Stats and inferencing

Activity - 04

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$$\hat{X} = h(y) = E[X|Y] = \begin{cases} y/2 & \text{for } 0 \le y \le \mu \\ y - \frac{\mu}{2} & \text{for } \mu \le y \le T \\ T + \frac{y-\mu}{2} & \text{for } T \le y \le T+\mu \end{cases}$$

Q) obtain the variance of the above mentioned estimator.

Ans//.

$$\sigma^2 X = E(\hat{x}^2) - (E\hat{x})^2$$

$$E\hat{x}_1 = \int h(y)\, f_y(y)\, dy \quad \text{for } 0 \le y \le \mu$$

$$= \int_0^{\mu} \frac{y}{2}\left(\frac{y}{\mu T}\right) dy$$

$$= \frac{1}{2\mu T}\int_0^{\mu} y^2\, dy \rightarrow \frac{1}{2\mu T}\left[\frac{y^3}{3}\right]_0^{\mu}$$

$$= \frac{1}{2\mu T}\left[\frac{u^3}{3}\right]$$

$$E\hat{x}_1 = \frac{\mu^2}{6T}$$

$$E\,\hat{X}_2 = \int h(y)\, f_Y(y)\, dy \quad \text{for } \mu \le y \le T$$

$$= \int_{\mu}^{T} y - \frac{\mu}{2} \cdot \frac{1}{T}\, dy$$

$$= \int_{\mu}^{T} \frac{2y - \mu}{2} \cdot \frac{1}{T}\, dy$$

$$= \frac{1}{2T} \int_{\mu}^{T} (2y - \mu)\, dy \rightarrow \frac{1}{2T} \left[\frac{\cancel{2}y^2}{\cancel{2}} - \mu y \right]_{\mu}^{T}$$

$$= \frac{1}{2T} \left[y^2 - \mu y \right]_{\mu}^{T}$$

$$= \frac{1}{2T} \left[T^2 - \mu(T) - \left(\mu^2 - \mu(\mu) \right) \right]$$

$$= \frac{1}{2T} \left[T^2 - \mu T - \mu^2 + \mu^2 \right]$$

$$= \frac{1}{2T} T \left[T - \mu \right] = \frac{T - \mu}{2}$$

$$E\,\hat{X}_3 = \int h(y)\, f_Y(y)\, dy \quad \text{for } T \le y \le T+\mu$$

$$= \int_T^{T+\mu} \frac{T+y-\mu}{2} \left(\frac{T-y+\mu}{\mu T} \right) dy$$

$$= \frac{1}{2\mu T} \int_T^{T+\mu} (T+y-\mu)(T-y+\mu)\, dy$$

$$= \frac{1}{2\mu T} \int_T^{T+\mu} T^2 - \cancel{Ty} + \cancel{T\mu} + \cancel{Ty} - y^2 + y\mu - \cancel{\mu T} + \mu y - \mu^2 \, dy$$

$$= \frac{1}{2\mu T} \int_T^{T+\mu} T^2 - y^2 - \mu^2 + 2y\mu \, dy$$

$$= \frac{1}{2\mu T} \left[T^2 y - \frac{y^3}{3} - \mu^2 y + \frac{2\mu y^2}{2} \right]_T^{T+\mu}$$

$$= \frac{1}{2\mu T} \left[T^2 (T+\mu) - \frac{(T+\mu)^3}{3} - \mu^2 (T+\mu) + \mu (T+\mu)^2 - \left(T^3 - T^3/3 - \mu^2 T + \mu T^2 \right) \right]$$

$$= \frac{1}{2\mu T} \left[T\mu^2 - \frac{\mu^3}{3} \right] = \frac{\mu}{2} - \frac{\mu^2}{6T}$$

now finding $E\,\hat{X}_1^2$

$$E\,\hat{X}^2 = \int_0^{\mu} h(y)^2 \, f_Y(y)\, dy$$

$$= \int_0^{\mu} \left(\frac{y}{2}\right)^2 \left(\frac{y}{\mu T}\right) dy$$

$$= \int_0^{\mu} \frac{y^2}{4} \cdot \frac{y}{\mu T} \, dy$$

$$= \frac{1}{4\mu T}\left[\frac{\mu^4}{4}\right] = \frac{\mu^3}{16T}$$

$$E\,\hat{X}_2^2 = \int h(y)^2 \, f_Y(y)\, dy \qquad \mu \le y \le T$$

$$= \int_{\mu}^{T} \left(y - \frac{\mu}{2}\right)^2 \left(\frac{1}{T}\right) dy$$

$$=$$

$$= \frac{1}{T}\left|\frac{y^3}{3} - \frac{y^2}{2}\mu + \frac{y\mu^2}{4}\right|_{\mu}^{T}$$

$$= \frac{-\mu^3 + 3T\mu^2 - 6T^2\mu + 4T^3}{12T}$$

$$E\,\hat{X}_3^{\,2} = \int h(y)^2 f_y(y)\,dy \qquad T+\mu \le y \le T$$

$$= \int_{T}^{T+\mu} \left(\frac{T+y-\mu}{2}\right)^2 \left(\frac{T-y+\mu}{\mu T}\right) dy$$

$$= \frac{1}{48}\,\mu \left(\frac{3\mu^2}{T} + 24T - 16\mu\right)$$

$$E\,\hat{x}^2 = E\hat{x}_1 + E\hat{x}_2 - E\hat{x}_3$$

$$= \frac{\mu^2}{6T} + \frac{T-\mu}{2} + \frac{\mu}{2} - \frac{\mu^2}{6T} = \frac{T}{2}$$

$$\Downarrow$$

Simplifying this gets

$$\sigma^2 = E\hat{x}^2 - [E\hat{x}]^2$$

$$\sigma^2 = \frac{\mu^3 + 8T^3 - 2\mu^2}{24T} - \frac{T^2}{4}$$

$$\sigma^2 = \frac{\mu^3 + 8T^3 - 2\mu^2 T - 6T^3}{24T}$$