

Academic Year 2023-24
Tutorial #06
PH100: Mechanics and Thermodynamics

1. The logarithmic decrement δ is defined to be the natural logarithm of the ratio of successive maximum displacements (in the same direction) of a free damped oscillator. Show that $\delta = \pi / Q$. Find the spring constant k and damping constant b of a damped oscillator having a mass of 5 kg, frequency of oscillation 0.5 Hz, and logarithmic decrement 0.02.
2. Two particles, each of mass M , are hung between three identical springs. Each spring is massless and has spring constant k . Neglect gravity. The masses are connected as shown to a dashpot of negligible mass. The dashpot exerts a force bv , where v is the relative velocity of its two ends. The force opposes the motion. Let x_1 and x_2 be the displacements of the two masses from equilibrium.
 - a. Find the equation of motion for each mass.
 - b. Show that the equations of motion can be solved in terms of the new dependent variables $y_1 = x_1 + x_2$ and $y_2 = x_1 - x_2$.
 - c. Show that if the masses are initially at rest and mass 1 is given an initial velocity v_0 , the motion of the masses after a sufficiently long time is $x_1 = x_2 = (v_0/2\omega) \sin(\omega t)$. Evaluate ω .
3. The motion of a damped oscillator driven by an applied force $F_0 \cos \omega t$ is given by $x_a(t) = A \cos(\omega t + \phi)$, where A and ϕ are given by Eq. (10.25). Consider an oscillator which is released from rest at $t = 0$. Its motion must satisfy $x(0) = 0$, $v(0) = 0$, but after a very long time, we expect that $x(t) = x_a(t)$. To satisfy these conditions we can take as the solution $x(t) = x_a(t) + X_b(t)$, where $X_b(t)$ is the solution to the equation of motion of the free damped oscillator, Eq. (10.8).
 - a. Show that if $x_a(t)$ satisfies the equation of motion for the forced damped oscillator, then so does $x(t) = x_a(t) + X_b(t)$, where $x_b(t)$ satisfies the equation of motion of the free damped oscillator, Eq. (10.25).
 - b. Choose the arbitrary constants in $X_b(t)$ so that $x(t)$ satisfies the initial conditions. [$x_b(t)$ is given by Eq. (10.9). Note that A and ϕ here are arbitrary.]
 - c. Sketch the resulting motion for the case where the oscillator is driven at resonance.
4. A particle of mass 50 g moves under an attractive central force of magnitude $4r^3$ dynes. The angular momentum is equal to $1,000 \text{ g}\cdot\text{cm}^2/\text{s}$.
 - a. Find the effective potential energy.
 - b. Indicate on a sketch of the effective potential the total energy for circular motion.
 - c. The radius of the particle's orbit varies between r_0 and $2r_0$. Find r_0 .