VIETNAM NATIONAL UNIVERSITY – HOCHIMINH CITY
INTERNATIONAL UNIVERSITY
SCHOOL OF INDUSTRIAL ENGINEERING & MANAGEMENT

A MULTI-TRIP SPLIT-DELIVERY VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND HETEROGENEOUS VEHICLES FOR INVENTORY REPLENISHMENT UNDER STOCHASTIC TRAVEL TIMES

Submitted in partial fulfilment of the requirements for the Degree of

Bachelor of Engineering in Logistics and Supply Chain Management

Student: PHAM QUANG THONG – IELSIU17073

Thesis advisor: DR. HA THI XUAN CHI

Ho Chi Minh city, Vietnam
July/2021

A MULTI-TRIP SPLIT-DELIVERY VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND HETEROGENEOUS VEHICLES FOR INVENTORY REPLENISHMENT UNDER STOCHASTIC TRAVEL TIMES

By

PHAM QUANG THONG

Submitted in partial fulfilment of the requirements for the Degree of
Bachelor of Engineering in Logistics and Supply Chain Management
International University, Ho Chi Minh City

JULY/ 2021

Signature of Student: ____
Pham Quang Thong

Certified by : ____
Dr. Ha Thi Xuan Chi
Thesis Advisor

Approved by : ____
Dr. Nguyen Van Hop
Dean of IEM School

ABSTRACT

Although Vehicle routing problem (VRP) have been studied for decades, little attention has been paid for frozen product distribution network as well as mathematical model for the problems related. As more start-up companies are trying finding ways to optimize their cost satisfied customer's demand while operating such limited resources, the more robust and applicable model to deal with many variances of real-life problem is always necessary. This study investigates the multi-trip split-delivery vehicle routing problem with soft time windows and heterogeneous fleet for daily planning under stochastic travel time. A mixed integer linear programming model is proposed for small scale solution and an algorithm logic concept is proposed as suggestion for further research. The math model is formulated and tested on IMB ILOG CPLEX Solver software. The approach proposed to handle stochastic travel times in the planning process is practical for a small-scale study. The possible outcomes of vehicles arrivals are systematically classified as their penalty for earliness or tardiness are initially estimated and processed during the data processing stage. To evaluate the performance, a convergence and sensitive analysis is carried out trying capture the tight of lower bound. The result shows the model works well with small set of retailers. However, for larger problems the model spends a lot of computational time. Future research can be suggested in many direction including developing integrated algorithm for better performance or research to improve the constraints.

Key words: Multi-trip split-delivery, Heterogenous fleet, Time windows, Mixed integer linear programming, Stochastic travel time

ACKNOWLEDGEMENT

Throughout the writing of this thesis, I have received a great deal of support and assistance.

First and foremost, I would like to express my special gratitude toward my supervisor, Dr. Ha Thi Xuan Chi, whose expertise was invaluable in coordinating the research questions and approach. As insightful feedback and guideline help shaping my experiment methods, which greatly contribute to my completion of this dissertation.

I would also like to extend my sincere thanks to my other committee member, M.Eng. Tran Van Ly and M.Sc. Duong Vo Nhi Anh who had definitely given out their most insightful and precious feedback as well as recommendation during my proposal defense session.

I would also like to acknowledge the Faculty of Industrial Engineering and Management for providing me with the opportunity to take this thesis research. Through this course, I find myself have been developed in terms of research and finding. It also sharpens my programming skills.

TABLE OF CONTENT

LIST OF TABLE

LIST OF FIGURE

CHAPTER 1 INTRODUCTION

1.1 Background

1.1.1 VRP

The assignment of designing delivery or pickup service in the area of transportation and supply chain is termed as VRP. VRP was first introduced as a generalization of Travel salesman problem (TSP) [1]. The main aim of VRP is to find the optimal routes as particular vehicles in given fleet traverse between the given sets of customers in order to service these nodes. The VRP was introduced as an approach to generate the set of travelling routes between a gasoline distribution terminal and several service stations in the gasoline delivery planning. In modern world, VRP plays a critical role in logistics and distributions for multiple field. Within literature involve optimization, VRP is among the most researched problems because of its substantial complexity and the appropriateness of its industry practice. Besides its capability of finding distinct routes, this feature is combined with many variants and constraints to generate the optimal model with different aim. Mostly, the VRP problem are used to optimize the total cost for the enterprise. However, this focus can be changed depends on the attribute of the industry focusing on. For example, in the case when customer experience is designated as the most important criteria, the problems can be designed its target on maximum satisfaction of the services.

1.1.2 Frozen food chain

In recent years, the require for fresh, refrigerated, and frozen food has increased persistently due to great demand for decent and healthy diets in urban fast-paced everyday living [2]. In

Vietnam it has no exception, with the improvement of people's income and life quality, people's awareness of healthy lifestyle and nutrition have also increased. Along with the raging Covid-19 pandemic, the increased demand for frozen, imported, and frozen foods that can be preserved for a long time signals a possibility of a new frozen consumption trend. The above factors correspondingly lead to the expansion of the market for low-temperature products. Compared with typical temperature commodity distribution, the cold product supply chain requires strict temperature and time control to preserve nourishment quality. Products have low thermal properties (such as ice cream) are more susceptible to any disruption in cold chain distribution [2]. Moreover, in distribution, like most of the product, the company usually has to deal with a large number of customers in scatter locations. It is obviously challenging to allocate customers to vehicles in the distribution network such that the costs are minimized while meeting the requirement of customer time factors. Because of its complexity, this type of problem is classified as NP hard problem [3]. As fresh or frozen product are needed to be delivered with such strict requirement, the risks are high, and a well-developed delivery system is needed to operate the replenishment efficiently. However, because of the fact that the demand is high rising, it contributes to the establishment of a lot of start-up logistics companies or small supermarkets franchise. The characteristics of start-ups are having limited resource, which explains for the small and heterogenous fleet of theirs. These vehicles are often assigned a large number of task during the working day as stocks in retail stores can be in short of supply. Because of limited resources and large-scale deliveries, it is optimal to use one vehicle for multiple trips a day and each retailer's demand can be met by multiple vehicles during planning horizon. Because it is moving in the urban area, the travel time will be surely stochastic, it is possible to arrive earlier or later than the time window of that retailer. Additionally, due to limited resources, it is impractical trying to completely avoid these cases, the best way is to estimate these additional costs into the total cost. To combine those elements efficiently, a detail, coherence and applicable planning is crucial. It will play important role in the operating problem since more of the cost are considered and also associated with more scenarios, the allocation of money flow will be more proper and help avoid losses both financially and reputation-wise.

1.2 Literature review

Vehicle routing problem (VRP) is defined as a combinatorial optimization problem aiming to generate a feasible set of routes for a fleet of vehicles delivering goods and services to satisfy a given set of customers [4]. Over decades VRP have been studied largely in the literature [5] and numerous major extension have been developed. Comprehensive review had been given out, providing a proper look into wide range of VRP and its extension, see [6]–[8]. Three main basics extension are considerately contributing for the research focus in VRP over five decades. These variants include VRP with time window (VRPTW), Capacitated VRP (CVRP) VRP with Multiple Trips (VRPMT).

The first major variants (CVRP) can be considered the most basics and the earliest extension being researched in VRP. Throughout the history, there are many approach and areas have been exploited. Exact algorithm based on branch-and-price for CVRP problem under stochastic time windows [9]. In the other filed, hybrid algorithm which proved to be effective solving CVRP [10].

The second primary extension, VRPTW investigates the delivery locations have time windows within which the deliveries (or visits) must be made within. In the case that delivery within time window is strictly applied and assigned for each locations such that they require an absolute service, the problem is called VRP with "hard time windows". A non-linear math model had been introduced by [11] as an attempt to minimize the time it production, delivering, and returning to the depot. The resolve of routes and schedules are also determined by the model. Furthermore, they also proposed two types of heuristics based on genetic algorithms to deal with large-scale problems. On the other hand, if the delivery can be made after the time windows exchanging for a penalty or any kind of incurred cost, the problem is concerning to "soft time windows". [12] developed the very first study about VRP with soft time window. In their research tardiness was allows and offset by penalty added in objective function. Tabu search heuristic was built to solve the problem. [13] depicted VRPTW as a multi-objective optimization problem as it tries to analyze both the number of vehicles and the total cost. In addition, they improved genetic algorithm as the proposed solving method for their problem.

Another major variant is multi-trip VRP (MTVRP). The problem makes allowance for vehicles to dispatch more than once during the planning horizon for more efficient and utilized of vehicles use. [14] initiated a theoretical problems on multi-trip vehicle with limited duration for each trip. In this study they proposed an algorithm as an improvement of the one proposed in an experienced load planner working on the complex, real-life problems of a case study called Burton's Biscuits: a study of the load planning operation at the new depot at Risley. M. Sc. Dissertation, Lancaster University in 1989. The algorithm was based on Tabu Search heuristic and had been proved that surpass the previous one. There are more literature on the study, [15], [16] can be considered as representative of MTVRP research.

As VRP are required to be more practical, many studies focusing on implementing mathematic model that capture real-world attributes aiming to provide more realistic plan for the networks of distribution. As in most real-world problems, the business who demand is high have great chances have to be served by different type of vehicles [17]. The heterogeneous vehicle routing problem (HVRP) has been studied considerably in the literature. [18] proposed a tabu search heuristics to solve HVRP with time window. [19] developed Hybrid Evolutionary Algorithm for the problem. Split-delivery is also one of major extension practically capture the real-world distribution requirement, in which a customer’s demand can be split among several vehicles as an effort to create more flexibility in distribution network [20]. [21] had proposed exact solution method for SDVRP. Based on branch-and-price algorithm, the problem is analyzed by decomposing the large model into possible routes, given the number of deliveries that generated as the subproblem. They considered both the case of an unlimited fleet and the case of a fleet limited to the minimum number of vehicles possible.

The VRP problem in distribution of frozen product has been viewed in a rather skewed manner or given little attention [2]. In their study they claimed that most of the study had been assumed frozen product can be treated with traditional constraints. These models may not be able to handle strict requirements and lead to management inefficiencies. Meanwhile, as is widely recognized, in the real-life situation, in order to operate the frozen delivery, the use of multiple vehicles to meet customers' quantity and time requirements are shown to be flexible and can fully exploit the capabilities of the fleet. Moreover, the travel time factor needs to be considered carefully and as a stochastic factors. Despite the fact that stochastic vehicle routing problems (SVRPs) have been studied for almost 50 years [22], SVRPs have received much less attention from the research compared to the deterministic counterpart. However, in the last 15 years, there have been an increasing tendency in the amount of work targeting stochastic versions of the VRP. Some review literature [23], [24] have mentioned the significant change in the amount of study in SVRPs. The growth in the amount of study targeting stochastic variants of VRP can be justified by the availability of more data contributing for the understanding of various random phenomena. Besides, advanced method for yielding solution and handling complex statistics problem have been more accessible. There are four phenomenon gained the most attention. The most studied stochastic phenomenon in VRP is demand [25], [26], next is travel time (or travel speed) of vehicle [27], [28], customer service time have also been studied widely [29] and finally, random occurrence of customers [30].

There was several approach to handle stochastics time in VRP. [31] used time-space network to with each customer at one time period is considered a node [3] consider travel times and transportation cost as normal distribution with mean and standard deviation is calculated from available data. There are also many approaches that have been proved to be effective solving (or dealing with) different variations of SVRP. Solution methods are divided into two halves, the exact methods in the first half and the heuristic methods in the other half. The travel times can be continuous or discrete random variables, which are associated with a probability distribution, e.g., uniform distribution, exponential distribution, and normal distribution, etc. there are some major study express different approach, for example [32] used Branch & Cut (B&C),. In the review of [33], they had pointed out tabu search has been the most popular and arguably the most effective method for solving VRP with stochastic factors.

Throughout the years, most research on the VRP has focused on searching heuristics, network scaling or algorithms. The improvement of the math model has received little attention. Still, there is still a lot of research focused on developing and improving the routing model. In current cold chain distribution, mathematical models used for VRP generally focus on optimizing total delivery costs including transportation costs, energy costs and sometimes cost of deterioration [2]. Mixed-Integer Programming (MIP) play important part as it is practical and fast for testing the efficiency of the model when the Routing Problem (RP) is not considered time sensitive or a recursive problem. The concept of Linear Programming (IP) was developed mainly to solve economic and non-time-oriented problems. The MIP idea is suitable for solving a relatively small set of input where the recursive problem is not the main problem. There are many study focus on MIP in VRP research [34]–[36]. However, when study must carry on the larger scale, exact algorithm become less of a preference, heuristics or meta heuristics have been developed as an attempt to solve a problem, learn, or discover that providing a proper and practical technique is unlikely to be optimal or perfect, but is still sufficient for direct goals [37]. Since the last three decades, many heuristic approaches have been widely implemented in this area.

1.3 Motivation

Along with the great demand for fresh food and frozen food, the demand for cold chain logistics is increasing day by day. At the same time, with more and more demanding customers and increasing constraints on the transportation network, higher requirements lead to higher logistics costs. Therefore, the important issue is how to control distribution costs. The improvement of the vehicle routing model has received little attention. There have been even less studies building mathematical models around and suitable for frozen products. As the demand is increasing rapidly in urban area. Exploiting this feature can set delivery industry to new heights and standards. And because of the strict conditions of the frozen chain and limited resource of most of the business. Variant elements in the combined VRP are needed. As mentioned, the VRP distribution system needs to be a combination of multi-trip split-delivery with time window and heterogenous fleet.

Among the various studies in the literature of VRP, [31] proposed a multi-trip split delivery vehicle routing problem with soft time windows (MTSDVRPTW), which is the only study that I know of considering the three major extensions of VRP in the same mathematic model. In some later years later, the extension of this study had considered a MTSDVRPTW under stochastic travel times [38]. The stochastic travel time is the most notable assumption in this problem. Based on the real-life, traffic conditions are frequently disturbed by various factors, this assumption is realistic and is more likely to generate accurate vehicle routing schedules. However, I note that the approach for this problem is rather too much complicated as the idea is to generate a matrix for vehicle at customer node in specific time period, which is 3-dimensional matrix. Although detail planning system can be generated, it can be a burden as collecting or generating data could require large time as well as a decent hardware facility to actually testing and analyze. Moreover, the formulation may infeasible if large amount of data is not consistence or positive definite during processing stage. A more simplified, set of data and less effort generation method is much needed as, to some extent, mathematical model is crucial for study that apparently small scale but more specific and combined many realistic factors. More importantly, a mixed integer linear formulation can serve a good foundation for further and larger study.

1.4 Objective

The focus on this paper is only on the mixed-integer programming modeling and formulation. Besides the aim is combine 3 major extension of VRPs: multi-trip, split-delivery, and time windows, two other factors as stochastic and heterogeneous fleet are also considered as an attempt to make the model more realistic and robust. (MTSDHVRPTW). The function's objective is to minimize the total transportation cost of transporting products from supplier to retailer to meet inventory replenishment needs in the cold chain.

The contribution of the study is to deliver MIP model solving the VRP problem that combine multi-trip, split-delivery, soft-time window, heterogeneous fleet and stochastic travel time. The important features and limitations of the model are also analyzed and represented. Future scope is also proposed for further study.

1.5 Scope

As the study focus on developing MIP model using for small scale for daily VRPs problem of frozen products, the scale of the data is also small. In this paper, the distribution of frozen product for Bach Hoa Xanh (BHX), a mini supermarket franchise is considered. As location data for retailer location is public, the study makes used of these data as the parameters for the model. Real location nodes based on their longitude and latitude focus on Tan Binh districts and district 10 in Ho Chi Minh city. With total of 30 retailer locations and one depot, this paper tries to find the shortest path between two nodes using ILOG CPLEX optimizer software for exact solution. As the aim is generate a feasible MIP formulations, the set of other data including fixed cost, variable cost, cost for scenarios. The planning horizon is divided into equally 40 periods. Each period is 10 minutes.

A sensitive analysis had been developed to evaluate the constraints. Also, since mixed integer linear programming have never been famous for effectively dealing with large scale problem. A Simulated Annealing algorithm then proposed aiming to evaluate the efficient of the mathematic model and play as the foundation for further research with larger set of sample. The work also discusses limitation of the model and a specify the future study direction.

1.6 Methodology

1.6.1 Research process

The study focusses on develop a feasible MIP model for MTSDHVRPTW evaluate the efficiency of the model in based on CPU time to find an optimal solution. The steps are as following.

- Specify the problem
- Indicate attribute of the problem
- Collecting and process data
- Formulate the model
- Solving model
- Data evaluation
- Increase the scale of the model
- Visualize and interpret the solution
- Suggestion

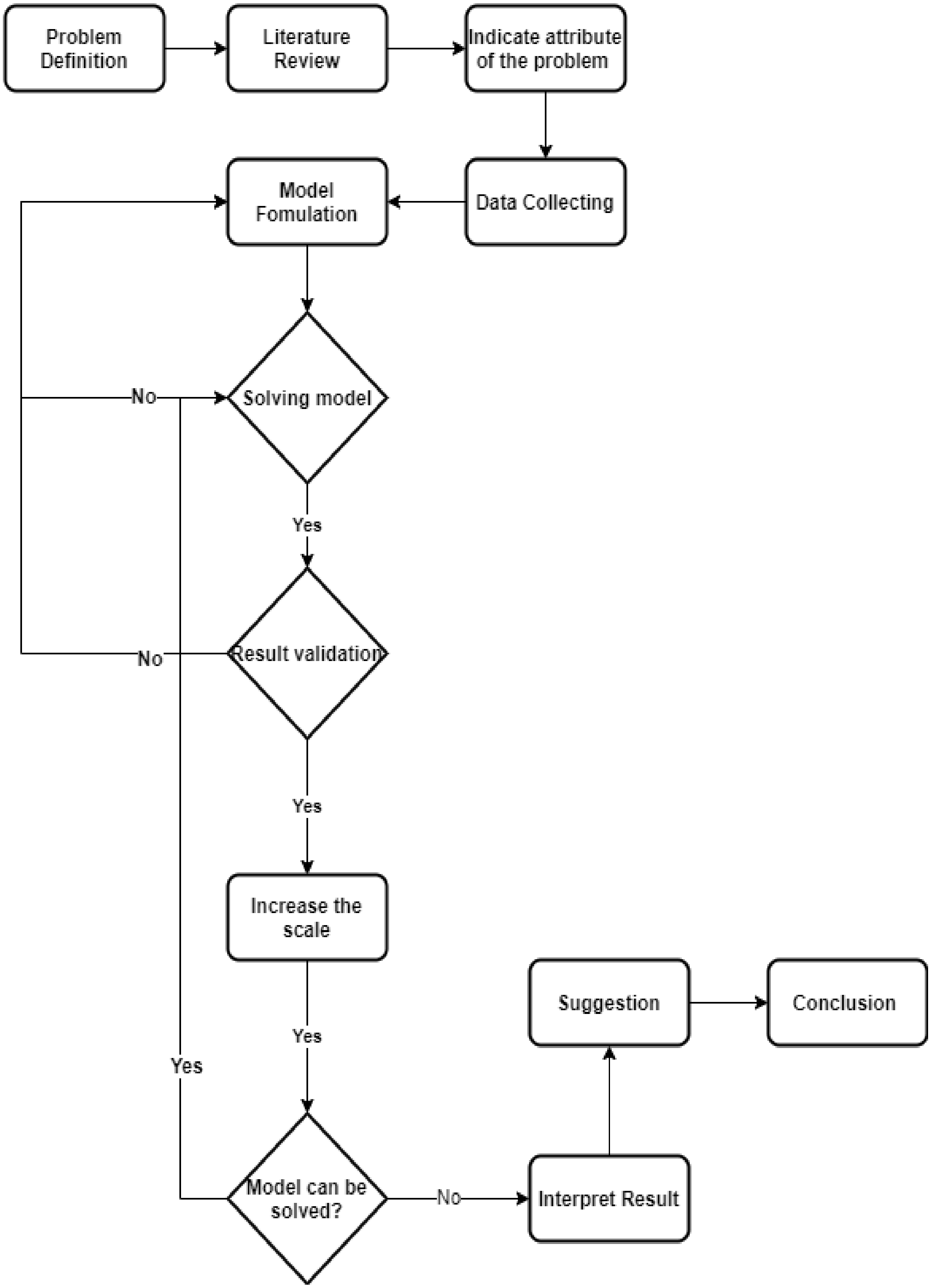

Figure 1: Flow chart of methodology

1.6.2 Specify the problem & Literature review

First the background of the problem is research, in this steps, it mostly involves literature reviewing. The whole idea of this steps is to determine the meaning in solving the problem and identify the limitation (or gap) of the current practice and existed study. The total of 39 literatures had been reviewed and concluded that there is little attention being paid for frozen products in VRP problems. The reason is lying at the misconception and assumption about the attributes of this type of distribution problems. Second, I try to identify why the study is needed. As the more and more start-up company is appears, these companies often have a limited number of resources, so the problem they have to deal with is small scale but when it comes to meeting a lot of conditions when working with frozen products. Furthermore, most of study have been focused on develop heuristics or algorithm, math model has gained less attention, since my study is trying to combine 5 factors multi-trip, split-delivery, time window, heterogenous fleet and stochastic travel time, which is as I know the first of its kind, a mixed integer programming model can be used as foundation for further study.

1.6.3 Determine the attribute of model.

The first need to be determined is the input for the model. As this is the VRP problem, there are certain parameters. Also, as my problem trying to combine another five factors, some specific statistic must also be specified. The major cost includes travel cost, which is depended on travel distance, holding cost or can be considered as the power cost for refrigerator keeping the frozen products at it demanding temperature inertia. Also, there are dispatching cost, incurred cost for stochastic, penalty or waiting cost and penalty for unsatisfied demand.

Every variants contribute to the feature of the model. The model is developed around the assumption:

This model is design for decision maker who will decide the route of the vehicles and order of services. This decision maker has thorough awareness about the demand of the retailers and detailed information of delivery time windows assigned for each retailers. The problem is soft time window, the additional cost and penalty is assigned for both scenarios when the vehicles arrive earlier or later than the time windows.

In urban areas the speed can be fluctuated lead to the travel speed in this problem is stochastics. This fluctuation can be explained by many factors. For example, interactions between neighboring vehicles and road section operations such as parking, pedestrian crossings, bus stops, etc. In rare cases, the travel time of one section of road may depend on another, such as when an overflow occurs due to severe congestion. However, it can be argued that the spillover is only expected during peak hours and is hardly observable in the daily inventory replenishment issue considered in this study. In other words, most of the travel time of a road segment is not affected by other road segments; For simplicity, the travel time through each road segment is assumed to be independent in this study. Furthermore, because the locations of suppliers and retailers are known, the travel time between these locations can easily be observed and we therefore assume that the distribution for travel time is available. However, in this study due to the lacking of observation, all the discrete distribution had been assumed. The probability will distribute mostly in the speed close to the planned travel speed (or average speed) and scatter in the rest.

As the focus of the model is not multi-product, a single product is assumed. In the future, the model can be extended to consider multiple types of product.

There are different type of vehicle, and the fleet size is fixed. Each vehicle has specific capacity, fuel consumption, dispatching cost and planned travel time.

The restriction on the maximum number of trips per day for a vehicle is equal to the number retailers who have been planned to be serviced. The logic behind is if the vehicle dispatch from the depot, it is expected to serve at least one retailer.

The times that one retailer can receive service is equal to number of planning vehicles at max. Or it can be said that one retailers can be serviced multiple time but not by the same vehicles.

The objective function tries to minimize all the cost which will be specified in together with the constraints.

1.6.4 Data collecting, processing and assumption

1.6.4.1 Distance

The distance is collected and choose from the address of 30 BHX in district 3, district 5, and district 10. The latitude and longitude of these location were collected one by one based on the address of these BHX retails, which are public. The distance between two retailers was then processed using Google Maps Formulas for Google Sheets developed by Amit Agarwal, a Google Developer Expert in Google Workspace and Google Apps Script. The code for these amazing formulars is free for public use and can be used with personal sheet. It helps generate distance matrix using Google Map without having registration to Google API. However, everyday there is a limit in quota request. Therefore, the task generate distance matrix with dimension 31x31 must be divided into 3 days. The latitude and longitude were used for geocoding, which means generate back the address and formatted as Google standard and can be read by the Google function. The “DISTANCE” and “INDEX” function have been used to generate distance matrix.

`=GOOGLEMAPS_REVERSEGEOCODE(B2;C2)`

A	B	C	D
	latitude	longitude	Google DIstance Formulars result
0	10.82536575	106.6350135	60 Hoàng Bật Đạt, Phường 15, Tân Bình, Thành phố Hồ Chí Minh, Vietnam
1	10.78753453	106.64715	210 Đường Hồng Lạc, Phường 11, Tân Bình, Thành phố Hồ Chí Minh, Vietnam
2	10.79536637	106.6405646	126 Trương Công Định, Phường 13, Tân Bình, Thành phố Hồ Chí Minh, Vietn
3	10.79321891	106.6653241	229/6A Bùi Thị Xuân, Phường 1, Tân Bình, Thành phố Hồ Chí Minh, Vietnam
4	10.78833039	106.6410241	724 Âu Cơ, Phường 13, Tân Bình, Thành phố Hồ Chí Minh, Vietnam
5	10.78375392	106.6475701	98 Đ. Ni Sư Huỳnh Liên, Phường 10, Tân Bình, Thành phố Hồ Chí Minh, Vie
6	10.80556684	106.6394325	73 Nhất Chi Mai, Phường 13, Tân Bình, Thành phố Hồ Chí Minh, Vietnam
7	10.80015754	106.6475182	116 Hoàng Hoa Thám, Phường 12, Tân Bình, Thành phố Hồ Chí Minh, Vietnam
8	10.79100374	106.6463613	108/4/27 Trần Mai Ninh, Phường 12, Tân Bình, Thành phố Hồ Chí Minh, Vie
9	10.80454838	106.6427029	70 Thân Nhân Trung, Phường 13, Tân Bình, Thành phố Hồ Chí Minh, Vietnam
10	10.81926549	106.6340464	33/32/19/2 Nguyễn Sỹ Sách, Phường 15, Tân Bình, Thành phố Hồ Chí Minh 7

Figure 2: Sample of Geocoding

`=GOOGLEMAPS_DISTANCE(index($E$2:$E$32;L$1+1;1);index($E$2:$E$32;$J2+1;1);"driving")`

	K	L	M	N	O	P	Q	R
	0	1	2	3	4	5	6	
0	1 m	5.7 km	4.3 km	6.8 km	4.7 km	5.9 km	3.3 km	4.6 km
1	6.7 km	1 m	2.7 km	4.1 km	1.9 km	0.7 km	4.7 km	3.1 km
2	4.4 km	1.4 km	1 m	4.1 km	0.8 km	2.0 km	2.2 km	1.3 km
3	7.8 km	4.7 km	5.0 km	1 m	5.4 km	4.8 km	4.8 km	4.1 km
4	4.9 km	1.0 km	0.8 km	4.6 km	1 m	1.2 km	2.7 km	1.8 km
5	6.1 km	0.8 km	2.0 km	3.7 km	1.2 km	1 m	3.6 km	2.2 km
6	3.5 km	3.3 km	2.0 km	3.8 km	2.7 km	3.8 km	1 m	1.6 km
7	4.7 km	1.7 km	1.3 km	3.0 km	1.8 km	2.2 km	1.7 km	1 m
8	5.3 km	0.5 km	1.0 km	3.3 km	1.1 km	1.0 km	2.7 km	1.0 km
9	4.1 km	3.2 km	1.9 km	3.7 km	2.6 km	3.8 km	0.4 km	1.4 km
10	1.0 km	4.8 km	3.5 km	6.0 km	3.9 km	5.1 km	2.5 km	3.7 km
11								

Figure 3: Sample of Distance data generation

1.6.4.2 Stochastic travel time

The planning model combining multi-trip, split-delivery and time window constraint can be categorized as a multi-commodity network flow problem (MCNF). Stochastic travel tine can be considered a side constraints, together with such complicated combination of VRPs extension, the problem is NP-hard in terms of optimization. In this study, stochastic travel time is considered as discreate random variable assumingly assign. The stochastic time will then be processed as cost and also a 3-dimensional matrix depending on vehicle and correlation matrix between retailer nodes.

Stochastic travel time depends on travel speed. Travel speed, in this study, is considered as discrete probability distributions. The stochastic cost is then calculated based on this distribution.

$$q_\tau^s = \sum_{w \in S_\tau, w \neq s} p_\tau^w f_\tau^{w,s} h_\tau^{w,s}$$

Where:

p_m^w the probability of travel time scenario w for vehicle m

$f_m^{w,s}$ the time inconsistency between the planned scenario s and realized scenario w for vehicle m, w does not equal s;

$h_m^{w,s}$ the penalty (per time unit per vehicle) if scenario s is planned but scenario w is realized for vehicle m, w does not equal s;

S_m the set of all travel time scenarios for vehicle m

There are two scenarios:

- Realized travel time scenario w is smaller than the planned scenario s for trip τ. There is cost for vehicle waiting ω. (Vehicle arrive earlier than planned)
- Realized travel time scenario w is larger than the planned scenario s for trip τ. There is penalty for schedule delay θ. (Vehicle arrive latter then planned)

For example:

In this example, for easier visualization the speed is assumed. In the real processing data sheet, the speed is considered and used to calculate the travel time, which is correlated with the approach mentioned above. For the best details, see attach Excel file.

The travel speed scenarios s of vehicle m with planned travel speed at **20km/h** of vehicle m are 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 with a probability of 2.19, 2.34, 2.48, 2.63, 2.77, 2.92, 3.07, 3.21, 3.36, 3.50,3.65, 3.80, 3.94, 4.09, 4.23, 4.38.

$$S_m=\{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$$

$p_m^w=$

$\{0.094, 0.059, 0.083, 0.052, 0.019, 0.145, 0.148,$,

$0.022, 0.042, 0.028, 0.070, 0.005, 0.138, 0.006, 0.012, 0.076\}$

$$f_m^{w,20}=\{5, 4, 3, 2, 10, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Therefore,

$q_\tau^{20}=5\times0.094\times\omega +4\times 0.059\times\omega +3\times0.083\times\omega +2\times0.052\times\omega +1\times0.019\times\omega$

$+0\times0.145\times\omega +1\times0.148\times\theta+2\times0.022\times\theta+3\times0.042\times\theta+4\times0.028\times\theta$

$+5\times0.070\times\theta +6\times 0.005\times\theta+7\times0.138\times\theta+8\times0.006\times\theta+9\times0.012\times\theta+10\times0.076\times\theta$

1.6.4.3 Data as 3-dimensional matrix

In this study, there are four parameters need to be represented in 3-dimensional matrix. There are travel cost, holding cost, travel time and stochastic incurred cost. All these four set of data depend on Vehicle and correlation matrix between retailer nodes. Since CPLEX have no support in reading 3-dimensional data, the parameters need to presented as 2-dimensional and use an intermediate array to read all the data, by calling the instruction SheetRead() in ILOG language. The data present can be found in .xlsx file and the code for processing reading data will be shown in the .mod and .dat file.

	Vehicle 1							Vehicle 2						
	0	1	2	3	4	5	sink	0	1	2	3	4	5	sink
0	0	7.182	5.418	8.568	6.048	7.434	0	0	5.586	4.214	6.664	4.704	5.782	0
1	8.442	0	3.402	5.166	2.268	0.882	8.442	6.566	0	2.646	4.018	1.764	0.686	6.566
2	5.544	1.764	0	5.166	1.134	2.52	5.544	4.312	1.372	0	4.018	0.882	1.96	4.312
3	9.828	5.922	6.3	0	6.804	6.048	9.828	7.644	4.606	4.9	0	5.292	4.704	7.644
4	6.174	1.134	1.134	5.67	0	1.386	6.174	4.802	0.882	0.882	4.41	0	1.078	4.802
5	7.686	1.008	2.52	4.662	1.386	0	7.686	5.978	0.784	1.96	3.626	1.078	0	5.978
sink	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	Vehicle 3							Vehicle 4	
	0	1	2	3	4	5	sink	0	1
0	0	4.104	3.096	4.896	3.456	4.248	0	0	5.13
1	4.824	0	1.944	2.952	1.296	0.504	4.824	6.03	0
2	3.168	1.008	0	2.952	0.648	1.44	3.168	3.96	1.26
3	5.616	3.384	3.6	0	3.888	3.456	5.616	7.02	4.23
4	3.528	0.648	0.648	3.24	0	0.792	3.528	4.41	0.81
5	4.392	0.576	1.44	2.664	0.792	0	4.392	5.49	0.72
sink	0	0	0	0	0	0	0	0	0

Figure 4: Sample of processed 3-dimensional matrix

1.6.5 Formulate the model

The MIP model has four main composition. The parameter can be considered as the input of model, the decision variables will provide the result, the objective function play as upper bound of the model, and the constraints. In this paragraph, certain indicators that are special and greatly affect the performance of the model will be mentioned. Along with that, the dimension handling of some large multidimensional parameters will be demonstrated.

1.6.5.1 Logical concept

The formulation based mostly on column generation. In this study, a trip long is defined as the travel of vehicle when it departs from depot until travel back to depot. Therefore, instead set up the number trip vehicles makes as a decision variable, I decide makes them as a set which the maximum value is equal to the number of retailer. The logic behind this is whenever the vehicle departs from depot, it has to serve at least one retailer, therefore, it requires no greater than number of trip equal the number of retailer needed to be served. The trips play as the index of some decision variables which includes the travel flow of the vehicles between retailer nodes. There will be constraints allow vehicle takes fewer trips than the number of trips allowed. As this is MIP, the linear factors need to be considered, to ensure the flow, the sink nodes are considered. The sink nodes will play as the signal of end trip in the formulation. Each trip will have one sink node, it also represents that the vehicle has traveled back to the depot whenever there is travel through it. The travel flow in one trip is dependent on many factors. First the load capacity of the vehicle, the vehicle must go back to depot if the delivered product is equal to the maximum capacity, or the remaining load is unable to satisfy the next retailers. Therefore, in formulation, in one specific trips, the sum of delivered product is unable to exceed the vehicle capacity. The travel flow is also affected by the time window and travel since there will be cost charging earliness or tardiness.

1.6.5.2 Objective function

The objective function tries to minimize travel cost, holding cost, stochastic travel time incurred cost, waiting cost and penalty for lateness.

1.6.5.3 Decision variables

Decision variables are the unknown in an optimization problem. It has a domain, which is a compact representation of the set of all possible values for the variable. Decision variable types are references to objects whose exact nature depends on the underlying optimizer of a model. In this problem the decision variables are designed to decide:

Whether the vehicle goes from one node to another during that trip;

The number unit of products will be service at specific retailer when arrive at that retailer;

The number unit of products will be failed to service to specific retailer;

The arrival time of vehicle at specific retailer;

The time that vehicle will start servicing its retailer;

The departure time of vehicle at specific retailer to move to another retailer.

1.6.5.4 Parameter

Due to many factors combine in this paper, the parameters must be associated with many indexes. The Travel cost and Product cost are designed as 3-dimentional matrix which depend on vehicles, and 2-dimensional matrix which represent correlation between retailers. Also, travel time is 3-dimentional matrix as well since this problem include heterogenous fleet factors, each vehicle will have different planned speed as it travels through location of retailer.

1.6.6 Solving the model

1.6.6.1 The use of ILOG CPLEX solver

The CPLEX solver from IBM ILOG is a high-performance solver for Linear Programming. It was originally developed by Robert E. Bixby and sold commercially from 1988 by CPLEX Optimization Inc. There are 3 competitive commercial solver for MIP problem, which are CPLEX, GuRoBi and XPRESS. The reason this study choose CPLEX as main solver for the model is several. CPLEX has been proved to be among the most efficient and high performance compared to the other solvers (Anand et al., 2017). Moreover, while being a commercial product, IBM have been very generous providing students, teacher or researcher at universities, institutes, or other academic facility. These factors make CPLEX the most accessible for undergraduate and become the best choice for this study.

1.6.6.2 CPLEX Algorithm solving MIP problem

When solving mixed integer programming (MIP) models, CPLEX uses branch-and-cut search algorithm.

The branch and cut can be considered as the hybrid of branch and bound and column generation. The branch-and-cut procedure involve a search tree which contains a set of nodes. Every node represents a linear programming (LP) and quadratic programming (QP) subproblem. These problem will be waited to be process. If they have not yet been processed these nodes are called active and become inactive vice versa. Until there are no active nodes are available or some limit has been reached (time limit, constraints, …) CPLEX will processes the remain available active nodes in the tree.

When two new nodes are created from a parent node it creates a branch. Ordinarily, a branch occurs when there is modification in the bounds on a single variable, with the new bounds still remain in effect for that new node and for any of its descendants. The solutions domain of the two new nodes will be completely different.

When a constraint is added to the model, it called the cut. The cut is played as the limitation of the size of continuous solution region of LP or QP problems. Therefore, the result is generate based on a reduction in the number of branches required to solve the MIP.

1.6.7 Result Validation

The model result is based on four factors:

- Accurate
- Descriptive Realistic
- Precise
- Robust
- Fruitful

1.6.7.1 Accuracy

To determine the accuracy of a mathematical model, the following need to be observed. After a set of functions has been built and solved, if the result of the decision variables generated by the equations equal (or are close to) real data is collected from real life data collected or from existed data-based system, then we can determine its accuracy. However, in this study, the MIP model solving MTSDHVRP, in my knowing, is the first of its kind, also as some sources of data involving in this problems are not accessible, there is a great number of parameters need to be assumed under some certain condition. Moreover, the set of inequality and the model are only "valid" can be proved by the validity of result. The result might have no chance close to the new world, however, as result can be generated by exact algorithm, the constraints and data input may have decent correlation and can be assumed the result is accurate.

1.6.7.2 Descriptive realistic

The model can be claimed to be realistic if the model based on assumptions is correct, depend on real-life facts, rules and logical. Since the assumption of this study is completely based on the previous research in literature, this study is realistic.

1.6.7.3 Precision

The precise of this model is totally depended on result of the solver and the scale of the problem. Since MIP model solving using exact algorithm, which is unfortunately famous for unstable dealing with large scale problem. Therefore, if there are exact solutions within the runtime, then the solution is precise, otherwise it is just accurate.

1.6.7.4 Robustness

A model is robust if it is relatively immune to errors in the input data. There is a natural tendency to build precise models, which can become theorems. A theorem is a well-proven tool: If the correct assumptions are this, one can prove that the result will be exactly the same. In real life, the three requirements which if not comply, the model can become a theorem. These requirement include the objectives are unclear or contradictory, the laws are unknown, the data are missing or corrupted. By definition, a robust mathematical model (RMM) is one that takes these uncertainties into account. It will work, it will deliver something, even if the goals are unclear, even if the rules are uncertain, even if the data is corrupt.

To evaluate the robustness of my model, many set of data have been tested parallelly and the scale of the problem also increase. The details of the test will be presented in **section 2.2.2**

1.6.7.5 Fruitfulness

The MIP model is definitely useful and critical for further study. Whether it is the improving of constraints or develop a search algorithm to solve larger scale problems, the logic behind

MIP is basics and can be used as a foundation for future study. Therefore, my study can be considered fruitful.

1.6.8 Increase the scale of the study

This can be considered as a steps to evaluate the model by trying to create many scenarios. The test will be test with 5 Retailers, 10 Retailers, 15 Retailers, 20 Retailers and 30 respectively.

1.6.9 Visualize and Analyzing model

The result after the CPLEX need to be displayed in order to support the observation and evaluation. Since CPLEX uses Brand and Cut algorithm which can be argued as the hybrid of branch and price and column generation algorithm.

Therefore the recognition about the how the column works is critical. The visualize attempt to display:

- Which vehicle was used;
- How many trips the vehicle made;
- Which route did it takes;
- Optimal total cost;

The following shows the logic behind my attempt to visualize and the result.

```
Begin

FOR Each m in number of vehicle

PRINT number of vehicle

FOR each t in number of maximum trip
```

```
FOR each j in number of all nodes
IF Route(m,t,0,j)=1 THEN
PRINT Route
l:=j
ENDIF
FOR each k in number of all nodes
IF Route(m,t,l,k)=1 THEN
PRINT Route
l:=k
ENDIF
PRINT("   ")
ENDFOR
End
```

Figure 5: Visualization pseudo code

1.6.10 Evaluate the model

The model will attempt to compare the computation time for each time the scale is increased. A convergence analysis and sensitive analysis will be carried out as an attempt to identify the pattern of the model as well as evaluate the performance.

1.6.11 Algorithm suggestion

Due to the complexity of the VRP problems, it is inefficient and sometimes infeasible to solve it with exact methods. Therefore, a metaheuristics integrated with the model or innovative random generation algorithm for initial solution may be the most effective approach. In this study, SA is proposed as a suggestive metaheuristics. There are studies that have been done on using this heuristics to solve optimization problems for various purposes. Therefore, it can be argued that SA is efficient and friendly when solving routing problems [39].

The proposed SA is proposed as a method to improve the solutions in future research. SA is a local search algorithm that possesses the ability to escape the local optimal level. This algorithm is very effective in solving convex or discrete optimization problems. Likewise, SA is used to solve integer programming problems. Moreover, the SA effectiveness can be explained by the simplicity of the implementation. Thanks to this, the convergence properties, as well as the hill climbing feature movements have contribute to the improvement of the initial solutions generated as these try to escape the local optimum trap in each iteration.

CHAPTER 2 MODELLING

2.1 Mathematical model

2.1.1 Index

N	*Number of Retailers*
M	*Number of Vehicles*
T	*Number of maximum Trips allowed*
i, j	*Index of Retailers* ($i, j = 1, 2, \ldots, N$)
m	*Index of Vehicles* ($m = 1, 2, \ldots, M$)

2.1.2 Set

T *Set of maximum trips allowed (t =1, 2, …, T)*

L *Set of trips in maximum trips allowed (L≤T)*

M *Set of heterogenous vehicles (m = 1, 2, …, M)*

V *Set of nodes in retailers (i = 1, 2, …, N)*

VD *Set of nodes include retailers and depot (i = 0, 1, …, N)*

VO *Set of nodes include retailers and sink nodes (i = 1, 2, …, N+1)*

VE *Set of nodes include retailers, depot and sink nodes (i = 0,1, …, N+1)*

VI *Set of nodes in depot*

VS *Set of sink nodes*

2.1.3 Parameters

M *A big number*

Vc_{ij}^{m} *Travel cost of vehicle m traveling from retailer i to j*

Sc_{ij}^{m} *Stochastic travel cost of vehicle m travelling from retailer i to j*

Pc_{ij}^{m} *Holding cost of vehicle m traveling from retailer i to j*

TR *Travel time*

E_i *Earliest time window of retailer i*

L_i *Latest time window of retailer i*

S_i *Service time needed at retailer i*

B_i *Cost of unsatisfied demand of retailer i*

D_i *Demand of retailer i*

Q_m *Capacity of vehicle m*

DV_m *Dispatching cost of vehicle m*

Ec_{mi} *Earliness waiting cost of vehicle m at retailer i*

Lc_i *Tardiness penalty cost of retailer i*

2.1.4 Decision variables

$v_{ij}^{m,t}$ *Equals 1 if vehicle m service retailer arc (i, j) at trip t, otherwise equals to 0*

$x_{ij}^{m,t}$ *The unit of product delivered at retailer arc (i, j) by vehicle m at trip t*

z_m *Equals 1 if vehicle m is used, otherwise equals to 0*

u_i *Flow of unsatisfied demand of retailer i*

$a_i^{m,t}$ *Arrival time at retailer i at trip t by vehicle m*

ge_i^m *Amount of early time arriving at retailer i by vehicle m*

gl_i^m *Amount of late time arriving at retailer i by vehicle m*

$pD_0^{m,1}$ *Departure time from depot at first trip by vehicle m*

$$Min\ z = \sum_{m\in M}\sum_{t\in T}\sum_{ij\in V}\left(Vc_{ij}^{m} + Sc_{ij}^{m} + Pc_{ij}^{m}\right)v_{ij}^{m,t} + \sum_{m\in M} Bu_i + \sum_{m\in M} DV_m\, z_m$$
$$+ \sum_{m\in M}\sum_{i\in V} Ec_i^m ge_i^m + \sum_{m\in M}\sum_{i\in V} Lc_m gl_i^m \tag{1}$$

2.1.5 Constraints

$$\sum_{j\in VO} v_{i,j}^{m,t} = 1 \qquad \forall i\ \in VI, \forall m \in M \tag{2}$$

$$\sum_{k\in VD} v_{i,j}^{m,t} = 1 \qquad \forall \mathrm{i}\ \in VS, \forall \mathrm{m} \in \mathrm{M} \tag{3}$$

$$\sum_{m\in M}\sum_{t\in T}\sum_{i\in VD} v_{i,j}^{m,t} \geq 0 \qquad \forall \mathrm{j} \in \mathrm{V} \tag{4}$$

$$\sum_{j\in VO} v_{i,j}^{m,t} - \sum_{k\in VD} v_{k,i}^{m,t} = 0 \qquad \forall i \in V, \forall m \in M, \forall t \in T \tag{5}$$

$$v_{k,i}^{m,t} - M\left(1 - v_{i,j}^{m,t}\right) \leq 0 \qquad \forall ij \in V, \forall m \in M, \forall t \in T \tag{6}$$

$$\sum_{i\in VE} v_{i,j}^{m,t} = 0 \qquad \forall j \in VD, \forall m \in M, \forall t \in T \tag{7}$$

$$\sum_{l\in L}\sum_{i\in VD} v_{i,j}^{m,l} + M\left(1 - v_{i,k}^{m,t}\right) \geq 0 \qquad \forall j \in V, k \in V, \forall m \in M, \forall t \in T\ (l \leq t) \tag{8}$$

$$\sum_{l\in L}\sum_{i\in VD} v_{i,j}^{m,l} - M\left(1 - v_{i,k}^{m,t}\right) \leq 0 \qquad \forall j \in V, k \in V, \forall m \in M, \forall t \in T\ (l \leq t) \tag{9}$$

$$a_0^{m,t} - a_i^{m,l} \geq TR_{i,0}^{m} \qquad \forall t \in T, \forall l \in T, \forall m \in M\ (t = l + 1) \tag{10}$$

$$a_0^{m,t} - a_i^{m,t} \geq TR_{0,i}^{m} \qquad \forall t \in T, \forall m \in M \tag{11}$$

$$a_j^{m,1} - pD_0^{m,1} - M\left(1 - v_{0,j}^{m,1}\right) \le TR_{0,j}^m \quad \forall j \in VO, \forall m \in M \tag{12}$$

$$a_j^{m,t} - a_i^{m,t} + M\left(1 - v_{i,j}^{m,t}\right) \ge S_j + TR_{i,j}^m \quad \forall i \in VD\ \forall j \in VS, \forall t \in T, \forall m \in M \tag{13}$$

$$ge_m^j \ge \sum_{t\in T}\sum_{i\in VD} v_{i,j}^{m,t} E_j - \sum_{t\in T} a_j^{m,t} \quad \forall j \in V, \forall m \in M \tag{14}$$

$$gl_m^j \ge \sum_{t\in T} a_j^{m,t} - L_j \quad \forall j \in V, \forall m \in M \tag{15}$$

$$\sum_{i\in VD, j\in V} x_{i,j}^{m,t} \le Q_m \quad \forall t \in T, \forall m \in M \tag{16}$$

$$\sum_{m\in M}\sum_{t\in T}\sum_{i\in VE} x_{i,j}^{m,t} + u_j = D_j \quad \forall j \in V \tag{17}$$

$$z_m + M\left(1 - \sum_{t\in T}\sum_{i\in VE} v_{i,j}^{m,t}\right) \ge 1 \quad \forall m \in M, \forall j \in V \tag{18}$$

$$v_{i,j}^{m,t} = 0,1 \quad \forall i,j \in VE, \forall t \in T, \forall m \in M \tag{19}$$

$$x_{i,j}^{m,t}, u_i, a_i^{m,t}, ge_m^j, gl_m^j, pD_0^{m,1} \ge 0 \quad and\ integral \tag{20}$$

2.1.6 Model explanation

(1) The objective function tries to minimize travel cost depend on distance between retailer nodes, stochastic cost depends on travel time discrete distribution, holding cost depend on travel time. Also, the function aims to minimize the unsatisfied demand, earliness, tardiness and vehicle dispatch. (2), (3) Make sure every trip makes by a vehicle, it will arrive from depot and back to the depot by sink node. (4) Imply there can be no visit make to retailer by any vehicles, if this happened, the cost for unsatisfied demand is high. (5), (6) Ensure each vehicle will departs from the node of a retailer after service complete and eliminate subtour. (7) Ensure no travel is made from sink nodes back to any nodes in one trip. (8), (9) Imply

the vehicles have no need use all the trips allowed. (10), (11) Ensure in every trip takes, arrival time from depot in previous trip will equal to the time start travel of next trip. (12) Generate start time from depot in first trip of a vehicle. (13) Ensure in one trip, the arrival time of the next travel is equal to the travel time plus the service time of the previous travel. (14), (15) Calculate the amount of earliness and tardiness of a vehicle at a retailer. (16) Ensure the total product delivered to retailers of a vehicle not exceed it capacity. (17) Total delivered product plus unsatisfied demand of a retailer is equal to its total demand. (18) Indicate whether which vehicle is used. (19), (20) define the type of variables.

2.2 Result Analysis

2.2.1 Statistics & Solving environment

The table below represent the statistics component of the model in different number of retailers. The statistics reflects the contents and the scale of the model.

Table 1: Statistics of model in different number of retailer

Number of Retailer	Constraints	Binary	Integer	nonnegative
5	5 070	3 224	1 169	18 089
10	53 540	18 244	6 334	189 174
20	724 680	119 684	40 664	2 418 344
30	3 501 420	376 324	126 994	11 343 514

The environment used for solving the problems is an Intel Core i5-6200U 2.30 GHz CPU with 8GB Ram and Microsoft Window 10. The version of ILOG CPLEX used is 12.6 and implemented with ILOG Interactive optimization studio. Because of capacity of CPU, the limitation of running time was set at 1 hour.

2.2.2 Performance evaluation

The table shows some performance indicators generated by CPLEX. The number of retailers is varying from 5 to 30. As show in table, the math model can generate exact solution on the scale with 5 retailers, the case with 10 retailers showed that with greater CPU run time the solution would be more optimal. The case with 20 and 30 retailers generates no solution within 1 hour running time. The gaps between upper bound (UB) and lower bound (LB) of the objective value in the case 5 retailers was 0.61% which can be considered fully optimized. In the case with 10 retailers, after 10 minutes solving the gaps was still high at 74.18%, however after 1 hour the gap was reduced to 42.41%. It can be estimated that the optimal solution can be generated within 3 or 4 hours. In the case with 15 retailers and higher the gaps after 1 hours are very high and there is no sign that the optimal solution can be generated within 24 hours.

Table 2: Result performance comparison with different number of retailer

Number of Retailer	**Vehicle used**	**Total cost**	**CPU runtime (second)**	**Total Unsatisfied Demand**	**Largest Early time**	**Largest Latest time**	**Gap**
5	2	536.621 **(Exact solution)**	47	0	3	0	0.61%
10	4	1914.06	603	0	264	29	74.18%
10	3	986.365	3600	2	120	23	42.41%
15	4	6887.23	3600	0	310	92	90.65%
20	N/A	N/A	3600	N/A	N/A	N/A	High
30	N/A	N/A	3600	N/A	N/A	N/A	High

2.2.2.1 Convergence analysis

The following shows the convergence curves obtained from proposed model instances, respectively. This visualization is available in CPLEX studio. In these figures, the yellow point represents a node wherever there is integer value. The green line is displayed as the evolution of best integer value processing. The red line shows a bound on the final solution. These figures are the correlation between objective function value and time. Through observation, it showed that the region of solution explored by proposed model is promising. The model converges towards the best solution. **Figure 6 and 7** shows the convergence curve for 5 and 10 retailer nodes. Throughout the process for both 5 and 10 retailer nodes, it can be observed that the slope of convergence curve is vary, the lower bound is always tight and close to the convergence curve. However, it is almost parallel and need a lot of time to reach the best solution, especially when dealing with larger scale with larger set of variables.

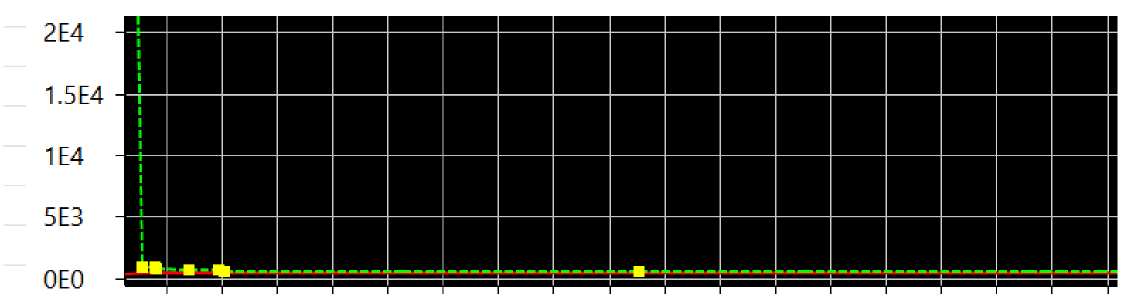

Figure 6: Convergence curve obtained from CPLEX with 5 retailer nodes

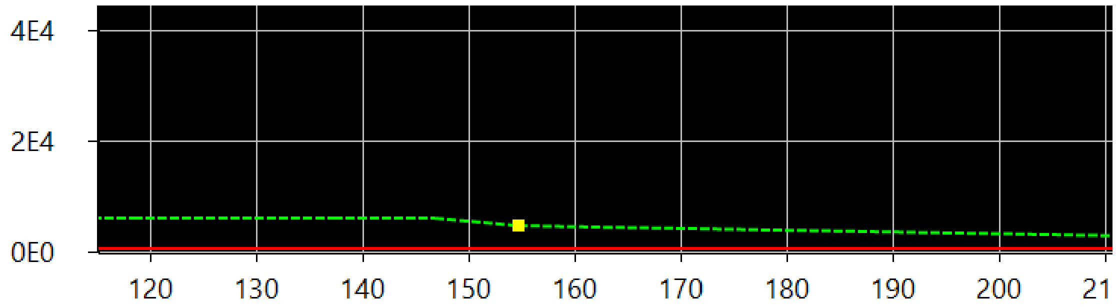

Figure 7: Convergence curve obtained from CPLEX with 10 retailer nodes

2.2.2.2 Sensitive analysis

It is obviously recognized from table the number of retailers need servicing is greatly impact the computational time. The other variables like earliness and tardiness and unsatisfied demand have also been affected, however, it can be argued that the change is not significant and can be optimal with longer run time. **Figure 8** show how the correlation between number of retailer affecting computational time and optimal total cost. **Figure 9** show the correlation between the gaps and computational time. **Figure 10** shows different increase in maximum capacity of vehicles. The 20, 40, 60 % of the original capacity have ben tested. Through the analysis, it is obvious that, by having more holding capacity, the optimal cost generated more optimal, the computational time is varied which indicate the vehicle capacity has no effect on solving time of the solver.

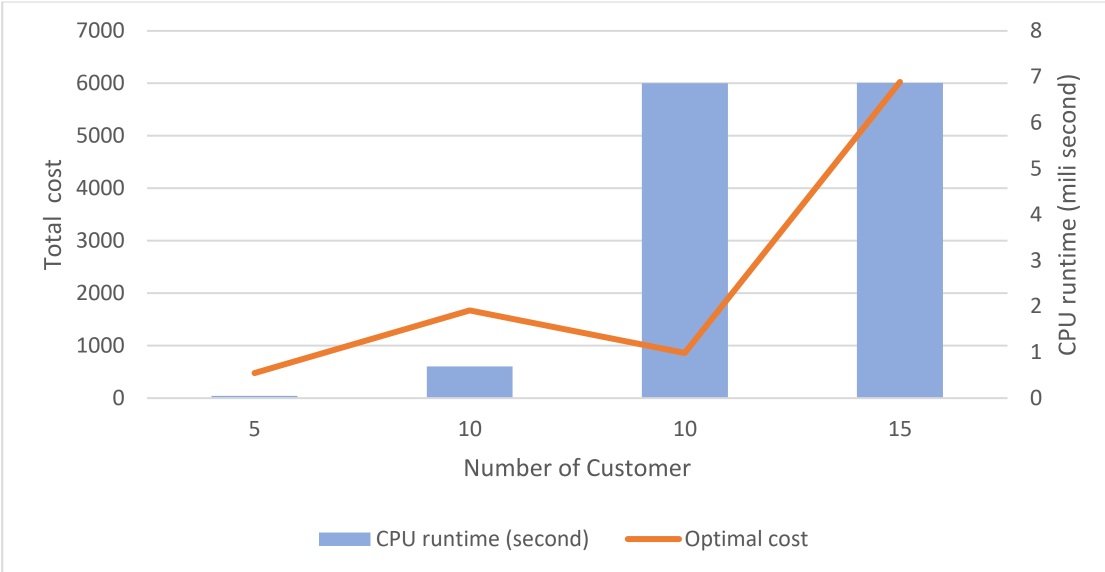

Figure 8: Comparison between retailer nodes with CPU runtime and computational solution

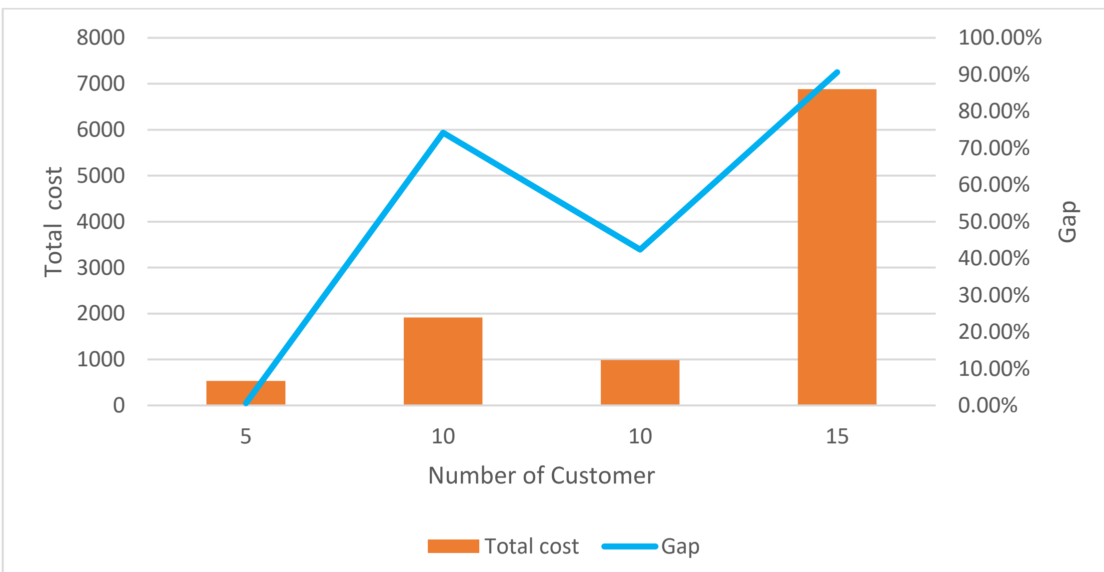

Figure 9: Comparison between retailer nodes with gaps and total cost

Capacity increased	0%	20%	40%	60%
Optimal cost	536.621	460.301	432.604	393.881
Solution time (second)	47	8	29	50
Vehicle dispatched	2	2	2	2

Figure 10: Sensitivity analysis for capacity of each vehicle

2.2.3 Logic concept of random generate algorithm

As mentioned, the exact algorithm by CPLEX is not efficient when dealing with larger set of data. The following is the proposed generator algorithm logical concept is innovative random. It is developed to generate the initial solutions; the steps to implement this method are as the following:

- **Step 1:** Choose a vehicle with the least dispatching cost.
- **Step 2:** Choose nodes according to the time window, priority (eligible) and the least distance to the depot among the remaining demand nodes, choose one on random and then go to **step 3**.
- **Step 3:** If there are still any remaining demand nodes eligible to be services and added to the vehicle's trip, go to step 4, otherwise, go to **step 5**.
- **Step 4:** The capacity of the vehicle is key, which make allowance for them to be chosen, select one of them randomly and then go to **step 3**. If there are no eligible demand nodes, go to **step 5** (the last node is considered eligible to be served only if the vehicle has capacity to serve it and then move back to the depot).
- **Step 5:** Go to the depot and then go to **step 6**.

- **Step 6:** If all demand nodes are satisfied, go to **step 7**, otherwise, update vehicle's capacity constraint. Generate another a trip from the depot to a demand node, go to **step 3**, otherwise, choose the next vehicle and go to **step 2**.
- **Step 7:** Stop the algorithm.

CHAPTER 3 CONCLUSION

3.1 Conclusion

This thesis tries to capture multi-trip, split-delivery, soft-time window vehicle routing problem with heterogenous fleet and under stochastic travel time as it focusses on dealing with frozen chain distribution problem in real life. A mixed-integer linear programming model have been proposed to solve the problem. This study suggests SA as a meta heuristics method for solving the problem in further study. A concepts of random generation algorithm has also been proposed. The mathematical model had been proved that worked and generate a promising solution region when dealing with such complicated combination of constraints and variables. However, the evaluation result showed that the model works best for small scale problem, which is the case with 5 retailers. When dealing with greater number retailers, although the lower bound is tight and close to the best solution, the slope is almost parallel to the best integer nodes. Sensitivity analysis for retailer nodes was studied according to the proposed approach, where it was found that the computation time and the optimal total cost are greatly depends on the number of retailers nodes scale expansion.

3.2 Discussion

The following tries to discuss three aspects can be affected by the application of the study.

3.2.1 Economics

With the foundation, the future of frozen product delivery could be promising and improve to higher levels. While the demand keeps increasing, the integrally development can always go with distortion. Therefore, this study as the basis for further study concerning the frozen products distribution.

3.2.2 Social

The consumer's consumption on frozen product is increasing, an improvement in logistics help create smooth and optimal distribution network. This help satisfied the demand of the consumer and create stability for food industry.

3.2.3 Environment

As the study tries to capture real-life problem in distribution, the goal is to generate the shortest path and minimalize the vehicle's consumption of fuel. The reduction in fuel consumption can help protect the environment.

3.3 Future scope

The study can be used as foundation and open several direction for future research. Firstly, a search algorithm or meta heuristics can be applied to solve this problem. Moreover, the development of metaheuristic integrated with the model to improve the performance and computational time would be accommodating. Also, the investigate on the optimized constraint satisfaction technique among the available method in literature is worth a study. Last but not least, the study on improving the constraints, or testing different constraints in different environment should also bring about valuable contribution.

REFERENCES

[1] J. H. Dantzig, G. B., and Ramser, “The truck dispatching problem,” *Manage. Sci.*, pp. 80–91., 1959.

[2] Y. Zhang and X. D. Chen, “An optimization model for the vehicle routing problem in multiproduct frozen food delivery,” *J. Appl. Res. Technol.*, vol. 12, no. 2, pp. 239–250, 2014, doi: 10.1016/S1665-6423(14)72340-5.

[3] V. Baradaran, A. Shafaei, and A. H. Hosseinian, “Stochastic vehicle routing problem with heterogeneous vehicles and multiple prioritized time windows: Mathematical modeling and solution approach,” *Comput. Ind. Eng.*, vol. 131, pp. 187–199, 2019, doi: 10.1016/j.cie.2019.03.047.

[4] H. Liu, Z. Guo, and Z. Zhang, “A hybrid multi-level optimisation framework for integrated production scheduling and vehicle routing with flexible departure time,” *Int. J. Prod. Res.*, vol. 0, no. 0, pp. 1–18, 2020, doi: 10.1080/00207543.2020.1821927.

[5] G. Laporte, “Fifty years of vehicle routing,” *Transp. Sci.*, vol. 43, no. 4, pp. 408–416, 2009, doi: 10.1287/trsc.1090.0301.

[6] B. Eksioglu, A. V. Vural, and A. Reisman, “The vehicle routing problem: A taxonomic review,” *Comput. Ind. Eng.*, vol. 57, no. 4, pp. 1472–1483, 2009, doi: 10.1016/j.cie.2009.05.009.

[7] E. Berhan, B. Beshah, D. Kitaw, and A. Abraham, “Stochastic Vehicle Routing Problem: A Literature Survey,” *J. Inf. Knowl. Manag.*, vol. 13, no. 3, 2014, doi: 10.1142/S0219649214500221.

[8] B. Golden, S. Raghavan, and E. Wasil, "The vehicle routing problem: Latest advances and new challenges," *Oper. Res. Comput. Sci. Interfaces Ser.*, vol. 43, 2008, doi: 10.1007/978-0-387-77778-8.

[9] C. H. Christiansen and J. Lysgaard, "A branch-and-price algorithm for the capacitated vehicle routing problem with stochastic demands," *Oper. Res. Lett.*, vol. 35, no. 6, pp. 773–781, 2007, doi: 10.1016/j.orl.2006.12.009.

[10] A. A. Juan, J. Faulin, R. Ruiz, B. Barrios, and S. Caballé, "The SR-GCWS hybrid algorithm for solving the capacitated vehicle routing problem," *Appl. Soft Comput. J.*, vol. 10, no. 1, pp. 215–224, 2010, doi: 10.1016/j.asoc.2009.07.003.

[11] C. Low, R. K. Li, and C. M. Chang, "Integrated scheduling of production and delivery with time windows," *Int. J. Prod. Res.*, vol. 51, no. 3, pp. 897–909, 2013, doi: 10.1080/00207543.2012.677071.

[12] E. Taillard, P. Badeau, and M. Gendreau, "A tabu search heuristic for the VRP with soft TW," vol. 31, no. 2. pp. 170–186, 1997.

[13] Z. Qinghua, L. Yao, C. Guoquan, W. Zhuan, H. Haiqin, and L. Kui, "Improved genetic algorithm for variable fleet vehicle routing problem with soft time window," *IEEE Int. Conf. Ind. Informatics*, pp. 233–238, 2008, doi: 10.1109/INDIN.2008.4618100.

[14] J. C. S. BrandãO and A. Mercer, "The multi-trip vehicle routing problem," *J. Oper. Res. Soc.*, vol. 49, no. 8, pp. 799–805, 1998, doi: 10.1057/palgrave.jors.2600595.

[15] F. Alonso, M. J. Alvarez, and J. E. Beasley, "A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility

restrictions," *J. Oper. Res. Soc.*, vol. 59, no. 7, pp. 963–976, 2008, doi: 10.1057/palgrave.jors.2602405.

[16] N. Azi, M. Gendreau, and J. Y. Potvin, "An exact algorithm for a single-vehicle routing problem with time windows and multiple routes," *Eur. J. Oper. Res.,* vol. 178, no. 3, pp. 755–766, 2007, doi: 10.1016/j.ejor.2006.02.019.

[17] A. Hoff, H. Andersson, M. Christiansen, G. Hasle, and A. Løkketangen, "Industrial aspects and literature survey: Fleet composition and routing," *Comput. Oper. Res.*, vol. 37, no. 12, pp. 2041–2061, 2010, doi: 10.1016/j.cor.2010.03.015.

[18] J. Jiang, K. M. Ng, K. L. Poh, and K. M. Teo, "Vehicle routing problem with a heterogeneous fleet and time windows," *Expert Syst. Appl.*, vol. 41, no. 8, pp. 3748–3760, 2014, doi: 10.1016/j.eswa.2013.11.029.

[19] Ç. Koç, T. Bektaş, O. Jabali, and G. Laporte, "A hybrid evolutionary algorithm for heterogeneous fleet vehicle routing problems with time windows," *Computers and Operations Research*, vol. 64. pp. 11–27, 2015, doi: 10.1016/j.cor.2015.05.004.

[20] C. Si, B. Golden, and E. Wasil, "The split delivery vehicle routing problem: applications, algorithms, test problems, and computational results," *Networks*, vol. 49, no. 4, pp. 318–329, 2007, doi: 10.1002/net.20181.

[21] C. Archetti, N. Bianchessi, and M. G. Speranza, "A column generation approach for the split delivery vehicle routing problem," *Networks*, vol. 58, no. 4, pp. 241–254, 2011, doi: 10.1002/net.20467.

[22] F. A. Tillman and R. W. Hering, "A study of a look-ahead procedure for solving the multiterminal delivery problem," *Transp. Res.*, vol. 5, no. 3, pp. 225–229, 1971,

doi: 10.1016/0041-1647(71)90023-2.

[23] V. Pillac, C. Guéret, and A. L. Medaglia, “A parallel matheuristic for the technician routing and scheduling problem,” *Optim. Lett.*, vol. 7, no. 7, pp. 1525–1535, 2013, doi: 10.1007/s11590-012-0567-4.

[24] M. Gendreau, O. Jabali, and W. Rei, “50th Anniversary invited article future research directions in stochastic vehicle routing,” *Transp. Sci.*, vol. 50, no. 4, pp. 1163–1173, 2016, doi: 10.1287/trsc.2016.0709.

[25] A. A. Juan, J. Faulin, J. Jorba, J. Caceres, and J. M. Marquès, “Using parallel & distributed computing for real-time solving of vehicle routing problems with stochastic demands,” *Ann. Oper. Res.*, vol. 207, no. 1, pp. 43–65, 2013, doi: 10.1007/s10479-011-0918-z.

[26] J. C. Goodson, “A priori policy evaluation and cyclic-order-based simulated annealing for the multi-compartment vehicle routing problem with stochastic demands,” *Eur. J. Oper. Res.*, vol. 241, no. 2, pp. 361–369, 2015, doi: 10.1016/j.ejor.2014.09.031.

[27] D. Taş, N. Dellaert, T. Van Woensel, and T. De Kok, “Vehicle routing problem with stochastic travel times including soft time windows and service costs,” *Comput. Oper. Res.*, vol. 40, no. 1, pp. 214–224, 2013, doi: 10.1016/j.cor.2012.06.008.

[28] J. Han, C. Lee, and S. Park, “A robust scenario approach for the vehicle routing problem with uncertain travel times,” *Transp. Sci.*, vol. 48, no. 3, pp. 373–390, 2014, doi: 10.1287/trsc.2013.0476.

[29] H. Lei, G. Laporte, and B. Guo, "Districting for routing with stochastic customers," *EURO J. Transp. Logist.*, vol. 1, no. 1–2, pp. 67–85, 2012, doi: 10.1007/s13676-012-0005-x.

[30] R. W. Bent and P. Van Hentenryck, "Scenario-based planning for partially dynamic vehicle routing with stochastic customers," *Oper. Res.*, vol. 52, no. 6, pp. 977–987, 2004, doi: 10.1287/opre.1040.0124.

[31] S. Yan, J. C. Chu, F. Y. Hsiao, and H. J. Huang, "A planning model and solution algorithm for multi-trip split-delivery vehicle routing and scheduling problems with time windows," *Comput. Ind. Eng.*, vol. 87, pp. 383–393, 2015, doi: 10.1016/j.cie.2015.05.034.

[32] A. S. Kenyon and D. P. Morton, "Stochastic vehicle routing with random travel times," *Transp. Sci.*, vol. 37, no. 1, pp. 69–82, 2003, doi: 10.1287/trsc.37.1.69.12820.

[33] J. Oyola, H. Arntzen, and D. L. Woodruff, "The stochastic vehicle routing problem, a literature review, Part II: solution methods," *EURO J. Transp. Logist.*, vol. 6, no. 4, pp. 349–388, 2017, doi: 10.1007/s13676-016-0099-7.

[34] G. A. Bula, F. A. Gonzalez, C. Prodhon, H. M. Afsar, and N. M. Velasco, "Mixed Integer Linear Programming Model for Vehicle Routing Problem for Hazardous Materials Transportation," *IFAC-PapersOnLine*, vol. 49, no. 12, pp. 538–543, 2016, doi: 10.1016/j.ifacol.2016.07.691.

[35] J. Sniezek and L. Bodin, "Using mixed integer programming for solving the capacitated arc routing problem with vehicle/site dependencies with an application

to the routing of residential sanitation collection vehicles," *Ann. Oper. Res.*, vol. 144, no. 1, pp. 33–58, 2006, doi: 10.1007/s10479-006-0006-y.

[36] S. Madankumar and C. Rajendran, "A mixed integer linear programming model for the vehicle routing problem with simultaneous delivery and pickup by heterogeneous vehicles, and constrained by time windows," *Sadhana - Acad. Proc. Eng. Sci.*, vol. 44, no. 2, 2019, doi: 10.1007/s12046-018-1048-y.

[37] M. L. Mutar, B. M. Aboobaider, and A. S. Hameed, "Review paper in vehicle routing problem and future research trend," *Int. J. Appl. Eng. Res.*, vol. 12, no. 22, pp. 12279–12283, 2017.

[38] J. C. Chu, S. Yan, and H. J. Huang, "A Multi-Trip Split-Delivery Vehicle Routing Problem with Time Windows for Inventory Replenishment Under Stochastic Travel Times," *Networks Spat. Econ.*, vol. 17, no. 1, pp. 41–68, 2017, doi: 10.1007/s11067-015-9317-3.

[39] E. Babaee Tirkolaee, P. Abbasian, M. Soltani, and S. A. Ghaffarian, "Developing an applied algorithm for multi-trip vehicle routing problem with time windows in urban waste collection: A case study," *Waste Manag. Res.*, vol. 37, no. 1_suppl, pp. 4–13, 2019, doi: 10.1177/0734242X18807001.

APENIDIX

Code in ILOG CPLEX

```
int NumberOfVehicle=4;
int NumberOfRetailer=5;
int NumberOfTrip=NumberOfRetailer;
range Trip=1..NumberOfTrip;
range Vehicle=1..NumberOfVehicle;
range RetailerD=0..NumberOfRetailer;
range RetailerOnly=1..NumberOfRetailer;
range RetailerExtra=0..NumberOfRetailer+1; //Depot + out
range RetailerOut=1..NumberOfRetailer+1;


float nb1[0..NumberOfRetailer+1,
1..NumberOfVehicle*(NumberOfRetailer+2)] = ...;
float Vcost[p in Vehicle, m in RetailerExtra ,s in RetailerExtra] =
nb1[m,s+1+(NumberOfRetailer+2)*(p-1)];
float nb2[0..NumberOfRetailer+1,
1..NumberOfVehicle*(NumberOfRetailer+2)] = ...;
float Pcost[p in Vehicle, m in RetailerExtra ,s in RetailerExtra] =
nb2[m,s+1+(NumberOfRetailer+2)*(p-1)];
float nb3[0..NumberOfRetailer+1,
1..NumberOfVehicle*(NumberOfRetailer+2)]=...;
float TravelTime[p in Vehicle, m in RetailerExtra ,s in RetailerExtra] =
nb3[m,s+1+(NumberOfRetailer+2)*(p-1)];
float nb4[0..NumberOfRetailer+1,
1..NumberOfVehicle*(NumberOfRetailer+2)]=...;
float Scost[p in Vehicle, m in RetailerExtra ,s in RetailerExtra] =
nb4[m,s+1+(NumberOfRetailer+2)*(p-1)];
{int} Depot={0};
{int} TripSt={1};

int M=1000000;

//Maxium running time setting

 execute Setting {
 cplex.epgap=0.1;
 cplex.tilim =10*60;
}
// parameters
int E[RetailerOnly]=...;
int L[RetailerOnly]=...;
float S[RetailerExtra]=...;// time of service of customer i by vehicle k
int bcost[RetailerOnly]=...; //unit cost of unsastisfied demand for dth
retailer
int Demand[RetailerExtra]=...; // Total demand for dth retailer;
int Q[Vehicle]=...;
int VehicleCost[Vehicle]=...;
float
CostEarly[Vehicle][RetailerOnly]=[[0.5,0.5,0.5,0.5,0.5],[0.3,0.3,0.3,0.3
,0.3],[0.2,0.2,0.2,0.2,0.2],[0.2,0.2,0.2,0.2,0.2]];
float CostLate[RetailerOnly]=...;
```

```
//decision variable
dvar boolean z[Vehicle];//if vehicle used
dvar boolean v[Vehicle][Trip][RetailerExtra][RetailerExtra];
dvar int+ x[Vehicle][Trip][RetailerExtra][RetailerExtra];
dvar int+ u[RetailerExtra];// ;flow of unsatisfied demand
dvar int+ a[Vehicle][Trip][RetailerExtra];// arrival time;
dvar int+ p[Vehicle][Trip][RetailerExtra];// departure time;
dvar int+ gLate[Vehicle][RetailerExtra];//Tardiness
dvar int+ gEarly[Vehicle][RetailerExtra];//Earliness
dvar int+ pDepot[Vehicle][TripSt][Depot];

dexpr float Totalcost =
sum(t in Trip, m in Vehicle, i in RetailerExtra, j in
RetailerExtra)(Vcost[m][i][j]+Pcost[m][i][j]+Scost[m][i][j])*v[m][t][i][
j]
+sum(d in RetailerOnly)bcost[d]*u[d]
+sum(m in Vehicle)z[m]*VehicleCost[m]
+sum(m in Vehicle, i in RetailerOnly)gEarly[m][i]*CostEarly[m][i]
+sum(m in Vehicle, i in RetailerOnly )gLate[m][i]*CostLate[i];
minimize Totalcost;

subject to
{
//Multi-trip
forall(t in Trip, m in Vehicle)sum(i in RetailerOut)v[m][t][0][i]==1;
forall(t in Trip, m in Vehicle)sum(i in
RetailerD)v[m][t][i][NumberOfRetailer+1]==1;
forall(j in RetailerOnly)sum(m in Vehicle, t in Trip, i in
RetailerD)v[m][t][i][j]>=0;//Retailer are not necessary all serviced
forall(t in Trip, m in Vehicle,i in RetailerOnly)sum(j in
RetailerOut)v[m][t][i][j]-sum(k in RetailerD)v[m][t][k][i]==0;
forall(t in Trip, m in Vehicle,k in RetailerOnly, i in RetailerOnly,j in
RetailerOnly)v[m][t][k][i]-(1-v[m][t][i][j])*M<=0;// subtour
forall(t in Trip, m in Vehicle)sum( i in
RetailerExtra)v[m][t][NumberOfRetailer+1][i]==0;// no going back from
sink node
forall(m in Vehicle, t in Trip, j in RetailerOnly)sum(l in
t..NumberOfTrip,i in RetailerD)(v[m][l][i][j])+(1-
v[m][t][0][NumberOfRetailer+1])*M>=0;// not necessary use all trip
forall(m in Vehicle, t in Trip, j in RetailerOnly)sum(l in
t..NumberOfTrip,i in RetailerD)(v[m][l][i][j])-(1-
v[m][t][0][NumberOfRetailer+1])*M<=0;
forall(t in Trip, m in Vehicle,i in RetailerExtra, j in RetailerExtra:
j==i)v[m][t][i][j]==0;
//Time window
forall(m in Vehicle,t in 1..NumberOfTrip-1, i in RetailerD)a[m][t+1][0]-
a[m][t][NumberOfRetailer+1]>=TravelTime[m][NumberOfRetailer+1][0];//Time
start from next trip must equal or greater than the previous
forall(m in Vehicle,t in Trip)a[m][t][NumberOfRetailer+1]-
a[m][t][0]>=TravelTime[m][0][NumberOfRetailer+1];
forall(m in Vehicle,t in
Trip)v[m][t][0][NumberOfRetailer+1]==1=>a[m][t][NumberOfRetailer+1]==a[m
][t][0];
forall(m in Vehicle, j in RetailerOut)a[m][1][j]-pDepot[m][1][0]-(1-
v[m][1][0][j])*M<=TravelTime[m][0][j];
```

```
forall(m in Vehicle, t in Trip, i in RetailerD, j in
RetailerOut)a[m][t][j]-a[m][t][i]+(1-v[m][t][i][j])*M
>=S[j]+TravelTime[m][i][j];
forall(m in Vehicle, t in Trip, j in RetailerOnly)sum(i in
RetailerD)v[m][t][i][j]==0 =>a[m][t][j]==0;
forall(m in Vehicle,j in RetailerOnly)gEarly[m][j]>=sum(t in Trip,i in
RetailerD)v[m][t][i][j]*E[j]-sum(t in Trip)a[m][t][j];
forall(m in Vehicle,j in RetailerOnly)gLate[m][j]>=sum(t in
Trip)a[m][t][j]-L[j] ;
forall(m in Vehicle,t in Trip)v[m][t][0][6]==1=>a[m][t][6]==a[m][t][0];
//Load
forall( m in Vehicle, t in Trip,i in RetailerExtra, j in
RetailerExtra)v[m][t][i][j]==0 => x[m][t][i][j]==0;
forall(m in Vehicle, t in Trip)sum(i in RetailerD, j in
RetailerOnly)x[m][t][i][j]<=Q[m];
forall(d in RetailerOnly) sum(m in Vehicle,t in Trip, i in
RetailerExtra)x[m][t][i][d]+u[d]==Demand[d];
forall(m in Vehicle, j in RetailerOnly)z[m]+(1-sum(t in Trip,i in
RetailerD)v[m][t][i][j])*M>=1;// use of vehicle

}
//Display result
execute Output1 {
writeln(cplex.getObjValue());

   writeln("-----------------------");
   writeln("Route:       ");
for (var m in Vehicle)
      {
      writeln("Vehicle ",m);
      for (var t in Trip)
      for (var j in RetailerExtra)
            if ( v[m][t][0][j]==1)
            {     if(j==NumberOfRetailer+1){
                  write("Trip ",t," :","stay at Depot"," ");
                  }
                  else
                        write("Trip ",t," :",j," (at period
",a[m][t][j],") ");

                  var l=j;
            for (var k in RetailerExtra)
                  {
                  if ( v[m][t][l][k]==1)
                        {if(k==NumberOfRetailer+1){
                        write("Depot"," (at period ",a[m][t][k],") ");

                        }
                        else
                              write(k," (at period ",a[m][t][k],") ");
                              l=k;
                         }

                  }
                               writeln("   ");
            }
      }
```

```
}
execute Output2{
      writeln("----------------------");
      write("Vehicle used: ");
for (var m in Vehicle)
if (z[m]==1)
 write(m," ");

}
```