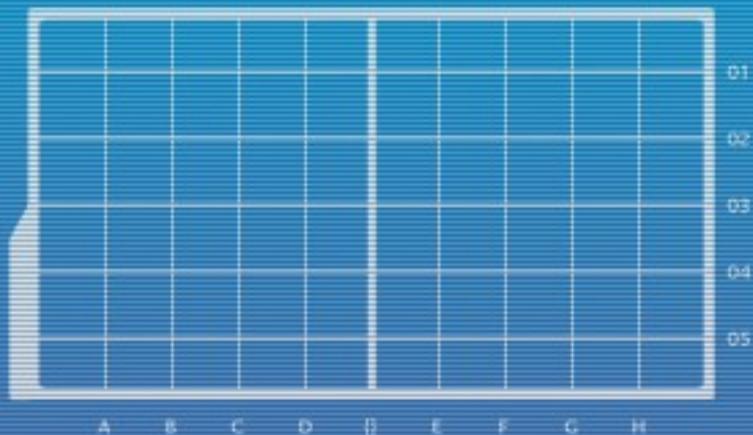




# DEPARTMENT OF INFORMATION SYSTEMS AND COMPUTER SCIENCE



```
001011101010001110101110010011101010101001000101
1101010110101010000101010101001010101010101010
101001010010010010101010101010101010101010101010
11100001111010110000000111101010101010000010101
111010101110010100010010111010100010100100111010
10101001010010010010000101010110101010101010010111
0010101001010100101010000001010101001111101000011001
1000110010000111100110101011000100110101010000101010
1100101010101000010011001010100010010101010101010
10100101001001001010101010101010101010101010101010
11100001111010110000000111101010101010000010101
0010010101001010010010100100010101010101001010010
10010100100001010100100101010010100101001010010010
1001010010101001010010101001010010101001001001001
100101010101001010101010100101010101001010101010
```



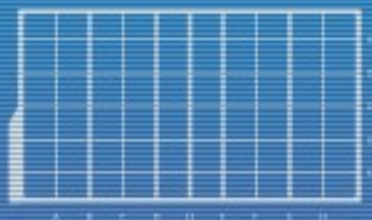
# Pipelining Circuits

Or How to Make Your Circuit  
Look Like a Factory

# Lecture Time!

- Registers: Ideal and Non-Ideal
- Pipelining: Basics
- Performance: Throughput and Latency

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010100101010010101010101010101



DISCS

# Registers

- Since registers (edge-triggered flip-flops) are required for pipelining, it is important to remember that they are physical devices and have limitations.
  - These limitations are expressed in their timing attributes, namely  $t_{pdCQ}$ ,  $t_{cdCQ}$ ,  $t_s$ , and  $t_h$ .
- Ideal registers (which don't exist in real life) are registers with  $t_{pdCQ} = t_{cdCQ} = t_s = t_h = 0$  (zero).
- Non-ideal registers are registers that are not ideal ~~(duh)~~ and exist in real life.
  - At least one of their stats is non-zero.

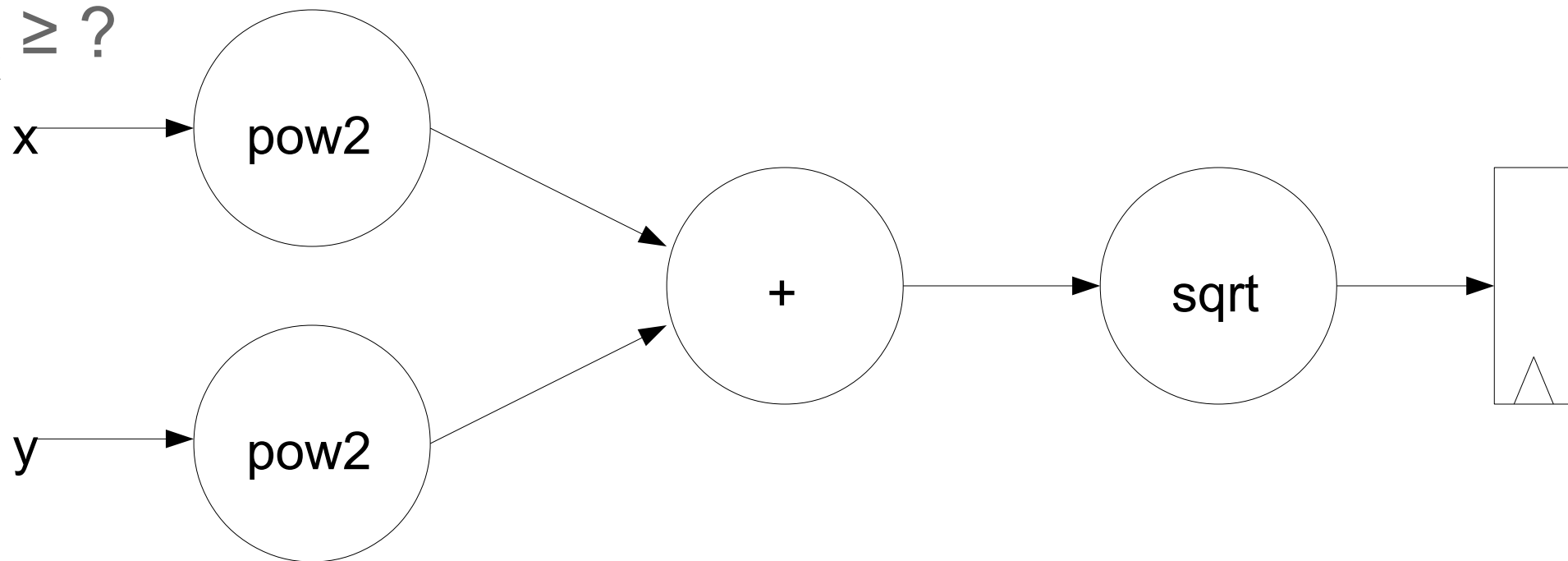
00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101001010101





# Example: Pythagorean Circuit

- Assume ideal registers.
- Assume that inputs are also coming from registers.
- pow2:  $t_{pd} = 5 \text{ ns}$ ,  $t_{cd} = 3 \text{ ns}$
- adder:  $t_{pd} = 2 \text{ ns}$ ,  $t_{cd} = 1 \text{ ns}$
- sqrt:  $t_{pd} = 6 \text{ ns}$ ,  $t_{cd} = 1 \text{ ns}$
- $t_{clk} \geq ?$

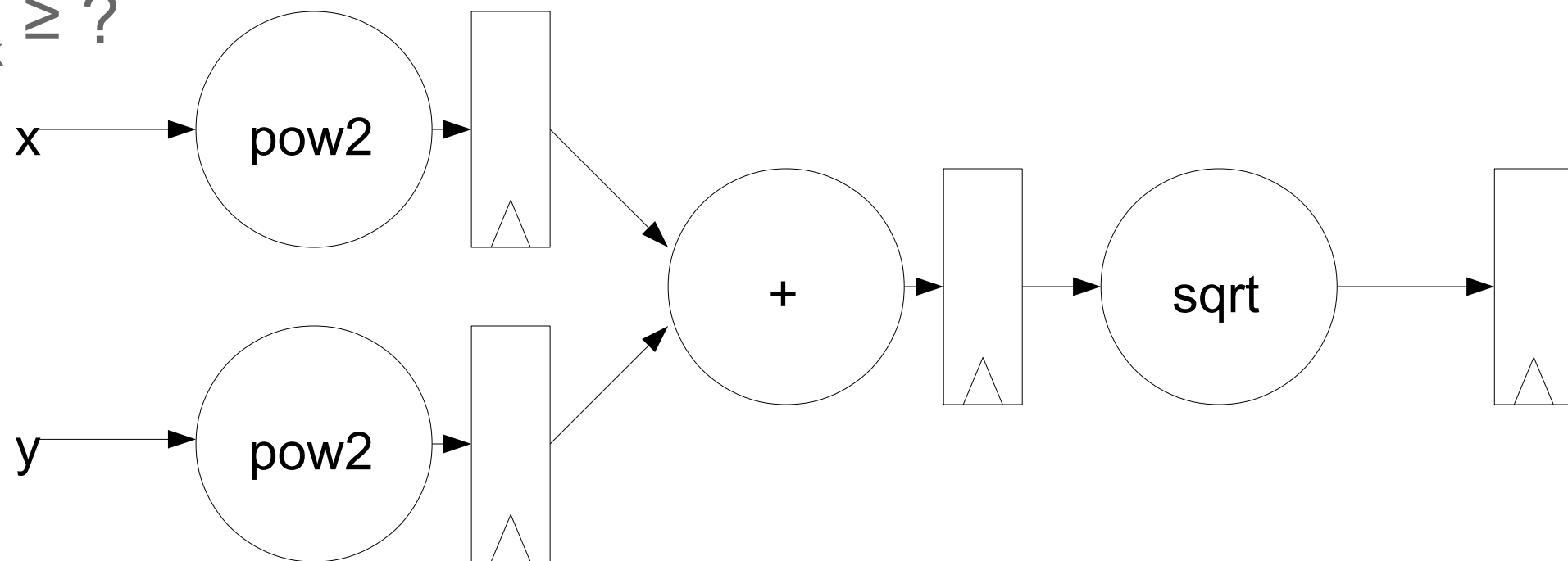


00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101010010101



# Pipelining

- Pipelining means dividing the circuit into shorter stages, resulting in a lower minimum clock period!
- pow2:  $t_{pd} = 5\text{ ns}$ ,  $t_{cd} = 3\text{ ns}$
- adder:  $t_{pd} = 2\text{ ns}$ ,  $t_{cd} = 1\text{ ns}$
- sqrt:  $t_{pd} = 6\text{ ns}$ ,  $t_{cd} = 1\text{ ns}$
- $t_{clk} \geq ?$

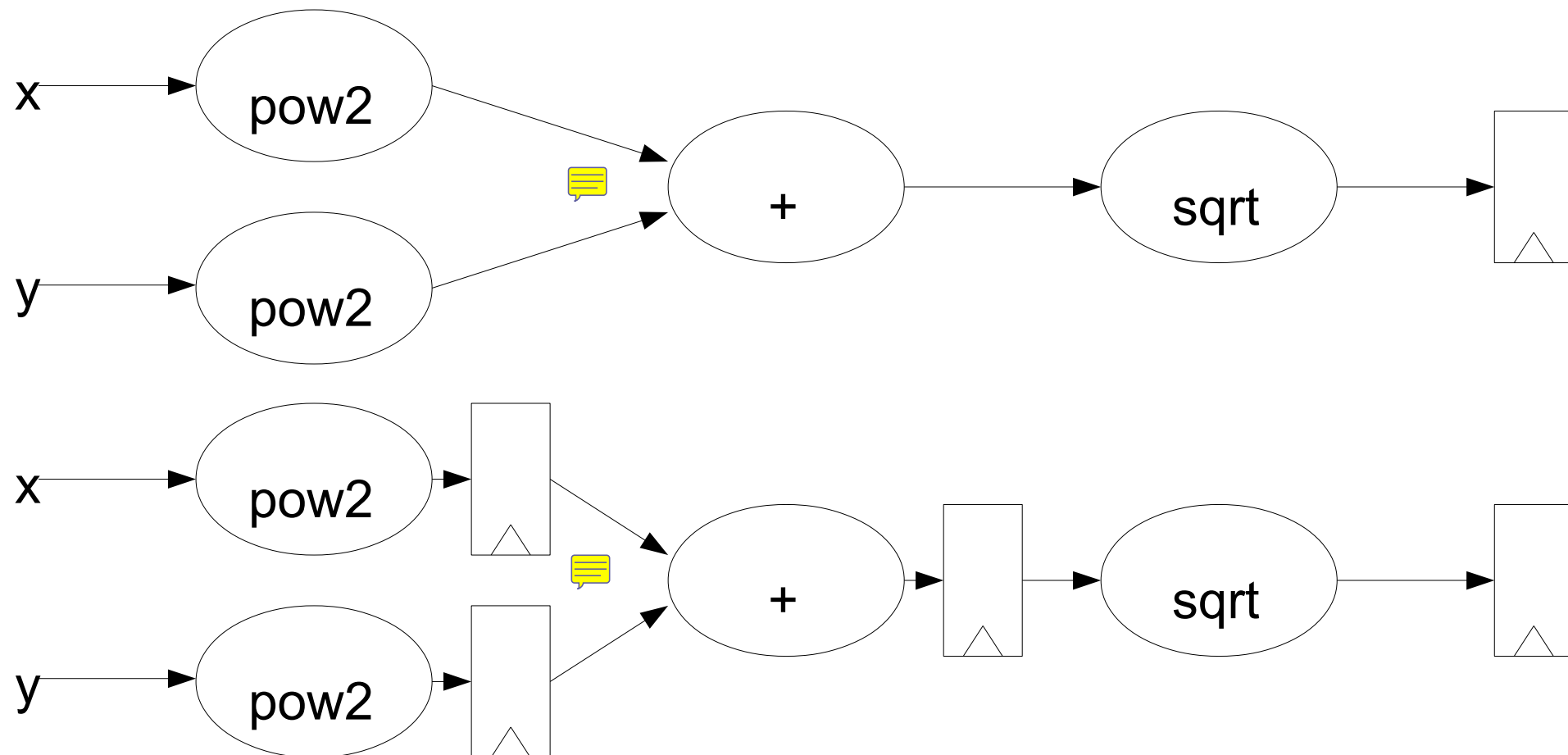


00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101010010101

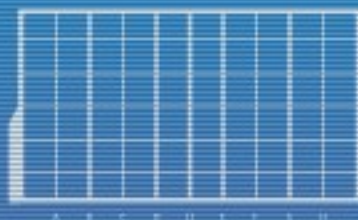


# Measuring Performance

- Compare the two circuits.
  - When both of them are started up, how long until the first output emerges?
  - After any given output, how long until the next output?



00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101001010101





# Throughput and Latency

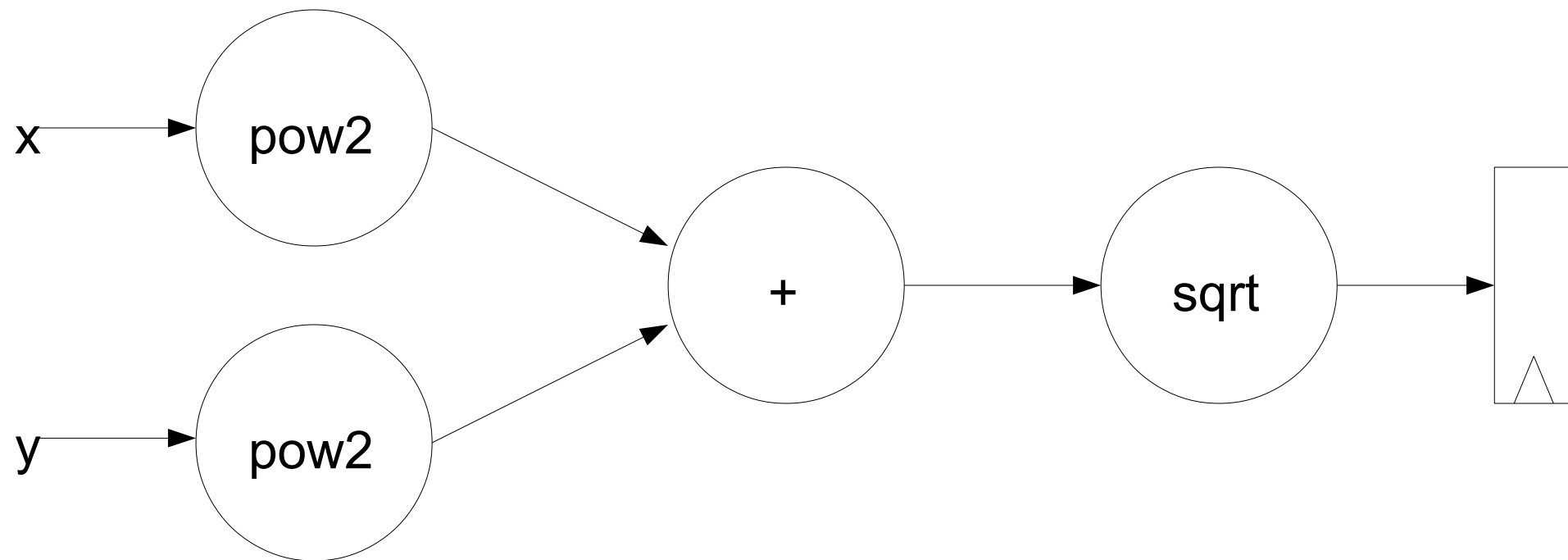
- Throughput = # of outputs /  $t_{\text{clk\_min}}$ 
  - # of outputs is usually 1 (one).
  - This represents the rate at which outputs emerge once all stages in the system have data.
    - In other words, once the pipeline is full.
- Latency = # of stages \*  $t_{\text{clk\_min}}$ 
  - # of stages is 1 (one) in an unpipelined system.
  - This represents the time for the system to generate an output for a given set of inputs.

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010100101010010101010101010101

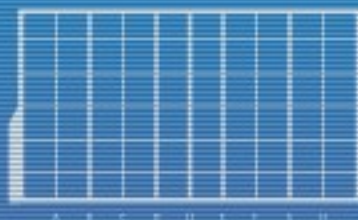


# Unpipelined

- $t_{\text{clk}} \geq 13 \text{ ns}$
- Throughput = ? 
- Latency = ? 



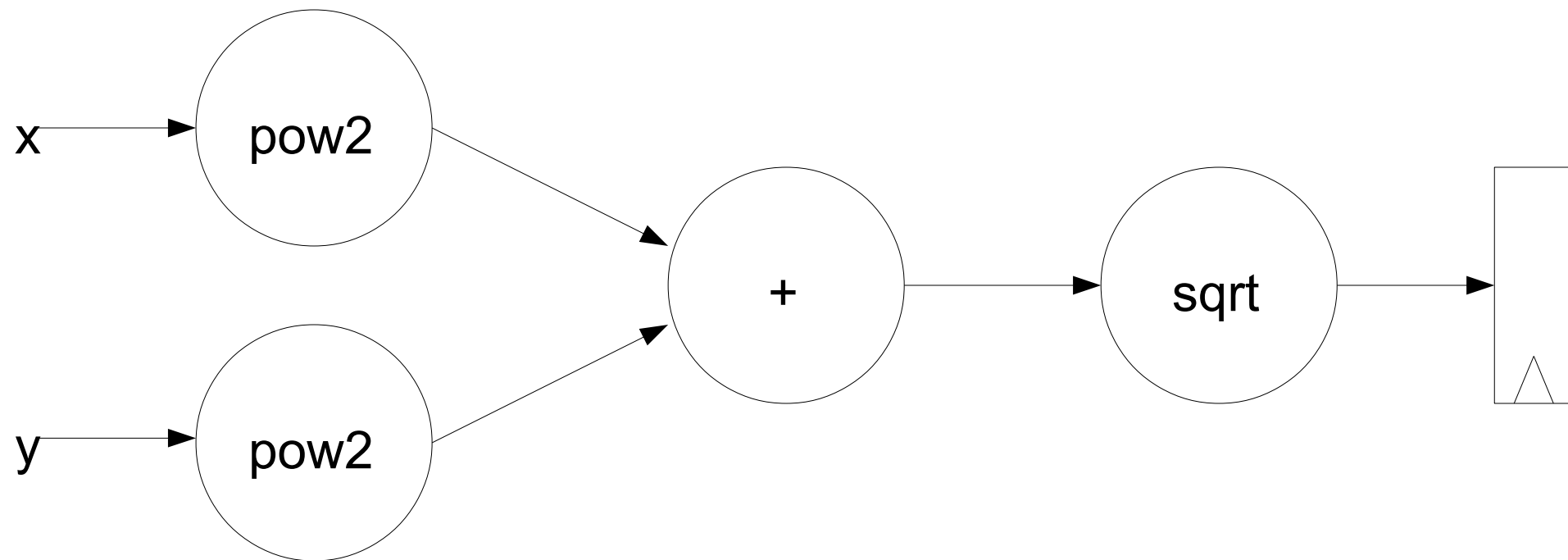
00101010010101000011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010100101010010101010101010101





# Unpipelined

- $t_{\text{clk}} \geq 13 \text{ ns}$
- Throughput =  $1 / t_{\text{clk}} = 1 / 13 \text{ ns} = 77 \text{ MHz}$
- Latency =  $1 * t_{\text{clk}} = 1 * 13 \text{ ns} = 13 \text{ ns}$

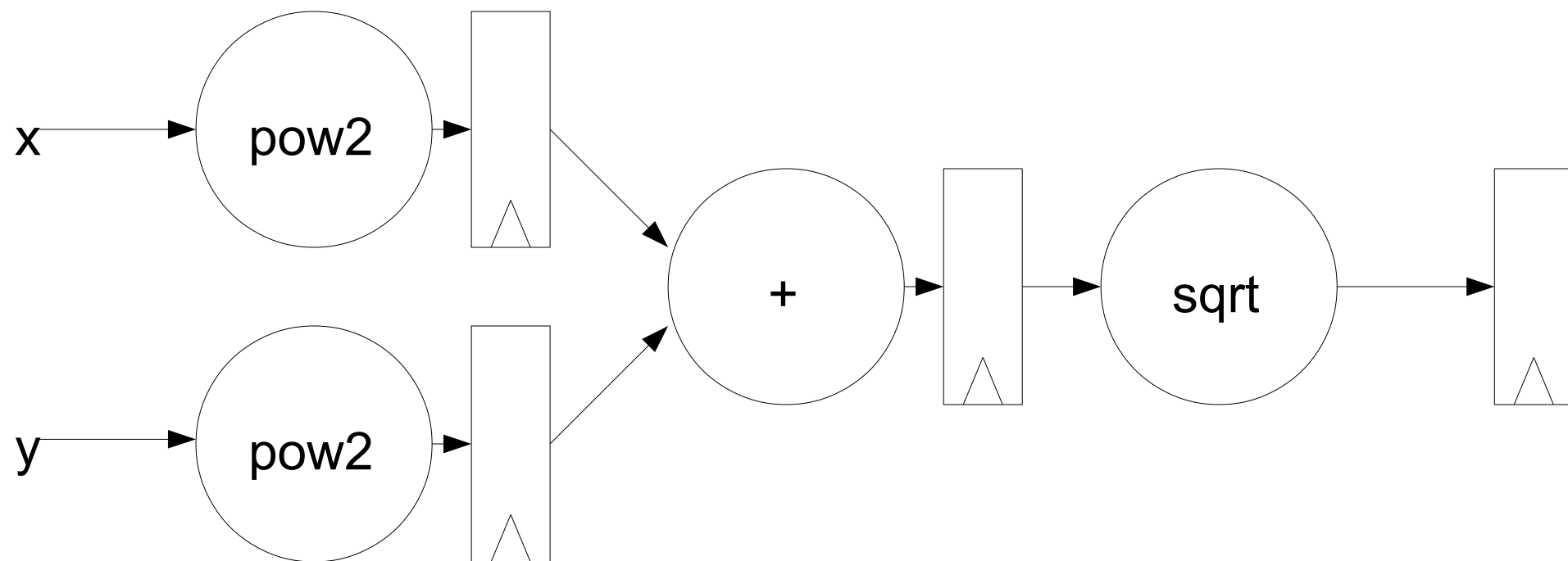


00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101010010101

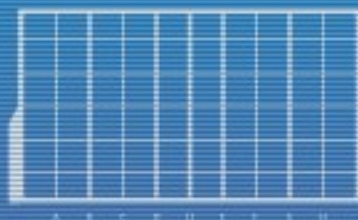


# Pipelined

- $t_{\text{clk}} \geq 6 \text{ ns}$
- Throughput = ?
- Latency = ?

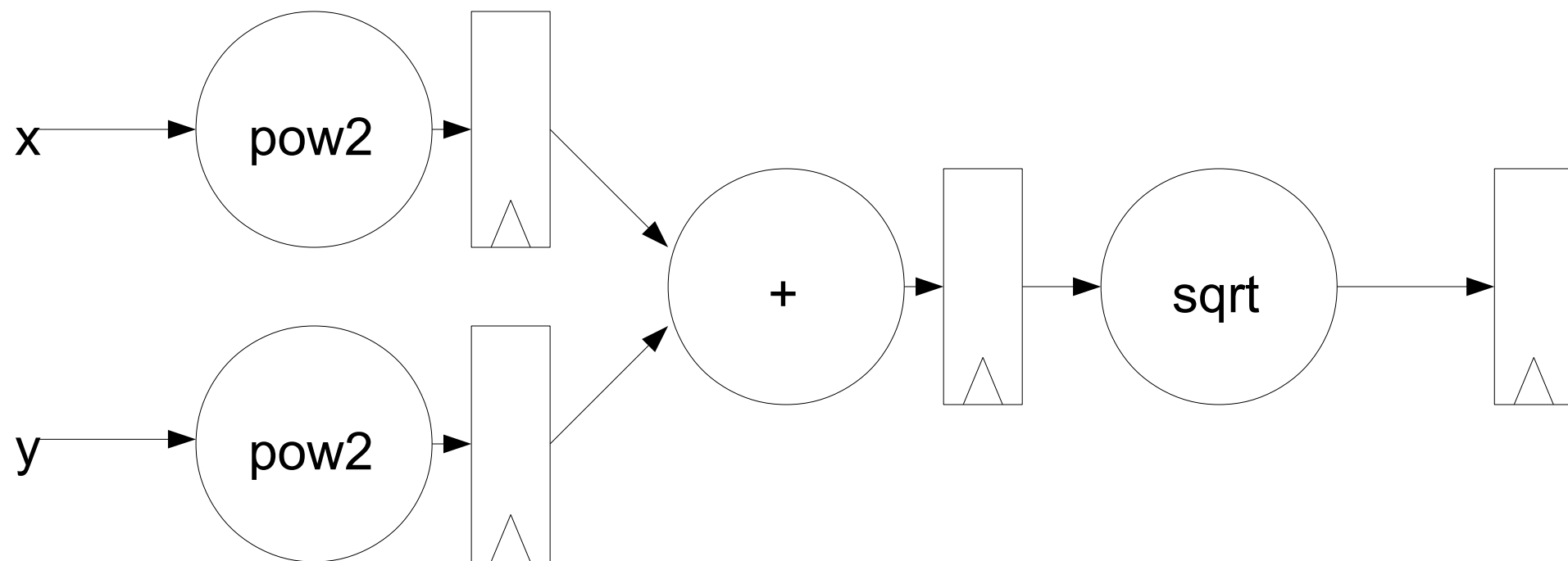


00101010010101000011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101001010101

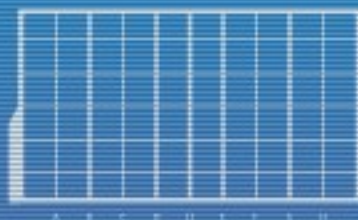


# Pipelined


- $t_{\text{clk}} \geq 6 \text{ ns}$
- Throughput =  $1 / t_{\text{clk}} = 1 / 6 \text{ ns} = 166 \text{ MHz}$
- Latency =  $3 * t_{\text{clk}} = 3 * 6 \text{ ns} = 18 \text{ ns}$

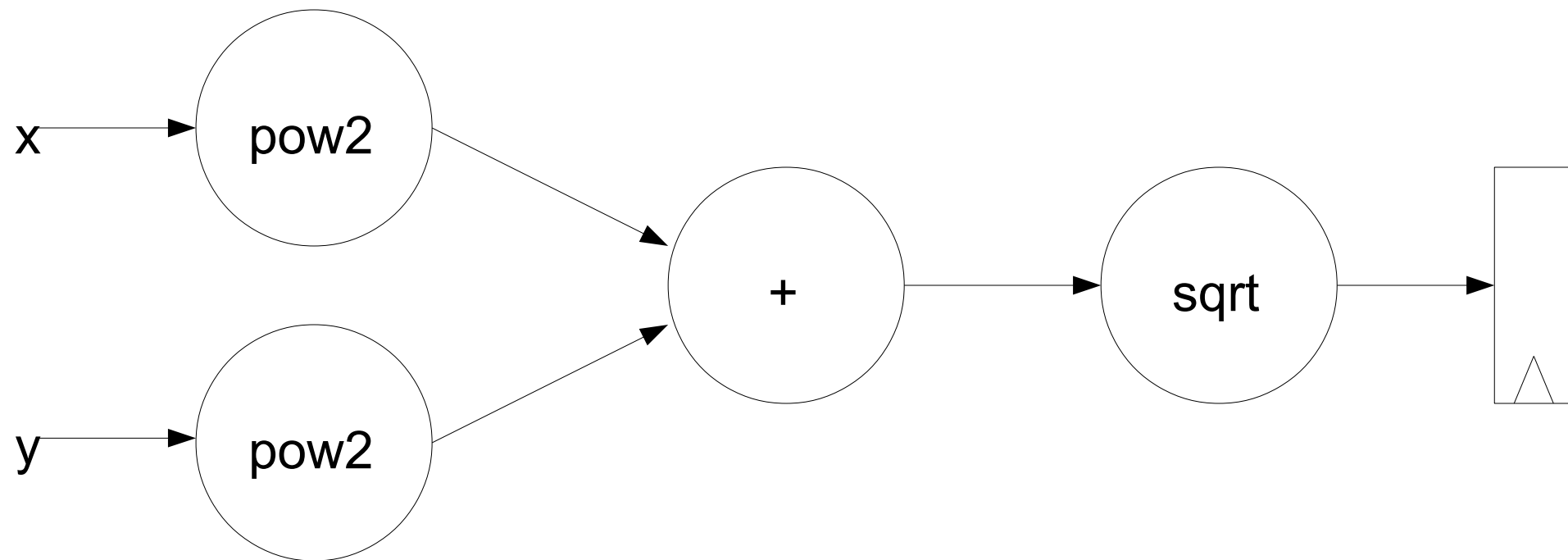


00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010100101001010101010101010101



# What About Non-Ideal Registers?

- $t_{pdCQ} = 3 \text{ ns}$ ,  $t_{cdCQ} = 1 \text{ ns}$ ,  $t_s = 2 \text{ ns}$ ,  $t_h = 1 \text{ ns}$  
- pow2:  $t_{pd} = 5 \text{ ns}$ ,  $t_{cd} = 3 \text{ ns}$
- adder:  $t_{pd} = 2 \text{ ns}$ ,  $t_{cd} = 1 \text{ ns}$
- sqrt:  $t_{pd} = 6 \text{ ns}$ ,  $t_{cd} = 1 \text{ ns}$
- $t_{clk} \geq ?$ , Throughput = ?, Latency = ?



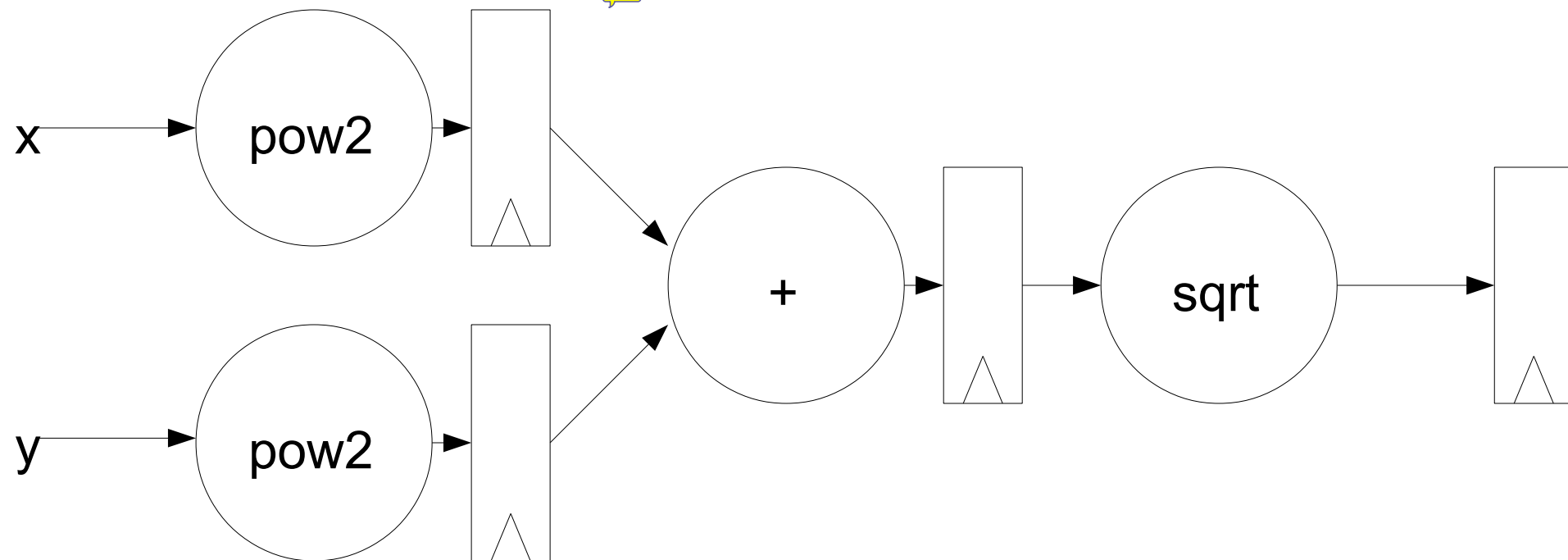
00101010010101000011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010100101010010101010101010101





# What About Non-Ideal Registers?

- $t_{pdCQ} = 3 \text{ ns}$ ,  $t_{cdCQ} = 1 \text{ ns}$ ,  $t_s = 2 \text{ ns}$ ,  $t_h = 1 \text{ ns}$
- pow2:  $t_{pd} = 5 \text{ ns}$ ,  $t_{cd} = 3 \text{ ns}$
- adder:  $t_{pd} = 2 \text{ ns}$ ,  $t_{cd} = 1 \text{ ns}$
- sqrt:  $t_{pd} = 6 \text{ ns}$ ,  $t_{cd} = 1 \text{ ns}$
- $t_{clk} \geq ?$ , Throughput = ?, Latency = ?



00101010010101000011110100001100  
10001100100001111001101010010101  
11001010101010100001001100101010100  
100101001001001010101010101010101  
11100001111010110000000111101001  
00100101010010100100100100100110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101010010101



# Unpipelined VS Pipelined

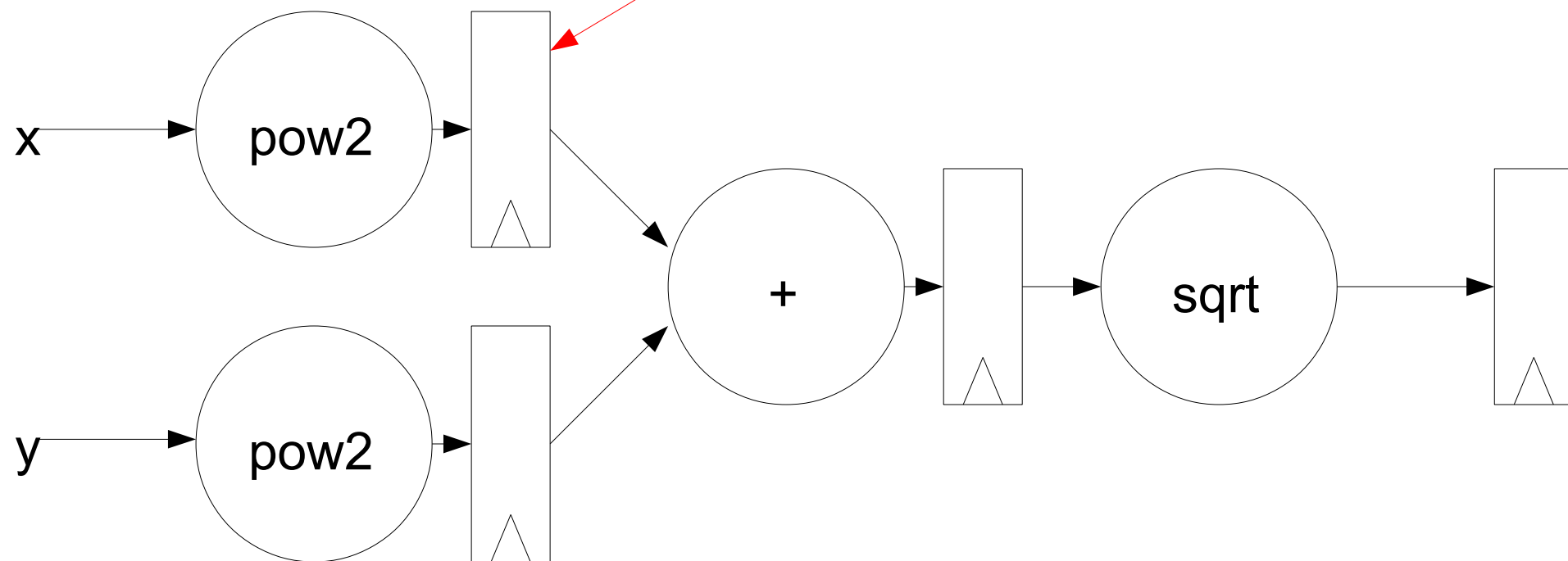
- In general, pipelining will:
  - Increase throughput...
  - ... at expense of slightly longer latency per object.
- Is this a good tradeoff?
  - Will the circuit most likely encounter several inputs in a row?
  - How many lines of code does an average program have?

```
00101010010101000011110100001100
10001100100001111001101010010101
110010101010100001001100101010100
1001010010010010101010101010101
11100001111010110000000111101001
001001010100101001001010010010110
10010100100001010100100101001010
10010100101010010100101010010101
10010100101010010101010101010101
```



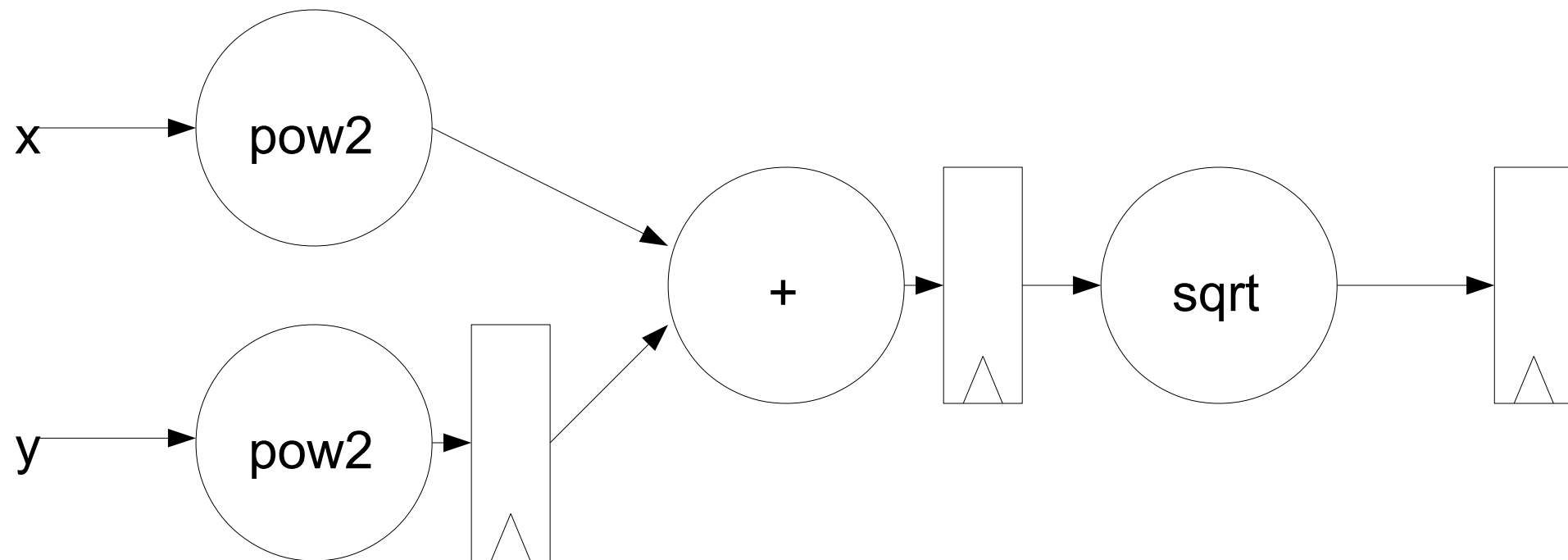
# Pipelining Guideline #1

- The circuit should contain the same number of registers along any path from any input to any output.
  - This insures that every computational unit in the circuit sees inputs in phase!
  - What if I took out this register?



# Ill-formed Pipelines

- Adder will get x and y from different pairs, resulting in an incorrect calculation!
  - x and y pairs become unsynchronized due to the unequal number of registers in the paths leading to the adder.



00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
0010010101001010010010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101001010101

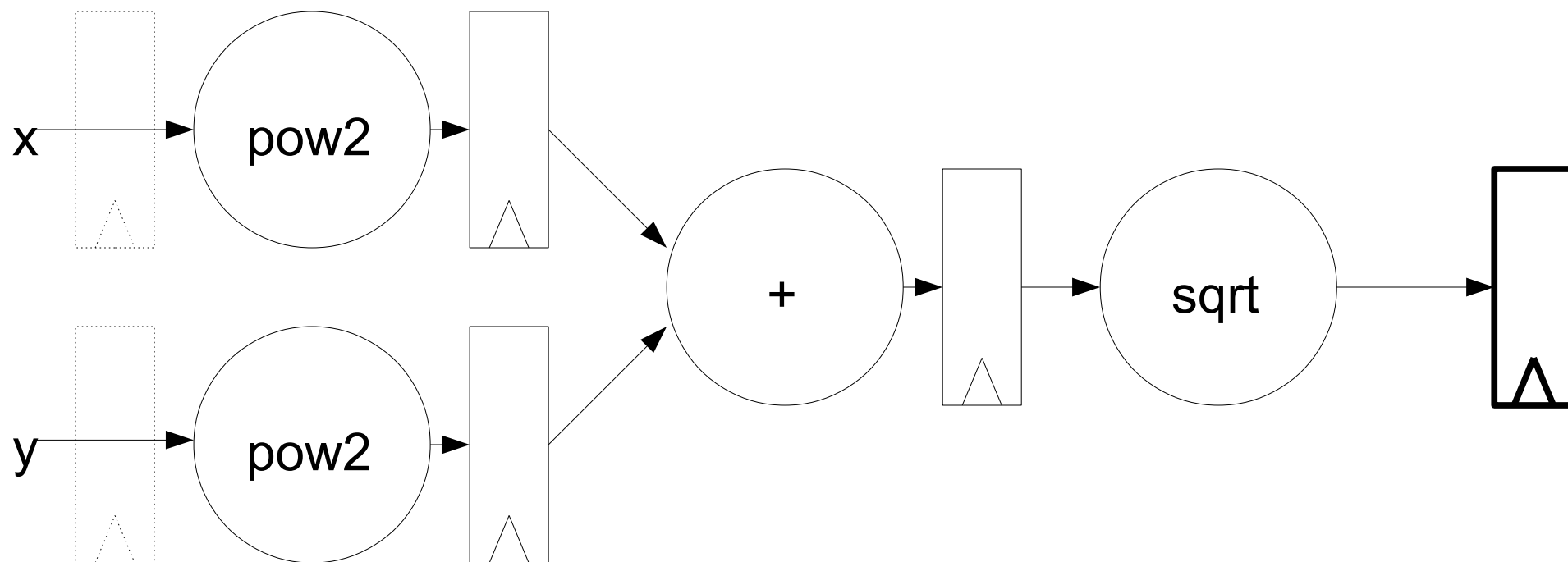


DISCS



# Pipelining Guideline #2

- There must ALWAYS be a register on each output.
- Inputs are assumed to be coming from registers (perhaps from another pipelined circuit) and therefore must not be drawn.



# Pipelining Guideline #3

- $t_{pdCQ}$  and  $t_s$  are constant.
  - This is the overhead caused by FFs that CANNOT be eliminated (except by removing the FFs) and will therefore limit throughput!
  - Even if we increase the number of stages, we cannot bring  $t_{clk\_min}$  lower than  $t_{pdCQ} + t_s$ .
  - We can however, still increase throughput. How?
    - Anyone here know what SMP means?
    - For now, we will assume that this solution is unavailable to us due to cost issues.

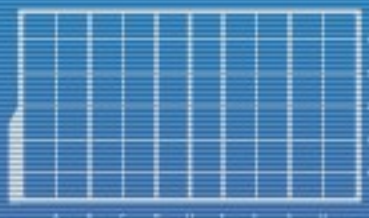
00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
10010100101010010101010101010101



# Pipelining Guideline #4

- When pipelining a circuit, these are your priorities:
  - First, *maximize throughput!* (minimize clock period)
  - Then, *minimize number of stages for that max throughput!* (minimize latency)
  - Finally, *minimize number of registers for a given throughput and latency!* (minimize cost)

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101010010101



# Pipelining Guideline #4

- Given a circuit, throughput is usually limited by a device that has the longest  $t_{pd}$ .
  - So find that first and isolate it using registers!
  - Don't forget that there are already registers present in the outermost areas of the circuit!
    - Where input/s come from and where output/s emerge!
- Once you've done that, you can divide the rest of circuit into stages, using the isolated device/s'  $t_{pd}$  as a guide.
  - Example / exercise time!

00101010010101000011110100001100  
10001100100001111001101010010101  
110010101010100001001100101010100  
1001010010010010101010101010101  
11100001111010110000000111101001  
001001010100101001001010010010110  
10010100100001010100100101001010  
10010100101010010100101010010101  
1001010010101001010101010010101

