Univariate Volatility Modeling

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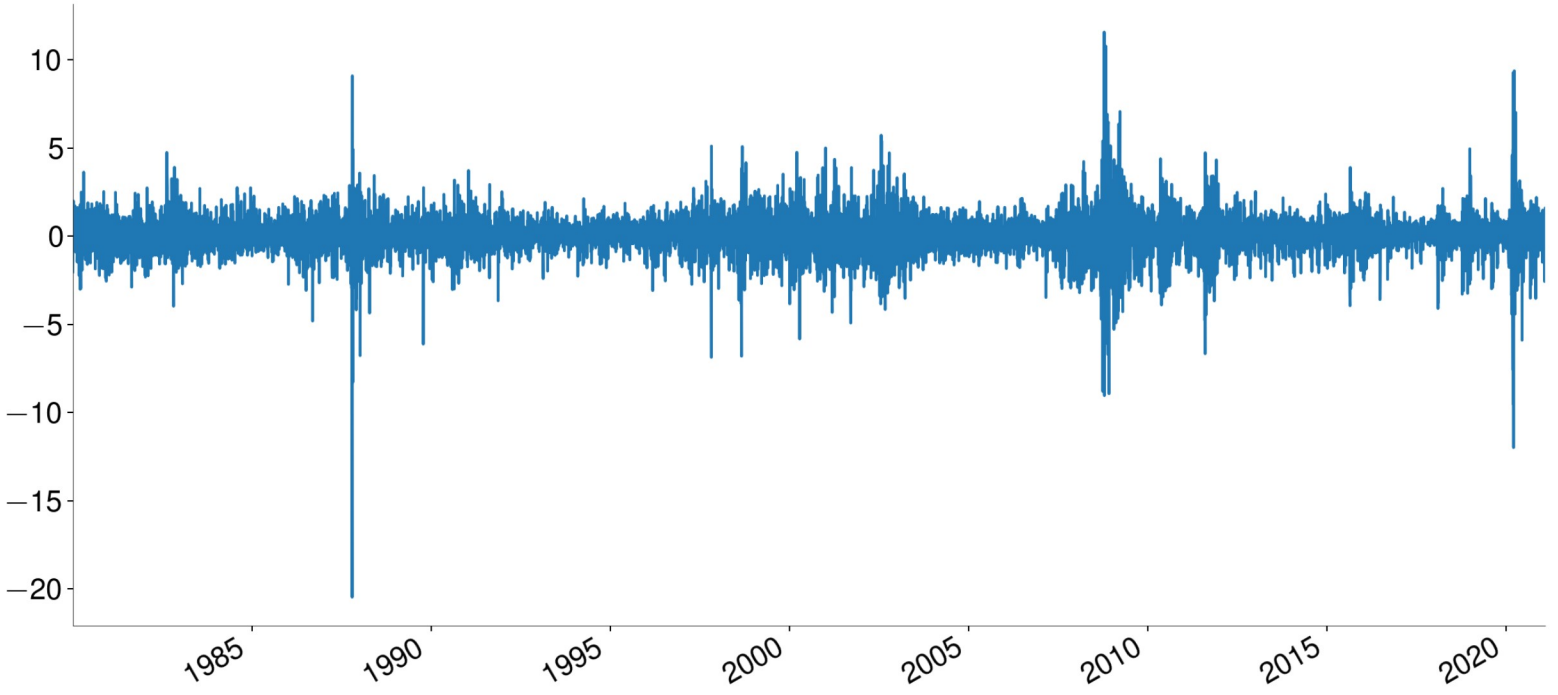
Topic Outline

- Visualizing Conditional Volatility
- Testing for Conditional Volatility
- ARCH Models
- GARCH Models
- Asymmetric Extensions
- Model Building
- Specification Checking
- Alternative Distributional Assumptions

Data

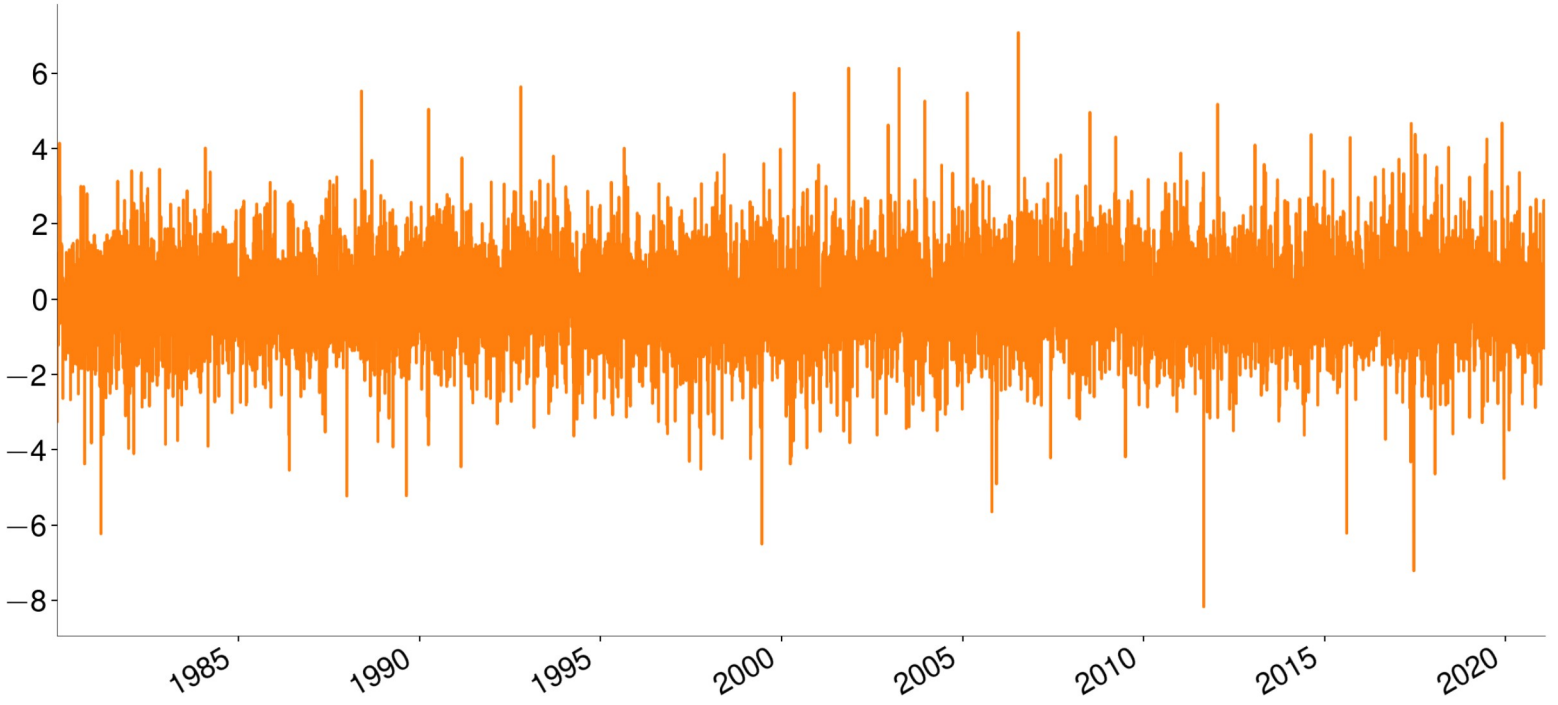
S&P 500 Returns

In [3]:
```
plot(sp500)
```



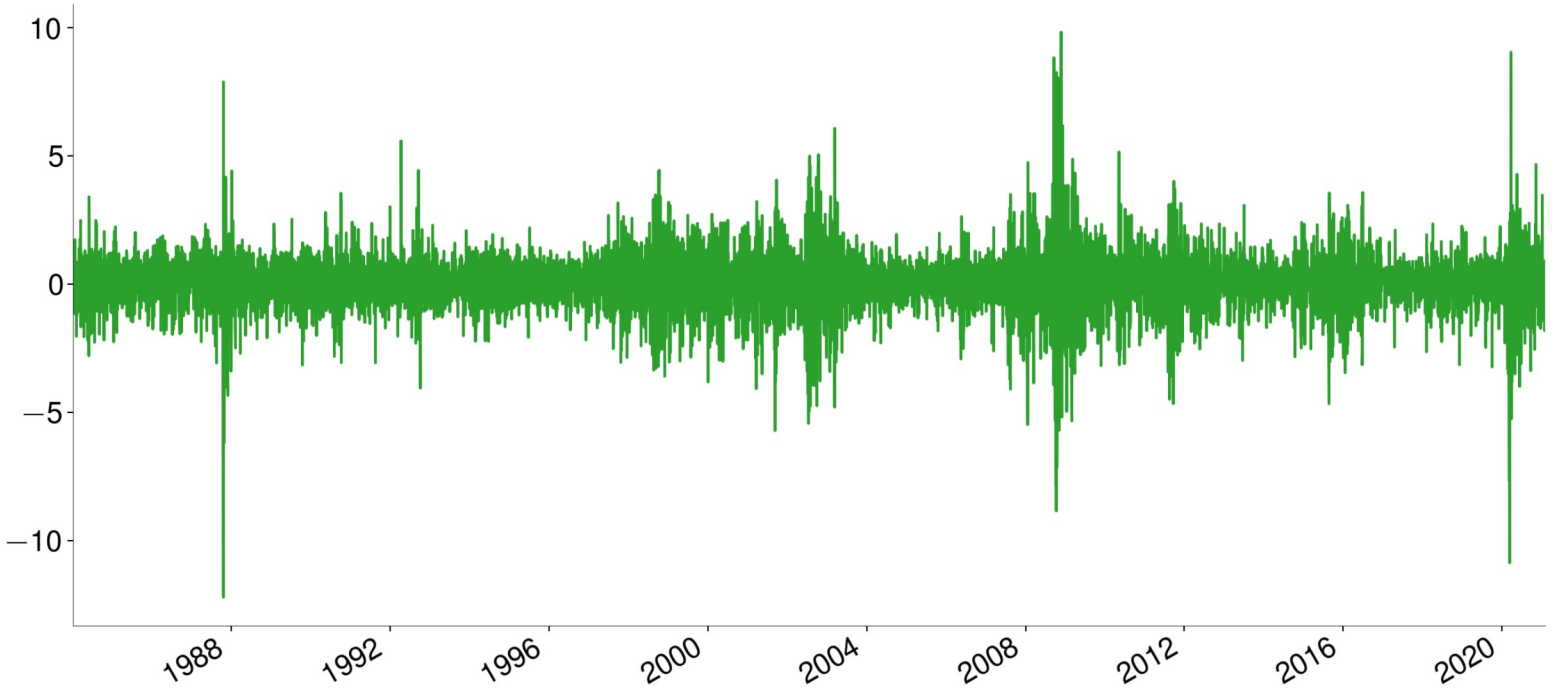
Simulated IID t_6 Data

In [4]:
```
plot(iid)
```



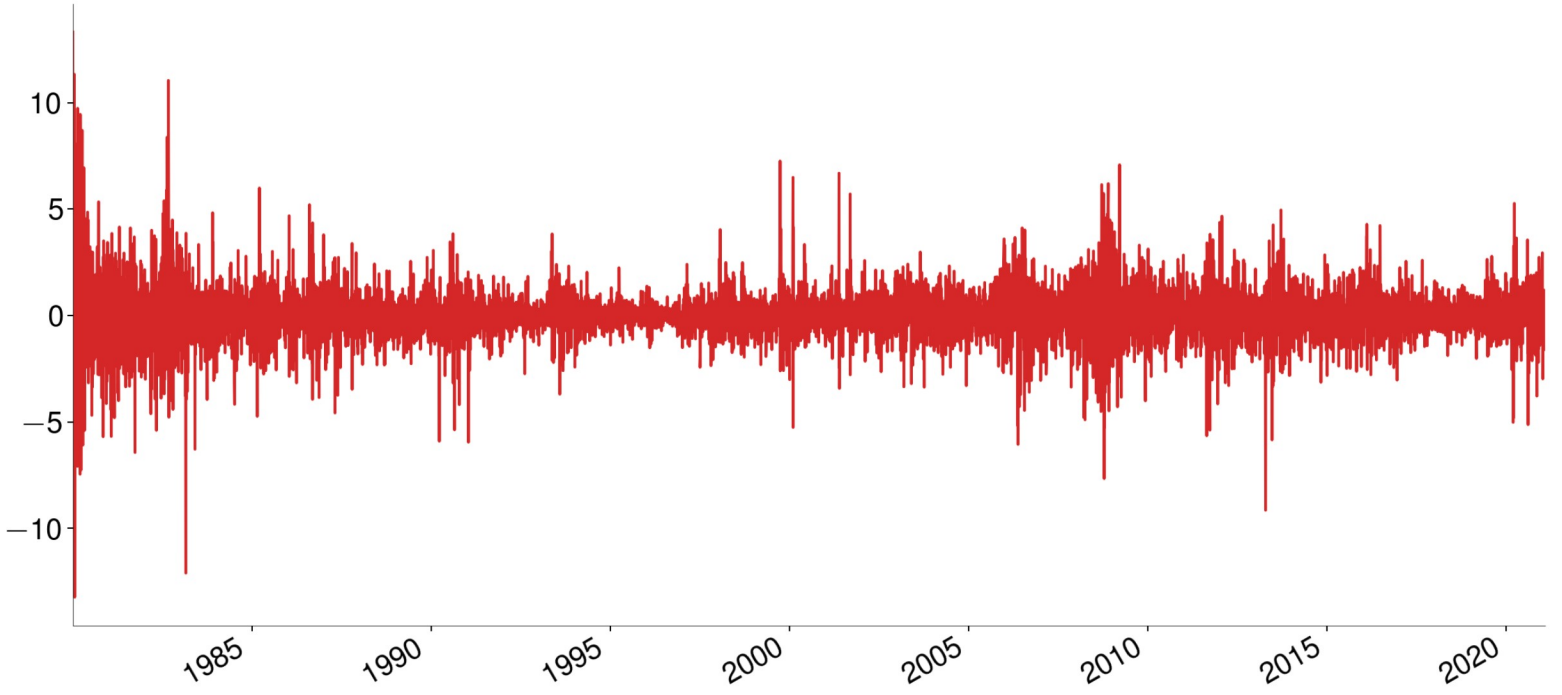
FTSE 200

In [5]:
```
plot(ftse100)
```



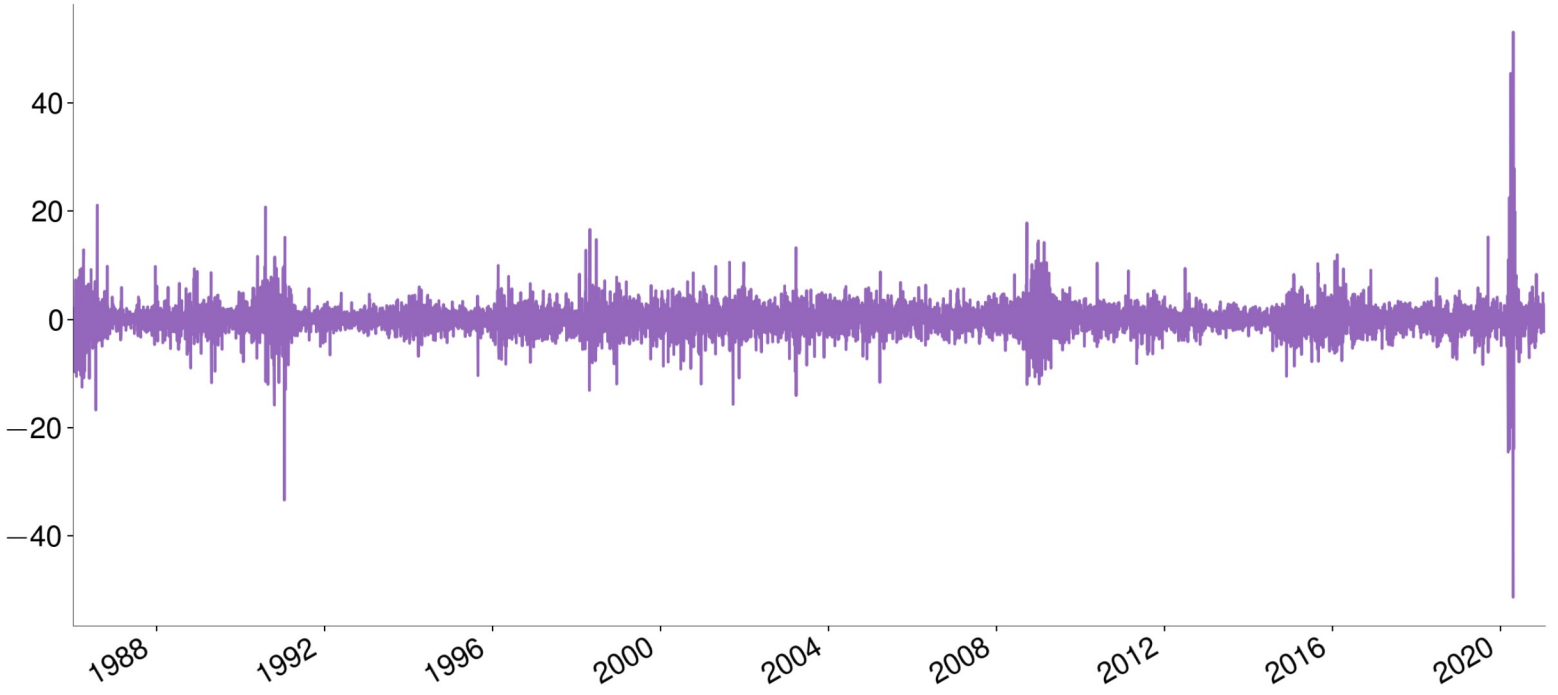
GOLD Fixing Price

In [6]:
```
plot(gold)
```



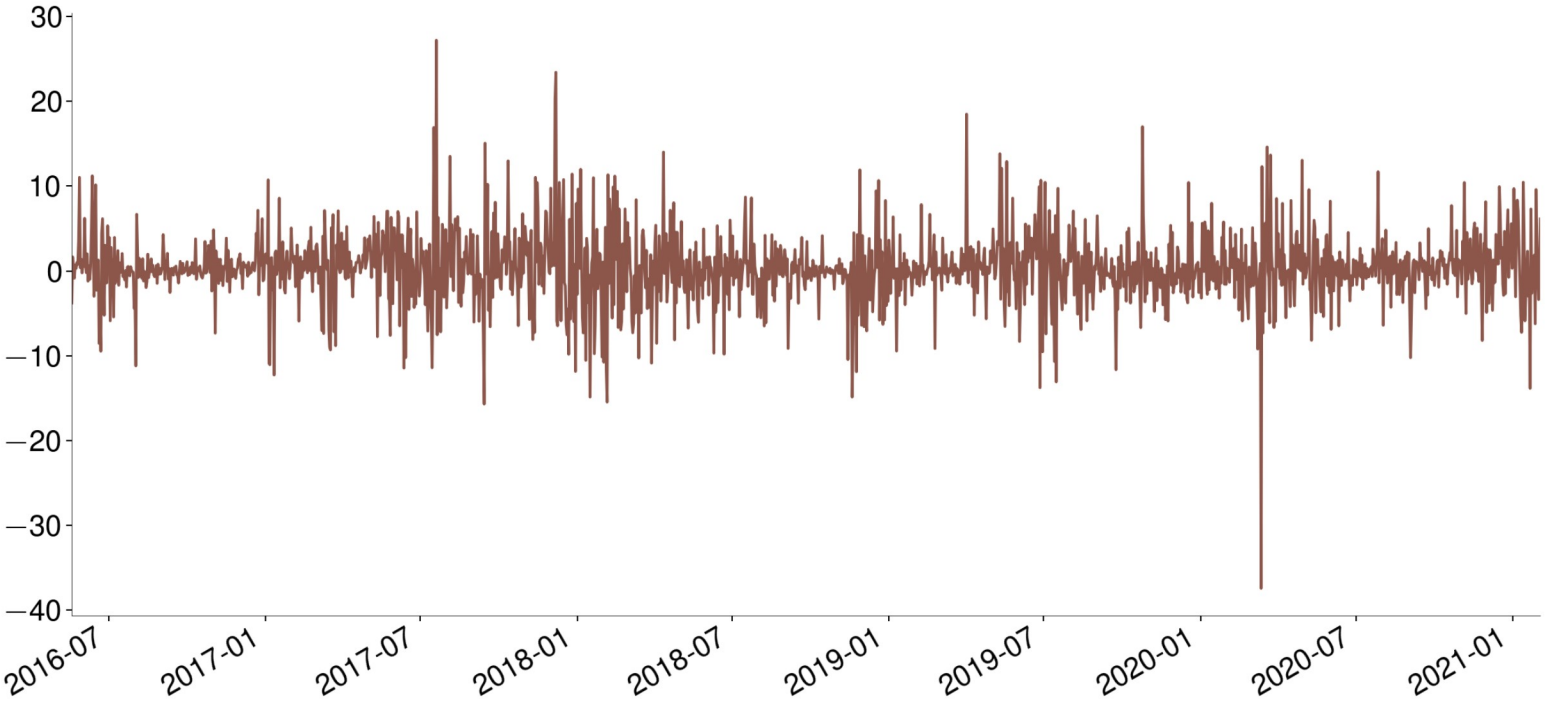
West Texas Intermediate Crude

In [7]:
```
plot(wti)
```



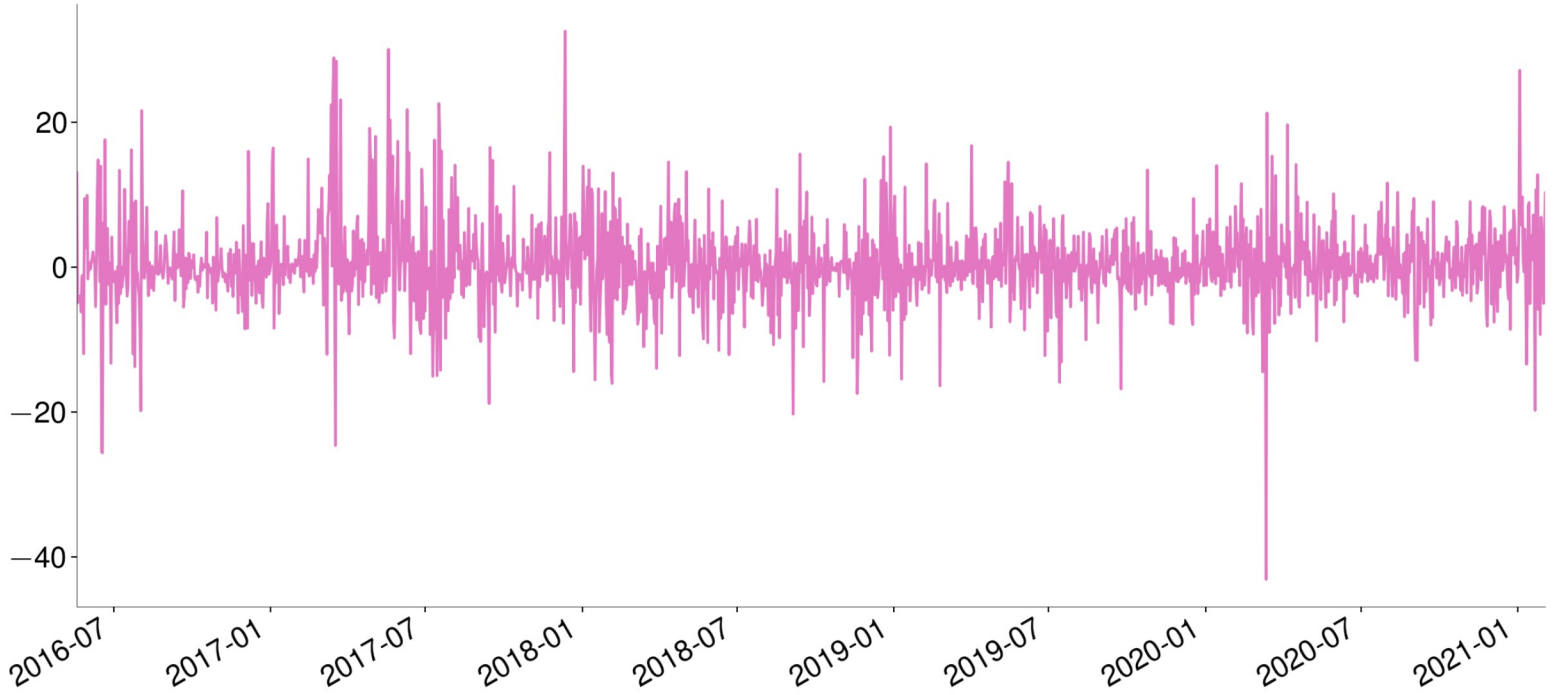
Coinbase BitCoin

In [8]:
```
plot(btc)
```



Coinbase Etherium

In [9]:
```
plot(eth)
```

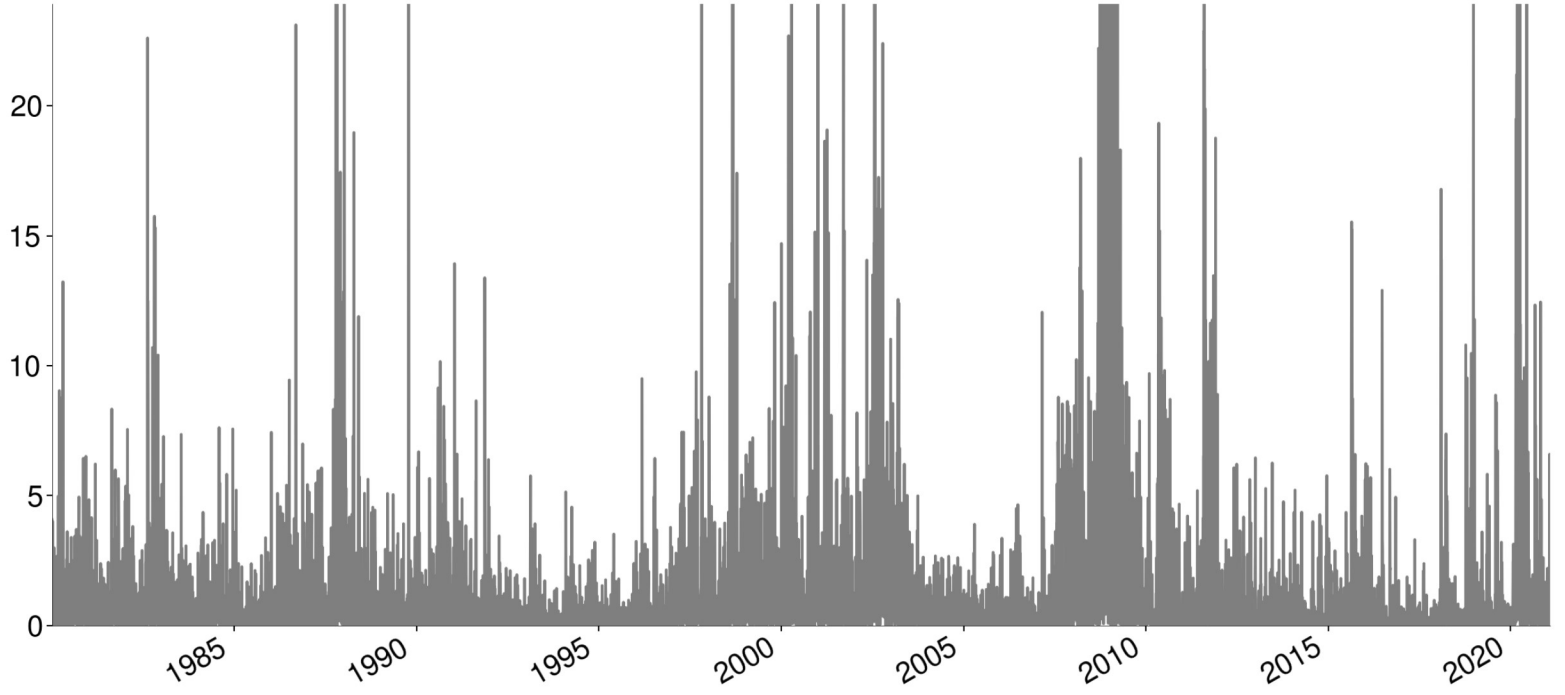


Visual Inspections for Volatility

- Often sufficient to inspect return directly
- Natural proxy for *variance* is r_t^2
    - Noisy
    - Squares distort plot (truncate at 99.5%)
- Better proxy $|r_t|$ which is closely related to *volatility*

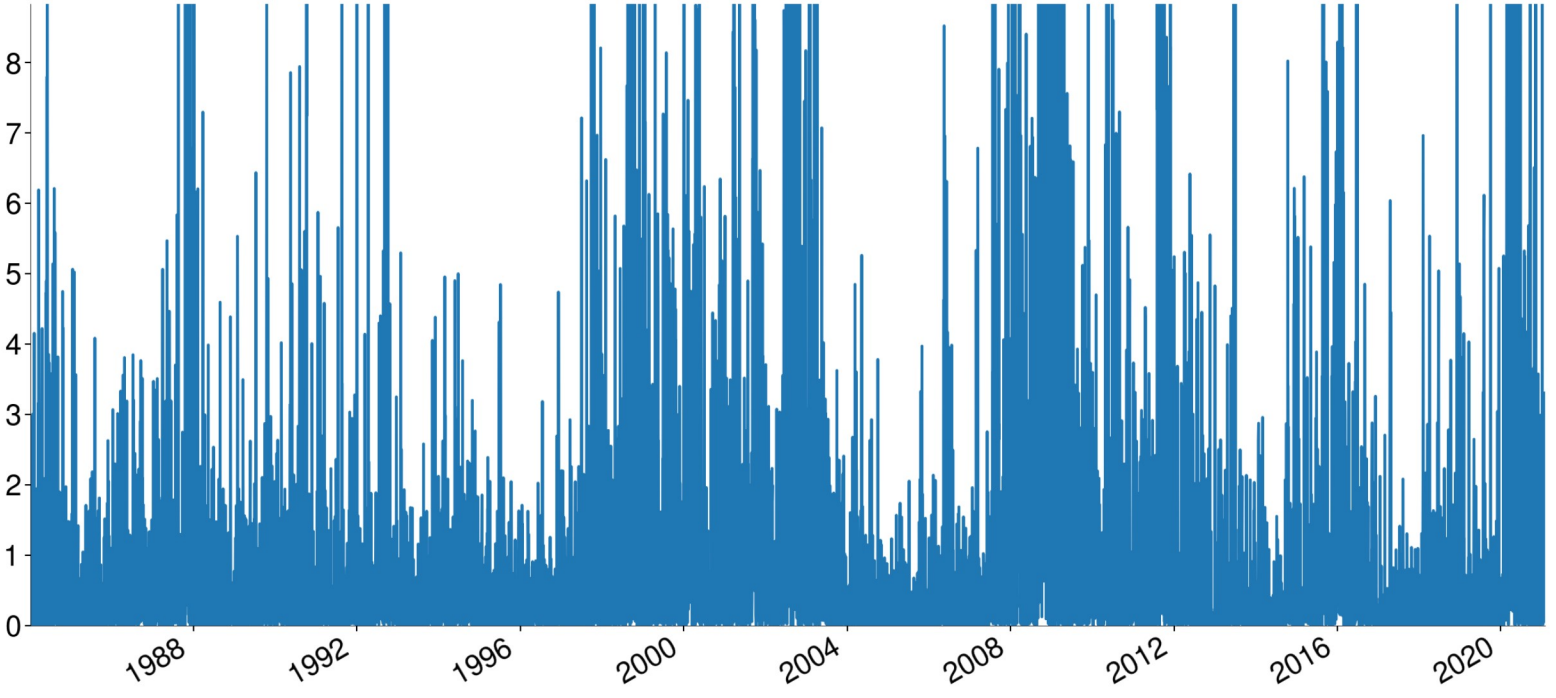
S&P 500 Squared Returns

In [10]:
```
truncated_plot(sp500 ** 2, 99.5)
```



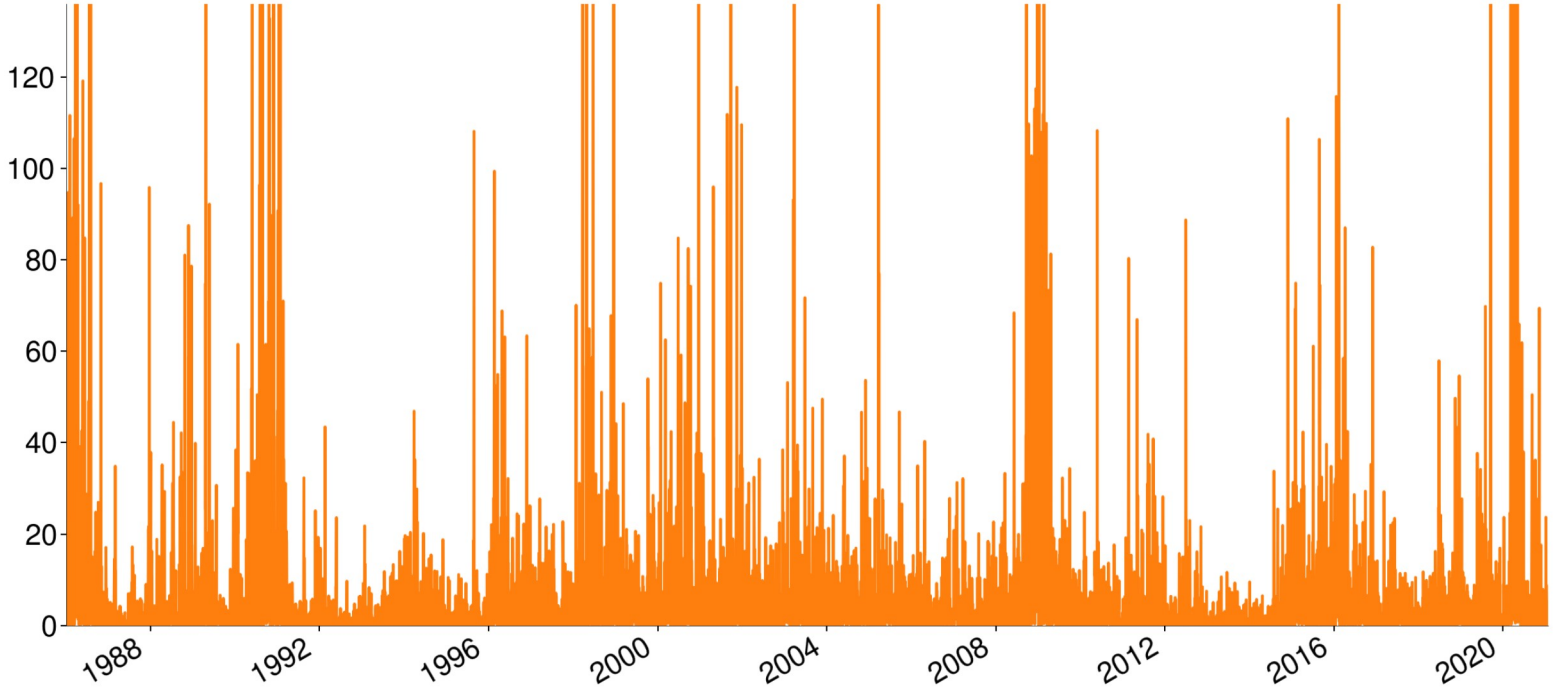
t_6 IID Simulated Returns Squared

In [11]:
```
truncated_plot(ftse100 ** 2, 98)
```



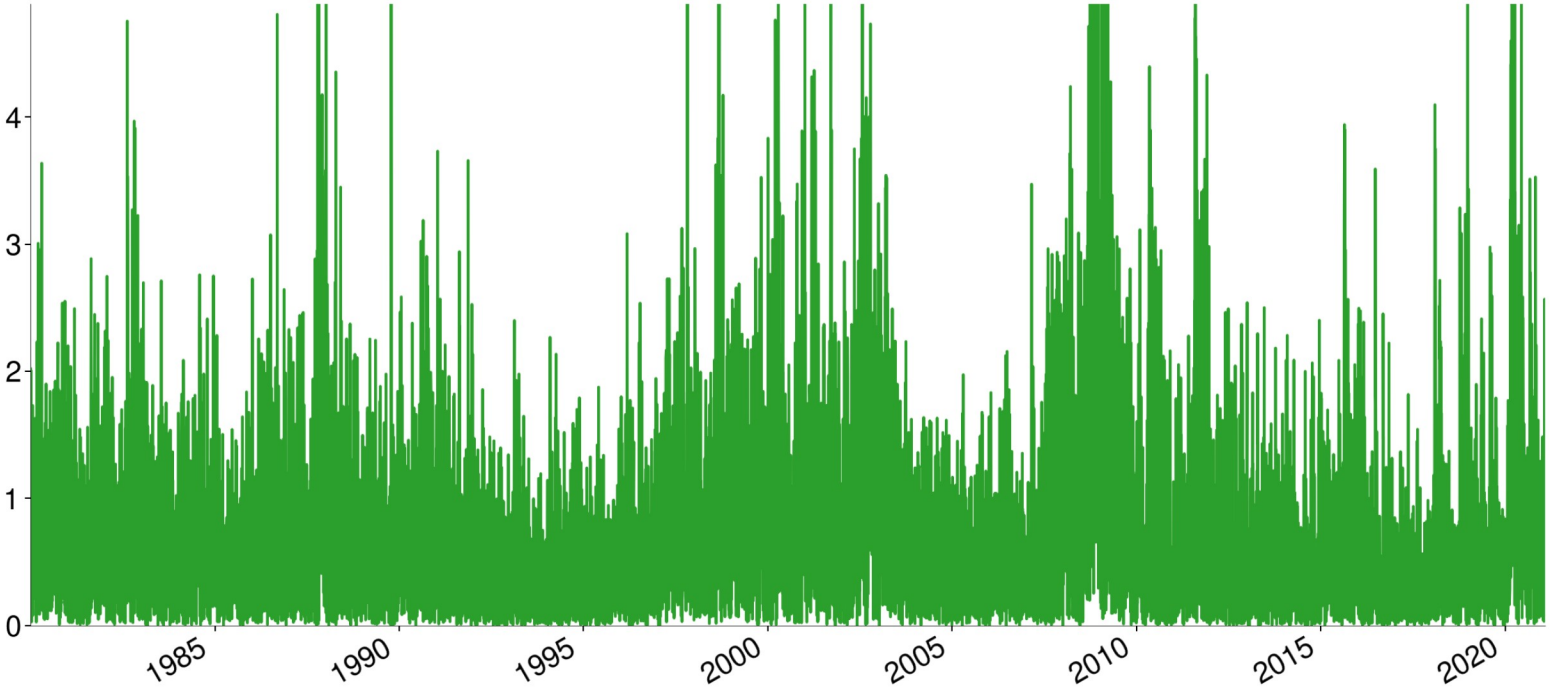
West Texas Intermediate Crude Squared Returns

In [12]:
```
truncated_plot(wti ** 2, 99.5)
```



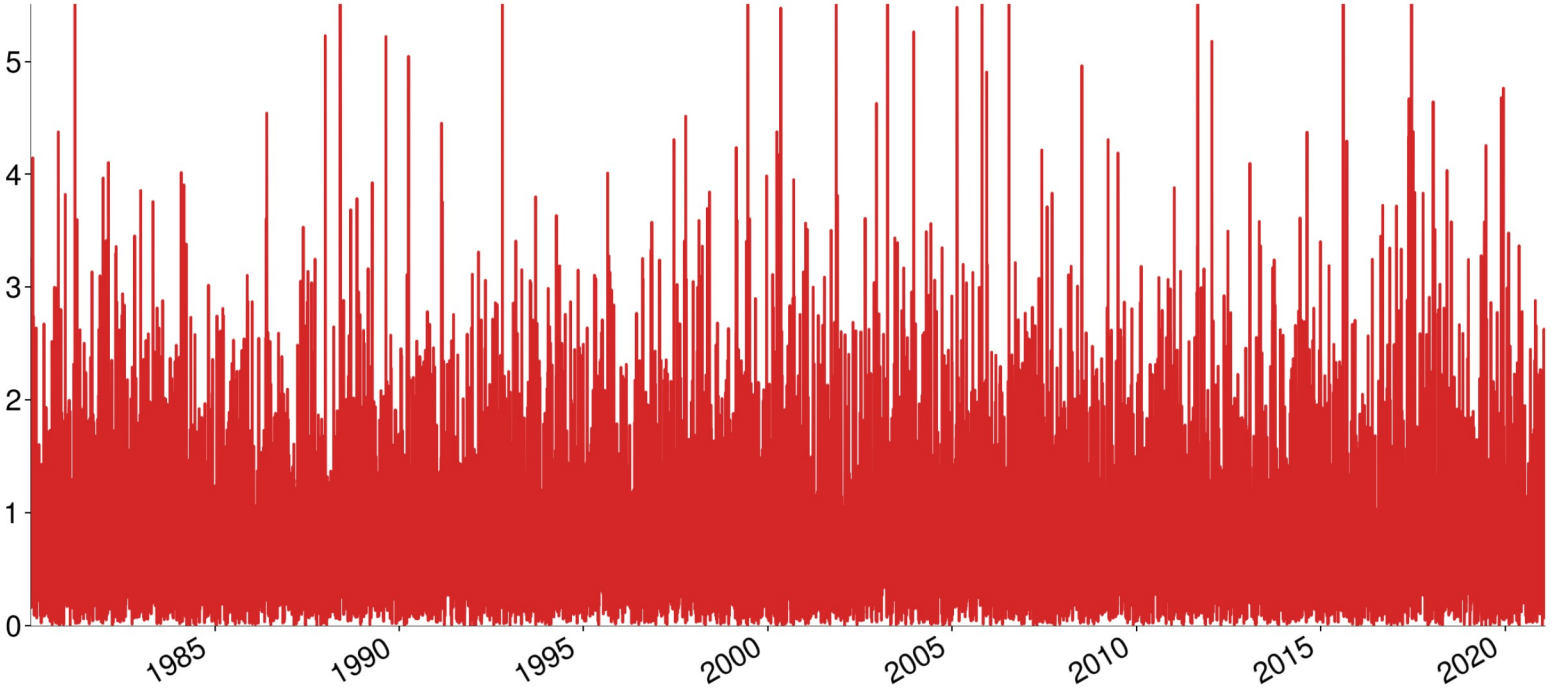
S&P 500 Absolute Values

In [13]:
```
truncated_plot(np.abs(sp500), 99.5)
```



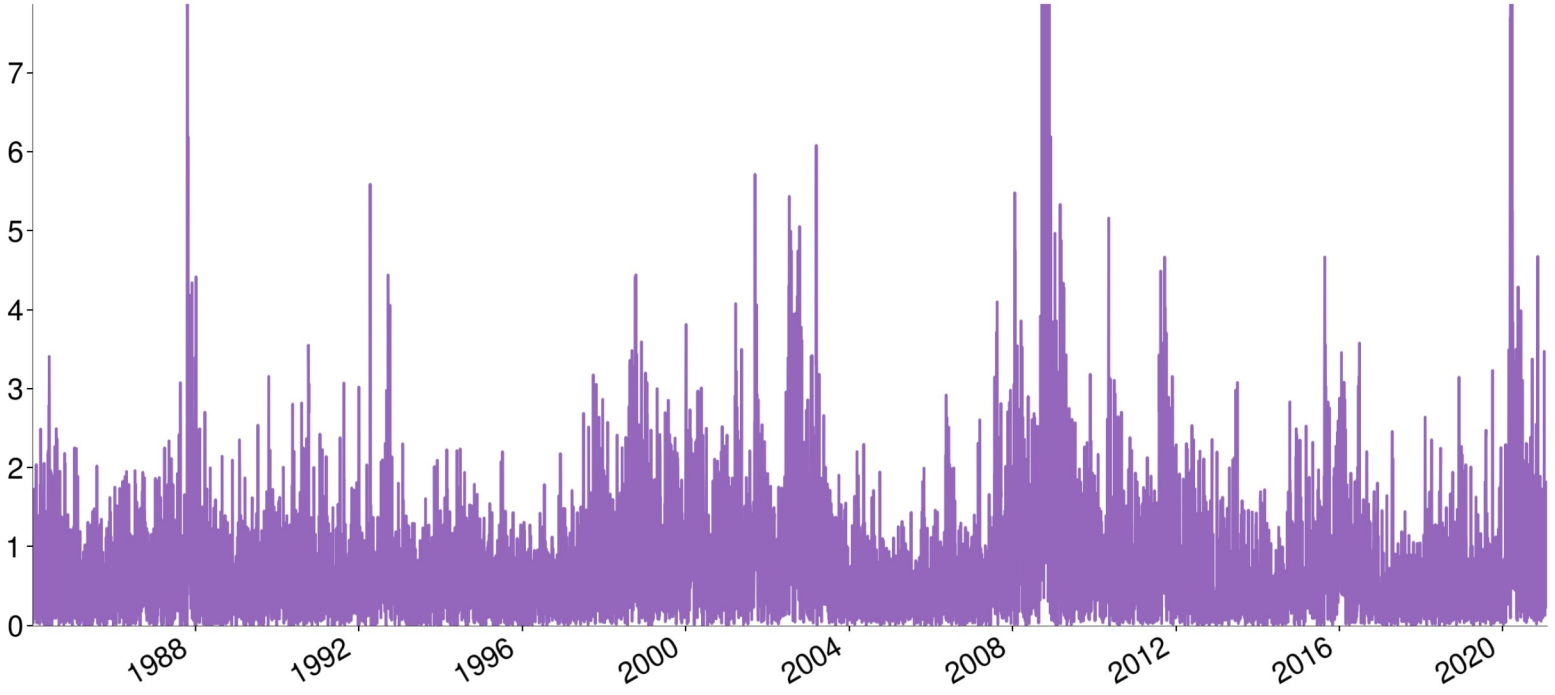
t_6 IID Simulated Absolute Returns

In [14]:
```
truncated_plot(np.abs(iid), 99.9)
```



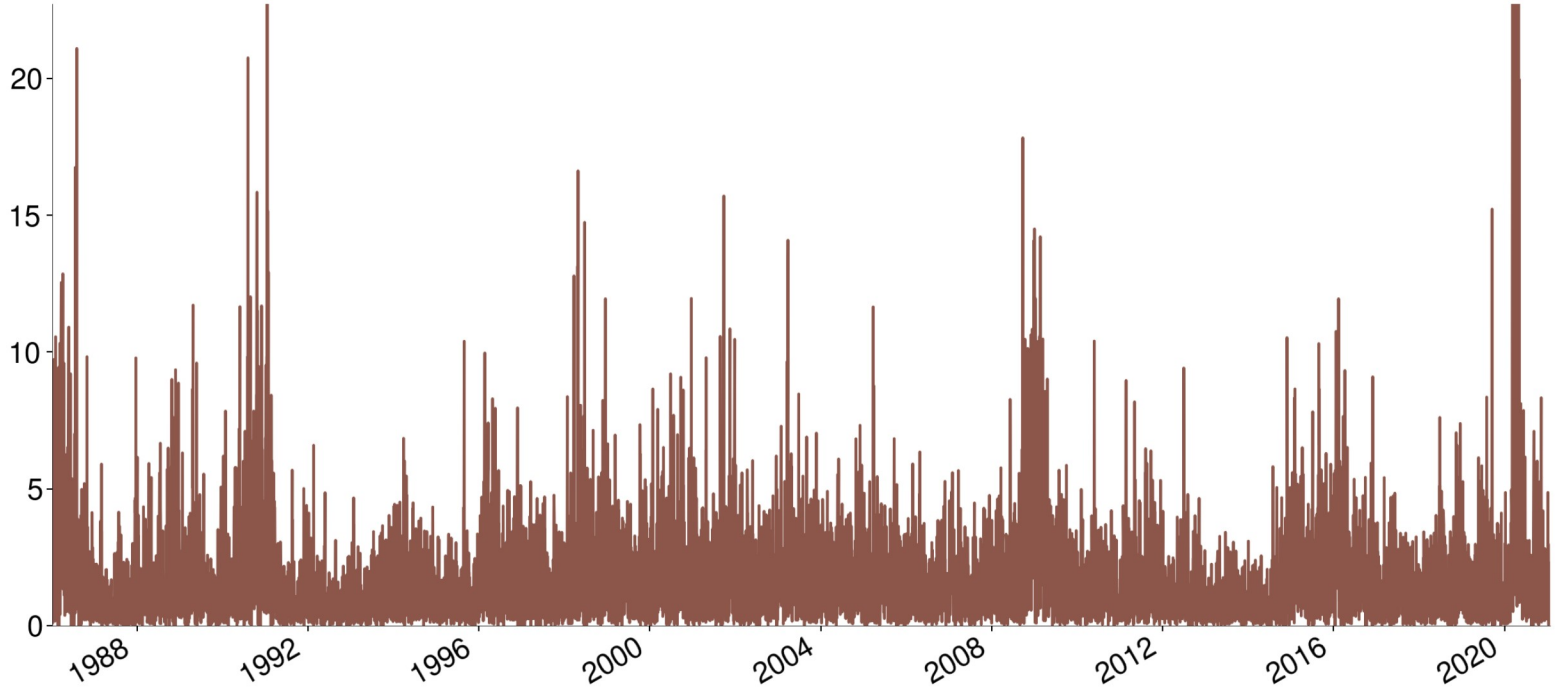
FTSE 100 Absolute Returns

In [15]:
```
truncated_plot(np.abs(ftse100), 99.9)
```



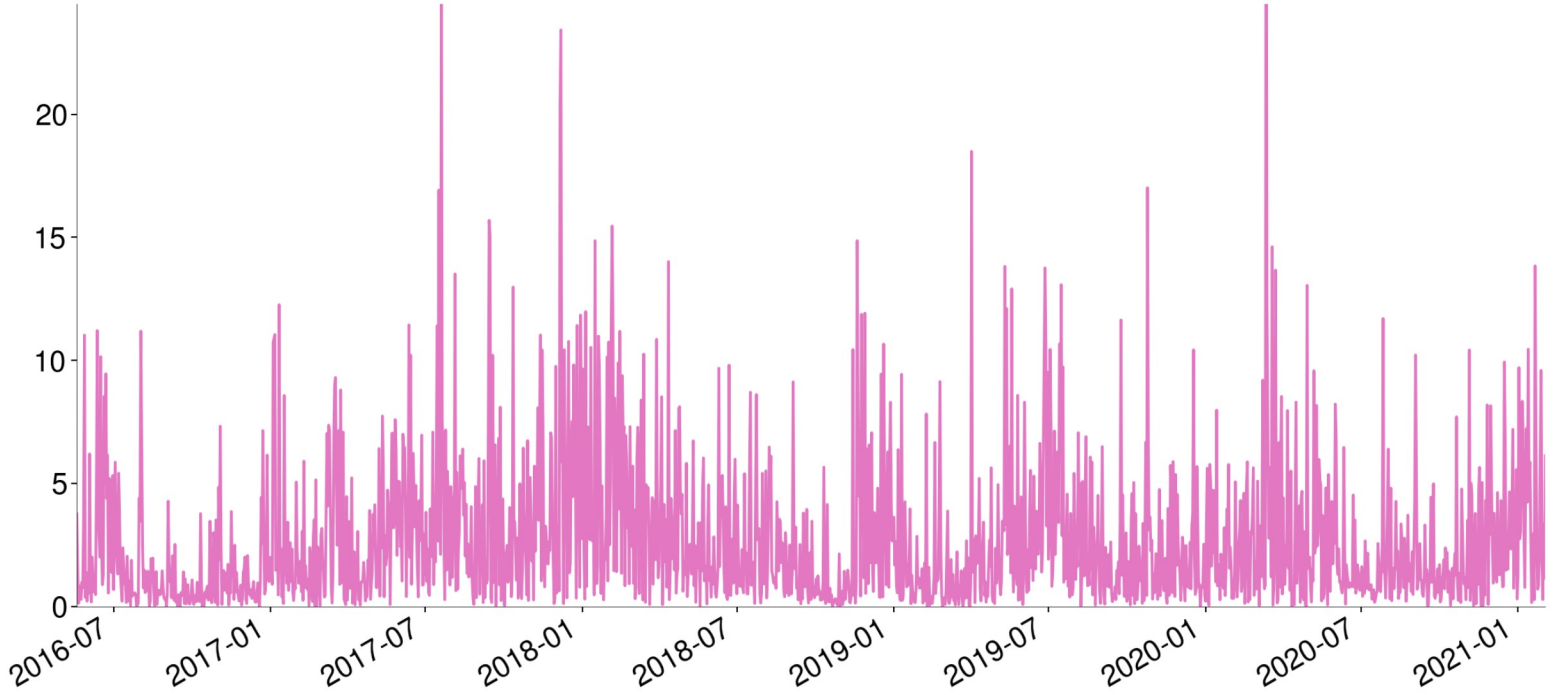
WTI Absolute Returns

In [16]:
```
truncated_plot(np.abs(wti), 99.9)
```



BitCoin Absolute Returns

In [17]:
```
truncated_plot(np.abs(btc), 99.9)
```



Formal Testing for Arch

- ARCH-LM test

$$\hat{\epsilon}_t = r_t - E_{t-1}[r_t]$$

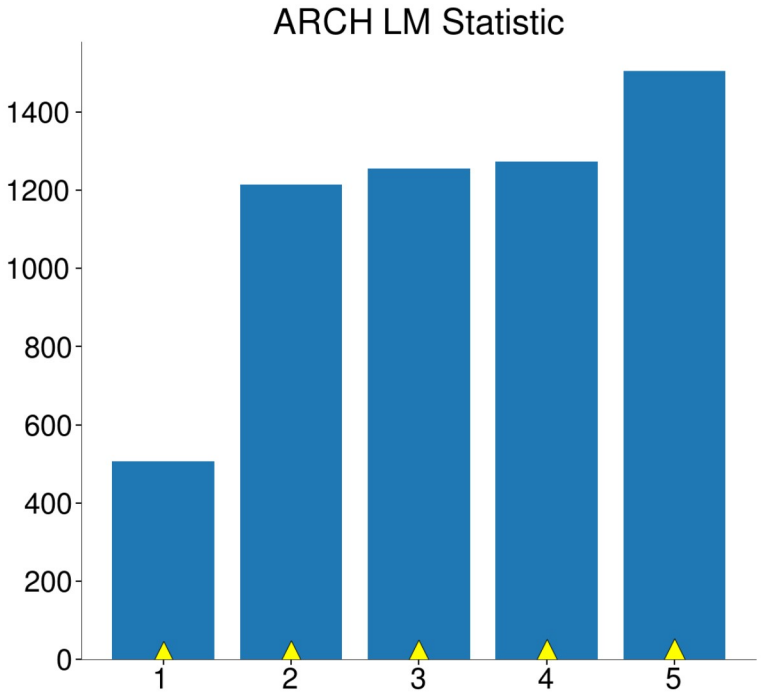
- Constant $E_{t-1}[r_t] = \mu$ here

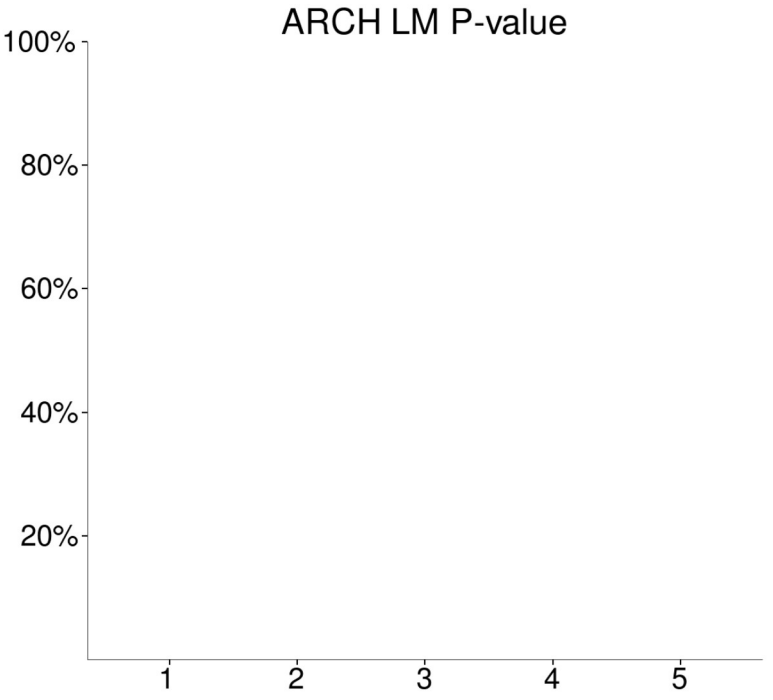
$$\hat{\epsilon}_t^2 = \omega + \alpha_1 \hat{\epsilon}_{t-1}^2 + \alpha_2 \hat{\epsilon}_{t-2}^2 + \ldots + \alpha_p \hat{\epsilon}_{t-p}^2 + \eta_t$$

- Test statistic is $T \times R^2$
- χ_p^2 distribution
- Virtually identical to White's test

Testing the S&P 500

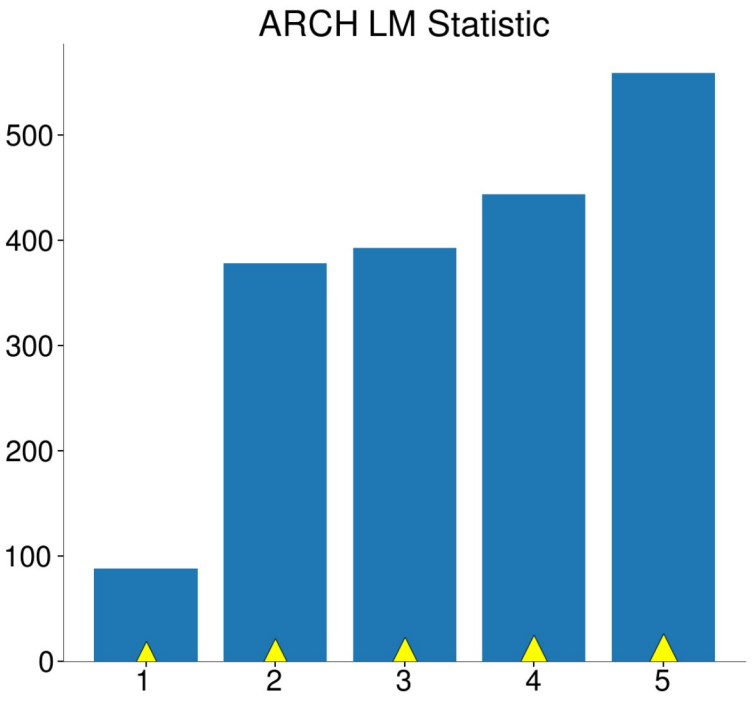
In [19]:
```
arch_lm(sp500, 5)
```

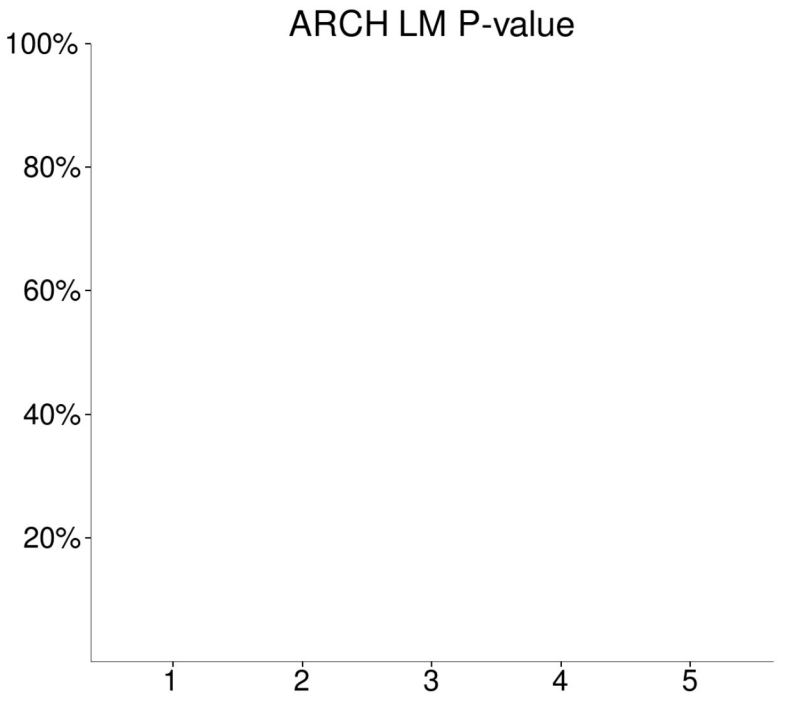




S&P 500 between 2000 and 2009

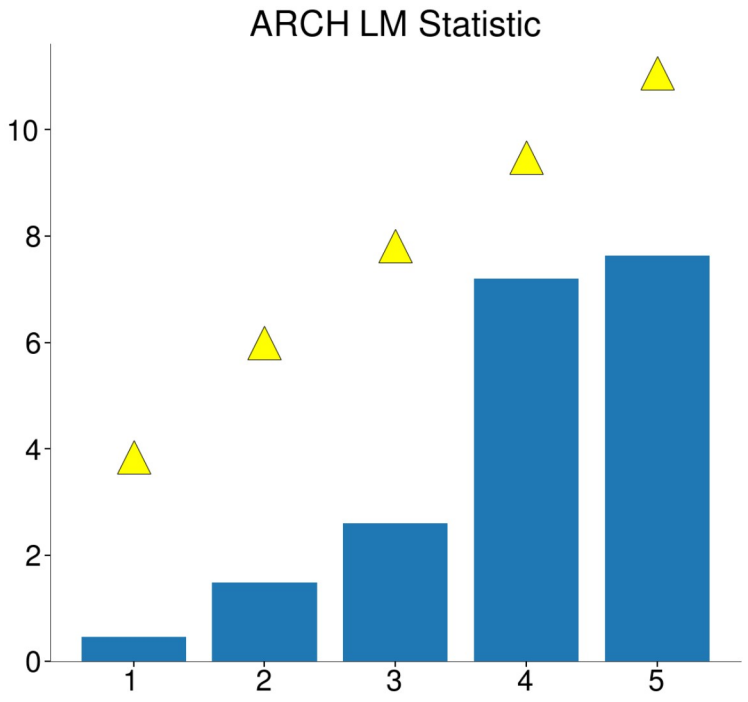
In [20]:
```
arch_lm(sp500["2000":"2009"], 5)
```

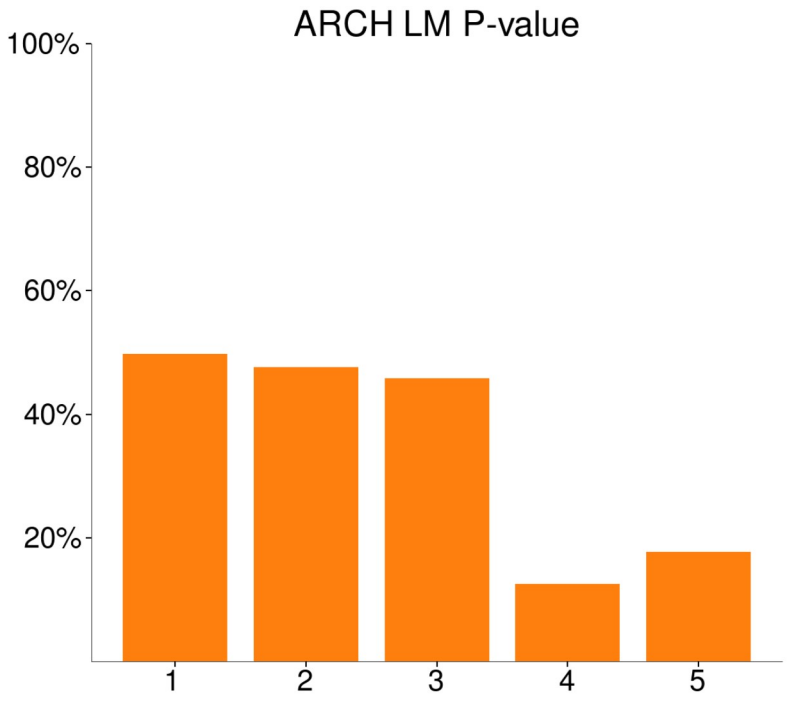




Testing the S&P 500 in 2006

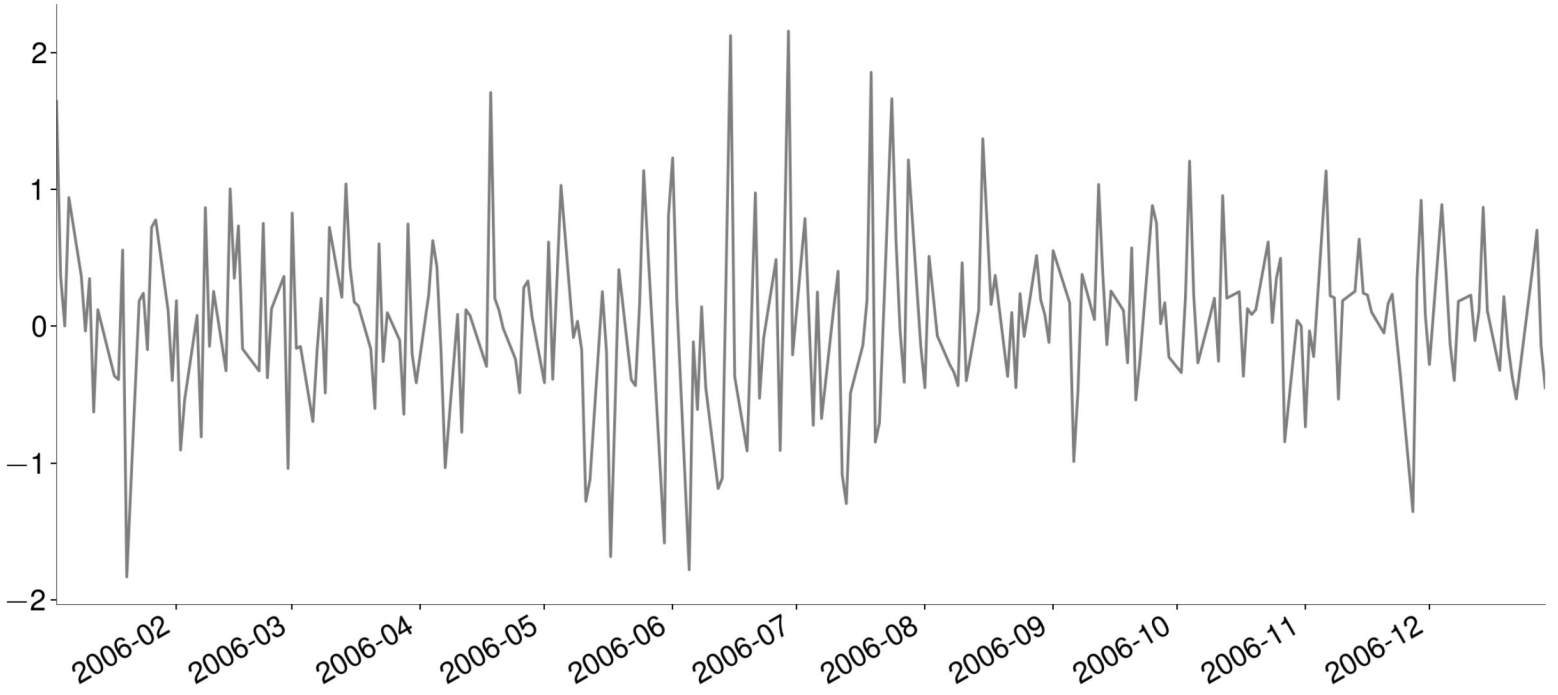
In [21]:
```
arch_lm(sp500["2006":"2006"], 5)
```





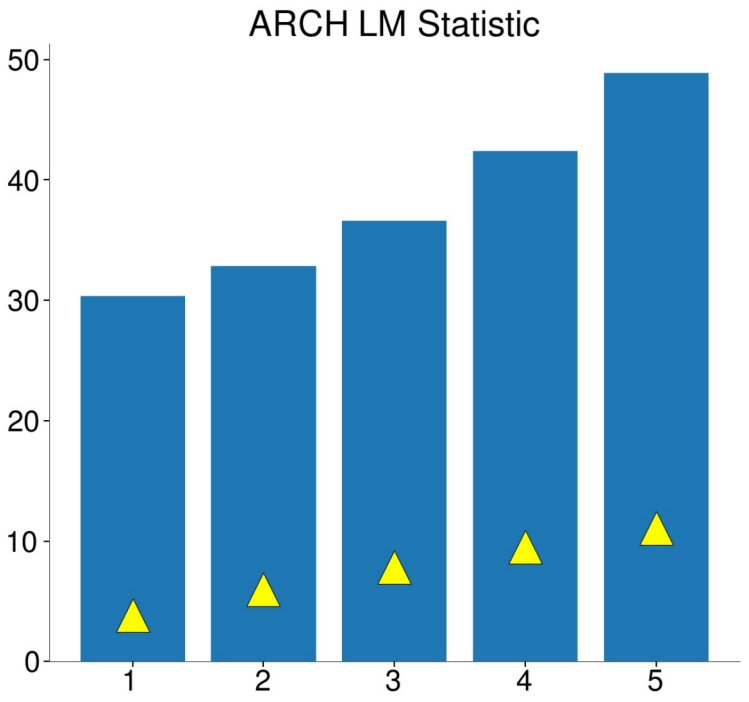
S&P 500 in 2006

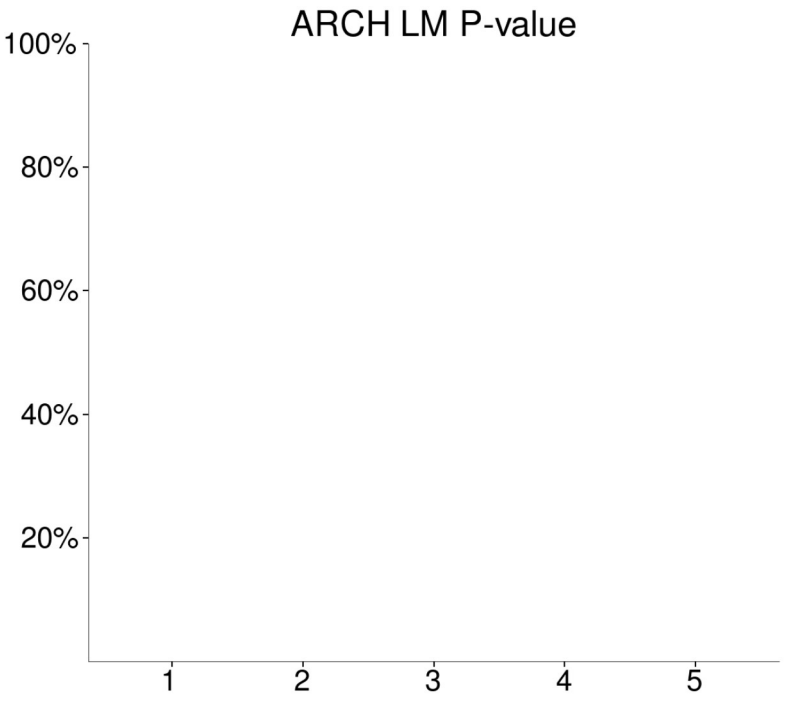
In [22]:
```
plot(sp500["2006":"2006"])
```



BitCoin

In [23]:
```
arch_lm(btc, 5)
```





ARCH Models

- Initial effort at volatility modeling
- Basic ARCH(1)

$$\begin{aligned}\epsilon_t &= r_t - \mu_t \\ \sigma_t^2 &= \omega + \alpha_1 \epsilon_{t-1}^2 \\ \epsilon_t &= \sigma_t e_t \\ e_t &\overset{iid}{\sim} N(0,1)\end{aligned}$$

- μ_t can be any mean model as long as $\in \mathcal{F}_{t-1}$
  - ARMA(P,Q)
- Complete ARCH(P)

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \ldots + \alpha_P \epsilon_{t-P}^2$$

- In practice needs many lags

ARCH(1)

In [25]:
```
summary(arch_model(ftse100, vol="arch", p=1).fit(disp="off"))
```

Out[25]:

Constant Mean - ARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-13702.5
Vol Model:	ARCH	**AIC:**	27411.0
Distribution:	Normal	**BIC:**	27432.4

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0419	1.094e-02	3.825	1.306e-04	[2.041e-02,6.330e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.8039	3.470e-02	23.172	8.784e-119	[0.736, 0.872]
alpha[1]	0.3505	3.970e-02	8.828	1.066e-18	[0.273, 0.428]

ARCH(5)

In [26]:
```
summary(arch_model(ftse100, vol="arch", p=5).fit(disp="off"))
```

Out[26]:

Constant Mean - ARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12834.2
Vol Model:	ARCH	**AIC:**	25682.4
Distribution:	Normal	**BIC:**	25732.4

Mean Model

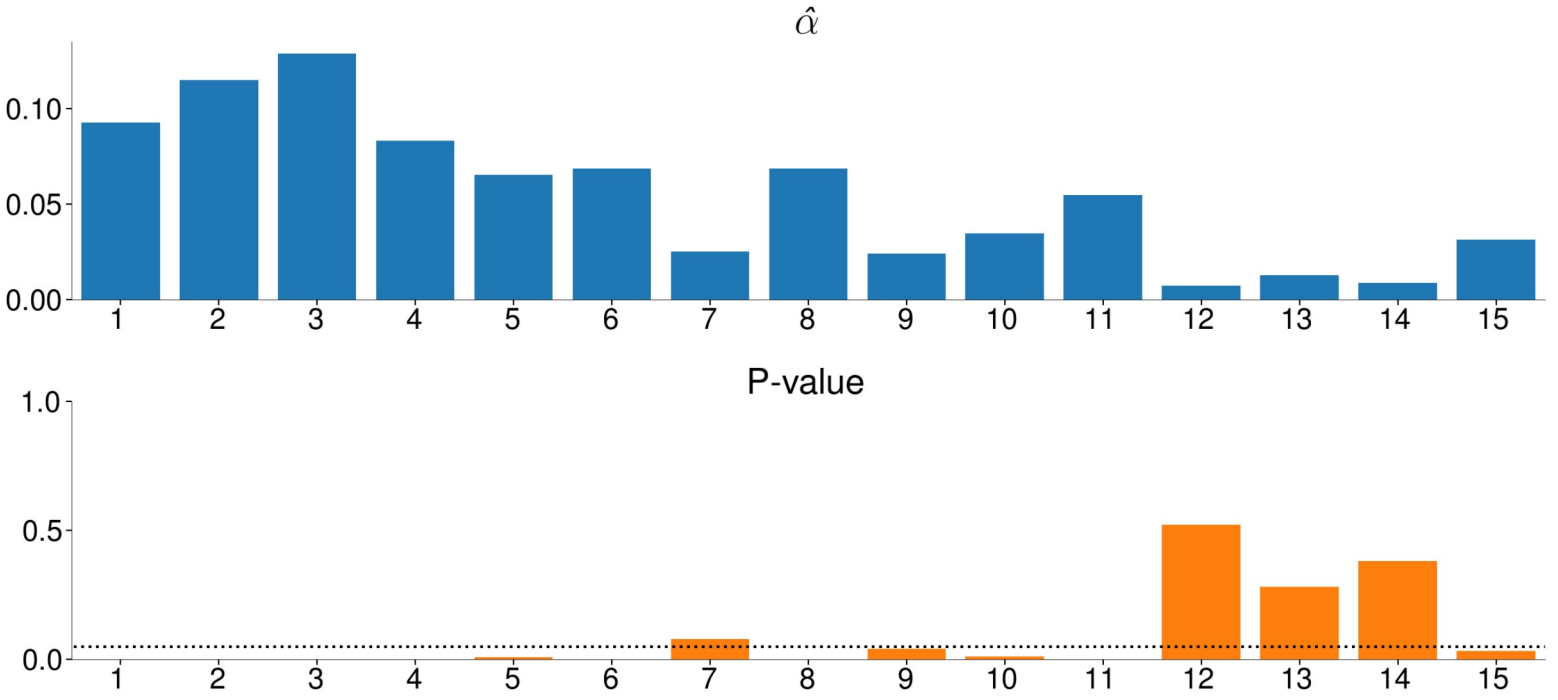
	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0511	9.097e-03	5.614	1.971e-08	[3.325e-02,6.891e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.3335	1.650e-02	20.219	6.670e-91	[0.301, 0.366]
alpha[1]	0.1202	1.927e-02	6.240	4.362e-10	[8.248e-02, 0.158]
alpha[2]	0.1685	1.788e-02	9.426	4.275e-21	[0.133, 0.204]
alpha[3]	0.1683	2.238e-02	7.519	5.504e-14	[0.124, 0.212]
alpha[4]	0.1429	1.717e-02	8.319	8.888e-17	[0.109, 0.177]
alpha[5]	0.1314	2.069e-02	6.349	2.173e-10	[9.082e-02, 0.172]

ARCH(15)

In [28]:
```
plot_arch(15)
```



HARCH Model

- Compact parameterization of long-lag ARCH
- Define

$$\overline{r^2_{t-1:j}} = j^{-1} \sum_{i=1}^{j} r^2_{t-i}$$

- HARCH is parameterized in terms of $\overline{r^2_{t-1:j}}$
- Standard model: HARCH(1,5,22)

$$\sigma_t^2 = \omega + \alpha_1 r^2_{t-1} + \alpha_5 \overline{r^2_{t-1:5}} + \alpha_{22} \overline{r^2_{t-1:22}}$$

- Compute effective coefficient by expanding

HARCH(1, 5, 22) Model

In [29]:
```
summary(arch_model(ftse100, vol="harch",p=[1,5,22]).fit(disp="off"))
```

Out[29]:

Constant Mean - HARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12679.1
Vol Model:	HARCH	**AIC:**	25368.1
Distribution:	Normal	**BIC:**	25403.9

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0459	8.614e-03	5.329	9.880e-08	[2.902e-02,6.278e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.1870	1.860e-02	10.053	8.955e-24	[0.151, 0.223]
alpha[1]	8.5708e-15	2.450e-02	3.498e-13	1.000	[-4.802e-02,4.802e-02]
alpha[5]	0.3609	6.262e-02	5.764	8.232e-09	[0.238, 0.484]
alpha[22]	0.4859	4.299e-02	11.302	1.281e-29	[0.402, 0.570]

Extended HARCH Model

In [30]:
```
summary(arch_model(ftse100, vol="harch",p=[1,5,22, 66]).fit(disp="off"))
```

Out[30]:

Constant Mean - HARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12654.9
Vol Model:	HARCH	**AIC:**	25321.8
Distribution:	Normal	**BIC:**	25364.8

Mean Model

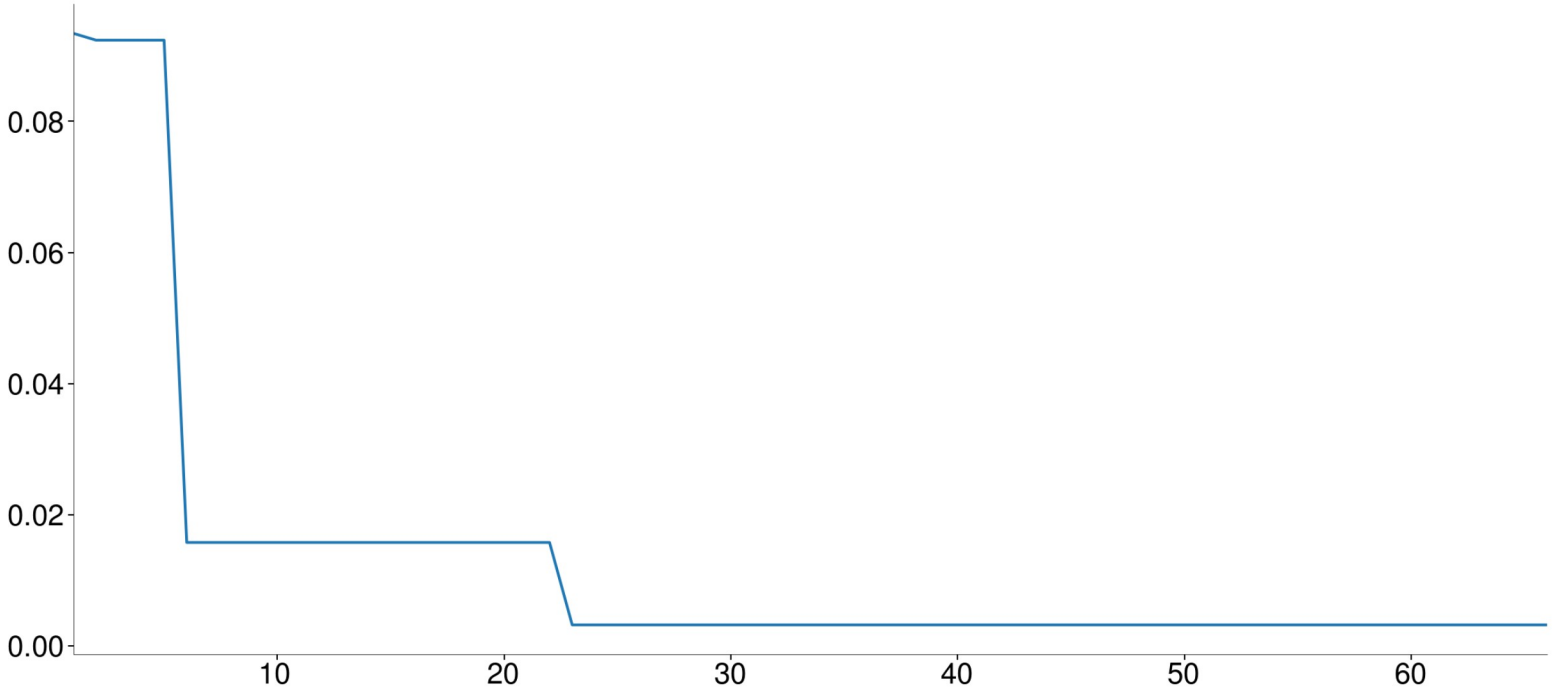
	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0473	8.620e-03	5.487	4.081e-08	[3.041e-02,6.420e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.1482	1.930e-02	7.678	1.615e-14	[0.110, 0.186]
alpha[1]	1.0355e-03	2.369e-02	4.372e-02	0.965	[-4.539e-02,4.746e-02]
alpha[5]	0.3830	6.023e-02	6.358	2.037e-10	[0.265, 0.501]
alpha[22]	0.2765	6.112e-02	4.524	6.058e-06	[0.157, 0.396]
alpha[66]	0.2122	4.437e-02	4.781	1.740e-06	[0.125, 0.299]

HARCH(1,5,22,66) expanded coefficients

In [32]:
```
harch_weights()
```



The GARCH Model

- Simple but effective generalization of ARCH

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

- ARCH(∞) in disguise

$$\sigma_t^2 = \frac{\omega}{1-\beta} + \alpha_1 \sum_{i=1}^{\infty} \beta_1^{i-1} \epsilon_{t-i}^2$$

- Similar to ARMA(1,1)
  - $\alpha + \beta$ is the AR coefficient
  - $-\beta$ is the MA coefficient

GARCH Model Results

In [33]:
```
garch = arch_model(ftse100, vol="garch").fit(disp="off")
summary(garch)
```

Out[33]:

Constant Mean - GARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12665.8
Vol Model:	GARCH	**AIC:**	25339.7
Distribution:	Normal	**BIC:**	25368.3

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0483	8.961e-03	5.395	6.854e-08	[3.078e-02,6.591e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0203	4.160e-03	4.868	1.126e-06	[1.210e-02,2.840e-02]
alpha[1]	0.1008	1.198e-02	8.418	3.821e-17	[7.736e-02, 0.124]
beta[1]	0.8824	1.324e-02	66.656	0.000	[0.856, 0.908]

The Conditional Mean

- All models have used a constant mean $\mu_t = \mu$
- Plausible for liquid financial assets
- For high-frequency data, mean makes little difference
- Can instead use $\mu_t = 0$

Zero mean

In [34]:
```
zero = arch_model(ftse100, vol="garch", mean="zero").fit(disp="off")
summary(zero)
```

Out[34]:

Zero Mean - GARCH Model Results

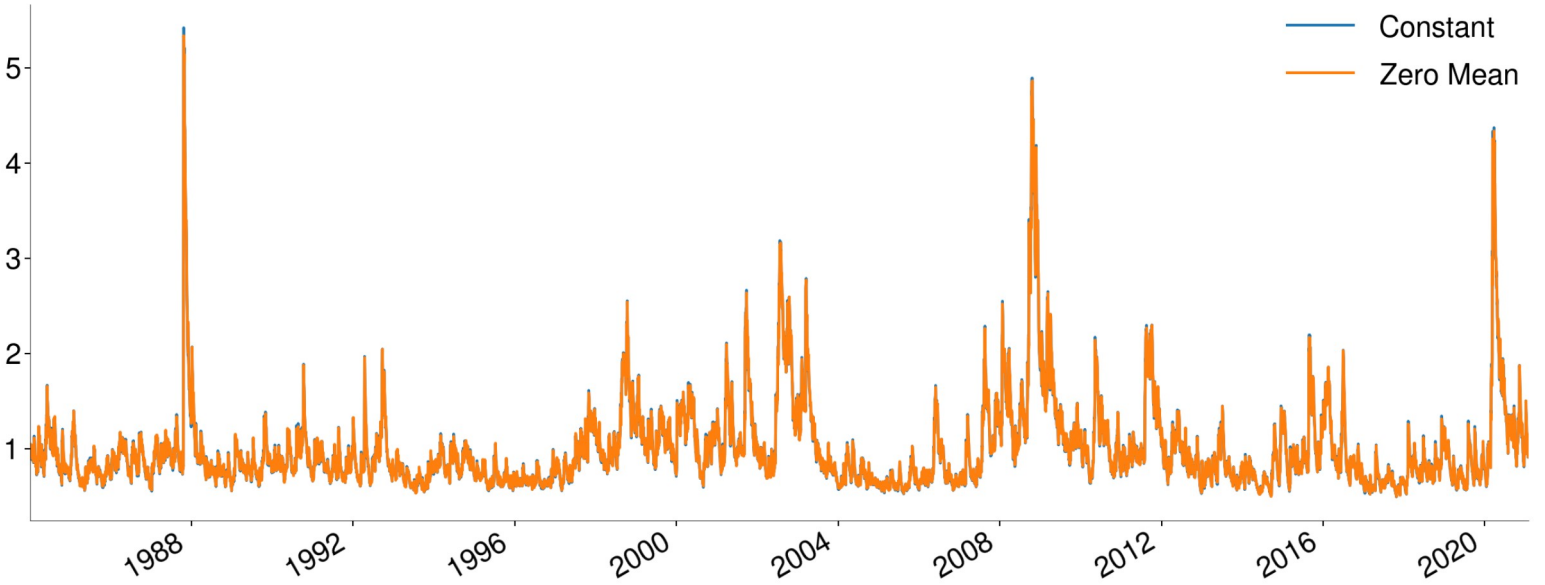
Mean Model:	Zero Mean	**Log-Likelihood:**	-12682.3
Vol Model:	GARCH	**AIC:**	25370.7
Distribution:	Normal	**BIC:**	25392.1

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0196	3.966e-03	4.929	8.244e-07	[1.178e-02,2.733e-02]
alpha[1]	0.0977	1.111e-02	8.793	1.456e-18	[7.589e-02, 0.119]
beta[1]	0.8861	1.250e-02	70.883	0.000	[0.862, 0.911]

Zero vs. Non-zero Mean

In [36]:
```
compare_mean_models()
```



Exponentially Weighted Moving Average (EWMA) Variance

- Restricted version of GARCH

$$\omega = 0$$
$$\alpha + \beta = 1 \Rightarrow \alpha = 1 - \beta$$

- Usually parameterize with λ

$$\sigma_t^2 = (1-\lambda)\epsilon_{t-1}^2 + \lambda\sigma_{t-1}^2$$

- Simpler if $\mu_t = 0$

$$\sigma_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\sigma_{t-1}^2$$

- Common to fix λ
  - Daily: 0.94
  - Weekly: 0.97
  - Monthly: 0.99

EWMA Results

In [37]:
```
ewma_var = arch.univariate.volatility.EWMAVariance(0.94)
summary(arch.univariate.ConstantMean(ftse100, volatility=ewma_var).fit(disp="off"))
```

Out[37]:

Constant Mean - EWMA/RiskMetrics Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12757.7
Vol Model:	EWMA/RiskMetrics	**AIC:**	25517.4
Distribution:	Normal	**BIC:**	25524.6

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0426	1.015e-02	4.196	2.721e-05	[2.269e-02,6.247e-02]

Asymmetries in Conditional Variance

GJR-GARCH

- Adds asymmetry to GARCH model

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \gamma_1 I_{[\epsilon_t - 1 < 0]} \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

GJR-GARCH(1,1,) Model

In [38]:
```
summary(arch_model(ftse100, vol="garch", o=1).fit(disp="off"))
```

Out[38]:

Constant Mean - GJR-GARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12574.3
Vol Model:	GJR-GARCH	**AIC:**	25158.6
Distribution:	Normal	**BIC:**	25194.4

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0245	8.442e-03	2.907	3.646e-03	[7.997e-03,4.109e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0207	3.938e-03	5.262	1.424e-07	[1.300e-02,2.844e-02]
alpha[1]	0.0263	6.720e-03	3.910	9.234e-05	[1.310e-02,3.945e-02]
gamma[1]	0.1126	1.802e-02	6.246	4.216e-10	[7.724e-02, 0.148]
beta[1]	0.8973	1.184e-02	75.758	0.000	[0.874, 0.921]

The TARCH Model

- Changes model from variance to volatility

$$\sigma_t = \omega + \alpha_1 |\epsilon_{t-1}| + \gamma_1 I_{[\epsilon_t - 1 < 0]} |\epsilon_{t-1}| + \beta_1 \sigma_{t-1}$$

TARCH(1,1,1)

In [39]:
```
summary(arch_model(ftse100, vol="garch", o=1, power=1).fit(disp="off"))
```

Out[39]:

Constant Mean - TARCH/ZARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12566.9
Vol Model:	TARCH/ZARCH	**AIC:**	25143.9
Distribution:	Normal	**BIC:**	25179.6

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0204	8.517e-03	2.401	1.636e-02	[3.754e-03,3.714e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0230	4.398e-03	5.223	1.763e-07	[1.435e-02,3.159e-02]
alpha[1]	0.0409	7.306e-03	5.602	2.124e-08	[2.660e-02,5.524e-02]
gamma[1]	0.0980	1.332e-02	7.359	1.850e-13	[7.192e-02, 0.124]
beta[1]	0.9070	1.098e-02	82.584	0.000	[0.885, 0.929]

Exponential GARCH

$$\ln \sigma_t^2 = \omega + \alpha_1 \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right| + \gamma_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln \sigma_{t-1}^2$$

EGARCH(1,1,1)

In [40]:
```
summary(arch_model(ftse100, vol="egarch", o=1).fit(disp="off"))
```

Out[40]:

Constant Mean - EGARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12565.9
Vol Model:	EGARCH	**AIC:**	25141.8
Distribution:	Normal	**BIC:**	25177.5

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0214	5.552e-03	3.861	1.128e-04	[1.056e-02,3.232e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	7.3006e-04	1.948e-03	0.375	0.708	[-3.089e-03,4.549e-03]
alpha[1]	0.1605	1.612e-02	9.953	2.447e-23	[0.129, 0.192]
gamma[1]	-0.0817	9.921e-03	-8.238	1.748e-16	[-0.101,-6.229e-02]
beta[1]	0.9794	3.723e-03	263.034	0.000	[0.972, 0.987]

Asymmetric Power ARCH

- Parameterizes power in model

$$\sigma_t^\delta = \omega + \alpha_1(|\epsilon_{t-1}| + \gamma_1\epsilon_{t-1})^\delta + \beta_1\sigma_{t-1}^\delta$$

APARCH(1,1,1) Model

In [41]:
```
from arch.univariate import ConstantMean, APARCH
summary(ConstantMean(ftse100, volatility=APARCH()).fit(disp="off"))
```

```
C:\Anaconda\lib\site-packages\scipy\optimize\optimize.py:282: RuntimeWarning: Value
s in x were outside bounds during a minimize step, clipping to bounds
  warnings.warn("Values in x were outside bounds during a "
```

Out[41]:

Constant Mean - APARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-12559.6
Vol Model:	APARCH	**AIC:**	25131.3
Distribution:	Normal	**BIC:**	25174.2

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0216	8.536e-03	2.527	1.151e-02	[4.838e-03,3.830e-02]

Volatility Model

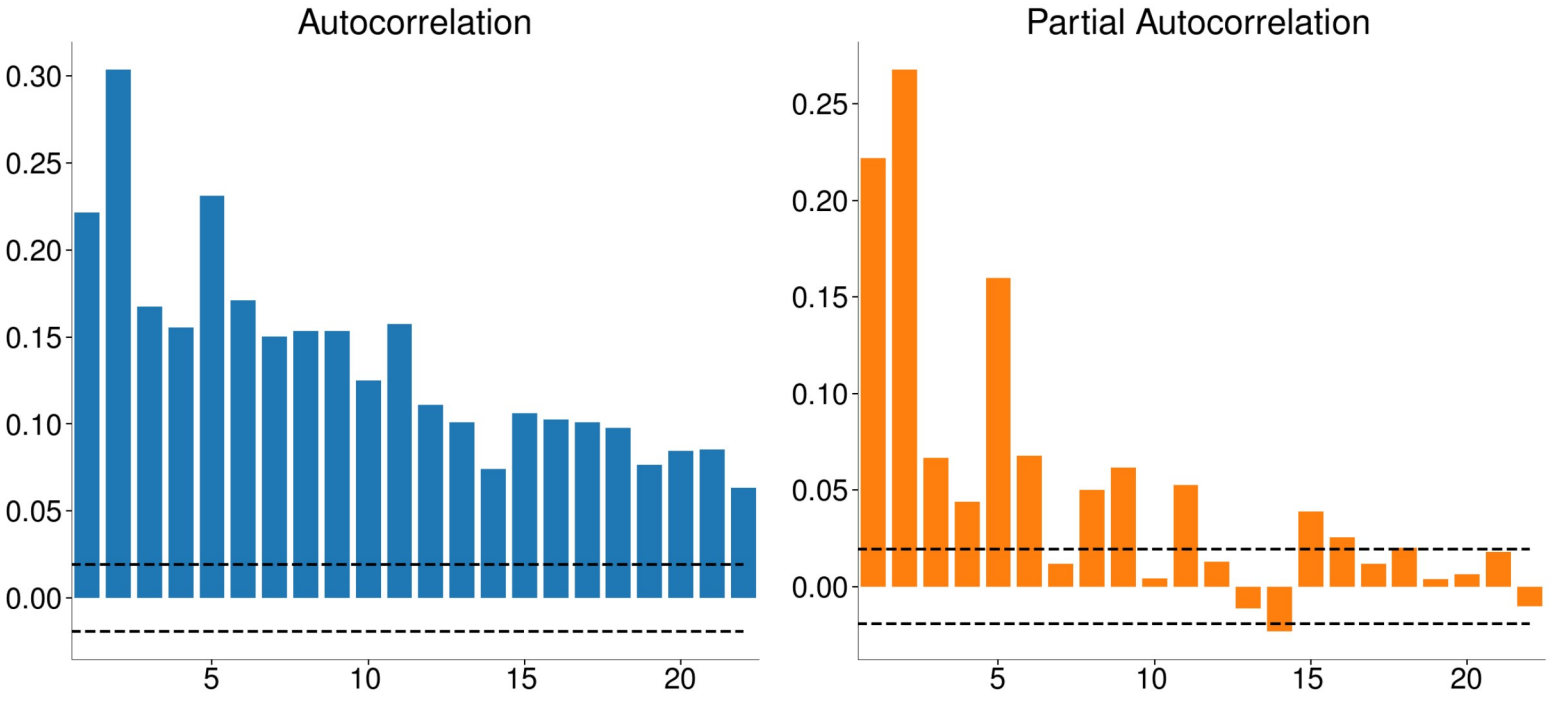
	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0220	4.132e-03	5.325	1.009e-07	[1.391e-02,3.010e-02]
alpha[1]	0.0854	9.696e-03	8.805	1.314e-18	[6.637e-02, 0.104]
gamma[1]	0.4937	6.798e-02	7.262	3.816e-13	[0.360, 0.627]
beta[1]	0.9048	1.096e-02	82.593	0.000	[0.883, 0.926]
delta	1.3226	0.158	8.384	5.124e-17	[1.013, 1.632]

ACF and PACF of ϵ_t^2

- Time-varying volatility appears through persistence in ϵ_t^2
  - Square is essential
- Standard tool is to plot ACF and PACF of $\hat{\epsilon}_t^2$
  - Estimate mean without time-varying volatility
- ACF indicates ARCH-like terms needed
- PACF indicates GARCH-like terms needed

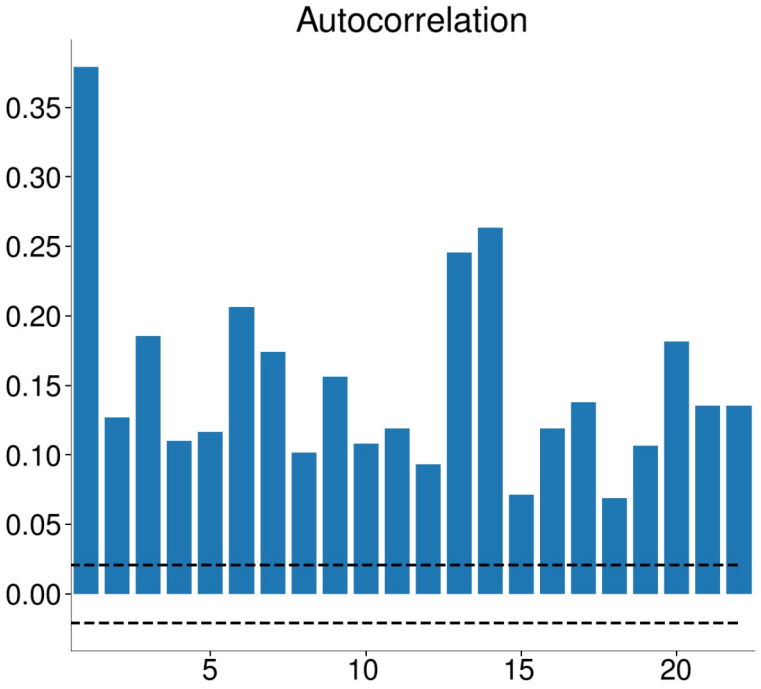
S&P 500

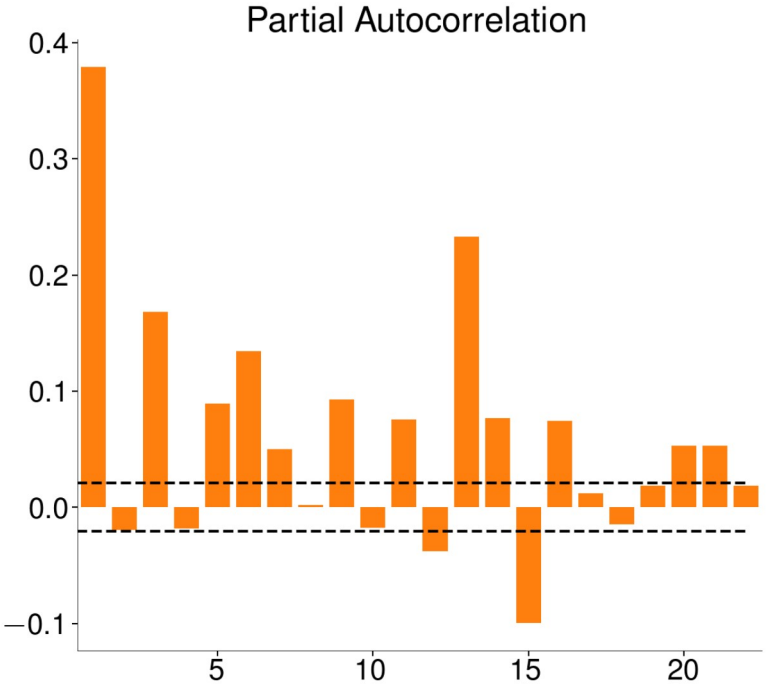
In [43]:
```
acf_pacf_plot(sp500**2, 22)
```



West Texas Internediate Crude

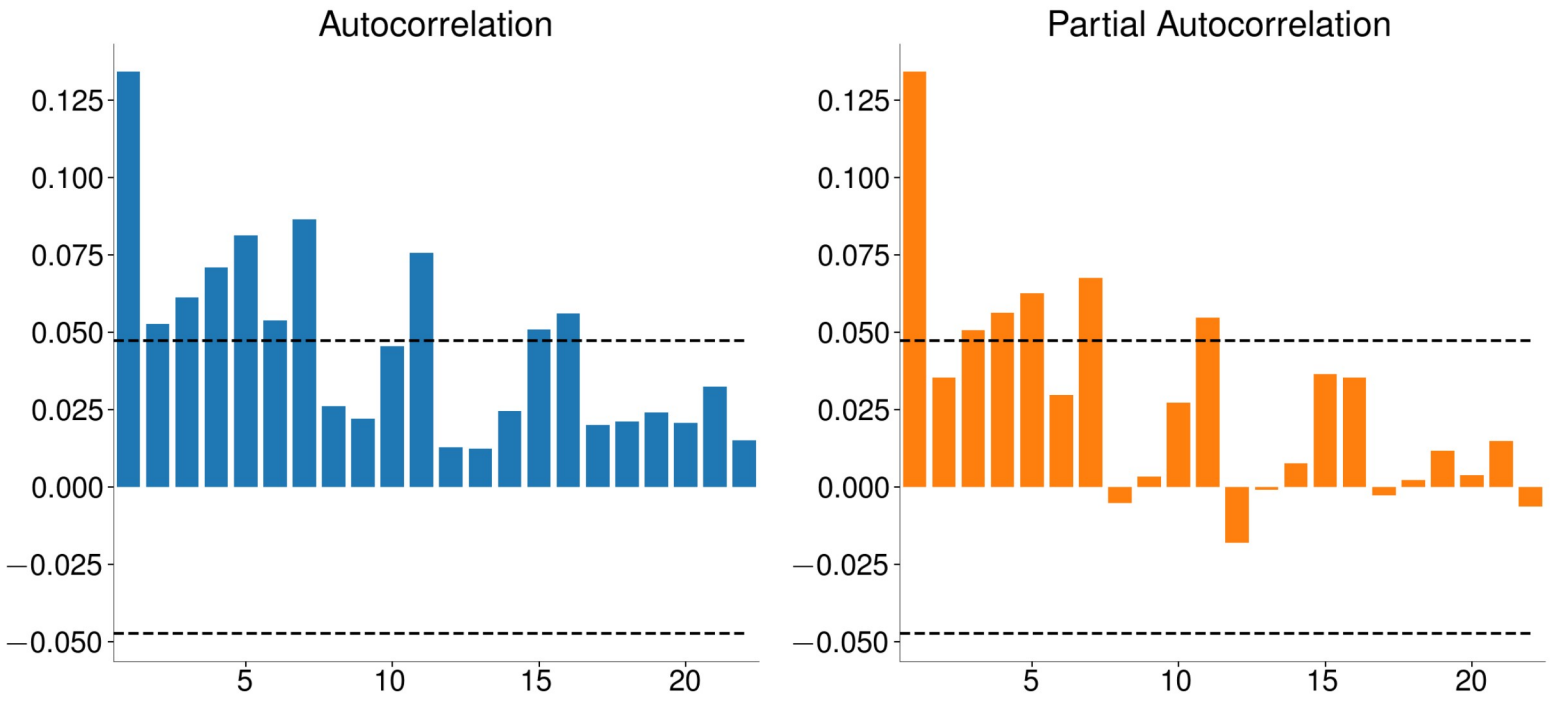
In [44]:
```
acf_pacf_plot(wti**2, 22)
```





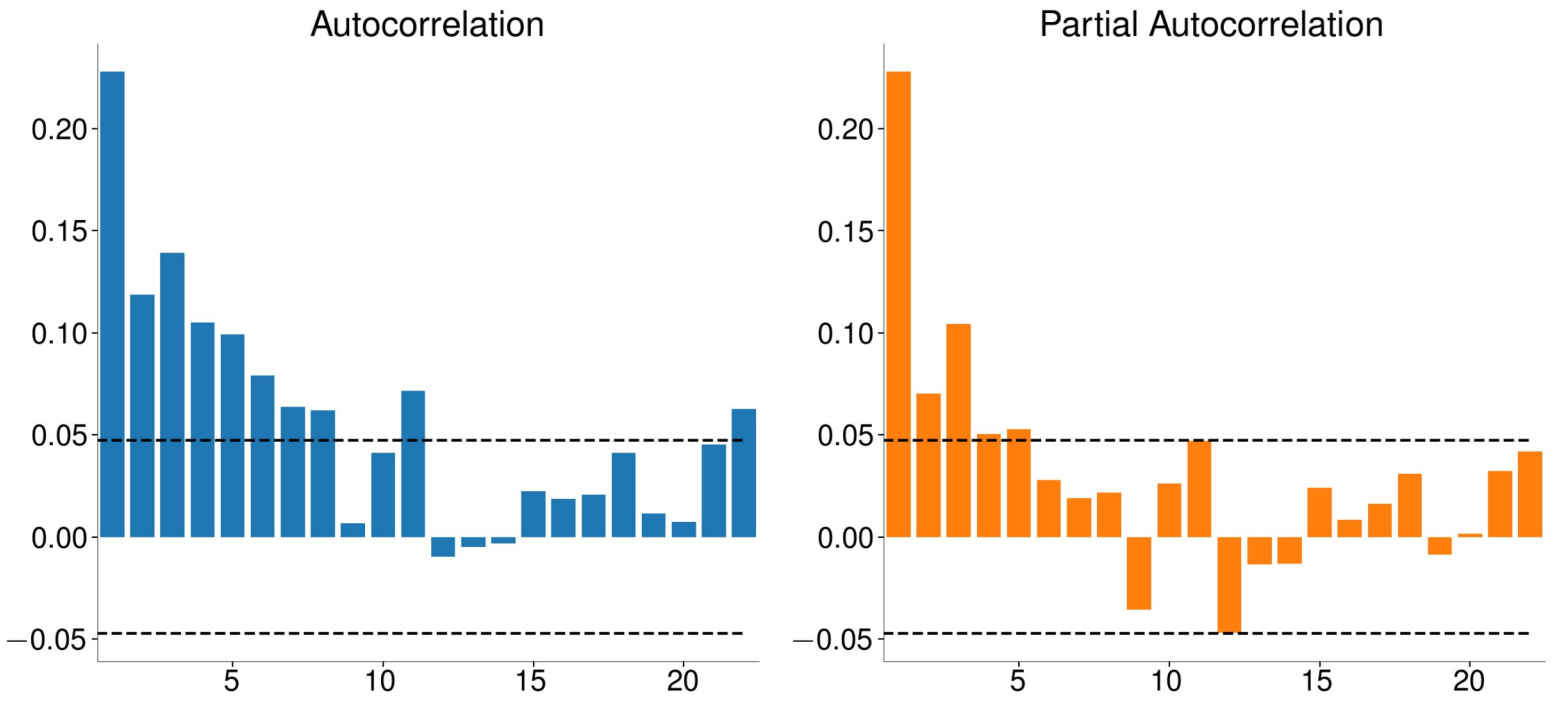
Bitcoin

In [45]:
```
acf_pacf_plot(btc**2, 22)
```



Etherium

In [46]:
```
acf_pacf_plot(eth**2, 22)
```



Model Building

- Usually begin with GARCH
- Check lag specification using StG
- Check for asymmetries using GJR-GARCH
- Check for functional form using TARCH and EGARCH

In [47]:
```
garch11 = arch_model(eth).fit(disp="off")
summary(garch11)
```

Out[47]:

Constant Mean - GARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5315.65
Vol Model:	GARCH	**AIC:**	10639.3
Distribution:	Normal	**BIC:**	10661.1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.2984	0.113	2.643	8.227e-03	[7.709e-02, 0.520]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.3878	1.542	2.197	2.804e-02	[0.365, 6.410]
alpha[1]	0.1739	4.966e-02	3.503	4.606e-04	[7.661e-02, 0.271]
beta[1]	0.7337	8.177e-02	8.973	2.897e-19	[0.573, 0.894]

In [48]:
```
garch21 = arch_model(eth, p=2).fit(disp="off")
summary(garch21)
```

Out[48]:

Constant Mean - GARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5315.65
Vol Model:	GARCH	**AIC:**	10641.3
Distribution:	Normal	**BIC:**	10668.6

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.2984	0.114	2.623	8.708e-03	[7.546e-02, 0.521]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.3876	1.974	1.716	8.613e-02	[-0.481, 7.257]
alpha[1]	0.1740	5.461e-02	3.185	1.445e-03	[6.692e-02, 0.281]
alpha[2]	8.3744e-16	7.523e-02	1.113e-14	1.000	[-0.147, 0.147]
beta[1]	0.7337	0.114	6.441	1.185e-10	[0.510, 0.957]

In [49]:

```
garch12 = arch_model(eth, q=2).fit(disp="off")
summary(garch12)
```

Out[49]:

Constant Mean - GARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5312.01
Vol Model:	GARCH	**AIC:**	10634.0
Distribution:	Normal	**BIC:**	10661.3

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.2985	0.112	2.657	7.890e-03	[7.828e-02, 0.519]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.8957	1.679	2.320	2.036e-02	[0.604, 7.187]
alpha[1]	0.2135	5.746e-02	3.715	2.031e-04	[0.101, 0.326]
beta[1]	0.4137	0.109	3.812	1.381e-04	[0.201, 0.626]
beta[2]	0.2677	8.408e-02	3.184	1.451e-03	[0.103, 0.433]

In [50]:
```
garch22 = arch_model(eth, p=2, q=2).fit(disp="off")
summary(garch22)
```

Out[50]:

Constant Mean - GARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5312.01
Vol Model:	GARCH	**AIC:**	10636.0
Distribution:	Normal	**BIC:**	10668.7

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.2985	0.113	2.644	8.186e-03	[7.724e-02, 0.520]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.8950	3.776	1.031	0.302	[-3.506, 11.296]
alpha[1]	0.2135	7.039e-02	3.033	2.422e-03	[7.552e-02, 0.351]
alpha[2]	2.6050e-15	0.190	1.369e-14	1.000	[-0.373, 0.373]
beta[1]	0.4137	0.530	0.780	0.435	[-0.625, 1.453]
beta[2]	0.2678	0.286	0.935	0.350	[-0.294, 0.829]

In [51]:

```
gjrgarch112 = arch_model(eth, q=2, o=1).fit(disp="off")
summary(gjrgarch112)
```

Out[51]:

Constant Mean - GJR-GARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5311.95
Vol Model:	GJR-GARCH	**AIC:**	10635.9
Distribution:	Normal	**BIC:**	10668.6

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.2892	0.115	2.520	1.174e-02	[6.427e-02, 0.514]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.9218	1.660	2.362	1.818e-02	[0.667, 7.176]
alpha[1]	0.2087	6.364e-02	3.279	1.041e-03	[8.396e-02, 0.333]
gamma[1]	0.0142	7.067e-02	0.201	0.841	[-0.124, 0.153]
beta[1]	0.4136	0.108	3.827	1.295e-04	[0.202, 0.625]
beta[2]	0.2657	8.858e-02	3.000	2.703e-03	[9.210e-02, 0.439]

In [52]:
```
tarch112 = arch_model(eth, q=2, o=1, power=1.0).fit(disp="off")
summary(tarch112)
```

Out[52]:

Constant Mean - TARCH/ZARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5311.64
Vol Model:	TARCH/ZARCH	**AIC:**	10635.3
Distribution:	Normal	**BIC:**	10668.0

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.3063	0.128	2.401	1.634e-02	[5.629e-02, 0.556]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.5484	0.247	2.216	2.671e-02	[6.330e-02, 1.033]
alpha[1]	0.1965	4.514e-02	4.353	1.344e-05	[0.108, 0.285]
gamma[1]	-4.9817e-03	4.319e-02	-0.115	0.908	[-8.964e-02,7.967e-02]
beta[1]	0.4471	9.875e-02	4.528	5.958e-06	[0.254, 0.641]
beta[2]	0.3178	9.509e-02	3.342	8.324e-04	[0.131, 0.504]

In [53]:
```
egarch112 = arch_model(eth, vol="EGARCH", q=2).fit(disp="off")
summary(egarch112)
```

Out[53]:

Constant Mean - EGARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5308.34
Vol Model:	EGARCH	**AIC:**	10626.7
Distribution:	Normal	**BIC:**	10653.9

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.3003	0.121	2.482	1.305e-02	[6.319e-02, 0.537]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.3951	0.149	2.658	7.863e-03	[0.104, 0.686]
alpha[1]	0.3671	7.099e-02	5.171	2.324e-07	[0.228, 0.506]
beta[1]	0.5440	8.495e-02	6.404	1.514e-10	[0.377, 0.710]
beta[2]	0.3488	8.511e-02	4.098	4.169e-05	[0.182, 0.516]

In [54]:
```
egarch102 = arch_model(eth, vol="EGARCH", o=0, q=2).fit(disp="off")
summary(egarch102)
```

Out[54]:

Constant Mean - EGARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-5308.34
Vol Model:	EGARCH	**AIC:**	10626.7
Distribution:	Normal	**BIC:**	10653.9

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.3003	0.121	2.482	1.305e-02	[6.319e-02, 0.537]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.3951	0.149	2.658	7.863e-03	[0.104, 0.686]
alpha[1]	0.3671	7.099e-02	5.171	2.324e-07	[0.228, 0.506]
beta[1]	0.5440	8.495e-02	6.404	1.514e-10	[0.377, 0.710]
beta[2]	0.3488	8.511e-02	4.098	4.169e-05	[0.182, 0.516]

Summary

In [56]:
```
models()
```

Out[56]:

	omega	alpha[1]	alpha[2]	gamma[1]	beta[1]	beta[2]	LLF	AIC	BIC
GARCH(1,1)	3.388	0.174			0.734		-5315.7	10639.3	10661.1
	2.197	3.503			8.973				
GARCH(1,2)	3.896	0.213			0.414	0.268	-5312.0	10634.0	10661.3
	2.320	3.715			3.812	3.184			
GARCH(2,1)	3.388	0.174	0.000		0.734		-5315.7	10641.3	10668.6
	1.716	3.185	0.000		6.441				
GARCH(2,2)	3.895	0.213	0.000		0.414	0.268	-5312.0	10636.0	10668.7
	1.031	3.033	0.000		0.780	0.935			
GJR(1,1,2)	3.922	0.209		0.014	0.414	0.266	-5311.9	10635.9	10668.6
	2.362	3.279		0.201	3.827	3.000			
TARCH(1,1,2)	0.548	0.196		-0.005	0.447	0.318	-5311.6	10635.3	10668.0
	2.216	4.353		-0.115	4.528	3.342			
EGARCH(1,1,2)	0.395	0.367			0.544	0.349	-5308.3	10626.7	10653.9
	2.658	5.171			6.404	4.098			
EGARCH(1,0,2)	0.395	0.367			0.544	0.349	-5308.3	10626.7	10653.9
	2.658	5.171			6.404	4.098			

Specification Checking

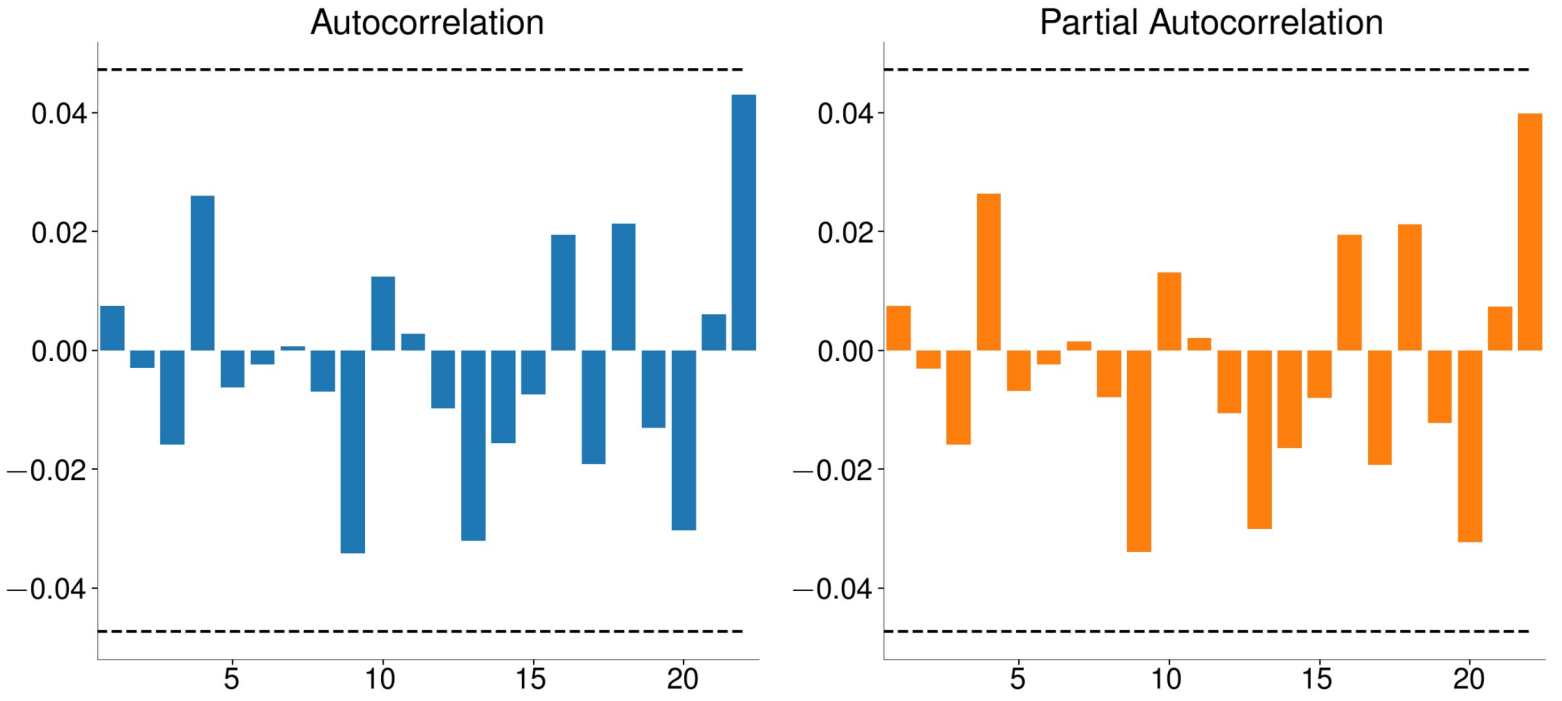
- Conceptually similar to mean modeling
- Emphasis of different form of residual

$$\hat{e}_t^2 = \frac{\hat{\epsilon}_t^2}{\hat{\sigma}_t^2}$$

- Check ACF/PACF for White Noise appearance

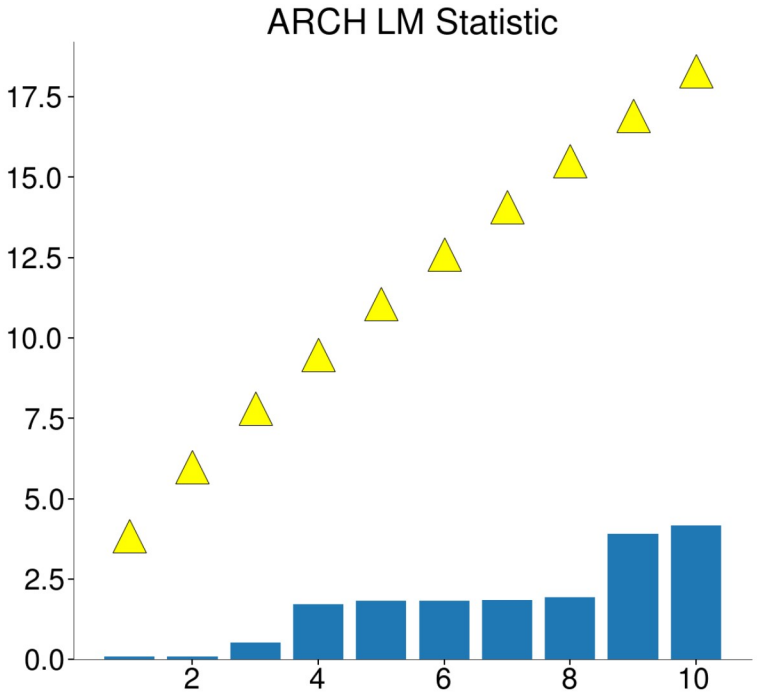
ACF and PACF of $\hat{e}_t^2$

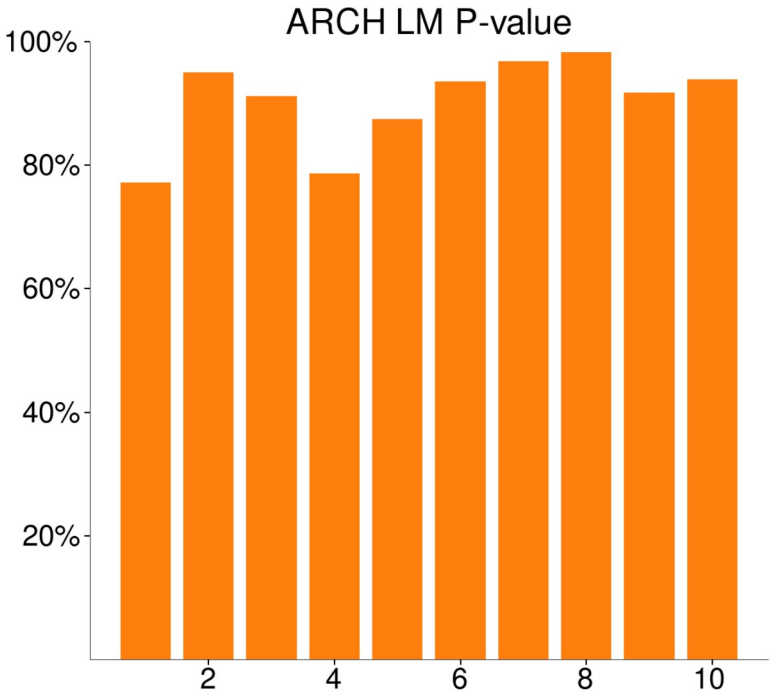
In [57]:
```
acf_pacf_plot(egarch102.std_resid ** 2, 22)
```



ARCH-LM test

In [58]:
```
arch_lm(egarch102.std_resid, 10)
```





Alternative Distributional Assumptions

- Have assumed $e_t \overset{iid}{\sim} N(0,1)$
- Can relax this assumption
- Common alternatives:
    - Student's t
    - Generalized Error Distribution
    - Hansen's Skew t

Baseline Normal

In [59]:
```
normal = arch_model(sp500, o=1, vol="egarch").fit(disp="off")
summary(normal, mean=False)
```

Out[59]:

Constant Mean - EGARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-13681.8
Vol Model:	EGARCH	**AIC:**	27373.6
Distribution:	Normal	**BIC:**	27409.8

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	2.8410e-03	2.112e-03	1.345	0.179	[-1.299e-03,6.981e-03]
alpha[1]	0.1587	1.692e-02	9.380	6.584e-21	[0.126, 0.192]
gamma[1]	-0.1040	1.256e-02	-8.280	1.229e-16	[-0.129,-7.935e-02]
beta[1]	0.9757	4.241e-03	230.054	0.000	[0.967, 0.984]

Standardized Students's t

- Student's t normalized to always have variance 1
- ν controls tail thickness

In [60]:
```
studt = arch_model(sp500, o=1, vol="egarch", dist="t").fit(disp="off")
summary(studt, mean=False)
```

Out[60]:

Constant Mean - EGARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-13408.6
Vol Model:	EGARCH	**AIC:**	26829.2
Distribution:	Standardized Student's t	**BIC:**	26872.7

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	1.8400e-03	1.820e-03	1.011	0.312	[-1.728e-03,5.407e-03]
alpha[1]	0.1489	1.025e-02	14.524	8.578e-48	[0.129, 0.169]
gamma[1]	-0.1040	8.646e-03	-12.027	2.562e-33	[-0.121,-8.704e-02]
beta[1]	0.9825	2.586e-03	379.952	0.000	[0.977, 0.988]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	6.4775	0.436	14.865	5.549e-50	[5.623, 7.332]

Generalized Error Distribution

- Nests the normal when $\nu = 2$
- Heavy tailed for $\nu < 2$

In [61]:
```
ged = arch_model(sp500, o=1, vol="egarch", dist="ged").fit(disp="off")
summary(ged, mean=False)
```

Out[61]:

Constant Mean - EGARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-13426.3
Vol Model:	EGARCH	**AIC:**	26864.6
Distribution:	Generalized Error Distribution	**BIC:**	26908.0

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	1.3248e-03	1.841e-03	0.720	0.472	[-2.284e-03,4.933e-03]
alpha[1]	0.1514	1.178e-02	12.848	8.805e-38	[0.128, 0.174]
gamma[1]	-0.1025	9.533e-03	-10.752	5.811e-27	[-0.121,-8.381e-02]
beta[1]	0.9805	2.975e-03	329.522	0.000	[0.975, 0.986]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	1.3358	3.350e-02	39.879	0.000	[1.270, 1.401]

Hansen's Skew t

- Extendeds the t with a skewness parameter λ

In [62]:
```
skewt = arch_model(sp500, o=1, vol="egarch", dist="skewt").fit(disp="off")
summary(skewt, mean=False)
```

Out[62]:

Constant Mean - EGARCH Model Results

Mean Model:	Constant Mean	**Log-Likelihood:**	-13394.4
Vol Model:	EGARCH	**AIC:**	26802.9
Distribution:	Standardized Skew Student's t	**BIC:**	26853.6

Volatility Model

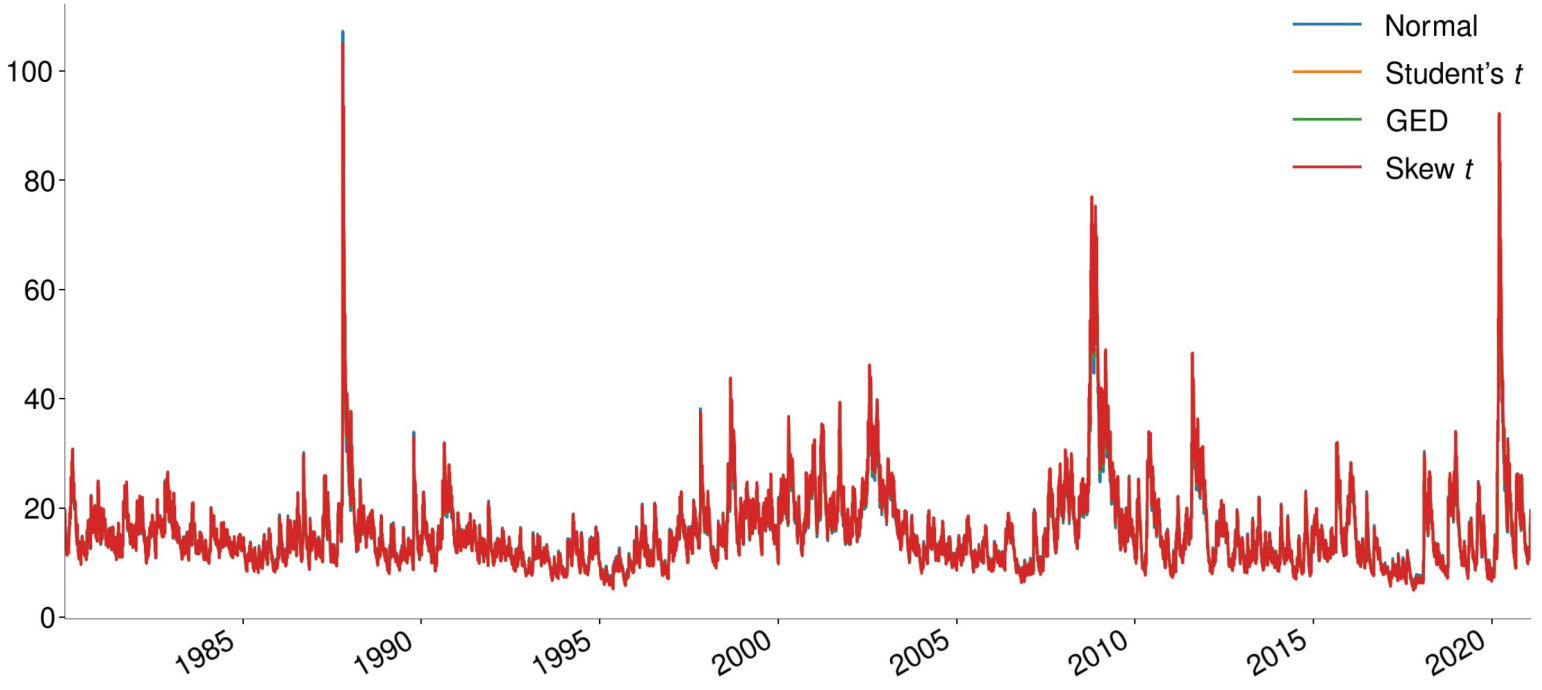
	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.1877e-03	1.928e-03	1.653	9.825e-02	[-5.911e-04,6.966e-03]
alpha[1]	0.1494	1.012e-02	14.757	2.755e-49	[0.130, 0.169]
gamma[1]	-0.1058	8.618e-03	-12.271	1.296e-34	[-0.123,-8.886e-02]
beta[1]	0.9815	2.622e-03	374.307	0.000	[0.976, 0.987]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	6.6500	0.452	14.718	4.944e-49	[5.764, 7.536]
lambda	-0.0705	1.241e-02	-5.683	1.323e-08	[-9.487e-02,-4.621e-02]

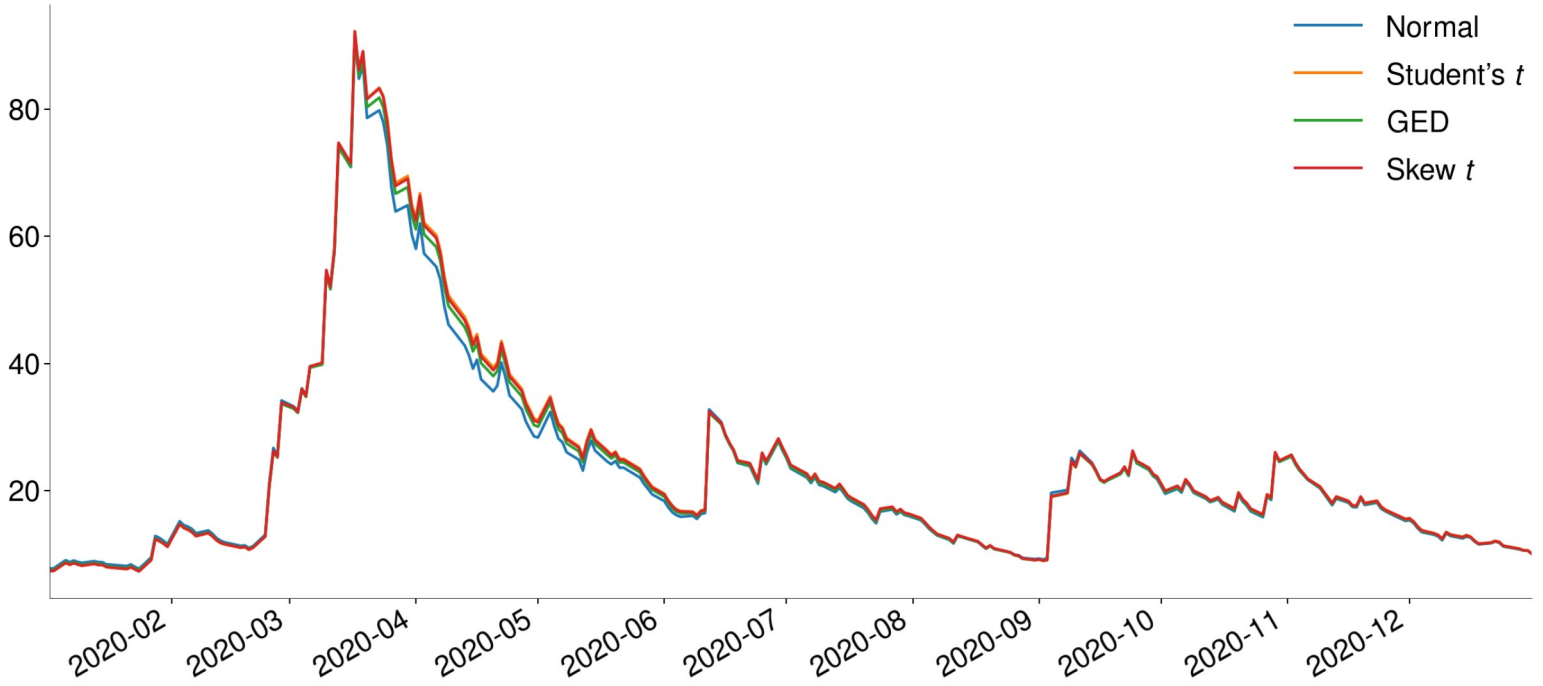
Comparing Volatility

In [64]:
```
compare_volatility(annualize=True)
```



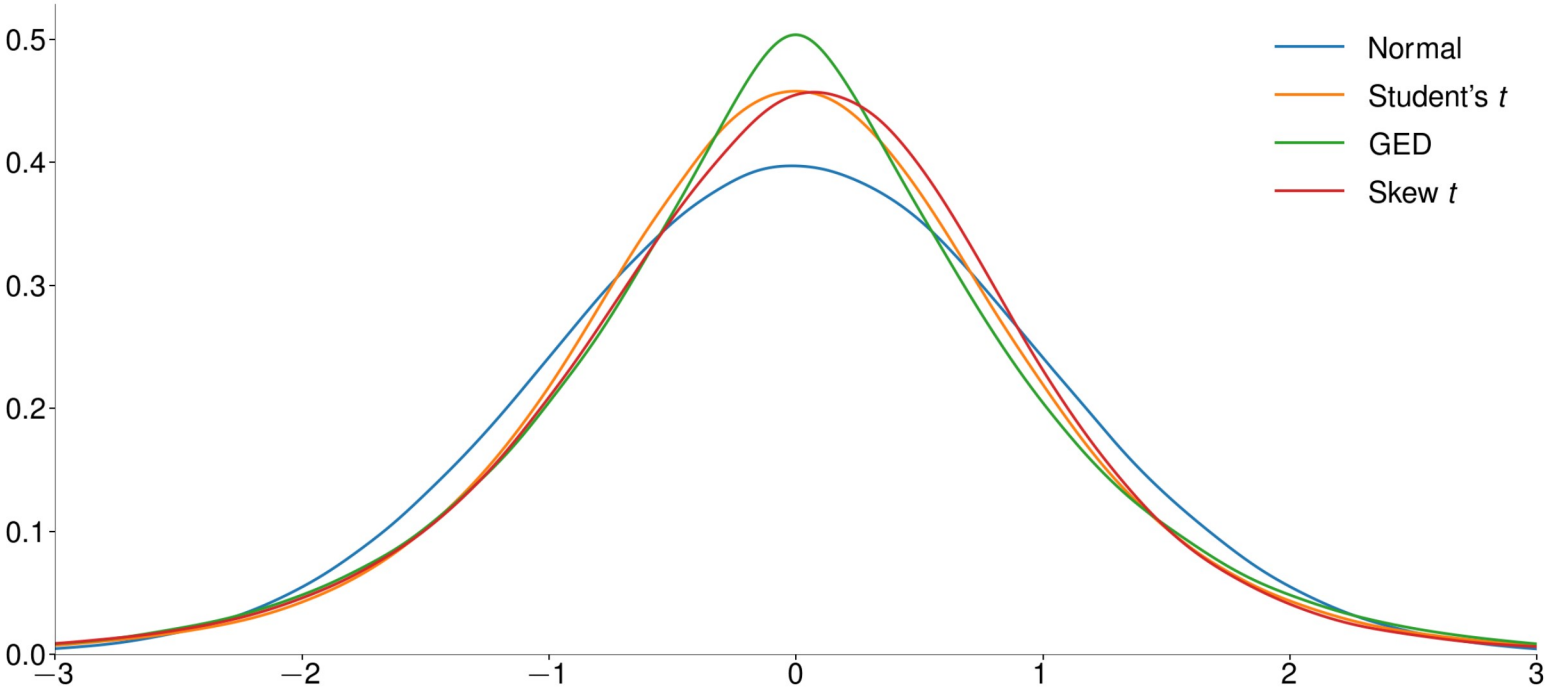
Comparing Volatility in 2020

In [65]:
```
compare_volatility(annualize=True, span="2020")
```



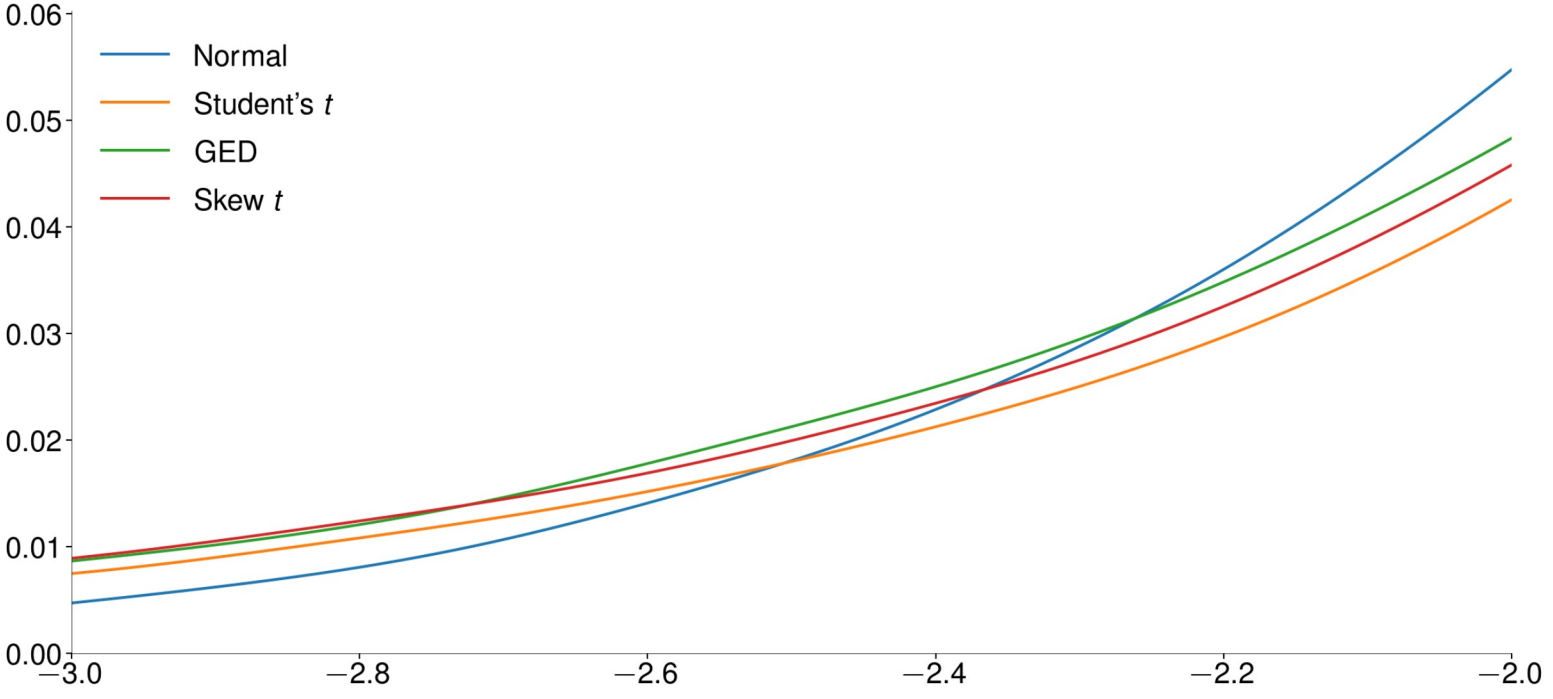
Comparing the estimated distributions

In [68]:
```
compare_dist()
```



Comparing the Tails

In [69]:
```
compare_dist(tails=True)
```



Next Week

Univariate Volatility Modeling

- Forecasting
- Forecast Evaluation
- Realized Variance

Happy New Year