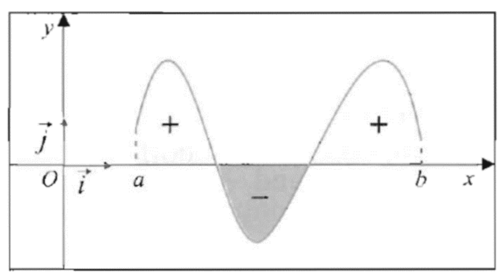
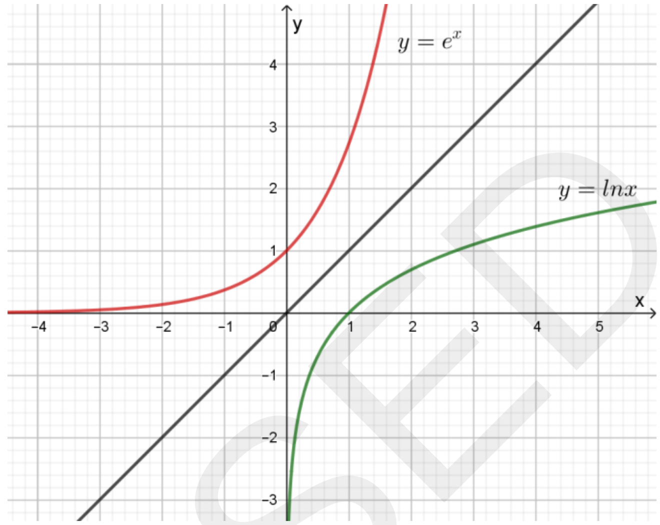
Integrals (or Primitives or Anti-derivatives):

* A function $F(x)$ is an anti-derivative of a function $g(x)$ when $F'(x) = g(x)$.
* If $F(x) = \int g(x)dx$, then, $F'(x) = g(x)$.
* $\left(\int f(x)dx\right)' = f(x)$ and $\int f'(x)dx = f(x)$ (plus a constant)
* $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
* $\int dx = x + C$; $\int x dx = \dfrac{x^2}{2} + C$; $\int \dfrac{1}{x^2}dx = -\dfrac{1}{x} + C$; $\int \sqrt{x}dx = \dfrac{2}{3}x\sqrt{x} + C$
* $\int a dx = ax + C \quad (a \in \mathbb{R})$; $\int \dfrac{1}{x}dx = \ln|x| + C$; $\int e^x dx = e^x + C$
* $\int (u+v)dx = \int u dx + \int v dx$; $\int k\, u dx = k \int u dx$
* $\int u'v\, dx = uv - \int v'u\, dx$ (integration by parts)
* Changing variable : $u(x), u' = \dfrac{du}{dx}$; $u' dx = du$; $\int u' f(u)dx = \int f(u)du$.
* If F is an anti $-$ derivative of f over I, then $\displaystyle\int_a^b f(x)dx = F(b) - F(a)$.
* $\int_a^b f(x)dx$ = algebraic area of the domain limited by the representative curve of *f*, the *x*-axis and the two straight lines of equations $x = a$ and $x = b$.
* Mean value of a function (average value) over an interval $[a; b]$ is $\overline{f} = \frac{1}{b-a}\int_a^b f(x)dx$.



* $\int_b^a f(x)dx = -\int_a^b f(x)dx$; $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (Chaseles' rule)
* $\int_a^b k dx = k(b-a)$ and $\int_a^b dx = b - a$ and $\int_a^a dx = 0$.
* Given the cost function : $C_T(x)$, then marginal cost function : $C_m(x) = C_T{}'(x)$
* Given marginal cost function : $C_m(x)$, then total cost function : $C_T(x) = \int C_m(x)dx + K$.

$\ln(x)$ and $\exp(x)$ summary:



- $(\ln|x|)' = \frac{1}{x}$; $(\ln|u|)' = \frac{u'}{u}$.
- $\ln(a \times b) = \ln a + \ln b$, where $a > 0$ and $b > 0$.
- $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$, where $a > 0$ and $b > 0$.
- $\ln\left(\frac{1}{a}\right) = -\ln a$, where $a > 0$.
- $\ln a^n = n \ln a$, where $a > 0$ and n is real.
- $\ln 1 = 0$; $\ln e = 1$; $\ln e^2 = 2$
- $\ln e^n = n$; $\ln\frac{1}{e} = -1$.
- $a = \ln e^a = e^{\ln a}$.
- $\ln x = a \Leftrightarrow x = e^a$.
- $\ln a = \ln b \Leftrightarrow a = b$, where $a > 0$ and $b > 0$.
- $\ln a < \ln b \Leftrightarrow a < b$, where $a > 0$ and $b > 0$.
- $\ln a > \ln b \Leftrightarrow a > b$, where $a > 0$ and $b > 0$.
- $\lim\limits_{x\to 0^+} \ln x = -\infty$; $\lim\limits_{x\to +\infty} \ln x = +\infty$.
- $\lim\limits_{x\to 0^+} x\ln x = 0^-$; $\lim\limits_{x\to +\infty}\left(\frac{\ln x}{x}\right) = 0$; $\lim\limits_{x\to +\infty}\left(\frac{x}{\ln x}\right) = +\infty$.
- $\lim\limits_{x\to 0^+} x^n \ln x = 0^-$; $\lim\limits_{x\to +\infty}\left(\frac{\ln x}{x^n}\right) = 0^+$ where $n > 0$.

L'Hopital's Rule for limits of indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

If $\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$ exists, then, $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ exists, and $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$.

- The function: $x \mapsto \ln x$ has domain $=]0; +\infty[$ and range $=]-\infty; +\infty[$.
- The function: $x \mapsto e^x$ has domain $=]-\infty; +\infty[$ and range $=]0; +\infty[$.
- These two functions are inverses of each other. Their graphs are symmetrical with respect to the first bisector ($y = x$).

- $\ln(e^x) = x; e^{\ln x} = x$; $y = \ln x \Leftrightarrow x = e^y$.
- $e^a = e^b \Leftrightarrow a = b$; $e^a < e^b \Leftrightarrow a < b$; $e^a > e^b \Leftrightarrow a > b$.
- $e^a \cdot e^b = e^{a+b}$; $\frac{e^a}{e^b} = e^{a-b}$; $(e^a)^b = e^{ab}$; $e^{-a} = \frac{1}{e^a}$.
- $(e^x)' = e^x$; $(e^u)' = e^u \cdot u'$.
- $\lim\limits_{x\to -\infty} e^x = 0^+$; $\lim\limits_{x\to +\infty} e^x = +\infty$; $\lim\limits_{x\to +\infty}\frac{e^x}{x} = +\infty$; $\lim\limits_{x\to +\infty}\frac{x}{e^x} = 0$;
  $\lim\limits_{x\to 0}\frac{e^x - 1}{x} = 1$; $\lim\limits_{x\to +\infty} e^{-x} = 0$; $\lim\limits_{x\to +\infty}\frac{e^x}{x^n} = +\infty$; $\lim\limits_{x\to -\infty} x^n e^x = 0$. $(n \in \mathbb{Z}^+)$