# PH100: Mechanics and Thermodynamics

## Lecture 5

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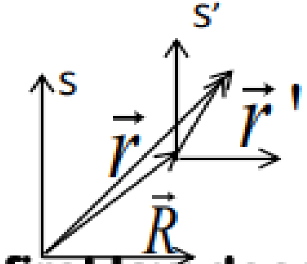
# Newton's first law of motion

1. **Gives a definition of (zero) force**
2. **Defines an *inertial frame*.**

**Zero Force:** When a body moves with *constant velocity in a straight line,* either there are no forces present or the net force acting on the body is zero $\sum_i \vec{F}_i = 0$ If the body changes its velocity, then there must be an acceleration, and hence a total non-zero force must be present. Velocity can change due to change in its magnitude or due to change in its direction or change in both.

**Inertial frame:** If the relative velocity between the two reference frames is constant, then the relative acceleration between the two reference frames is zero, $\vec{\mathbf{A}} = \frac{d\vec{\mathbf{V}}}{dt} = \vec{0}$ and the reference frames are considered to be *inertial reference frames*. **The inertial frame is then simply a frame of reference in which the first law holds.**




$$\vec{r}\,' = \vec{r} - \vec{v}t, \quad \vec{v} = \frac{d\vec{R}}{dt}$$

Galilean transformation

**Is Earth an inertial frame?**

The first law does *not hold in an arbitrary frame. For example, it fails in the frame of a rotating* turntable.

# Newton's Second law of motion:

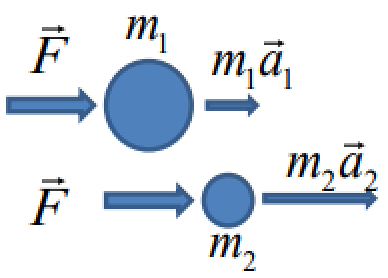
**If any force generates a change in motion, a double force will generate double change in the motion, a triple force will correspond to triple change in the motion, whether that force is impressed altogether and at once or gradually or successively.**

Change of motion is described by the change in momentum of body. For a point mass particle, the momentum is defined as $\vec{p} = m\vec{v}$

Suppose that a force is applied to a body for a time interval $\Delta t$ . *The impressed force or impulse produces a change in the momentum of the body,*

$$\vec{\bar{\mathbf{I}}} = \vec{\bar{\mathbf{F}}}\,\Delta t = \Delta\vec{\mathbf{p}}$$

**The instantaneous action of the total force acting on a body at a time *t is defined by taking the mathematical limit as the time interval Δt becomes smaller and smaller,***




$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

$$\vec{\mathbf{F}}^{\text{total}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{p}}}{\Delta t} \equiv \frac{d\vec{\mathbf{p}}}{dt} \qquad \vec{F}^{\text{total}} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt}$$

**Inertial mass**

$$\vec{\mathbf{F}}^{\text{total}} = m\vec{\mathbf{a}}$$

Inertial mass $\equiv$ Gravitational mass

# Newton's third law of motion:

**Consider two bodies engaged in a mutual interaction. Label the bodies 1 and 2 respectively. Let $\vec{F}_{1,2}$ be the force on body 1 due to the interaction with body 2, and $\vec{F}_{2,1}$ be the force on body 2 due to the interaction with body 1.**


$$\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$$

Gravitational force: $\vec{F}_{12} = -G\dfrac{m_1 m_2}{r^2}\hat{r}_{12}$ $\quad \hat{r}_{12} = -\hat{r}_{21}$

Coulomb force: $\vec{F}_{12} = k\dfrac{q_1 q_2}{r^2}\hat{r}_{12}$ $\quad \vec{F}_{12} = -\vec{F}_{21}$

**All real Forces arise due to interaction!**

**If the acceleration of a body is the result of an outside force, then somewhere in the universe there must be an equal and opposite force acting on another body. The interaction may be a complicated one, but as long as the forces are equal and opposite, Newton's laws are satisfied.**

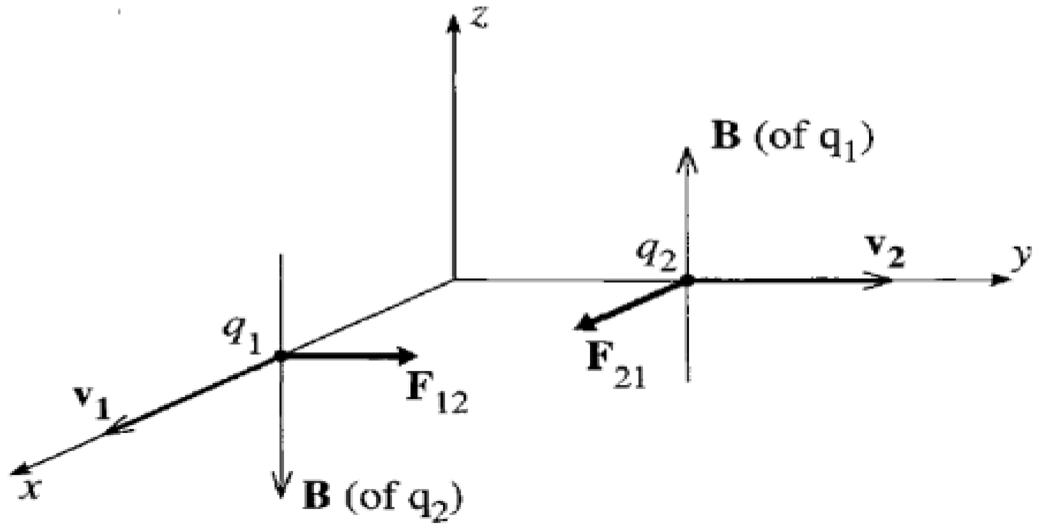
**Newton's 3$^{rd}$ law emphasizes Conservation of Momentum**

# Validity of Newton's laws

- **Validity of the first two laws**
  - The first law is always valid (add a pseudo force).
  - The second law **F** = $\dot{p}$ holds but **F** and ***p*** have different expressions in the relativistic limit.

- **The 3rd law is not valid in the relativistic limit. Why????**

# Consider two positive charges




Each of the positive charges $q_1$ and $q_2$ produces a magnetic field that exerts a force on the other charge. The resulting magnetic forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ do not obey Newton's third law.

**Momentum conservation is not valid**

# Application of Newton's laws: Prescription

**Step 1**: Divide a composite system into constituent systems each of which can be treated as a point mass.

**Step 2**: Draw free body force diagrams for each point mass.

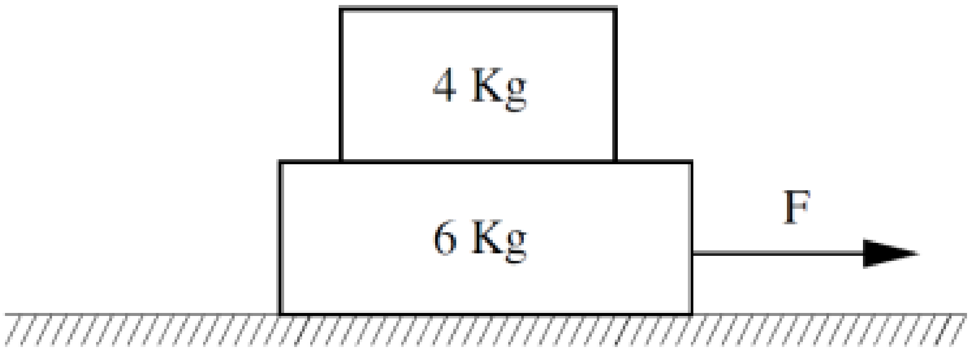
**Step 3**: Introduce a coordinate system, the inertial frame, and write the equations of motion.

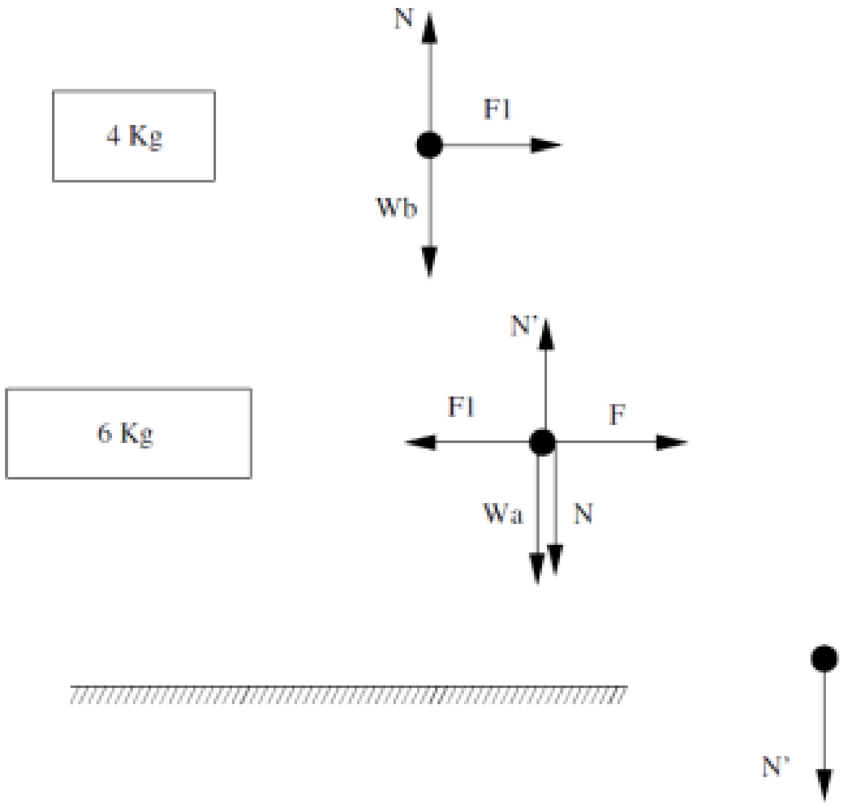
**Step 4**: Motion of a body may be constrained to move along certain path or plane. Express each constraint by an equation called constraint equation.

**Step 6**: Identify the number of unknown quantities. There must be enough number of equations ( Equations of motion + constraint equations) to solve for all the unknown quantities.

# Example 1

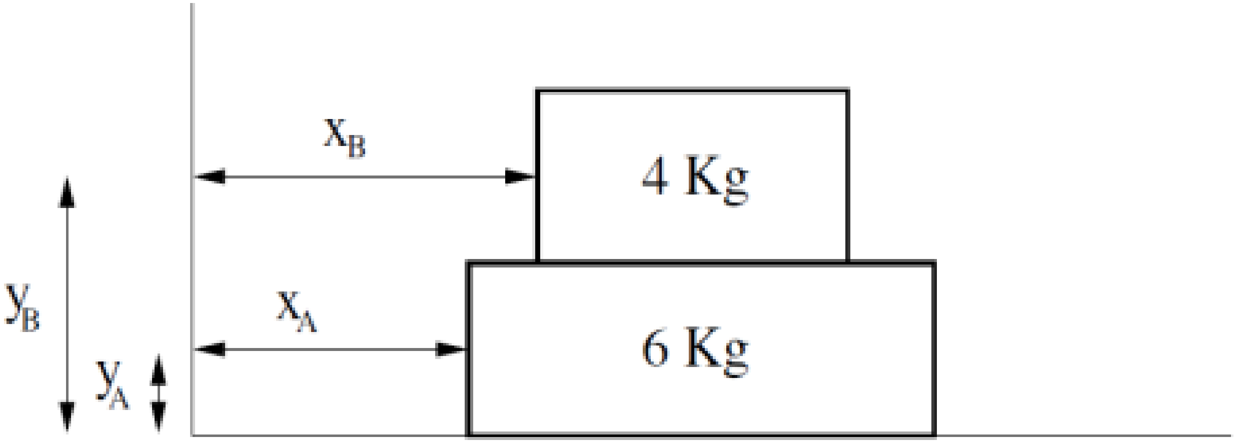
A 4 Kg block rests on top of a 6 Kg block, which rests on a frictionless table. Coefficient of friction between blocks is $0.25$. A force $F = 10N$ is applied to the lower block.



# Identify the constraints

Fix the coordinate system to the table.




$$
\begin{aligned}
y_A &= const \\
y_B &= const \\
x_A &= x_B + const
\end{aligned}
$$

# EOM in x and y-directions

Equations of Motion in Y direction.

$$
\begin{aligned}
m_A \ddot{y_A} &= N' - W_A - N \\
m_B \ddot{y_B} &= N - W_B
\end{aligned}
$$

Constraints

$$
\begin{aligned}
\ddot{y_A} &= 0 \\
\ddot{y_B} &= 0
\end{aligned}
$$

Solution

$$
\begin{aligned}
N' &= W_A + W_B \\
N &= W_B
\end{aligned}
$$

Equations of Motion in X direction.

$$
\begin{aligned}
m_A \ddot{x_A} &= F - F_1 \\
m_B \ddot{x_B} &= F_1
\end{aligned}
$$

Constraints

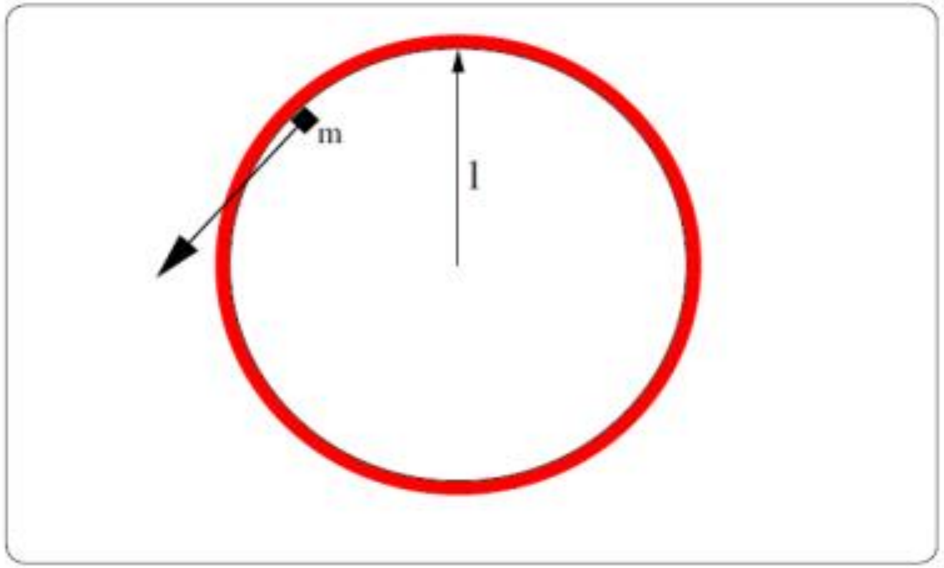
$$
\ddot{x_A} = \ddot{x_B}
$$

Solution

$$
\begin{aligned}
\ddot{x_A} = \ddot{x_B} &= \frac{F}{m_A + M_B} = 1\mathrm{m/s}^2 \\
F_1 &= m_B \ddot{x_B} = 4N
\end{aligned}
$$

The force $F_1 < \mu N = 10$ N, the maximum frictional force between the blocks. Hence the solution is consistent with assumption.

A block of mass $m$ slides on a frictionless table. It is constrained to move move inside a ring of radius $l$ fixed to the table. At $t = 0$ the block is touching the ring and has a velocity $v_0$ in tangential direction.




Find the velocity of the mass at subsequent times.

Constraint Equation is $r = l$, that is $\dot{r} = \ddot{r} = 0$.

Equations of Motion

$$
\begin{aligned}
m\left(\ddot{r} - r\dot{\theta}^2\right) = -ml\dot{\theta}^2 &= -N \\
m\left(r\ddot{\theta} - 2\dot{r}\dot{\theta}\right) = mr\ddot{\theta} &= -f
\end{aligned}
$$

Eliminating $N$, we get

$$
\begin{aligned}
\ddot{\theta} &= -\mu\dot{\theta}^2 \\
v(t) &= l\dot{\theta}
\end{aligned}
$$

# Forces of Nature

## Fundamental Forces

Gravitational Forces

Electromagnetic Forces

Weak Nuclear Forces

Strong Nuclear Forces

- Electrostatic force on two electrons is $10^{36}$ larger the gravitational force.
- Strong forces are nucleonic forces that is responsible for the stability of the nuclei. The magnitude is very large and does not decay as inverse square of the distance. It is a short range force.
- In large atoms weak forces play a key role in phenomenon like radioactivity. It is about $10^{25}$ stronger than the gravitation force, but $10^{11}$ weaker than the electromagnetic force.

**A unified theory for the common origin of all the forces is sought.**

# Everyday forces: Contact Forces

## Force arises from interaction between two bodies.

**By contact forces we mean the forces which are transmitted between bodies by short-range atomic or molecular interactions.**

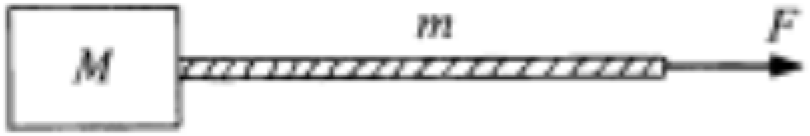
**Examples: push, pull, tension of a string, normal force, the force of friction, etc.**

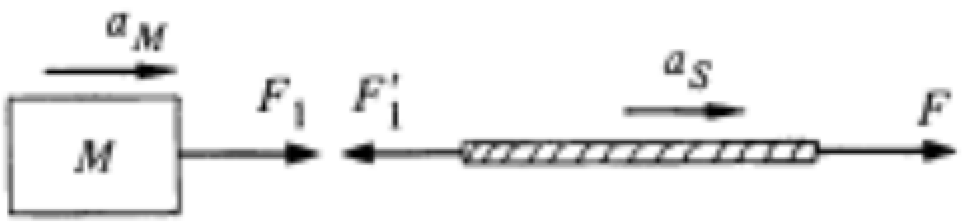
The origin of these forces can be explained in terms of the fundamental properties of matter. However, our approach will emphasize the properties of these forces and the techniques for dealing with them in physical problems, not worrying about their microscopic origins.

# Tension in a string: Most common example

A string consists of long chains of atoms. When a string is pulled, we say it is under tension. The long chains of molecules are stretched, and inter-atomic forces between atoms in the molecules prevent the molecules from breaking apart. To illustrate the behaviour of strings under tension:

Consider a block of mass *M* pulled by a string of mass *m*. A force *F* is applied to the string. What is the force that the string "transmits" to the block?




$$F_1 = M a_M$$

$$F - F_1' = m a_S.$$

String is inextensible: $a_S = a_M$

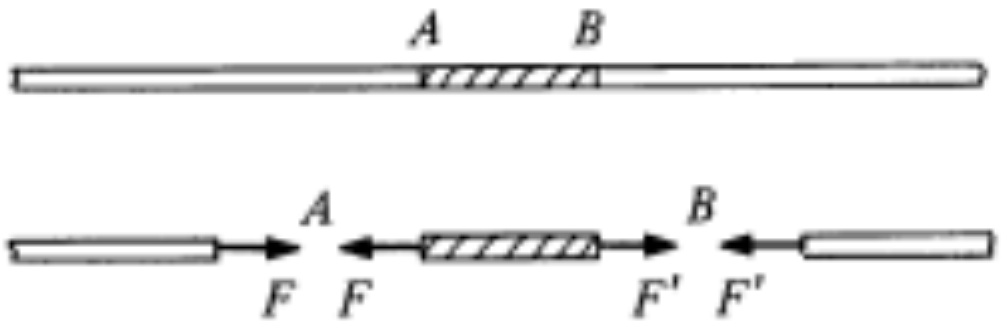
By Newton's third law: $F_1 = F_1'$

$$a = \frac{F}{M+m},$$

$$F_1 = F_1' = \frac{M}{M+m} F.$$

The force on the block is less than *F*. The string does not transmit the full applied force. If the mass of the string is negligible compared with the block, $F_1 = F$ to good approximation.

A string is composed of short sections interacting by contact forces. Each section pulls the sections to either side of it, and by Newton's third law, it is pulled by the adjacent sections. The magnitude of the force acting between adjacent sections is called **Tension**. **There is no direction associated with tension.** In the sketch, the tension at *A* is *F* and the tension at *B* is *F'*.




- Although a string may be under considerable tension, if the tension is uniform, the net string force on each small section is zero and the section remains at rest unless external forces act on it.
- If there are external forces on the section, or if the string is accelerating, the tension generally varies along the string.

# Thank You