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Truth table of $\neg$:
Let p be a proposition.

p	$\neg$ p
T	F
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Find Converse, Inverse, Contrapositive of above statement.

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Check: $p \Rightarrow q$ and $\neg q \Rightarrow \neg p$ are equivalent.

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Similarly prove that $\neg(p \vee q) = \neg p \wedge \neg q$.

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$$\forall x(Q(x)), \text{Domain of } x\text{: } \mathbb{R} \text{ and } Q(x) : x^2 + 1 > 0$$

read as "$Q(x)$ for all values of x in the domain."
Existential Quantifiers:
$P(x)$: There exists a real number x such that $x^2 + 1 = 0$.

$$\exists x(Q(x)), \text{Domain of } x : \mathbb{R} \text{ and } Q(x) : x^2 + 1 = 0$$

(1) For every real number x there exists a real number y such that $x + y = 0$.

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There exists a student in the class who does not own a computer and his all friends do not own a computer.

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Domain of both quantifiers: $\mathbb{R}$

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Rules of Inference: Method to check validity of arguments

"All human beings have two legs."
" All men have two legs."

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" If you attend all classes then you can appear for surprise test."
"You attended all classes."
Therefore,
" You can appear for surprise test."

is a valid argument.

“ If you attend all classes then you can appear for surprise test.”

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Conclusion is not true.

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Valid argument form: Conclusion is always true, if premises are all true and no matter what propositions are substituted for propositional variables.

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$$\begin{array}{c} P \Rightarrow Q \\ Q \\ \therefore P \end{array}$$

Exercise: Verify using truth table.

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7. Premises: $p \vee q$. $\neg p \vee r$.
   $\therefore q \vee r$.

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$s, s \Rightarrow t, \therefore t.$

Rules of Inference for Quantified Statements

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$$\therefore \exists x P(x).$$

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Resolvant of C_1 and $C_2 := Q \vee R$.

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Verify $(C_1 \wedge C_2) \Rightarrow R(C_1, C_2)$ is a Tautology.

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Resolution Principle: Given a set S of clauses, a Resolution of C from S is a finite sequence $C_1, C_2, \ldots, C_k$ of clauses such that each C_i either is a clause in S or a resolvant of clauses preceding C_i and $C_k = C$.
A Resolution of empty clause $\Box$ is called a proof of S.

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If above is a resolution of $\square$, then we get a proof (valid argument).

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$C_1 = P \vee \neg Q$
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$C_3 = \neg(P \vee R) \equiv C_3' = \neg P,\ C_3'' = \neg R$ (Negation of Conclusion)
$C_4 =$ Resolvant of $C_1, C_2 = P \vee R$
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Rules of Inference for propositions and Quantifiers:

$\forall x(P(x) \Rightarrow Q(x))$

$P(a)$ for some element a in the domain.

$\therefore Q(a)$ is a valid argument.

Normal Forms

Propositional logic

Recall:

Literal: variable or negation of a propositional variable.

Disjunction ($\vee$) of literals is called sum/clause.

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$p \wedge \Rightarrow q$ is not a formula.

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$$p \Leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$
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$\therefore$ DNF is not unique.

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minterms of p and q: $p \wedge q, \neg p \wedge q, p \wedge \neg q, \neg p \wedge \neg q$.

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Find PDNF of $p \Rightarrow q$:
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Principal Conjunctive Normal Form (PCNF): product of maxterms only.
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To prove $(p_1 \vee p_2 \vee \cdots \vee p_n) \Rightarrow q$ tautology
we prove $p_i \Rightarrow q$ is tautology, for each i.

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Example: Prove that only pair of consecutive integers not exceeding 100 each of which are perfect powers is (8, 9).
Proof by examining each n, $1 \leq n \leq 100$,
n is a perfect power, then checking $n = 1$ is also a perfect power.

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Proofs of such statement can be given by

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Example (non-constructive proof): Show that there exists irrationals x, y such that x^y is rational.
Take $x = y = \sqrt(2)$. If x^y is irrational then we are done.
If not then take $x = \sqrt(2)^{\sqrt(2)}$ and $y = \sqrt(2)$ to get $x^y = 2$.

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