



DEPARTMENT OF INFORMATION SYSTEMS AND COMPUTER SCIENCE



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										02
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										04
										05
A	B	C	D	E	F	G	H			

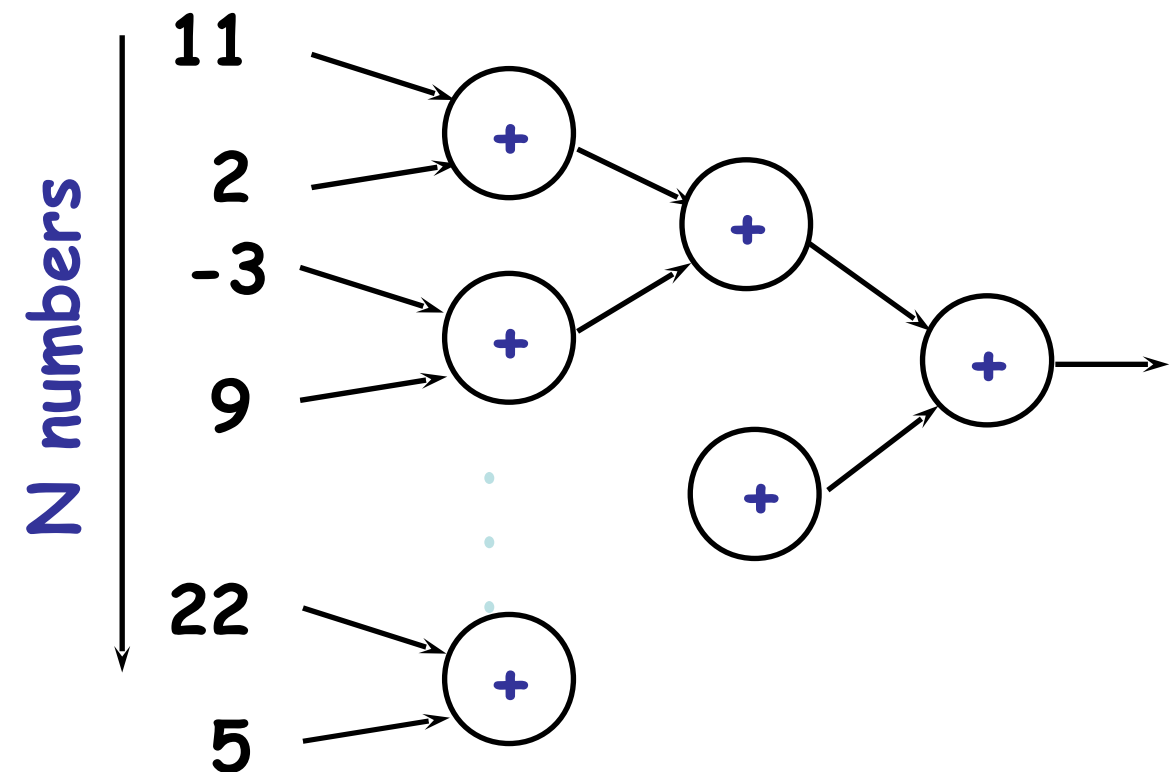
Feedback, State, and Memory

Cyclic Circuits – Can They Work?

Adding a column of N numbers

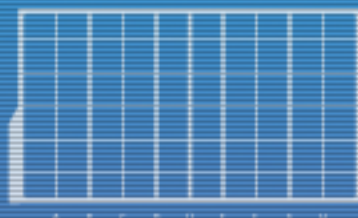
- ▶ Form tree of adders?
- ▶ Delay is $O(\log_2 N)$
- ▶ Size grows as $O(N)$
 - ▶ $(N/2) + (N/4) + (N/8) \dots = N-1$
 - ▶ or, $1 + 2 + 4 + 8 + 16 + \dots + N/2 = N-1$
 - ▶ or, think of it as a tennis tournament: each player loses *exactly* once except for champion. Therefore, $N-1$ matches.

Combinational



- ▶ **Not scalable to arbitrary N!**
 - ▶ **Need infinite hardware!**
- ▶ **Not practical for general CPU.**

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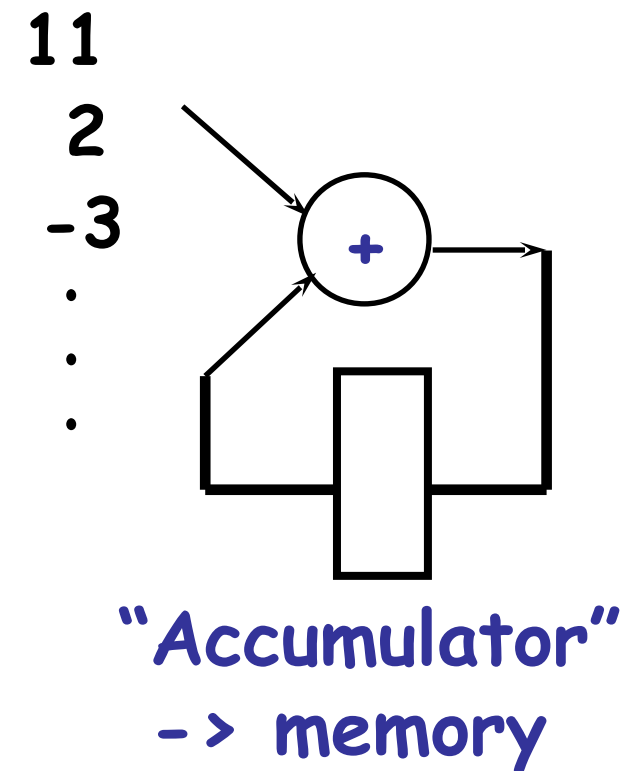


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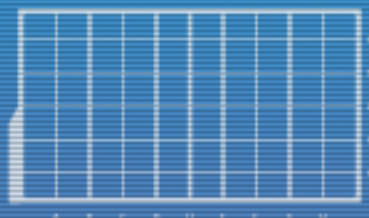
Adding a column of N numbers

- ▶ Use an “accumulator”?
 - ▶ *Remember* previous value and feed it to next input: NEED MEMORY
- ▶ Called “Sequential”
 - ▶ Because things happen “in sequence” over time.
- ▶ Size is constant!
- ▶ Delay is $O(N)$ “cycles”
- ▶ A bit slower, but definitely more practical and scalable to arbitrary N.

Sequential



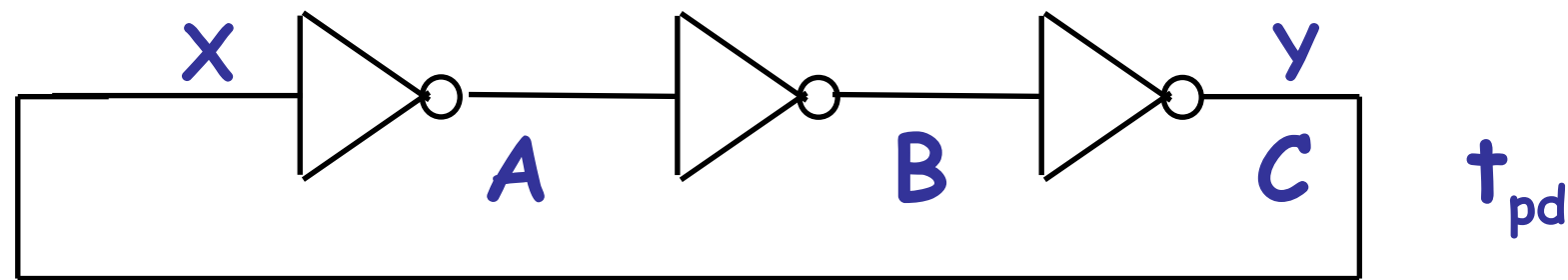
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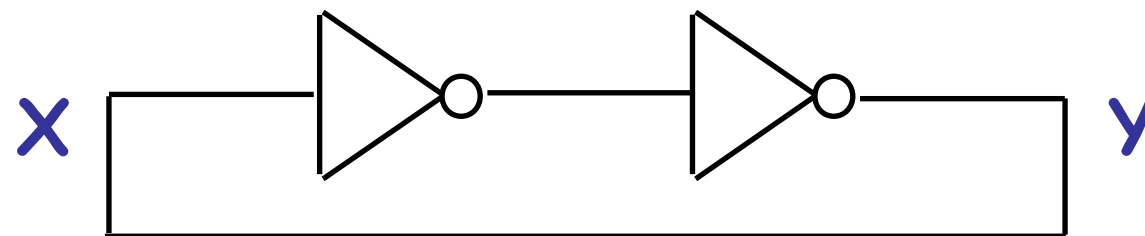
Implementing Memory: Inverter Ring?

- ▶ Odd-number cycle --- unstable: values always change



$$V_y = \overline{\text{old } V_x} = \text{new } V_x \quad \leftarrow \text{oscillates!}$$

- ▶ Even-number cycle --- stable (?): values don't change

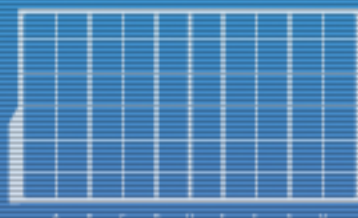


$$V_y = \text{INVERT}(\text{INVERT}(V_x))$$

$$V_y = V_x$$

(potentially) stable

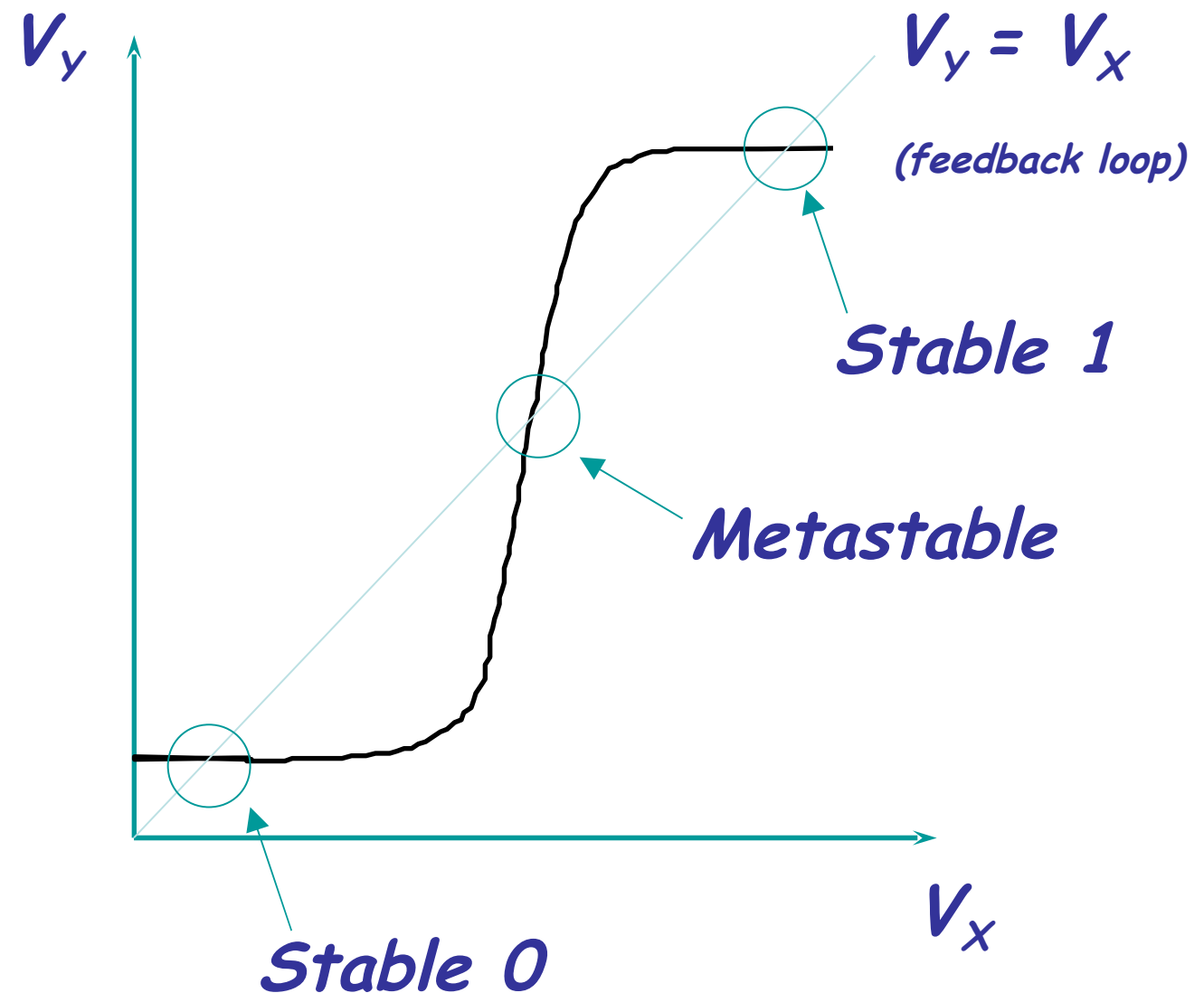
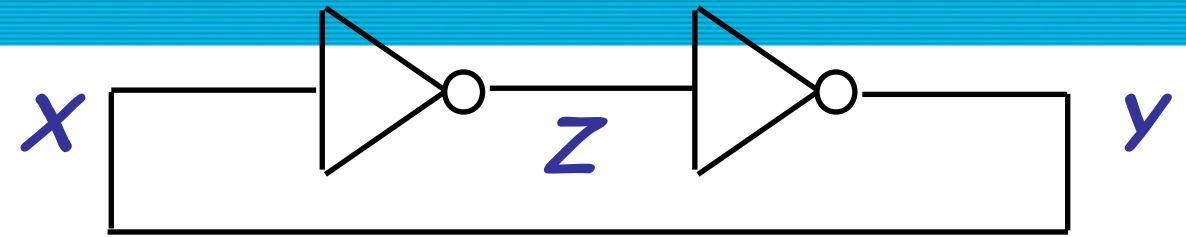
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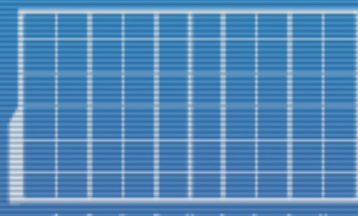
DISCS

A Closer Look

- ▶ 3 points on output curve where $V_Y = V_X$
- ▶ Stable 0 and 1 (2 points)
 - ▶ Slight deviation away from stable point will get pushed back by high-gain part.
 - ▶ Value will hold. (Slight deviations will correct themselves.)
- ▶ Metastable state (3rd point)
 - ▶ Slight deviation will push it quickly to either 0 or 1.



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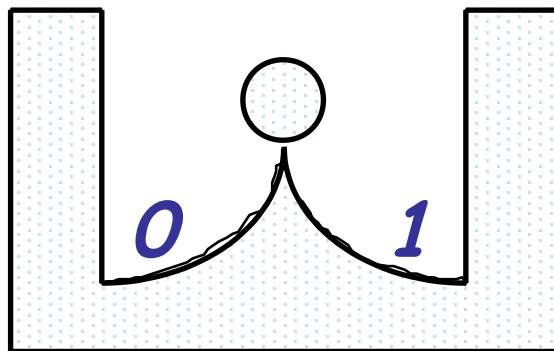


DISCS

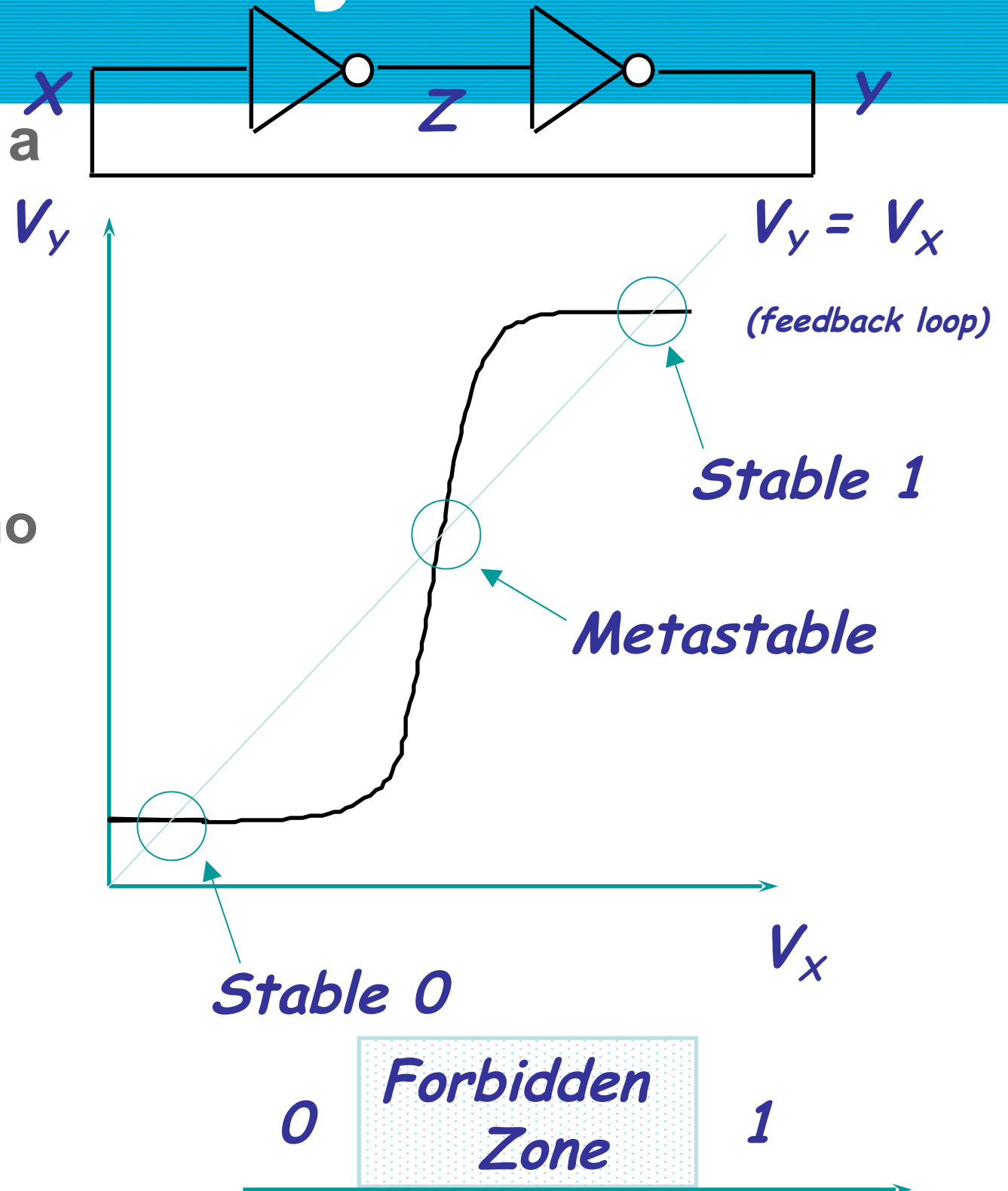
Metastability

- ▶ Like having a ball balanced on a hill:

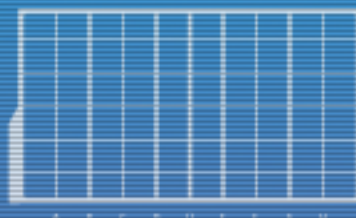
- ▶ Probability of staying metastable gets exponentially small in time, but always non-zero.
- ▶ There is always a tiny chance that it will remain metastable no matter how much time passes.



- ▶ Caused by feeding forbidden values into loop.
 - ▶ Therefore, avoid doing so.



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100101001001001010101010101010101
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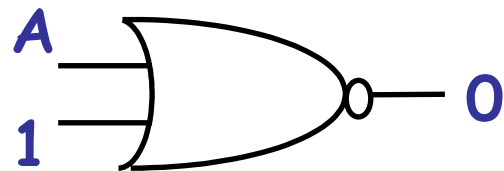
DISCS

Setting 0 or 1: the SR Latch

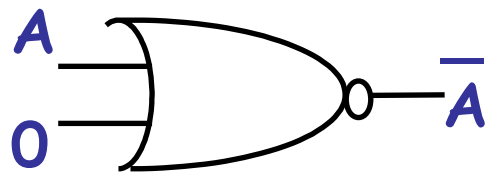
- ▶ Problem with Inverter Ring: No inputs!
- ▶ Even if it works:
 - ▶ Need “controllable” inverters!

▶ NOR

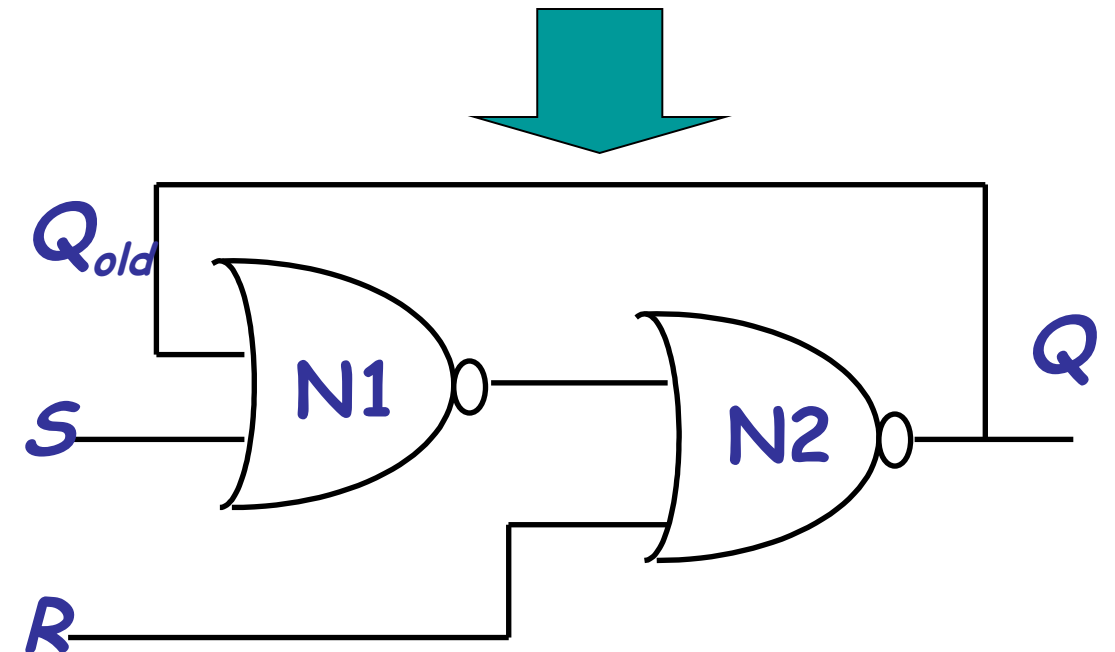
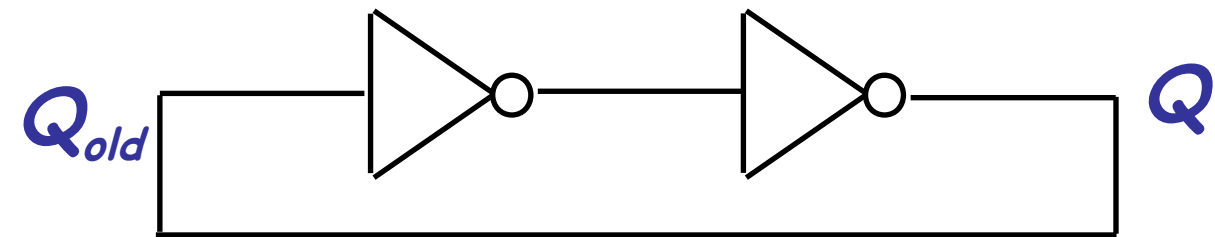
- ▶ 1 -> always 0



- ▶ 0 -> inverter

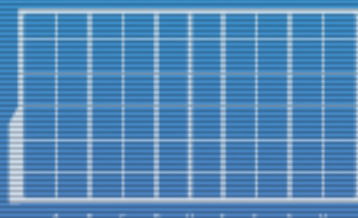


- ▶ Control configuration via S and R.



S	R	Q
0	0	Q_{old}
0	1	0
1	0	1
1	1	0

00101010010101000011110100001100
10001100100001111001101010010101
11001010101010100001001100101010100
100101001001001010101010101010101
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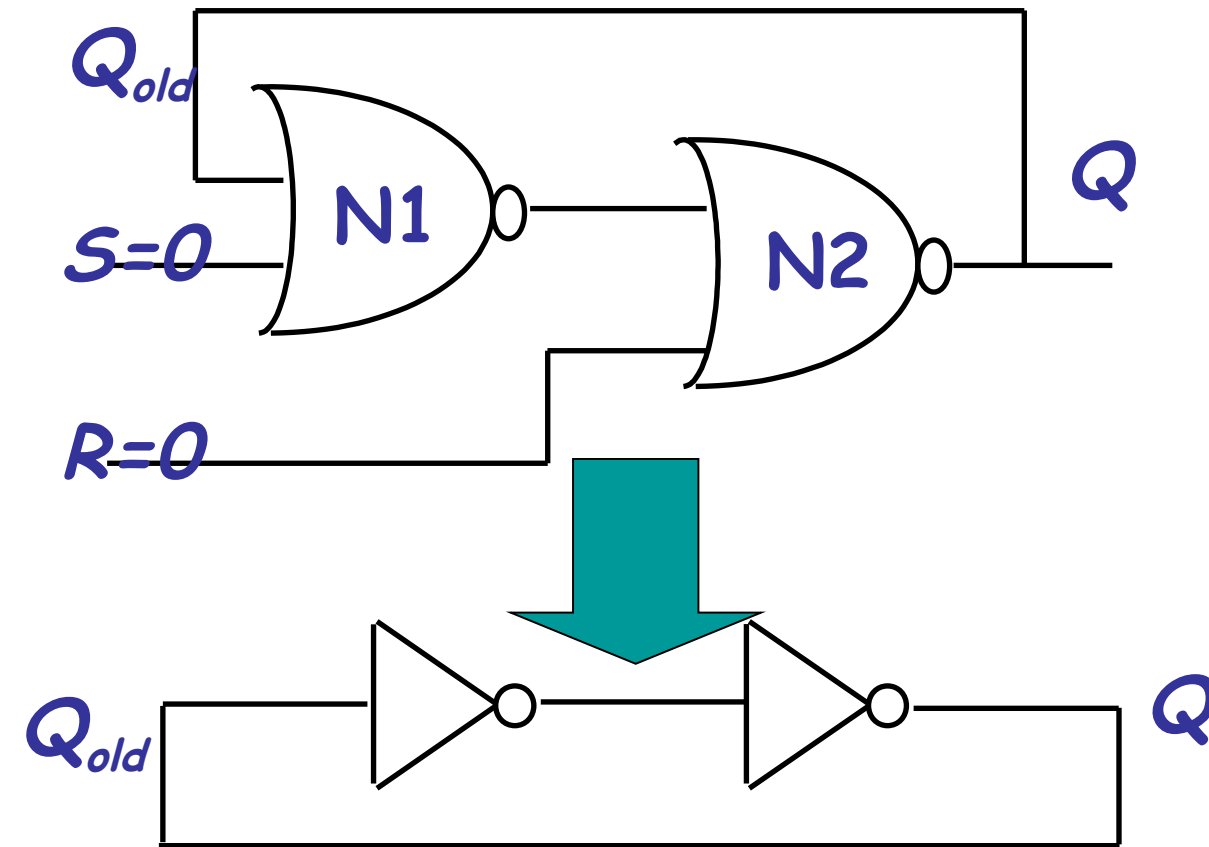
DISCS

NOR-NOR SR Latch

► $S=0, R=0 \rightarrow$ “Latch”

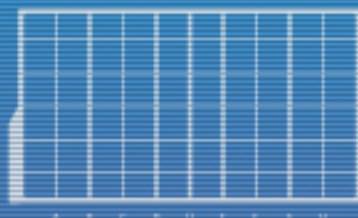
► Inverter loop

► $Q = Q_{old}$ (keep old value)



S	R	Q
0	0	Q_{old}
0	1	0
1	0	1
1	1	0

00101010010101000011110100001100
10001100100001111001101010010101
11001010101010100001001100101010100
100101001001001010101010101010101
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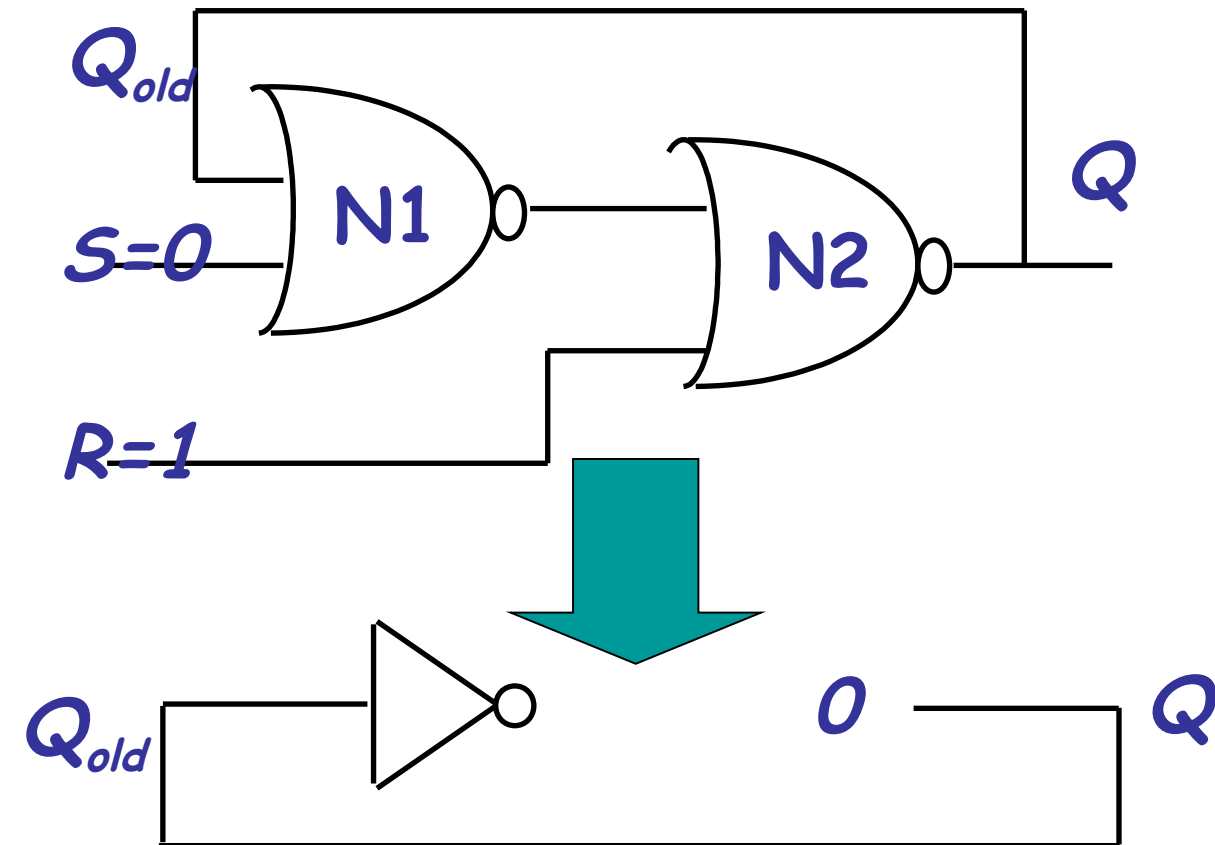
DISCS

NOR-NOR SR Latch

► $S=0, R=1 \rightarrow$ “Reset”

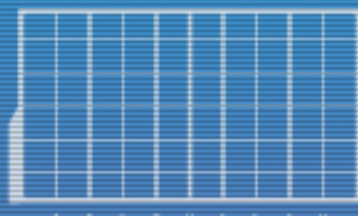
► N2 output always 0

► $Q = 0$



S	R	Q
0	0	Q_{old}
0	1	0
1	0	1
1	1	0

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10001100100001111001101010010101
110010101010101000010011001010100
1001010010010010101010101010101
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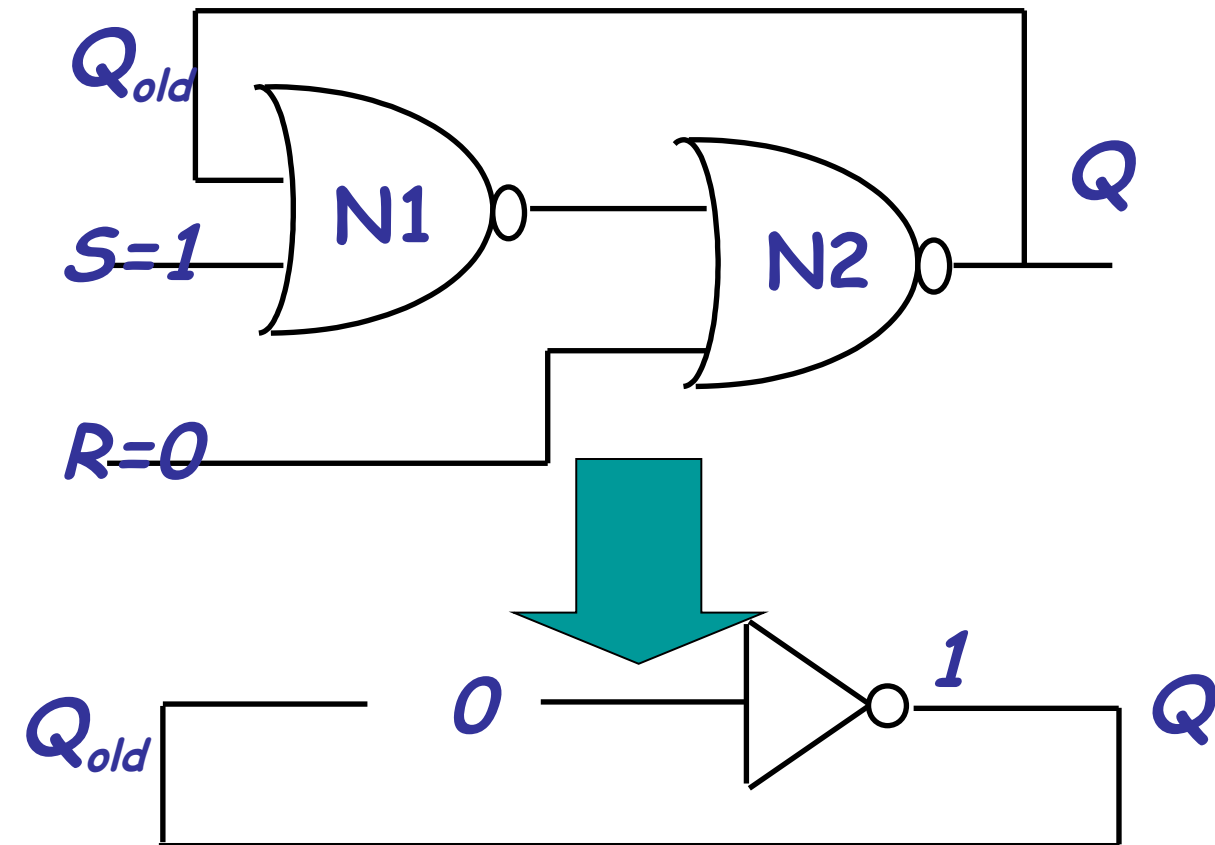


DISCS

NOR-NOR SR Latch

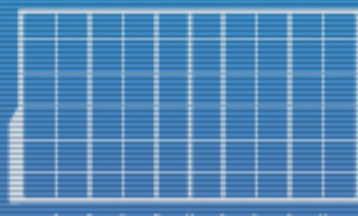
► $S=1, R=0 \rightarrow$ “Set”

► N1 output always 0, N2 inverts $\rightarrow Q = 1$



S	R	Q
0	0	Q_{old}
0	1	0
1	0	1
1	1	0

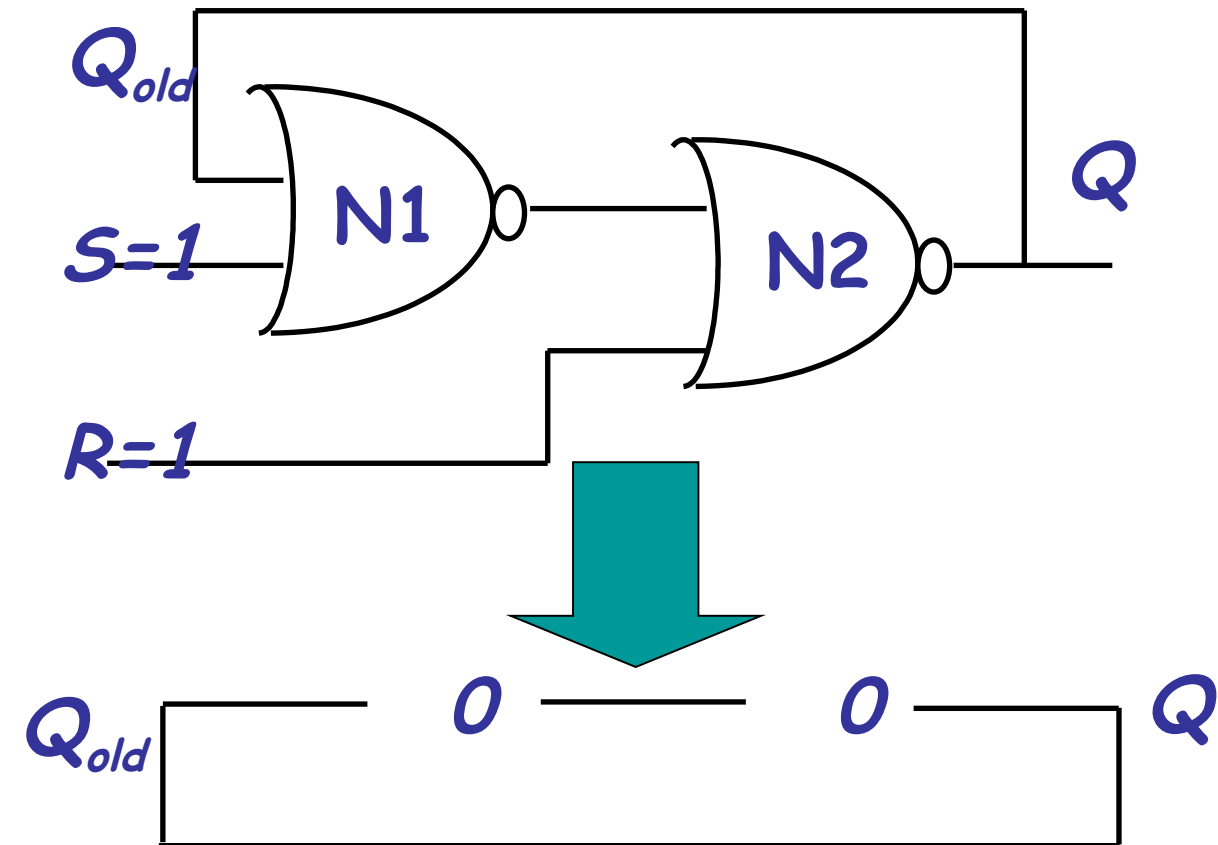
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DISCS

NOR-NOR SR Latch

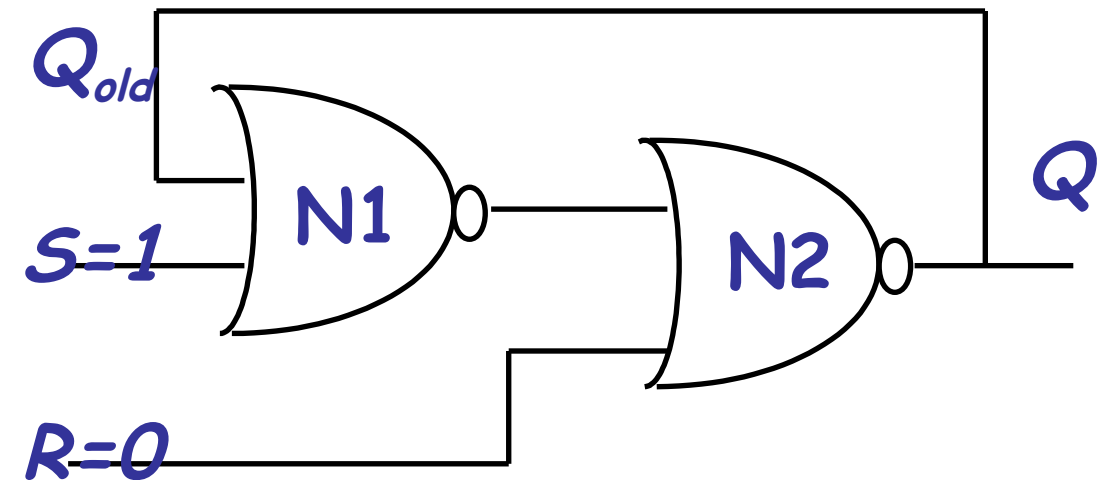
- ▶ **S=1, R=1 (avoid)**
 - ▶ results in $Q = 0$, BUT, cannot *safely* switch back to $S=0, R=0$



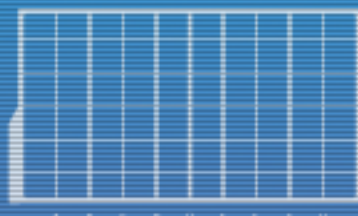
S	R	Q
0	0	Q_{old}
0	1	0
1	0	1
1	1	0

NOR-NOR SR Latch

- ▶ To use, either **SET** ($S=1, R=0$), or **RESET** ($S=0, R=1$).
- ▶ Then, hold.
 - ▶ Long enough for signals to propagate around loop.
- ▶ Then, **LATCH** by setting $S=0, R=0$.
- ▶ **NEVER** go from $S=1, R=1$ to $S=0, R=0$!
 - ▶ Both S and R are changing.
 - ▶ No way to know which one will get latched.
 - ▶ Possible metastable state if caught at forbidden values!

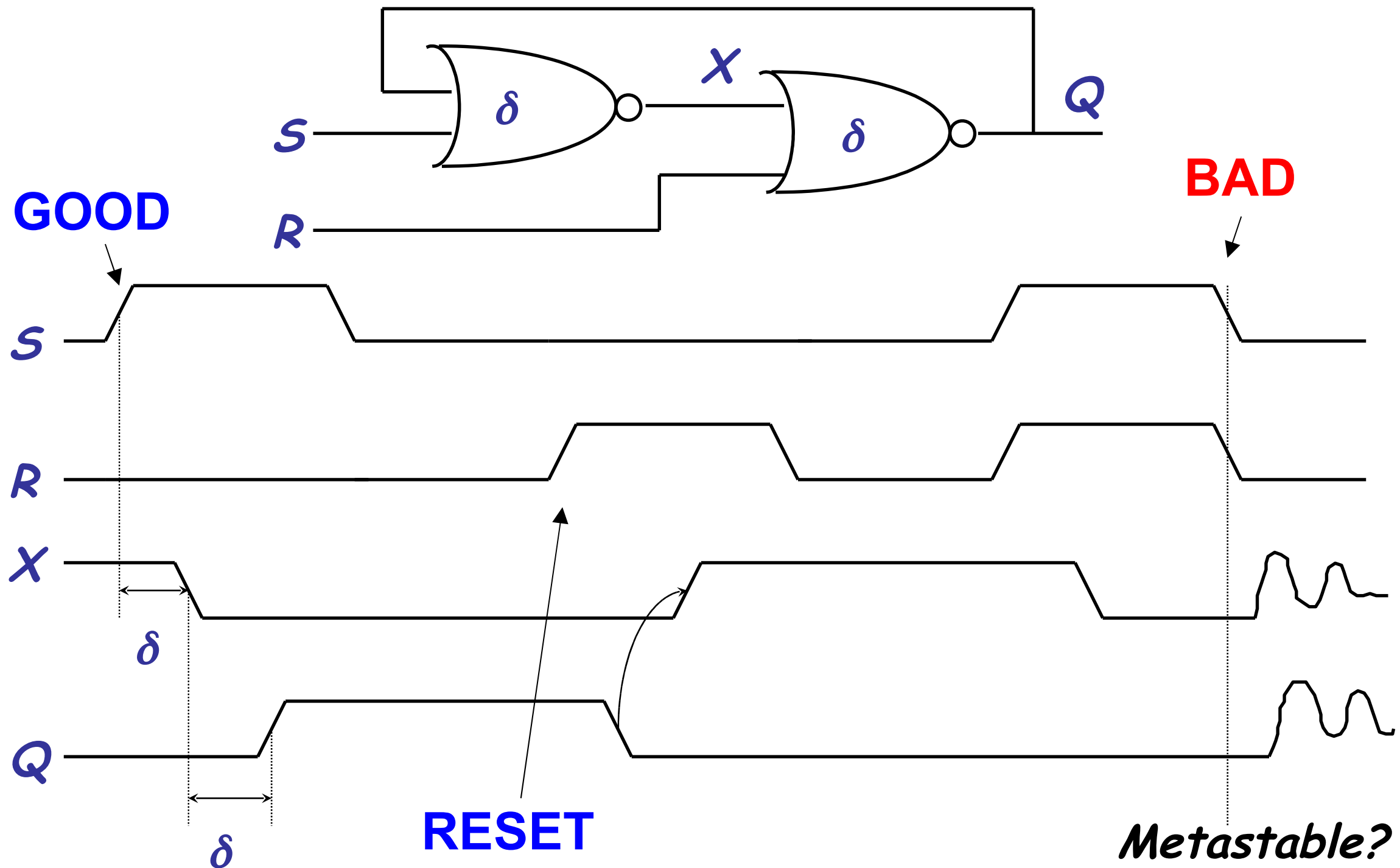


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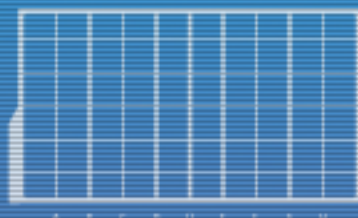


DISCS

S-R Latch Timing



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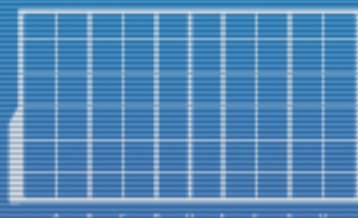


DISCS

Timing Rules

- ▶ As much as possible, only one input can change “at the same time”.
 - ▶ Single input change or SIC rule.
- ▶ We also should allow input transitions only after state has settled!
 - ▶ We risk corrupting output if input changes while state/output is not yet stable.
 - ▶ This constraint will be quantified in terms of setup time and hold time.

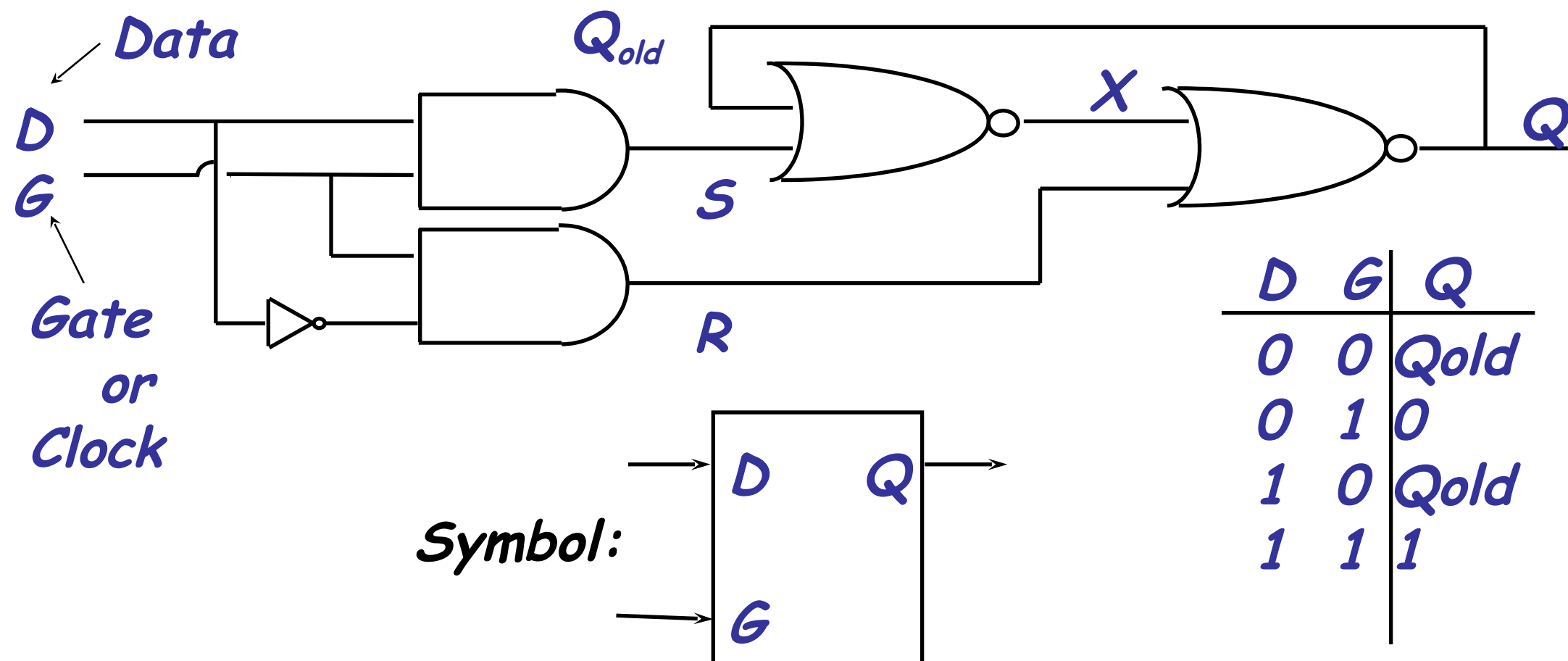
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DISCS

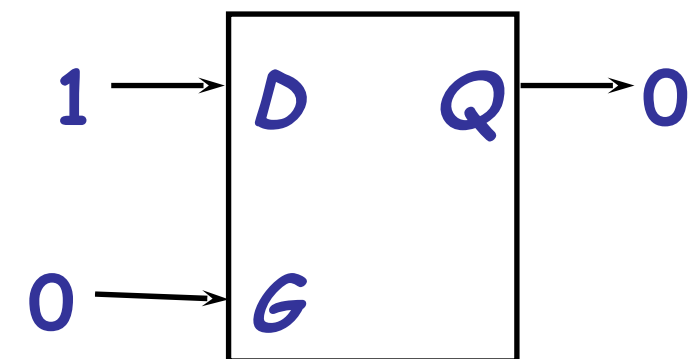
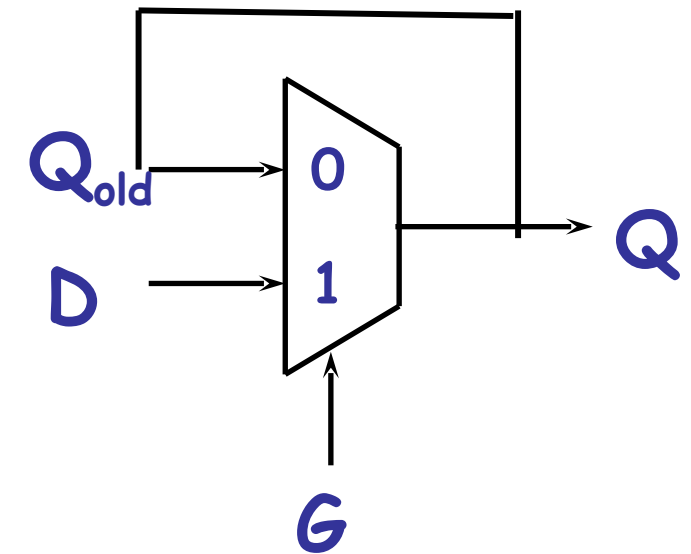
D-Latch or Transparent Latch

- ▶ If Gate is 0, then both S and R = 0.
- ▶ If Gate is 1, then $S = D$, $R = \sim D$.
 - ▶ Also, $Q = D$, hence the name.

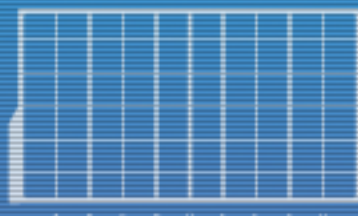


Another implementation of D-Latch

- ▶ Can be seen as a mux loop:
 - ▶ $Q = D$, if $G=1$ “transparent”.
 - ▶ $Q = Q_{old}$ if $G=0$ “latched”.
- ▶ Or, can be seen as a “gate”:
 - ▶ $G=1$ “open”
 - ▶ $G=0$ “closed”
- ▶ Different from CMOS transmission gate because it has memory.
 - ▶ Transmission gate outputs hi-Z (disconnected) when closed .
 - ▶ D-latch outputs old value.

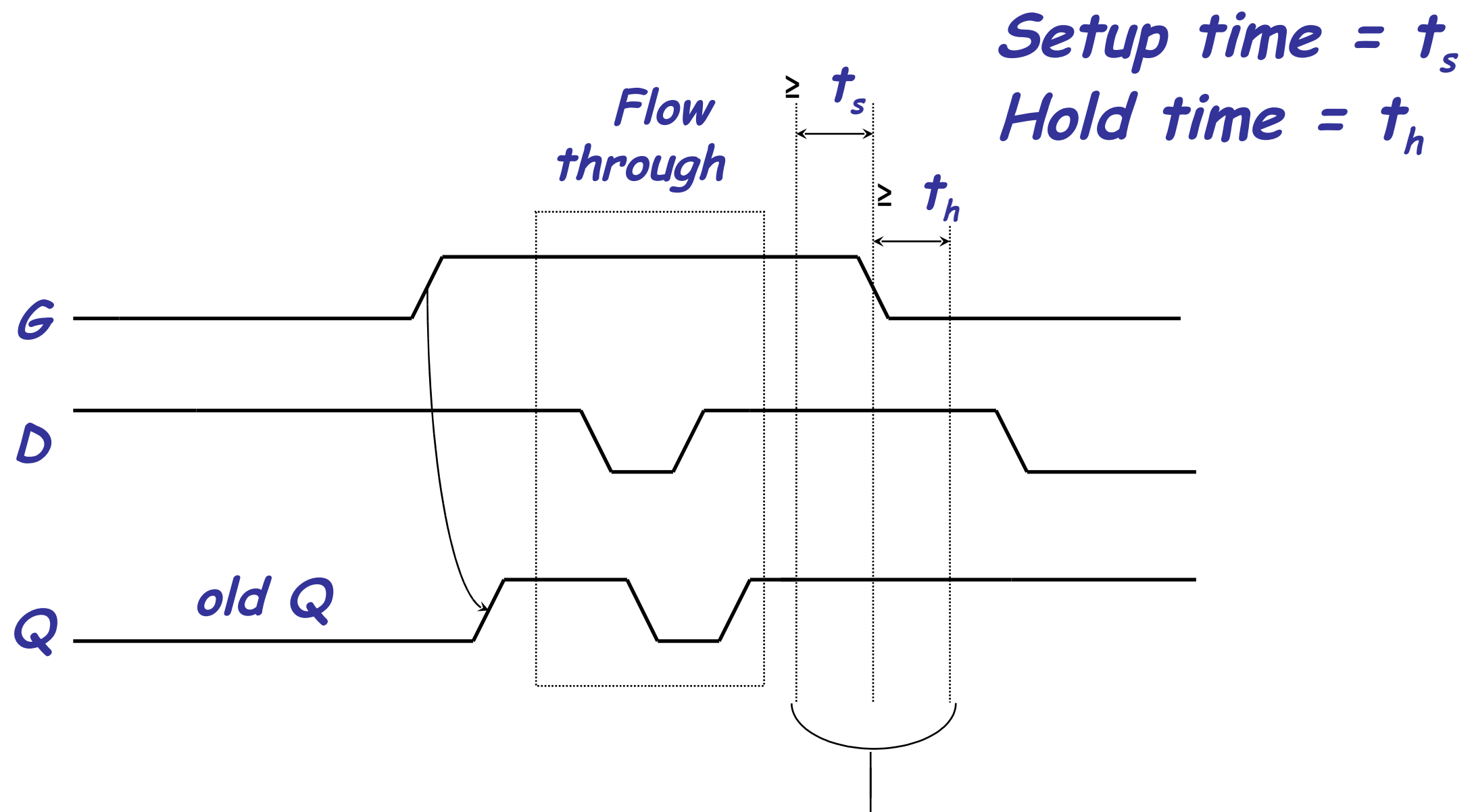


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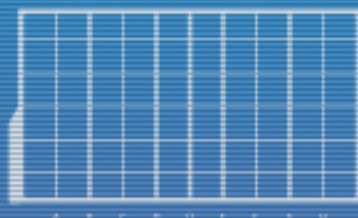
DISCS

Timing of D-Latch



D must not change in this region

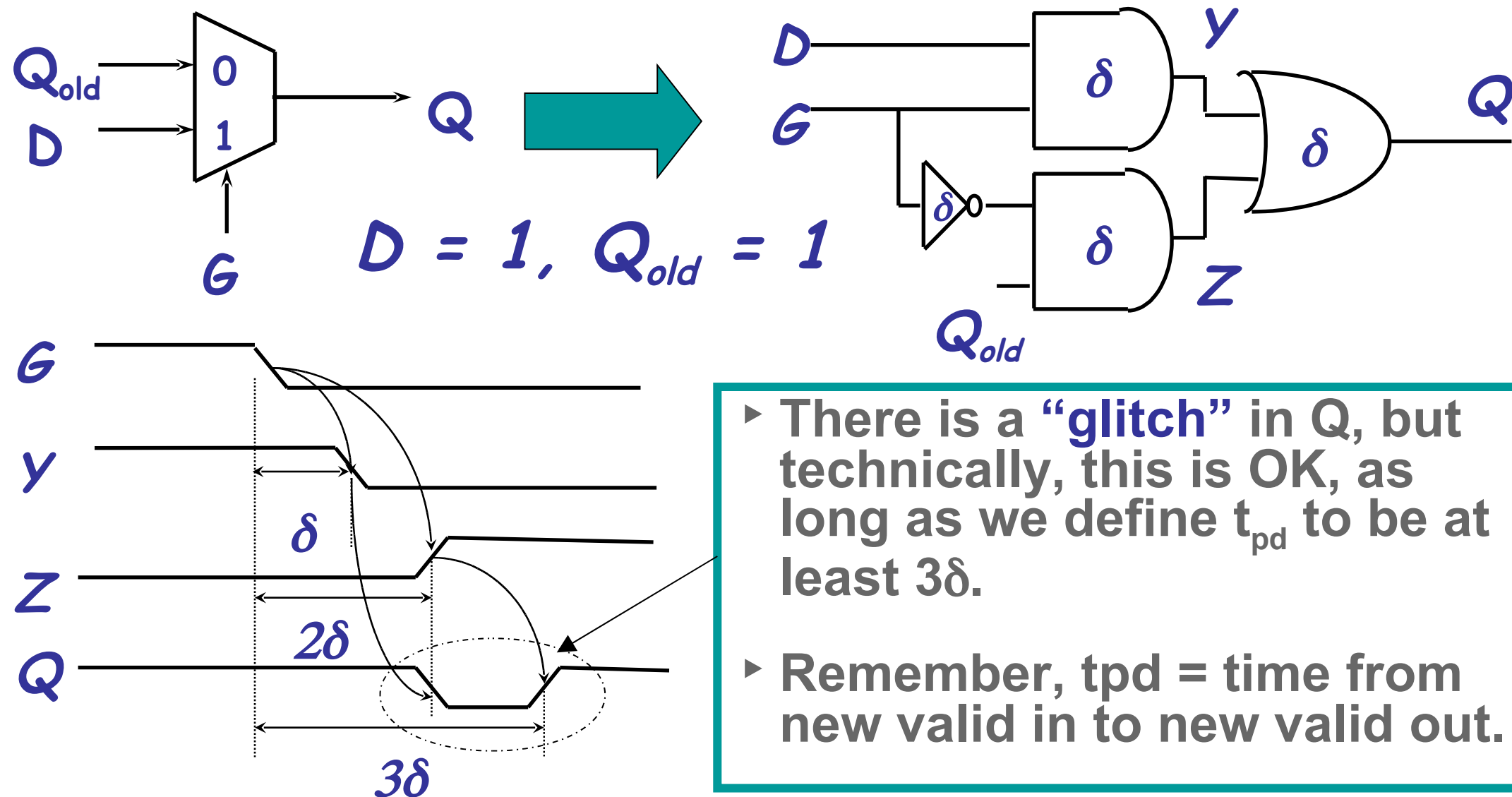
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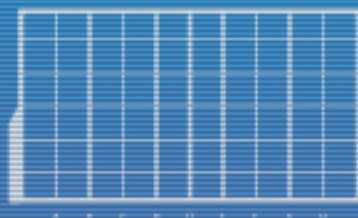
DISCS

Reality Bites: Hazards

- In combinational logic, we don't care what happens between t_{cd} and t_{pd} .



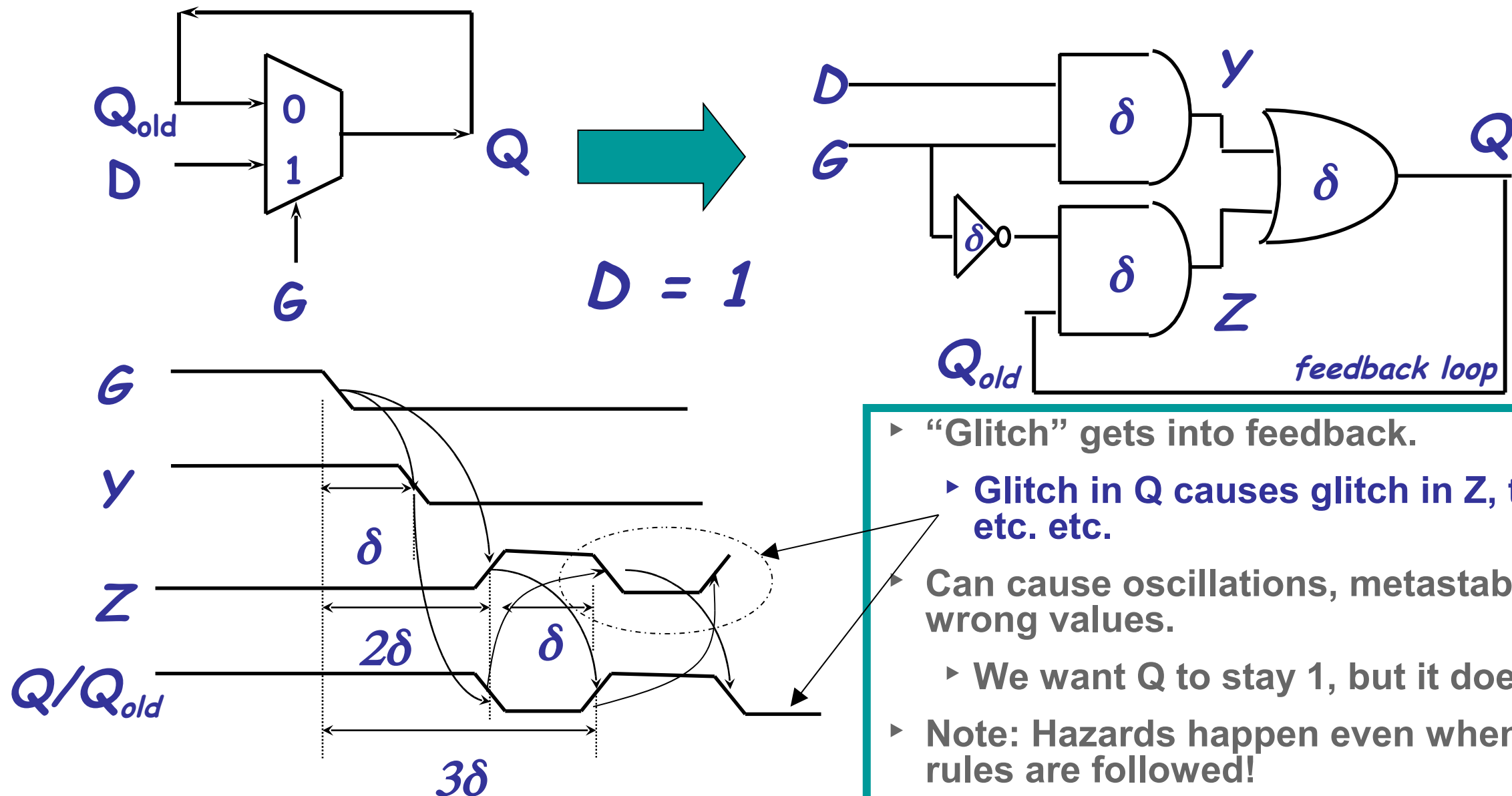
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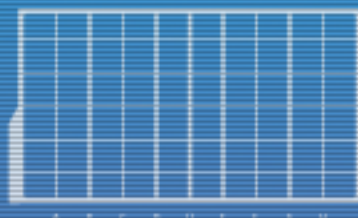
DISCS

Reality Bites: Hazards

- But what happens when we add a feedback loop? (e.g., mux loop D-latch)



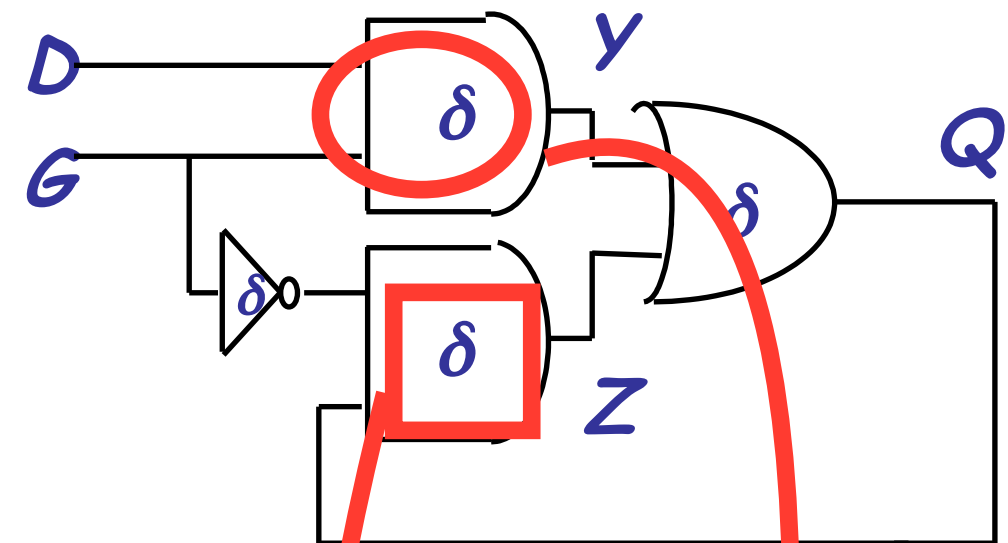
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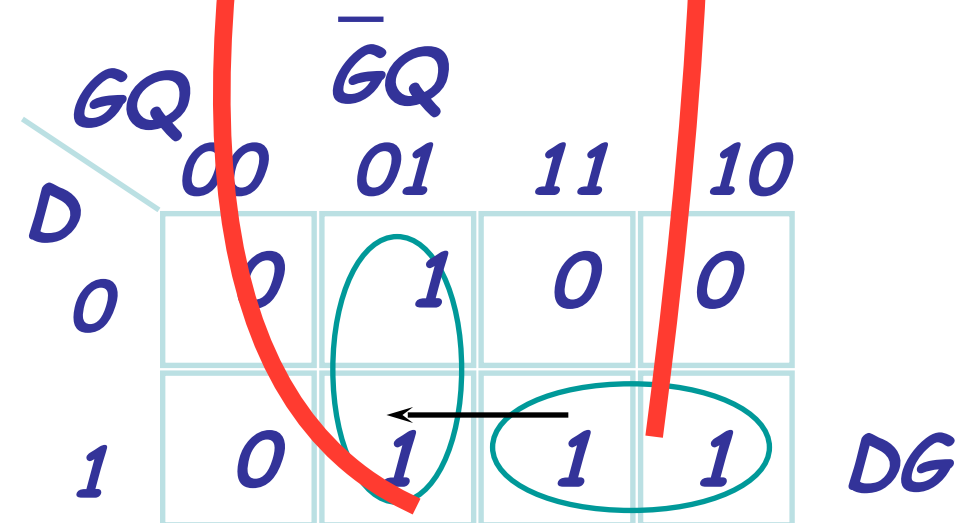
DISCS

Fixing Hazards

- ▶ Hazard due to input moving from DG to GQ.
 - ▶ Inputs to OR *momentarily* both 0, due to delays.
 - ▶ “Glitch” in OR output.
- ▶ Think of patches as “umbrellas”.
 - ▶ We “get wet” crossing from patch to patch.
- ▶ Solution: Add patches to cover gaps!

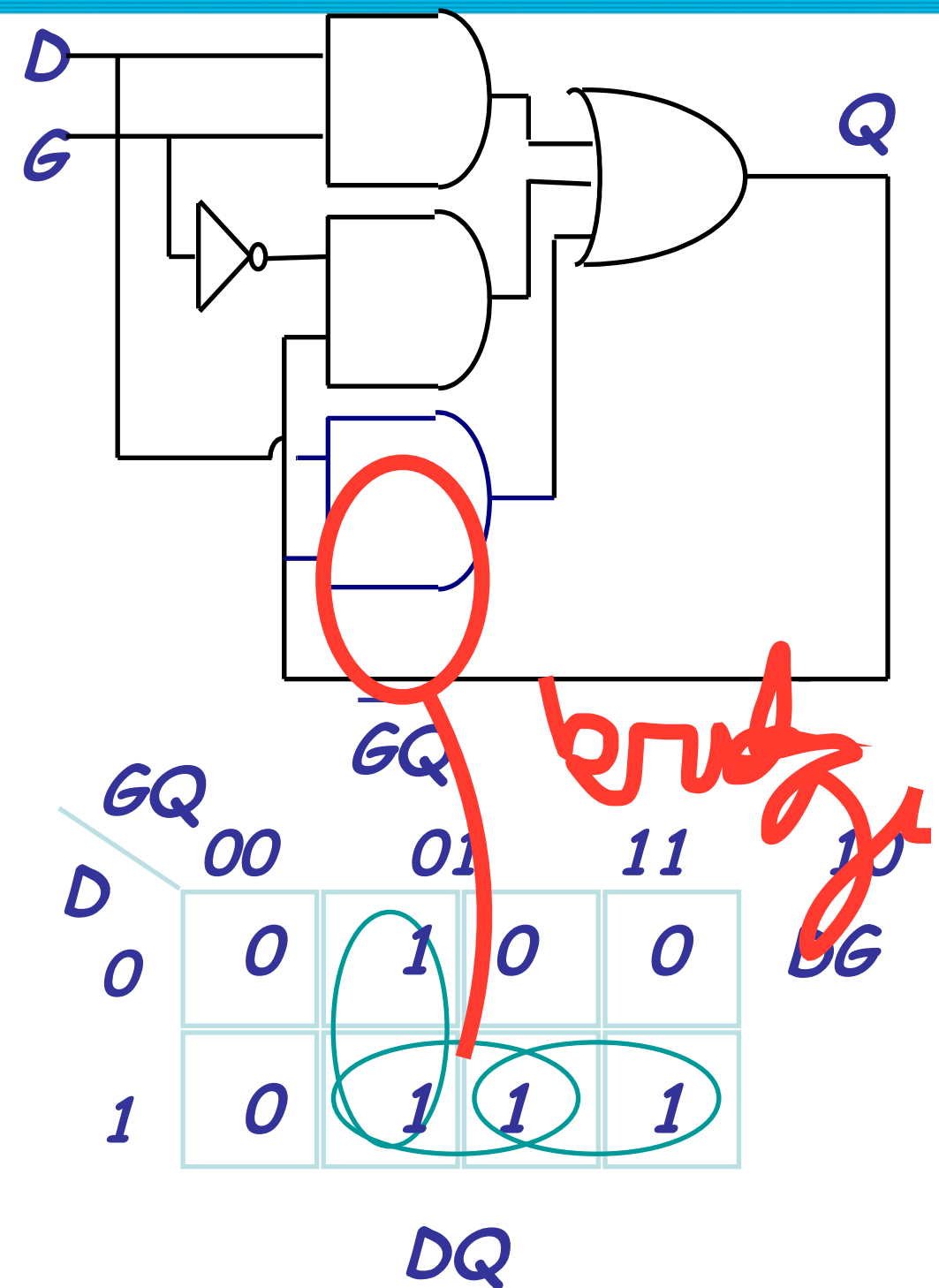


Karnaugh Map:



Fixing Hazards

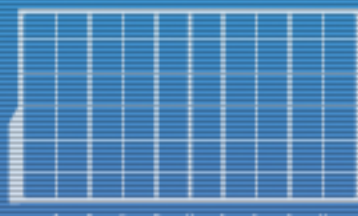
- ▶ Add patches to cover gaps in K-map.
 - ▶ Now, OR gate output is held high by DQ when crossing from DG to GQ.
- ▶ Assumes *hazard-free (glitch-free) gates*.
 - ▶ No glitches due to irrelevant inputs.
 - ▶ No glitches due to SIC.
 - ▶ e.g., OR output stable at 1, as long as at least 1 input is 1 – regardless of how the other 2 inputs change.



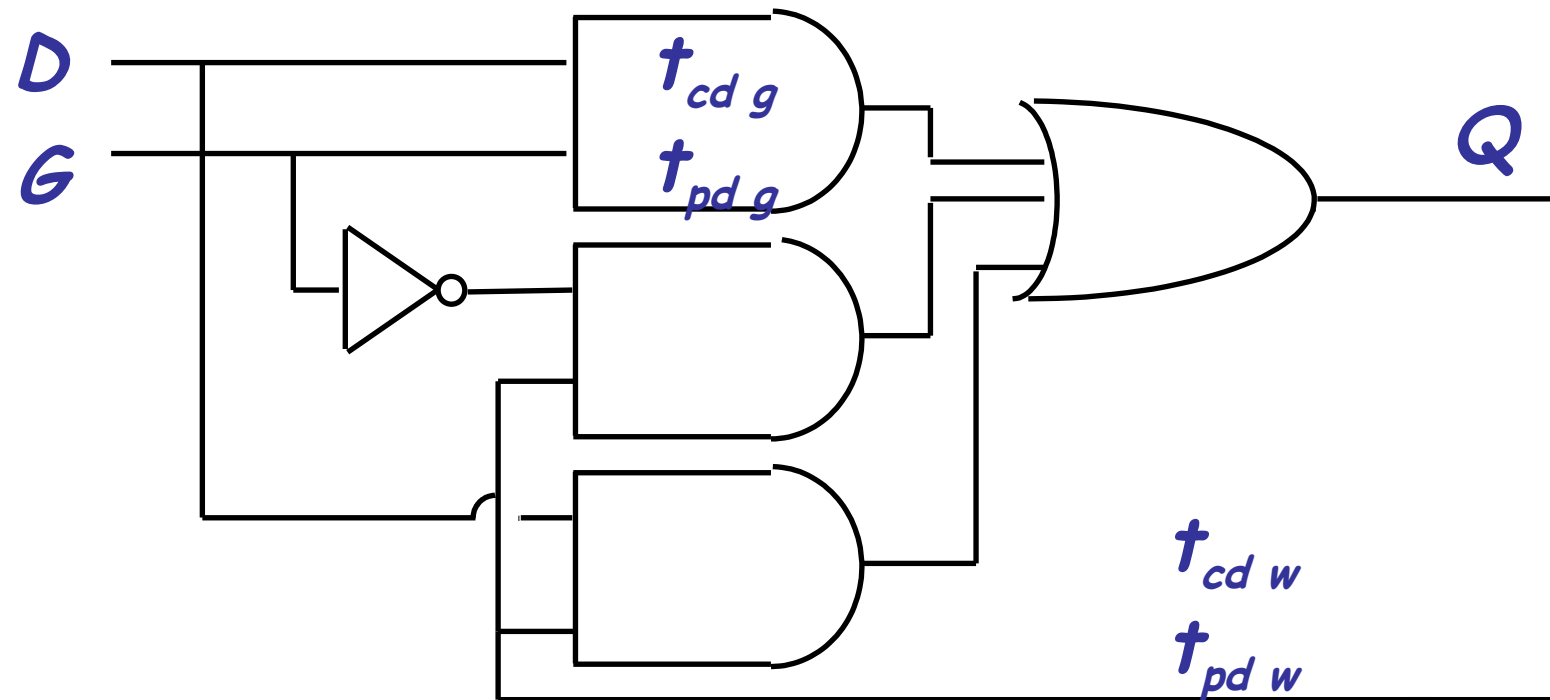
Hazards Synopsis

- ▶ **Hazards:** “Glitches” in output behavior *even if we follow SIC rule.*
 - ▶ If SIC is not followed, then glitches are hard to avoid because of race conditions (think of the inputs as starting positions).
 - ▶ Hazards are glitches that occur *even if only one input is changed.*
- ▶ **Problem:** If glitches are part of feedback loop, then bad values propagate!
 - ▶ Results include oscillations, metastability, wrong values, etc.
- ▶ **Solution:** Cover gaps in K-map.
 - ▶ Add gates to connect all patches .
 - ▶ Protects against SIC hazards only.

```
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11001010101010100001001100101010100
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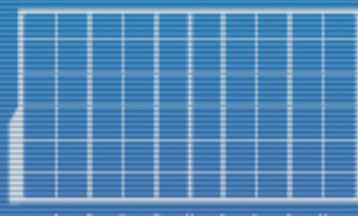


Reality Bites (Again)



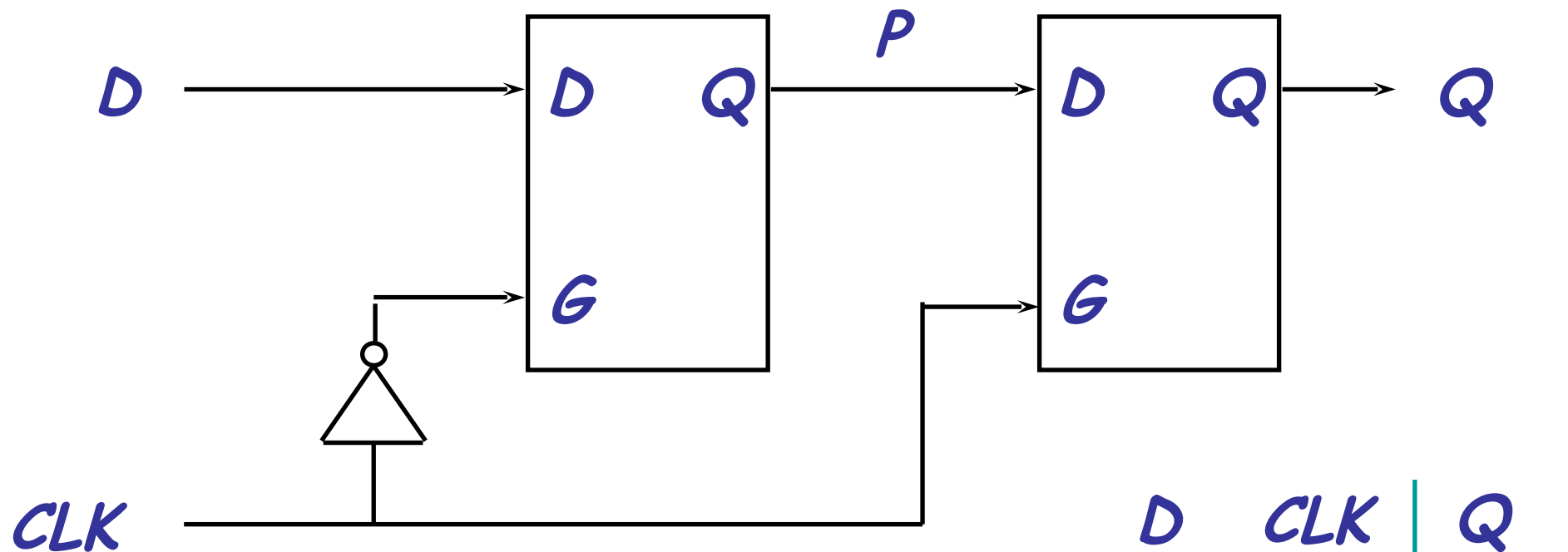
- In reality, setup and hold times are not only dependent on contamination and propagation delays of gates, but also of wires.
- Exhaustive transistor-level simulation is required to determine setup and hold times accurately – But we won't worry about that.

00101010010101000011110100001100
10001100100001111001101010010101
110010101010100001001100101010100
1001010010010010101010101010101
11100001111010110000000111101001
001001010100101001001010010010110
10010100100001010100100101001010
10010100101010010100101010010101
10010101010101010101010101010101

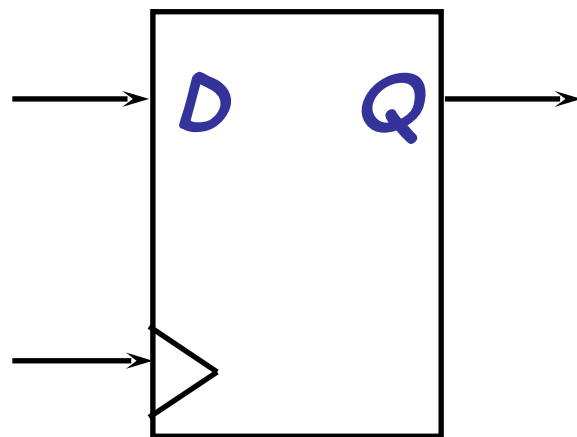


DISCS

Edge-Triggered Flip-Flop



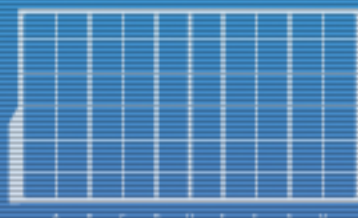
Symbol:



*aka register or
"Positive-edge-
triggered FF"*

D	CLK	Q
$\{0, 1\}$	0	Q_{old}
$\{0, 1\}$	1	Q_{old}
D	\uparrow	D
$\{0, 1\}$	\downarrow	Q_{old}

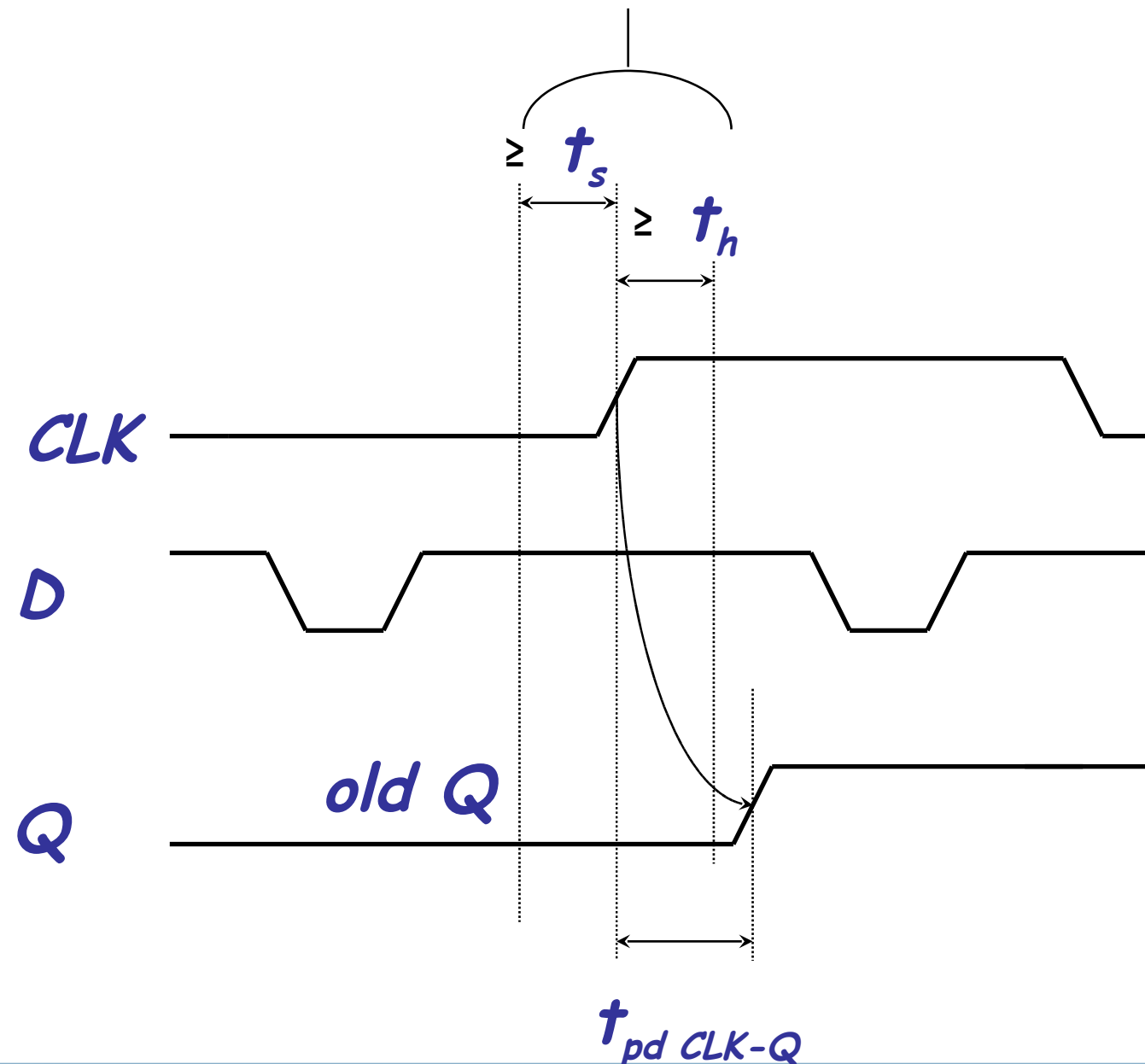
0010101001010100011110100001100
10001100100001111001101010010101
110010101010100001001100101010100
1001010010010010101010101010101
11100001111010110000000111101001
001001010100101001001010010010110
10010100100001010100100101001010
10010100101010010100101010010101
10010100101010010100101010010101



DISCS

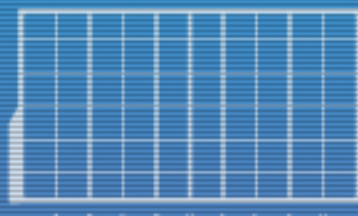
Edge-Triggered Flip-Flop

D must not change in this region



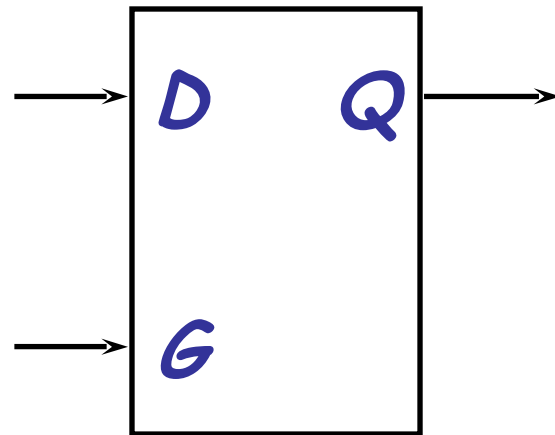
Setup time = t_s
Hold time = t_h

00101010010101000011110100001100
10001100100001111001101010010101
11001010101010100001001100101010100
100101001001001010101010101010101
11100001111010110000000111101001
001001010100101001001010010010110
10010100100001010100100101001010
10010100101010010100101010010101
10010101010101010101010101010101

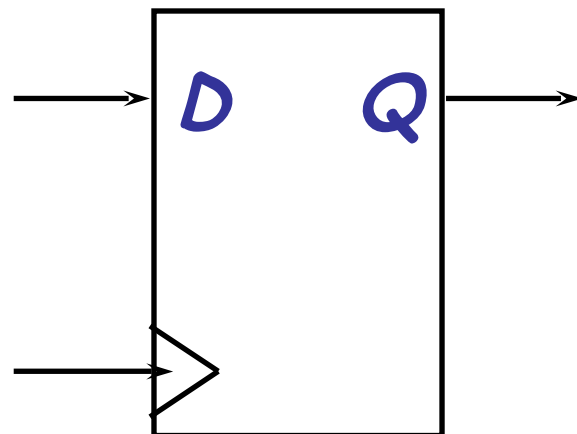


DISCS

Latches and Flip-Flops

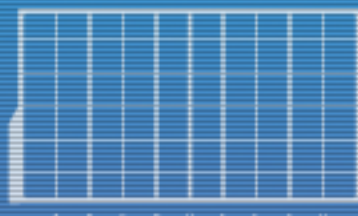


D-Latch: like a toll-gate, transparent when $G=1$.



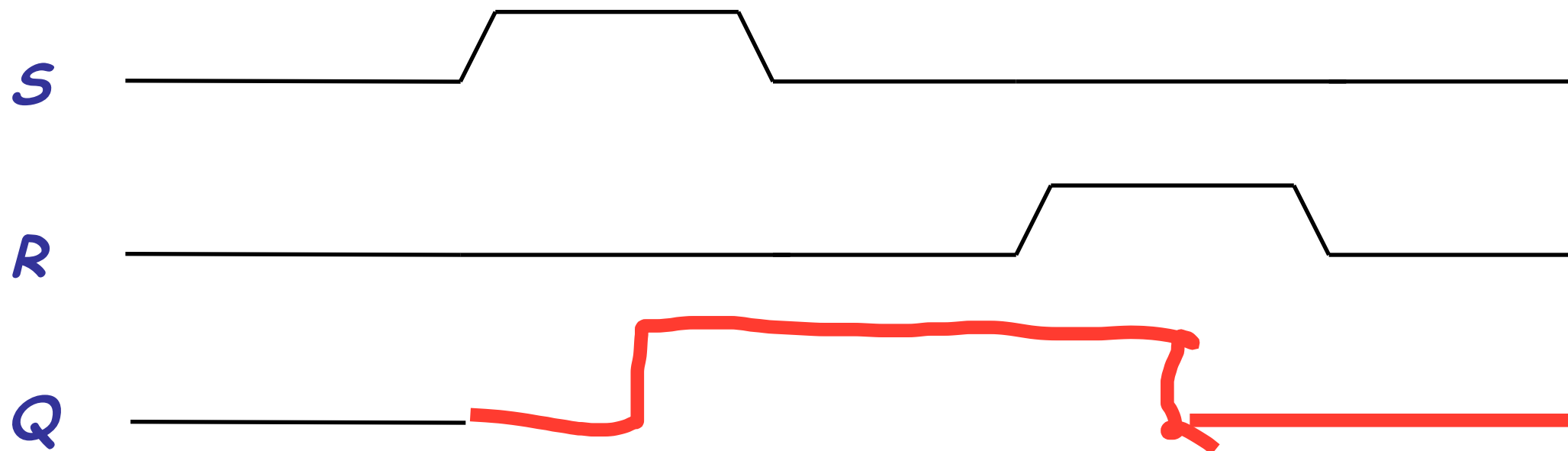
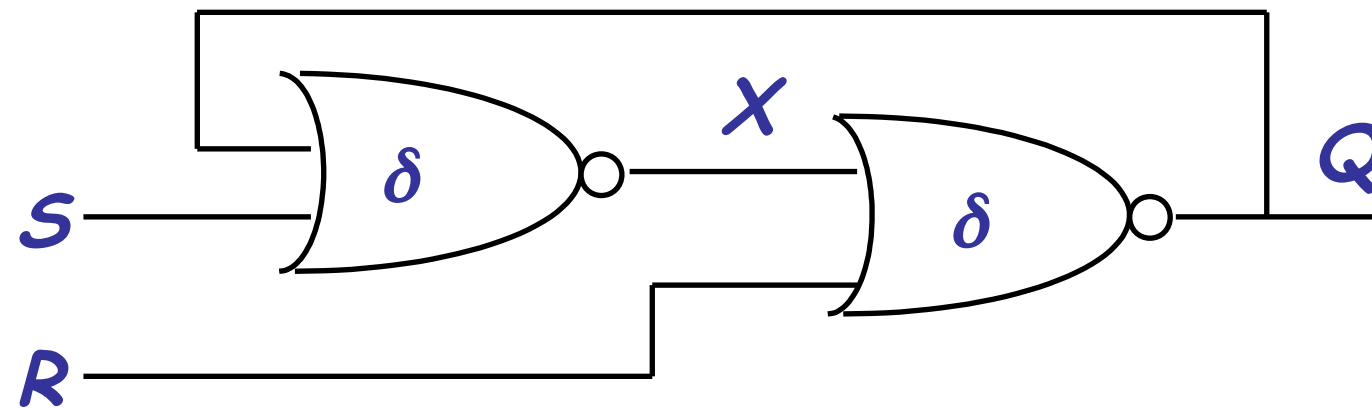
FF: like an air-lock, never transparent except at clock edge.

00101010010101000011110100001100
10001100100001111001101010010101
11001010101010100001001100101010100
100101001001001010101010101010101
11100001111010110000000111101001
001001010100101001001010010010110
10010100100001010100100101001010
10010100101010010100101010010101
10010101010101010101010101010101

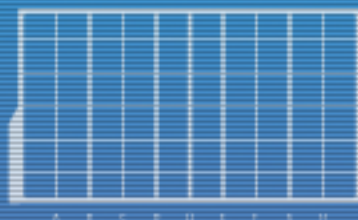


DISCS

S-R Latch

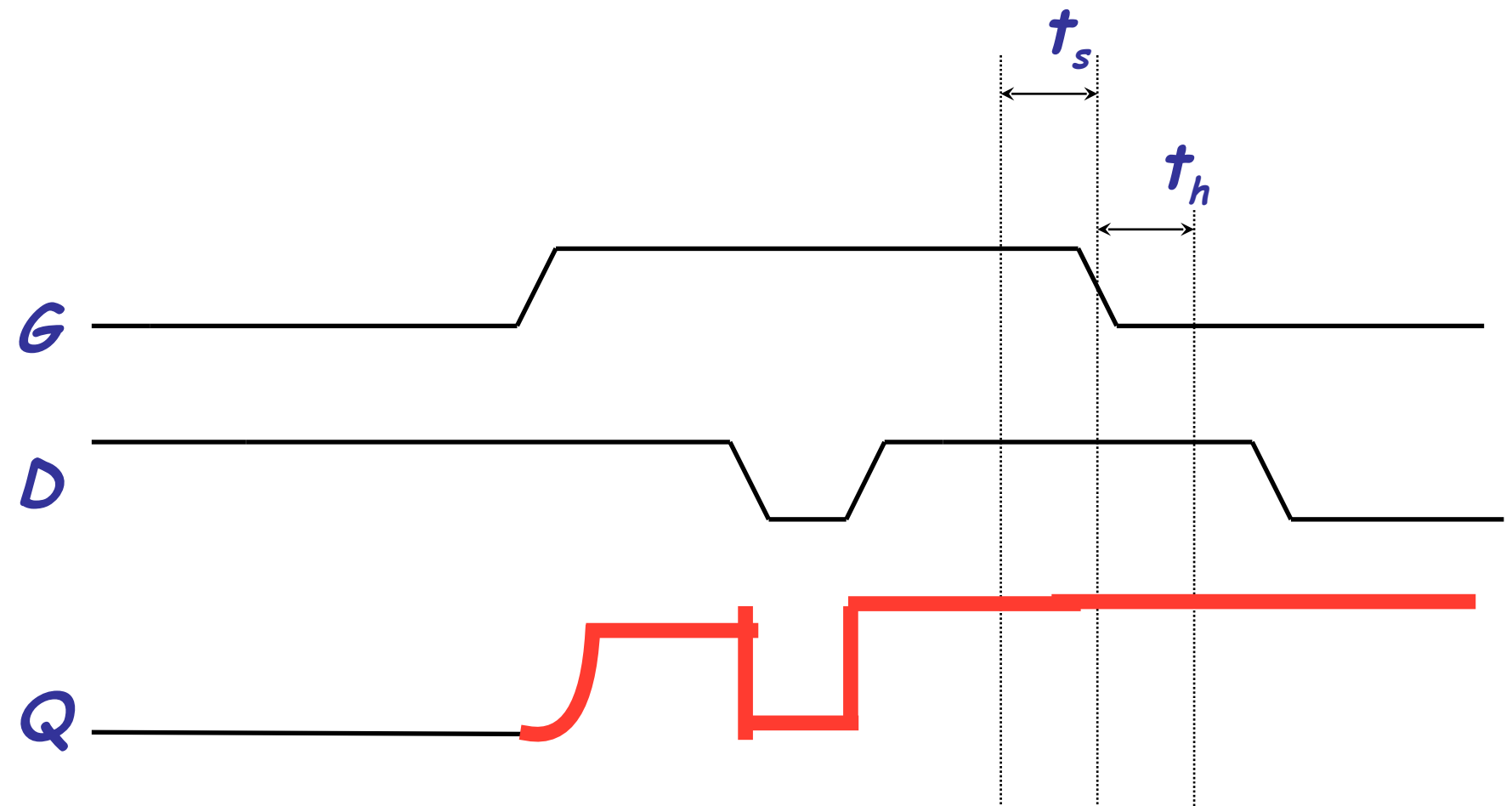
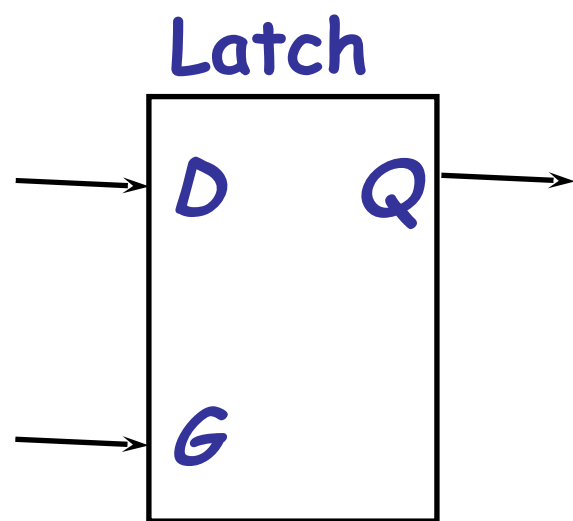


00101010010101000011110100001100
10001100100001111001101010010101
11001010101010100001001100101010100
100101001001001010101010101010101
11100001111010110000000111101001
001001010100101001001010010010110
10010100100001010100100101001010
10010100101010010100101010010101
100101010101001010101010010101

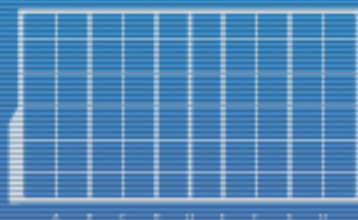


DISCS

D-Latch

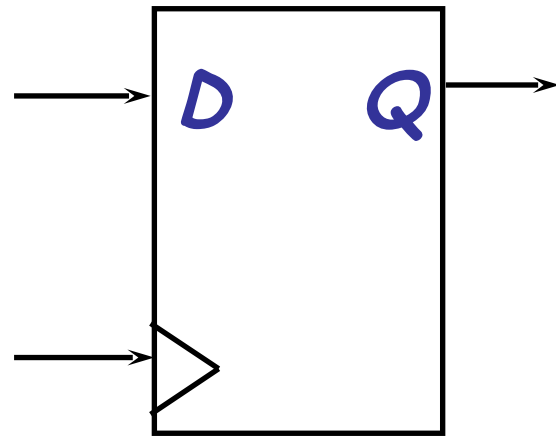


00101010010101000011110100001100
10001100100001111001101010010101
11001010101010100001001100101010100
100101001001001010101010101010101
11100001111010110000000111101001
001001010100101001001010010010110
10010100100001010100100101001010
10010100101010010100101010010101
10010101010101010101010101010101



DISCS

Edge-Triggered Flip-Flop



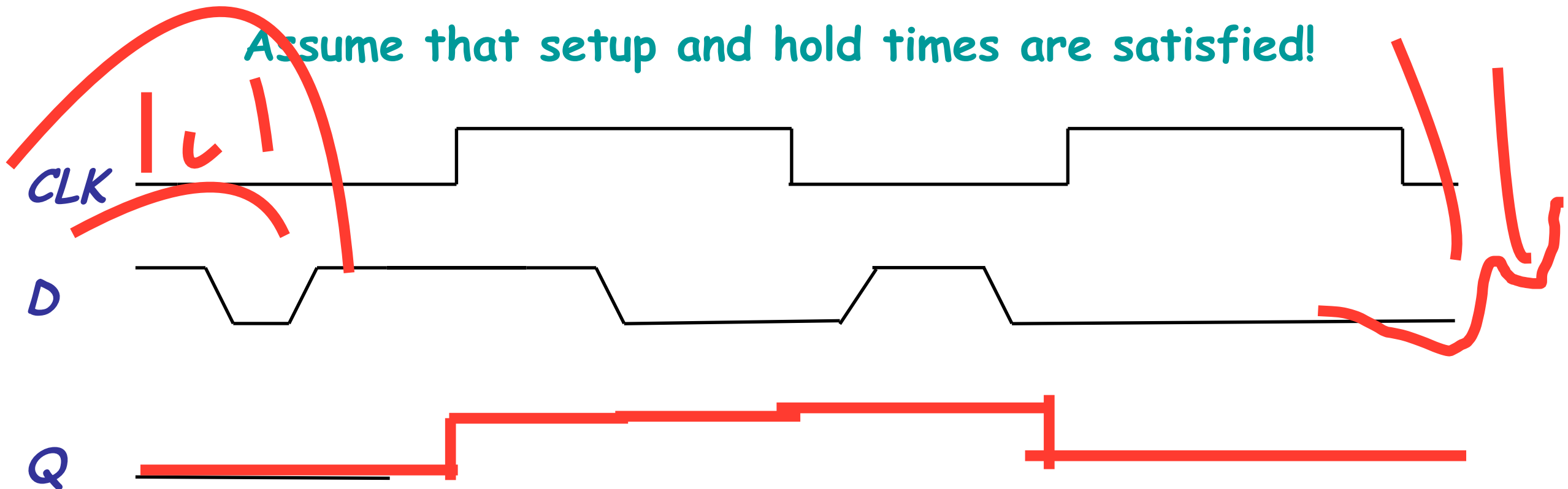
Setup time = t_s

Hold time = t_h

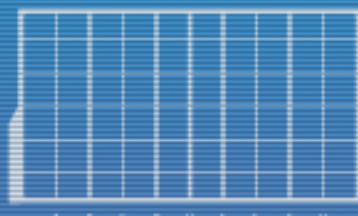
Clk-Q prop. = t_{pdC-Q}

Clk-Q cont. = t_{cdC-Q}

Assume that setup and hold times are satisfied!



00101010010101000011110100001100
10001100100001111001101010010101
11001010101010100001001100101010100
100101001001001010101010101010101
11100001111010110000000111101001
00100101010010100100100100100110
10010100100001010100100101001010
10010100101010010100101010010101
10010100101010010100101010010101



DISCS