

In Activity 02, you were assigned the following estimation problem:

Consider  $X \sim \mathcal{U}[0, T]$ , and also Consider a sensor measurement  $Y$  made on  $X$  such that

$$Y | X \sim \mathcal{U}[X, X + \mu]$$

where  $\mu \in \mathbb{R}^+$ . Therefore,  $Y$  conditioned on  $X$  is uniformly distributed in the range  $[X, X + \mu]$ . Although it is not necessary, we may assume that  $\mu$  is much smaller than  $T$  to make the sensor reasonably acceptable.

For the given measurement  $Y = y$ , you obtained the following MSE estimator:

$$\hat{X} = h(Y) = E[X | Y] = \begin{cases} \frac{y}{2} & \text{for } 0 \leq y \leq \mu \\ y - \frac{\mu}{2} & \text{for } \mu \leq y \leq T \\ \frac{T + y - \mu}{2} & \text{for } T \leq y \leq T + \mu. \end{cases}$$

### Question 01:

Obtain the variance of the abovementioned estimator.