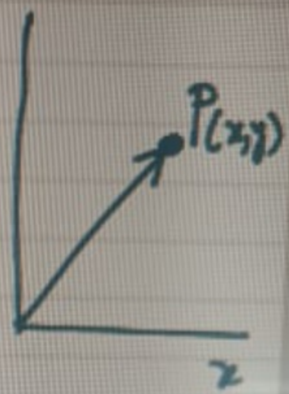
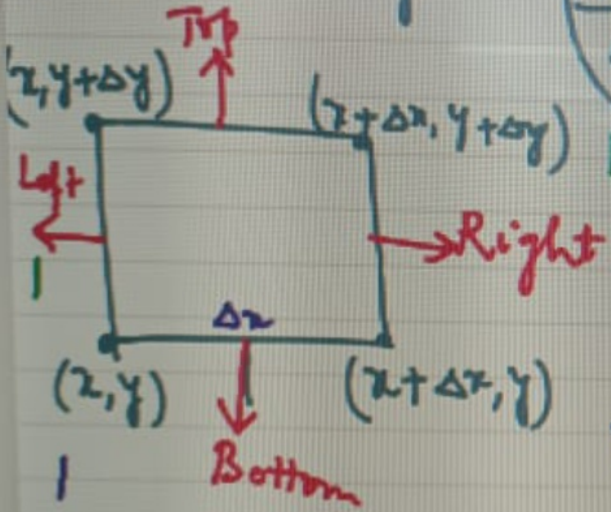
$\vec{A}(x,y) = A_x(x,y)\,\hat{i} + A_y(x,y)\,\hat{j}$

$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\dot{P}(t)$
$\downarrow$
$P(t)$



$\vec{\nabla}\cdot\vec{A} = \text{Divergence} = \frac{\text{Flux}}{\text{Area}} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}$



Bottom ~~Top~~ : $\left[A_x(x,y)\hat{i} + A_y(x,y)\hat{j}\right](-\Delta x)\hat{j}$

$= -A_y(x,y)\,\Delta x$

Top : $\left[A_x(x,y+\Delta y)\hat{i} + A_y(x,y+\Delta y)\hat{j}\right] \cdot \Delta x\,\hat{j}$

Top + Bottom $= \left[A_y(x,y+\Delta y) - A_y(x,y)\right]\Delta x$

$\approx \frac{\partial A_y}{\partial y}\Delta x\,\Delta y$

Left ~~& Right~~ : $\left[A_x(x,y)\hat{i} + A_y(x,y)\hat{j}\right]\cdot \Delta y\,(-\hat{i})$

Right : $\left[A_x(x+\Delta x,y)\hat{i} + A_y(x+\Delta x,y)\hat{j}\right]\cdot \Delta y\,\hat{i}$

$= \left[A_x(x+\Delta x,y) - A_x(x,y)\right]\Delta y$

$\approx \frac{\partial A_x}{\partial x}\Delta x\,\Delta y$

$\nabla \times \vec{F} = 0$, $\nabla T = \hat{i}\dfrac{\partial T}{\partial x} + \hat{j}\dfrac{\partial T}{\partial y} + \hat{k}\dfrac{\partial T}{\partial z}$.

$\vec{F} = \nabla T$

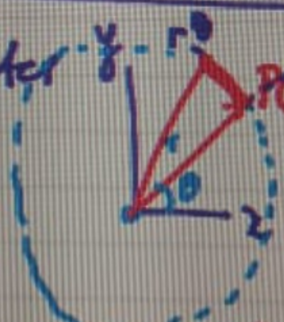
$\vec{E} = -\dfrac{\partial V}{\partial x}$

$\vec{F} = -\dfrac{dU}{dx}$

$$\nabla \times \nabla T = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{\partial T}{\partial x} & \dfrac{\partial T}{\partial y} & \dfrac{\partial T}{\partial z} \end{vmatrix} = 0$$

Plane polar coordinates

r, θ



$x\hat{i} + y\hat{j}$

$$\vec{r} = P(x,y) = x(t)\hat{i} + y(t)\hat{j}$$
$$= r(t)\cos(\theta(t))\hat{i} + r(t)\sin\theta(t)\hat{j}$$

t = time

$$\vec{r} = r(t)\left[\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j}\right] = r\hat{r}$$

$x^2 + y^2 = r^2$

$\dfrac{y}{x} = \tan\theta$.

$$\hat{r} = \frac{\partial\vec{r}/\partial r}{\left|\partial\vec{r}/\partial r\right|} = \cos\theta(t)\hat{i} + \sin\theta(t)\hat{j}$$

$$\hat{\theta} = \frac{\partial\vec{r}/\partial\theta}{\left|\partial\vec{r}/\partial\theta\right|} = \frac{\cancel{r(t)}\left[-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}\right]}{\cancel{r(t)}}$$

Circulation

$$L+R+T+B = \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right] \Delta x \Delta y$$

$$\frac{\text{Circulation}}{\text{Area}} = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}.$$

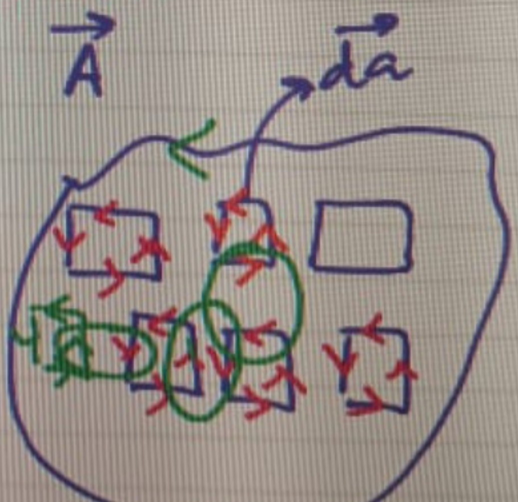
$$\underline{\underline{\nabla \times \vec{A}}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

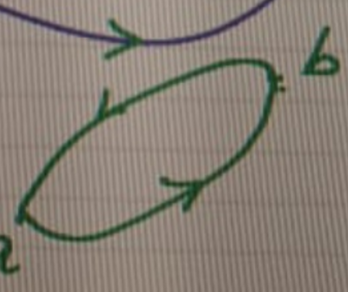
Curl

$$\oint \vec{\nabla} \times \vec{A} \cdot d\vec{a}$$

$$= \oint_C \underbrace{\vec{A} \cdot d\vec{l}}_{\to 0}$$

$$\vec{A} = \vec{F}$$

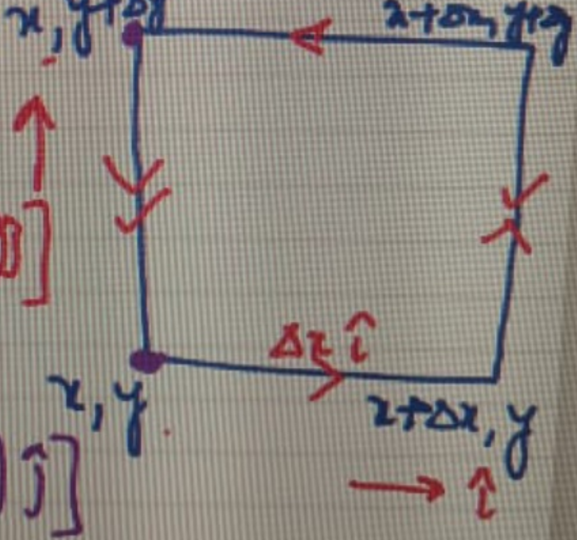




$$\underbrace{L+R+B+T}_{\text{Total flux of rectangle}} = \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}\right] \Delta x \, \Delta y$$

$$\frac{\text{Flux}}{\text{Area}} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}$$

$$\vec{\nabla} \cdot \vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y}\right) \cdot \left(A_x \hat{i} + A_y \hat{j}\right)$$



Circulation

$$\text{Top}\left[A_x(x, y+\Delta y)\hat{i} + A_y(x, y+\Delta y)\hat{j}\right] \cdot (-)\Delta x \, \hat{i}$$

$$+$$

$$\text{Bottom}\left[A_x(x,y)\hat{i} + A_y(x,y)\hat{j}\right] (\Delta x)\hat{i}$$

$$= \left[-A_x(x, y+\Delta y) + A_x(x,y)\right] \Delta x$$

$$= -\frac{\partial A_x}{\partial y} \Delta x \, \Delta y \qquad \Big| \quad L+R = \frac{\partial A_y}{\partial x} \Delta x \Delta y$$