the system's non-linearity and is akin to a surge of energy that moves it to a state far from equilibrium—a state that Ilya Pregogine has demonstrated can result in new structures and even new order.[7] Applied to Pregogine's dissipative structures, these bifurcations are points at which initial conditions (a system's history)

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[6] Numpy and Matplotlib. Code configured by the author is available in Appendix I.
[7] Capra, 136-137.

determine whether the structure breaks down or breaks through to one of several new states of order.[8] From that point on, the map displays a period doubling behavior where *x* values progressively oscillate between 2, 4, 16, and 32 periods. That is, until λ reaches its so-called "accumulation point" of 3.57, and the oscillations become totally chaotic and unpredictable. Though as Pregogine predicted, pushing the system far from equilibrium can yield more nuanced forms of order. Pushed beyond the point of accumulation, one might be tempted to write the system off as forever chaotic, but counterintuitively, order returns. Figure 3 below shows that when λ reaches 3.83, the system returns to a stable period of 3, and figure 4 provides a zoomed-in look (refer to scale) at this unexpected return to periodicity.[9]

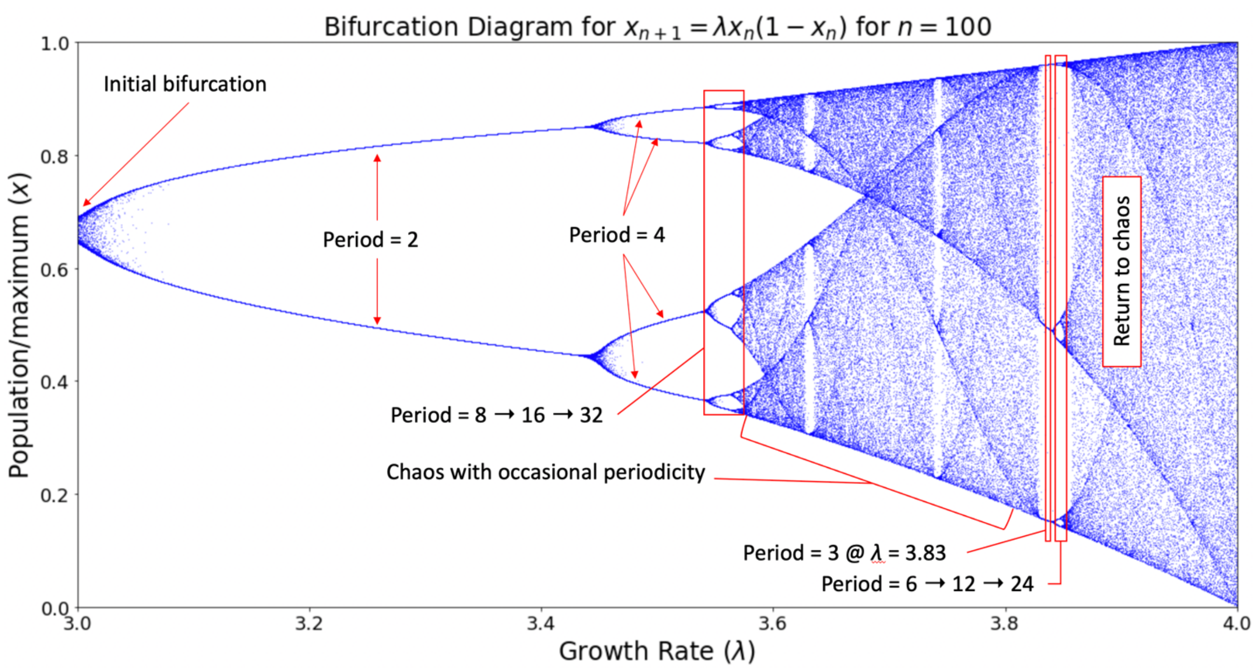

Figure 3

[8] Capra, 191.
[9] At this scale, the matoplotlib library fails to render the graph with sufficient detail. The author switched to the Pynamical Python library to achieve a higher fidelity. The associated configured code used by the author is included in Appendix I.

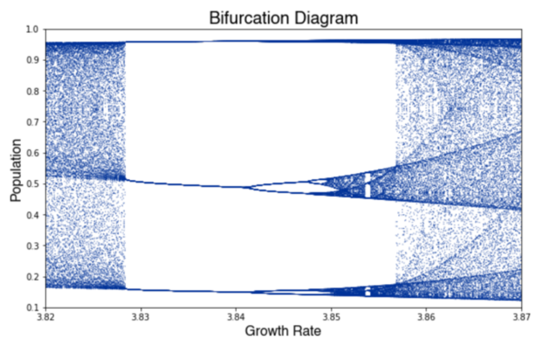

Figure 4

Recalling the aim of this paper is to suggest parallels between psychological development as it relates to faith crises illuminates a few conclusions about one-dimensional iterative functions. First, increasing the value of λ eventually pushes a system beyond equilibrium subjecting it to chaos. Second, points of extreme instability or "bifurcations" begin to appear as the function exhibits period doubling. In dissipative structures, the value of x is sensitive to the system's history. Third, the logistic map is one of many "mutual inhibition equations" that describe how competing influences work themselves out. Without something constraining the system, reflected in the logistic function as (1-x_n), the population explodes to the max at $\lambda = 1$ and no bifurcations and oscillations occur as shown in figure 5. Finally, the chaotic oscillations between values of x does not occur forever; counter to our expectations, periodicity and order return. At critical bifurcation points, the dissipative structures can descend into chaos or emerge into novel states of order. Visualizing the one-dimensional logistic map in phase space enables us to grasp a pattern that seems to inform the values of x at every λ. A pattern illustrating the values toward which the system tends to evolve is called an attractor, and understanding how attractors work will help associate them to psycho-spiritual processes later in the paper.

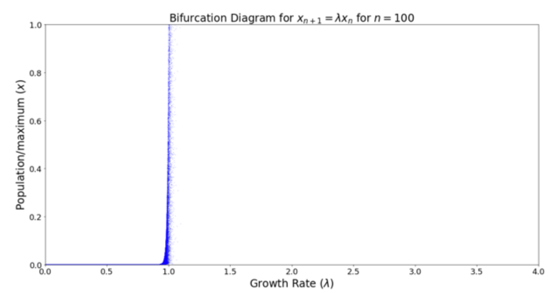

Figure 5

The fact that chaos is deterministic and patterned strikes us as paradoxical, because many of us conventionally understand it as mere "randomness." When researchers abstract their models to a few central variables in phase space, points appear to orbit in harmony with a governing structure. Of course, using the term “attractor” seems to infuse the shape with supernatural abilities although it does not function causally like Teilhard de Chardin’s “Omega Point” or Terrence McKenna’s “Transcendental Object at the End of Time.” We would not assign any causality to the *shape* of iron filings around a magnet, and we at least have difficulty ascribing causality to attractors in phase space, though we might be able to use them to deduce the systems dynamical properties. Attractors appear to us in at least three forms depending on whether a system is stable, periodic, or chaotic. The logistic map contains more than one of these. When the value of λ is less than 3, the graph reveals a point attractor showing a stable population equilibrium. Increasing that value beyond 3 reveals behavior characteristic of a periodic attractor that describes regular oscillatory activity in phase space. Beyond the point of accumulation, chaotic values of x nevertheless organize around the so-called “basin of attraction” or area around which points orbit in phase space. This last set of attractors embody a deep structural and universal paradox: they are ultimately unpredictable at any point along their trajectory, but viewed from “above” in phase space, they are infinitely recognizable.[10] Figures 6 and 7 respectively show a static view of the Lorenz (strange) attractor and an animated version showing 100 points with near identical initial conditions.[11] As with other dynamical systems sensitive to starting conditions, orbits around the Lorenz attractor basins diverge chaotically in phase space. In summary, strange attractors highlight the ordering principles of chaotic systems. Points along an orbit are chaotic, sensitive to initial conditions, and

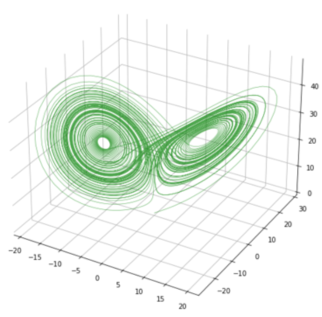

Figure 6

Figure 7

[10] Van Eenwyk, 54.

[11] The code configured by the author, which is based on open-source Python libraries, is available in Appendix I.

yet adhere to a non-causal structure called the basin of attraction. The list below reviews key concepts from iterative bifurcation diagrams and strange attractors that will be important for the discussion below about criticality and chaos in the psyche.

1. Infusing a dynamical system with energy (i.e., increasing the value λ in the logistic map) pushes a system beyond equilibrium.
2. Far from equilibrium, points of extreme instability or “bifurcations” begin to appear.
3. In dissipative structures, the value of x taken at bifurcation points is sensitive to the system’s initial conditions and history.
4. Mutual inhibition characterizes dynamical system that model competing influences; the constraints are important.
5. Chaotic oscillations beyond the point of accumulation occasionally give way to periodicity and order before returning to chaos.
6. Chaotic dynamics represented in phase space are “drawn” to basins of attraction that give us insight ordering principles.

Applying metaphors from chaos theory to the human psyche requires a psychological framework for analysis and comparison. Jungian psychology offers a rich set of agents and dynamics that allow robust overlap between the two disciplines. What follows is an outline of selected Jungian dynamics of the psyche; a metaphorical mapping of Jungian concepts to those in the list immediately above; and suggestions for further analysis.

Jung’s Dynamics of the Psyche

In its broadest sense, the analytical methods of Carl Jung divide the self between conscious and unconscious domains. The former includes our ego and personality while the latter plays host to inaccessible individual and collective aspects of our psyche. In that chthonic realm, our ego has a foil in the form of a shadow, and because it is part of the unconscious, it can never be directly addressed by the ego. Its presence has a vast impact on consciousness and so its effects can be observed. The shadow is a container for all the "dark aspects of the personality" that our psyche has (e.g., our initial conditions) removed from consciousness.[12] The presence of a force diametrically opposed to the ego creates a perpetual tension between the conscious and unconscious parts of the self. It is the integration of these split-off shadow aspects—bringing them into consciousness and acknowledging them as parts of our Self—that brings us closer to wholeness. Jung calls this process is called individuation, and it is a notoriously challenging journey aptly summarized in his formulation of an earlier quote from Neitzsche: "no tree can grow to heaven, it is said, unless its roots reach down to hell."[13] Ken Wilber, too, speaks of this dichotomy and the salvation available to us if we open ourselves up to its dynamic. What Jung calls the conscious ego and the unconscious shadow, Wilber calls Eros and Thanatos respectively.[14] Eros exists primarily to reduce uncertainty and preserve the integrity of the ego. Thanatos, by contrast, stands in as the chthonic realm of the unconscious shadow. Jung and Wilber agree that shadow/Thanatos can overwhelm the ego/Eros and thereby bring it to a point of criticality where a path bifurcates into differentiation or transformation/integration. And as is the case with all chaotic dynamics, the path taken depends on the historical conditions of the psyche. The next several paragraphs aim to demonstrate that chaos theory may provide insight into the interplay between the unconscious and consciousness, because this interaction is governed by chaotic dynamics.

Why did humans evolve this seemingly Sisyphean, dialectical mechanism in the first place? Fritjof Capra reminds us that nature is imbued with a measure of creativity that generates

[12] Jung, C. G. 1979. *Aion: Researches into the Phenomenology of the Self* (version Second edition ; first Princeton/Bollingen paperbook print.) Second edition ; first Princeton/Bollingen paperbook print ed. Bollingen Series, 20. Princeton, N.J.: Princeton University Press. 8.
[13] Jung, *Aion*, 43.
[14] Wilber, 78.

novelty and increasingly complex forms.[15] One hypothesis for the emergence of novel structures can be seen in the Nobel Prize-winning work of Ilya Pregogine who, in the late twentieth century, described how systems far from equilibrium can generate order from chaos—novel structures emerging from higher entropy states. Might it be that the tension between consciousness and unconsciousness, between the ego and the shadow, is the appearance of this universal dynamic in the human psyche? If so, we are invited to orient to an experience of integration that incorporates rather than rejects chaos. As John Van Eenwyk, a Jungian psychiatrist at the University of Washington, has observed, the chaos generated when the opposing forces of the conscious and unconscious meet in the psyche can lead to increased psychological functioning and creativity in the same way that deterministic chaos can create greater complexity and order than existed at the onset.[16]

The introductory framing of Jungian dynamics above is generally sufficient for the purposes of this paper, but further excavation of the processes responsible for propelling the psyche to higher levels of organization and coherence will aid in aligning the process with relevant elements of chaos theory. For Jung, the unconscious mind circumambulates discrete forces called archetypes that mobilize psychic energy into a kind of oscillating current between the unconscious and conscious mind.[17] These archetypes contain broadly recognizable and coherent forms (e.g., mother, trickster, shadow) that show up mythologically across national and cultural boundaries. Though they themselves are not representable in consciousness, they invest images and symbols with a power that commands our attention. Symbols can be thought of as a manifold that enables visualization of high dimensional objects (archetypes/weather) in low

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[15] Capra, 222.
[16] Van Eenwyk, 63, 98, 112.
[17] Van Eenwyk, 28.

dimensional terms (images/Lorenz attractor). In short, archetypes create basins of attraction that draw in our thoughts and actions, mostly unconsciously.

The archetypes also perform an important balancing role. When the ego has invested too much energy in a particular point of view, the archetypes incline that energy toward other, underemphasized.[18] Most of the time, we do not acknowledge that this opposing view in fact lives within us as shadow, and so we project those dark elements onto others or the world so that we can keep the forces at bay. As the potential psychic energy difference between ego and shadow increases, the psyche is brought to a point of criticality far from equilibrium. Perceiving irreconcilable dualities, the psyche plunges into the very essence of paradox and must address the oscillations between ego and shadow.[19] Whether the person is able to integrate the shadow at the root of the eruption of psychic energy is one matter; what is important to note about this process is that it is inherently *iterative*. As noted above, all human beings innately strive for wholeness by navigating the oscillations between unconscious and consciousness contents, an interminable process that can never be exhausted.[20] When shadow is integrated into consciousness, the "function" produces a new value of the ego state that is then fed in as a new input, further demonstrating the iterative nature of the dynamic. Since this iterative, oscillating process repeats, it is called synchronic, and it is responsible for generating the energy that fuels psychological novelty and development. Synchronic dynamics are particularly acute in so-called "double bind" scenarios where feedback between perception or intellect and experience is paralyzing.[21] Faced with this influx of energy that has pushed the system far from equilibrium, bifurcation points begin to appear that offer paths to chaos or increased order. In dynamic system

[18] Van Eenwyk, 25.
[19] Van Eenwyk, 25.
[20] Jung says that "the unconscious as we know it can never be 'done with' once and for all." Jung, *Aion*, 20.
[21] Van Eenwyk, 51.

theory, bifurcations caused by inordinately high λ that do not stabilize they can transition to a Hopf bifurcation, which leads an otherwise well-behaved system ineluctably cascading to the brink of chaos.[22] Synchronic dynamics energize growth and are responsible for creating novel structures in the psyche,[23] but a second, concurrent process is responsible for *development*: the diachronic dynamic.

If synchronic dynamics provide fuel and charge to the engine, diachronic ones engage the clutch and steer from one developmental point to another. In this way, the two-dimensional approach to psychological change mirrors Ken Wilber's transformation/translation dynamic. In Jung's framework, the shadow can overwhelm the ego and lead to integration or further differentiation. For Wilber, "when Thanatos exceeds Eros, translation [or horizontal movement with a developmental stage] fails and transformation ensues."[24] Diachronic dynamics create highly individualized "stories" in a psyche, so they are less prone to pattern than the synchronic processes that feed them. Should the diachronic process slow down in a particular psyche, the unconscious addresses the one-sidedness by interjecting a tension of opposites, activating the synchronic aspects of individuation.[25] Clearly, the two processes are interdependent, one pushing the λ through tension of opposites and the other acting on the energy for psychological development. Table 1 below draws analogs between the properties of the dynamical systems

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[22] Van Eenwyk, 61.
[23] Van Eenwyk, 43.
[24] Wilber, 78.
[25] Van Eenwyk, 88.

examined in the first few pages of this paper and Jung's two-pronged dynamics of the psyche just discussed.

Dynamical System Properties	Jungian Dynamics
When λ increases…	When the energy differential between the shadow and ego increases…
…bifurcations and chaos appear in iterated, non-linear functions.	…the psyche is pushed to criticality…
Values of x oscillate at sufficiently high values of λ.	…characterized by oscillations between the opposites in the conscious and unconscious.
Order returns periodically past the point of accumulation.	Painful as they are, chaos enables periodic returns to higher functioning.
In phase space, chaotic orbits sensitive to initial conditions organize around attractors.	The psyche orients to archetypes, which charge images with symbolic power, initiating synchronic dynamics

Table 1

Chaos and the Modern Faith Crisis

The phenomenon of the modern faith crisis supplies a prototype for testing these fused dynamics. The crisis is *modern*, because it pits post-enlightenment rationality and science against spirituality and matters of faith. The experience of Leo Tolstoy aptly represents the major facets of such a crisis. Although, raised in the Eastern Orthodox Church, he lost his erstwhile faith during his education. Though this estranged him from the church, this first falling away did not push him to a critical bifurcation point. It was, instead, an existential desperation that besieged him later in life which neither science nor philosophy could satisfy. Seeing the blissful state of his peasants who were deeply rooted in religious faith, he grew more and more convinced that "without belief in god he could not live, yet he could not believe in god."[26] He was deeply torn between the belief that faith provides the answers to life and the irrational, decidedly unscientific position it appeared to force on him.[27] Although his crisis was accented by deep suicidality and

[26] Tolstoy, Leo and Aylmer Maude. 2005. *A Confession*. Mineola, NY: Dover Publications. iv.
[27] Of this experience, Tolstoy said, "Faith still remained to me as irrational as it was before, but I could not but admit that it alone gives mankind a reply to the questions of life…" See Tolstoy, 46.

depression, he nevertheless allowed the oscillations between opposite positions to play out naturally in his mind, embodying Jung's later suggestion that "if the ego does not interfere with its irritating rationality, the opposites, just *because* they are in conflict, will gradually draw together, and…settle down in a latent state of concord (original emphasis)."[28] At a certain point in Tolstoy's faith crisis, the pronounced oscillations seemed to simply relax into a "state of latent concord." He was so taken by the imperceptible and gradual nature of the transition that in his memoirs he simply wrote, "when and how this change occurred I could not say."[29] Could it be, as Van Eenwyk proposes, that "by submitting to the chaos of the unconscious that it becomes deterministic, progressing from order to chaos and up to higher levels of order?"[30]

During catastrophe events like a personal, Tolstoy-style, modern faith crisis if the ego can allow itself to oscillate between the viewpoints of ego and shadow, the chaos can become deterministic. This often happens in contemporary American religion when congregants, long having focused consciousness on a single set of polarities (e.g., "good," "god," "inerrancy of scripture"), experience the break-through of a highly charged shadow to challenge the status quo. The phase space trajectory at these highly energized λ levels could engender Hopf bifurcations, which, according to Van Eenwyk, can occur in religious contexts "when discontinuities within a text interrupt our harmony within it, forcing us to search for consistency. The bifurcation sets off a cascade of oscillations among possibilities for completing the text."[31] Here, we can substitute any body of doctrine for "text" and see that discontinuities made apparent to consciousness by the shadow that long held them unconsciously (until the disparity was too great for the psyche to

[28] Jung, Carl. 1953-1979. "Mysterium Coniunctionis" in *The Collected Works*, vol. 14. Princeton: Princeton University Press. Par. 505.
[29] Tolstoy, 60.
[30] Van Eenwyk, 100.
[31] Van Eenwyk, 76.

contain and the shadow broke through). Those undergoing such a crisis might benefit from the attitude of the Desert Fathers who intentionally broke with the conventional order so they could mentally inhabit an “irrational void.”[32] This is also in line with Jung’s recommendation for the ego to step back and acknowledge its impotence and “[let] the furious battle of psychic powers go its own way.”[33] Another important take-away involves the element of mutual inhibition represented in the logistic map as the constraining expression $(1-x_n)$. Constraints are necessary prerequisites to healthy chaotic development, not barriers to it. Western religion has little tolerance for the presence of “evil,” which means that the archetype of the trickster (e.g., who brings disorder) lacks expression in conscious life. In the religion of the Iroquois, harmony requires the presence of the trickster archetype who overturns convention and crosses boundaries. To have one without the other would create disharmony in the world.[34] Recognizing the chaotic dynamics of faith crisis as the necessary interaction of psychologically opposed forces can help reduce the associated anxiety.

Conclusions

Chaos remains a pariah in modern psychology, though its utility is expanding in modes of scientific inquiry. The essence of Jungian individuation is precisely this process by which the archetypes draw attention of the ego into cyclical and diachronic dynamics that lead to psychological development and evolution of consciousness. The ego need only relax its grip on what it thinks it knows to be drawn into the chaotic world of the possible. Many theorists and researchers who work with chaotic dynamics have stressed the importance of introducing

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[32] Van Eenwyk, 74.
[33] Jung, *Mysterium Coniunctionis*, par. 505.
[34] F. David Peat. 2002. *Blackfoot Physics*. York Beach: Red Wheel. 174-176.

dynamical systems theory and chaos principles at earlier stages of education. Robert May, who brought the logistic map to bear in the field of biology in the mid 1970s stated, “we would be better off is more people realized the simple…systems do not necessarily possess simple…properties.” In some areas of science, researchers are proposing complete paradigmatic shifts away from deterministic thinking to theories that start from chaos. For example, Robert Pool tells us that “healthy systems don’t want homeostasis. They want chaos.”[35] One avenue for carrying a chaos-based inquiry into consciousness would be to validate the suggestion here that iterated quadratic equations like the logistic map can describe the activity of the psyche, notably at catastrophe points of bifurcation and subsequent oscillations. From there, determining whether applying the Feigenbaum constant can help inform responses to and techniques for navigating crises, religious or otherwise.

[35] Pool, Robert. 1989. “Is It Chaos, or Is It Just Noise?” *Science* 243 (4887): 25–25. In Van Eenwyk p. 167.

Appendix I: Python Code

To develop the diagrams in this paper, I drew on open-source Python libraries and publicly available snippets to configure and run code in a local Jupyter Notebook. Figures 1, 2, 3, and 5 were generated from the same code with parameters shifted to achieve the desired effect. I will first print the base code that generates a logistic map over 100 iterations and then describe the specific configurations for each of the figures.

```
[1]: # Configured for PARP 6756 Final Project (Ben Thomas)
```

```
[2]: # import libraries
     import numpy as np
     import matplotlib.pyplot as plt
```

```
[3]: # Create a logistic map function
     def logistic(λ, x):
         return λ * x * (1 - x)
```

```
[4]: n = 1000000  # points plotted
     λ = np.linspace(0.0, 4.0, n)
     iterations = 100
```

```
[5]: x = np.random.uniform(0.0, 1.0, n)  # starting value
     for _ in range(iterations):  # iterate that function
         x = logistic(λ, x)

     fig = plt.figure(figsize=(20, 10))
     ax = plt.axes()

     ax.set_title(f"Bifurcation Diagram for $x_{{n + 1}} = λx_{{n}}(1-x)$ for $n=${iterations}", fontsize=24)
     ax.set_xlabel('Growth Rate ($λ$)', fontsize=24)
     ax.set_ylabel('Population/maximum ($x$)', fontsize=24)
     ax.tick_params(axis='both', which='major', labelsize=18)
     ax.set_xlim(0.0, 4.0)
     ax.set_ylim(0.0, 1.0)

     ax.plot(λ, x, ',b', alpha=.5)
```

Figure 1
To depicts the logistic map without iteration, block 4 is updated to `iterations=1`

Figure 2
This illustrates the logistic map run over 100 iterations and matches the configuration above (i.e., `iterations = 100`).

Figure 3
This figure is a simple zoom achieved by setting the min values of the x axis in block 5: `ax.set_xlim(3.0, 4.0)`

Figure 5
The goal in this image is to show the behavior of the function without the constraint expression. To achieve this, we simply remove `* (1-x)` from block 3.

Appendix I: Python Code

Due to the resolution quality on further zooming, I decided to use the Pynamical library to illustrate the logistic map at close range in **Figure 4**.[36] Although this required some additional local setup using a Docker instance, the Pynamical library abstracts everything required to graph the logistics map in matplotlib and numpy behind a single block of code:

```
[1]: from pynamical import logistic_map, simulate, bifurcation_plot
     pops = simulate(model=logistic_map, num_gens=100, rate_min=3.7, rate_max=3.9, num_rates=1000, num_discard=100)
     bifurcation_plot(pops, xmin=3.7, xmax=3.9)
```

For the static (**Figure 6**), I used the following code:

```
[1]: # Configured for PARP 6756 Final Project (Ben Thomas)

[2]: %matplotlib inline
     import numpy as np, matplotlib.pyplot as plt, matplotlib.font_manager as fm, os
     from scipy.integrate import odeint
     from mpl_toolkits.mplot3d.axes3d import Axes3D

[3]: font_family = 'Blender Pro'
     title_font = fm.FontProperties(family=font_family, style='normal', size=12, weight='normal', stretch='normal')

[4]: # define the initial conditions (i.e., x, y, z positions in space)
     initial_state = [1.0, 2, 3]

     # set the system parameters
     sigma = 10.
     rho   = 28.
     beta  = 8./3.

     # set the time points
     start_time = 0
     end_time = 100
     time_points = np.linspace(start_time, end_time, end_time*100)

[5]: # define the lorenz model
     # x, y, and z constitute the system state; t = time; and sigma, rho, beta are system params
     def lorenz_system(current_state, t):

         # x, y, z in space at the current time point
         x, y, z = current_state

         # the 3 lorenz equations!
         dx_dt = sigma * (y - x)
         dy_dt = x * (rho - z) - y
         dz_dt = x * y - beta * z

         # return a list of the equations describing the system
         return [dx_dt, dy_dt, dz_dt]

[6]: # relying on odeint() to solve the Lorenz equations
     xyz = odeint(lorenz_system, initial_state, time_points)

     # get the individual arrays of x, y, and z values from the array of arrays
     x = xyz[:, 0]
     y = xyz[:, 1]
     z = xyz[:, 2]

[7]: # plot the lorenz attractor in three-dimensional phase space
     fig = plt.figure(figsize=(12, 9))
     ax = fig.add_subplot(projection='3d')
     ax.xaxis.set_pane_color((1,1,1,1))
     ax.yaxis.set_pane_color((1,1,1,1))
     ax.zaxis.set_pane_color((1,1,1,1))
     ax.plot(x, y, z, color='g', alpha=0.7, linewidth=0.6)
     ax.set_title('The Lorenz Attractor in Phase Space', fontproperties=title_font)

     plt.show()
```

[36] See Boeing, G. 2016. "Visual Analysis of Nonlinear Dynamical Systems: Chaos, Fractals, Self-Similarity and the Limits of Prediction." Systems, 4 (4), 37. doi:10.3390/systems4040037.

Finally, **Figure 7**, which is an animated Lorenz attractor I configured and ran in a Jupyter notebook based on code located in a publicly available GitHub repository.[37]

[37] Droin, Colas. 2019. *Jupyter-notebooks*. GitHub Repository. https://github.com/ColasDroin/Jupyter-notebooks