BLG 354E Homework - 1

Due 13.03.2020 23:59

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Policy:

- Cheating is highly discouraged. It will be punished by a negative grade. Also disciplinary actions will be taken. Please do your homework on your own. Team work is not allowed. Pattern of your solutions must belong to only you.
- Upload your solutions through Ninova. Homeworks sent via e-mail and late submissions will not be accepted.
- You should write all your codes in Python language using Jupyter notebook. You can install Jupyter Notebook by following these steps on this documentation. If you are not familiar with Jupyter Notebook, you can check this tutorial.
- Prepare a report including all your solutions, codes and their results.
- You do not have to use Latex for the report but if you use Latex, you will get 20% more points. You can use this Latex template for the report.
- If you do not use Latex, the handwritten parts of the solutions must be presented on white paper legibly and scanned clearly. 10% penalty will be applied for illegible reports.

Chapter 1: Introduction

1. **[10 points]**

   Answer these questions.

   (a) From signals and systems perspective, draw a simple block diagram for a car parking sensor system. The system is capable of measuring the distances to nearby objects via its sensors and giving sound alert of obstacles. (Note: You do not have to draw all the details, just give a rough diagram.)

   (b) Give an example for 1D, 2D, 3D and 4D signals.

Chapter 2: Sinusoids

2. **[15 points]**

Let

$$z_1(t) = \mathrm{Re}\{3e^{j(\omega t - \frac{2}{3}\pi)}\}$$
$$z_2(t) = \frac{e^{j(\omega t - \frac{1}{2}\pi)} + e^{-j(\omega t - \frac{1}{2}\pi)}}{4}$$
$$z_3(t) = 4\sin(\omega t - \frac{2}{3}\pi)$$

$x(t)$ is defined as follows:

$$x(t) = z_1(t) + 2z_2(t) + z_3(t)$$

(a) Express $x(t)$ in the form $x(t) = A\cos(\omega t + \phi)$ by finding the numerical values of A and ϕ. Use complex phasor manipulations to obtain the answer.

(b) Plot all the phasors used to solve the problem in (a) in the complex plane.

(c) Write a script that will plot the signal $x(t)$ using Python 3.6+. Please select suitable sampling space that makes the curve a faithful representation of the cosine function (Select suitable ω).

Chapter 3: Spectrum Representation

3. **[15 points]**

Let

$$z_1(t) = j(e^{j(\frac{11}{12}\pi)} + e^{-j(\frac{11}{12}\pi)})(e^{j(\frac{1}{12}\pi)} - e^{-j(\frac{1}{12}\pi)})$$
$$z_2(t) = \sin(180\pi t - \frac{1}{4}\pi)$$
$$z_3(t) = 3\cos(500\pi(t - 10^{-3}))$$
$$z_4(t) = 2\cos(250\pi t + \frac{3}{8}\pi)$$

$x(t)$ is defined as follows:

$$x(t) = 4z_1(t) - 4z_2(t) + z_3 + 2z_+(t)$$

(a) Sketch the spectrum of this signal, indicating the complex size of each frequency component. Make separate plots for real/imaginary or magnitude/phase of the complex amplitudes at each frequency. (Draw graphs by hand)

(b) Is $x(t)$ periodic? If so, what is the period?

(c) What is the fundamental frequency of this signal? Which harmonics does $x(t)$ contain?

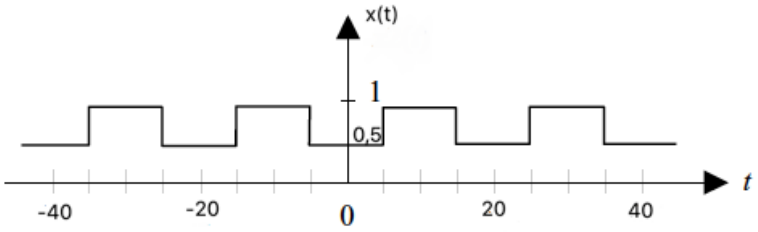
4. **[10 points]**

For the functions

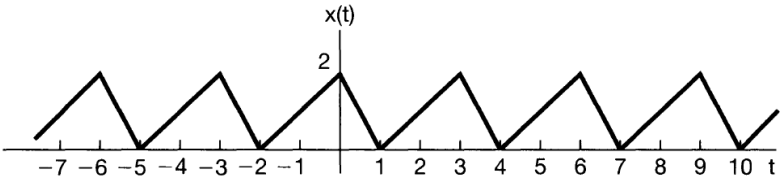
$$x_1(t) = e^{-3|t|}$$
$$x_2(t) = 1 - Ae^{-9|t|}$$

find the value of A such that functions are orthogonal over the interval $[-\infty, \infty]$.

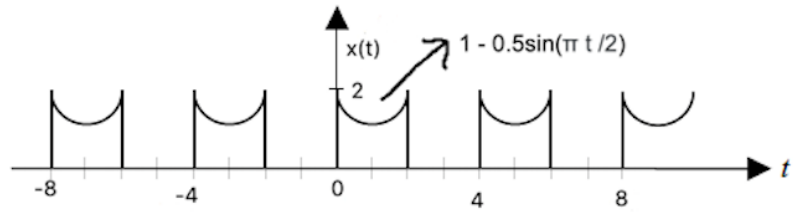
5. **[40 points]**



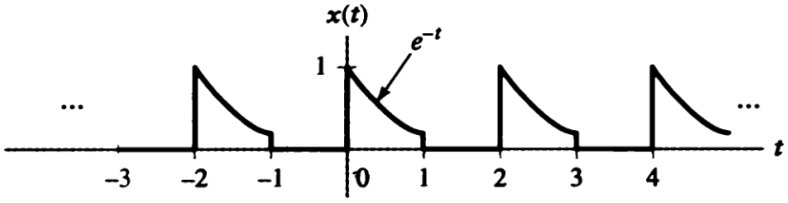
(i)



(ii)



(iii)



(iv)

(a) Find the Fourier series coefficients for given continuous-time signals by hand.

(b) Using the Fourier series coefficients formula found in (a) and the coefficients between

i. 1 - 10
ii. 1 - 50
iii. 1 - 100
iv. 1 - 200
v. (-100) - 100

obtain the each signals again using Python 3.6+. Compare and briefly explain the graphical results.

6. **[10 points]**

$$a_k = \begin{cases} jk, & |k| < 3 \\ 0, & \textit{otherwise} \end{cases}$$

Find the signal $x(t)$ for given Fourier series coefficients. Period of this continuous-time signal is 4.