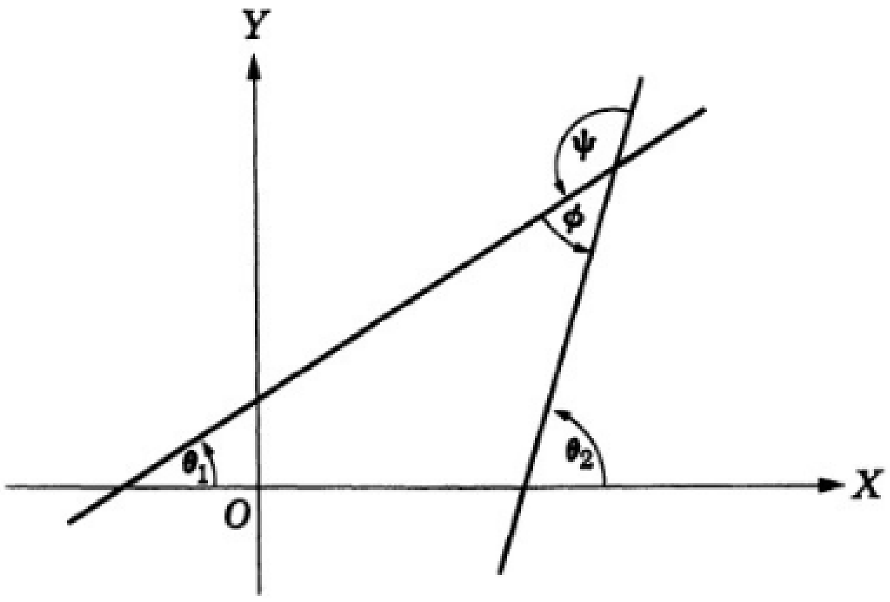
Lecture 5

Angle between two lines

- Two intersecting lines form four angles.



$$\phi + \theta_1 = \theta_2 \qquad \text{or} \qquad \phi = \theta_2 - \theta_1.$$

$$\tan\phi = \tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \tan\theta_2}.$$

$$\tan\phi = \frac{m_2 - m_1}{1 + m_1 m_2},$$

$$\text{Let } y = m_1x + c_1 \text{ and}$$
$$y = m_2x + c_2$$

be the equations of two straight lines and let these two lines make angles θ_1 and θ_2 with x- axis.

$$\text{Then } m_1 = \tan\theta_1 \text{ and}$$
$$m_2 = \tan\theta_2$$

If ϕ (phi) is the angle between these two straight lines, then

$$\phi = \theta_2 - \theta_1 \Rightarrow \tan\phi = \tan(\theta_2 - \theta_1)$$
$$\Rightarrow \tan\phi = \frac{m_2 - m_1}{1 + m_2m_1}$$
$$\Rightarrow \phi = \tan^{-1}\frac{m_2 - m_1}{1 + m_2m_1}$$

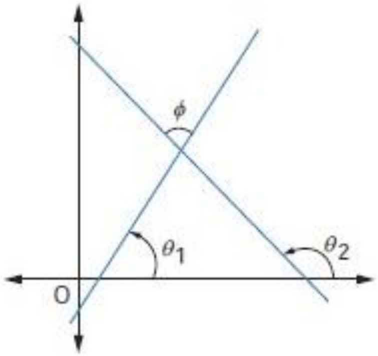


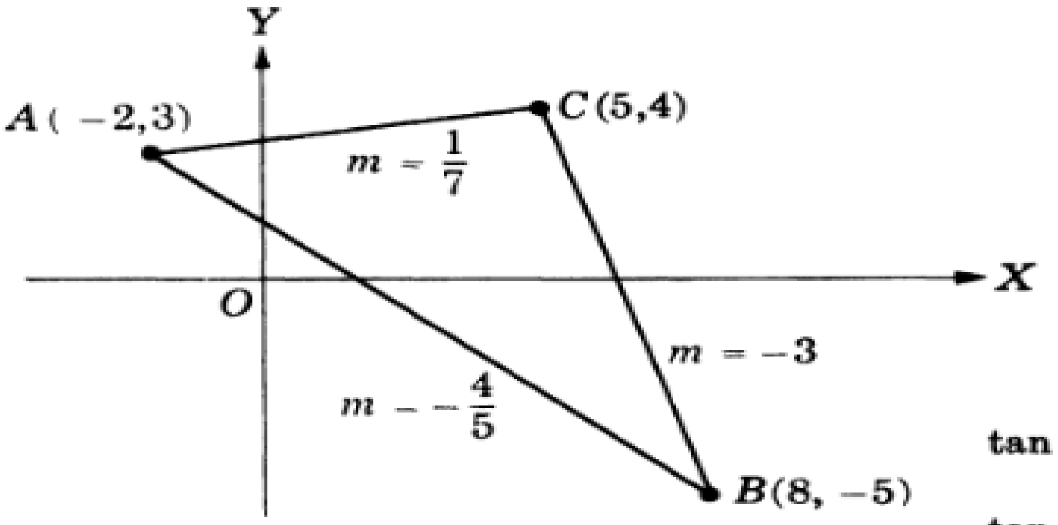
Figure 6.34

If $\frac{m_2 - m_1}{1 + m_2m_1}$ is positive then ϕ is the acute angle and if it is negative ϕ is the obtuse angle

between the two lines

Example:

Find the tangents of the angles of the triangle whose vertices are $A(-2,3)$, $B(8,-5)$, and $C(5,4)$.



$$\tan A = \frac{\frac{1}{7} - \left(-\frac{4}{5}\right)}{1 + \left(-\frac{4}{5}\right)\left(\frac{1}{7}\right)} = \frac{33}{31} = 1.06, \qquad A = 47°,$$

$$\tan B = \frac{-\frac{4}{5} - (-3)}{1 + (-3)\left(-\frac{4}{5}\right)} = \frac{11}{17} = .647, \qquad B = 33°,$$

$$\tan C = \frac{-3 - \frac{1}{7}}{1 + \left(\frac{1}{7}\right)(-3)} = \frac{-22}{4} = -5.5, \qquad C = 100°.$$

Example:

If $(2, 1)$ and $(-5, 0)$ are endpoints of a diameter of a circle, find the center and radius of the circle.

Example:

Find the general equation of the line given a slope equal to -1 and x-intercept equal to 6.

Example:

Find the general equation of line L passing through the point (-7,-5) and perpendicular to the line given by

$$3x + 4y - 19 = 0$$

Solution:

$$3x + 4y - 19 = 0$$

$$4y = 19 - 3x$$

$$y = \frac{19}{4} - \frac{3}{4}x$$

$$y = -\frac{3}{4}x + \frac{19}{4}$$

$$m = -\frac{3}{4}; c = \frac{19}{4}$$

Let the equation of line be $y = mx + c$

Passing through the point $(-7, -5)$

$$-5 = m(-7) + c$$

With slope $\perp\ to\ m = -\frac{3}{4}; m = \frac{4}{3}$

$$-5 = \left(\frac{4}{3}\right)(-7) + c$$

$$-5 = -\frac{28}{3} + c$$

$$c = -5 + \frac{28}{3}$$

$$c = \frac{-15 + 28}{3}$$

$$c = \frac{13}{3}$$

Hence equation of the required line is

$$y = \frac{4}{3}x + \frac{13}{3}$$

Or

$$3y = 4x + 13$$

Or

$$3y - 4x - 13 = 0$$

The line through (6, -4) and (-3,2) is parallel to the line through (2, 1) and (0, y). Find y

Q10. ~~The line through~~

Slope the line l_1 is $m_1 = \frac{2+4}{-3-6} = \frac{6}{-9} = -\frac{2}{3}$

" " l_2 : $m_2 = \frac{y-1}{0-2} = \frac{y-1}{-2}$

Since $l_1 \parallel l_2$ so, $m_1 = m_2$

hence

$$\frac{y-1}{-2} = -\frac{2}{3}$$

$$y-1 = -\frac{2}{3} \times -2$$

$$y-1 = \frac{4}{3}$$

$$y = \frac{4}{3} + 1 = \frac{7}{3}$$

so ~~$y = \frac{7}{3}$ Ans.~~

$y = \frac{7}{3}$ Ans.

Using slopes, prove that (6,5), (-3,0) and (4,-2) are the vertices of a right triangle.

Let $A(6,5)$, $B(-3,0)$ and $C(4,-2)$

We find slopes of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$.

Slope of $\overline{AB}$ is $m_1 = \frac{0-5}{-3-6} = \frac{-5}{-9} = \frac{5}{9}$

Slope of $\overline{AC}$ is $m_2 = \frac{-2-5}{4-6} = \frac{-7}{-2} = \frac{7}{2}$

Slope of $\overline{BC}$ is $m_3 = \frac{-2-0}{4+3} = -\frac{2}{7}$

Since $m_2 \cdot m_3 = \left(\frac{7}{2}\right)\left(-\frac{2}{7}\right) = -1$

~~So, the given vertices are the veri~~

So, $\overline{AC} \perp \overline{BC}$, hence the given vertices are the vertices of right triangle. **Proved.**

Find measure of the angle from the line through (-3,1) and (4,3) to the line (1,-2) and (6,7)

Since slope of line l_1 from $(-3,1)$ to $(4,3)$ is:

$$m_1 = \frac{3-1}{4+3} = \frac{2}{7}$$

Slope of line l_2 from $(1,-2)$ to $(6,7)$ is:

$$m_2 = \frac{7+2}{6-1} = \frac{9}{5}$$

Formula of angle from l_1 to l_2 is

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan\theta = \frac{\frac{9}{5} - \frac{2}{7}}{1 + \left(\frac{2}{7}\right)\left(\frac{9}{5}\right)} = \frac{\frac{63-10}{\cancel{35}}}{\frac{35+18}{\cancel{35}}} = \frac{\cancel{53}}{\cancel{53}} = 1$$

$$\theta = \tan^{-1}(1) = 45^\circ \ \text{Ans.}$$

The measure of the angle from a line l_1 with slope $-\frac{1}{3}$ to a line l_2 is $135°$. Find the slope of the line l_2.

Slope of line l_1 is $m_1 = -\frac{1}{3}$

slope of line l_2 is $m_2 = ??$

Angle b/w l_1 & l_2 is $\theta = 135°$

we have.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan 135° = \frac{m_2 - (-\frac{1}{3})}{1 + (-\frac{1}{3})m_2}$$

The measure of the angle from a line l_1 with slope $-\frac{1}{3}$ to a line l_2 is $135°$. Find the slope of the line l_2.

$$\tan 135^\circ = \frac{m_2 + \frac{1}{3}}{1 - \frac{1}{3}m_2}$$

$$-1 = \frac{3m_2 + 1}{3 - m_2}$$

$$-3 + m_2 = 3m_2 + 1$$

$$3m_2 - m_2 = -3 - 1$$

$$2m_2 = -4$$

$$m_2 = -2 \quad \underline{\underline{\text{Ans.}}}$$