# Experiment 1

**Objective:** Time and Frequency response of RC circuits.

**Theory:**

All electrical or electronic circuits or systems suffer from some form of "time-delay" between its input and output, when a signal or voltage, either continuous (DC) or alternating (AC) is firstly applied to it. This delay is generally known as the **time delay** or **Time Constant** of the circuit and it is the time response of the circuit when a step voltage or signal is firstly applied. The resultant time constant of any electronic circuit or system will mainly depend upon the reactive components either capacitive or inductive connected to it and is a measurement of the response time with units of τ.

When an increasing DC voltage is applied to a discharged Capacitor, the capacitor draws a charging current and "charges up", and when the voltage is reduced, the capacitor discharges in the opposite direction. Because capacitors are able to store electrical energy they act like small batteries and can store or release the energy as required.

The charge on the plates of the capacitor is given as: $Q = CV$. This charging (storage) and discharging (release) of a capacitors energy is never instant but takes a certain amount of time to occur with the time taken for the capacitor to charge or discharge to within a certain percentage of its maximum supply value (in between 10% to 90%) being known as its **Time Constant** (τ).

If a resistor is connected in series with the capacitor forming an RC circuit, the capacitor will charge up gradually through the resistor until the voltage across the capacitor reaches that of the supply voltage. The time also called the transient response, required for the capacitor to fully charge is equivalent to about **5 time constants** or 5τ.

This transient response time τ, is measured in terms of $\tau = R \times C$, in seconds, where R is the value of the resistor in ohms and C is the value of the capacitor in Farads. This then forms the basis of an RC charging circuit were 5τ can also be thought of as "5RC".

## RC Charging Circuit

The Figure 1 below shows a capacitor, ($C$) in series with a resistor, ($R$) forming a **RC Charging Circuit** connected across a DC battery supply ($V_s$) via a mechanical switch. When the switch is closed, the capacitor will gradually charge up through the resistor until the voltage across it reaches the supply voltage of the battery. The manner in which the capacitor charges up is also shown in Figure 1.

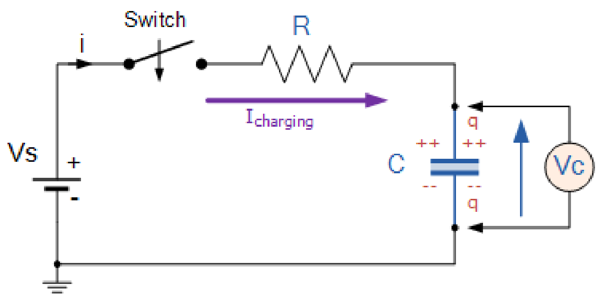


Fig.1

Let us assume above, that the capacitor, $C$ is fully "discharged" and the switch (S) is fully open. These are the initial conditions of the circuit, then $t = 0$, $i = 0$ and $q = 0$. When the switch is closed the time begins at $t = 0$ and current begins to flow into the capacitor via the resistor.

Since the initial voltage across the capacitor is zero, ($V_c = 0$) the capacitor appears to be a short circuit to the external circuit and the maximum current flows through the circuit restricted only by the resistor $R$. Then by using Kirchhoff's voltage law (KVL), the voltage drops around the circuit are given as:

$$V_s = R\, i(t) + V_c\,(t)$$

The capacitor now starts to charge up as shown in Figure 2, with the rise in the RC charging curve steeper at the beginning because the charging rate is fastest at the start and then tapers off as the capacitor takes on additional charge at a slower rate.

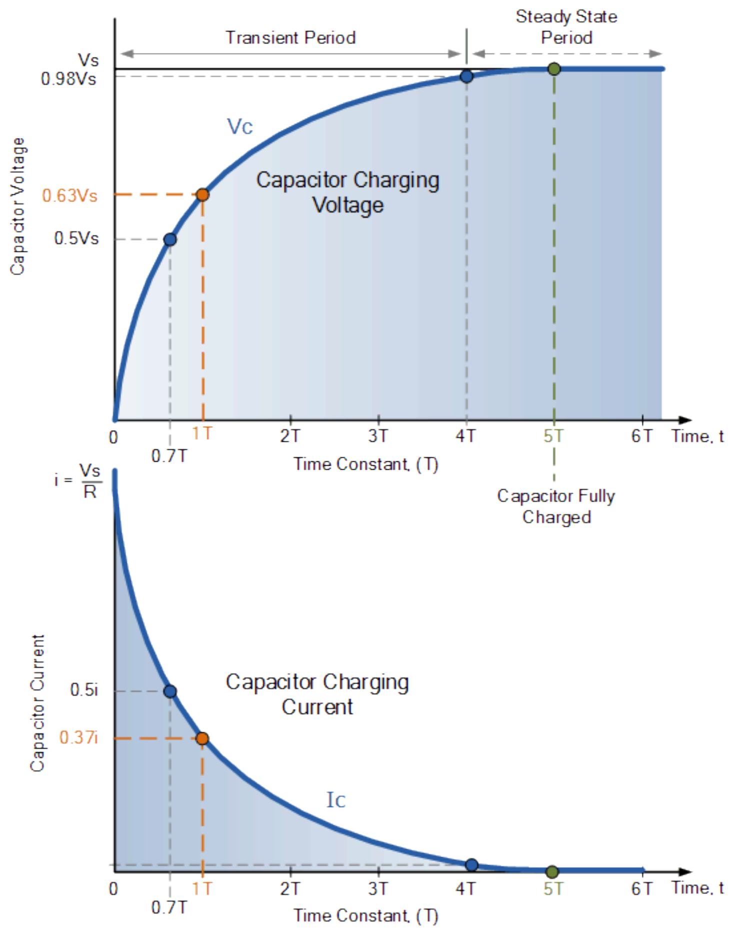


Fig.2

As the capacitor charges up, the potential difference across its plates slowly increases with the actual time taken for the charge on the capacitor to reach 63% of its maximum possible voltage, in our curve 0.63Vs being known as one Time Constant, ($\tau$).

This 0.63Vs voltage point is given the abbreviation of $1\tau$, (one time constant).

The capacitor continues charging up and the voltage difference between $V_s$ and $V_c$ reduces, so to does the circuit current, $i$. Then at its final condition greater than five time constants ($5\tau$) when the capacitor is said to be fully charged, $t = \infty$, $i = 0$, $q = Q = CV$. Then at infinity the current diminishes to zero, the capacitor acts like an open circuit condition therefore, the voltage drop is entirely across the capacitor.

Since voltage $V$ is related to charge on a capacitor given by the equation, $V_c = Q/C$, the voltage across the value of the voltage across the capacitor ($V_c$) at any instant in time during the charging period is given as:

$$V_c(t) = V_s\,[1\text{-exp}\,(-t/RC)]$$

After a period equivalent to 4 time constants, ($4\tau$) the capacitor in this $RC$ charging circuit is virtually fully charged and the voltage across the capacitor is now approx. 98% of its maximum value, $0.98V_s$. The time period taken for the capacitor to reach this $4\tau$ point is known as the **Transient Period**.

After a time of 5T the capacitor is now fully charged and the voltage across the capacitor, ($V_c$) is equal to the supply voltage, ($V_s$). As the capacitor is fully charged no more current flows in the circuit. The time period after this $5\tau$ point is known as the **Steady State Period**.

**RC Integrator Circuit:**

For a passive RC integrator circuit, the input is connected to a resistance while the output voltage is taken from across a capacitor. The capacitor charges up when the input is high and discharges when the input is low. The RC network is nothing more than a resistor in series with a capacitor, that is a fixed resistance in series with a capacitor that has a frequency dependent reactance which decreases as the frequency across its plates increases. Thus at low frequencies the reactance, $X$c of the capacitor is high while at high frequencies its reactance is low due to the standard capacitive reactance formula of $X\text{c} = 1/(2\pi f\text{C})$, and we saw this effect as Passive Low Pass Filters.

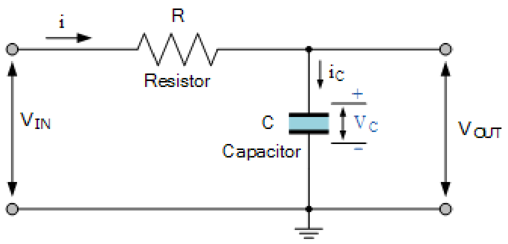


Fig. 3

If the input signal is a sine wave, an **RC integrator** will simply act as a simple low pass filter (LPF) with a cut-off or corner frequency that corresponds to the RC time constant ($\tau$) of the series network and whose output is reduced above this cut-off frequency point. Thus when fed with a pure sine wave an RC integrator acts as a passive low pass filter.

For an RC integrator circuit shown in Figure 3, the input signal is applied to the resistance with the output taken across the capacitor, then $V_{OUT}$ equals $V_C$. As the capacitor is a frequency dependent element, the amount of charge that is established across the plates is equal to the time domain integral of the current. That is, it takes a certain amount of time for the capacitor to fully charge as the capacitor cannot charge instantaneously only charge exponentially.

$$V_{OUT} = V_C = Q/C = 1/C \int i\, dt = 1/C \int V_{IN}/R\, dt = 1/RC \int V_{IN}\, dt$$

$$V_{OUT} = 1/RC \int V_{IN}\, dt$$

So in other words, the output from an RC integrator circuit, which is the voltage across the capacitor is equal to the time integral of the input voltage, $V_{IN}$ weighted by a constant of 1/RC. Where RC represents the time constant, τ.

### Single Pulse RC Integrator:

When a single step voltage pulse is applied to the input of an RC integrator, the capacitor charges up via the resistor in response to the pulse. However, the output is not instant as the voltage across the capacitor cannot change instantaneously but increases exponentially as shown in Figure 4 as the capacitor charges at a rate determined by the RC time constant, τ = RC.

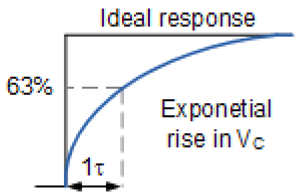


Fig.4

### RC Integrator Circuit Example:

The time constant, τ of the RC integrator circuit shown in Figure 5 is: RC = 100kΩ x 1uF = 100ms.

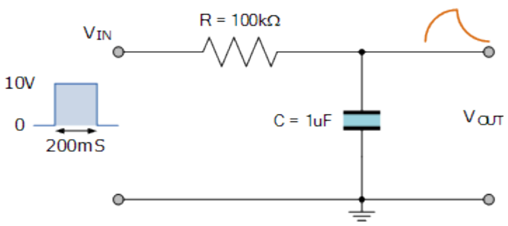


Fig.5

So if we apply a step voltage pulse to the input with a duration of say, two time constants (200ms), then the capacitor will charge to 86.4% of its fully charged value. If this pulse has an amplitude of 10 volts, then this equates to 8.64 volts before the capacitor discharges again back through the resistor to the source as the input pulse returns to zero.

If we assume that the capacitor is allowed to fully discharge in a time of 5 time constants, or 500ms before the arrival of the next input pulse, then the graph of the charging and discharging curves would look something like shown in Figure 6.

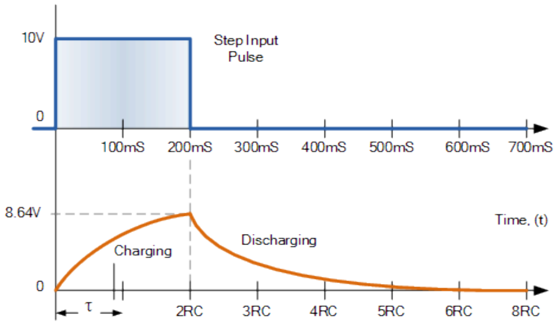


Fig.6

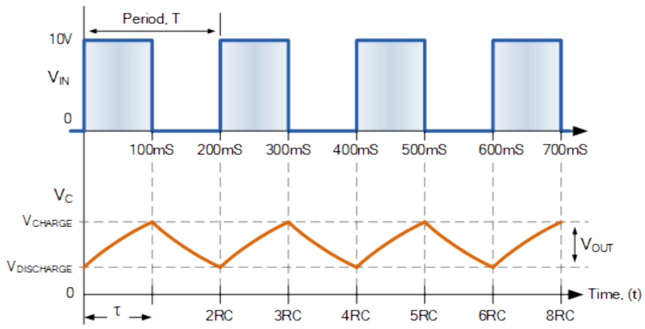


Fig.7

Note that the capacitor is discharging from an initial value of 8.64 volts (2 time constants) and not from the 10 volts input.

We can also see that as the RC time constant is fixed, any variation to the input pulse width will affect the output of the RC integrator circuit. If the pulse width is increased and is equal to or greater than 5RC, then the shape of the output pulse will be similar to that of the input as the output voltage reaches the same value as the input.

If, however, the pulse width is decreased below 5RC, the capacitor will only partially charge and not reach the maximum input voltage resulting in a smaller output voltage because the capacitor cannot charge as much resulting in an output voltage that is proportional to the integral of the input voltage. A typical output is shown in Figure 7. So if we assume an input pulse equal to one time constant, that is 1RC, the capacitor will charge and discharge not between 0 volts and 10 volts but between 63.2% and 38.7% of the voltage across the capacitor at the time of change. Note that these values are determined by the RC time constant.

So for a continuous pulse input, the correct relationship between the periodic time of the input and the RC time constant of the circuit, integration of the input will take place producing a sort of ramp up, and then a ramp down output. But for the circuit to function correctly as an integrator, the value of the RC time constant has to be large compared to the inputs periodic time. That is RC $\gg$ T, usually 10 times greater.

This means that the magnitude of the output voltage (which was proportional to 1/RC) will be very small between its high and low voltages severely attenuating the output voltage. This is because the capacitor has much less time to charge and discharge between pulses but the average output DC voltage will increase towards one half magnitude of the input and in our pulse example above, this will be 5 volts (10/2).

**RC Differentiator Circuit:**

For a passive RC differentiator circuit, the input is connected to a capacitor while the output voltage is taken from across a resistance. A passive RC differentiator is nothing more than a capacitance in series with a resistance that is a frequency dependent device which has reactance in series with a fixed resistance (the opposite to an integrator). Just like the integrator circuit, the output voltage depends on the circuit’s RC time constant and input frequency.

Thus at low input frequencies the reactance, $X_c$ of the capacitor is high blocking any d.c. voltage or slowly varying input signals. While at high input frequencies the capacitors reactance is low allowing rapidly varying pulses to pass directly from the input to the output.

This is because the ratio of the capacitive reactance ($X_c$) to resistance ($R$) is different for different frequencies and the lower the frequency the less output. So for a given time constant, as the frequency of the input pulses increases, the output pulses more and more resemble the input pulses in shape.

If the input signal is a sine wave, an **RC differentiator** will simply act as a simple high pass filter (HPF) with a cut-off or corner frequency that corresponds to the RC time constant ($\tau$) of the series network.

Thus when fed with a pure sine wave an RC differentiator circuit acts as a simple passive high pass filter due to the standard capacitive reactance formula of $X_c = 1/(2\pi f C)$.

But a simple RC network can also be configured to perform differentiation of the input signal. We know that the current through a capacitor is a complex exponential given by: $i_C = C(dV_c/dt)$. The rate at which the capacitor charges (or discharges) is directly proportional to the amount of resistance and capacitance giving the time constant of the circuit. Thus the time constant of a RC differentiator circuit is the time interval that equals the product of R and C. Consider the basic RC series circuit in Figure 8.

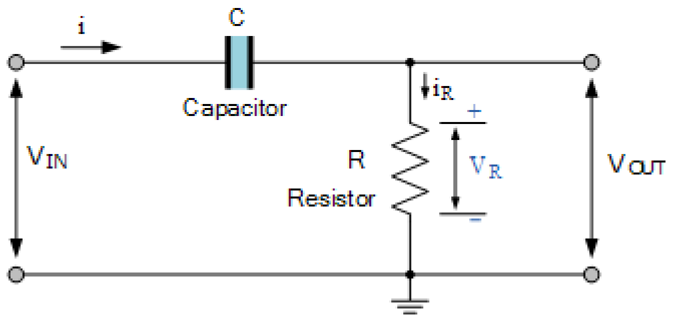


Fig. 8

The input signal is applied to one side of the capacitor with the output taken across the resistor, then $V_{OUT}$ equals $V_R$. As the capacitor is a frequency dependent element, the amount of charge that is established across the plates is equal to the time domain integral of the current. That is it takes a certain amount of time for the capacitor to fully charge as the capacitor cannot charge instantaneously only charge exponentially.

As $V_{OUT}$ equals $V_R$ where $V_R$ according to ohms law is equal too: $i_R$ x $R$. The current that flows through the capacitor must also flow through the resistance as they are both connected together in series. Thus:

$$V_{OUT} = V_R = R \text{ x } i_R = R \text{ x } i_C = R \text{ x } C(dV_{IN}/dt)$$

$$V_{OUT} = RC\ (dV_{IN}/dt)$$

Then we can see that the output voltage, $V_{OUT}$ is the derivative of the input voltage, $V_{IN}$ which is weighted by the constant of RC.

As we saw in case of RC Integrators that when a single step voltage pulse is applied to the input of an RC integrator, the output becomes a saw-tooth waveform if the RC time constant is long enough. The RC differentiator will also change the input waveform but in a different way to the integrator.

**Single Pulse RC Differentiator**

When a single step voltage pulse is firstly applied to the input of an RC differentiator, the capacitor “appears” initially as a short circuit to the fast changing signal. This is because the slope dv/dt of the positive-going edge of a square wave is very large (ideally infinite), thus at the instant the signal appears, all the input voltage passes through to the output appearing across the resistor.

After the initial positive-going edge of the input signal has passed and the peak value of the input is constant, the capacitor starts to charge up in its normal way via the resistor in response to the input pulse at a rate determined by the RC time constant, $\tau = RC$.

As the capacitor charges up, the voltage across the resistor, and thus the output decreases in an exponentially way until the capacitor becomes fully charged after a time constant of 5RC, resulting in zero output across the resistor. Thus the voltage across the fully charged capacitor equals the value of the input pulse as: $V_C = V_{IN}$ and this condition holds true so long as the magnitude of the input pulse does not change.

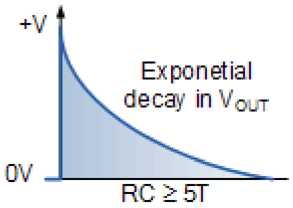


Fig. 9(a)

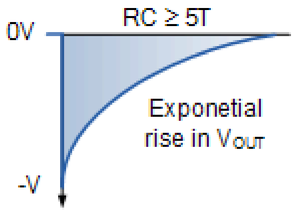


Fig. 9(b)

If now the input pulse changes and returns to zero, the rate of change of the negative-going edge of the pulse pass through the capacitor to the output as the capacitor cannot respond to this high dv/dt change. The result is a negative going spike at the output as shown in Figure 9.

After the initial negative-going edge of the input signal, the capacitor recovers and starts to discharge normally and the output voltage across the resistor, and therefore the output, starts to increases exponentially as the capacitor discharges.

Thus whenever the input signal is changing rapidly, a voltage spike is produced at the output with the polarity of this voltage spike depending on whether the input is changing in a positive or a negative direction, as a positive spike is produced with the positive-going edge of the input signal, and a negative spike produced as a result of the negative-going input signal.

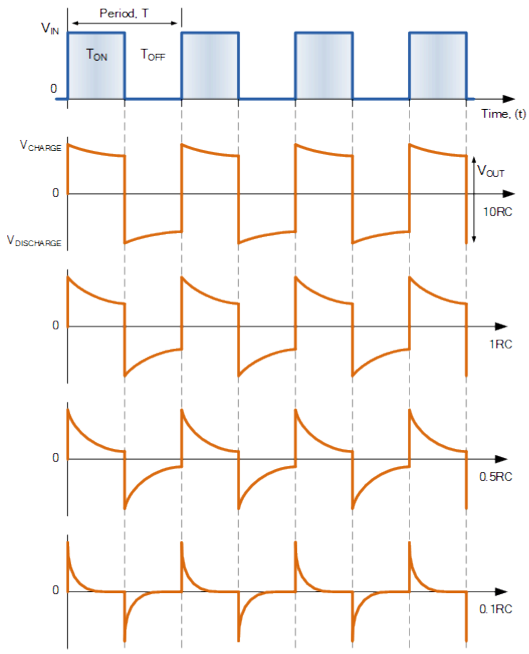


Fig. 10

Thus the RC differentiator output is effectively a graph of the rate of change of the input signal which has no resemblance to the square wave input wave, but consists of narrow positive and negative spikes as the input pulse changes value.

By varying the time period, T of the square wave input pulses with respect to the fixed RC time constant of the series combination, the shape of the output pulses will change as shown in Figure 10.

Then we can see that the shape of the output waveform depends on the ratio of the pulse width to the RC time constant. When RC is much larger (greater than 10RC) than the pulse width the output waveform resembles the square wave of the input signal. When RC is much smaller (less than 0.1RC) than the pulse width, the output waveform takes the form of very sharp and narrow spikes as shown in Figure 10.

So by varying the time constant of the circuit from 10RC to 0.1RC we can produce a range of different wave shapes. Generally, a smaller time constant is always used in RC differentiator circuits to provide good sharp pulses at the output across R. Thus the differential of a square wave pulse (high dv/dt step input) is an infinitesimally short spike resulting in an RC differentiator circuit. Thus, a differentiator circuit is used to produce trigger or spiked typed pulses for timing circuit applications.

Lets assume a square wave waveform has a period, T of 20ms giving a pulse width of 10ms (20ms divided by 2). For the spike to discharge down to 37% of its initial value, the pulse width must equal the RC time constant, that is RC = 10ms. If we choose a value for the capacitor, C of 1uF, then R equals 10kΩ.

For the output to resemble the input, we need RC to be ten times (10RC) the value of the pulse width, so for a capacitor value of say, 1uF, this would give a resistor value of: 100kΩ. Likewise, for the output to resemble a sharp pulse, we need RC to be one tenth (0.1RC) of the pulse width, so for the same capacitor value of 1uF, this would give a resistor value of: 1kΩ, and so on.

**Experimental Procedure:**

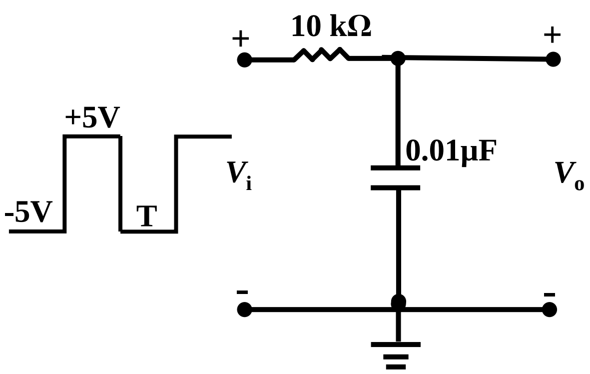


Fig. 11 RC Integrator Circuit

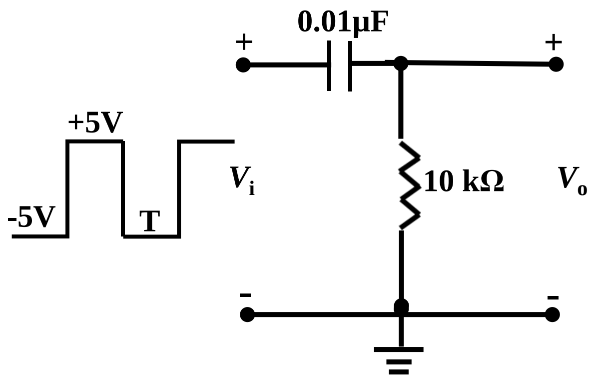


Fig. 12 RC Differentiator Circuit

**A. Transient Response of RC Circuits**

**A.1 RC Integrator Circuit**

Wire the circuit of Figure 11. Connect signal from the OUTPUT socket of the function generator (FG) to the RC circuit, and also to the CH-1 input of the cathode ray oscilloscope (CRO). Choose square wave signal and adjust the amplitude control to obtain a waveform going from −5 V to +5 V. Connect the output of the RC circuit to CH-2 input of the CRO. Be sure to choose the DC mode for both CH-1and CH-2 inputs so as to observe the dc levels of the signals. Use CRO to set the frequency of FG.

(i) **Time response when time period of the input signal << time constant τ=RC:** Observe and sketch $V_i$ and $V_o$ with respect to time. Note down the salient features of $V_o$.

(ii) **Time response when time period of the input signal ≈ time constant τ=RC:** Observe and sketch $V_i$ and $V_o$ with respect to time. Note down the salient features of $V_o$. Choose any two convenient points on the rising and falling parts of $V_o$ and measure the corresponding voltages and the time intervals. From these readings, obtain the time constant τ of the circuit. Compare the result with that obtained using the values of the components (R and C) used in the circuit.

(iii) **Time response when time period of the input signal >> time constant τ=RC :** Observe and sketch $V_i$ and $V_o$ with respect to time.

**A.2 RC Differentiator Circuit**

Wire the circuit of Figure 12. As in the case of the RC integrator circuit, obtain time response of this circuit for the following three cases. Sketch $V_i$ and $V_o$ each case.

(i) **Step response when time period of the input signal >> time constant τ=RC**.

(ii) **Step response when time period of the input signal >> time constant τ=RC**.

(iii) **Step response when time period of the input signal >> time constant τ=RC**.

**B. Steady-State Response of RC Circuit**

1. Connect up the circuit in Figure 13 (this is also called an RC low pass filter).
2. Set up the CRO to display on both channels (I and II) and connect the points indicated in Figure 13 to the respective channels.
3. Apply a sine wave from the MAIN output of the FG to the input point of the circuit. Ensure that input coupling mode for both channels is ac (both ac/dc coupling buttons pressed OUT). Let the sine wave be 2V peak-to-peak with zero dc offset.

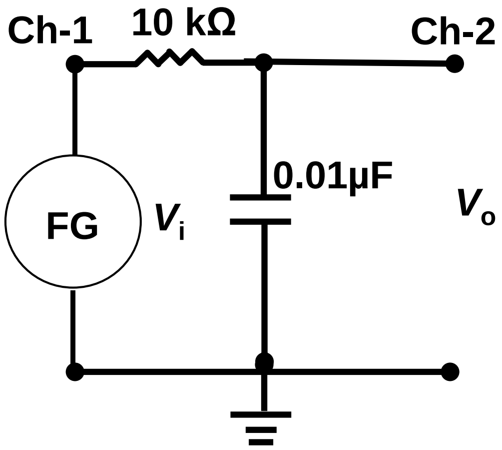


Fig. 13

4. At frequencies of 200Hz, 500Hz, 1kHz, 2kHz, 5kHz, 10kHz, 20kHz, 50kHz, 100kHz, 200kHz, note the amplitudes of $V_i$ and $V_o$. In the range 1kHz to 2kHz, take readings every 100Hz.
5. From the table of voltage amplitudes obtained in step 4 above, calculate the logarithmic gain $A_v = 20 \log[V_o / V_i]$ and plot against frequency on semilog graph paper.

Name:……………………………………………..Roll No…………………………

**Laboratory Report (to be submitted on the same day)**

**Objective:** Study of Time and Frequency response of RC circuit.

**1. Circuit Diagram:**

| RC Integrator Circuit | RC Differentiator Circuit |
|---|---|
| | |

**2.** Chosen R =……………………… , C =………………….. , τ (theoretical) =……………………

**3. Time Response:** Paste the traced response for a given square wave of ±5V.

| RC Integrator Circuit | | |
|---|---|---|
| **T << τ** | **T ≈ τ** | **T >> τ** |
| | | |
| RC Differentiator Circuit | | |
| **T << τ** | **T ≈ τ** | **T >> τ** |
| | | |

**4.** Frequency Response of low pass filter:

| Frequency (in Hz) | Input Voltage, $V_i$ (in V) | Output Voltage, $V_o$ (in V) | Voltage Gain (in dB) |
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5. **Plot Gain (in dB) vs Frequency:** (Attach the plot drawn on semi-log graph paper)
6. **Results:** Discuss the results obtained in the experiment.