



DEPARTMENT OF  
INFORMATION SYSTEMS  
AND COMPUTER SCIENCE



```
00101110101000111010111100100111010101101001000101
1101010111010101000010101010100101010101010101010
10100101001001001001010101010101010101010101101010101010
1110000111101011000000011110101010101010000010101
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1000110010000111100110101011000100110101010000101010
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1001010010000101010010010101001010010101010010010
1001010010101001010010101001010010101001001001001
100101010101001010101010010101010101010010101010
```

										01
										02
										03
										04
										05
A	B	C	D	E	F	G	H			

# More CMOS Gates

Review of Digital Logic

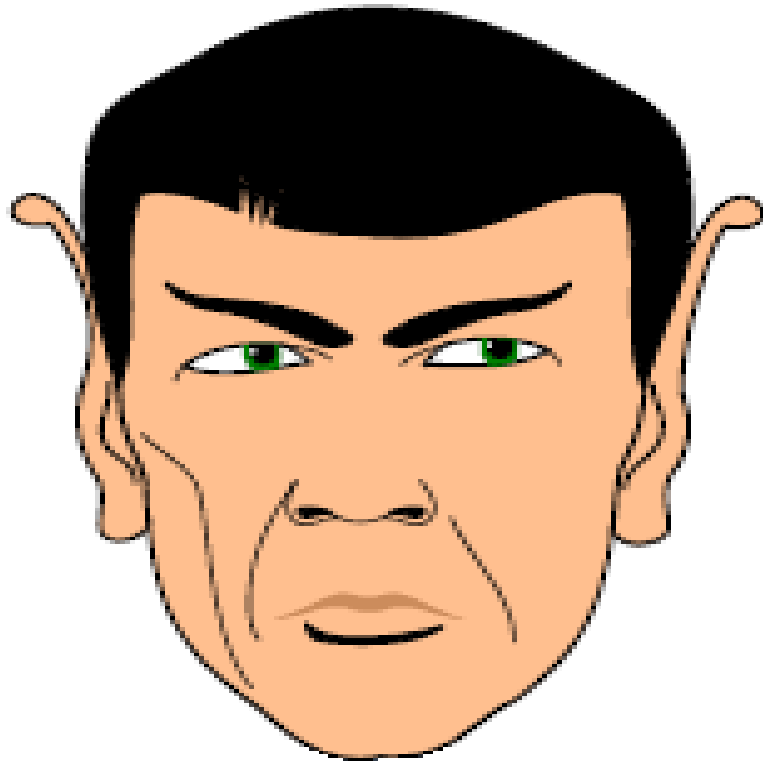
# Joke Time!



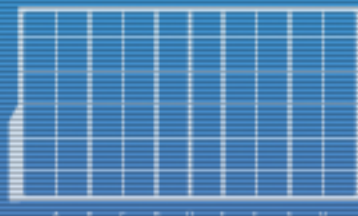
=



+



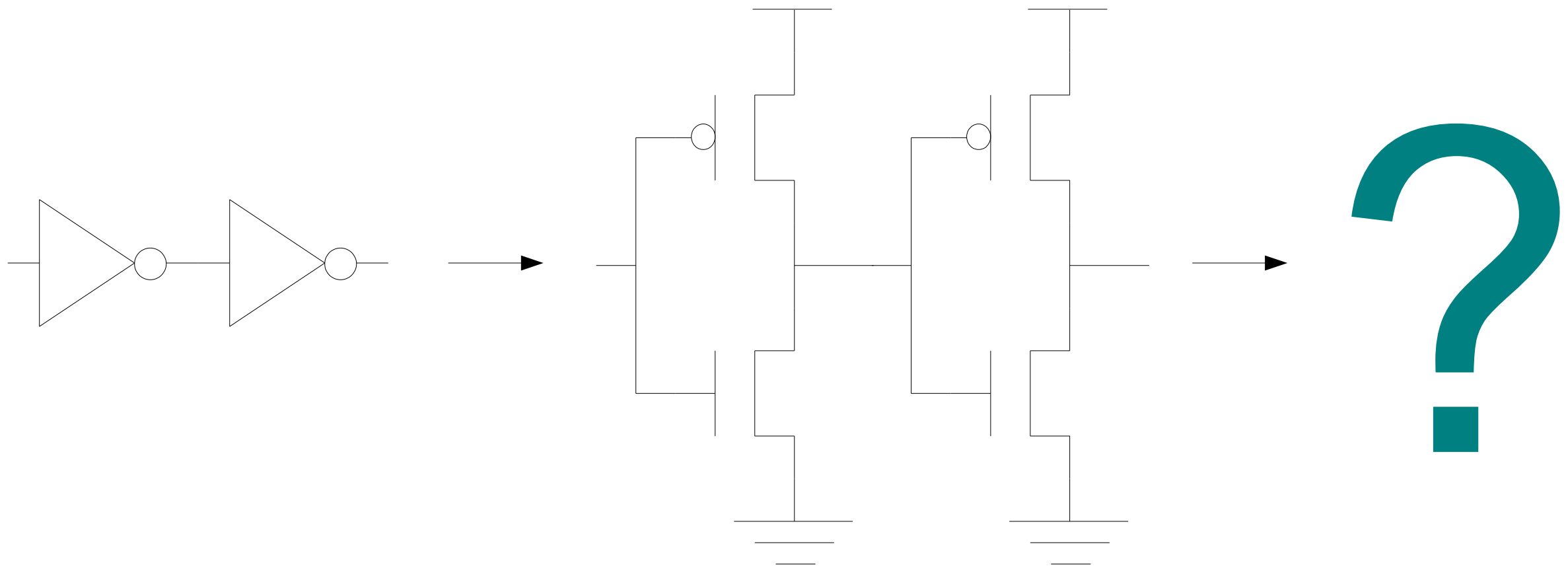
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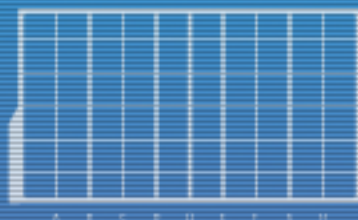
DISCS

# Transistor Limitations

- What happens if you chain 2 CMOS inverters?



00101010010101000011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
100101001001001010101010101010101  
11100001111010110000000111101001  
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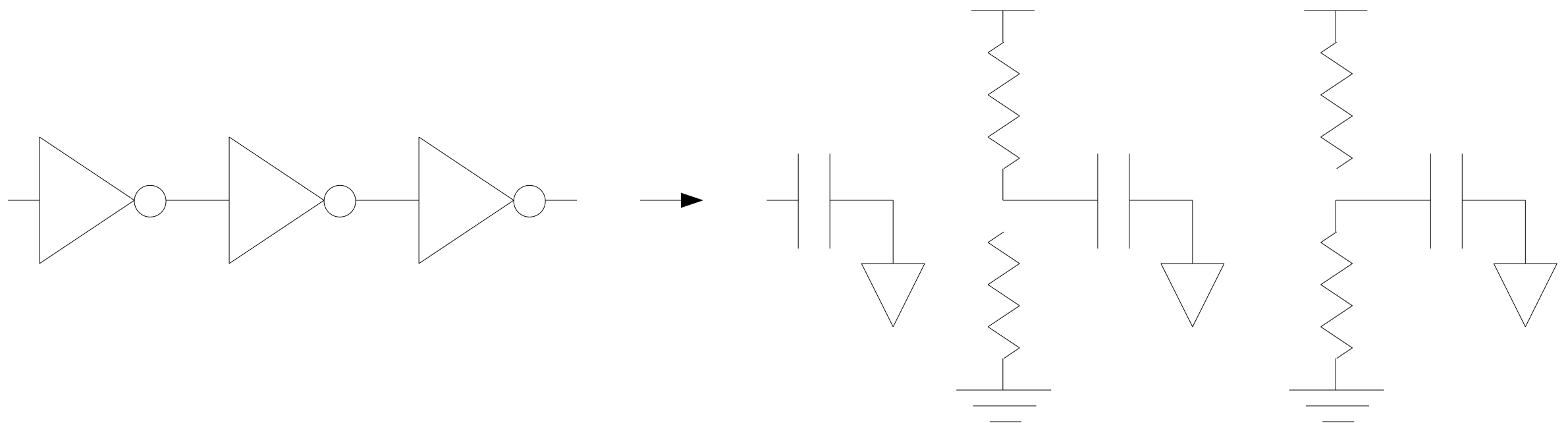


DISCS

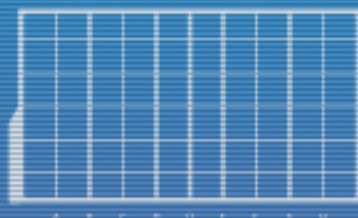


# Delays

- Logic gates have:
  - Output resistance
  - Input capacitance
- They form an RC circuit when chained together!

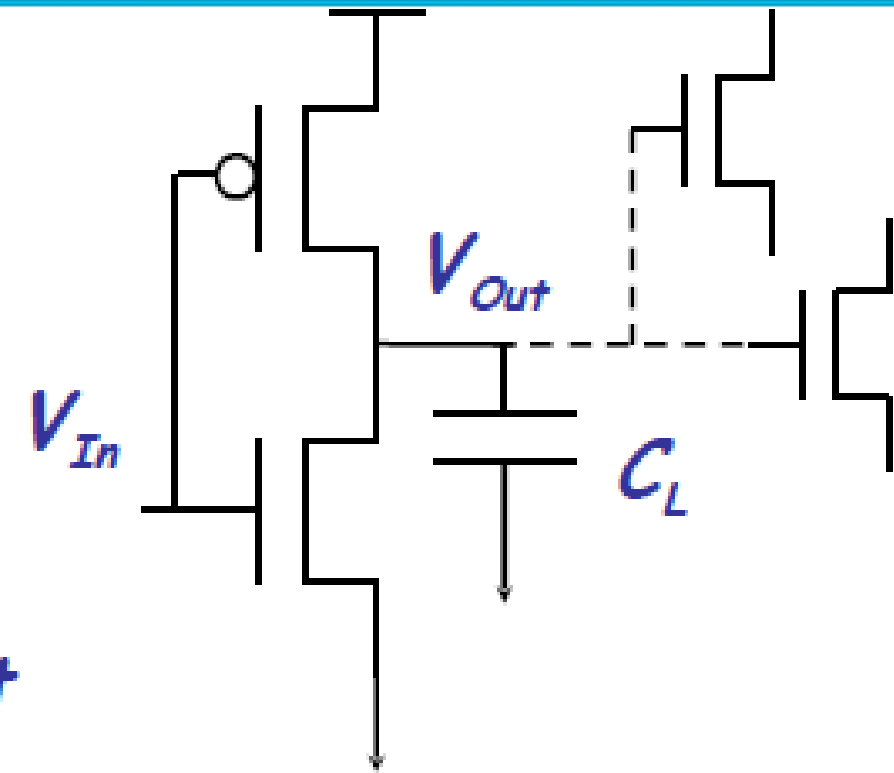
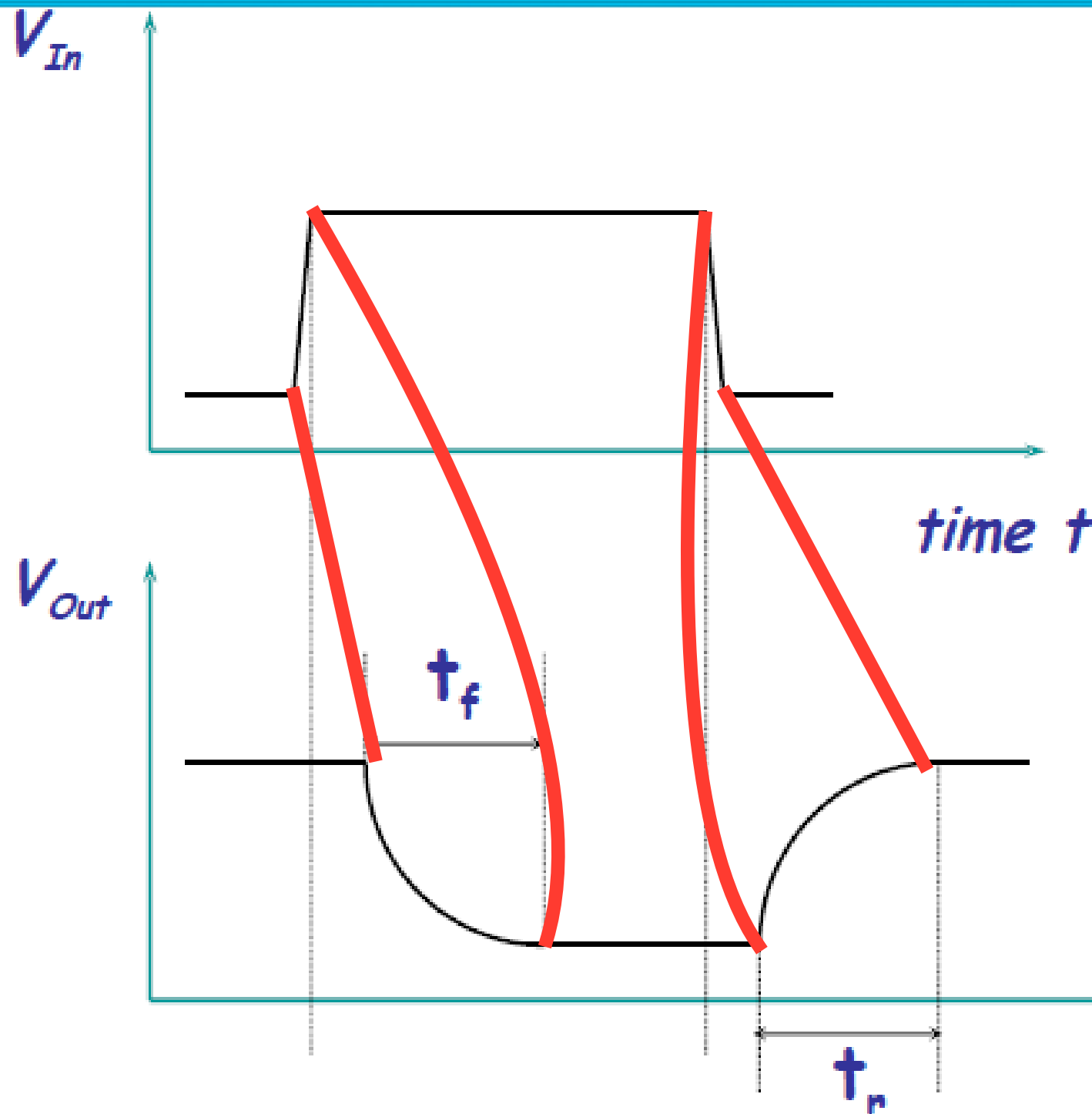


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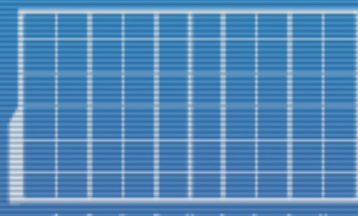
DISCS

# Rise Time and Fall Time



Time constant  
of  $t_f$ ,  $t_r$  equals  
 $R_{ON} C_L$

00101010010101000011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
100101001001001010101010101010101  
11100001111010110000000111101001  
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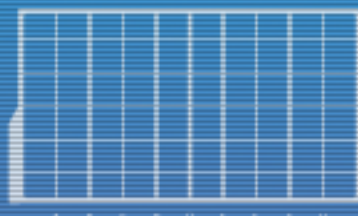


DISCS

# Propagation Delay

- Propagation delay ( $t_{pd}$ ) is the upper bound between new valid inputs and new valid outputs, or how long it takes for the entire system to “settle” after inputs are changed.
  - In plain English, when an input is changed and becomes its new valid value, what's the maximum wait time before a corresponding valid output emerges?
  - Let's try to determine  $t_{pd}$  from the graph in the previous slide.
    - It should include the time it takes for the transistor to turn on/off, as well as the rise/fall time.

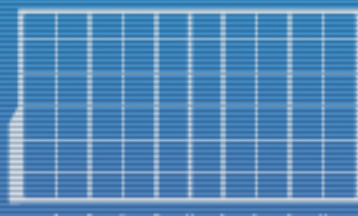
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1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101



# Contamination Delay

- Contamination delay ( $t_{cd}$ ) is lower bound between invalid inputs and invalid output, or how long it takes for one invalid signal to mess up the result.
  - In plain English, when an input is changed and enters the forbidden zone during transition, what's the minimum wait time before an output passes through the forbidden zone too?
  - $t_{cd}$  is also known as minimum  $t_{pd}$  or  $t_{pd-min}$ 
    - A device's  $t_{pd}$  cannot go below its  $t_{cd}$  ... why not?
  - If not known, assume  $t_{cd} = 0$ , the worst-case scenario.
    - As soon as an invalid input enters, output becomes invalid.

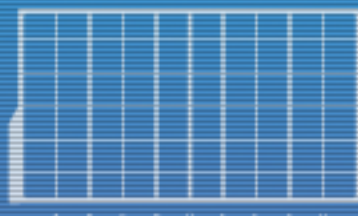
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1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101



# Definitions

- Propagation delay  $t_{pd}$ : Worst-case delay from new valid inputs to new valid outputs.
- Contamination delay  $t_{cd}$ : Best-case delay from invalid inputs to invalid outputs.
  - This is over all input combinations, over all input-output paths!
    - That's a lot of testing / measurements!
  - Delays are additive: Device  $t_{pd}$  and  $t_{cd}$  is sum of its parts'  $t_{pd}$ s and  $t_{cd}$ s.

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
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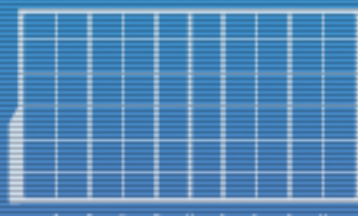




# Definitions

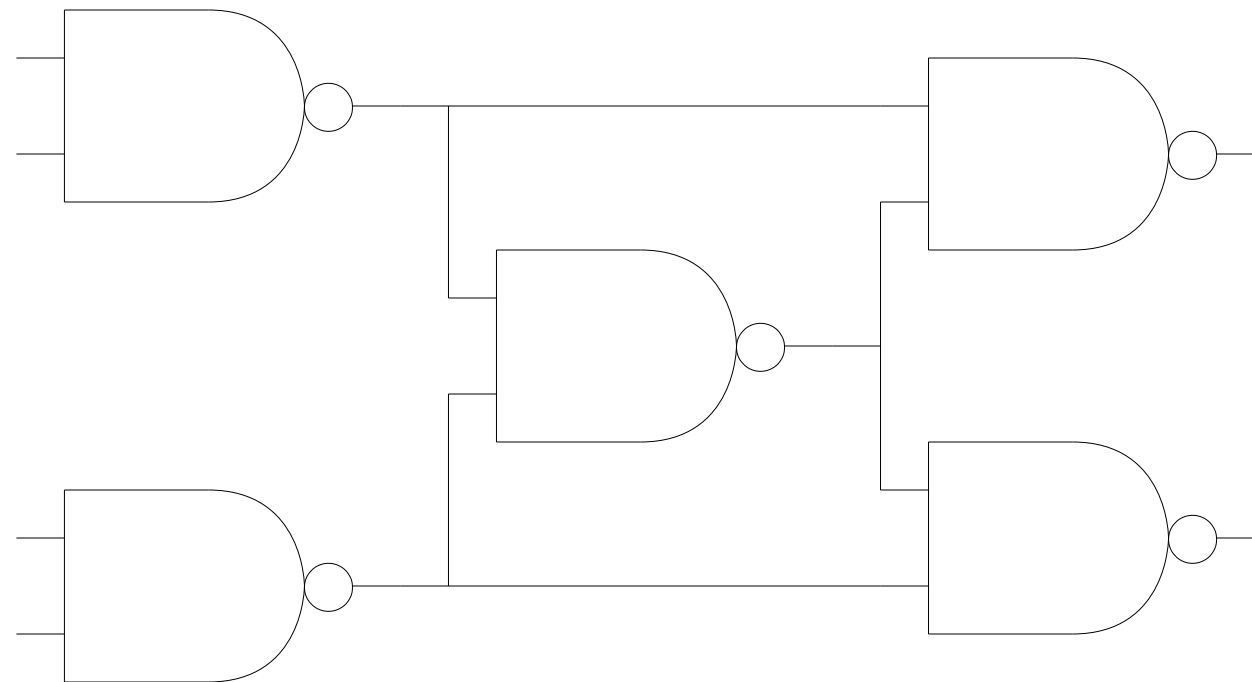
- Rise time  $t_r$ : Output signal transition time from invalid low to valid high.
- Fall time  $t_f$ : Output signal transition time from invalid high to valid low.
  - While not quite true, we can assume  $t_r$  and  $t_f$  to be properties of only the last stage (in an acyclic circuit).
    - After all, final output comes from the last stage of any given acyclic circuit, so rise/fall time measurement comes from there BUT remember that these times ARE STILL affected by the rise/fall times of the preceding stages!

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
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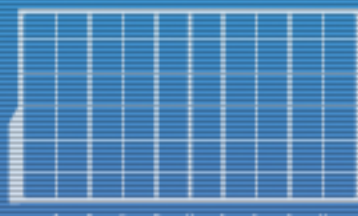


# Delays in Combinational Circuits

- Static discipline dictates that upon receiving a logically valid A and B, the circuit below should output a logically valid C (after some delay).
- Assuming each gate has a  $t_{pd} = 5\text{ns}$  and  $t_{cd} = 1\text{ns}$ , what's the  $t_{pd}$  and  $t_{cd}$  of the circuit below?



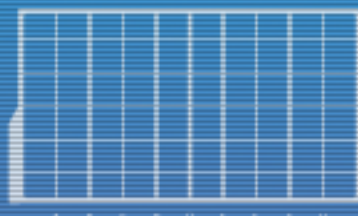
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11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101



# CMOS is cool, but...

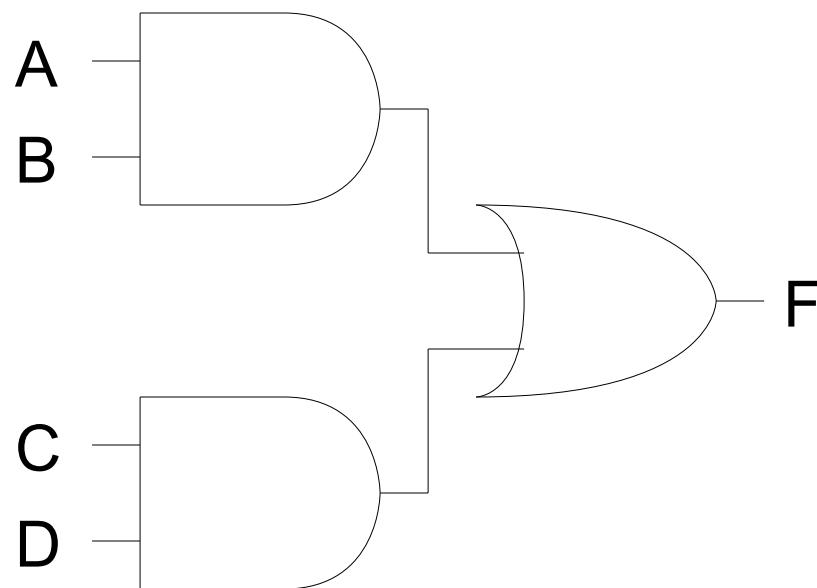
- We can't make customized gates every time we want a Boolean function implemented!
- Why not use what's already available?
- Already have 1-input to 4-input gates!
- We can make anything entirely out of NANDs or NORs!
  - Lab exercises in optimization, yo.
- Boolean functions consist of NOTs (inverters), ANDs, and ORs anyway!

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
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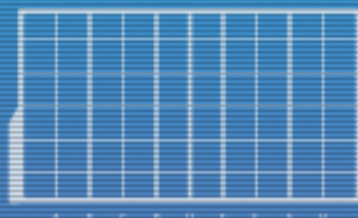


# Simplistic Viewpoint

- All we need are two-level circuits!
  - Inverters for inputs not included.
- Example:  $F = AB + CD$



00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
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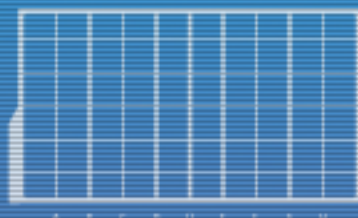
DISCS



# Simplistic Viewpoint

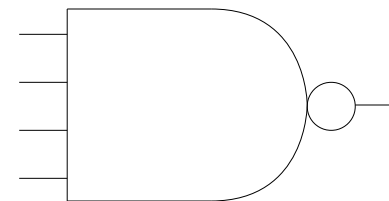
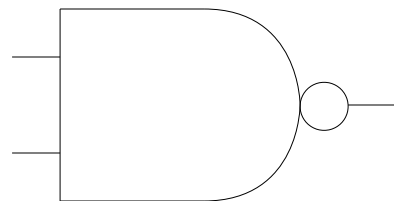
- Two-level circuits are fast because transitions only propagate through at most two levels of logic.
  - Again, not including input inverters.
- Make two-level circuits for:
  - $F = (!A)BC + DE$
  - $F = !A + BC$
  - $F = AB + C( D + E )$
  - $F = A( B + C( D + E ) )$
  - $F = !( A ( B + C( D + E ) ) )$

00101010010101000011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
100101001001001010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
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10010101010101010101010101010101

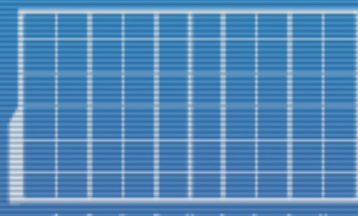


# Circuit Delay

- **Fan-in** refers to the number of digital inputs a logic gate has.
- Since we're talking about making two-level circuits, wouldn't it be nice to have 128-input versions of our current logic gates?

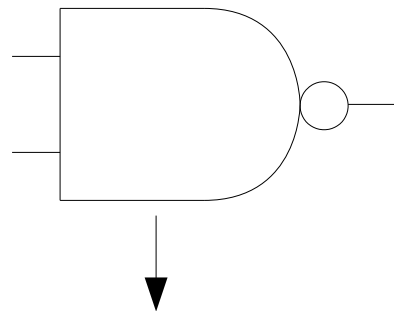


00101010010101000011110100001100  
10001100100001111001101010010101  
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11100001111010110000000111101001  
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100101010101001010101010010101

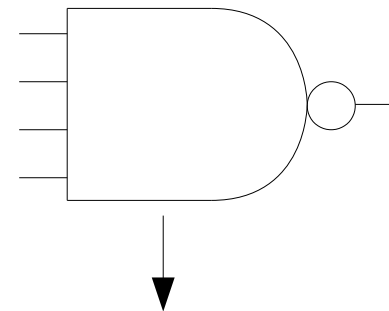


# Circuit Delay

- Higher fan-in may mean more transistors in series.
- This means higher resistance encountered.
- Since gates are often connected to each other, this means greater **RC** delays.

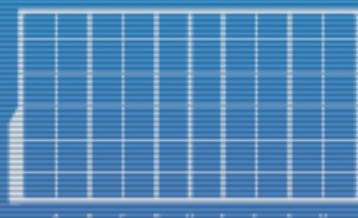


2 NFETs  
in series  
in CMOS



4 NFETs  
in series  
in CMOS

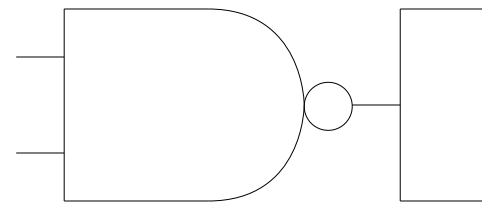
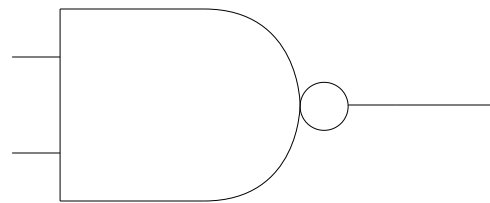
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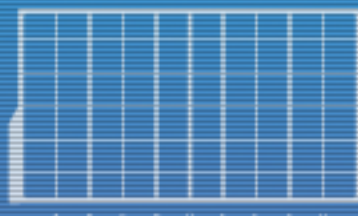
DISCS

# Circuit Delay

- **Fan-out** refers to the maximum number of input nodes that a logic gate can output to.
- Chip makers usually specify this (and voltage and current) to guarantee their advertised  $t_{pd}$ .



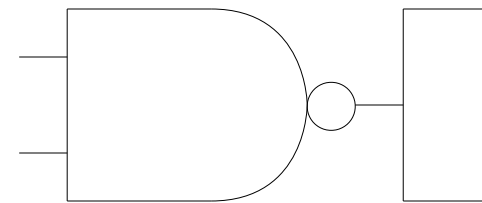
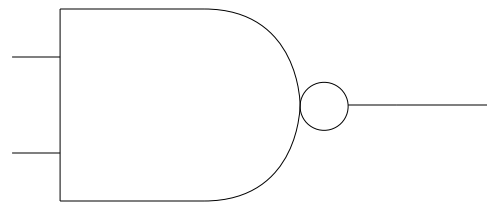
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10010100101010010100101010010101  
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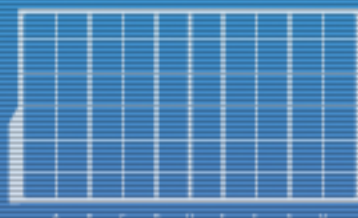
# Circuit Delay

- Higher fan-out means higher capacitance encountered.
- This means greater RC delays.



higher load  
capacitance

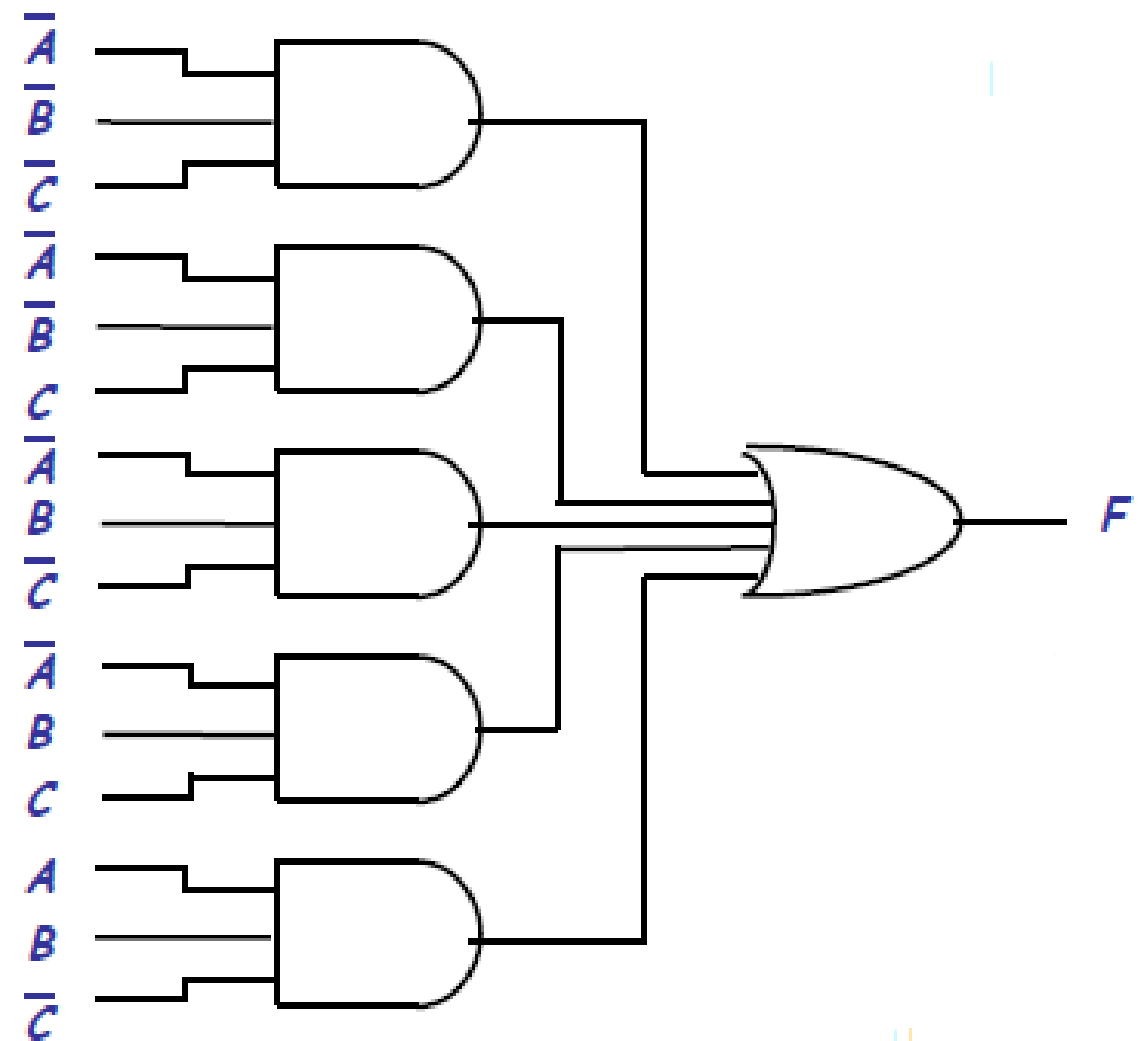
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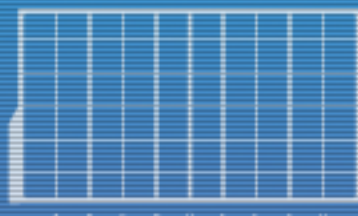
# Truth Tables and Logic Circuits

- Truth tables define Boolean functions...
- ... which can be implemented as 2-level logic circuits.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0



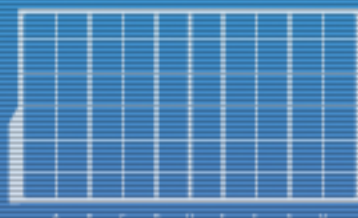
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 11100001111010110000000111101001  
 0010010101001010010010010010110  
 10010100100001010100100101001010  
 10010100101010010100101010010101  
 10010101010101010101010101010101



# Can We Do Better?

- ANDs tend to be either NANDs with an output inverter or NORs with input inverters.
- ORs tend to be either NORs with an output inverter or NANDs with input inverters.
- This will affect transistor count!

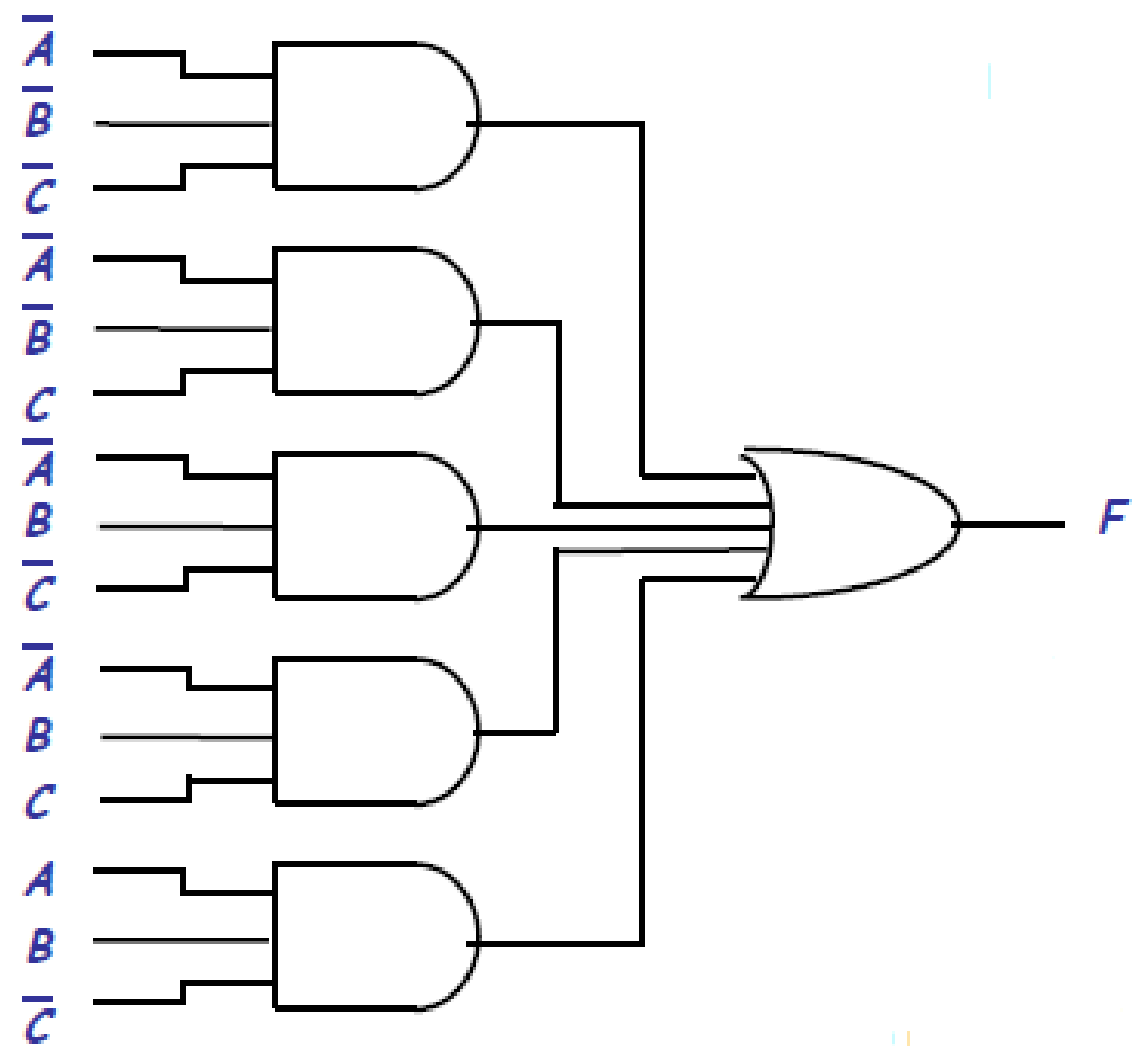
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10010101010101010101010101010101



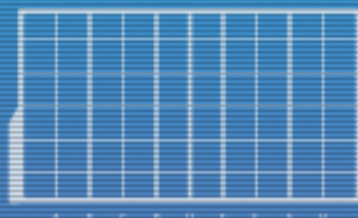
DISCS

# Expensive AND Slow

- Note: Assume !A, !B, and !C are already available.
- 3-input AND gate  
= 3-input NAND + inverter  
=  $6 + 2 = 8$  transistors
- 5 of those, so...  
=  $5 * 8 = 40$  transistors
- 5 input OR gate  
= 5-input NOR + inverter  
=  $10 + 2 = 12$  transistors
- Total  
=  $40 + 12 = 52$  transistors



0010101001010100011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
1110000111101010000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101

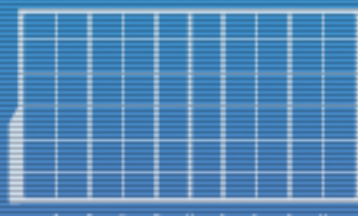




# Does it have to be AND-OR?

- DeMorgan's Law states that an OR gate with inverted inputs is a NAND gate.
  - While we are *logically* adding inverters, we are actually converting the OR to a NAND and therefore *physically* removing inverters.
- We can't just add an inverter to each input wire connecting to the OR gate – that will change the actual Boolean formula.
  - If I add an inverter, what can I add to cancel it out?

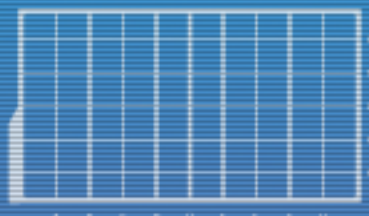
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10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101



# Does it have to be AND-OR?

- We add two inverters to each input wire connecting to the OR gate!
  - This will allow us to replace the OR with NAND (one inverter from each input wire used)
- What about the second inverter?
  - Each input wire of the OR gate is the output wire of what gate?

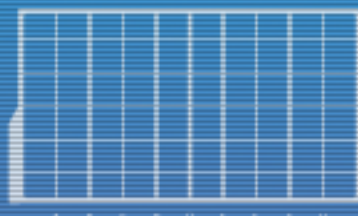
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001001010100101001001010010010110  
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10010101010101010101010101010101



# Does it have to be AND-OR?

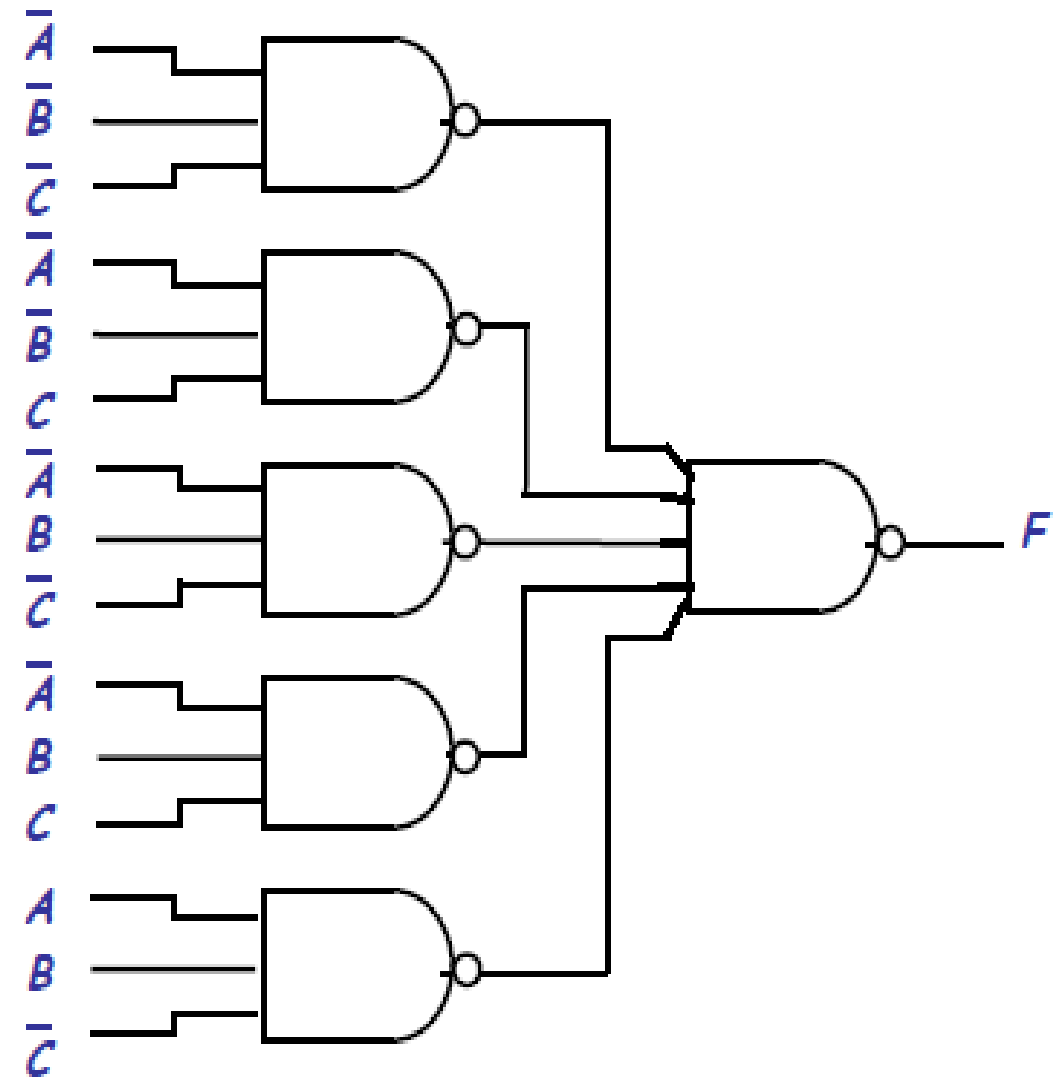
- An AND gate with an inverter at its output is simply a NAND gate.
  - We can replace the ANDs with NANDs (another inverter from each input wire used)
- We *logically* added inverters...
- ... to *physically* remove inverters...
- ... and *improve performance* (reduce  $t_{pd}$  /  $t_{cd}$ )!

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
100101010101001010101010010101

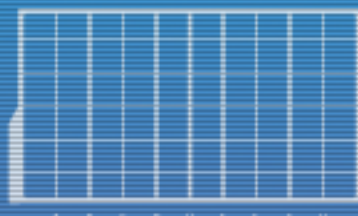


# ... Still Expensive

- Note: Assume !A, !B, and !C are already available.
- 3-input NAND gate  
= 6 transistors
- 5 of those, so...  
=  $5 * 6 = 30$  transistors
- 5 input NAND gate  
= 10 transistors
- Total  
=  $30 + 10 = 40$  transistors



0010101001010100011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101

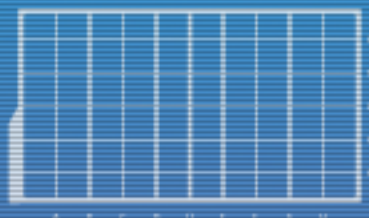




# Can We Do Better?

- Do you remember how to simplify logic equations?
  - To be reviewed.
- Do you remember how to use Karnaugh maps?
  - Not to be reviewed, but you can use them during quizzes/exercises/tests if you want to.

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101



DISCS

# Review: Simplifying Logic

- Given logic functions  $f$ ,  $g$ , and  $h$ :

$$(f) (f) = f \qquad f + f = f$$

$$(f) (1) = f \qquad f + 1 = 1$$

$$(f) (0) = 0 \qquad f + 0 = f$$

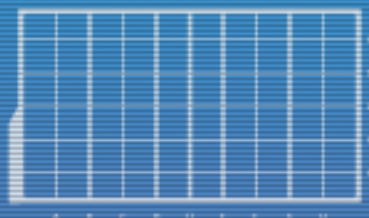
$$(f) (!f) = 0 \qquad f + !f = 1$$

- Distributive Laws

$$f (g + h) = fg + fh$$

$$(f + g) (f + h) = f + gh$$

0010101001010100011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
100101001001001010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101



DISCS

# Review: Simplifying Logic

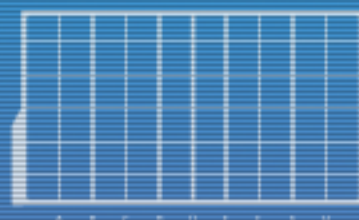
$$F = (!A!B!C) + (!A!B C) + (!A B!C) + (!A B C) + ( A B!C)$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + ( A B!C)$$

$$= \underline{\hspace{2cm}} + ( A B !C)$$

$$= \underline{\hspace{2cm}}$$

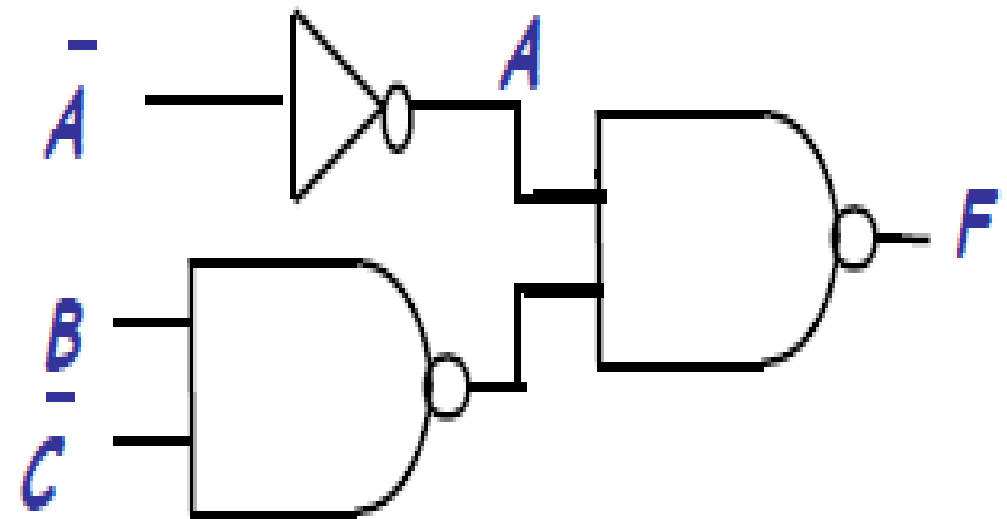
00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
100101010101001010101010010101



DISCS

# Cheap AND Fast

- Note: Assume  $\bar{A}$ ,  $\bar{B}$ , and  $\bar{C}$  are already available.
- Inverter (1-input NAND?)  
= 2 transistors
- 2-input NAND  
= 4 transistors
- Another 2 input NAND  
= 4 transistors
- Total  
=  $2 + 4 + 4 = 10$  transistors



00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010101010101010101010101010101

