Numerical Methods for Optimization and Control Theory

Assignment 1 - Daniel Kuknyo

Tasks assigned: 2, 5, 7, 13

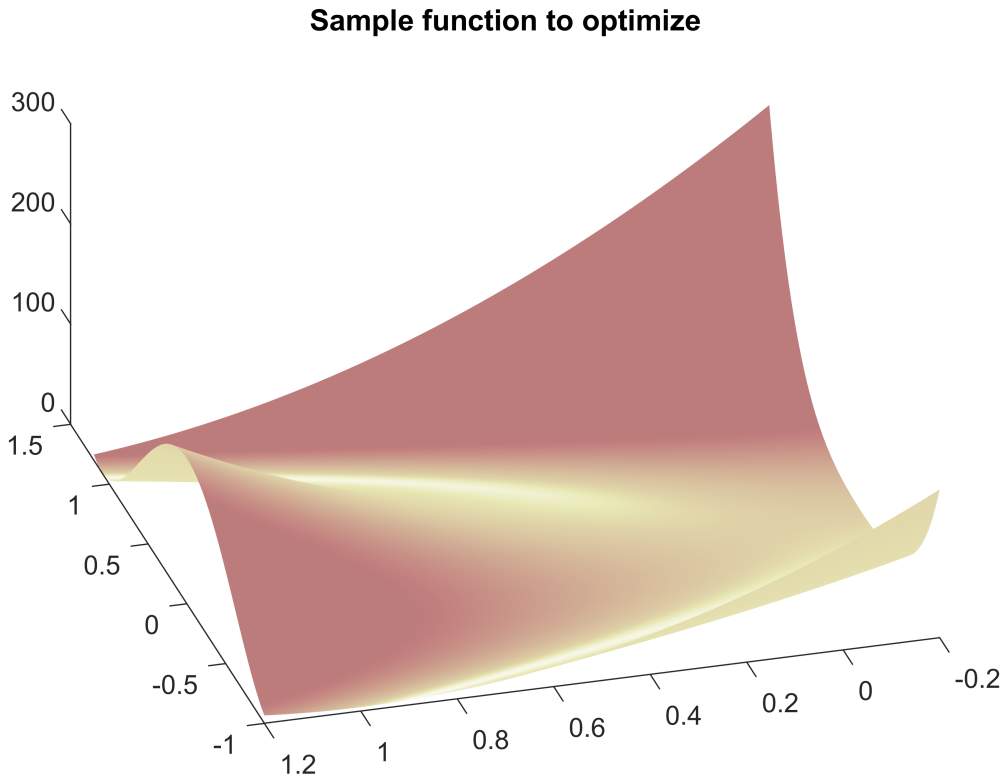
5. Modify the implementation of the Armijo Gradient descent method, such that the plots also include the further points examined by the line search method underneath, and also the examined line should be visualised at each iteration.

Define the function

```
% Rosenbrock function
rr = [-1.0 1.2 -0.2 1.2];
startpt = [0; 1];
f=@(x) 100.*(x(2)-x(1).^2).^2+(1-x(1)).^2;
df=@(x) [400.*x(1).^3-400.*x(1).*x(2)+2.*x(1)-2;200.*(x(2)-x(1).^2)];
ddf=@(x) [1200.*x(1).^2-400.*x(2)+2 -400.*x(1);-400.*x(1) 200];
```

Plot the Rosenbrock function

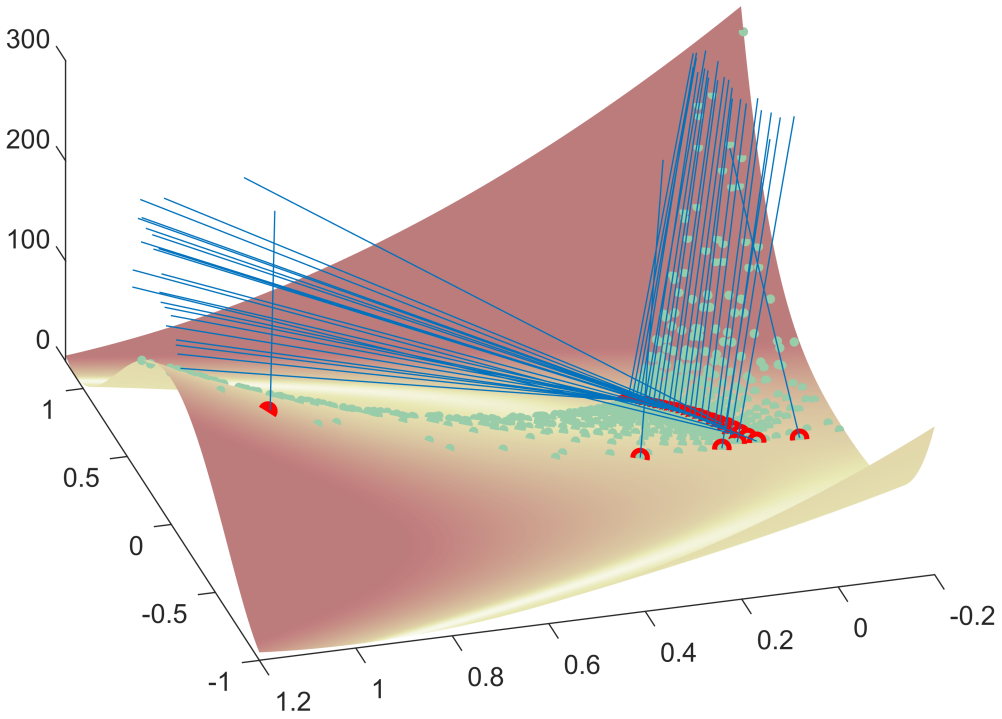
```
a = rr(1);
b = rr(2);
c = rr(3);
d = rr(4);
res = 256;

X = linspace(a,b,res);
Y = linspace(c,d,res);
[X,Y] = meshgrid(X,Y);
Z = zeros(res,res);

for i=1:res
    for j=1:res
        Z(i,j) = f([X(i,j),Y(i,j)]);
    end
end

figure; hold on;
surfl(X,Y,Z);
colormap("pink");
shading interp;
title("Sample function to optimize");
view([-106 46]);
hold off;
```



Let's plot!

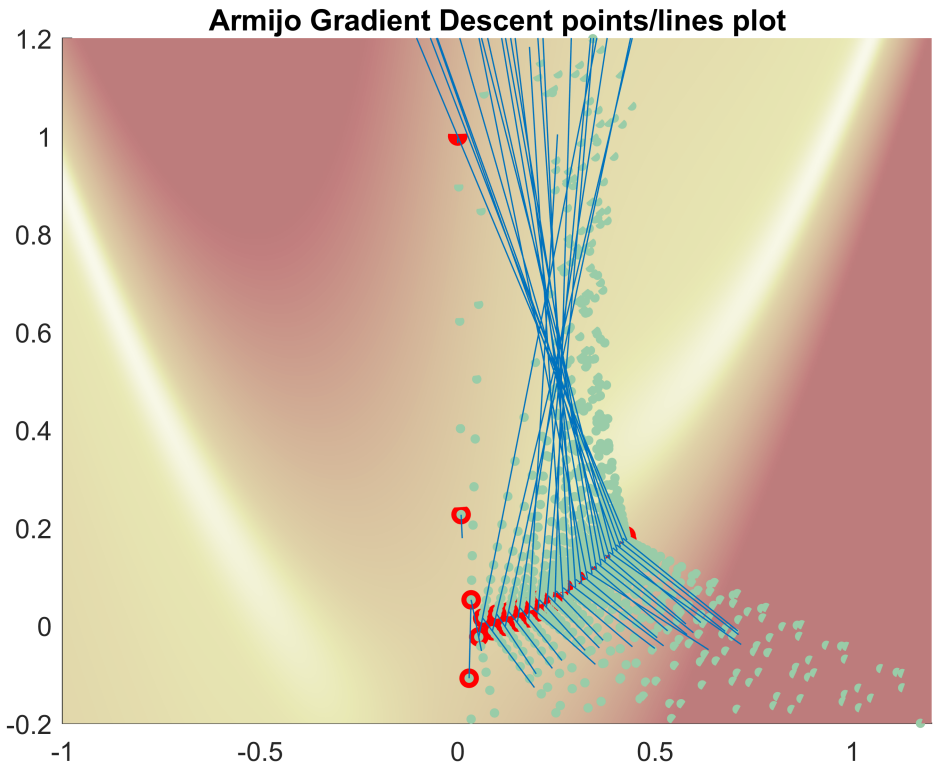
On the following diagram you can see the Gradient Descent points in red, Armijo LS points in green and the lines inspected by the algorithm with blue.

```
plot_Armijo_GD(@grad_descent, f, df, ddf, 50, ...
    startpt, rr(1), rr(2), rr(3), rr(4), 256, [-106 46]);
```



A different viewing angle

```
plot_Armijo_GD(@grad_descent, f, df, ddf, 50, ...
    startpt, rr(1), rr(2), rr(3), rr(4), 256, [0 90]);
```



Functions

```
function [ x ] = plot_Armijo_GD(G, f, df, ddf, testpts, startpt, a, b, c, d, res, viewAngle)
    % Get the general function
    w = 0;
    v = 300;
    X = linspace(a,b,res);
    Y = linspace(c,d,res);
    [X,Y] = meshgrid(X,Y);
    Z = zeros(res,res);

    for i=1:res
        for j=1:res
            Z(i,j) = f([X(i,j), Y(i,j)]);
        end
    end

    % x->points of GD, pts->points of Amijo LS, lns->Lines of Armijo
    [x, pts, lns] = grad_descent(f, df, 0, startpt, testpts);

    % Plot
    figure;
    hold on;

    % Plot the surface
```

```
    surfl(X,Y,Z);

    % Plot the GD points
    for i=1:size(x, 2)
        px = x(1, i);
        py = x(2, i);
        pz = f([px, py]);
        if(~isOutofBounds(a,b,c,d,[px;py;pz]))
            plot3(px, py, pz,  '.', ...
                'MarkerSize', 25, ...
                'MarkerEdgeColor', 'red', ...
                'MarkerFaceColor', [1 .6 .6]);
        end
    end

    % Plot the Armijo points
    for i=1:size(pts, 2)
        px = pts(1, i);
        py = pts(2, i);
        pz = pts(3, i);
        if(~isOutofBounds(a,b,c,d,[px;py;pz]))
            plot3(px, py, pz,  '.', ...
                'MarkerSize', 12, ...
                'MarkerEdgeColor', [.6 .8 .66], ...
                "MarkerFaceColor", [.38 .51 .31]);
        end
    end

    % Plot the Armijo lines
    x(:, end) = [];
    for i=1:size(lns, 2)
        px = x(1, i);
        py = x(2, i);
        pz = f([py, py]);
        pt = [px, py, pz];

        lx = lns(1, i);
        ly = lns(2, i);
        lz = f([lx, ly]);
        lt = [lx, ly, lz];

        pconn = [pt; lt];
        pconn = [lt; pt];
        line(pconn(:,1), pconn(:,2), pconn(:,3));
    end

    % Settings for the plot
    colormap("pink");
    shading interp;
    title("Armijo Gradient Descent points/lines plot");
```

```
    view(viewAngle);
    xlim([a b]);
    ylim([c d]);
    zlim([w v]);
    hold off;
end

% Is a point out of the bounds? 1 -> we don't plot it (aesthetic purposes)
function isOut = isOutofBounds(a, b, c, d, x)
    isOut = false;
    if(x(1)<a | x(1)>b | x(2)<c | x(2)>d | x(3)>300 | x(3)<0)
        isOut = true;
    end
end

function [x, pts, lns] = grad_descent(f, df, ~, x0, iter)
    rho = 0.8;
    c = 0.2;
    alpha = 1;
    x_k = x0; % Starting point
    x = [x0]; % Points of Gradient Descent
    lns = [];
    pts = [];

    for k = 1:iter
        p = -df(x_k); % Direction of (steepest) descent
        [gamma, pts_k] = Armijo_LS(f, df, p, x_k, alpha, rho, c); % Step length
        x_k = x_k + gamma*p;
        x = [x x_k];
        lns = [lns p]; % These lines will be plotted
        pts = [pts pts_k];
    end
end

function [alpha, pts_k] = Armijo_LS(f, df, p, x0, alpha, rho, c)
    f0 = f(x0);
    g0 = df(x0);
    x = x0 + alpha .* p;
    fk = f(x);
    dp = (g0'*p);
    pts_k = [];

    while fk < f0 + c * alpha * dp
      alpha = alpha / rho;
      x = x0 + alpha * p;
      fk = f(x);
      fxk = [x; fk];
      pts_k = [pts_k fxk];
    end
    while fk > f0 + c * alpha * dp
```

```
      alpha = rho * alpha;
      x = x0 + alpha * p;
      fk = f(x);
      fxk = [x; fk];
      pts_k = [pts_k fxk];
    end
end
```