Exercise Manual: Signal Filtering using FIR Filters

Objective

This exercise manual will guide you through designing and implementing different types of Finite Impulse Response (FIR) filters using various windowing techniques. You will generate a composite signal, analyze its frequency content, and apply low-pass, high-pass, band-pass, and band-stop filters. The goal is to understand signal filtering concepts and their impact on frequency components.

Task 1: Signal Generation

Introduction

In this task, you will generate a composite signal consisting of three sinusoidal components with different frequencies. The objective is to create a signal that contains both low and high-frequency components, which we will filter in later tasks.

Basic Parameters

- Sampling frequency: $f_s = 1000$ Hz
- Signal duration: $t_{\max} = 3$ seconds
- Time step: $dt = \frac{1}{f_s}$
- Time vector: $t = dt : \frac{1}{f_s} : t_{\max}$
- Number of samples: $N = \text{length}(t)$
- Low-frequency component: $f_{\text{low}} = 50$ Hz
- Mid-frequency component: $f_{\text{mid}} = 200$ Hz
- High-frequency component: $f_{\text{high}} = 300$ Hz

Instructions

1. Define the sampling frequency and time duration.
2. Generate three sinusoidal signals of different frequencies.
3. Sum the signals to create a composite signal.
4. Compute and plot the magnitude spectrum of the signal using the Fast Fourier Transform (FFT).

Task 2: Low-Pass Filtering with Hanning Window

Introduction

In this task, you will design a low-pass FIR filter using the Hanning window method. A low-pass filter allows low-frequency components to pass while attenuating high-frequency components.

Instructions

1. Define the cutoff frequency and derive filter parameters.
2. Compute the ideal impulse response of the low-pass filter.
3. Apply the Hanning window to the filter coefficients.
4. Filter the composite signal.
5. Compute and plot the magnitude spectrum of the filter itself.
6. Compare the unfiltered and filtered signals in the frequency domain.

Task 3: High-Pass Filtering with Hanning Window

Introduction

A high-pass filter allows high-frequency components to pass while attenuating low-frequency components. You will design a high-pass FIR filter using the Hanning window method.

Basic Method Used

- The high-pass filter is derived from the low-pass filter by subtracting the low-pass impulse response from an all-pass impulse response.
- Mathematically, the high-pass filter impulse response is calculated as:

$$h_{\text{high}}(n) = \text{ap} - h_{\text{low}}(n)$$

- The Hanning window is then applied to smooth the transition.
- You will use the following equations:

$$\text{ImpLo} = \frac{\sin(\Omega_c(n - M + \epsilon))}{\pi(n - M + \epsilon)}$$

$$\text{ap} = \frac{\sin(\pi(n - M + \epsilon))}{\pi(n - M + \epsilon)}$$

Instructions

1. Compute the high-pass filter response using the complement of the low-pass filter response.
2. Apply the Hanning window.
3. Filter the composite signal.
4. Compute and plot the magnitude spectrum of the filter itself.
5. Compare the unfiltered and filtered signals in the frequency domain.

Task 4: Band-Pass Filtering with Hanning Window

Introduction

A band-pass filter allows frequencies within a specific range to pass while attenuating frequencies outside this range.

Basic Method Used

- A band-pass filter is created by subtracting two low-pass filters with different cutoff frequencies.
- Mathematically, the impulse response is computed as:

$$h_{\text{bp}}(n) = h_{\text{low}}(f_{\text{high}}, n) - h_{\text{low}}(f_{\text{low}}, n)$$

- The Hanning window is applied to ensure smooth transitions in frequency response.

Instructions

1. Define the lower and upper cutoff frequencies.
2. Compute the band-pass filter response.
3. Apply the Hanning window.
4. Filter the composite signal.
5. Compute and plot the magnitude spectrum of the filter itself.
6. Compare the unfiltered and filtered signals in the frequency domain.

Task 5: Band-Stop Filtering with Hanning Window

Introduction

A band-stop filter attenuates frequencies within a specific range while allowing other frequencies to pass.

Basic Method Used

- A band-stop filter is derived from the complement of a band-pass filter.
- Mathematically, the impulse response is computed as:

$$h_{\text{bs}}(n) = h_{\text{all-pass}}(n) - h_{\text{bp}}(n)$$

- The Hanning window is applied to ensure a smooth response.
- You will use the following equations:

$$\text{ImpLoWide} = \frac{\sin(\Omega_c(n - M + \epsilon))}{\pi(n - M + \epsilon)}$$

$$\text{ImpLoNarrow} = \frac{\sin(\Omega_{c2}(n - M + \epsilon))}{\pi(n - M + \epsilon)}$$

$$\text{ImpBand} = \text{ImpLoWide} - \text{ImpLoNarrow}$$

Instructions

1. Define the stopband frequencies.
2. Compute the band-stop filter response.
3. Apply the Hanning window.
4. Filter the composite signal.
5. Compute and plot the magnitude spectrum of the filter itself.
6. Compare the unfiltered and filtered signals in the frequency domain.

Post Lab Exercise

Task 1:

1. Design a high-pass filter using the Hamming window with a cutoff frequency of 250 Hz.
2. Choose your own composite signal values.
3. Compute and plot the magnitude spectrum of the filter itself.
4. Compare the unfiltered and filtered signals in the frequency domain.

Task 2:

1. Design a band-reject filter using the Blackman window with a cutoff frequency of 250 Hz.
2. Choose your own composite signal values.
3. Compute and plot the magnitude spectrum of the filter itself.
4. Compare the unfiltered and filtered signals in the frequency domain.

Task 3: Kaiser Window Low-Pass Filter Design

Introduction

In this task, you will design a low-pass FIR filter using the Kaiser window method and analyze how the filter's attenuation affects its frequency response.

Basic Parameters

- Sampling frequency: 1000 Hz
- Cutoff frequency: 100 Hz
- Different attenuation values: 10, 40, 60

Instructions

1. Compute the transition width based on the cutoff frequency.
2. Compute the filter length and Kaiser beta parameter based on the attenuation.
3. Design and apply the Kaiser window.
4. Compute and plot the magnitude response of the filters.
5. Compare the effects of different attenuation values on the filter response.

Observations & Questions

1. How does the attenuation affect the filter length?
2. What impact does increasing attenuation have on the transition width?
3. How does the Kaiser window compare to the Hanning window in terms of filter performance?
4. What happens to the stopband attenuation as the attenuation increases?