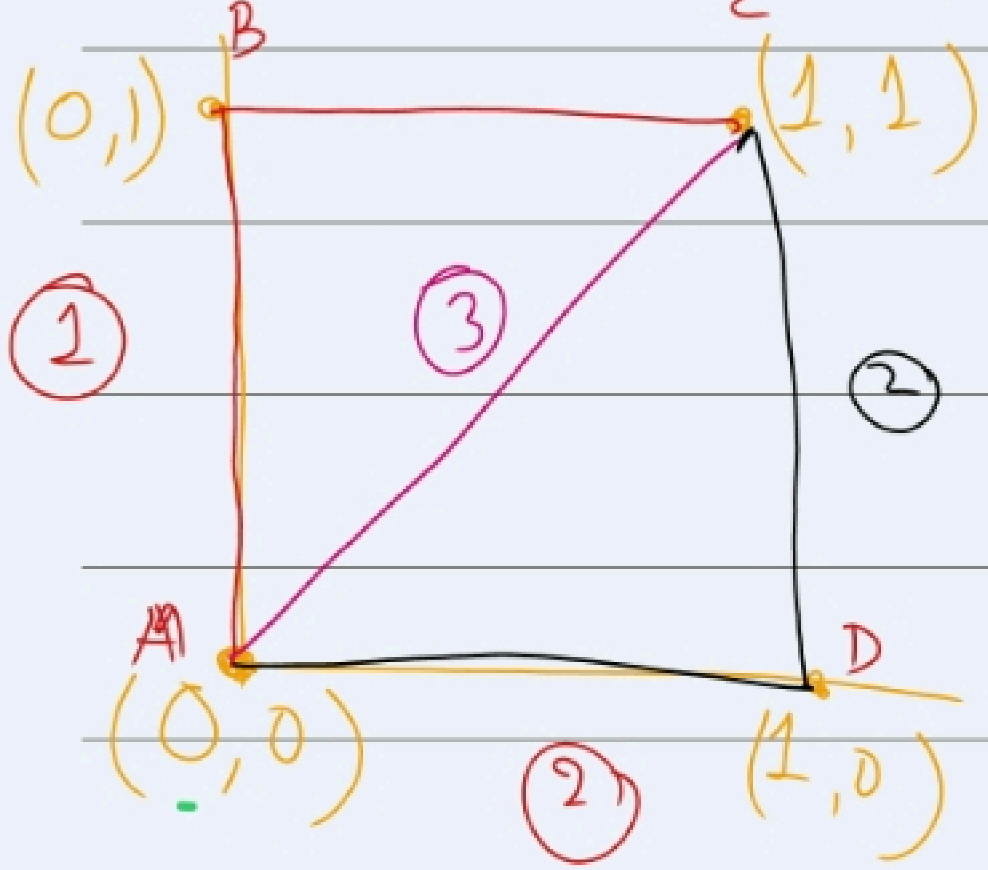
Conservation of Momentum :→ $F_{ext} = \frac{d\vec{p}}{dt}$.

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{r} = -W_{ba}$$

Work & Energy :→



$\vec{F} = xy\hat{i} + y^2\hat{j}$, $d\vec{r} = dx\hat{i} + dy\hat{j}$

$$① \left[\int_A^B (xy\hat{i} + y^2\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) = \int_{A,0}^{B,1} \underbrace{xy\,dx}_{0} + y^2 dy = \frac{1}{3} \right.$$

$x = 0$

$$\int_B^C xy\,dx + \underbrace{y^2 dy}_{0} = \frac{1}{2}$$

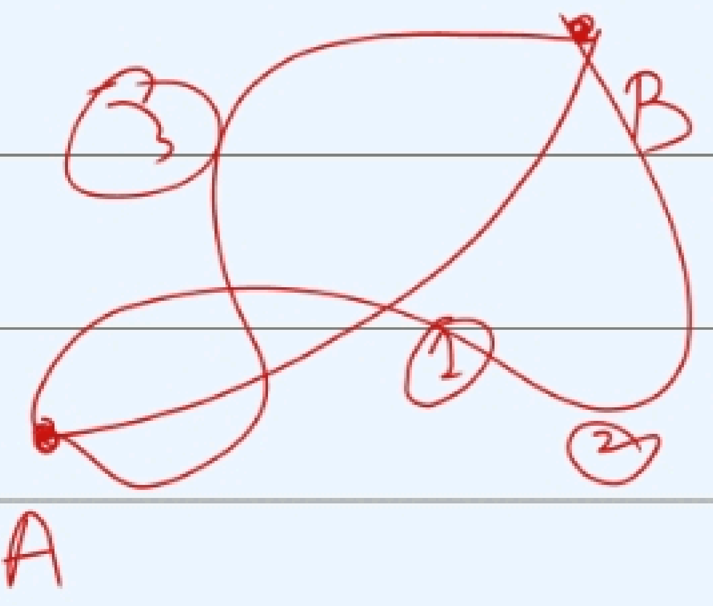
$y = 1, dy = 0$

$W_{AC} = 5/6$

06/10/2023

Work & Energy :-

$F = \frac{d\vec{p}}{dt} = m\vec{a}$



$$W = \int_A^B \vec{F} \cdot d\vec{r}$$

$W_1 \neq W_2 \neq W_3$

$$W = \int_A^B m\left(\frac{d\vec{v}}{dt} \cdot \vec{v}\right) dt \qquad [m = \text{constant}]$$

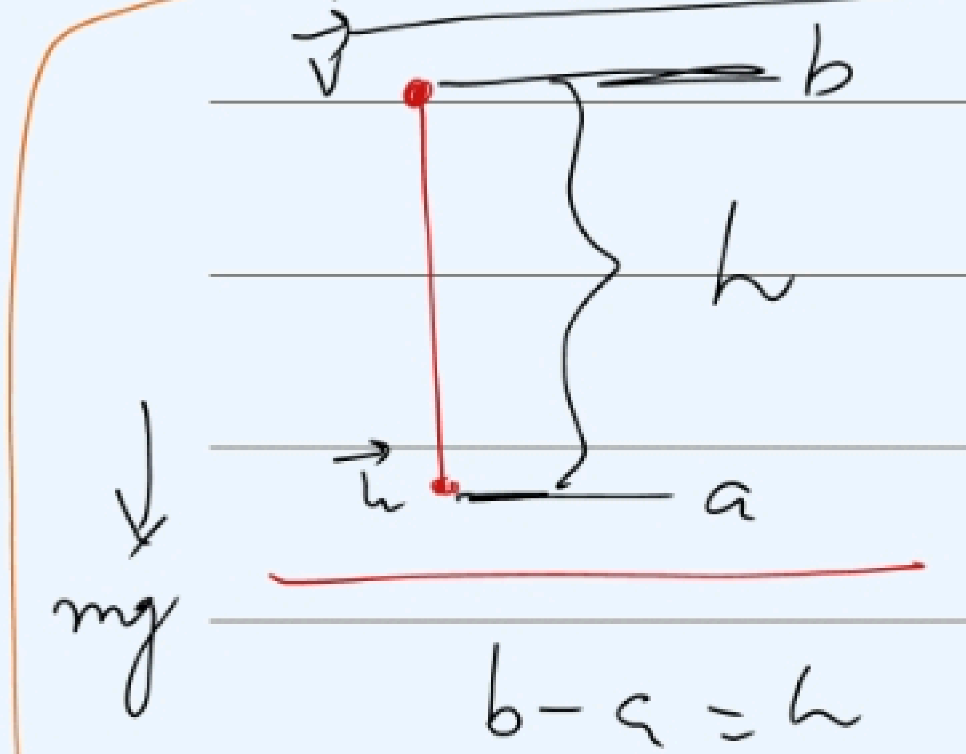
$$\frac{d}{dt}(v^2) = \frac{d}{dt}(\vec{v} \cdot \vec{v}) = 2\,\frac{d\vec{v}}{dt} \cdot \vec{v}$$

Work Energy Theorem

$$W = \int_A^B \frac{m}{2}\left(\frac{d}{dt}v^2\right) dt = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = K.E(B) - K.E(A)$$

one-particle

$$\boxed{W = K\cdot E(B) - K\cdot E(A)}$$



$$W = \int \vec F \cdot d\vec r = -\int_a^b (mg)\, dr = -mgh$$

$$\frac{1}{2} m \left[v^2 - u^2 \right] = \frac{1}{2}m \left[2gh \right]$$

$b - a = h$

$$K\cdot E(B) - K\cdot E(A) = -mgh$$

System of Particles $\quad W = \frac{1}{2} M \dot{X}_B^2 - \frac{1}{2} M \dot{X}_A^2 \qquad X_{A,B} = $ C.M of System.

Conservative force $\Leftrightarrow$ 1D $\quad$ Central force

Central force

$$W = \int_A^B f(r)\,\hat{r}\cdot \underbrace{d\vec{r}}_{(dr\,\hat{r} + r\,d\theta\,\hat{\theta})} = \int_A^B f(r)\,dr$$

$$W\big|_{ba} = K.E(b) - K.E(a) = -\underbrace{U(b) + U(a)}_{dU} = \int \vec{F}\cdot d\vec{r}$$

$$U(b) - U(a) = -\int \vec{F}\cdot d\vec{r}$$

1D: $d\vec{r} = dx\,\hat{i}$

$\vec{F} = f\,\hat{i}$

$$\Rightarrow \boxed{F = -\frac{dU}{dx}}$$

② : $(0,0)-(1,0)-(1,1)$

$W_{AC} \to W_{AD} - W_{DC}$

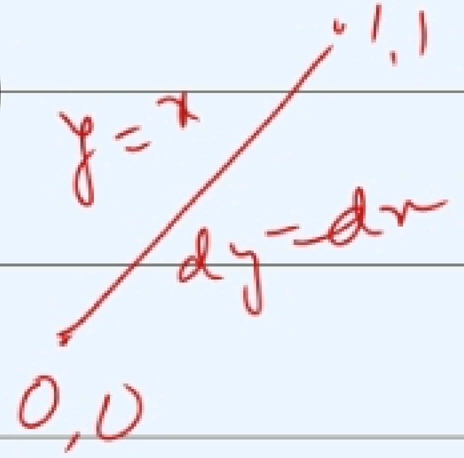
$$W_{AD} = \int_A^D xy\,dx + y^2\,dy \to 0$$

$y = 0,\ dy \to 0$

$$W_{DC} = \int_D^C \underbrace{\overset{0}{xy\,dx}}_{x=1} + y^2\,dy \to 1/3$$

W ① $\neq W$ ②

③



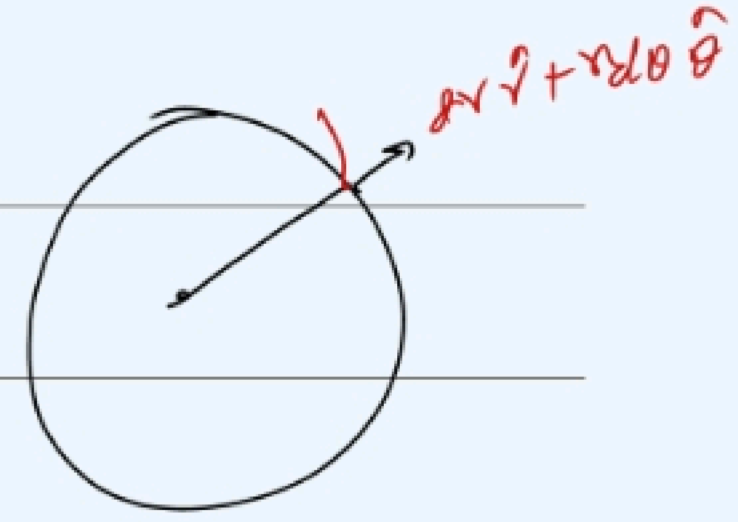
$$W_3 = \int_0^1 xy\,dx + y^2\,dy = 2/3$$

Central forces:

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{r^2}\hat{r} = f(r)\hat{r}$$

$$\vec{F} = q\vec{E} = K\frac{q_1 q_2}{r^2}\hat{r}$$

$$W = \int_a^b \vec{F}\cdot d\vec{r}$$

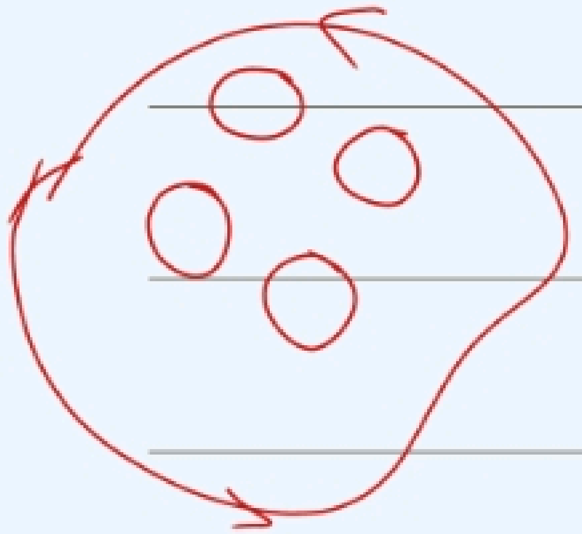


$$\int_a^a \vec{F}\cdot d\vec{r} = 0$$

$$= \int_a^b f(r)\,\hat{r}\cdot\left(dr\,\hat{r} + r\,d\theta\,\hat{\theta}\right)$$

$$= \int_a^b f(r)\,dr = f(b) - f(a)$$

Stokes Theorem:



$$\oint \vec{F}\cdot d\vec{l} = \underline{\iint (\nabla\times\vec{F})\cdot d\vec{a}}$$

$$0$$

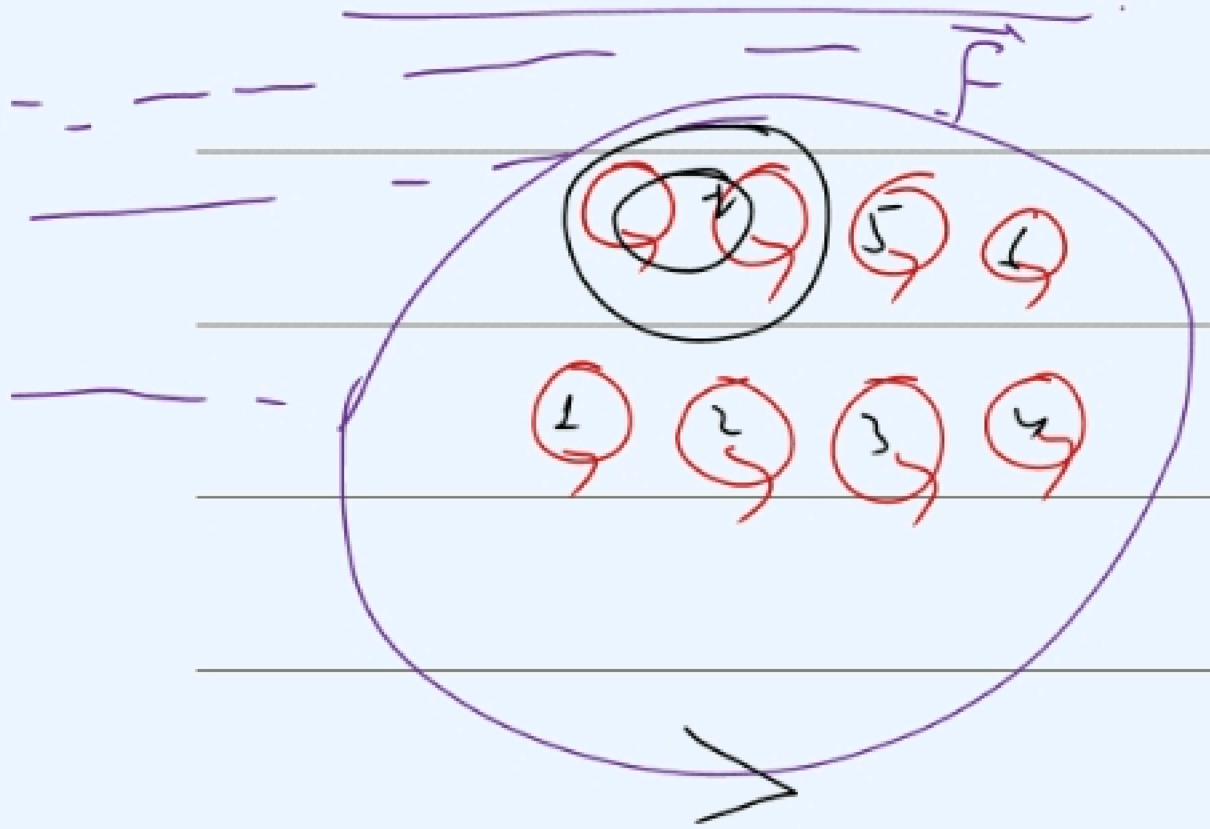
$$\underline{\nabla\times\nabla f}$$

$$0$$

$$U_b - U_a = -\int_a^b \vec{F}\cdot dr$$

Conservative force ⇒ Path Independent (work done)

Stoke's Theorem ⇒



$$\oint \vec{F}\cdot d\vec{l} = \underbrace{\iint (\nabla\times\vec{F})\cdot d\vec{a}}_{0}$$

$$\vec{F} = -\nabla f$$

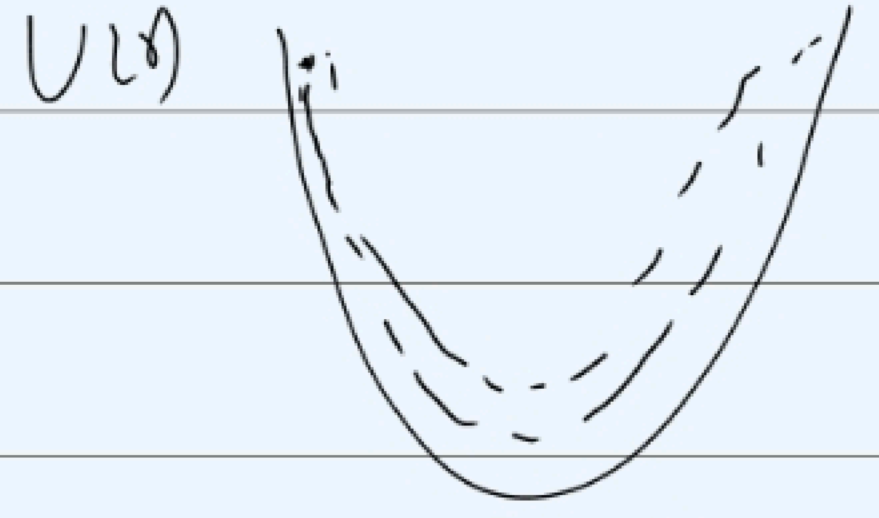
$$= -\left(\hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}\right)$$

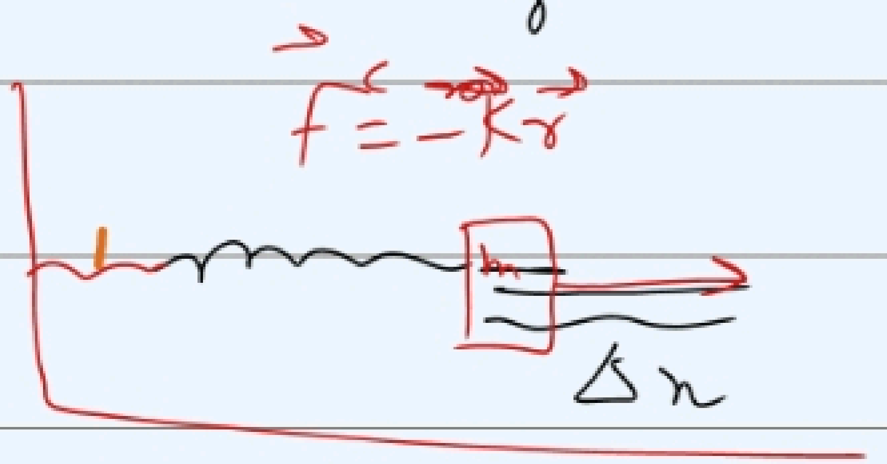
1D: $\vec{F} = -\hat{i}\frac{\partial f}{\partial x}$

$$U(r) = \frac{1}{2} K \left[x^2 + y^2 + z^2 \right] \qquad \begin{matrix} 1D \\ y = 0 \\ z = 0 \end{matrix}$$

$$\vec{F} = -\nabla U = -K \left[x\hat{i} + y\hat{j} + z\hat{k} \right]$$

$$\boxed{\vec{F} = -K\vec{r}}$$





Task $\Rightarrow$ $U(r) = -\frac{G m_1 m_2}{r}$, $\vec{F}$

$$W = \int \vec{F} \cdot d\vec{r} \,, \quad \vec{F} = F^{C} + F^{NC}$$

$$W = K.E(b) - K.E(a) = -U(b) + U(a)$$

$$W = E(b) - E(a) + Q \longrightarrow \text{Heat}$$

Particle collisions and conservation Laws :→

$$K.E_i = K.E_f$$

① Elastic :→ $\Delta p = 0 \Rightarrow p_i = p_f$

② Inelastic :→ $K.E_i \neq K.E_f$

$$K.E_i = K.E_f + Q$$

Elastic Collision

m_1

$m_2 = 3m_1$ || Both have ($\vec{v}$) equal & opposite velocities || Final velocity

Momentum: $m_1 v - 3m_1 v = m_1 v_1' + 3m_1 V_2'$

$$\Rightarrow \boxed{V_1' = -2V - 3V_2'}$$

K.E: $K.E_i = K.E_f$:

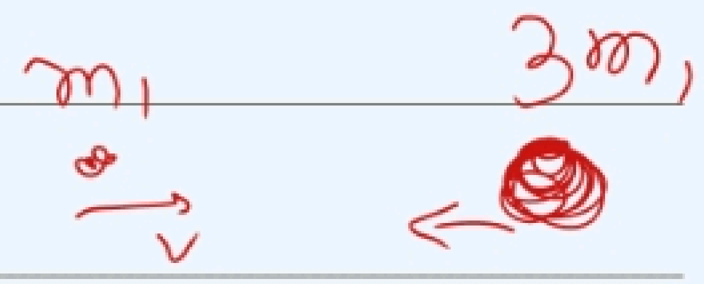
K.E:

$$\cancel{\tfrac{1}{2}}\cancel{m_1}v^2 + \cancel{\tfrac{1}{2}}3\cancel{m_1}v^2 = \cancel{\tfrac{1}{2}}\cancel{m_1}{v_1'}^2 + \cancel{\tfrac{1}{2}}3\cancel{m_1}{v_2'}^2$$

$$v_1' = -2v - 3v_2'$$

$m_1 \; \| v \| \; v_1'$

$3m_1 \; \| v \| \; v_2'$



$$4v^2 = {v_1'}^2 + 3{v_2'}^2$$

$$= \left[-2v - 3v_2'\right]^2 + 3{v_2'}^2$$

$$\cancel{4v^2} = \cancel{4v^2} + 9{v_2'}^2 + 12vv_2' + 3{v_2'}^2$$

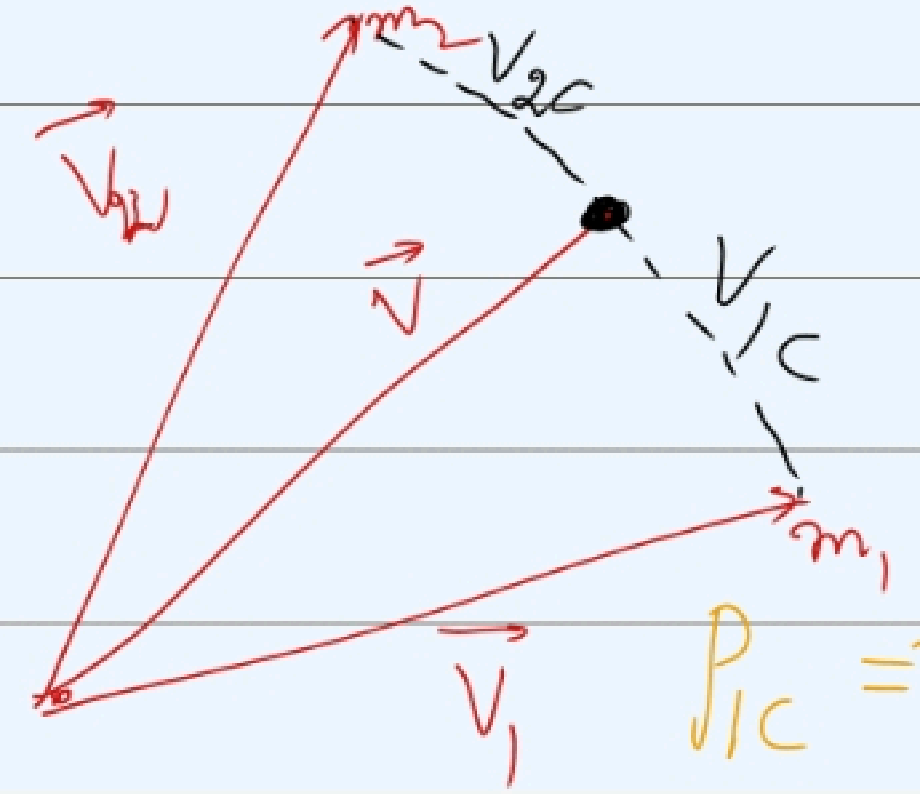
$$12\left[{v_2'}^2 + vv_2'\right] = 0$$

$v_2' = 0$

$v_2' = -v$

Collisions and C.M. Coordinates:→

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$



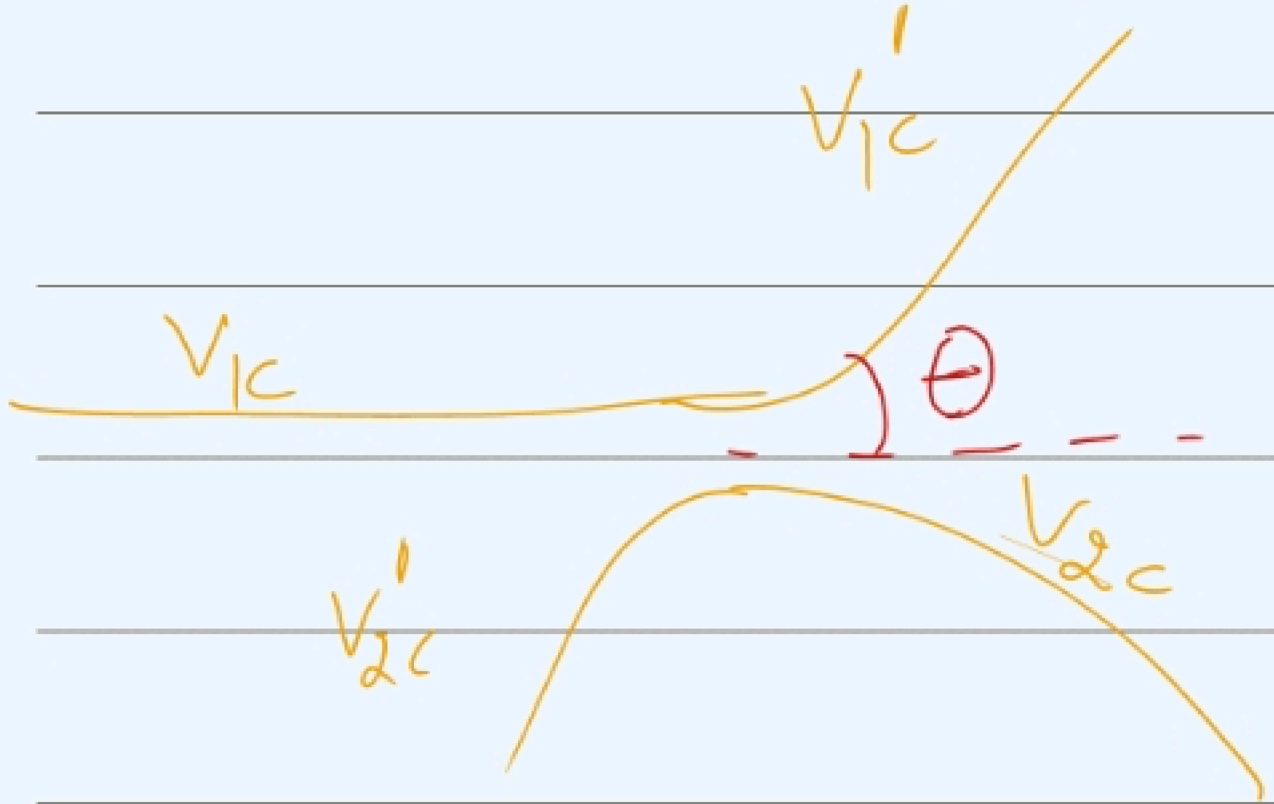
$$\vec{V}_{1c} = \vec{V}_1 - \vec{V} = \vec{V}_1 - \left[\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right]$$

$$P_{1c} = m_1\vec{V}_{1c} = m_1\left[\frac{\cancel{m_1\vec{V}_1} + m_2\vec{V}_1 - (\cancel{m_1\vec{V}_1} + m_2\vec{V}_2)}{m_1+m_2}\right] = \frac{m_1 m_2(\vec{V}_1 - \vec{V}_2)}{m_1+m_2}$$

$$\vec{P}_{2c} = m_2\vec{V}_{2c} = \vec{V}_2 - \vec{V} = -\frac{m_1 m_2}{m_1+m_2}(\vec{V}_1 - \vec{V}_2)$$

$$\vec{P} = 0$$

Elastic Collision & C.M. frame of reference :-



$$m_1 V_{1c} - m_2 V_{2c} = 0 \Rightarrow V_{1c} = \frac{m_2}{m} V_{2c}$$

$$m_1 V_{1c}' - m_2 V_{2c}' = 0 \Rightarrow V_{1c}' = \frac{m_2}{m_1} V_{2c}'$$

$$K.E_i = K.E_f$$

$$\Rightarrow \frac{1}{2} m_1 V_{1c}^2 + \frac{1}{2} m_2 V_{2c}^2 = \frac{1}{2} m_1 V_{1c}'^2 + \frac{1}{2} m_2 V_{2c}'^2$$

$$V_{2c}^2 = V_{2c}'^2$$

$$\Rightarrow \cancel{\frac{1}{2}} \cancel{m_1} \left[\frac{m_2^2}{m_1^2} V_{2c}^2\right] + \cancel{\frac{1}{2}} m_2 V_{2c}^2 = \cancel{\frac{1}{2}} m_1 \left[\frac{m_2^2}{m_1^2} V_{2c}'^2\right] + \cancel{\frac{1}{2}} m_2 V_{2c}'^2$$

$$\Rightarrow \left[m_2 + \cancel{\frac{m_2^2}{m_1}}\right] V_{2c}^2 = \left[\cancel{\frac{m_2^2}{m_1}} + m_2\right] V_{2c}'^2 \Rightarrow \boxed{V_{2c} = V_{2c}'}$$

Laboratory & C.M frame scattering Angle :→

Correlation scattering angle
in L-frame & c.m-frame.