Numerical Methods for Optimization and Control Theory

Daniel Kuknyo - Assignment 2

Assigned tasks: 1, 5, 10, 12

5. * Implement a constrained barrier method with logarithmic barrier. Make it able to explicitly compute the gradients and Hessians of the modified f_ρ objectives based on the exact gradients and Hessians of the g_i contraint functions, given in the input. Use Newton's method as the unconstrained subroutine. You may use the code for the Carrol barrier (`barrier_path.m`) as reference. You may implement the method with approximated Hessians for partial score.

Defining the functions

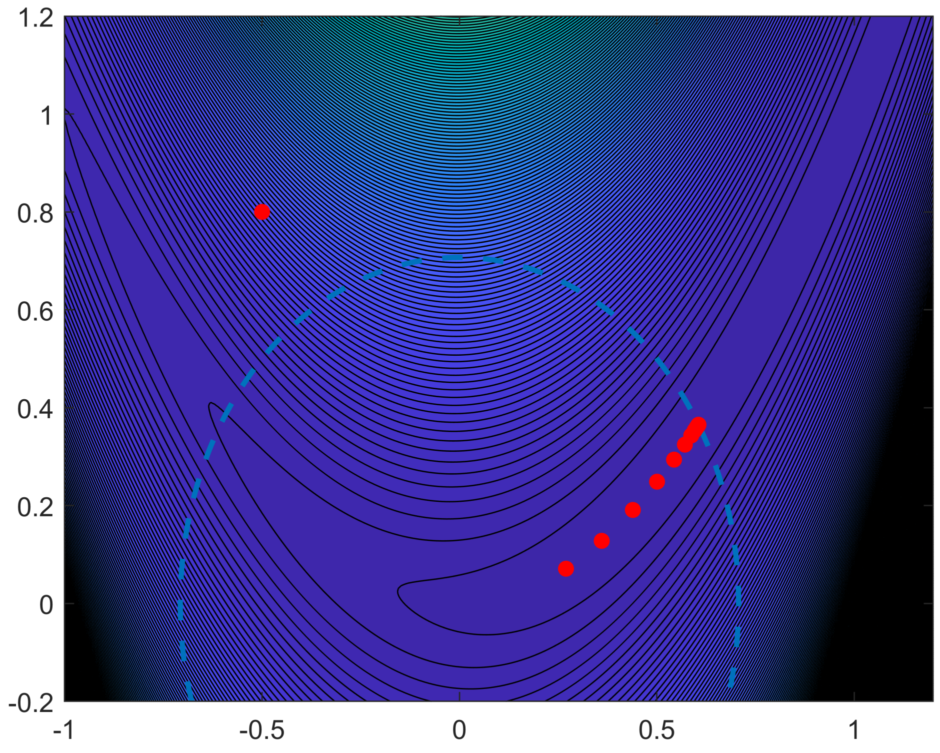
```
% Rosenbrock function
clc; clear all;
rr =  [-1.0, 1.2, -0.2, 1.2]; % Recommended region
  f = @(x) 100.*(x(2)-x(1).^2).^2+(1-x(1)).^2;
 df = @(x) [400.*x(1).^3-400.*x(1).*x(2)+2.*x(1)-2;200.*(x(2)-x(1).^2)];
ddf = @(x) [1200.*x(1).^2-400.*x(2)+2 -400.*x(1);-400.*x(1) 200];

% Circle constraint function
  g = @(x) x(1)^2 + x(2)^2 - 0.5;
 dg = @(x) [2*x(1) 2*x(2)];
ddg = @(x) [2 0; 0 2];
```

Running the optimization

```
x = test_constrained_method(@barrier_path,f, df, ddf, ... % Objective function
                                          g, dg, ddg, ... % Constraint function
                                          0, 0, 0, 20, ... % Test points
                                          [-0.5; 0.8], ... % Starting point
                                          rr(1),rr(2),rr(3),rr(4),256); % Region
```

```
Warning: Function behaves unexpectedly on array inputs. To improve performance, properly vectorize your
function to return an output with the same size and shape as the input arguments.
```



Functions

```
function [ x ] = test_constrained_method(G,f,df,ddf,g,dg,ddg,h,dh,ddh,...
                                         testpts,startpt,a,b,c,d,res)
    X = linspace(a,b,res);
    Y = linspace(c,d,res);
    Z = zeros(res,res);
    for i=1:res
        for j=1:res
            Z(i,j)=f([X(i);Y(j)]);
        end
    end

    x = zeros(testpts,2);
    x(1,:) = startpt;
    for j=2:testpts
        x(j,:) = G(f,df,ddf,g,dg,ddg,h,dh,ddh,[x(1,1);x(1,2)],j-1);
    end

    contourf(X,Y,Z',200); hold on
    ithcomp = @(expr,ind) expr(ind);
    try
        k = size(g(startpt),1);
        for j=1:k
            compfun = @(x,y) ithcomp(g([x;y]),j);
            fimplicit(compfun, [a,b,c,d], '--', 'LineWidth', 2.0);
        end
    catch
    end
    try
        k = size(h(startpt),1);
        for j=1:k
            compfun = @(x,y) ithcomp(h([x;y]),j);
            fimplicit(compfun, [a,b,c,d], '--', 'LineWidth', 2.0);
        end
    catch
    end
    scatter(x(:,1),x(:,2),'r','filled');
    hold off;
end

function [ x ] = barrier_path(f,df,ddf,g,dg,ddg,~,~,~,x0,iter)
    rho = 1; % 1
    mul = 2; % 2
    internal_limit = 50; % 20-50

    x = x0;
    for l = 1:iter
        frho = @(x) f(x) + 1/rho * log_barrier(g,x);
        dfrho = @(x) df(x) + 1/rho * log_barrier_grad(g,dg,x);
        ddfrho = @(x) ddf(x) + 1/rho * log_barrier_hessian(g, dg, ddg, x);

        x = Newton_barrier(frho,dfrho,ddfrho,x,internal_limit,g);
        rho = rho * mul;
    end
end

function gx = log_barrier(g, x) % Value of the logarithmic barrier
    gx = -sum(log(-g(x)));
end

function dgx = log_barrier_grad(g, dg, x) % Gradient of the logarithmic barrier
    dgx = -dg(x)' * (1./g(x));
end

function [ ddgx ] = log_barrier_hessian(g,dg, ddg, x) % Hessian of the barrier
    ddgx = zeros(size(ddg));

    gx = g(x);
    dgx = dg(x);
    ddgx = ddg(x);

    for k=1:size(dgx, 1)
        temp_hess = zeros(size(ddg));
        for i=1:size(dgx, 2)
           for j=1:size(dgx, 2)
               temp_hess(i, j) = -((ddgx(i, j) * gx(k) - dgx(k, i) * dgx(k, j)) / gx(k)^2);
           end
        end

        ddgx = ddgx + temp_hess;
    end
end

function [ x ] = Newton_barrier(f, df, ddf, x0, iter, g)
    rho = 0.5; % Original value: 0.5
    c = 0.2;
    x = x0;
    delta = 1e-9; % Correction constant to ensure p is a descent direction

    for k = 1:iter
        H = ddf(x);

        % Required to check if finite differencing the Hessian
        if sum(isinf(H)) > 0
            break;
        end

        [Q,D] = eig(H);
        for i = 1:size(D(:,1))
            if D(i,i) < delta
                D(i,i) = 1/delta;
            else
                D(i,i) = 1/D(i,i);
            end
        end
        p = -Q * D * Q' * df(x);
        alpha = norm(p);

        % Have to ensure we are searching in int G !!
        while ~all(g(x+alpha*p)<0)
            alpha = alpha*rho;
            p = -p;
        end

        gamma = Armijo_Barrier(f, df, p, x, alpha, rho, c, g);
        x = x + gamma * p;
    end
end

function [ alpha ] = Armijo_Barrier(f, df, p, x0, alpha, rho, c, g)
    f0 = f(x0);
    g0 = df(x0);
    x = x0 + alpha .* p;
    fk = f(x);
    dp = (g0'*p); % Directional derivative

    % Repeat until the Armijo conditions are satisfied
    while fk < f0 + c * alpha * dp && all(g(x0+alpha/rho*p)<0)
      alpha = alpha / rho;
      x = x0 + alpha * p;
      fk = f(x);
    end
    while fk > f0 + c * alpha * dp && all(g(x0+alpha*rho*p)<0)
      alpha = rho * alpha;
      x = x0 + alpha * p;
      fk = f(x);
    end
end
```