Indian Institute of Information Technology Vadodara
MA 101: Introduction to Discrete Mathematics
Tutorial 7

1. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes a) are there in total?
   b) contain exactly two heads?
   c) contain at most three tails?
   d) contain the same number of heads and tails?

2. How many bit strings of length 10 either begin with 3 zeros or end with 3 ones?

3. How many ways are there to select 12 countries in the United Nations to serve on a security council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries?

4. What is the minimum number of students, each of whom comes from one of the 35 states, who must be enrolled in a university to gurantee that there are at least 100 students who come from same state.

5. A bowl contains 10 red balls and 10 white balls. How many balls must a woman select to be sure of having at least three balls of same colour.

6. Give a formula for the coefficient of x^k in the expansion of $(x + 1/x)^{100}$, where $k \in \mathbb{N}$.

7. A circular r-permutation of n people is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.
   a) How many circular n-permutation of n people are possible, where n = 3, 4 and in general?
   b) How many circular 3-permutation of 5 people are possible?
   c) How many circular r-permutation of n people are possible?

8. How many non-negative integer solutions are there to the equation $x_1+x_2+...+x_5 = 21$ with $x_1, x_2 \geq 1, x_3 = 3, x_4 \geq 2, x_5 \geq 0$?

9. A book publisher has 3000 copies of a discrete mathematics book. How many ways are there to store these books in their three warehouses if the copies of the book are indistinguishable?

10. How many ways are there to distribute six indistinguishable balls into nine distinguishable bins?

11. Find the coefficient of $x^3y^2z^5$ in $(x + y + z)^{10}$.

12. Let n be a positive integer. Show that in any set of n consecutive integers, there is exactly one divisible by n.

13. Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.