Habib University

EE/CE 453/352: Digital Signal Processing - Spring 2024
Saad Baig

Time = 120 minutes | **Final Exam (30-4-2024)** | Max Points: 40

Instructions:

i. **Smart watches, laptops, and similar electronics are strictly NOT allowed.**
ii. **Answer sheets should contain all steps, working, explanations, and assumptions.**
iii. Attempt the exam with black/blue ink.
iv. Print your name and HU ID on all sheets.
v. This is a closed-book examination but you are allowed a double-sided A4 sized cheat sheet.
vi. You are not allowed to ask/share your method or answer with your peers. The work submitted by you is solely your own work. Any violation of this will be the violation of HU Honor code and proper action will be taken as per university policy if found to be involved in such an activity.

CLO Assessment:

This quiz will assess students for the following course learning outcomes.

Course Learning Outcome		Learning Domain Level	Questions
CLO 2	Analyze discrete-time signals and systems in transform domain using z-Transform, DTFT, and DFT.	Cog-4	1
CLO 3	Design various types of digital filters to meet given specifications.	Cog-4	2, 3
CLO 4	Apply appropriate digital signal processing techniques to an adequately-explained domain-specific scenario.	Cog-4	4

Undertaking:

I hereby affirm that I have read the instructions. I am fully aware of the HU honor code and the repercussions of its violation, and hereby pledge that the work I am going to submit is clearly my own.

Signature: ______________________

Name: INSTRUCTOR SOLUTION **HU ID:** ______________

TABLE 7.2 Properties of the DFT

Property	Time Domain	Frequency Domain
Notation	$x(n),\ y(n)$	$X(k),\ Y(k)$
Periodicity	$x(n) = x(n+N)$	$X(k) = X(k+N)$
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(k) + a_2X_2(k)$
Time reversal	$x(N-n)$	$X(N-k)$
Circular time shift	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular frequency shift	$x(n)e^{j2\pi ln/N}$	$X((k-l))_N$
Complex conjugate	$x^*(n)$	$X^*(N-k)$
Circular convolution	$x_1(n) \circledN x_2(n)$	$X_1(k)X_2(k)$
Circular correlation	$x(n) \circledN y^*(-n)$	$X(k)Y^*(k)$
Multiplication of two sequences	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k) \circledN X_2(k)$
Parseval's theorem	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$

Property	Time Domain	z-Domain	ROC
Notation:	$x(n)$	$X(z)$	ROC: $r_2 < \lvert z\rvert < r_1$
	$x_1(n)$	$X_1(z)$	ROC_1
	$x_2(n)$	$X_1(z)$	ROC_2
Linearity:	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(z) + a_2X_2(z)$	At least $ROC_1 \cap ROC_2$
Time shifting:	$x(n-k)$	$z^{-k}X(z)$	ROC, except $z = 0$ (if $k > 0$) and $z = \infty$ (if $k < 0$)
z-Scaling:	$a^n x(n)$	$X(a^{-1}z)$	$\lvert a\rvert r_2 < \lvert z\rvert < \lvert a\rvert r_1$
Time reversal	$x(-n)$	$X(z^{-1})$	$\frac{1}{r_1} < \lvert z\rvert < \frac{1}{r_2}$
Conjugation:	$x^*(n)$	$X^*(z^*)$	ROC
z-Differentiation:	$n\,x(n)$	$-z\frac{dX(z)}{dz}$	$r_2 < \lvert z\rvert < r_1$
Convolution:	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$	At least $ROC_1 \cap ROC_2$

TABLE 4.1 *Some common z-transform pairs*

Sequence	*Transform*	*ROC*
$\delta(n)$	1	$\forall z$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-b^n u(-n-1)$	$\frac{1}{1-bz^{-1}}$	$\lvert z\rvert < \lvert b\rvert$
$[a^n \sin\omega_0 n]\,u(n)$	$\frac{(a\sin\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$[a^n \cos\omega_0 n]\,u(n)$	$\frac{1-(a\cos\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-nb^n u(-n-1)$	$\frac{bz^{-1}}{(1-bz^{-1})^2}$	$\lvert z\rvert < \lvert b\rvert$

Question 1 [10 pts]: Answer the following questions for the digital system shared below:

$$y[n] = 0.7y[n-1] - 0.12y[n-2] + x[n-1] + x[n-2]$$

a) [4 pts] Is the system stable?
b) [6 pts] Compute the response of the system to the input $x[n] = nu[n]$.

Solution:

$$Y(z) = 0.7z^{-1}Y(z) - 0.12z^{-2}Y(z) + z^{-1}X(z) + z^{-2}X(z)$$

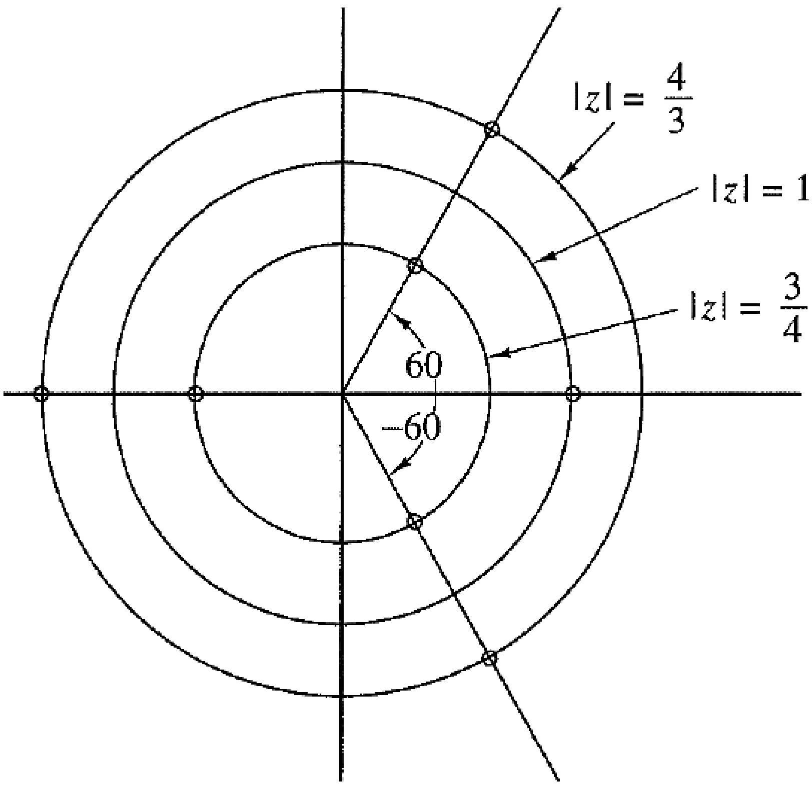
$$\frac{Y(z)}{X(z)} = H(z) = \frac{z^{-1} + z^{-2}}{1 - 0.7z^{-1} + 0.12z^{-2}} = \frac{z+1}{z^2 - 0.7z + 0.12} = \frac{z+1}{(z-0.3)(z-0.4)}$$

System is stable because poles are inside the unit circle: $z = 0.3$ & $z = 0.4$.

$$x[n] = nu[n], \qquad X(z) = \frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$$

$$Y(z) = X(z)H(z) = \frac{z(z+1)}{(z-0.3)(z-0.4)(z-1)^2}$$

Question 2 [16 pts]: Consider the pole-zero plot shown below:



a) [6 pts] What is the magnitude of the frequency response at $\omega = \pi/2$?
b) [6 pts] Draw a direct-form realization that exploits all symmetries to minimize the number of multiplications.
c) [4 pts] What kind of filter does it represent in terms of allowed bandwidth of frequencies (LPF, HPF etc.) and the impulse response (FIR or IIR)?

Solution:

$$H(z) = \left(z + \frac{4}{3}\right)\left(z + \frac{3}{4}\right)(z-1)(z - 0.38 - j0.65)(z - 0.38 + j0.65)(z - 0.67 - j1.15)(z - 0.67 + j1.15)$$

Replacing $z = e^{j\omega} = e^{j\frac{\pi}{2}} = j$:

$$H(e^{j\omega}) = \left(j + \frac{4}{3}\right)\left(j + \frac{3}{4}\right)(j-1)(j-0.38-j0.65)(j-0.38+j0.65)(j-0.67-j1.15)(j-0.67+j1.15)$$

$$H(e^{j\omega}) = 5.12 - j3.69, \qquad |H(e^{j\omega})| = 6.31$$

Direct form implementation is too time consuming and complex for the exam duration, though by calculation we can obtain the following coefficients: [1, 2.72, 5.26, 9.63]

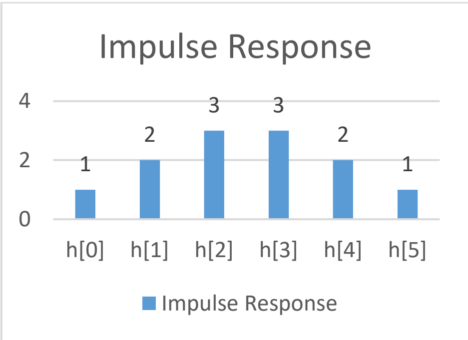
Filter seems to be allowing specific frequencies towards the higher sides, some sort of specialized bandpass filter. It should be FIR as explicit poles are missing.

Question 3 [4 pts]: A linear phase FIR filter of length 6 has been designed. Its impulse response is $h(n)$. Given the following information:

$$h(0) = 1, \qquad h(1) = 2, \qquad h(2) = 3, \qquad h(3) = 3$$

a) [2 pts] Sketch the complete impulse response of this filter.
b) [2 pts] What type of FIR filter is this? Type I, II, III or IV?

Solution:



This is an Even Symmetric FIR filter (Type II).

Question 4 [5pts]: Describe the process and structure involved in designing and implementing filters for a sampling rate conversion system.

Sample Answer: Filter design for a sampling rate conversion (SRC) system is crucial for maintaining signal integrity. Two key considerations are:

a) **Anti-Aliasing (for Downsampling):** When reducing the sampling rate (downsampling), an anti-aliasing filter is required to remove high-frequency components that could cause aliasing. The filter's cutoff frequency is typically set to half the new sampling rate to ensure that no aliases are produced.
b) **Reconstruction (for Upsampling):** When increasing the sampling rate (upsampling), a reconstruction filter is needed to smooth the interpolated signal and remove high-frequency artifacts introduced by the upsampling process. The filter's cutoff frequency is usually set to the original Nyquist frequency (half the original sampling rate) to eliminate high-frequency components.

The filter design process involves selecting an appropriate filter type (e.g., FIR or IIR), determining the filter order and coefficients, and ensuring that the filter meets the desired specifications for passband ripple, stopband attenuation, and transition bandwidth.

Question 5 [5 pts]: Write a few lines about the algorithm and application of any two of the following:

a) Hough Transform
b) Kalman Filter
c) Wavelet Transform
d) Particle Swarm Optimization

Sample Answer:

a) **Hough Transform:** The Hough Transform is an algorithm used in image processing and computer vision for detecting shapes, primarily lines and curves. It works by converting the problem of detecting these shapes into a parameter space, where each point in the parameter space corresponds to a possible shape in the image. By identifying peaks in this parameter space, the algorithm can robustly detect lines and curves, even in the presence of noise and partial occlusion.

b) **Kalman Filter:** The Kalman Filter is an algorithm used for state estimation in systems with noise. It is widely used in various fields such as control systems, navigation, and signal processing. The filter recursively estimates the state of a dynamic system based on noisy measurements over time. It combines predictions from a mathematical model of the system's behavior with measurements to provide an optimal estimate of the system's state, taking into account the uncertainties in both the model and the measurements.

c) **Wavelet Transform:** The Wavelet Transform is a mathematical tool used for analyzing and processing signals and images. Unlike the Fourier Transform, which uses sinusoidal basis functions, the Wavelet Transform uses wavelets, which are small waves of varying frequency and duration. This allows the Wavelet Transform to provide a time-frequency representation of a signal, capturing both high-frequency and low-frequency components with high resolution in time or frequency, depending on the application.

d) **Particle Swarm Optimization (PSO):** Particle Swarm Optimization is a population-based stochastic optimization technique inspired by the social behavior of bird flocking or fish schooling. In PSO, a population of candidate solutions, called particles, moves through the search space to find the optimal solution. Each particle adjusts its position based on its own experience and the experiences of neighboring particles. PSO is commonly used in optimization problems where the objective function is continuous and differentiable, and it has been applied in various fields, including engineering, economics, and machine learning.