PH100: Mechanics and Thermodynamics

Lecture 6

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Friction

- Mostly encounter two kinds of friction:

(i) Sliding friction – Comes into play when two bodies slide on one another. All surfaces are rough at the atomic level.

(ii) Fluid/Air friction – At very small velocities, air friction is absent. Usually at moderate velocities, it is proportional to velocities (such as a ball moving through viscous fluids). At very large velocities (such as an aeroplane where the air swirls around as the plane moves), friction may be proportional to square of velocity or even higher powers.

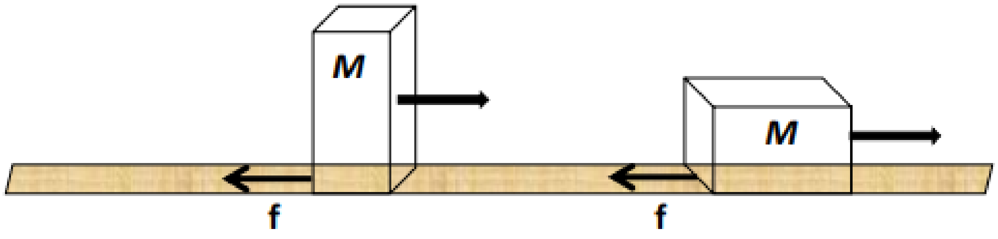
Sliding Friction

- Friction arises when the surface of one body moves, or tries to move, along the surface of a second body.

- The maximum value of the friction is $f_{friction} = \mu N$
  where N is the normal force and μ is the coefficient of friction.

- When a body slides across a surface, the friction force is directed opposite to the instantaneous velocity and has magnitude μN. The force of sliding friction is slightly less than the force of static friction, but for the most part we shall neglect this effect.

- For two given surfaces, the force of sliding friction is independent of the area of contact.



Important features of Friction

- Friction is independent of the area of contact because the actual area of contact on an atomic scale is a minute fraction of the total surface area.

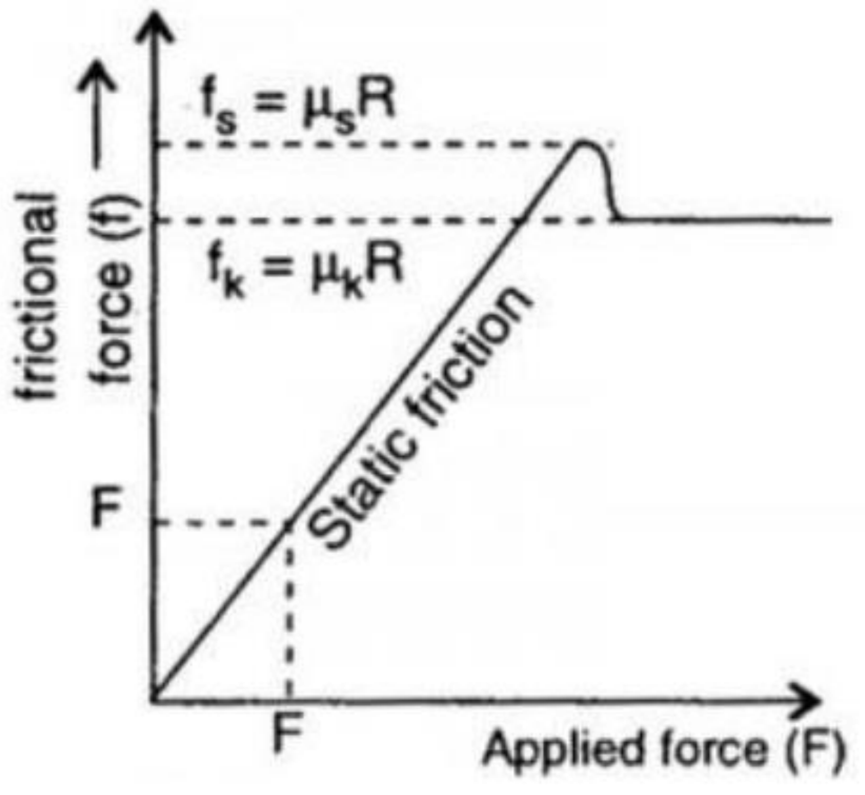
- Friction occurs because of the interatomic forces at these minute regions of atomic contact.

- Non rigid bodies, like automobile tires, are more complicated. A wide tire is generally better than a narrow one for good acceleration and braking.

- Frictional force is also independent of relative velocity between two surfaces.

- This is approximately true for a wide range of low speeds, as the speed increases and air friction come into play, it is found that friction not only depends on the speed, but upon the square and sometimes higher powers of the speed.

Applied force vs Frictional force



Laws of Friction vs Newton's laws

- Two laws of friction:

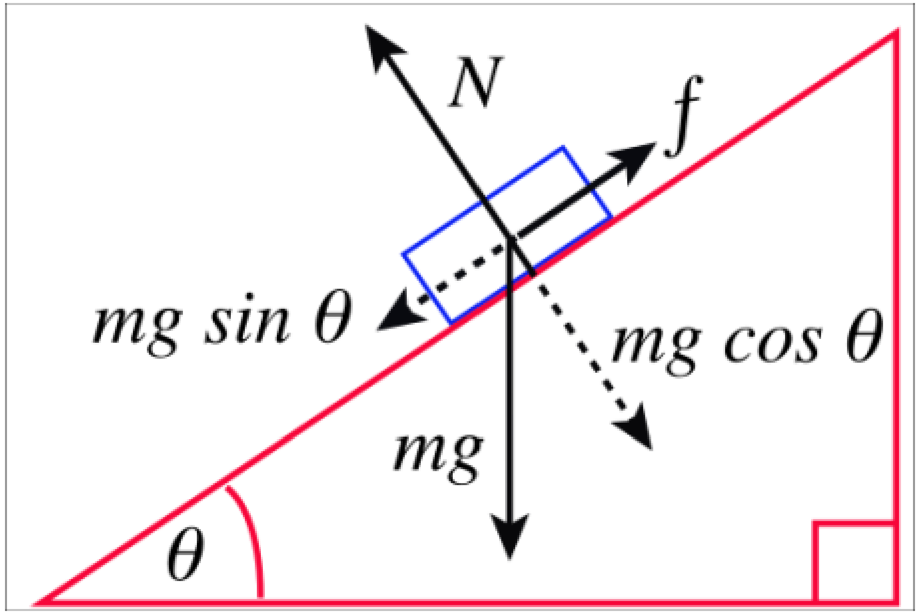
$$F = \mu N \quad \text{(for sliding)}$$

$$F = cv^{\alpha} \quad \text{(for fluid friction); } \alpha = 1,2,\ldots$$

Distinguish it with $F = ma$!!

Newton's laws are real laws, while laws of friction are empirical laws.

How to experimentally determine friction or test F = μN?



At the verge of sliding

$$mg\ sin\theta = \mu N = \mu mg\ cos\theta$$

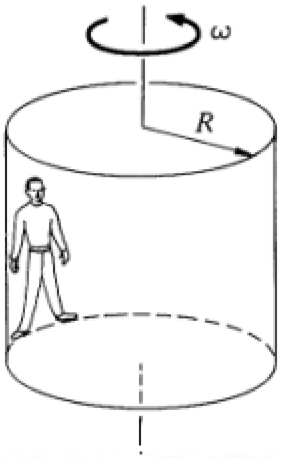
$$\mu = \tan\theta$$

An object will start to slude at a given inclination. If the same block is loaded by providing extra weight, it will still be sliding at the given angle. Coefficient of friction is constant for a given angle.

In fact if this experiment is performed by continuously varying the angle, then at the correct angle, the block begins to slide, **but not steadily.** Thus μ being constant Is only roughly true.

The spinning terror

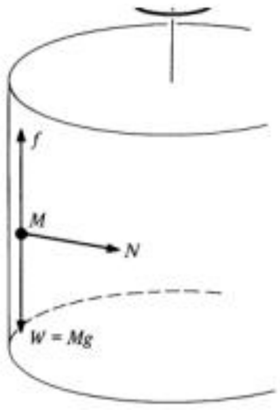
The Spinning Terror is an amusement park ride—a large vertical drum which spins so fast that everyone inside stays pinned against the wall when the floor drops away. What is the minimum steady angular velocity ω which allows the floor to be dropped away safely?



$$N = MR\omega^2.$$

By the law of static friction,

$$f \leq \mu N = \mu MR\omega^2.$$

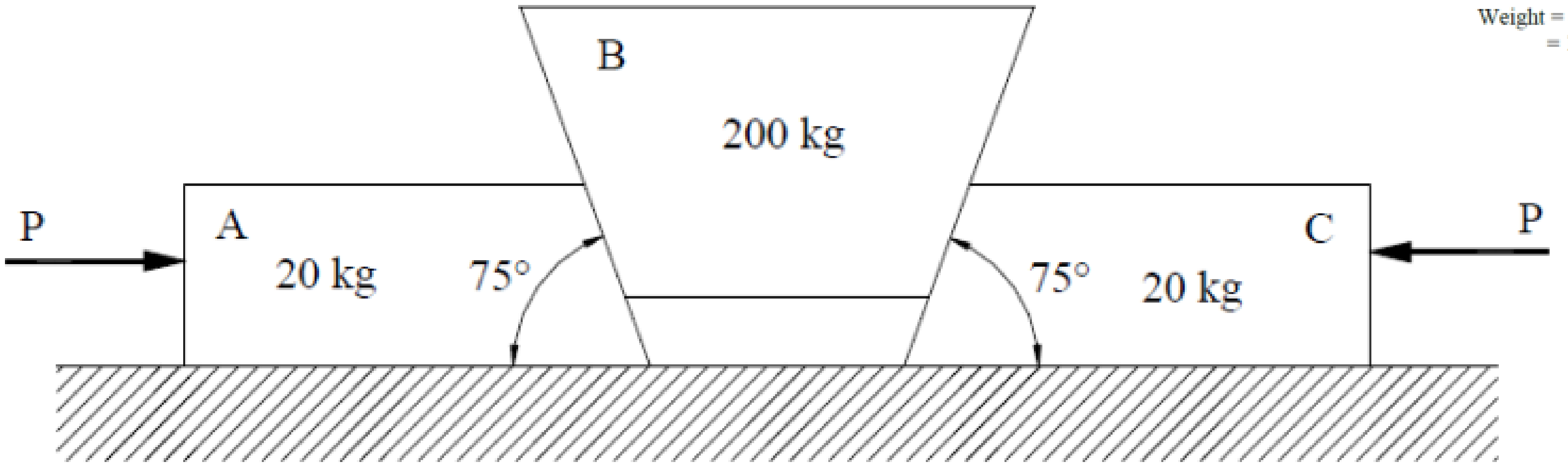


Since we require M to be in vertical equilibrium, $f = Mg,$

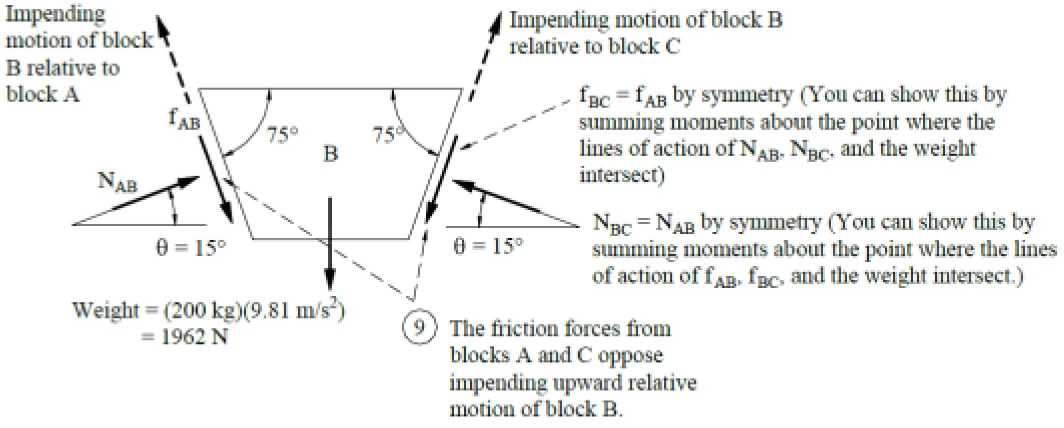
Therefore, $Mg \leq \mu MR\omega^2$ or $\omega^2 \geq \frac{g}{\mu R}.$ or $\omega_{\min} = \sqrt{\frac{g}{\mu R}}.$

The blocks and friction

. If the coefficient of static friction for all surfaces of contact is 0.25, determine the smallest value of the forces P that will move wedge B upward.



(8) Free-body diagram of block B



(10) Equations of equilibrium

$$\xrightarrow{+} \Sigma F_x = 0:\ N_{AB}\cos 15° - N_{AB}\cos 15° + f_{AB}\cos 75° - f_{AB}\cos 75° = 0 \qquad (5)$$

(Note that this equation reduces to 0 = 0. This happens because we have assumed symmetry to conclude that $f_{BC} = f_{AB}$ and $N_{BC} = N_{AB}$.)

$$+\uparrow \Sigma F_y = 0:\ N_{AB}\sin 15° + N_{AB}\sin 15° - f_{AB}\sin 75° - f_{AB}\sin 75° - 1962\text{ N} = 0 \qquad (6)$$

(6) Equations of equilibrium

$$\xrightarrow{+} \Sigma F_x = 0:\ P - f_A - f_{AB}\cos 75° - N_{AB}\cos\theta = 0 \qquad (1)$$

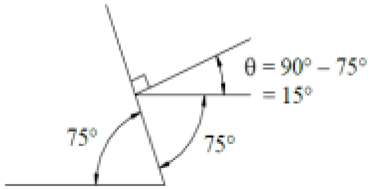
$$+\uparrow \Sigma F_y = 0:\ N_A - 196.2\text{ N} + f_{AB}\sin 75° - N_{AB}\sin\theta = 0 \qquad (2)$$

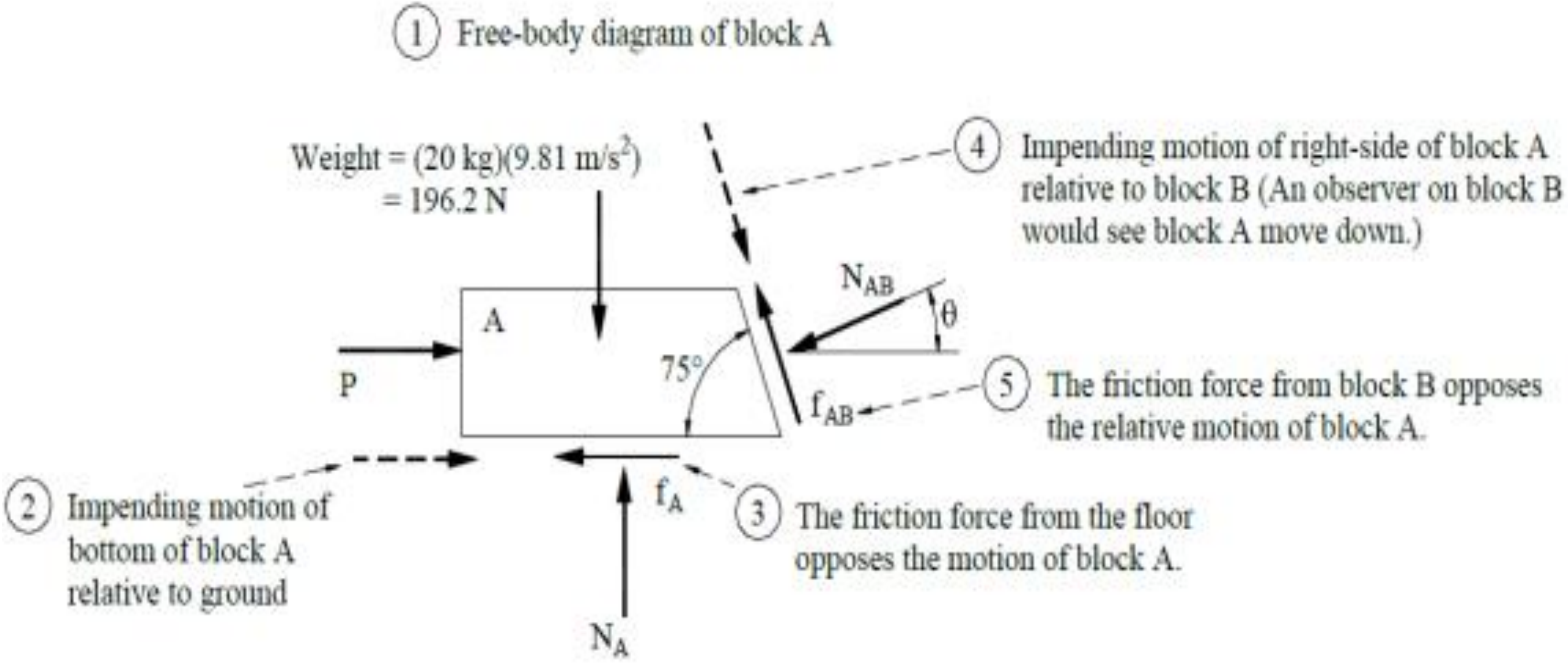
Slip impends so,

$$f_A = f_{A\text{-max}} \equiv \mu N_A = 0.25 N_A \qquad (3)$$

$$f_{AB} = f_{AB\text{-max}} \equiv \mu N_{AB} = 0.25 N_{AB} \qquad (4)$$

(7) Geometry




1 Free-body diagram of block A
Weight = (20 kg)(9.81 m/s²)
= 196.2 N
A
P
75°
N_AB
θ
f_AB
f_A
N_A
4 Impending motion of right-side of block A relative to block B (An observer on block B would see block A move down.)
5 The friction force from block B opposes the relative motion of block A.
2 Impending motion of bottom of block A relative to ground
3 The friction force from the floor opposes the motion of block A.

Conservation of Momentum

Momentum conservation for a system of particles

- So far we talked about point particles. There is a need to:

  (a) generalize it to extended bodies

  (b) to deal with variable mass problem

1.Momentum ($p = mv$) is a more fundamental quantity than m & v separately.

2. Newton's 2nd law should be written as F = $\dot{p}$ instead of 'ma' (for variable m).

3. For a system of particles, **an external Force** causes change of **total momentum** of the system. The internal forces cancel each other.

It will be useful to locate a point for a system of particles where all the mass may be concentrated at. Then the single particle EOM will continue.

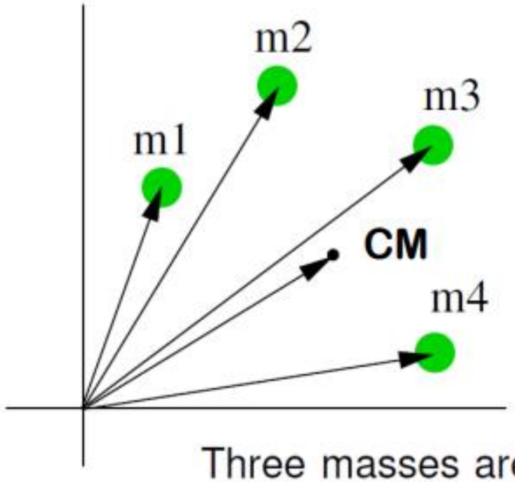
Center of Mass

A system has n particles with masses and positions given by

$$m_1,\ m_2, \ldots,\ m_n$$
$$\mathbf{r}_1,\ \mathbf{r}_2, \ldots,\ \mathbf{r}_n$$

Define a **Center of Mass** as

$$\mathbf{R}_{CM} = \frac{1}{M}\left(\sum_i m_i \mathbf{r}_i\right)$$



Example

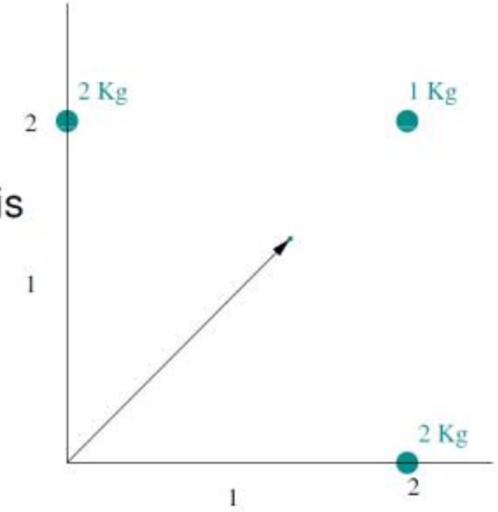
Three masses are kept in a plane as shown in the figure.

$m_1 = 2$ Kg, $m_2 = 2$ Kg and $m_3 = 1$ Kg

$r_1 = 2\mathbf{j}$, $r_2 = 2\mathbf{i}$ and $r_3 = 2\mathbf{i} + 2\mathbf{j}$

Total Mass is 5 Kg. Then Center of Mass is given by

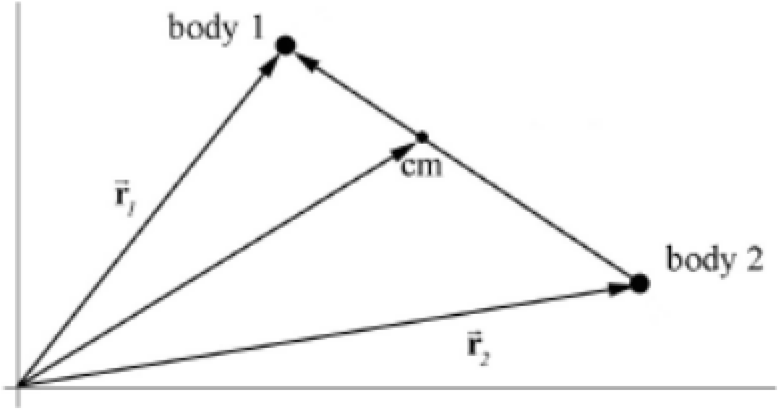
$$\begin{aligned}\mathbf{R}_{cm} &= \frac{1}{5}\left(2\mathbf{r_1} + 2\mathbf{r_2} + \mathbf{r_3}\right) \\ &= \frac{6}{5}\left(\mathbf{i} + \mathbf{j}\right)\end{aligned}$$



Center of mass

The center of mass vector, $\mathbf{R}_{cm}$, of the two-body system

$$\vec{\mathbf{R}}_{cm} = \frac{m_1 \vec{\mathbf{r}}_1 + m_2 \vec{\mathbf{r}}_2}{m_1 + m_2}.$$



For a **continuous rigid body**, each point-like particle has mass dm and is located at the position r'. The center of mass is then defined as an integral over the body,

$$\vec{\mathbf{R}}_{cm} = \frac{\int_{\text{body}} dm\,\vec{\mathbf{r}}'}{\int_{\text{body}} dm}.$$

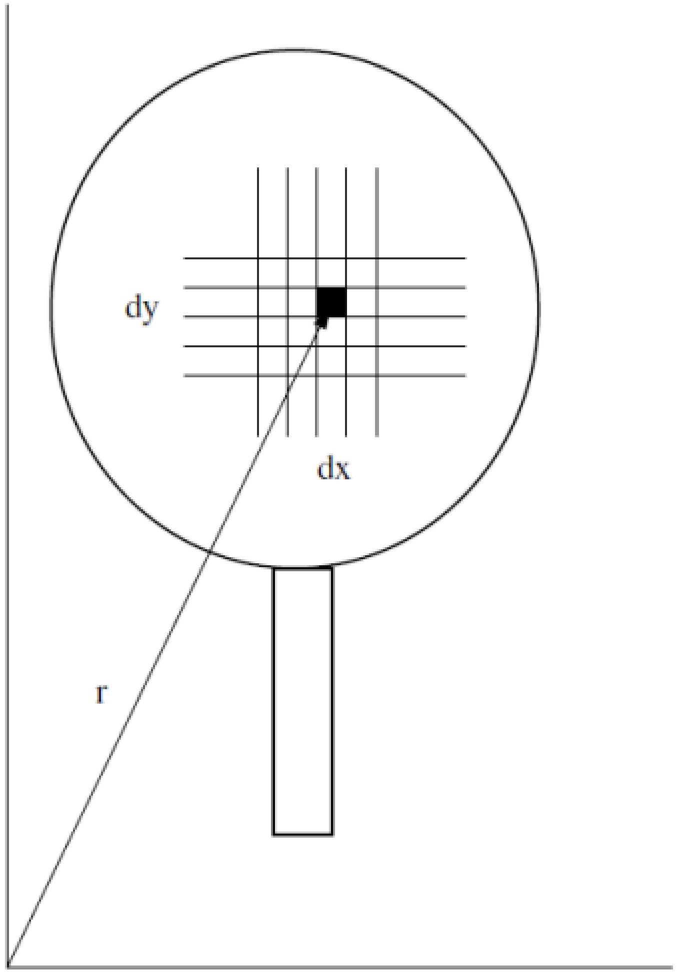
Planar Continuous Bodies

The density is given by $\rho(\mathbf{r})$. An element at $\mathbf{r}$ and of area $dxdy$ has a mass
$dm = \rho(\mathbf{r})dxdy$.

$$\begin{aligned}\mathbf{R}_{cm} &= \frac{1}{M}\sum \mathbf{r}dm \\ &= \frac{1}{M}\int \mathbf{r}\rho(\mathbf{r})dxdy\end{aligned}$$

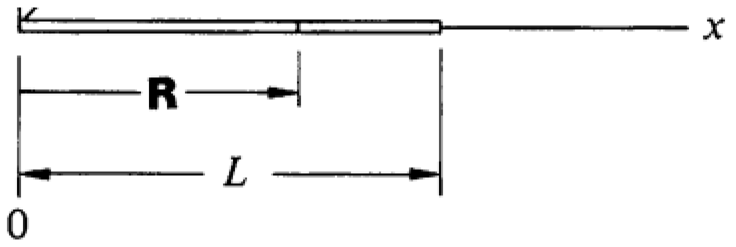
And

(1) $$M = \int \rho(\mathbf{r})dxdy$$



Examples

The mass per unit length λ of a rod of length L varies as $\lambda = \lambda_0(x/L)$, where λ_0 is a constant and x is the distance from the end marked O. Find the center of mass.



$$M = \int dm = \int_0^L \frac{\lambda_0 x\, dx}{L} = \tfrac{1}{2}\lambda_0 L.$$

$$\mathbf{R}_{cm} = \frac{2}{\lambda_0 L}\int_0^L (x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}})\,\frac{\lambda_0 x\, dx}{L}$$

$$= \frac{2}{L^2}\,\frac{\hat{\mathbf{i}}}{3}\,x^3\Big|_0^L = \tfrac{2}{3}L\hat{\mathbf{i}}.$$

Equations of Motion

Now, by definition,

$$\begin{aligned} M\mathbf{R}_{cm} &= \sum m_i \mathbf{r}_i \\ M\ddot{\mathbf{R}}_{cm} &= \sum m_i \ddot{\mathbf{r}}_i \end{aligned}$$

But for each particle, labled by i,

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i = \mathbf{F}_i^{ext} + \mathbf{F}_i^{int}$$

Hence,

$$M\ddot{\mathbf{R}}_{cm} = \sum \mathbf{F}_i^{ext} + \sum \mathbf{F}_i^{int}$$

What are CM coordinates good for?

But by Newton's third law, all internal forces appear in pairs and are equal and opposite. Thus in the following summation all internal forces cancel each other out.

$$\sum_i \mathbf{F}_i^{int} = 0$$

Thus Equation of Motion for Center of Mass of any system

$$M\ddot{\mathbf{R}}_{CM} = \mathbf{F}^{ext} = \sum_i \mathbf{F}_i^{ext}$$

One point $\mathbf{R}_{cm}$ traces the same motion as that of a single particle of mass M under the influence of a force $\mathbf{F}^{ext}$

Translational Motion of the Center of Mass

The velocity of the center of mass is given by $\vec{\mathbf{V}}_{\mathrm{cm}} = \frac{1}{m^{\mathrm{total}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i = \frac{\vec{\mathbf{p}}^{\mathrm{total}}}{m^{\mathrm{total}}}.$

The total momentum is then expressed in terms of the velocity of the center of mass by

$$\vec{\mathbf{p}}^{\mathrm{total}} = m^{\mathrm{total}} \vec{\mathbf{V}}_{\mathrm{cm}}.$$

The total external force is equal to the change of the total momentum of the system,

$$\vec{\mathbf{F}}^{\mathrm{total}}_{\mathrm{ext}} = \frac{d\,\vec{\mathbf{p}}^{\mathrm{total}}}{dt} = m^{\mathrm{total}} \frac{d\,\vec{\mathbf{V}}_{\mathrm{cm}}}{dt} = m^{\mathrm{total}} \vec{\mathbf{A}}_{\mathrm{cm}}, = m^{total} \ddot{\vec{R}}_{cm}$$

where $\vec{\mathbf{A}}_{\mathrm{cm}}$, is the acceleration of the center of mass.

The system behaves as if all the mass is concentrated at the center of mass and all the external forces act at that point. This is an over simplification. The shape of the body and the point of application of force matters.

The same force on the same mass with different shape may lead to different types of motion.

$$\vec{F}^{total}_{ext} = m^{total} \ddot{\vec{R}}_{cm}$$

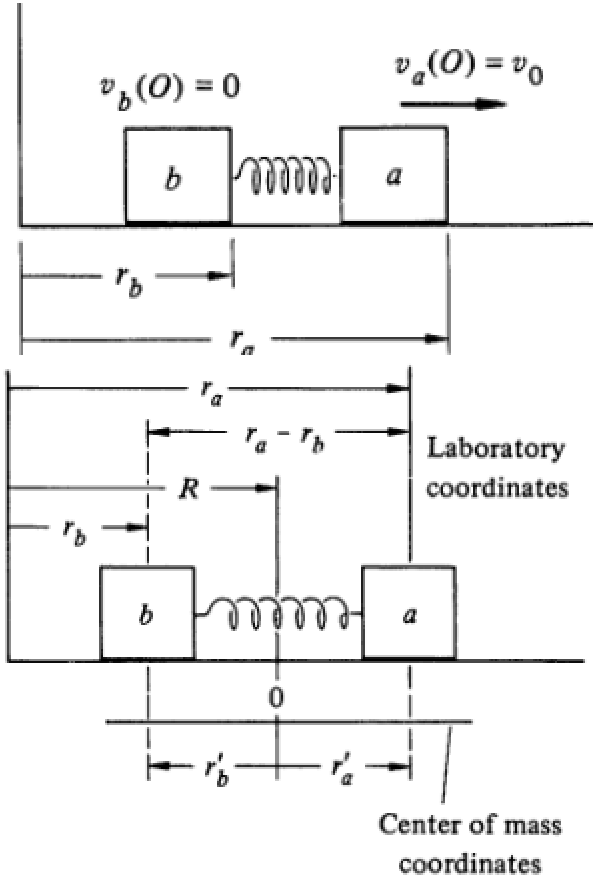
Note EOM describes translational motion.

Center of mass Theorem

The center of mass of a system of particles (rigid or non-rigid) moves as if the entire mass were concentrated at that point and all external forces act there.

An example

Two identical blocks *a* and *b* both of mass *m* slide without friction on a straight track. They are attached by a spring of length *l* and spring constant *k*. Initially they are at rest. At *t* = 0, block *a* is hit sharply, giving it an instantaneous velocity v_0 to the right. Find the velocities for subsequent times.



$m_1\ddot{r}_a=-k(r_a-r_b+l)$, $m_2\ddot{r}_b=-k(r_b-r_a-l)$.

$$R = \frac{mr_a + mr_b}{m+m} = \frac{1}{2}(r_a + r_b).$$

$$r'_a = r_a - R = \tfrac{1}{2}(r_a - r_b)$$

$$r'_b = r_b - R = -\tfrac{1}{2}(r_a - r_b) = -r'_a.$$

$$r_a - r_b - l = r'_a - r'_b - l,$$

$$m\ddot{r}'_a = -k(r'_a - r'_b - l)$$

$$m\ddot{r}'_b = +k(r'_a - r'_b - l),$$

EOM In CM frame

$$m(\ddot{r}'_a - \ddot{r}'_b) = -2k(r'_a - r'_b - l).$$

Letting $u = r'_a - r'_b - l,\ \ m\ddot{u} + 2ku = 0.$

$u = A \sin \omega t + B \cos \omega t,$ where $\omega = \sqrt{2k/m}.$

Difference in CM coordinates executes SHM

Applying initial conditions:

$$t = 0,\ u(0) = 0 \qquad B = 0.$$

Since $u = r_a' - r_b' - l = r_a - r_b - l,$

At t=0, $\dot{u}(0) = v_a(0) - v_b(0) = A\omega \cos(0) = v_0, \qquad A = v_0/\omega$

Therefore, $u = (v_0/\omega)\sin \omega t.$

Since $v_a' - v_b' = \dot{u}$, and $v_a' = -v_b'$, we have $v_a' = -v_b' = \frac{1}{2}v_0 \cos \omega t.$

The laboratory velocities are:

$$v_a = \dot{R} + v_a' \qquad \dot{R} = \tfrac{1}{2}[v_a(0) + v_b(0)]$$

$$v_b = \dot{R} + v_b'. \qquad = \tfrac{1}{2}v_0.$$

$$v_a = \frac{v_0}{2}(1 + \cos \omega t) \quad v_b = \frac{v_0}{2}(1 - \cos \omega t).$$

The masses move to the right on the average, but they alternately come to rest in a push me-pull-you fashion.

Thank You