Numerical Methods for Optimization and Control Theory

Assignment 1 - Daniel Kuknyo

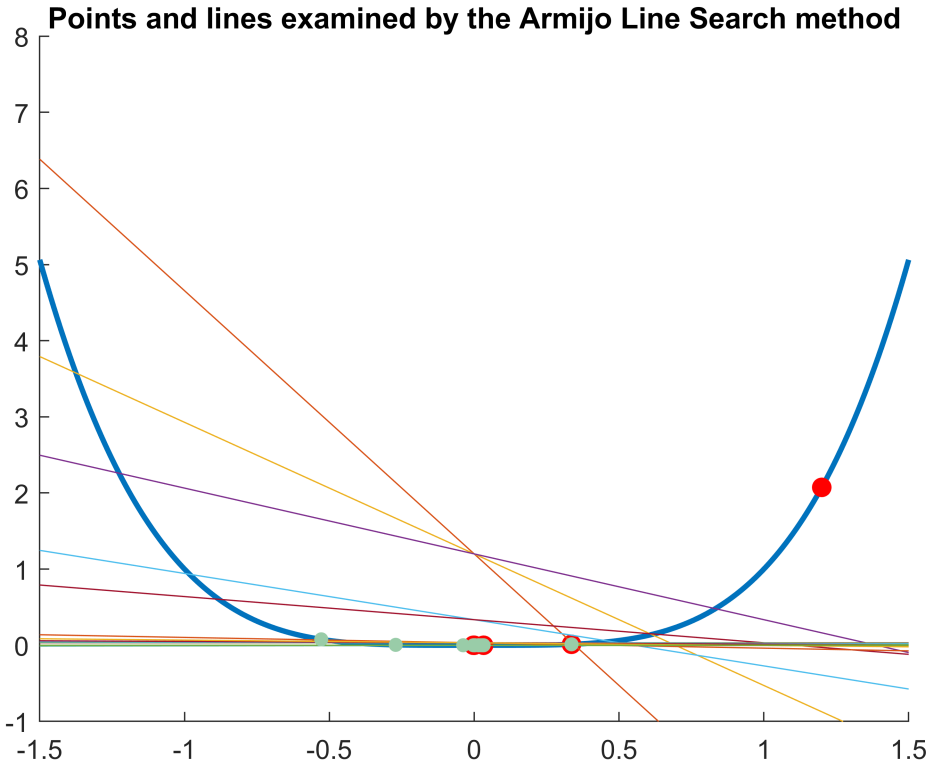
Tasks assigned: 2, 5, 7, 13

5. Modify the implementation of the Armijo Gradient descent method, such that the plots also include the further points examined by the line search method underneath, and also the examined line should be visualised at each iteration.

On the following plot, the green dots will correspond to the points inspected by the method, and the colorful lines are the inspected lines.

```
f = @(x) x.^4;
df = @(x) 4*x.^3;
ddf = @(x) 12*x.^2;

x = plot_method_armijo(@Armijo_Gradient_Descent, f, df, ddf, 20, 1.2, -1.5, 1.5, 256);
```



Functions

```
function x = plot_method_armijo(G, f, df, ddf, testpts, startpt, a, b, res)
    X = linspace(a, b, res);
    Y = f(X);
    x = [startpt];
```

```
    pts = [];
    lns = [];

    % Collect the points, examined points and examined points
    for k=1:testpts
        [xk, points, lines] = G(f,df,ddf,startpt,a,b,k);
        x = [x xk];
        pts = [pts; points];
        lns = [lns; lines];
    end

    figure; hold on;
    plot(X, Y, 'LineWidth', 2); % Plot the function itself
    plot(x, f(x), '.', 'MarkerSize', 25, 'MarkerEdgeColor', 'red', ...
        'MarkerFaceColor', [1 .6 .6]); % Plot the points

    % Plot the lines examined by the line search method
    for i=1:size(lns, 1)
        b0 = lns(i, 1);
        b1 = lns(i, 2);
        linefn = @(x, b0, b1) b0 + b1*x;
        plot(X, linefn(X, b0, b1));
    end

    % Plot the points examined by the line search method
    for i=1:size(pts, 1)
        plot(pts(i), f(pts(i)), '.', 'MarkerSize', 18, 'MarkerEdgeColor', [.6 .8 .66], ...
            "MarkerFaceColor", [.38 .51 .31]);
    end

    % Some extra settings for the plots
    xlim([a b]);
    ylim([-1 8]);
    title('Points and lines examined by the Armijo Line Search method');
    hold off;
end

function [x, points_t, lines_t] = Armijo_Gradient_Descent(f,df,~,x0,~,~,iter)
    rho = 0.5; % Step length multiplier
    c = 0.2;   % Sufficient decrease condition
    alpha = 1; % Initial step length

    points_t = [];
    lines_t = [];

    x = x0;
    for k = 1:iter
        p = -df(x);
        % Step length ---- Armijo LS examines further points
        [gamma, points, lines] = Armijo_LS(f, df, p, x, alpha, rho, c);
        x = x + gamma * p;
        points_t = [points_t; points];
        lines_t = [lines_t; lines];
    end
```

```
end

function [alpha, points, lines] = Armijo_LS(f, df, p, x0, alpha, rho, c)
    f0 = f(x0);
    g0 = df(x0);
    x = x0 + alpha .* p;
    fk = f(x);
    dp = (g0'*p); % Directional derivative at x0

    points = [];
    lines = [];

    % Repeat until the Armijo conditions are satisfied
    while fk < f0 + c * alpha * dp
      alpha = alpha / rho;
      x = x0 + alpha * p;
      fk = f(x);

      points = [points; x];
      lines = [lines; x0 alpha*p];
    end

    while fk > f0 + c * alpha * dp
      alpha = rho * alpha;
      x = x0 + alpha * p;
      fk = f(x);

      points = [points; x];
      lines = [lines; x0 alpha*p];
    end
end
```