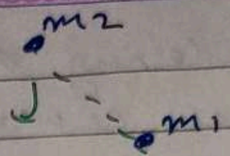
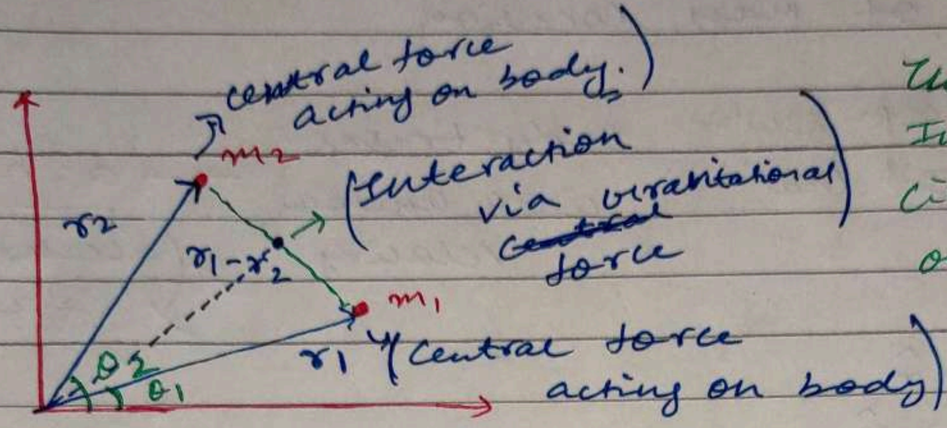
Central Force Motion:-

Central Force $\rightarrow f(r).\hat{r}$





Two mass are Interacting in circumstance of any force.

Equation of motions:- $\boxed{r = r_1 - r_2}$

$$m_1 \ddot{r}_1 = f(r)\,\hat{r}$$
$$m_2 \ddot{r}_2 = -f(r)\,\hat{r}$$

{ all possibility are possible

(i) $\vec{r}(t) = ?$

(ii) $\frac{dr}{dt} = ?$

(iii) $\theta(t) = ?$

(iv) $\frac{d\theta}{dt} = ?$

(v) $\frac{d\vec{r}}{d\theta} = ?$

~~Ther//~~

Problem:- there are two bodies m_1 and m_2 in which central forces are acting. then find $\vec{r}(t)$, $\vec{\theta}(t)$, $\dot{r}(t)$, $\dot{\theta}(t)$, $d\vec{r}/d\theta$.

Centre of mass: $R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

$\ddot{R} = 0$ (assuming that no external force is there)

$R = R_0 + Vt$

Here, central forces are the forces present Inside the system. forces are not

Considering as such that It will give Jerk to the centre of mass location.

$F_{ext} = 0$ $\xrightarrow[\text{Ist law}]{\text{from newton}}$ Body travel with uniform velocity $\longrightarrow a = 0$ (acceleration = zero)

$$m_1 \ddot{r}_1 = f(r)\,\hat{r}$$

$$m_2 \ddot{r}_2 = -f(r)\,\hat{r}$$

$$\underbrace{\ddot{r}_1 - \ddot{r}_2}_{\ddot{r}} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) f(r)\,\hat{r}$$

Reduced mass

$$\boxed{\mu = \frac{m_1 m_2}{m_1 + m_2}}$$

$$\ddot{r} = \frac{m_1 + m_2}{m_1 m_2} f(r)\,\hat{r}$$

$$\boxed{f(r)\cdot\hat{r} = \mu \ddot{r}}$$

converted 2-body problem

↓

1-body problem

for single body,

central force $= f(r)\cdot\hat{r}$

acceleration $= \ddot{r}$

Reduced mass $= \mu$

↳ But for 3 or more Bodies, we cannot go via this concept. this concept is valid only for two bodies.

- $\vec{r} = \vec{r}_1 - \vec{r}_2$
- $R = \dfrac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$
- $\vec{r}_2 = \vec{r}_1 - \vec{r}$

$$R = \frac{m_1 r_1 + m_2 (r_1 - r)}{m_1 + m_2}$$

$$= \frac{(m_1 + m_2) r_1}{m_1 + m_2} - \frac{m_2 r}{m_1 + m_2}$$

$$\Rightarrow R = r_1 - \frac{m_2 r}{m_1 + m_2}$$

$$\boxed{r_1 = R + \frac{m_2 r}{m_1 + m_2}}$$

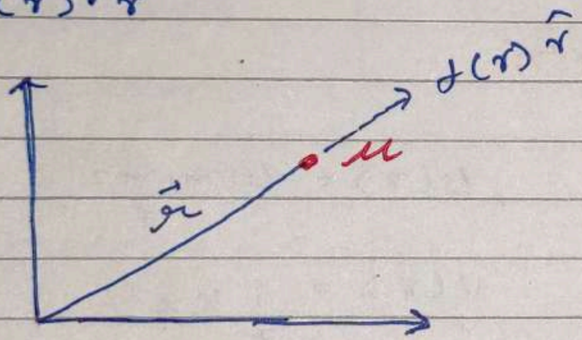
General properties of central force motion:-

↳ $f(r) \cdot \hat{r}$

$\vec{L} = \vec{r} \times \vec{p}$

↳ Angular momentum

↳



$\vec{\tau} = \vec{r} \times \vec{F} = 0$

↳ torque

↳ there is only central force acting on mass μ, so the net torque on mass μ will be zero.

$$\tau = (\vec{r} \times f(r) \cdot \hat{r}) = 0 \ \{(\hat{i} \times i = 0)\}$$

↳ If net torque is zero, then Angular momentum is ~~also~~ conserved or it will be constant. $(L_i = L_f)$

↳ $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times \mu \vec{v} = r \mu v_\theta$

$(\text{for } v_r \Rightarrow \hat{r} \times \hat{r} = 0)$

(9,16,17)

↳ In polar co-ordinate system, the acceleration will be

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

- $\mu(\ddot{r} - r\dot{\theta}^2)\hat{r} = f(r)\hat{r}$ (Radial)
- $\mu(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0$ (Tangential)

total energy = K.E + P.E

$$= \frac{1}{2}\mu v^2 + U(r)$$

↳ $\vec{r} = r\hat{r}$

$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$ $\boxed{v_\theta = r\dot{\theta}}$

$\vec{L} = \vec{r} \times \vec{p} = r\mu v_\theta = \mu r^2\dot{\theta}$

for Ex:- $f(r) = -\frac{Gm_1m_2}{r^2}$, $U(r) = \frac{Gm_1m_2}{r}$

$f(r) = -kx$ $U(r) = \frac{1}{2}kx^2$

$E = \frac{1}{2}\mu v^2 + U(r) = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) + U(r)$

$$= \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\mu r^2 \cdot \frac{L^2}{\mu^2 r^4} + U(r)$$

$$E(r) = \frac{1}{2}\mu\dot{r}^2 + \underbrace{\frac{1}{2}\frac{L^2}{\mu r^2} + U(r)}_{V_{eff}}$$

$$V_{eff} = \frac{L^2}{2\mu r^2} + U(r)$$

$$\frac{dr}{dt} = \sqrt{\frac{2(E - V_{eff})}{\mu}}$$

$\Downarrow$

$$\int_{r_0}^{r} \frac{dr}{\sqrt{\frac{2}{\mu}(E - V_{eff})}} = t - t_0$$

$$\dot{\theta} = \frac{l}{\mu r^2}$$

$$\frac{d\theta}{dt} = \frac{l}{\mu r^2}$$

$$\theta - \theta_0 = \int_{t_0}^{t} \frac{l}{\mu r^2} \cdot dt$$

$$\hookrightarrow \frac{dr}{d\theta} = \frac{dr}{dt} \cdot \frac{1}{d\theta/dt} = \sqrt{\frac{2}{\mu}(E - V_{eff})} \times \frac{1}{l/\mu r^2}$$

$\Downarrow$

$\left\{ r(t), \theta(t), \frac{dr}{d\theta} \right\} \rightarrow$ calculated.

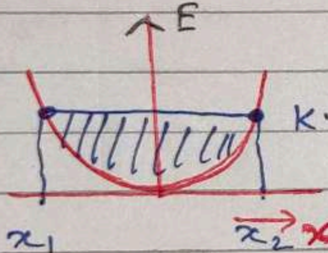
$\vec{F} = -Kx$

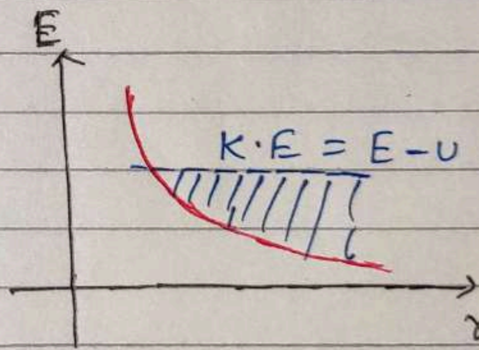
$U = \frac{1}{2}Kx^2$

(Bounded)

$\vec{F} = -A/r^2$ (unbounded)

$U = \frac{A}{r}$





If x_1 and x_2 are position where $E = U$, then body can't cross x_1 & x_2, because after that KE will become (-ve), which is not possible.

$\Downarrow$

System is Bounded. It can move from x_1 to x_2 only.

$\hookrightarrow$ unbounded system

Body can travel anywhere, there is no boundation.

$$U_{eff} = \frac{l^2}{2\mu r^2} + U(r) \qquad f(r) = \frac{Gm_1m_2}{r^2}, \quad U(r) = -\frac{Gm_1m_2}{r}$$