Econometric Model for Interest Rate in France

Bengusu Nar & Barbara Karwowska

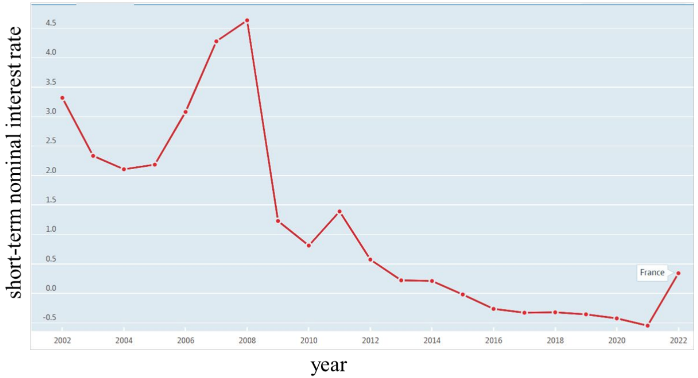
Table of contents

- Descriptive analysis of variables and Taylor rule
- Testing Taylor model on monthly and yearly data
- Testing alternative model specifications with exchange rates and past inflation rates
- Model of choice
- Final diagnostic checks on the chosen model and conclusions

Descriptive Analysis: Interest Rate

Nominal interest rate in France exhibited a downward trend in the years following the introduction of euro, going from 3.3% in 2002 down to 2.1% in 2004. This trend reflected the combined efforts of the European Central Bank and Banque de France to stimulate growth in the newly established eurozone. From then on, the policy rate kept increasing peaking at 4.6% in 2008. In the aftermath of the financial crisis of 2008, the economic recession forced the French central bank to implement an expansionary monetary policy to stimulate the economy. Consequently, the interest rate fell sharply to 1.2% in 2009 and reached its lowest value of 0.8% in 2010. It went slightly up in 2011, but afterwards it was gradually falling, hitting the zero lower bound in 2015. In the following years, it continued to go down assuming negative values and reaching a minimum of -0.5% in 2021. In 2022, due to inflation, it went back up again, increasing to 0.3%. In this work, we will investigate the variables determining the interest rate. We chose to focus on the period between 2002 and 2018 so as to include only the years when euro was the currency and exclude the years of the pandemic as this might lead to abnormalities (we still account for one crisis of 2008, though).

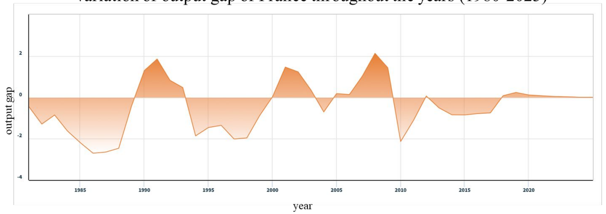
Variation of short-term nominal interest year in the period 2002-2022



Descriptive Analysis: Independent Variables

Output gap refers to the difference between the actual output of an economy and the maximum potential output expressed as a percentage of the potential gross domestic product.

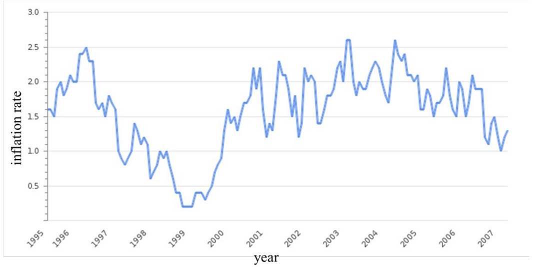
Variation of output gap of France throughout the years (1980-2023)



As shown on the graph to the left, France historically had varying output gaps.

Another variable described in the original model is the difference between inflation rate and the target inflation. Over the last twenty years, France was consistent in hitting its target of 2%, as shown below.

Variation of inflation rate of France throughout the years (1980-2023)



Descriptive Analysis: Taylor Rule

We will use the Taylor Model to examine the relationship between the short-term interest rate and variables affecting it according to the Taylor rule: output gap and deviation of inflation from its target. The corresponding formula for estimating the interest rate is as follows:

$$i_t = \alpha + \beta_\pi \left(\pi_t - \pi^*\right) + \beta_y y_t$$

Based on this model, our initial goal will be to estimate the coefficients β_π, β_y and α for given inflation rates π_t, and output gaps y_t between 2003 and 2018. In our regressions, π^* is replaced by 2%, which corresponds to the inflation target of France, which oscillated around that value throughout the period. (According to ECB No 2575 / July 2021 Working Paper, it was in the range 1.1%-2.1%, but due to unavailability of precise time-series data, we will assume it remained constant at 2%. In fact, under the assumption of constant inflation target, the exact value chosen will not affect the fit of our model.)

First, we will consider the original model using monthly and then yearly data. However, since the model was proposed in 1992 by studying the historical monetary policies of the US, its relevance for France in the period 2003-2018 may be limited. We will discuss it and propose an alternative specification.

Interpolation of the Output Gap

As opposed to other independent variables, the monthly data is not available for the output gap because the natural output and output are measured in yearly cycles. To overcome this, we interpolated output gap, assuming in our approach a constant rate of change throughout a given year. The interpolation function:

```r
interpolation <- function(df) {
  df$val_interp[df$month == 1] <- df$Value[df$month == 1]
  (acs <- rep(NA, 12))
  df$interp_lag <- c(df$val_interp[13:(length(df$val_interp))],acs);
  df$diff <- c(df$val_interp - df$interp_lag)

  for (i in min:(max-1)) {
    for (j in 2:13) {
      init = df$val_interp[df$month==1 & df$year == i]
      end=df$interp_lag[df$month==1 & df$year == i]
      record=which(df$month==j & df$year==i)
      record1=which(df$month==1 & df$year==i)
      if (init  < end) {
        df$val_interp[record]<- df$val_interp[record1] +
          abs(df$diff[record1])/12*(j-1)
      }
      else {
        df$val_interp[record] <-
          df$val_interp[record1] -
          abs(df$diff[record1])/12*(j-1)
      }
    }
  }

  return(df)
```

```r
(N <- sum(!is.na(output_gap$Value)))
(min <- min(output_gap$Time))
(max <- max(output_gap$Time))
month <- rep(1:12, each=1, times=N)
year <- rep(min:max, each=12, times=1)
(month_df <- data.frame(month, year))
output_gap_m <- merge(x = month_df, y = output_gap, by.x = "year",
                      by.y = "Time", all = TRUE)
output_gap_m <- output_gap_m[1:(N*12-11),]
output_gap_m <- interpolation(output_gap_m)
gap_m <- output_gap_m$val_interp
```

Testing the Taylor Model with Monthly Data

```
reg_m <- lm(i_m ~ delta_pi_m + gap_m)
```

We run the linear regression, regressing monthly short-term interest rate on the interpolated monthly output gap and the monthly deviation of the inflation rate from its target. As a result, we get very low p-values for individual coefficients corresponding to independent variables and the intercept. Moreover, the p-value corresponding to the F-statistic is low. However, the value of the coefficient of determination is moderate, which may be due to violated OLS assumptions.

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.15169    0.09182  23.434  < 2e-16 ***
delta_pi_m   0.68866    0.09535   7.222 1.44e-11 ***
gap_m        0.48655    0.05425   8.969 4.07e-16 ***
```

```
Residual standard error: 0.9984 on 178 degrees of freedom
Multiple R-squared:  0.5936,	Adjusted R-squared:  0.5891
F-statistic:   130 on 2 and 178 DF,  p-value: < 2.2e-16
```

To test OLS assumptions, we choose the RESET test for linearity and the Breusch-Godfrey test for serially uncorrelated errors. Since we do not assume normality of the error term, we run the studentized Breusch-Pagan test (White Test) for homocesdacity.

```
RESET = 39.011, df1 = 2, df2 = 176, p-value = 9.473e-15
```

```
Breusch-Godfrey test for serial correlation of order up to 1

data:  reg_m
LM test = 173.58, df = 1, p-value < 2.2e-16
```

At the 5% significance level, we fail all these tests. We estimate that this problem is occurring due to high correlation caused by interpolation. So from now on we will focus on the annual data.

```
studentized Breusch-Pagan test

data:  reg_m
BP = 6.9121, df = 2, p-value = 0.03155
```

Testing the Taylor Model with Annual Data

```
reg_y <- lm(i_y ~ delta_pi_y + output_gap_y)
```

If we consider the annual data, we obtain evidence for the statistical significance of the output gap, but the inflation loses its significance. Also the p-value corresponding to the F-statistic is low, but substantially higher than for the monthly data, for which all regressors were significant, reflecting worse overall significance. Yet, the value of the R squared is improved.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)    1.8042     0.3093   5.833 5.85e-05 ***
delta_pi_y     0.1730     0.3904   0.443  0.66488
output_gap_y   0.7350     0.1966   3.737  0.00249 **
```

```
Residual standard error: 0.9564 on 13 degrees of freedom
Multiple R-squared:  0.6883,	Adjusted R-squared:  0.6403
F-statistic: 14.35 on 2 and 13 DF,  p-value: 0.0005121
```

We run the tests for the OLS assumptions again. While we pass the test for homoscedasticity, the other tests fail. Failure in linearity might imply omitted variables, the problem which we will address in the following part.

```
data:  reg_y
LM test = 9.5682, df = 1, p-value = 0.00198
```

```
data:  reg_y
RESET = 16.547, df1 = 2, df2 = 11, p-value = 0.0004825
```

```
	studentized Breusch-Pagan test

data:  reg_y
BP = 0.36636, df = 2, p-value = 0.8326
```

Additional Variables: Past Inflation and Exchange Rates

With annual data, the coefficient of determination still takes relatively low values and the output of the RESET test implies lack of linearity of the model. This may suggest the omission of relevant variables. Since France is a very open economy, getting inspiration from the uncovered interest parity theory, we decided to include the nominal exchange rate between euro and US dollars as an additional explanatory variable. Moreover, we considered the exchange rates of two other major trading partners of France outside the EU and apart from the US, namely the UK and China.

Furthermore, our previous results undermined the significance of the inflation rate for explaining the nominal interest rate. However, this is in contrast with the economic theory. Indeed, given that the major purpose of the monetary policy of the Central Bank is to regulate the inflation, we expect that the policy rate is affected by inflation. We acknowledge the possibility of a non-linear relationship between the interest rate and the inflation rate. However, from another perspective, we are also aware that it takes time for the Central Bank to adjust its policy rate to the inflation. These considerations prompted us to treat a model with past interest rate as an explanatory variable. We will test two model specifications: one with the last year’s interest rate and another with the interest rate from two years ago.

Alternative Approach with Inflation & Exchange Rates

```
reg_exchanges <- lm(i_y ~ delta_pi_y + output_gap_y + usd + cny + gbp)
```

Regressing on the explanatory variables from the original model and all three exchange rates mentioned, we obtain rather insignificant results. This is most likely due to the abundance of variables.

Therefore, we decide to drop the least significant variable first, which is delta_pi_y, the deviation of the inflation rate from its target. We will regress on the combination of exchange rates and then keep dropping variables until we get higher significance. By this method, we aim at identifying the omitted variable to include in our alternative model specification.

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   6.789251   3.204836   2.118   0.0602 .
delta_pi_y   -0.001548   0.319396  -0.005   0.9962
output_gap_y  0.425292   0.218120   1.950   0.0798 .
usd          -7.487863   4.321468  -1.733   0.1138
cny         -23.593048  25.465889  -0.926   0.3760
gbp           2.670117   2.322348   1.150   0.2770
```

Alternative Approach with Exchange Rates

```
reg_exchanges2 <- lm(i_y ~ output_gap_y + usd + cny + gbp)
reg_exchange2 <- lm(i_y ~ output_gap_y + usd)
```

When we run the linear regression, including the output gap and all three euro/national currency exchange rates as explanatory variables, we do not obtain high significance. We suspect that this might be due to the correlation among exchange rates since they tend to move together. Thus, we decide to regress on the exchange rate (first with output_gap_y and then without; we display results of the latter as they are more significant) between euro and the currency of the biggest international trade partner of France outside the Eurozone, which is the US. The p-value corresponding to this regressor is also the lowest, indicating the highest statistical significance, which further supports our economic reasoning.

This regression passes all the OLS tests.

```
RESET = 1.1399, df1 = 2, df2 = 11, p-value = 0.3549
BP = 1.3589, df = 2, p-value = 0.5069
LM test = 1.1544, df = 1, p-value = 0.2826
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)    6.7830     2.7939   2.428   0.0335 *
output_gap_y   0.4247     0.1769   2.400   0.0352 *
usd           -7.4901     4.0971  -1.828   0.0947 .
cny          -23.5615    23.4759  -1.004   0.3371
gbp            2.6742     2.0671   1.294   0.2223

Residual standard error: 0.6673 on 11 degrees of freedom
Multiple R-squared:  0.8716,	Adjusted R-squared:  0.8249
F-statistic: 18.66 on 4 and 11 DF,  p-value: 7.252e-05
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)    8.3238     2.1238   3.919  0.00176 **
output_gap_y   0.7244     0.1151   6.295 2.77e-05 ***
usd           -8.3485     2.6752  -3.121  0.00812 **

Residual standard error: 0.7286 on 13 degrees of freedom
Multiple R-squared:  0.8191,	Adjusted R-squared:  0.7913
F-statistic: 29.43 on 2 and 13 DF,  p-value: 1.49e-05
```

Alternative Approach with Past Inflation

```
reg_lag1 <- lm(i_y ~ delta_piy_lag1 + output_gap_y)
reg_lag2 <- lm(i_y ~ delta_piy_lag2 + output_gap_y)
```

We consider two possible alternative model specifications, one with the inflation rate from the previous year and another with the inflation rate from two years ago. Hence, for delta_piy_lag1 we use historical data on inflation rates from 2002 to 2017, whereas for delta_piy_lag2 we use inflation data from 2001 to 2016.

The choice of this methodology is motivated by the fact that it takes time for the monetary policy to respond to the inflation.

As we used data with time lag, the significance of the inflation increased, slightly more for delta_piy_lag2. However, in both cases, the regressors still fail at 5% significance level.

Thus, we decided to continue with exchange rates as additional variables and we will disregard past inflation.

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	1.8042	0.3093	5.833	5.85e-05	***
delta_pi_y	0.1730	0.3904	0.443	0.66488	
output_gap_y	0.7350	0.1966	3.737	0.00249	**

Coefficients:

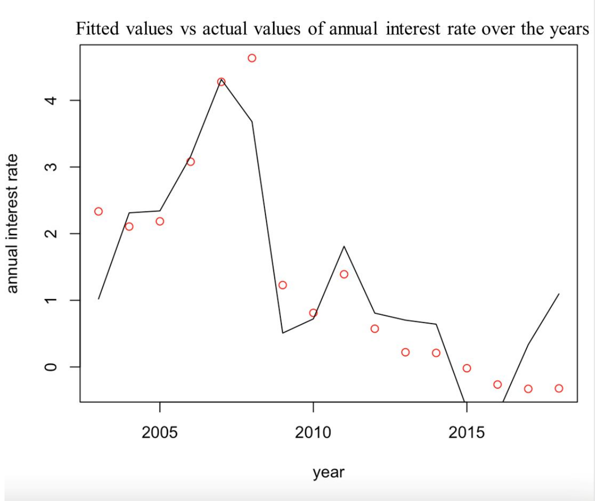
	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	1.9699	0.2758	7.144	7.55e-06	***
delta_piy_lag1	0.4532	0.2837	1.597	0.134207	
output_gap_y	0.7227	0.1434	5.039	0.000227	***

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	1.9962	0.2618	7.626	3.76e-06	***
delta_piy_lag2	0.5155	0.2672	1.929	0.075834	.
output_gap_y	0.7402	0.1345	5.502	0.000102	***

Model of Choice

After considering the original Taylor formula variables, multiple exchange rates, and inflation rates from past years, we concluded that USDEUR exchange rate and the output gap are more predictive for the short term interest rate. The regression on these variables also passes the tests for no correlation between the error terms, linearity and homoscedasticity.



The graph on the left depicts the prediction of the model and the actual data. We can observe that the fit is rather close. The suggested model is therefore:

$$i_t = \alpha + \beta E_{usd,t} + \gamma y_t + \epsilon_t$$

Here, i represents the short term interest rate, E represents the exchange rate USDEUR and y represents the output gap as percentage of potential GDP. Epsilon is the error term.

Diagnostic Checks and Conclusions

The new model passes all the suggested tests for weak OLS assumptions: Ramsey's RESET test for linearity, studentized Breusch-Pagan (White) test for homoscedasticity, and LM test (Breusch-Godfrey) for serial uncorrelation of the error.

```
RESET = 1.1399, df1 = 2, df2 = 11, p-value = 0.3549
LM test = 1.1544, df = 1, p-value = 0.2826
BP = 1.3589, df = 2, p-value = 0.5069
```

Moreover, obtained R squared statistic is higher than the one in the original Taylor model. So, considering these results, we conclude that updating the Taylor formula by discarding inflation and adding exchange rate could be a new null hypothesis. This hypothesis should be checked with more data from different countries and different periods.

```
Residual standard error: 0.6995 on 13 degrees of freedom
Multiple R-squared:  0.8333,    Adjusted R-squared:  0.8076
F-statistic: 32.48 on 2 and 13 DF,  p-value: 8.776e-06
```

Sources

- OECD. “OECD Statistics.” Oecd.org, 2015. https://stats.oecd.org/.
- Nasdaq.com. “Nasdaq Data Link,” 2023. https://data.nasdaq.com/.
- “Deutsche Bundesbank Monthly Report,” 1998. https://www.bundesbank.de/resource/blob/705614/4e13aafc17aaa48f62c88a031e19aff7/mL/1998-01-monetary-policy-data.pdf.
- Adam, Klaus, Erwan Gautier, Sergio Santoro, and Henning Weber. “The Case for a Positive Euro Area Inflation Target: Evidence from France, Germany and Italy.” *SSRN Electronic Journal*, 2021, p. 6. https://doi.org/10.2139/ssrn.3894752.
- Österholm, Pär. “The Taylor Rule: A Spurious Regression?*.” *Bulletin of Economic Research* 57, no. 3 (June 28, 2008): 217–47. https://doi.org/10.1111/j.0307-3378.2005.01220.x.
- Svensson, Lars E. O. “What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules.” *Journal of Economic Literature* 41, no. 2 (June 2003): 426–77. https://doi.org/10.1257/jel.41.2.426.
- Investopedia. “Output Gap: What It Means, Pros & Cons of Using It, Example,” 2023. https://www.investopedia.com/terms/o/outputgap.asp.