



Univerzitet u Beogradu - Elektrotehnički fakultet
Katedra za Signale i sisteme



13E053DOS - Digitalna obrada signala

Domaći zadatak

STUDENT

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PARAMETRI

$P = 3 \quad Q = 3 \quad R = 1 \quad S = 1$

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Zadatak 1 • Parametar $P = 3$

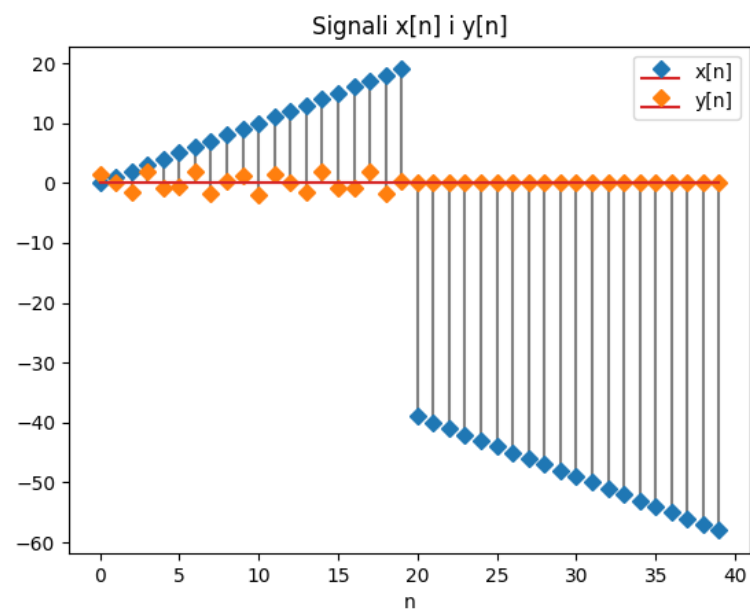
- Analitički oblik signala $x[n]$ i $y[n]$ i broj tačaka N

$$x[n] = \begin{cases} n & 0 \leq n \leq 19 \\ n - 19 & \text{inace} \end{cases}$$

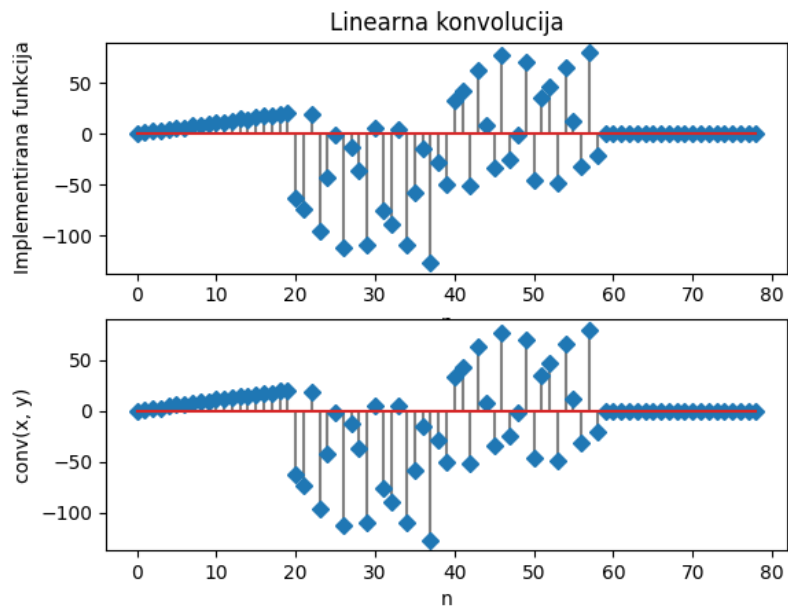
$$y[n] = \begin{cases} 2 \cos(4n + \frac{\pi}{4}) & 0 \leq n \leq 19 \\ 0 & \text{inace} \end{cases}$$

$$N = 40$$

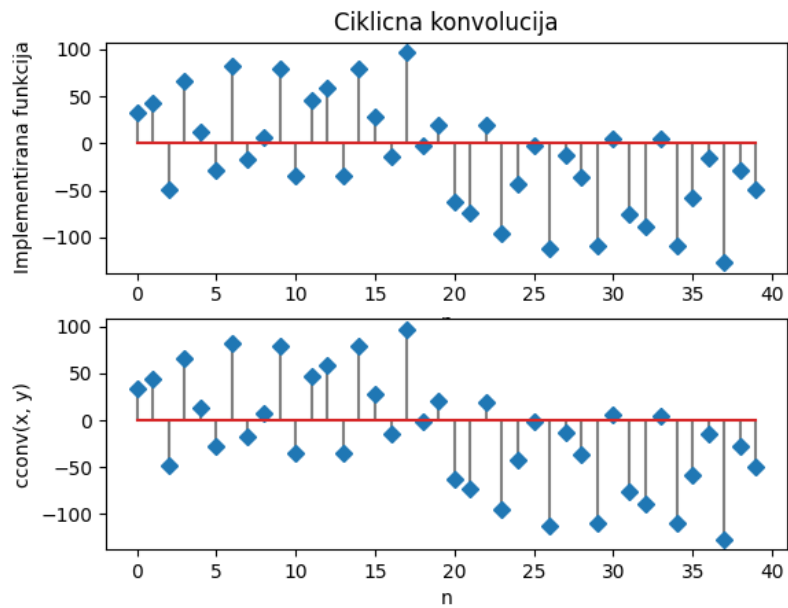
- Prikaz signala $x[n]$ i $y[n]$



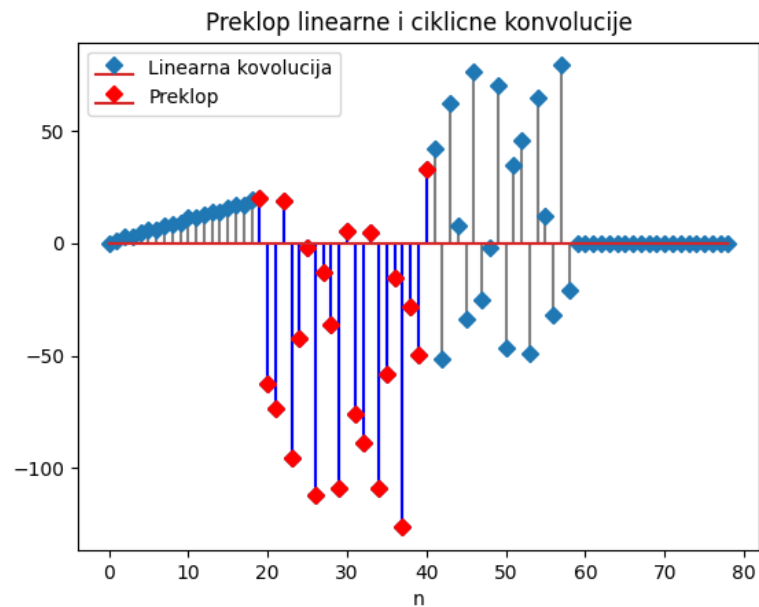
- Prikaz rezultata linearne konvolucije i rezultata funkcije conv



- *Prikaz rezultata ciklične konvolucije i rezultata funkcije cconv*



- *Linearna i ciklična konvolucija imaju iste vrednosti u sledećim odbircima*



- Programski kôd

```

1 import numpy as np
2 from matplotlib import pyplot as plt
3
4 # Postavke zadatka
5 P = 3
6 N = 10 * (P + 1)
7
8 def lin_conv(x, y):
9     """
10     Returns linear convolution of two discrete signals
11     """
12
13     N = len(x)
14     x = np.concatenate((x, np.zeros(N)))
15     y = np.concatenate((y, np.zeros(N)))
16
17     lin = np.zeros(2 * N - 1)
18     for n in range(2 * N - 1):
19         lin[n] = sum([x[k] * y[n - k] for k in range(n + 1)])
20     return lin
21
22 def circ_conv(x, y):
23     """
24     Returns circular convolution of two discrete signals
25     """
26
27     N = len(x)
28     circ = np.zeros(N)
29     for n in range(N):
30         circ[n] = sum([x[k] * y[(n - k) % N] for k in range(N)])
31     return circ
32
33 def cconv(x, y):
34     """ Imitation of MATLAB's cconv function """
35     return np.real(np.fft.ifft( np.fft.fft(x)*np.fft.fft(y) ))
36
37 if __name__ == "__main__":
38
39     # Definicija x[n] signala

```

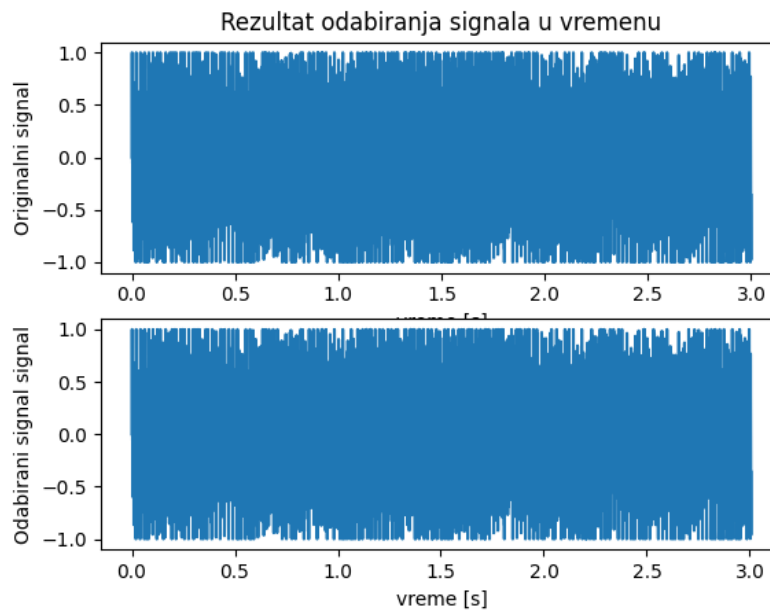
```

40 x = np.arange(N)
41 x[N//2:] = 1 - x[N//2:] - N/2
42
43 # Definicija y[n] signala
44 y = np.arange(N, dtype=float)
45 y[:N//2] = 2.0 * np.cos((P + 1)*y[:N//2] + np.pi/4)
46 y[N//2:] = 0
47
48 # Plot ulaznih signala
49 plt.stem(x, linefmt='gray', markerfmt='D', label='x[n]')
50 plt.stem(y, linefmt='gray', markerfmt='D', label='y[n]')
51 plt.xlabel('n')
52 plt.title('Signali x[n] i y[n]')
53 plt.legend()
54 plt.savefig('figures/zad1_signali.png')
55
56 # Linearna konvolucija
57 lin = lin_conv(x, y)
58
59 # Plot linearne konvolucije
60 plt.figure()
61 plt.subplot(2, 1, 1)
62 plt.title('Linearna konvolucija')
63 plt.stem(lin, linefmt='gray', markerfmt='D')
64 plt.xlabel('n')
65 plt.ylabel('Implementirana funkcija')
66
67 plt.subplot(2, 1, 2)
68 plt.stem(np.convolve(x, y), linefmt='gray', markerfmt='D')
69 plt.xlabel('n')
70 plt.ylabel('conv(x, y)')
71 plt.savefig('figures/zad1_linearna_konvolucija.png')
72
73 # Ciklična konvolucija
74 circ = circ_conv(x, y)
75
76 # Ciklična konvolucija
77 plt.figure()
78 plt.subplot(2, 1, 1)
79 plt.title('Ciklična konvolucija')
80 plt.stem(circ, linefmt='gray', markerfmt='D')
81 plt.xlabel('n')
82 plt.ylabel('Implementirana funkcija')
83
84 plt.subplot(2, 1, 2)
85 plt.stem(cconv(x, y), linefmt='gray', markerfmt='D')
86 plt.xlabel('n')
87 plt.ylabel('cconv(x, y)')
88 plt.savefig('figures/zad1_ciklicna_konvolucija_provera.png')
89
90 # Overlap dve funkcije
91 overlap = lin * np.isin(lin, circ)
92 overlap[overlap == 0] = None
93
94 plt.figure()
95 plt.title('Preklap linearne i ciklicne konvolucije')
96 plt.stem(lin, linefmt='gray', markerfmt='D', label='Linearna konvolucija')
97 plt.stem(overlap, linefmt='blue', markerfmt='rD', label='Preklap')
98 plt.xlabel('n')
99 plt.legend()
100 plt.savefig('figures/zad1_overlap.png')

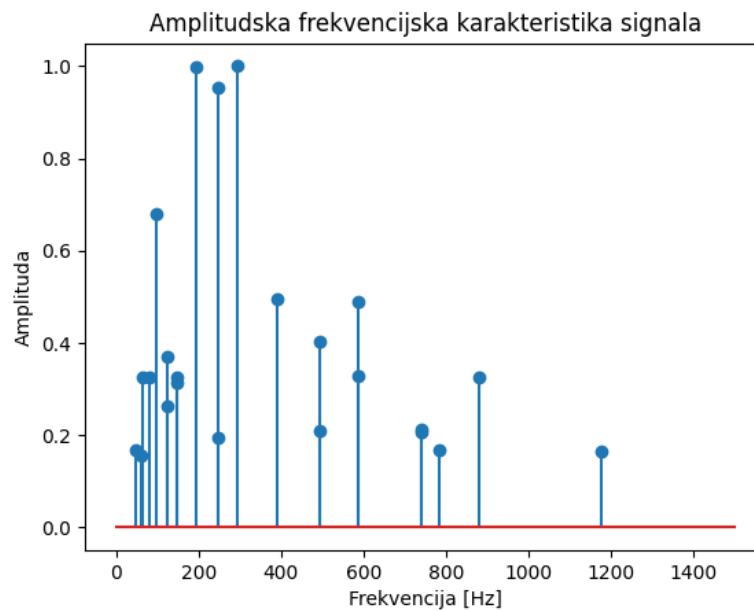
```

Zadatak 2 • Parametar $Q = 3$

- Prikaz originalnog signala i signala dobijenog odabiranjem sa f_s iz tabele



- Amplitudska frekventijska karakteristika originalnog signala



- Signal se sastoji od sledećih tonova

$G3, C4, E4$

- Programski kôd

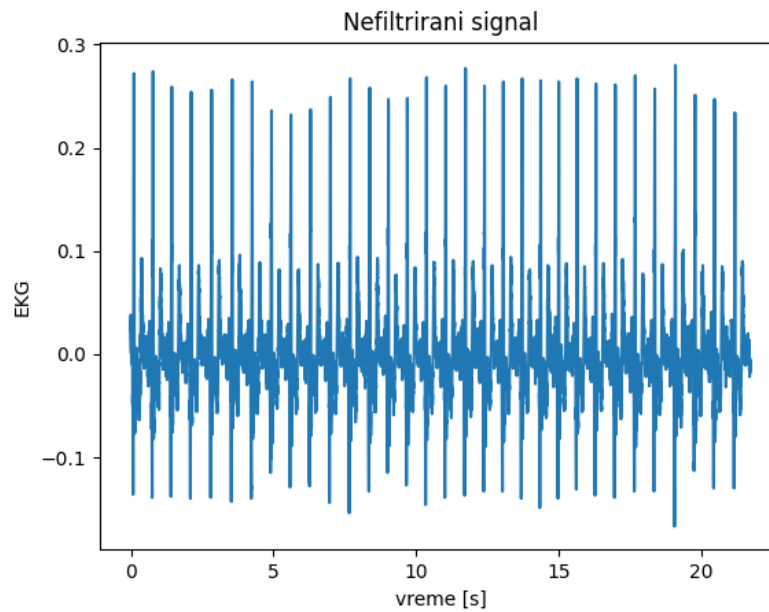
```

1 import numpy as np
2 from matplotlib import pyplot as plt
3 from scipy.io.wavfile import read
4
5 # Postavka zadatka
6 Q = 3
7 fs = 4400
8
9 if __name__ == "__main__":
10     # Ucitavanje originalnog signala
11     fs_original, x = read('sounds/audio' + str(Q) + '.wav')
12     time_original = np.arange(0, len(x) / fs_original, 1 / fs_original)
13
14     # Odabiranje signala zadatom frekvencijom
15     x_sampled = x[::fs_original // fs]
16     time_sampled = np.arange(0, len(x_sampled) / fs, 1 / fs)
17
18     # Prikaz vremenskih oblika odabranog i ne odabranog signala
19     plt.subplot(2, 1, 1)
20     plt.title('Rezultat odabiranja signala u vremenu')
21     plt.xlabel('vreme [s]')
22     plt.ylabel('Originalni signal')
23     plt.plot(time_original, x / max(x))
24     plt.subplot(2, 1, 2)
25     plt.xlabel('vreme [s]')
26     plt.ylabel('Odabrani signal')
27     plt.plot(time_sampled, x_sampled / max(x_sampled))
28     plt.savefig('figures/zad2_odabiranje.png')
29
30     # Furijeova transformacija originalnog signala
31     X = np.fft.fft(x) / len(x)
32     X = X[range(len(x) // 2)]
33     X = abs(X) / max(abs(X))
34
35     # Niz frekvencija za Furijeovu transformaciju
36     freq = np.arange(len(x) // 2 * fs_original / len(x))
37
38     # Poslednji prikaz u bitnom delu spektra
39     last_idx = np.where(freq <= 1500)[0][-1]
40
41     # Uklanjanje manje bitnih delova spektra
42     # radi lepseg plotovanja
43     X[X < 0.15] = None
44
45     # Prikaz spektra signala
46     plt.figure()
47     plt.title('Amplitudska frekvencijska karakteristika signala')
48     plt.xlabel('Frekvencija [Hz]')
49     plt.ylabel('Amplituda')
50     plt.stem(freq[:last_idx], X[:last_idx])
51     plt.savefig('figures/zad2-spektar.png')
52
53     # Trazeni tonovi
54
55     tones = ['D3', 'E3', 'F3', 'G3', 'A3', 'B3', 'C4', 'D4', 'E4', 'F4', 'G4', 'A4', 'B4',
56             'C5', 'D5']
57     freqs = np.array([146.83, 164.81, 174.61, 196.00, 220.00, 220.00, 246.94, 261.63,
58                     293.66, 329.63, 349.23, 392.00, 440.00, 493.88, 523.25, 587.33])
59     main_freq = [freq[idx] for idx, f in enumerate(X) if f > 0.8]
60
61     print([tones[np.argmin(abs(freqs - f))]] for f in main_freq)

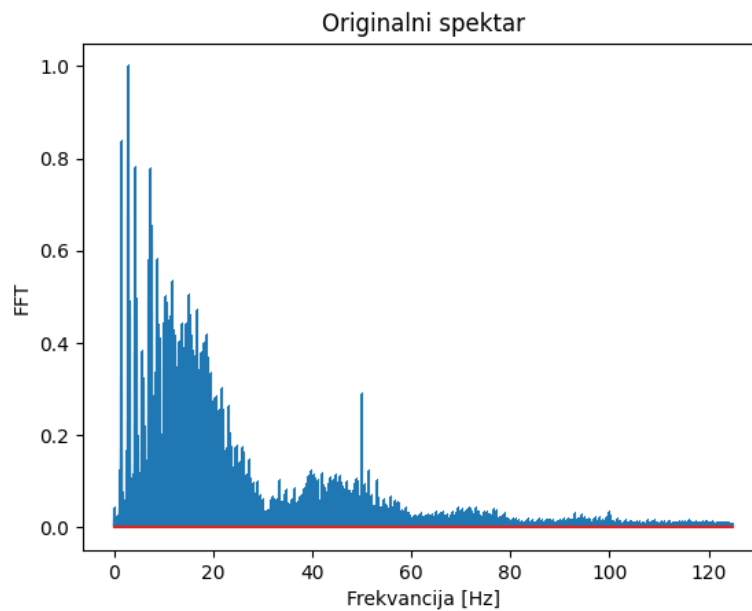
```

Zadatak 3 • Parametar $R = 1$

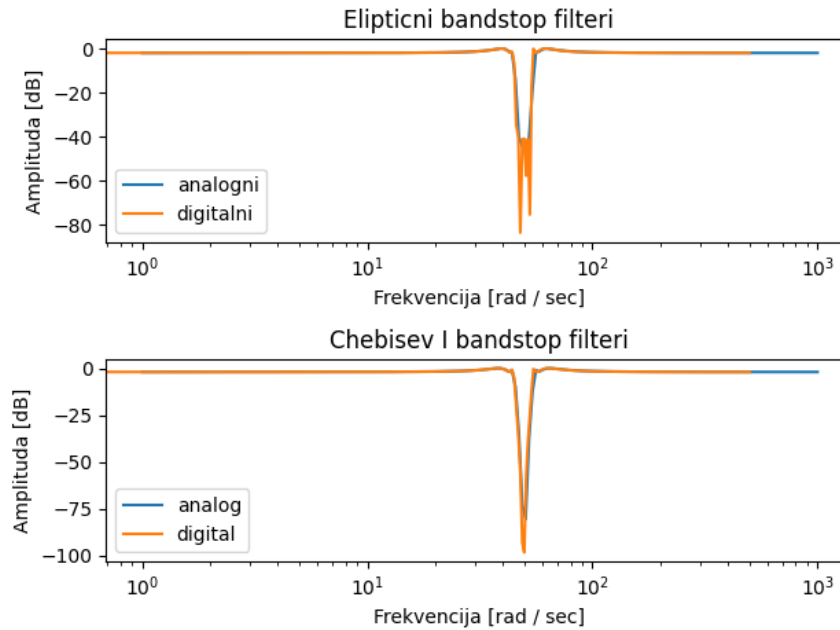
- EKG signal u vremenskom domenu



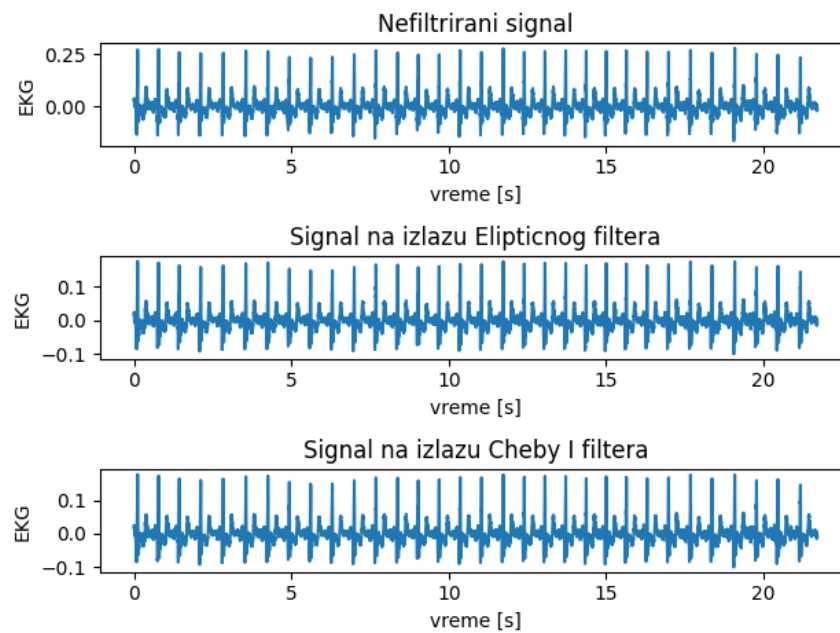
- Amplitudska frekventijska karakteristika EKG signala



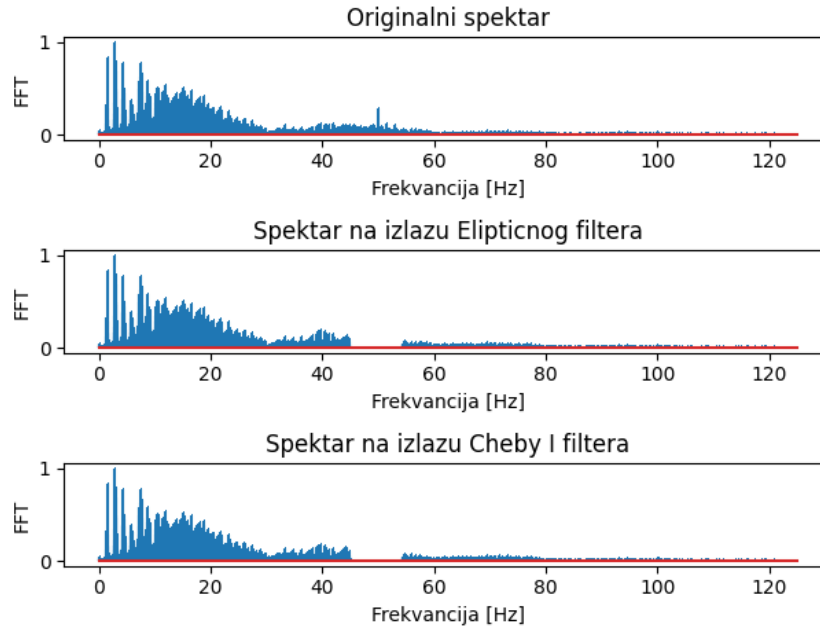
- Amplitudske frekventijske karakteristika analognih i digitalnih filtara



- *Originalni i filtrirani signali u vremenskom domenu*



- *Amplitudske frekvencijske karakteristika originalnog i filtriranih signala*



- Programski kôd

```

1 import numpy as np
2 from scipy import signal
3 from scipy.io import loadmat
4 from matplotlib import pyplot as plt
5
6 P = 3
7 R = 1 # Eliptic i Cheb I
8
9
10 if __name__ == "__main__":
11
12     ## Ucitavanje podataka
13
14     # Vremenski signal
15     data = loadmat('input/ekg' + str(R) + '.mat')
16     x = data['x'].flatten()
17     fs = 2 * data['fs']
18     t = (np.arange(len(x)) / fs).flatten()
19
20     # Spektar
21     X = np.fft.fft(x)
22     X = X[range(len(X) // 8)] / max(X)
23     freq = (np.arange(len(X)) * fs / len(x)).flatten()
24
25     ## Filtriranje
26
27     # Elliptic filter
28     be, ae = signal.ellip(4, 2, 40, [45, 55], btype='bandstop', analog=True)
29     bze, aze = signal.bilinear(be, ae, fs / (2 * np.pi))
30
31     # Filtriranje
32     x_ellip = signal.filtfilt(bze, aze, x)
33
34     # Racunanje spektra
35     X_ellip = np.fft.fft(x_ellip)
36     X_ellip = X_ellip[range(len(X_ellip) // 8)] / max(X_ellip)
37     freq = (np.arange(len(X_ellip)) * fs / len(x_ellip)).flatten()
38
39     # Cheby I filter

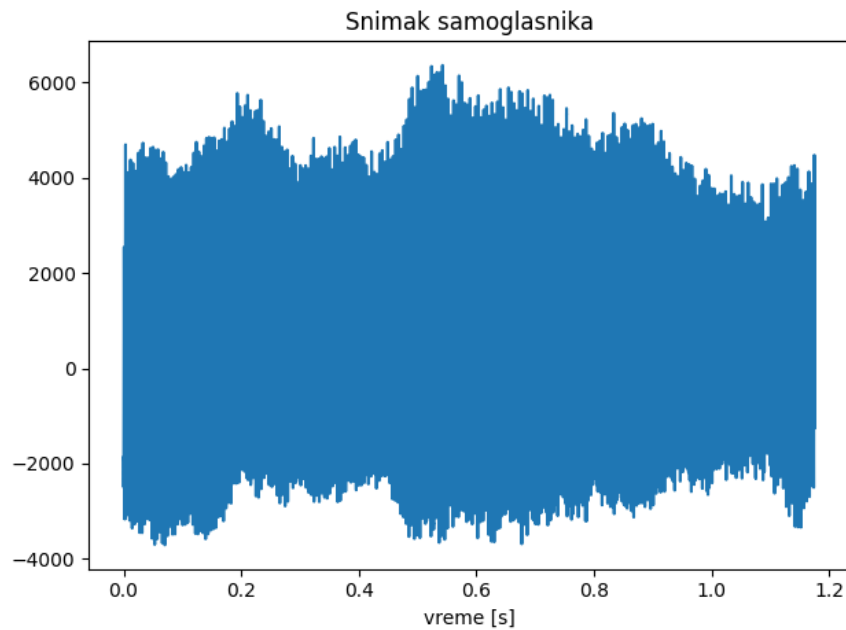
```

```
40 bc, ac = signal.cheby1(4, 2, [45, 55], btype='bandstop', analog=True)
41 bzc, azc = signal.bilinear(bc, ac, fs / (2 * np.pi))
42
43 # Filtriranje
44 x_cheby1 = signal.filtfilt(bzc, azc, x)
45
46 # Racunanje spektra
47 X_cheby1 = np.fft.fft(x_cheby1)
48 X_cheby1 = X_cheby1[range(len(X_cheby1) // 8)] / max(X_cheby1)
49
50 ## Racunanje broja otkucaja u minuti
51
52 # Thresholdovanje signala
53 trashhold = 0.8 * max(x_cheby1)
54 thr = x_cheby1 > trashhold
55
56 # Brojanje pikova
57 cnt = sum([1 for idx, _ in enumerate(thr[1:]) if thr[idx - 1] and not thr[idx]])
58
59 # Rezultat
60 print(f'Broj otkucaja u minuti je {int(cnt * (60 / t[-1]))}')
61
62
63 ## Plotovanje rezultata
64
65 # a) signal vreme
66 plt.xlabel('vreme [s]')
67 plt.ylabel('EKG')
68 plt.title('Nefiltrirani signal')
69 plt.plot(t, x)
70
71 plt.savefig('figures/zad3_signal_vreme.png')
72
73 # b) spektar signal
74 plt.figure()
75 plt.stem(freq, abs(), markerfmt=',')
76 plt.xlabel('Frekvencija [Hz]')
77 plt.ylabel('FFT')
78 plt.title('Originalni spektar')
79
80 plt.savefig('figures/zad3_signal_spektar.png')
81
82 # c) AFK filtera
83 plt.figure()
84
85 plt.subplot(2, 1, 1)
86 w, h = signal.freqs(bc, ac)
87 wz, hz = signal.freqz(bzc, azc)
88 plt.semilogx(w, 20 * np.log10(abs(h)), label='analogni')
89 plt.semilogx((wz * fs / (2 * np.pi)).T, 20 * np.log10(abs(hz)), label='digitalni')
90 plt.legend()
91 plt.title('Elipcticni bandstop filteri')
92 plt.xlabel('Frekvencija [rad / sec]')
93 plt.ylabel('Amplituda [dB]')
94
95 plt.subplot(2, 1, 2)
96 w, h = signal.freqs(bc, ac)
97 wz, hz = signal.freqz(bzc, azc)
98 plt.semilogx(w, 20 * np.log10(abs(h)), label='analog')
99 plt.semilogx((wz * fs / (2 * np.pi)).T, 20 * np.log10(abs(hz)), label='digital')
100 plt.legend()
101 plt.title('Chebisev I bandstop filteri')
102 plt.xlabel('Frekvencija [rad / sec]')
103 plt.ylabel('Amplituda [dB]')
104
105 plt.tight_layout()
106 plt.savefig('figures/zad3_filteri.png')
107
```

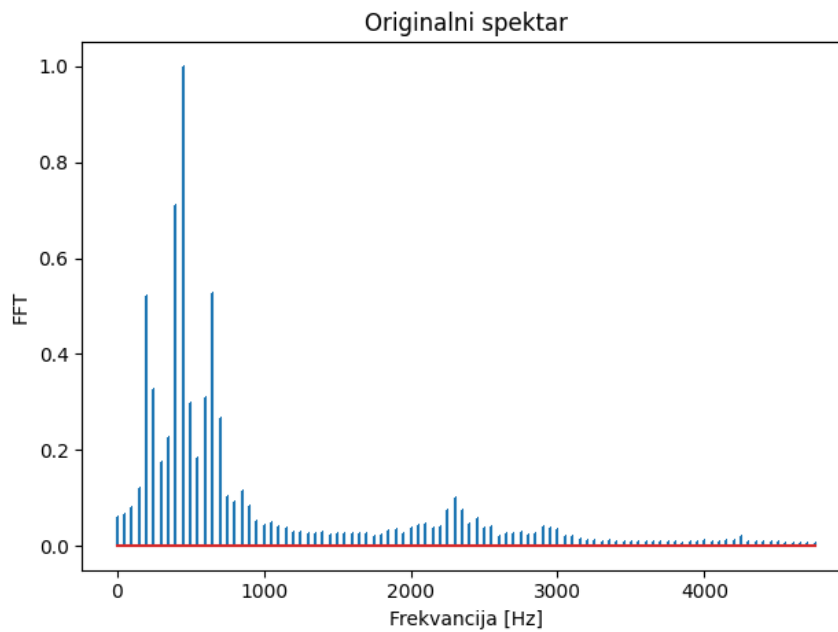
```
108 # d) svi signali u vremenu
109 plt.figure()
110
111 plt.subplot(3, 1, 1)
112 plt.xlabel('vreme [s]')
113 plt.ylabel('EKG')
114 plt.title('Nefiltrirani signal')
115 plt.plot(t, x)
116
117 plt.subplot(3, 1, 2)
118 plt.plot(t, x_ellip)
119 plt.xlabel('vreme [s]')
120 plt.ylabel('EKG')
121 plt.title('Signal na izlazu Elipticnog filtera')
122
123 plt.subplot(3, 1, 3)
124 plt.plot(t, x_cheby1)
125 plt.xlabel('vreme [s]')
126 plt.ylabel('EKG')
127 plt.title('Signal na izlazu Cheby I filtera')
128
129 plt.tight_layout()
130 plt.savefig('figures/zad3_filtrirani_vreme.png')
131
132 # e) AFK svih signala
133 plt.figure()
134 plt.tight_layout()
135
136 plt.subplot(3, 1, 1)
137 plt.stem(freq, abs(X), markerfmt=',')
138 plt.xlabel('Frekvencija [Hz]')
139 plt.ylabel('FFT')
140 plt.title('Originalni spektar')
141
142 plt.subplot(3, 1, 2)
143 plt.stem(freq, abs(X_ellip), markerfmt=',')
144 plt.xlabel('Frekvencija [Hz]')
145 plt.ylabel('FFT')
146 plt.title('Spektar na izlazu Elipticnog filtera')
147
148 plt.subplot(3, 1, 3)
149 plt.stem(freq, abs(X_cheby1), markerfmt=',')
150 plt.xlabel('Frekvencija [Hz]')
151 plt.ylabel('FFT')
152 plt.title('Spektar na izlazu Cheby I filtera')
153
154 plt.tight_layout()
155 plt.savefig('figures/zad3_filtrirani_spektar.png')
```

Zadatak 4 • Parametar $S = 1$

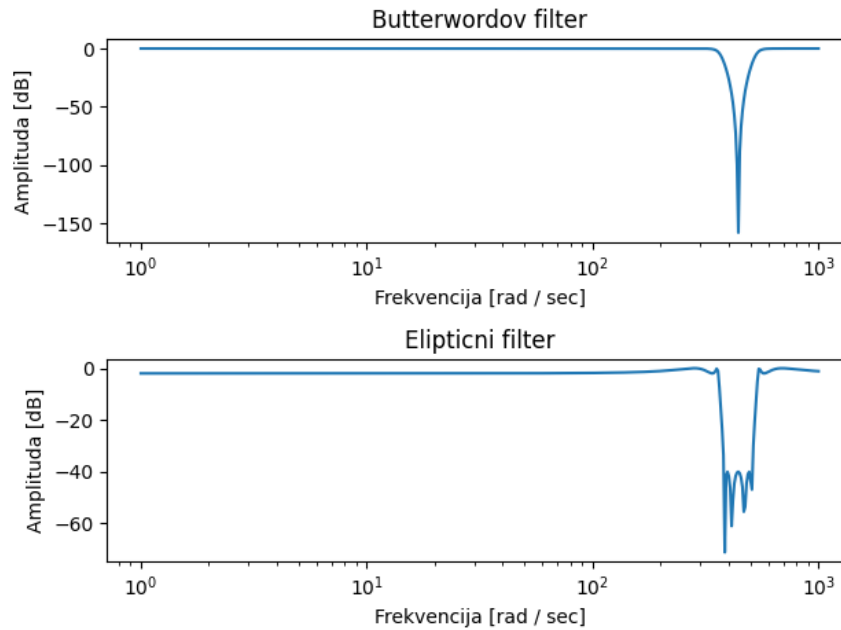
- Snimak samoglasnika u vremenskom domenu



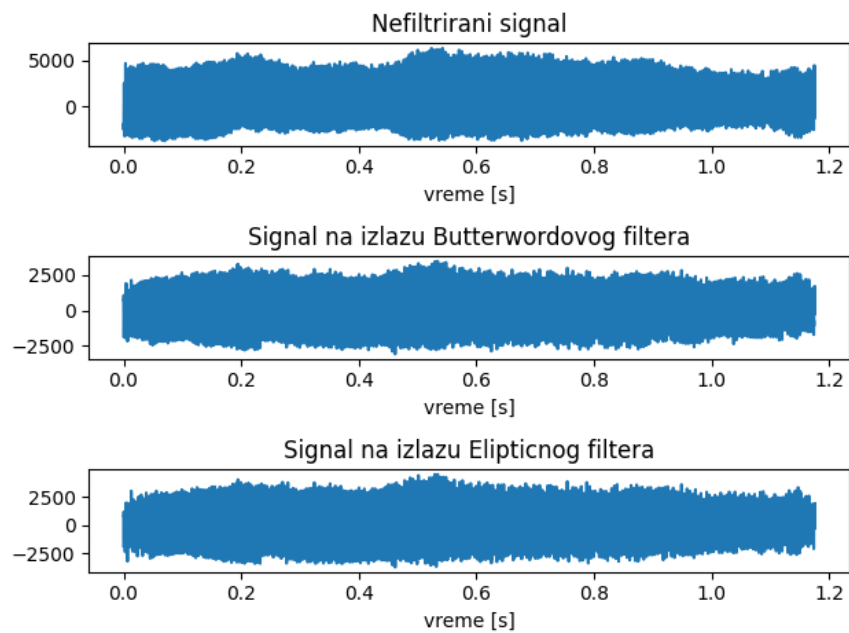
- Usrednjena amplitudska frekvencijska karakteristika snimljenog signala



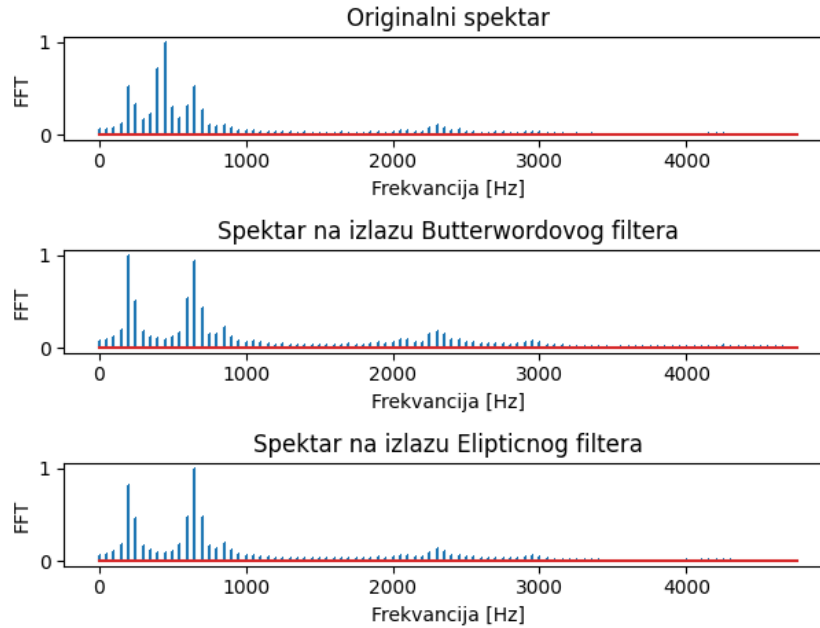
- Amplitudske frekvencijske karakteristike projektovanih filtara



- *Originalni i filtrirani signali u vremenskom domenu*



- *Amplitudske frekvencijske karakteristike originalnog i filtriranih signala*



- Programski kôd

```

1 import numpy as np
2 from scipy import signal
3 from scipy.io import loadmat
4 from matplotlib import pyplot as plt
5
6 P = 3
7 R = 1 # Eliptic i Cheb I
8
9
10 if __name__ == "__main__":
11
12     ## Ucitavanje podataka
13
14     # Vremenski signal
15     data = loadmat('input/ekg' + str(R) + '.mat')
16     x = data['x'].flatten()
17     fs = 2 * data['fs']
18     t = (np.arange(len(x)) / fs).flatten()
19
20     # Spektar
21     X = np.fft.fft(x)
22     X = X[range(len(X) // 8)] / max(X)
23     freq = (np.arange(len(X)) * fs / len(x)).flatten()
24
25     ## Filtriranje
26
27     # Elliptic filter
28     be, ae = signal.ellip(4, 2, 40, [45, 55], btype='bandstop', analog=True)
29     bze, aze = signal.bilinear(be, ae, fs / (2 * np.pi))
30
31     # Filtriranje
32     x_ellip = signal.filtfilt(bze, aze, x)
33
34     # Racunanje spektra
35     X_ellip = np.fft.fft(x_ellip)
36     X_ellip = X_ellip[range(len(X_ellip) // 8)] / max(X_ellip)
37     freq = (np.arange(len(X_ellip)) * fs / len(x_ellip)).flatten()
38
39     # Cheby I filter

```

```
40 bc, ac = signal.cheby1(4, 2, [45, 55], btype='bandstop', analog=True)
41 bzc, azc = signal.bilinear(bc, ac, fs / (2 * np.pi))
42
43 # Filtriranje
44 x_cheby1 = signal.filtfilt(bzc, azc, x)
45
46 # Racunanje spektra
47 X_cheby1 = np.fft.fft(x_cheby1)
48 X_cheby1 = X_cheby1[range(len(X_cheby1) // 8)] / max(X_cheby1)
49
50 ## Racunanje broja otkucaja u minuti
51
52 # Thresholdovanje signala
53 trashhold = 0.8 * max(x_cheby1)
54 thr = x_cheby1 > trashhold
55
56 # Brojanje pikova
57 cnt = sum([1 for idx, _ in enumerate(thr[1:]) if thr[idx - 1] and not thr[idx]])
58
59 # Rezultat
60 print(f'Broj otkucaja u minuti je {int(cnt * (60 / t[-1]))}')
61
62
63 ## Plotovanje rezultata
64
65 # a) signal vreme
66 plt.xlabel('vreme [s]')
67 plt.ylabel('EKG')
68 plt.title('Nefiltrirani signal')
69 plt.plot(t, x)
70
71 plt.savefig('figures/zad3_signal_vreme.png')
72
73 # b) spektar signal
74 plt.figure()
75 plt.stem(freq, abs(), markerfmt=',')
76 plt.xlabel('Frekvencija [Hz]')
77 plt.ylabel('FFT')
78 plt.title('Originalni spektar')
79
80 plt.savefig('figures/zad3_signal_spektar.png')
81
82 # c) AFK filtera
83 plt.figure()
84
85 plt.subplot(2, 1, 1)
86 w, h = signal.freqs(bc, ac)
87 wz, hz = signal.freqz(bzc, azc)
88 plt.semilogx(w, 20 * np.log10(abs(h)), label='analogni')
89 plt.semilogx((wz * fs / (2 * np.pi)).T, 20 * np.log10(abs(hz)), label='digitalni')
90 plt.legend()
91 plt.title('Elipicni bandstop filteri')
92 plt.xlabel('Frekvencija [rad / sec]')
93 plt.ylabel('Amplituda [dB]')
94
95 plt.subplot(2, 1, 2)
96 w, h = signal.freqs(bc, ac)
97 wz, hz = signal.freqz(bzc, azc)
98 plt.semilogx(w, 20 * np.log10(abs(h)), label='analog')
99 plt.semilogx((wz * fs / (2 * np.pi)).T, 20 * np.log10(abs(hz)), label='digital')
100 plt.legend()
101 plt.title('Chebisev I bandstop filteri')
102 plt.xlabel('Frekvencija [rad / sec]')
103 plt.ylabel('Amplituda [dB]')
104
105 plt.tight_layout()
106 plt.savefig('figures/zad3_filteri.png')
107
```



```

108 # d) svi signali u vremenu
109 plt.figure()
110
111 plt.subplot(3, 1, 1)
112 plt.xlabel('vreme [s]')
113 plt.ylabel('EKG')
114 plt.title('Nefiltrirani signal')
115 plt.plot(t, x)
116
117 plt.subplot(3, 1, 2)
118 plt.plot(t, x_ellip)
119 plt.xlabel('vreme [s]')
120 plt.ylabel('EKG')
121 plt.title('Signal na izlazu Elipticnog filtera')
122
123 plt.subplot(3, 1, 3)
124 plt.plot(t, x_cheby1)
125 plt.xlabel('vreme [s]')
126 plt.ylabel('EKG')
127 plt.title('Signal na izlazu Cheby I filtera')
128
129 plt.tight_layout()
130 plt.savefig('figures/zad3_filtrirani_vreme.png')
131
132 # e) AFK svih signala
133 plt.figure()
134 plt.tight_layout()
135
136 plt.subplot(3, 1, 1)
137 plt.stem(freq, abs(X), markerfmt=',')
138 plt.xlabel('Frekvencija [Hz]')
139 plt.ylabel('FFT')
140 plt.title('Originalni spektar')
141
142 plt.subplot(3, 1, 2)
143 plt.stem(freq, abs(X_ellip), markerfmt=',')
144 plt.xlabel('Frekvencija [Hz]')
145 plt.ylabel('FFT')
146 plt.title('Spektar na izlazu Elipticnog filtera')
147
148 plt.subplot(3, 1, 3)
149 plt.stem(freq, abs(X_cheby1), markerfmt=',')
150 plt.xlabel('Frekvencija [Hz]')
151 plt.ylabel('FFT')
152 plt.title('Spektar na izlazu Cheby I filtera')
153
154 plt.tight_layout()
155 plt.savefig('figures/zad3_filtrirani_spektar.png')

```

• Bonus - kôd i rezultati

```

1 import numpy as np
2 from matplotlib import pyplot as plt
3 from scipy import signal
4 from scipy.io.wavfile import read, write
5
6
7 def estimate_pitch(x, fs):
8     """ Returns fft of 20ms window """
9
10    step = int(fs * 20 * 1e-3)
11
12    spek = []
13
14    for idx in range(0, step * (len(x) // step), step):
15        X = np.fft.fft(x[idx:idx + step])
16        X = X[:len(X) // 2] / max(X)
17        X = abs(X)

```

```
18
19     if len(spek) == 0:
20         spek = X
21     else:
22         spek += X
23
24     spek /= len(x) / step
25     spek /= max(spek)
26     freq = (np.arange(len(spek)) * fs / step).flatten()
27
28     lowbound = np.nonzero(freq < 165)[-1][-1]
29     highbound = np.nonzero(freq < 255)[-1][-1]
30
31     return freq[lowbound + np.argmax(spek[lowbound:highbound+1])]
32
33
34
35 if __name__ == "__main__":
36     # Učitavanje originalnog signala
37     fs, x = read('sounds/in.wav')
38
39     print(f'Pitch frekvencija je {estimate_pitch(x, fs)}')
```

Pitch frekvencija je 200Hz