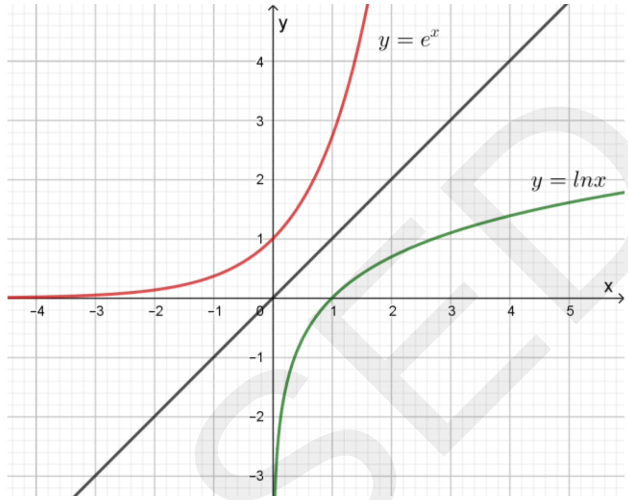
Mathematics: Chapter Summary and rules

ln(*x*) and exp(x) summary:



- $\frac{\mathrm{d}}{\mathrm{dx}}(\ln|x|) = \frac{1}{x}$; $\frac{d}{dx}\ln|u| = \frac{u'}{u}$
- $\ln(a \times b) = \ln a + \ln b$, where $a > 0$ and $b > 0$.
- $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$, where $a > 0$ and $b > 0$.
- $\ln\left(\frac{1}{a}\right) = -\ln a$, where $a > 0$.
- $\ln a^n = n\ln a$, where $a > 0$ and n is real.
- $\ln 1 = 0$; $\ln e = 1$; $\ln e^2 = 2$
- $\ln e^n = n$; $\ln\frac{1}{e} = -1$.
- $a = \ln e^a = e^{\ln a}$.
- $\ln x = a \Leftrightarrow x = e^a$.
- If $e^x = a$ $(a > 0)$, then $x = \ln a$
- $\ln a = \ln b \Leftrightarrow a = b$, where $a > 0$ and $b > 0$.
- $\ln a < \ln b \Leftrightarrow a < b$, where $a > 0$ and $b > 0$.
- $\ln a > \ln b \Leftrightarrow a > b$, where $a > 0$ and $b > 0$.

- $\lim_{x\to 0^+} \ln x = -\infty$; $\lim_{x\to+\infty} \ln x = +\infty$.
- $\lim_{x\to 0^+} x\ln x = 0^-$; $\lim_{x\to+\infty}\left(\frac{\ln x}{x}\right) = 0$; $\lim_{x\to+\infty}\left(\frac{x}{\ln x}\right) = +\infty$.
- $\lim_{x\to 0^+} x^n \ln x = 0^-$; $\lim_{x\to+\infty}\left(\frac{\ln x}{x^n}\right) = 0^+$ where $n > 0$.

L'Hopital's Rule for limits of indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

If $\lim_{x\to a}\frac{f'(x)}{g'(x)}$ exists, then, $\lim_{x\to a}\frac{f(x)}{g(x)}$ exists, and $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$.

- The function: $x \mapsto \ln x$ has domain $=]0;\ +\infty[$ and range $=]-\infty;\ +\infty[$.
- The function: $x \mapsto e^x$ has domain $=]-\infty;\ +\infty[$ and range $=]0;\ +\infty[$.
- These two functions are inverses of each other. Their graphs are symmetrical with respect to the first bisector ($y = x$).

- $\ln(e^x) = x; e^{\ln x} = x$; $y = \ln x \Leftrightarrow x = e^y$.
- $e^a = e^b \Leftrightarrow a = b$; $e^a < e^b \Leftrightarrow a < b$; $e^a > e^b \Leftrightarrow a > b$
- $e^a \cdot e^b = e^{a+b}$; $\frac{e^a}{e^b} = e^{a-b}$; $(e^a)^b = e^{ab}$; $e^{-a} = \frac{1}{e^a}$
- $(e^x)' = e^x$; $(e^u)' = e^u \cdot u'$

- $\lim_{x\to-\infty} e^x = 0^+$; $\lim_{x\to+\infty} e^x = +\infty$; $\lim_{x\to+\infty}\frac{e^x}{x} = +\infty$; $\lim_{x\to+\infty}\frac{x}{e^x} = 0$;
  $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$; $\lim_{x\to+\infty} e^{-x} = 0$; $\lim_{x\to+\infty}\frac{e^x}{x^n} = +\infty$; $\lim_{x\to-\infty} x^n e^x = 0$. $(n \in \mathbb{Z}^+)$

Integration and derivatives:

$*\ F(x)$ is an anti-derivative $g(x) \Leftrightarrow F'(x) = g(x)$.

$*\ F(x) = \int g(x)dx \ \Leftrightarrow\ F'(x) = g(x)$.

$*\ \dfrac{d}{dx}\left(\int f(x)dx\right) = f(x)$ and $\displaystyle\int f'(x)dx = f(x) + C$

$*\ \displaystyle\int x^n dx = \frac{x^{n+1}}{n+1} + C \ \ (n \neq -1)$

$*\ \displaystyle\int (u+v)dx = \int udx + \int vdx \ \ ; \quad \int kudx = k\int udx \ \ ; k \in \mathbb{R}$

$*\ \displaystyle\int u'v = uv - \int v'u$ (integration by parts)

$*\ \displaystyle\int u'f(u)dx = \int f(u)du$

$*\ f'_x = f'_u \times u'_x$ (chain rule: $\frac{df}{dx} = \frac{df}{du} \times \frac{du}{dx}$)

$*\ (u \pm v)' = u' \pm v'$

$*\ (uv)' = u'v + v'u$

$*\ \left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$*\ \left(\dfrac{1}{u}\right)' = -\dfrac{u'}{u}$

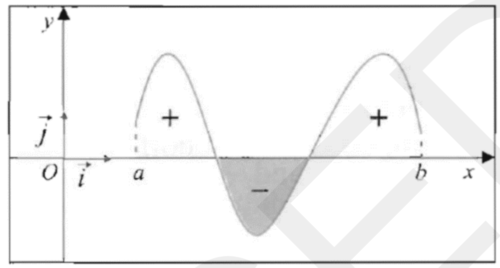
In this table : a and b are real numbers.

$\boldsymbol{f(x)}$	$\boldsymbol{f'(x)}$	$\int \boldsymbol{f(x)dx}$
1	0	x
x	1	x^2
a	0	ax
$ax+b$	a	$\frac{1}{2}ax^2 + bx$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\|x\|$
$\frac{1}{x^2}$	$-\frac{2}{x^3}$	$-\frac{1}{x}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\frac{2}{3}x\sqrt{x}$
$\frac{1}{\sqrt{x}}$	$-\frac{1}{2x\sqrt{x}}$	$2\sqrt{x}$
$\ln x$	$\frac{1}{x}$	$x\ln x - x$
e^x	e^x	e^x
$\sin x$	$\cos x$	$-\cos x$
$\cos x$	$-\sin x$	$\sin x$
$\tan x$	$1 + \tan^2 x$	$-\ln\|\cos x\|$

* If F is an anti-derivative of f over I, then $\int_a^b f(x)dx = F(b) - F(a)$.

* $\int_a^b f(x)dx$ = algebraic area of the domain limited by the representative curve of f, the x-axis and the two straight lines of equations $x = a$ and $x = b$.

* Mean value of a function (average value) over an interval $[a; b]$ is $\overline{f} = \frac{1}{b-a}\int_a^b f(x)dx$.



* $\int_a^b \big(f(x) - g(x)\big)dx =$ area of the domain limited by the representative curves of f and g ($f(x) > g(x)$ over $[a, b]$) and the two straight lines of equations $x = a$ and $x = b$.

* $\int_b^a f(x)dx = -\int_a^b f(x)dx$

* $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

* $\int_a^b kdx = k(b-a)$ and $\int_a^b dx = b - a$ and $\int_a^a dx = 0$.

* If $f(x)$ is an odd function, then $\int_{-a}^a f(x)dx = 0$, $a \in \mathbb{R}$.
* If $f(x)$ is an even function, then $\int_{-a}^a f(x)dx = 2 \times \int_0^a f(x)dx$, $a \in \mathbb{R}$.
* If $f(x) \leq g(x)$, then $\int f(x)dx \leq \int g(x)dx$

* If $H(x) = \int_a^{u(x)} f(t)dt$, then $H'(x) = f\big(u(x)\big) \times u'(x)$, $a \in \mathbb{R}$

Special economics:

* Given the cost function : $C_T(x)$, then marginal cost function : $C_m(x) = C_T{}'(x)$

* Given marginal cost function : $C_m(x)$, then total cost function : $C_T(x) = \int C_m(x)dx + K$.