Machine Learning

Choice of Linear Model and Regularization principles

Thomas Milcent - 2025

Regularization key principles

Reduce overfitting

- Highly complex models can fit the training data too well, capturing noise instead of underlying patterns.
- Regularization **adds a penalty** to **limit model complexity**, helping it generalize better to new data.

Balance bias and variance

- Regularization helps **reduce variance** (sensitivity to data fluctuations) at the cost of a **slight increase in bias**.
- The goal is a better bias-variance tradeoff, so the model performs well on unseen data.

Add a constraint on model parameters via a penalty (L1 or L2)

- L1 (Lasso): encourages sparsity in coefficients (some become exactly 0).
- L2 (Ridge): prevents coefficients from becoming too large.

Regularization key principles

Minimize

Prediction error + λ penalty on model complexity

- **Biased estimator** when $\lambda \neq 0$.
- Trade bias for a smaller variance.
- λ can be set by cross-validation.

- Simpler model ≈ fewer parameters

→ **shrinkage**: drive the coefficients of the parameters towards 0.

Ridge Regression – L2 Penalty

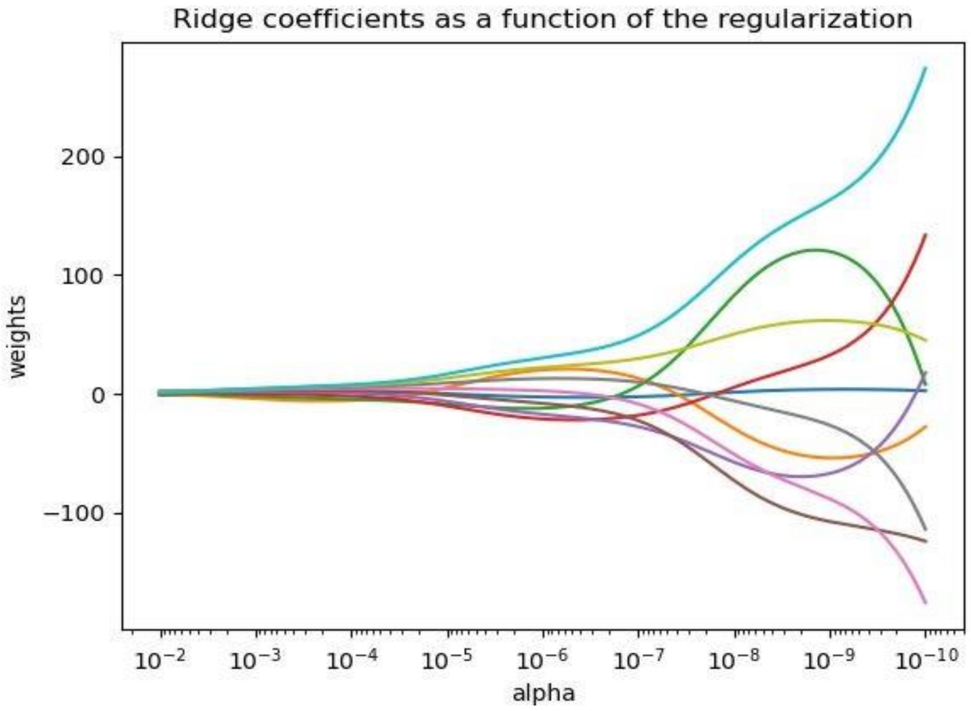
Sum-of-squares penalty $$\hat{\beta}_{\text{ridge}} = \arg\min_{\beta} ||y - X\beta||_2^2 + \lambda||\beta||_2^2$$

Ridge regression estimator $$\hat{\beta}_{\text{ridge}} = (X^\top X + \lambda I)^{-1} X^\top \boldsymbol{y}$$

$$\text{if } (X^\top X + \lambda I) \text{ invertible}$$

Solution Path

[Plot Ridge coefficients as a function of the regularization — scikit-learn 1.3.2 documentation]



Decreasing value of lambda (“alpha” in this example)

Ridge Regression – L2 Penalty

Grouped selection:

- correlated variables get similar weights
- identical variables get identical weights

Ridge regression shrinks coefficients towards 0 but does not result in a **sparse model**.

Sparsity:

- many coefficients get a weight of 0
- they can be eliminated from the model.

Lasso regression – L1 Penalty

L1 penalty

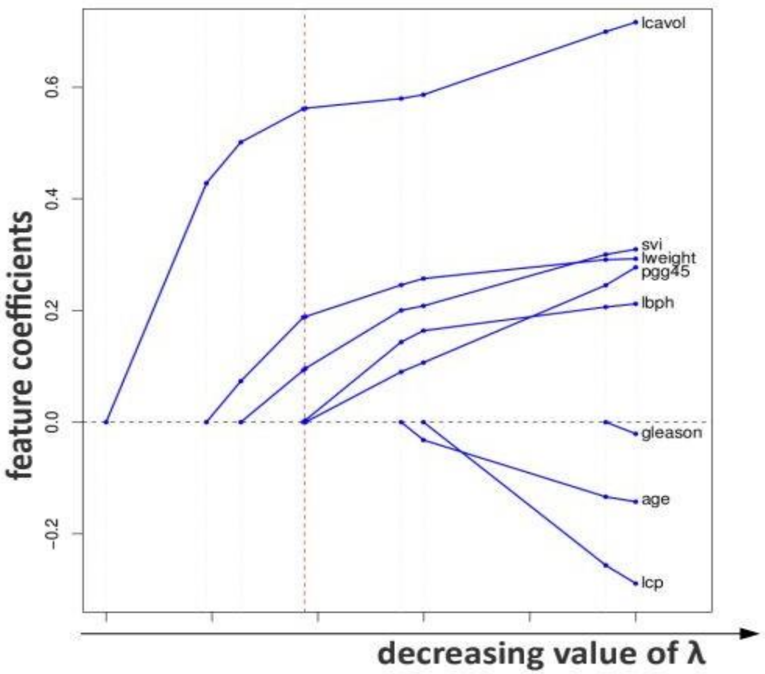
$$\hat{\beta}_{\text{lasso}} = \arg\min_{\boldsymbol{\beta}} ||y - X\boldsymbol{\beta}||_2^2 + \lambda||\boldsymbol{\beta}||_1$$

$$||\boldsymbol{\beta}||_1 = \sum_{j=1}^{p} |\beta_p|$$

Tends to give sparse coefficients

No explicit solution: quadratic form under linear constraints to solve this

Solution path



Elastic Net – Mix of L1 and L2 penalty

Combine Lasso & Ridge regression

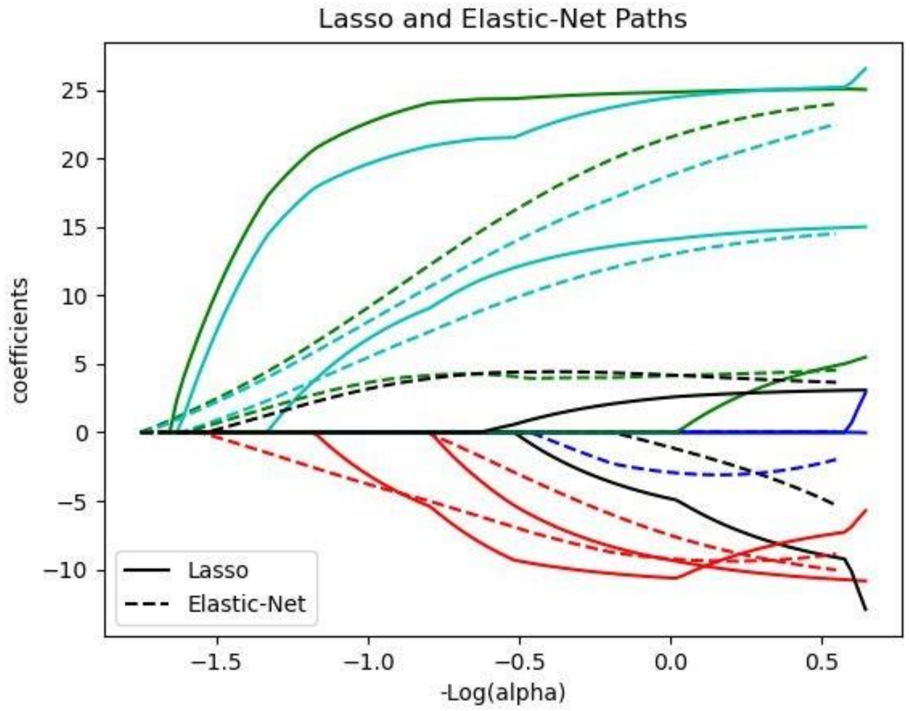
$$\hat{\beta}_{\text{enet}} = \arg\min_{\boldsymbol{\beta}} ||y - X\boldsymbol{\beta}||_2^2 + \lambda \left(\alpha ||\boldsymbol{\beta}||_2^2 + (1-\alpha)||\boldsymbol{\beta}||_1 \right)$$

The best of the two approach
Selects variables like the Lasso regression
Shrinks together coefficients of correlated variables like the Ridge regression

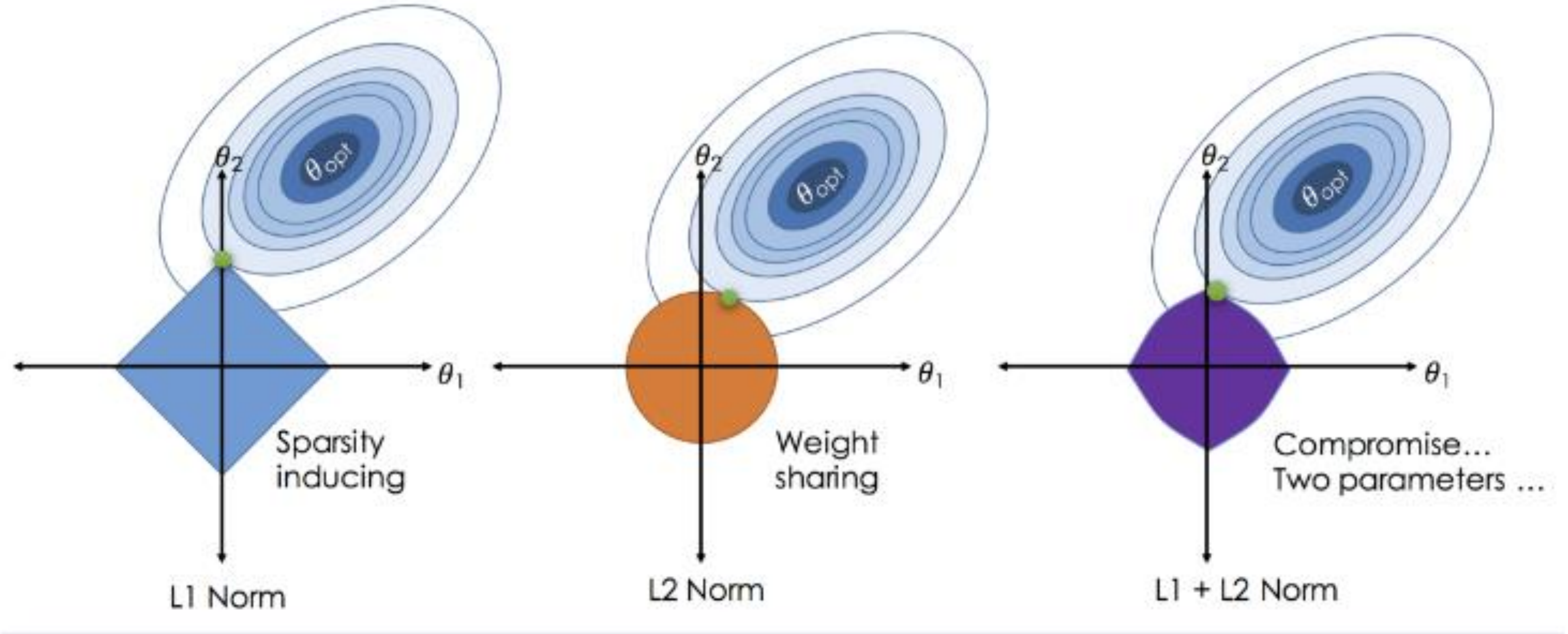
Elastic Net – Mix of L1 and L2 penalty

Example

[Lasso and Elastic Net — scikit-learn 1.3.2 documentation]



To Sum Up : Lasso vs Ridge vs Elastic Net



Play with Lasso and Ridge here : https://www.interactive-ml.com/regularization.html?

To Sum Up : Lasso vs Ridge vs Elastic Net

	Ridge (L2)	**Lasso (L1)**	**Elastic Net (L1 + L2)**
Penalty type	L2: sum of squared coefficients	L1: sum of absolute coefficients	Combination of L1 and L2 penalties
Effect on coefficients	Shrinks coefficients but **never to zero**	Can shrink coefficients **exactly to zero**	Shrinks coefficients and can set some to zero (depending on the mix)
Model type	Dense (keeps all features)	Sparse (feature selection)	Semi-sparse: balances sparsity with stability
Behavior with correlated predictors	Shares weight across correlated variables (“grouping effect”)	Tends to pick **one** and drop the others	Encourages groups of correlated variables, but can still drop some
When it works best	Many small/medium effects; multicollinearity	Only a few features matter; interpretability needed	Many predictors, possibly correlated; feature selection + stability
Stability of solution	Very stable	Less stable when predictors are correlated	More stable than Lasso, more flexible than Ridge
Bias–variance behavior	More bias than OLS, much lower variance	More bias, variance reduction + sparsity	Tunable trade-off between L1 and L2 effects
Computational notes	Closed-form solution exists	Requires iterative algorithms	Requires iterative algorithms
Typical hyperparameters	λ (regularization strength)	λ (strength)	λ (strength) + α (mix between L1 and L2)

Machine Learning - Generalized Linear Regression

Recap

Understand regularization as a means to control model complexity

Define Lasso, ridge regression, elastic net

Understand the role of the l1 and l2 norms in regularization

Interpret solution paths for Lasso and ridge regression