Habib University

EE/CE 453/352: Digital Signal Processing - Spring 2025
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Time = 40 minutes	**Quiz 03 SOLUTION**	Max Points: 30

Instructions:

i. **Smart watches, laptops, and similar electronics are strictly NOT allowed.**
ii. **Answer sheets should contain all steps, working, explanations, and assumptions.**
iii. Attempt the quiz with black/blue ink.
iv. Print your name and HU ID on all sheets.
v. This is a closed-book examination but you are allowed a single-sided A4 sized cheat sheet.
vi. You are not allowed to ask/share your method or answer with your peers. The work submitted by you is solely your own work. Any violation of this will be the violation of HU Honor code and proper action will be taken as per university policy if found to be involved in such an activity.

CLO Assessment:

This quiz will assess students for the following course learning outcomes.

Course Learning Outcome		Learning Domain Level
CLO 2	Design various types of digital filters to meet given specifications.	Cog-5

Undertaking:

I hereby affirm that I have read the instructions. I am fully aware of the HU honor code and the repercussions of its violation, and hereby pledge that the work I am going to submit is clearly my own.

Signature: ______________________________

Name: ______________________________ **HU ID:** ________________

In both questions below, α = sum of HU ID digits. E.g. if HU ID = 01425, $\alpha = 12$.

Question 1 [15 pts]: Transform the single-pole high pass Butterworth filter with system function:

$$H_a(s) = \frac{s}{s + \Omega_c}$$

into a digital filter with by means of the bilinear transform. Use $T = 0.5$ ms and digital cutoff frequency $\omega_c = 0.\alpha\,\pi$ rad.

e.g. if HU ID = 01425, $\omega_c = 0.12\pi$ rad.

Solution:

Let $\omega_c = 0.4\pi$ rad.

$$\Omega_c = \frac{2}{T} tan \frac{0.4\pi}{2} = \frac{2}{T}(0.73) \text{ rad/s}$$

$$G(z) = H_a(s)\Big|_{s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{\frac{2}{T}\left(\frac{z-1}{z+1}\right)}{\frac{2}{T}\left(\frac{z-1}{z+1}\right) + \frac{2}{T}(0.73)} = \frac{\frac{(z-1)}{(z+1)}}{\frac{(z-1)}{(z+1)} + 0.73} = \frac{\frac{(z-1)}{(z+1)}}{\frac{z-1+0.73(z+1)}{(z+1)}}$$

$$G(z) = \frac{z-1}{1.73z - 0.27}$$

Question 2 [15 pts]: Design an FIR digital filter to approximate an ideal lowpass filter with passband gain of unity, cutoff frequency of π/α radians and windowing function of length $M = 5$. You are allowed to truncate or round up to 1 decimal place.

a) Use a Rectangular window.
b) Use a Bartlett window.

<u>Hints:</u>

An ideal lowpass filter is given by:

$$H(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

The impulse response is given by:

$$h(n) = \begin{cases} \frac{\omega_c}{\pi} & n = 0 \\ \frac{\omega_c}{\pi}\frac{\sin(\omega_c n)}{\omega_c n} & n \ne 0 \end{cases}$$

Bartlett window

$$w(n) = \begin{cases} \frac{2n}{M-1}, & 0 \le n \le \frac{M-1}{2} \\ 2 - \frac{2n}{M-1}, & \frac{M-1}{2} \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

Solution:

Let cutoff frequency $\omega_c = 0.4\pi$

$$h_a[n] = \left\{\frac{0.4\pi}{\pi}, \frac{0.4\pi}{\pi}\frac{\sin(0.4\pi)}{0.4\pi}, \frac{0.4\pi}{\pi}\frac{\sin(0.4\pi \times 2)}{0.4\pi \times 2}, \frac{0.4\pi}{\pi}\frac{\sin(0.4\pi \times 3)}{0.4\pi \times 3}, \frac{0.4\pi}{\pi}\frac{\sin(0.4\pi \times 4)}{0.4\pi \times 4}\right\}$$

$$h_a[n] = \{0.4, 0.3, 0.09, -0.06, -0.08\}$$

$$w_b(n) = \left\{0, \frac{2}{5-1}, \frac{4}{5-1}, 2 - \frac{6}{5-1}, 2 - \frac{8}{5-1}\right\} = \{0, 0.5, 1, 0.5, 0\}$$

$$h_b[n] = h_a[n] \times w_b[n] = \{0, 0.15, 0.09, -0.03, 0\}$$