EE/CE 453/352: Digital Signal Processing

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Homework 1 SOLUTION

Question 1 [3 pts]: Use the following input sequences to compute the requested sequence:

$$y[n] = b[3-n] + c[n]$$

For indices where the input sequences are not specified, consider values to be zero. Clearly specify the index range of nonzero values for each output.

$$b[n] = \{2\ \ 4\ \ 3\ \ \text{-}5\ \ \text{-}2\ \ 1\ \ 6\ \ 1\ \ \text{-}3\ \ \text{-}2\ \ 2\} \qquad -8 \le n \le 2$$
$$c[n] = \{\text{-}1\ \ 2\ \ \text{-}3\ \ 4\ \ \text{-}1\ \ 2\ \ \text{-}3\ \ 4\} \qquad -6 \le n \le 1$$

Solution:

n	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
b[-n]					2	-2	-3	1	6	1	-2	-5	3	4	2			
b[3-n]								2	-2	-3	1	6	1	-2	-5	3	4	2
c[n]	-1	2	-3	4	-1	2	-3	4										
y[n]	-1	2	-3	4	-1	2	-3	6	-2	-3	1	6	1	-2	-5	3	4	2

Question 2 [5 pts]: Consider the following system properties discussed in class:

1) Memoryless
2) Causality
3) Linearity
4) Time Invariance
5) Stability

Determine which of these properties hold and which do not hold for each of the following systems (justify your answers):

a) $y[n] = x[2-n]$
b) $y[n] = nx[-n]$

Solution:

Property	System (a)	System (b)
Memory	System has memory: $y[n] = x[-n+2]$	System has memory: $y[n] = nx[-n]$
Causality	Non-causal because if $n = -1$: $y[-1] = x[1+2] = x[3]$	Non-causal because if $n = -1$: $y[-1] = -x[1]$

	Which means there is a value before $n = 0$.	Which means there is a value before $n = 0$.
Linearity	System is linear: $\alpha y_1[n] + \beta y_2[n] = \alpha x_1[-n+2] + \beta x_2[-n+2]$	System is linear: $\alpha y_1[n] + \beta y_2[n] = \alpha n x_1[-n] + \beta n x_2[-n]$
Time-invariance	System is time-variant: $y[n-k] \neq x[-n+2-k]$ $\rightarrow y[n-k] = x[-(n-k)+2] = x[-n+2+k]$	System is time-variant: $y[n-k] \neq nx[-n-k]$ $\rightarrow y[n-k] = (n-k)x[-(n-k)] = (n-k)x[-n+k]$
Stability	System is stable because time reversal or shifting does not change the max and min value of signal: $\lvert x[-n+2] \rvert < B$	System is unstable: If $x[n] = u[n]$ then: $y[n] = nu[-n]$

Question 3 [2 pts]: Consider the following system properties discussed in class:

1) Causality
2) Stability

Determine which of these properties hold and which do not hold for each of the LTI systems whose impulse response is given below (justify your answers):

a) $h[n] = \delta[n] - \delta[n-1] + \delta[n+1]$
b) $h[n] = (0.2)^n u[n]$

Solution:

Property	**System (a)**	**System (b)**
Causality	Non-causal because of $\delta[n+1]$. $h[n] \neq 0, \quad n < 0$	System is causal (step function starts at 0).
Stability	System is stable (completely summable).	System is stable as $(0.2)^n$ converges. $\sum_{n=-\infty}^{\infty} h[n] = \frac{0.2}{1-0.2} = \frac{1}{4} < \infty$

Question 4 [5 pts]: A LTI system has an impulse response:

$$h[n] = \delta[n] + 0.5\delta[n-1] + 0.25\delta[n-2]$$

Compute and hand-sketch the output $y[n]$ for the following inputs:

a) $x[n] = \delta[n] + 2\delta[n-4] - 0.5\delta[n-6]$
b) $x[n] = \delta[n] - 0.5\delta[n-1]$

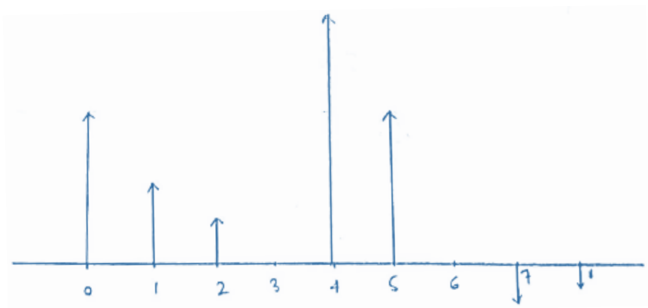
Solution:

a) $x[n] = \delta[n] + 2\delta[n-4] - 0.5\delta[n-6]$

$$y[n] = x[n] + 0.5x[n-1] + 0.25x[n-2]$$

$$y[n] = \{\delta[n] + 2\delta[n-4] - 0.5\delta[n-6]\} + 0.5\{\delta[n-1] + 2\delta[n-5] - 0.5\delta[n-7]\} \\ +0.25\{\delta[n-2] + 2\delta[n-6] - 0.5\delta[n-8]\}$$

$$y[n] = \delta[n] + 0.5\delta[n-1] + 0.25\delta[n-2] + 2\delta[n-4] + \delta[n-5] + 0.5\delta[n-6] \\ - 0.25\delta[n-7] - 0.125\delta[n-8]$$

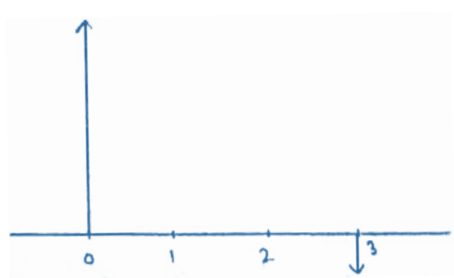


b) $x[n] = \delta[n] - 0.5\delta[n-1]$

$$y[n] = x[n] + 0.5x[n-1] + 0.25x[n-2]$$

$$y[n] = \delta[n] - 0.5\delta[n-1] + 0.5\{\delta[n-1] - 0.5\delta[n-2]\} + 0.25\{\delta[n-2] - 0.5\delta[n-3]\}$$

$$y[n] = \delta[n] - 0.125\delta[n-3]$$



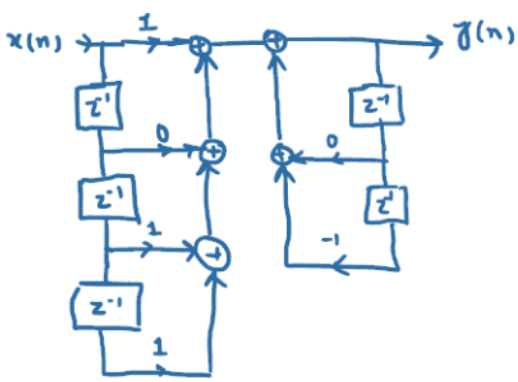
Question 5 [15 pts]: Consider the systems represented by the following Linear, Constant Coefficient Difference Equations (LCCDEs):

1) $y[n] + y[n-2] = x[n] + x[n-2] + x[n-3]$
2) $y[n] - 2x[n-2] = x[n-1] + 2x[n]$
3) $y[n] + 2y[n-2] = x[n] + 3x[n-2] + y[n-3]$

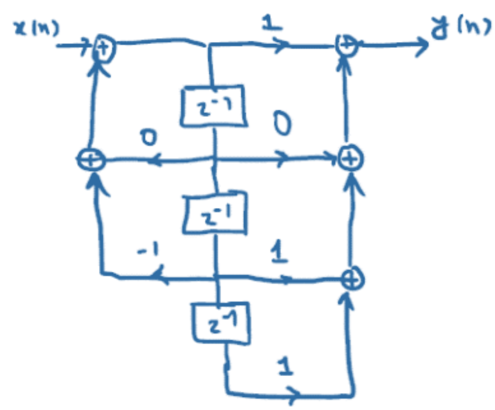
a) Identify which of these systems are FIR and IIR systems.
b) For IIR systems, draw the block diagrams for Direct Form I and Direct Form II realizations.
c) In part (b), make an attempt to minimize the number of adder blocks needed. (Hint: notice the coefficients of LCCDE).

Solution:

a) Systems 1 and 3 are IIR, and system 2 is FIR
b) For system 1:

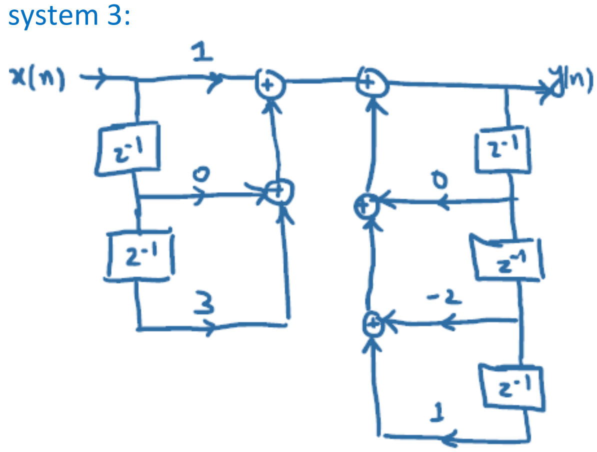


DIRECT FORM I

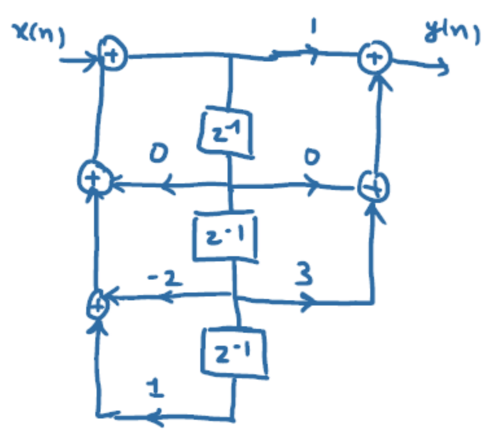


DIRECT FORM II

For system 3:

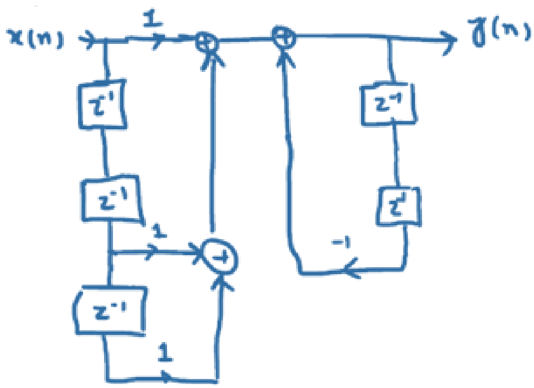


DIRECT FORM I

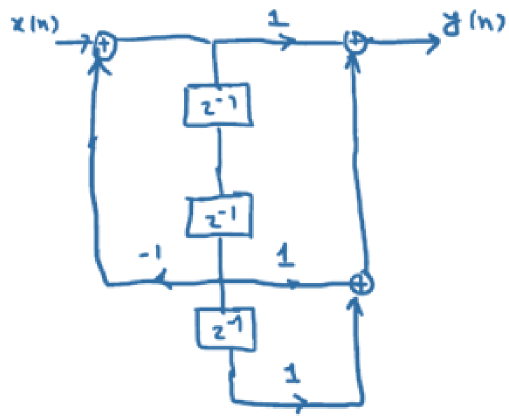


DIRECT FORM II

c) For system 1:

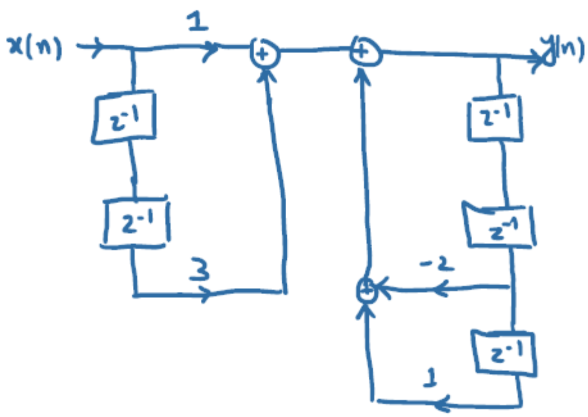


DIRECT FORM I

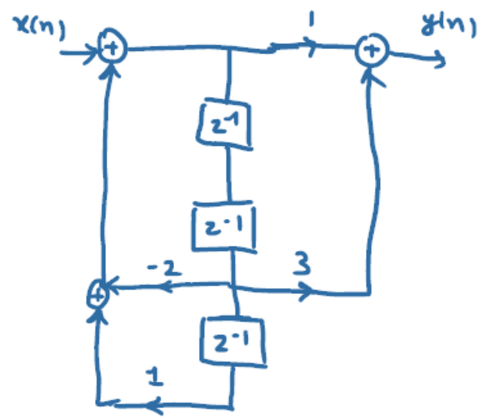


DIRECT FORM II

For system 3:



DIRECT FORM I



DIRECT FORM II