Midterm-I Exam (Part A)
CE 362: Statistics and Inference
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Timing: 02:25 PM - 03:40 PM
Oct 26, 2022
Duration: 75 min

Q1 [25 points]: A random sample of 51 items is taken. The data is shown below (and also shared in a separate file). Use this data to test the following hypotheses, assuming you want to take only a 1% risk of committing a Type I error and that the data is known to be normally distributed.

$$\mathrm{H_0}: \quad \mu = 60$$
$$\mathrm{H_a}: \quad \mu < 60$$

59.37	61.56	61.66	58.57
55.45	61.02	58.02	57.52
62.15	62.82	54.04	50.74
54.74	61.99	59.74	58.39
52.86	52.40	62.43	54.60

Q2 [25 points]: Suppose you are testing $\mathrm{H_0}: p = 0.3$ versus $\mathrm{H_a}: p \neq 0.3$. A random sample of 740 items shows that 205 have this characteristic. With a 0.05 probability of committing a Type-I error, test the hypothesis.

(a) Using p-value method, find the probability of the observed z value for this problem. What is your decision about the hypothesis test?

(b) If you had used the critical value method, what would the two critical values be?

(c) How do the sample results compare with the critical values?

Q3 [25 points]: A savings and loan averages about $\$100,000$ in deposits per week. However, because of the way pay periods fall, seasonality, and erratic fluctuations in the local economy, deposits are subject to a wide variability.

In the past, the variance for weekly deposits has been about $\$199,996,164$. In terms that make more sense to managers, the standard deviation of weekly deposits has been $\$14,142$ (which is simply the square root of $\$199,996,164$).

Shown here are data from a random sample of 15 weekly deposits for a recent period. Assume weekly deposits are normally distributed. Use these data and $\alpha = 0.10$ to test to determine whether the variance for weekly deposits has changed from its past value $\$199,996,164$.

$\$95,000$	$135,000$	$115,000$
$70,000$	$45,000$	$105,000$
$130,000$	$140,000$	$130,000$
$110,000$	$95,000$	$70,000$
$85,000$	$100,000$	$120,000$

Q4 [25 points]: Suppose a hypothesis states that the mean is exactly 60. If a random sample of 30 items is taken to test this hypothesis, what is the value of Type-II error probability, β, if the population standard deviation is 8 and the alternative mean is 65? Use Type-I error probability, $\alpha = 0.01$

Q1 Solution:

$n = 20$. $\alpha = 0.01$ (Type-I error)

$\bar{X} = 58$ (using Matlab)

$S^2 = 14.298$ (using Matlab).

$H_0 : \mu = 60$

$H_a : \mu < 60$

One-tailed problem:
We use t-statistic because variance is not given, assuming it to be unknown.

$df = n-1 = 20-1 = 19$.

$t_{\alpha, df} = t_{0.01, 19} = -2.5395$
(minus sign to be used)

$$t = \frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{58-60}{\sqrt{\frac{14.298}{20}}} = -2.3654$$

Observed $t = -2.3654 > t_{0.01,19} = -2.539$

The decision is to fail to reject the null hypothesis.

Q2. Solution:

$$H_0: \quad p = 0.3$$
$$H_a: \quad p \neq 0.3$$

A two-tailed problem. $\alpha = 0.05$.

$$n = 740$$
$$n_0 = 205$$
$$\hat{p} = \frac{205}{740} = 0.277$$

We use z-statistic for this proportion problem.

$$\text{observed } Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.277 - 0.3}{\sqrt{\frac{0.3 \times 0.7}{740}}} = -1.365.$$

For two-tail, $\alpha/2 = 0.025$

$$Z_{critical} = Z_{0.025} = \pm 1.96$$

Since, observed $Z = -1.365 > -1.96 = Z_c$

The decision is to fail to reject the null hypothesis.

Q2. <u>p-value method.</u>

observed $Z = -1.365$

from the table of z-statistic, we obtain

$$\Phi(-1.365) = \Phi(z) = \text{Area} \cong \frac{\Phi(-1.36) + \Phi(-1.37)}{2}$$

$$\cong \frac{0.0869 + 0.0853}{2}$$

$$\cong 0.0861 \text{ (using interpolation)}$$

A true value, however, may be computed using computer

$$\Phi(-1.365) = 1 - \frac{1}{2}\operatorname{erfc}\left(\frac{-1.365}{\sqrt{2}}\right)$$

$$= 0.086126525$$

The simple interpolation (as shown above) provides a pretty good approximation.

(4)

Since the p-value = 0.0861 > $\frac{\alpha}{2}$ = 0.025, the decision is to fail to reject the null hypothesis.

Critical values method :

$$Z_c = \frac{\hat{p}_c - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$\pm 1.96 = \frac{\hat{p}_c - 0.3}{\sqrt{\frac{0.3 \times 0.7}{740}}}$$

$$\hat{p}_c = 0.3 \pm 0.03301$$

0.267 & 0.333 are the critical values.

Since $\hat{p}$ = 0.277 (the observed value) is not outside critical values in tails, the decision is to fail to reject the null hypothesis.

Q3 Solution:

$$H_0: \sigma^2 = \$\,199{,}996{,}164$$

$$H_a: \sigma^2 \neq \$\,199996{,}164$$

This is a two-tailed problem.

$\alpha = 0.1$ (given) $\Rightarrow \frac{\alpha}{2} = 0.05$

$n = 15$ (values given).

$df = n-1 = 14$

$S^2 = \text{var(given-data-values)}$

$= 738571428.57$

This is variance test problem, we use χ^2-statistic.

$$\chi^2_{0.05,14} = 23.6848$$

$$\chi^2_{0.95,14} = 6.5706$$

The observed statistic is

$$\chi^2 = \frac{(15-1)\,738571428.57}{199996164} = 51.7$$

Since $\chi^2 = 51.7 > 23.684$
The decision is to REJECT the null hypothesis. The variance has changed.

Q4 Solution.

$$H_0 : \mu = 60$$
$$H_a : \mu \neq 60$$

This is two-tailed problem.

$n = 30$,

$\sigma^2 = 8$,

$\alpha = 0.01 \Rightarrow \alpha/2 = 0.005$

For β, assume $\mu_a = 65$.

$Z_{0.005} = \pm 2.5758$

Let us find critical values of sample mean.

$$Z_c = \frac{\bar{X}_c - \mu}{\sigma/\sqrt{n}}$$

$$\bar{X}_c = \mu + Z_c \frac{\sigma}{\sqrt{n}}$$

$$\bar{X}_c = 60 \pm 2.5758 \times \frac{8}{\sqrt{30}}$$

56.2378 and 63.7622

Finding critical Z-values for alternate hypothesis.

$$Z_1^u = \frac{63.7622 - 65}{8/\sqrt{30}} = -0.8474$$

$$Z_1^L = \frac{56.2378 - 65}{8/\sqrt{30}} = -5.999 = -6$$

$$\beta = \Phi(-0.8474) - \underbrace{\Phi(-6)}_{\approx 0}$$

$$\beta = 0.\underbrace{19838610225 0215}_{\text{using Matlab.}}$$

$$1 - \frac{1}{2}\operatorname{erfc}(-0.8474/\sqrt{2})$$

The probability of Type-II error is 19.84%.