Title

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Abstract

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1 Introduction

2 Procedure

3 Design of Experiments

We design the experiment to efficiently determine the influence of the parameter a, b, and d on the mean travel time and the mean jerk experienced during the journey. The complex nature of the problem does not allow to make assumptions on the model underlying the experiments. We therefore choose to investigate several models. We begin with a simple linear model of the form

$$y_a = a_0 + \sum_{i=1}^{3} a_i x_i + \varepsilon_a \quad \text{and} \tag{1a}$$

$$y_b = b_0 + \sum_{i=1}^{3} b_i x_i + \varepsilon_b, \tag{1b}$$

where y_a and y_b are the responses (mean travel time and mean jerk), a_i and b_i are the effects of the ith parameter on the travel time and the jerk respectively, x_1, x_2 and x_3 are the values taken by the parameters A, B and C respectively. Since the results of the analysis suggest that the influence of parameter A on the travel time is negligible (cf. **??**), we adjust our model by omitting this parameter. The nature of the algorithm suggest that interactions could play an important role. Hence, we will also investigate a linear model allowing for interactions between the parameters as described in eq. 3.

$$y_a = a_0 + \sum_{i=1}^{3} a_i x_i + \sum_{i,j=1, i\neq j}^{3} a_{ij} x_i x_j + \varepsilon_a \quad \text{and} \tag{2a}$$

$$y_b = b_0 + \sum_{i=1}^{3} b_i x_i + \sum_{i,j=1, i\neq j}^{3} b_{ij} x_i x_j + \varepsilon_b \tag{2b}$$

As for the linear model, we found that the influence of parameter A and it's interactions is negligible and omit the parameter for the travel time. We also test a quadratic model with the following equations:

$$y_a = a_0 + \sum_{i=1}^{3} a_i x_i + \sum_{i,j=1}^{3} a_{ij} x_i x_j + \varepsilon_a \quad \text{and} \tag{3a}$$

$$y_b = b_0 + \sum_{i=1}^{3} b_i x_i + \sum_{i,j=1}^{3} b_{ij} x_i x_j + \varepsilon_b, \tag{3b}$$

which has additionally the quadratic factors a_{ii}. Due to the limited number of experiments, we choose not to test higher order models.

4 Results

We estimate the effects (coefficients of the model) by minimizing the the residue in the sense of least square as expressed by eq. 4.

$$\hat{\alpha} = \arg\min_{\alpha} \left(\sum_{k=1}^{N} (y_{ak} - X_k \alpha)^2\right) \tag{4}$$

$\hat{\alpha}$ are the estimated half effects, N the number of experiments, y_{ak} is the result of the kth experiment for response a and X_k is the kth row of the model matrix X containing the values for each parameter as well as the corresponding effects and α are the half effects. We define the dispersion matrix D of the model as

$$D = (X^T X)^{-1}. \tag{5}$$

The solution to eq. 4 can be computed as

$$\hat{\alpha} = D X^T Y. \tag{6}$$

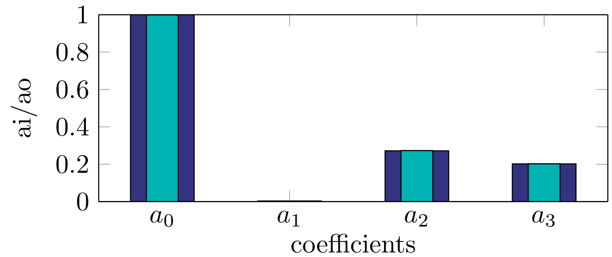
4.1 Linear Model

First, we will discuss the results for the linear model. Fig. 1 shows the half effects for both the travel time and the jerk in blue. It can be seen that the influence of parameter A (coefficients a_1 and b_1) on both the travel time and the jerk is negligible compared to the other effects.

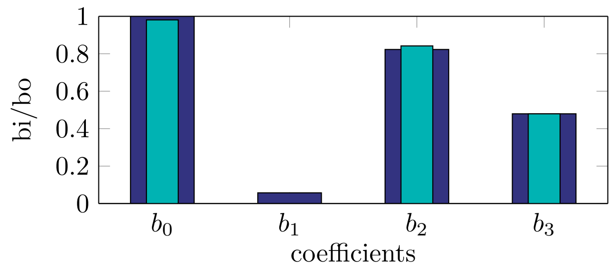
The ANOVA table is shown in tbl. 1 for the mean travel time and in tbl. 2 for the mean experienced jerk. The p-value for the travel time model (2.35e-7) is significantly lower than the p-value for jerk model (4.72e-2). This suggest that the travel time model fits much better the experimental data.

Fig. **??** shows the experimental results for the travel time as well as the planes representing the interpolation using the effects that we have found. The colors correspond to different values of parameter C, i.e., the dots should be close to the plane of the same color. Fig. **??** shows the results in terms of jerk. Note that here the colors correspond to the value of the parameter A. It can be seen that the points for measurements with the same values for parameter B and C are close by, encouraging the assumption that the influence of parameter A is negligible.

We therefore adjust both our models by omitting parameter A. We expect the residue to increase slightly since we remove a degree of freedom. The removal of a degree of freedom should however increase F-factor and therefore reduce the p-value, since the residue gains a degree of freedom. The resulting estimation of the half effects is shown in fig. 1 in



(a) travel time



(b) jerk

Figure 1: Relative half effects on the mean travel time (a) and the mean experienced jerk (b) using a linear model without interactions; In blue results for using all parameters and in turquoise if omitting parameter A (coefficients $a1$ and $b1$)

Source	SS	df	MS	F	p
Constant	34.4	4	8.61	95	2.35e-07
Residue	0.815	9	0.0906		

Source	SS	df	MS	F	p
Constant	34.4	3	11.5	141	1.78e-08
Residue	0.815	10	0.0815		

Table 1: ANOVA table of the **linear model** for the mean **travel time**; Top: full linear model; Bottom: omitting parameter A (coefficient $a1$); It shows the sums of squares (SS), the degrees of freedom (df), the mean square of the error (MS), the Fisher coefficient (F) and the p-value (p), signifying the probability that this result occurs at random;

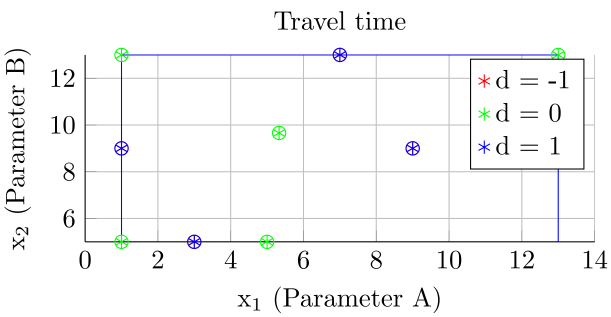


Figure 2: Insert caption

turquoise. It can be seen that the values of the half effects are similar to the previous results.

When comparing the ANOVA tables for the travel time (tbl. 1), it can be seen that the p value decreases by a factor of approximately 13, suggesting that omitting parameter A is improving our model. For the model of the jerk, the p-value decreases by a factor of approximately 2.7 (cf. tbl. 2).

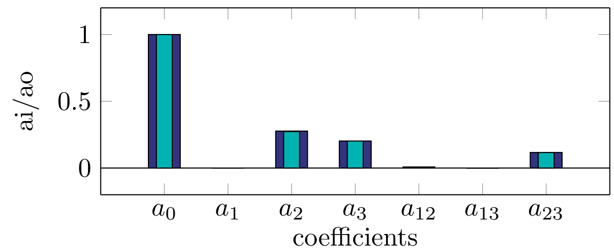
Source	SS	df	MS	F	p
Constant	2.41e+03	4	601	3.72	0.0472
Residue	1.46e+03	9	162		

Source	SS	df	MS	F	p
Constant	2.4e+03	3	801	5.5	0.0172
Residue	1.46e+03	10	146		

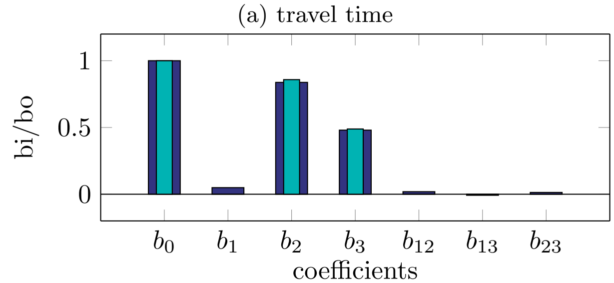
Table 2: ANOVA table of the **linear model** for the mean experienced **jerk**; Top: full linear model; Bottom: omitting parameter A (coefficient $b1$)

4.2 Linear Model with interactions

Fig. 3 shows the effects for this model. For both response variables we can again see that the influences of parameter A is negligible as it was the case for the purely linear model. Furthermore, the interactions appear to be negligible in the case of the jerk model. Considering the ANOVA table for this model (tbl. 3 and tbl. 4), it can be seen that the p value is significantly higher than for the linear model. This suggests that the addition of the interactions does not improve the model. This is due to the fact that the number of degrees of freedom of the models increase significantly were as whereas the residues remain approximately the same as for the linear models. While removing parameter A from the travel time model reduces it's p-value, it still remains inferior to the linear model.



(a) travel time



(b) jerk

Figure 3: Relative half effects on the mean travel time (a) and the mean experienced jerk (b) using a **linear model with interactions**; In blue results for using all parameters and in turquoise if omitting coefficients $a1$, $a12$ and $a13$ and $b1$, $b12$, $b13$, and $b23$)

Source	SS	df	MS	F	p
Constant	34.6	7	4.94	43.4	0.000101
Residue	0.682	6	0.114		

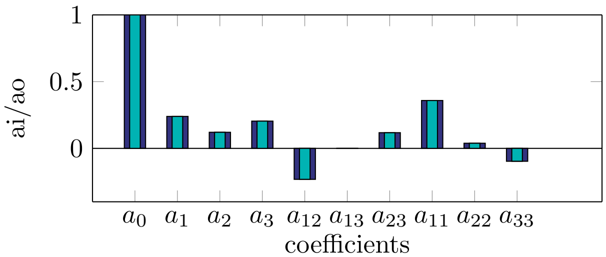
Source	SS	df	MS	F	p
Constant	33.6	4	8.4	46.8	5.04e-06
Residue	1.62	9	0.18		

Table 3: ANOVA table of the **linear model with interactions** for the mean **travel time**; Top: full linear model with interactions; Bottom: omitting parameter A (coefficients $a1$, $a12$ and $a13$)

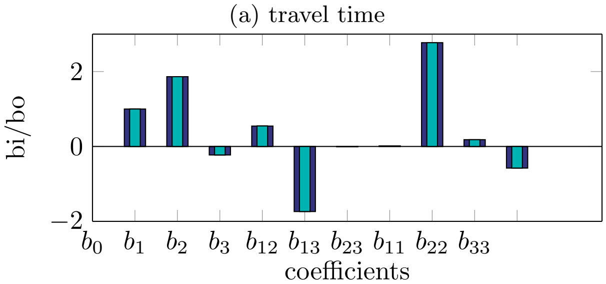
Source	SS	df	MS	F	p
Constant	2.41e+03	7	344	1.42	0.344
Residue	1.46e+03	6	243		

Table 4: ANOVA table of the **linear model with interactions** for the mean experienced **jerk**

4.3 Quadratic Model



(a) travel time



(b) jerk

Figure 4: Relative half effects on the mean travel time (a) and the mean experienced jerk (b) using a **quadratic model**; In blue results for using all parameters and in turquoise if omitting coefficients $a13$ and $b13$ and $b23$

Fig. 4 shows the effects for the quadratic model. Interestingly, in this model parameter A has the largest influence on both the mean travel time and the mean experienced jerk. Fig. **??** shows the experimental values as well as the quadratic interpolation for the mean arrival time. It can be seen, that the fitted surface is much closer to the measurement points. However, this is mainly due to the fact of introducing additional degrees of freedom. Tbl. 5 and tbl. 6 shows that for both responses, the p-value is higher than for the linear model, suggesting that the quality of model decreases by introducing quadratic terms.

Source	SS	df	MS	F	p
Constant	35.1	10	3.51	81.7	0.00198
Residue	0.129	3	0.043		

Source	SS	df	MS	F	p
Constant	35.1	9	3.9	121	0.000164
Residue	0.129	4	0.0322		

Table 5: ANOVA table of the **quadratic** model for the mean **travel time**; Top: full quadratic model; Bottom: omitting coefficient $a12$

Source	SS	df	MS	F	p
Constant	3.54e+03	10	354	3.3	0.178
Residue	322	3	107		

Source	SS	df	MS	F	p
Constant	3.54e+03	9	393	4.88	0.0706
Residue	322	4	80.6		

Table 6: ANOVA table of the **quadratic** model for the mean **travel time**; Top: full quadratic model; Bottom: omitting coefficient $b12$ and $b13$

5 Conclusion

In this work, we investigated the influence of three parameters on the performance of a collision avoiding algorithm design to allow collision free navigation of autonomous aerial vehicles. We measured the performance of the algorithm in terms of the mean travel time as well as the mean jerk that would be experienced by a passenger in a vehicles during the travel.

Due to technical issues on the platform, we decided to perform the experiment in simulation. We fitted three different models on the experimental data: A purely linear model, a linear model allowing for interactions and a quadratic model allowing for linear interactions.

By comparing the p-value of the models, the linear model has been found to be the best model for modelling both the mean travel time and the mean experienced jerk. We found that the influence of parameter A is negligible for both responses. Therefore, we adjusted the model by removing parameter A. This improves the confidence of the fit and reduces the model's complexity. The travel time can be expressed as

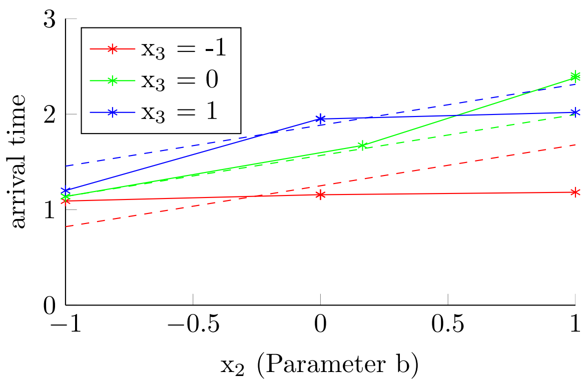


Figure 5: travel time

Fig. 5 shows the experimental values for the mean travel time and the linearly interpolated data. The linear fit for the jerk can be seen in fig. 6. It can be seen that the fit for the travel time is more accurate than for the jerk. This is encouraged by a comparison of the p-value. While the travel time model has a p-value of 1.78e-8, the jerk model as a p-value of 0.0172. While the model for the travel time has an acceptable p-value, the reliability of the model for the jerk is less convincing.

We conclude that parameter A is negligible for both the travel time and the jerk. Furthermore, both responses are assumed to depend linearly on parameters B and C.

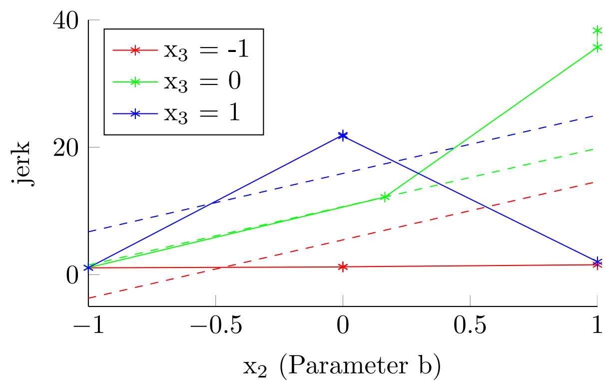


Figure 6: jerk