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Homework 01
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CE/MATH 362/322-L1

Course: Statistics and Inferencing
Due Date: Oct. —, 2024
Points: –

Problem 1: Suppose that X is a discrete random variable with

$$P(X=0)=\frac{2}{3}\theta \qquad P(X=1)=\frac{1}{3}\theta$$
$$P(X=2)=\frac{2}{3}(1-\theta) \qquad P(X=3)=\frac{1}{3}(1-\theta)$$

where $0 \leq \theta \leq 1$ is a parameter.
The following 10 independent observations were taken from such a distribution: $(3,0,2,1,3,2,1,0,2,1)$.
a. Find the method of moments estimate of θ.
b. Find the mean and variance of MoM estimator.
c. Find an approximate standard error for your estimate.
d. What is the maximum likelihood estimate of θ ?
e. What is an approximate standard error of the maximum likelihood estimate?
f. If the prior distribution of Θ is uniform on $[0,1]$, what is the posterior density? Plot it. What is the mode of the posterior?

Problem 2: Suppose that X is a discrete random variable with
$P(X=1)=\theta$ and $P(X=2)=1-\theta$.
Three independent observations of X are made: $x_1=1, x_2=2, x_3=2$.
a. Find the method of moments estimate of θ.
b. What is the likelihood function?
c. What is the maximum likelihood estimate of θ?
d. If Θ has a prior distribution that is uniform on $[0,1]$, what is its posterior density?

Problem 3: Suppose that $X \sim \mathrm{Bin}(n,p)$.
a. Show that the MLE of p is $\hat{p}=X/n$.
b. Show that MLE of part (a) attains the Cramér-Rao lower bound.
c. If $n=10$ and $X=5$, plot the log likelihood function.

Problem 4: Suppose that X follows a geometric distribution,

$$P(X = k) = p(1 - p)^{k-1}$$

and assume an i.i.d. sample of size n.
a. Find the method of moments estimate of p.
b. Find the MLE of p.
c. Find the asymptotic variance of the MLE.
d. Let p have a uniform prior distribution on $[0, 1]$. What is the posterior distribution of p? What is the posterior mean?

Problem 5: Consider an i.i.d. sample of random variables with density function

$$f(x \mid \sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x|}{\sigma}\right)$$

a. Find the method of moments estimate of σ.
b. Find the maximum likelihood estimate of σ.
c. Find the asymptotic variance of the MLE.
d. Find a sufficient statistic for σ.