Consider $X \sim U[0,T]$, and also Consider an observation Y made on X such that

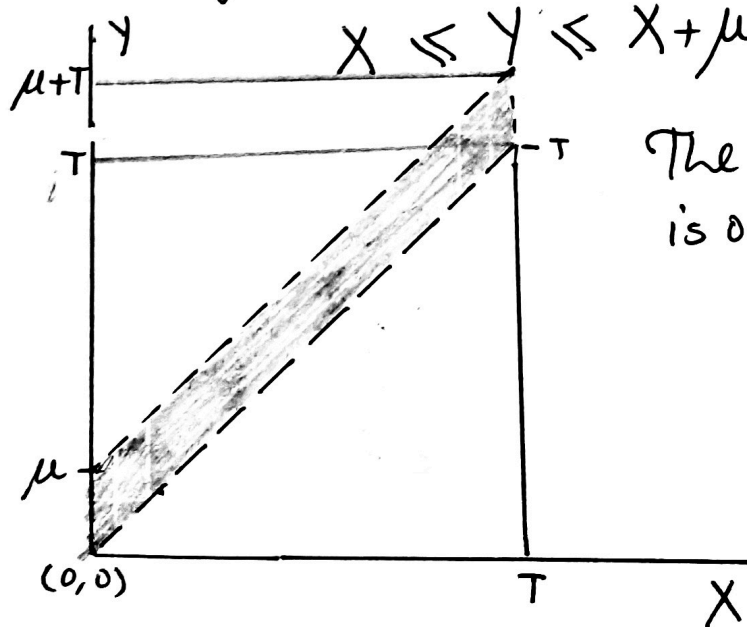$$Y|X \sim U[X, X+\mu]$$

where $\mu \in \mathbb{R}^+$.

Therefore, Y conditioned on X is uniformly distributed in the range $[X, X+\mu]$.

Although, it is not necessary but we may assume that μ is far smaller than T. to make sensor reasonably acceptable.

Your task is to design an M.S.E. estimator of X given Y.

Hint: Consider the sample space, we must recognize the region where

$$X \leqslant Y \leqslant X+\mu \quad \text{is true.}$$

The shaded region is our sample space.

Show that the estimator is obtained as follows:

$$\hat{X} = E[X|Y] = \begin{cases} \dfrac{y}{2} & \text{for } 0 \leq y \leq \mu \\[3mm] y - \dfrac{\mu}{2} & \text{for } \mu \leq y \leq T \\[3mm] \dfrac{T+y-\mu}{2} & \text{for } T \leq y \leq T+\mu. \end{cases}$$