BLG 354E Homework - 2

Due 05.05.2020 23:59

Abdullah Ekrem Okur
Yusuf H. Sahin

Policy:

- Cheating is highly discouraged. It will be punished by a negative grade. Also disciplinary actions will be taken. Please do your homework on your own. Team work is not allowed. Pattern of your solutions must belong to only you.
- Upload your solutions through Ninova. Homeworks sent via e-mail and late submissions will not be accepted.
- You should write all your codes in Python language using Jupyter notebook. You can install Jupyter Notebook by following these steps on this documentation. If you are not familiar with Jupyter Notebook, you can check this tutorial.
- Prepare a report including all your solutions, codes and their results.
- You do not have to use Latex for the report but if you use Latex, you will get 20% more points. You can use this Latex template for the report.
- If you do not use Latex, the handwritten parts of the solutions must be presented on a paper legibly and scanned clearly. 10% penalty will be applied for illegible reports.

For your questions: Abdullah Ekrem Okur (okurabd@itu.edu.tr)

1. **[15 points]**

   Determine whether each of the following systems are (a) linear, (b) time-invariant, (c) causal, and (d) stable.

   i

   $$y(t) = 5x^2(t) + 2x(t) + 4$$

   ii

   $$y(t) = \begin{cases} 0, & t < 0 \\ x(t) - x(t-4), & t \geq 0 \end{cases}$$

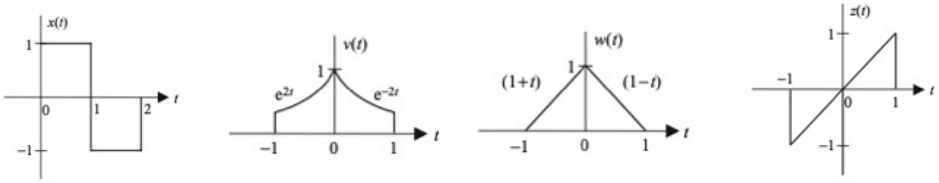
iii

$$y[n] = \sum_{m=n-2}^{n+2} x[m] - 2|x[n]|$$

iv

$$y[n] + 5y[n-1] + 9y[n-2] + 5y[n-3] + y[n-4] = 2x[n] + 4x[n-1] + 2x[n-2]$$

2. **[20 points]**



(a) For the CT signals above, determine the following convolutions using graphical methods.

i

$$y_1(t) = x(t) * z(t)$$

ii

$$y_2(t) = x(t) * v(t)$$

iii

$$y_3(t) = w(t) * w(t)$$

(b) For the DT signals below, determine the following convolutions using graphical methods. (You do not need to draw separate graphics for all n values.)

i

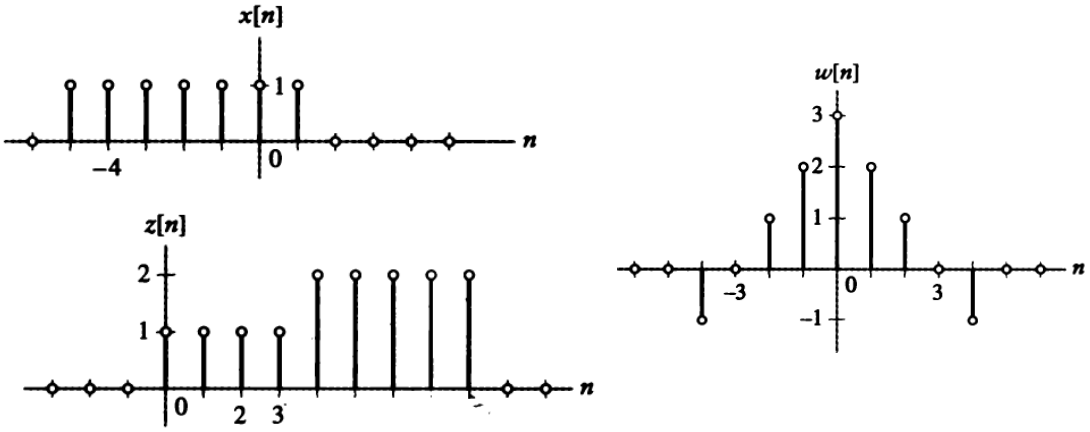
$$y_1[n] = x[n] * z[n]$$

ii

$$y_2[n] = w[n] * x[n]$$

iii

$$y_3[n] = w[n] * z[n]$$



NOTE: You can draw graphics on a paper by hand.

3. **[15 points]**

Determine the following convolutions.

i

$$y_1[n] = \big(u[n+10] - 2u[n] + u[n-4]\big) * u[n-2]$$

ii

$$y_2[n] = \cos\left(\frac{\pi}{2}n\right) * \left(\left(\frac{1}{2}\right)^n u[n-2]\right)$$

iii

$$y(t) = (2\delta(t+1) + \delta(t-5)) * u(t-1)$$

iv

$$y_3(t) = u(t) * h(t)$$

where

$$h(t) = \begin{cases} e^2 t, & t < 0 \\ e^{-3t}, & t \geq 0 \end{cases}$$

4. **[10 points]**

A discrete time signal $x[n]$ is given input to a linear time-invariant (LTI) FIR filter with an impulse response of $h[n]$ and the corresponding output signal is $y[n]$, where $x[n]$ and $h[n]$ are as follows:

$$x[n] = \delta[n] - \delta[n-5] + 4\delta[n+3], \tag{1}$$

$$h[n] = \delta[n] - \delta[n+2] - 7\delta[n-3].$$

(a) Write down the difference equation for this signal, and find $y[n]$ by substituting (1) for input in the difference equation.

(b) Find the output signal $y[n]$ using Python. Plot $x[n]$, $h[n]$ and $y[n]$.

5. **[25 points]**

In the homework two audio files with names "suphi_clean.wav" and "suphi_noisy.wav" are given. In this part of the homework, you are expected to obtain a denoised audio file from "suphi_noisy.wav" and obtain a clean audio similar to "suphi_clean.wav" using filtering techniques.

(a) **[15 points]**
Use

- Running Average filter
- Median filter [1]

with different parameters on "suphi_noisy.wav". To do this, you should write your own convolution function. When implementing convolution, instead of using many for loops try to adapt numpy operations to faster the process.

(b) **[5 points]** Implement a function to calculate Mean Square Error value between your cleansed audio and "suphi_clean.wav". Please do not use built-in MSE functions. Which method is better? Try to minimize the MSE for both methods. Visualize graphics of your denoised audio and the original clean audio for both filters.

[1] In median filtering, the data inside the window are ordered and the middle element is selected as filter output.

(c) **[5 points]** What are the difference between running average filter and median filter in terms of linearity? Explain and show whether each of them are linear.

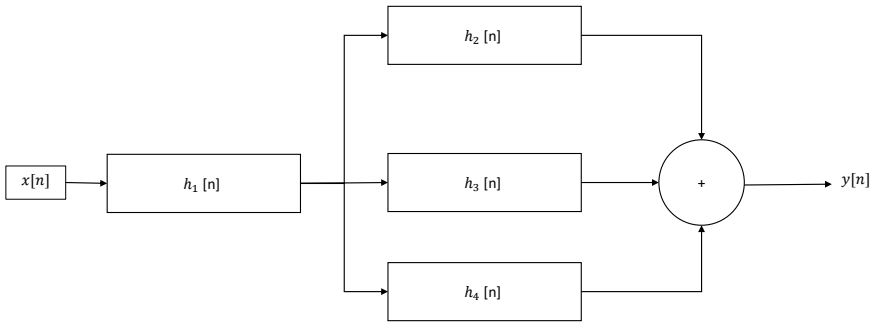
BONUS: Challenge [25 points]
You are free to design a filter or system to denoise the audio. For the first 20 students according to MSE scores, bonus points will be given.

You should upload your denoised data to the following Kaggle challenge to get a place in the leaderboard. If the values are not reproducible by the code in your HW files, your rank will be eliminated.

https://www.kaggle.com/t/13d2ec7124734fa4a69d977eae12c0d8

If you have any question about question 5, you can send an e-mail to sahinyu@itu.edu.tr.

6. **[15 points]**



For the interconnected system given above h_1, h_2, h_3 and h_4 are defined as:

$$h_1[n] = \alpha^n u[n]$$
$$h_2[n] = u[n]$$
$$h_3[n] = u[n+3] - u[n]$$
$$h_4[n] = \delta[n+1]$$

Let the overall impulse response for the system relating $y[n]$ to $x[n]$ is denoted as $h[n]$. Find $h[n]$ using the properties of convolution operators. Show that the same output can also be obtained without using these properties.