# Indian Institute of Information Technology Vadodara
# MA 101: Introduction to Discrete Mathematics
# Tutorial 5

1. What time does a 12 hour clock read 45 hours before it reads 1:00pm?
2. What is -101 mod(13)?
3. Show that $2^{340} \equiv 1 \text{mod}(31)$.
4. Find the last digit of $333^{555}$.
5. State and prove divisibility test of 7. Can you design divisibility test of 13?
6. What is the remainder obtained when $2^{70} + 3^{70}$ is divided by 13?
7. Find the multiplicative inverse of each non-zero element of $\mathbb{Z}_{11}$ to verify $\mathbb{Z}_{11}$ is a field.
8. Find all integers $x$ such that $2x \equiv 3 \bmod 5, 3x \equiv 4 \bmod 7, x \equiv 5 \bmod 11$.
9. Use Fermat's little theorem to compute $5^{2023}$ mod 7, $5^{2023}$ mod 11, and $5^{2023}$mod 13. Use Chinese remainder theorem to find $5^{2023}$ mod 1001.
10. Prove that if n is a positive integer such that the sum of the divisors of n is n + 1, then n is prime. Is converse true?
11. Show that Strong Goldbach conjecture, which states that every even integer greater than 2 is the sum of two primes, implies the statement- (Weak Goldbach conjecture) every odd integer greater than 5 is the sum of three primes.