Habib University

EE/CE 453/352: Digital Signal Processing - Spring 2025
Saad Baig

Time = 70 minutes | **Midterm SOLUTION** | Max Points: 40

Instructions:

i. **Smart watches, laptops, and similar electronics are strictly NOT allowed.**
ii. **Answer sheets should contain all steps, working, explanations, and assumptions.**
iii. Attempt the quiz with black/blue ink.
iv. Print your name and HU ID on all sheets.
v. This is a closed-book examination but you are allowed a single-sided A4 sized cheat sheet.
vi. You are not allowed to ask/share your method or answer with your peers. The work submitted by you is solely your own work. Any violation of this will be the violation of HU Honor code and proper action will be taken as per university policy if found to be involved in such an activity.

CLO Assessment:

This quiz will assess students for the following course learning outcomes.

Course Learning Outcome		Learning Domain Level	Questions
CLO 1	Analyze discrete-time signals and systems in time domain.	Cog-4	1 & 2
CLO 2	Analyze discrete-time signals and systems in transform domain using z-Transform, DTFT, and DFT.	Cog-4	3 & 4

Undertaking:

I hereby affirm that I have read the instructions. I am fully aware of the HU honor code and the repercussions of its violation, and hereby pledge that the work I am going to submit is clearly my own.

Signature: ______________________

Name: **INSTRUCTOR SOLUTION** **HU ID:** ______________

TABLE 7.2 Properties of the DFT

Property	Time Domain	Frequency Domain
Notation	$x(n),\ y(n)$	$X(k),\ Y(k)$
Periodicity	$x(n) = x(n+N)$	$X(k) = X(k+N)$
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(k) + a_2X_2(k)$
Time reversal	$x(N-n)$	$X(N-k)$
Circular time shift	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular frequency shift	$x(n)e^{j2\pi ln/N}$	$X((k-l))_N$
Complex conjugate	$x^*(n)$	$X^*(N-k)$
Circular convolution	$x_1(n) \ \textcircled{N}\ x_2(n)$	$X_1(k)X_2(k)$
Circular correlation	$x(n) \ \textcircled{N}\ y^*(-n)$	$X(k)Y^*(k)$
Multiplication of two sequences	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k) \ \textcircled{N}\ X_2(k)$
Parseval's theorem	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$

Property	Time Domain	z-Domain	ROC
Notation:	$x(n)$	$X(z)$	ROC: $r_2 < \|z\| < r_1$
	$x_1(n)$	$X_1(z)$	ROC_1
	$x_2(n)$	$X_1(z)$	ROC_2
Linearity:	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(z) + a_2X_2(z)$	At least $ROC_1 \cap ROC_2$
Time shifting:	$x(n-k)$	$z^{-k}X(z)$	ROC, except $z = 0$ (if $k > 0$) and $z = \infty$ (if $k < 0$)
z-Scaling:	$a^nx(n)$	$X(a^{-1}z)$	$\|a\|r_2 < \|z\| < \|a\|r_1$
Time reversal	$x(-n)$	$X(z^{-1})$	$\frac{1}{r_1} < \|z\| < \frac{1}{r_2}$
Conjugation:	$x^*(n)$	$X^*(z^*)$	ROC
z-Differentiation:	$n\,x(n)$	$-z\frac{dX(z)}{dz}$	$r_2 < \|z\| < r_1$
Convolution:	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$	At least $ROC_1 \cap ROC_2$

TABLE 4.1 *Some common z-transform pairs*

Sequence	*Transform*	*ROC*
$\delta(n)$	1	$\forall z$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$a^nu(n)$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-b^nu(-n-1)$	$\frac{1}{1-bz^{-1}}$	$\|z\| < \|b\|$
$[a^n \sin\omega_0 n]\,u(n)$	$\frac{(a\sin\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
$[a^n \cos\omega_0 n]\,u(n)$	$\frac{1-(a\cos\omega_0)z^{-1}}{1-(2a\cos\omega_0)z^{-1}+a^2z^{-2}}$	$\|z\| > \|a\|$
$na^nu(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-nb^nu(-n-1)$	$\frac{bz^{-1}}{(1-bz^{-1})^2}$	$\|z\| < \|b\|$

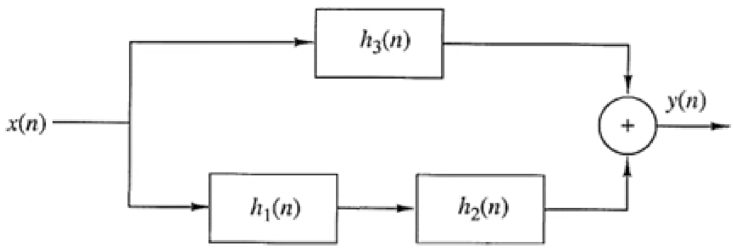
Question 1 [10 pts]: For the system defined below, state with proof whether or not it is dynamic, linear, causal, stable and/or time invariant.

$$y[n] = x[n]\sin(\omega_0 n)$$

Solution:

Property	Status
Memory	System is **static**. There are no advanced or delayed versions if we test it with different values of n. $y[0] = x[0]\sin(0)$
Linearity	System is **linear**: $\alpha y_1[n] = \alpha x_1[n]\sin(\omega_0 n)$ $\beta y_2[n] = \beta x_2[n]\sin(\omega_0 n)$ $\alpha y_1[n] + \beta y_2[n] = \alpha x_1[n]\sin(\omega_0 n) + \beta x_2[n]\sin(\omega_0 n)$
Causality	System is **causal**. There are no advanced versions if we test it with different values of n. $y[0] = x[0]\sin(0)$
Stability	System is **stable.** Bounded input will return a bounded output amplified by the sinusoidal value.
Time Invariance	System is time **variant**, because: $y[n-k] \neq x[n-k]\sin(\omega_0 n - k)$ $y[n-k] = x[n-k]\sin(\omega_0 n - \omega_0 k)$

Question 2 [10 pts]: Consider the following causal LTI system:



$$h_1[n] = 2\delta[n-2] - 3\delta[n+1] \qquad\qquad h_2[n] = \delta[n-1] + 2\delta[n+2]$$

$$h_3[n] = 5\delta[n-5] + 7\delta[n-3] + 2\delta[n-1] - \delta[n] + 3\delta[n+1]$$

a) Determine $h[n]$, where $h[n]$ is a combination of $h_1[n]$, $h_2[n]$ and $h_3[n]$.
b) Draw the block diagram representation of $h[n]$ complete with adder, multiplier and delay blocks.

Solution:

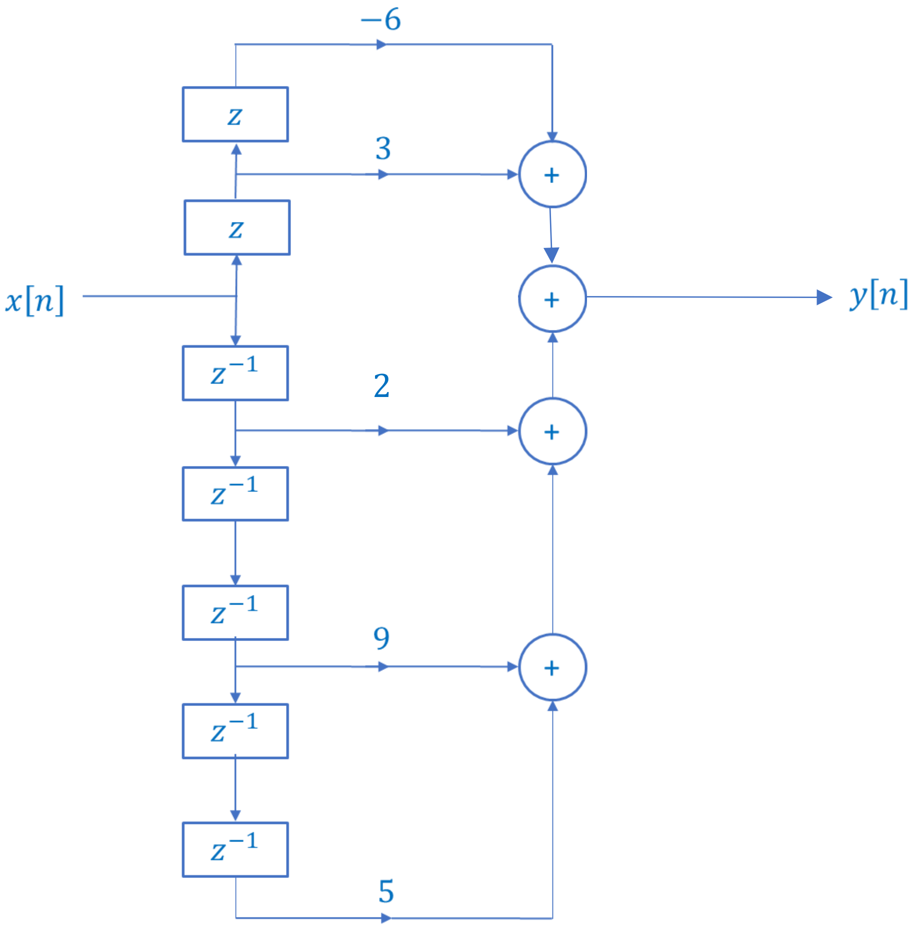
$$h[n] = (h_1[n] * h_2[n]) + h_3[n]$$

$$h_1[n] = \{-3,\underset{\uparrow}{0},0,2\}, \qquad h_2[n] = \{2,0,\underset{\uparrow}{0},1\}, \qquad h_3[n] = \{3,\underset{\uparrow}{-1},2,0,7,0,5\}$$

$h_2[n]\backslash h_1[n]$	-3	**0**	0	2
2	-6	0	0	4
0	0	0	0	0
0	0	**0**	0	0
1	-3	0	0	2

$$h_1[n] * h_2[n] = \{-6,0,0,1,0,0,2\}$$

$$h[n] = \{-6,0,0,1,0,0,2\} + \{3,-1,2,0,7,0,5\} = \{-6,0,3,0,2,0,9,0,5\}$$



Question 3 [16 pts]: Let $x(n) = (0.8)^n u(n)$, $h(n) = (-0.9)^n u(n)$, and $y(n) = x(n) * h(n)$. Assuming a causal system:

a) Determine $Y(z)$ using the z-transform (with RoC).

b) Determine $y(n)$ using the inverse z-transform.

Solution:

$$X(z) = \frac{1}{1-0.8z^{-1}}, \qquad |z| > 0.8, \qquad H(z) = \frac{1}{1+0.9z^{-1}}, \qquad |z| > 0.9$$

$$\boxed{Y(z) = X(z)H(z) = \frac{1}{(1-0.8z^{-1})(1+0.9z^{-1})}, \qquad |z| > 0.9}$$

$$Y(z) = \frac{z^2}{(z-0.8)(z+0.9)}$$

Applying partial fractions method:

$$\frac{Y(z)}{z} = \frac{z}{(z-0.8)(z+0.9)} = \frac{A}{(z-0.8)} + \frac{B}{(z+0.9)} \rightarrow ①$$

Multiplying and dividing both sides by $(z-0.8)$ in eq. ①, $\quad \frac{z}{(z+0.9)} = A + \frac{B(z-0.8)}{(z+0.9)}$

Setting $z = 0.8$, $\quad \frac{0.8}{0.8+0.9} = A \approx 0.47$

Multiplying and dividing both sides by $(z+0.9)$ in eq. ①, $\quad \frac{z}{(z-0.8)} = \frac{A(z+0.9)}{(z-0.8)} + B$

Setting $z = -0.9$, $\quad \frac{-0.9}{-0.9-0.8} = B \approx 0.53$

$$\frac{Y(z)}{z} = \frac{0.47}{(z-0.8)} + \frac{0.53}{(z+0.9)}, \qquad Y(z) = \frac{0.47z}{(z-0.8)} + \frac{0.53z}{(z+0.9)} = \frac{0.47}{(1-0.8z^{-1})} + \frac{0.53}{(1+0.9z^{-1})}$$

$$\boxed{y[n] = [0.47(0.8)^n + 0.53(-0.9)^n]u[n]}$$

Question 4 [4 pts]: Calculate the number of computations (addition and multiplication) required for direct DFT and radix-2 decimation in time FFT algorithm for sequences of length 2048.

Solution:

<u>For DFT</u>:

Additions: $N(N-1) = 4{,}192{,}256$

Multiplications: $N^2 = 4{,}194{,}304$

<u>For FFT (radix-2 DIT)</u>:

Additions: $N \log_2 N = 22{,}528$

Multiplications: $N \log_2 N = 22{,}528$