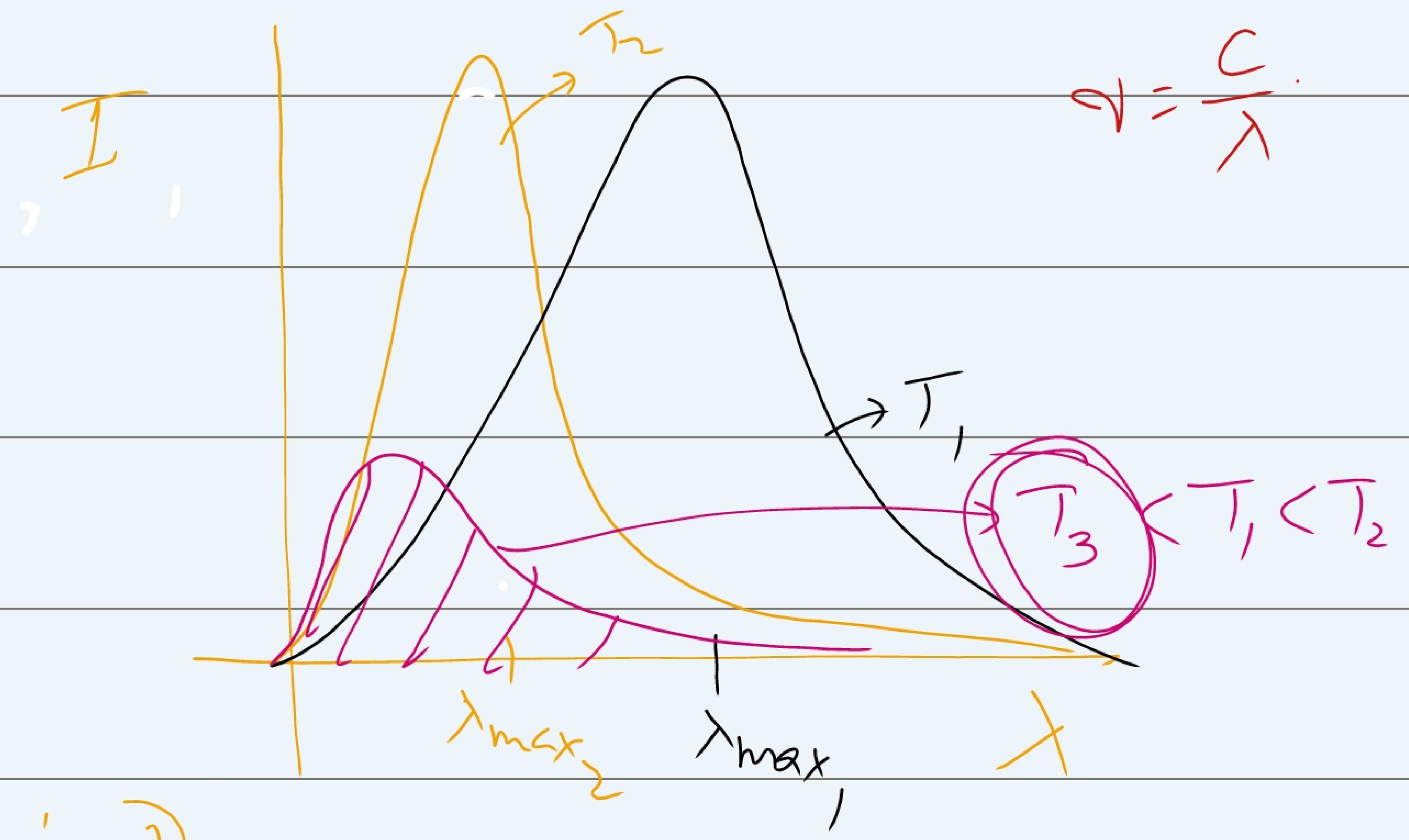
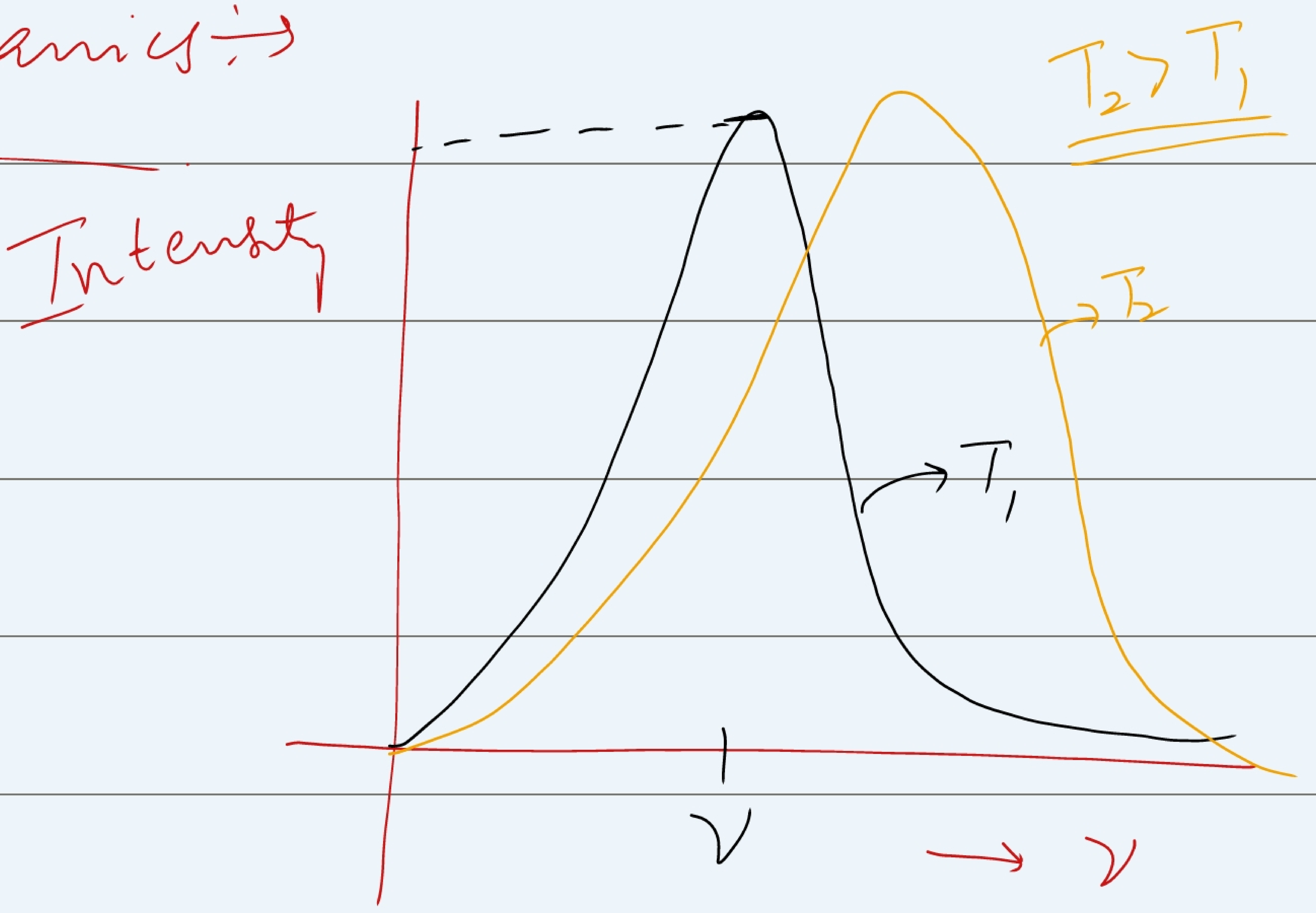
01/11/2023

Failures of classical Mechanics :→



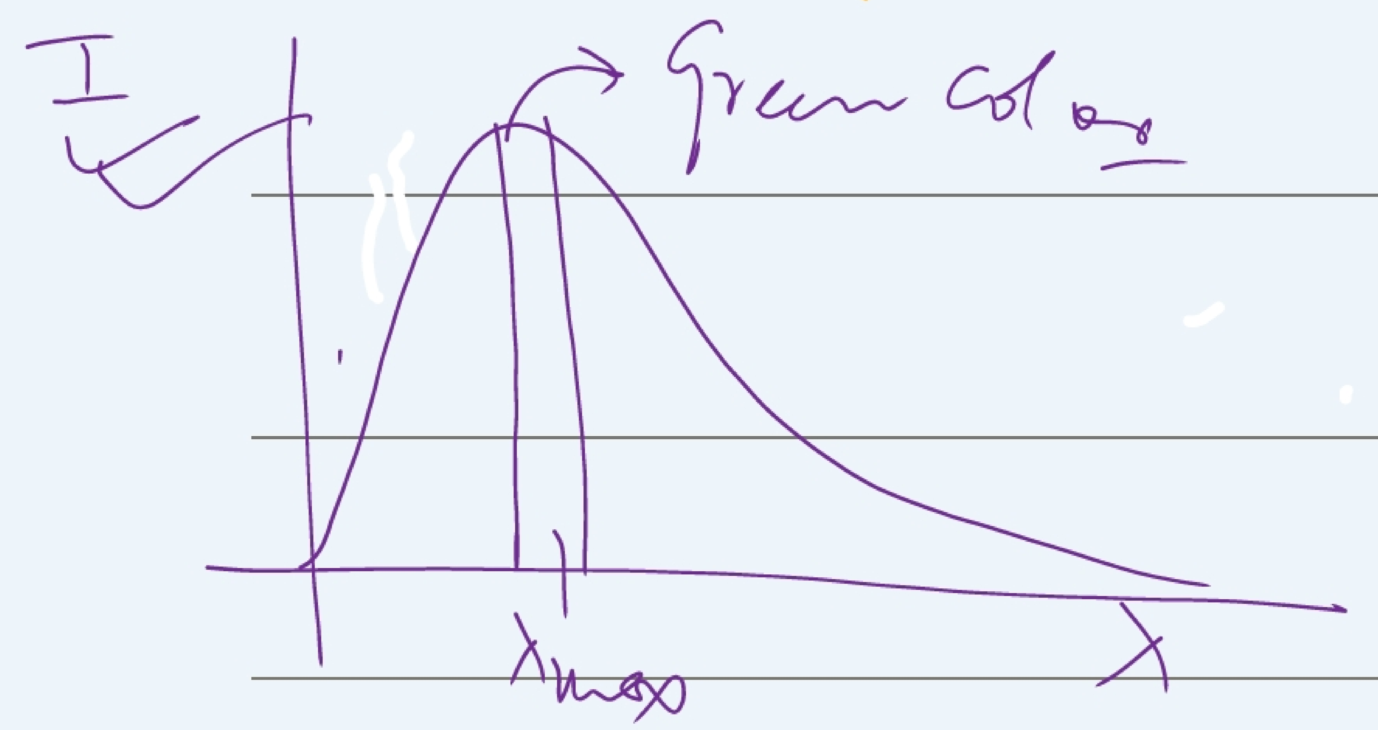


Wien's Displacement

$$\lambda_{max} T = 2.898 \times 10^{-3} \, mK$$

Stefan Boltzmann Law

$$R(T) = \epsilon \sigma T^4$$

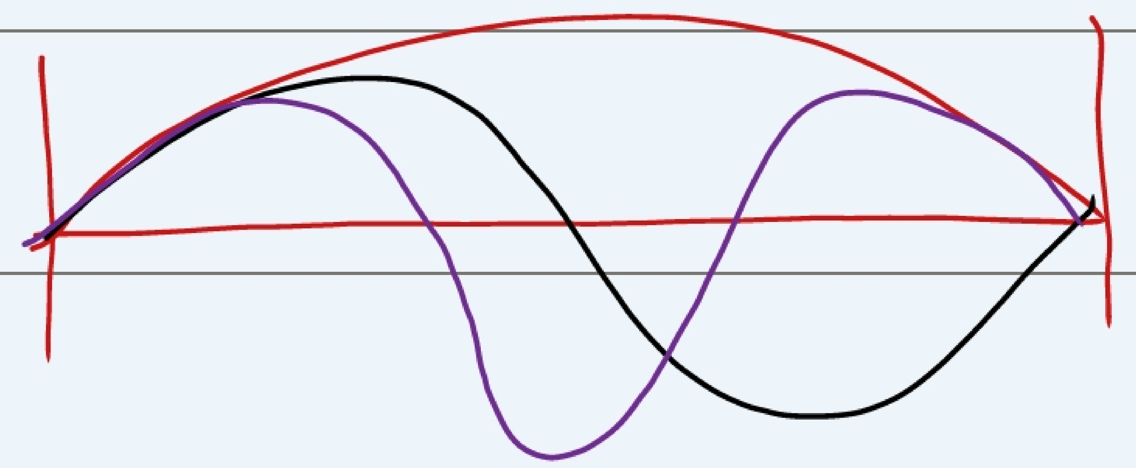


$$\lambda_{max} \cdot 6000K = 2.898 \times 10^{-3} \, mK$$

$$\lambda_{max} \simeq 480 \times 10^{-9} \, m$$

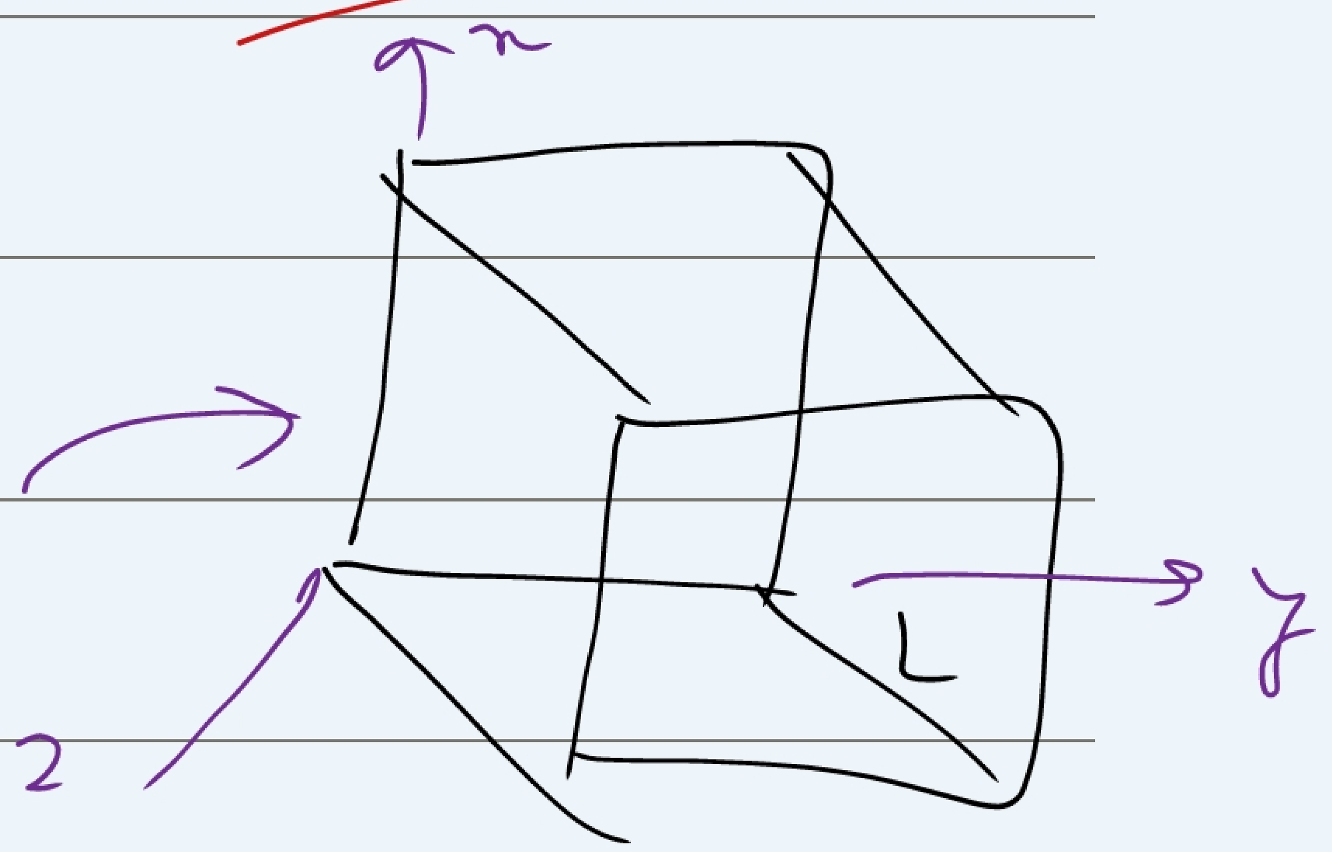
Rayleigh-Jeans Law :→

$$\frac{c}{\nu_n} = \lambda_n = \frac{2L}{j}$$



$$\sqrt{j_x^2 + j_y^2 + j_z^2}^{\,2}$$

$\epsilon = 1$ $\quad L^3$



1 mode $\quad \frac{\lambda}{2} = L$

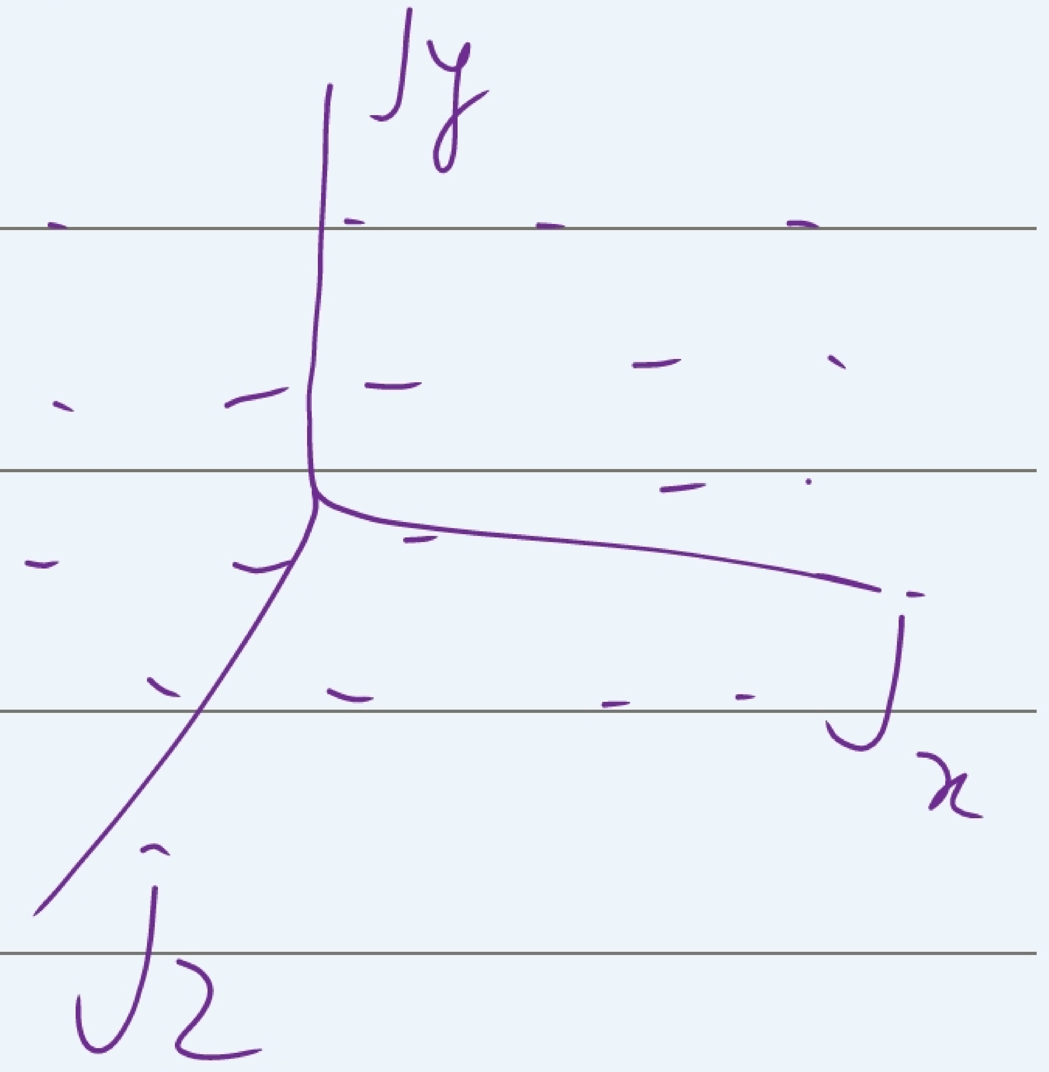
$\lambda = L$

$\frac{3\lambda}{2} = L$

$\Rightarrow \boxed{j_x \lambda = 2L}$, $\quad j = 1, 2, 3, \ldots$

$j_y \lambda = 2L$

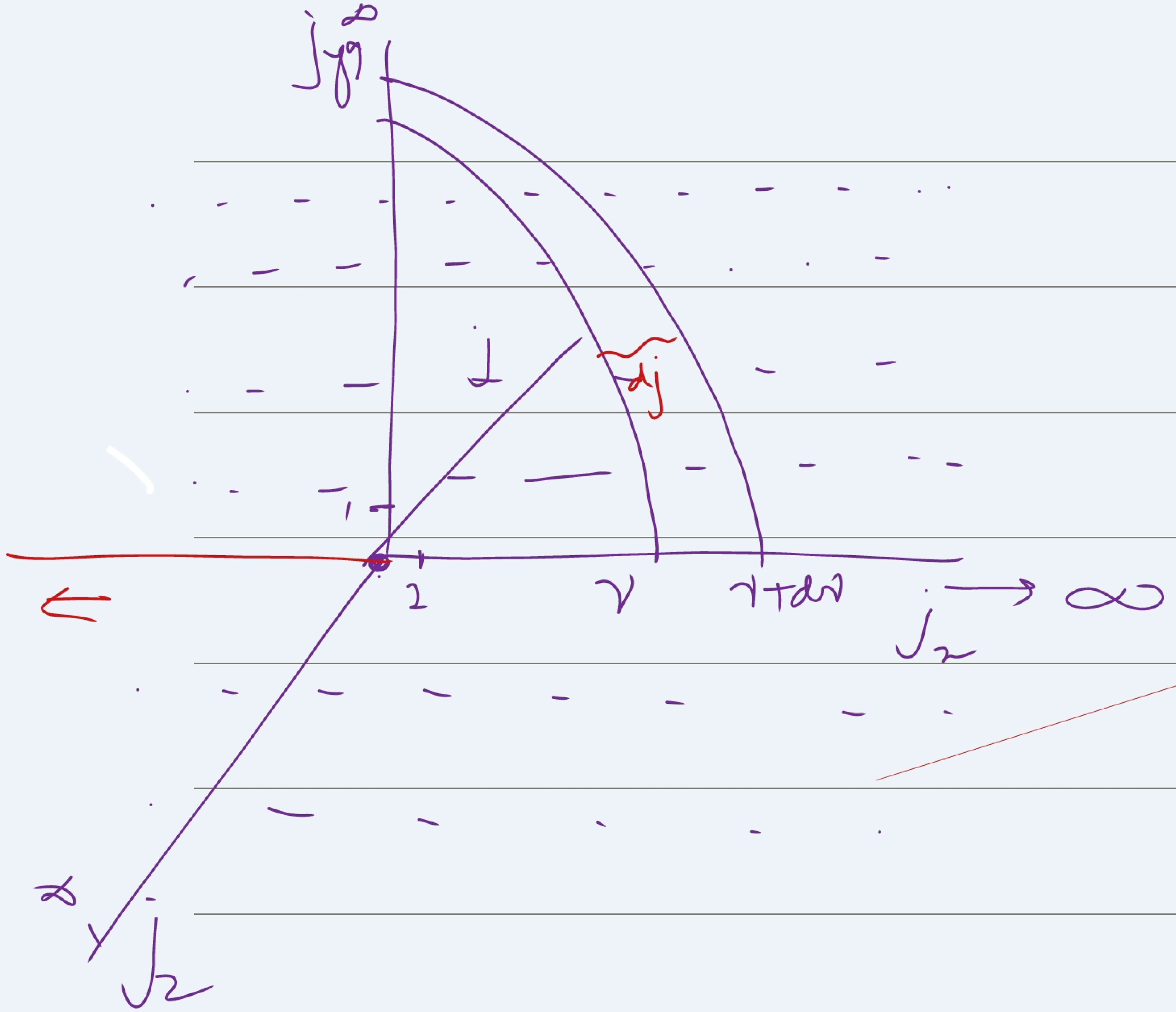
$j_z \lambda = 2L$



For any arbitrary Direction, Standing wave

$$= \sqrt{j_x^2 + j_y^2 + j_z^2}$$

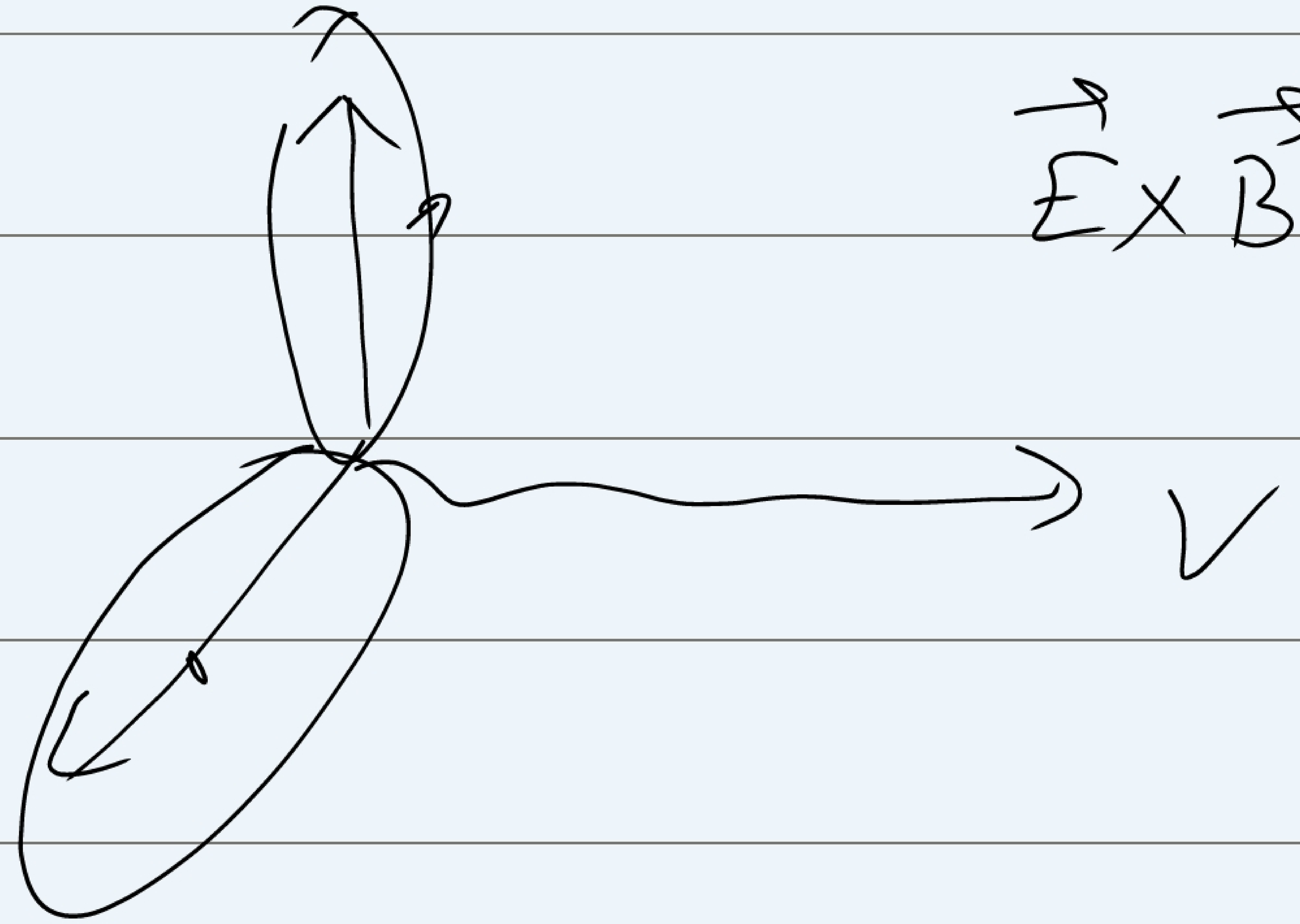
$j_x = \frac{2L}{\lambda}, \; j_y = \frac{2L}{\lambda}, \; j_z = \frac{2L}{\lambda}$



Volume of Standing (No. of Standing) Wave

$g(\nu)\,d\nu$

$= \frac{4\pi j^2\, dj \times \frac{1}{8} \times 2}{} = \pi j^2\, dj$



$\vec{E} \times \vec{B} =$

$$g(\nu)\,d\nu = \pi j^2\,dj \qquad j = \frac{2L}{\lambda} = \frac{2L\nu}{c}$$

$$= \pi\left(\frac{2L\nu}{c}\right)^2 \cdot \left(\frac{2L}{c}\right) d\nu \qquad dj = \frac{2L}{c}\,d\nu$$

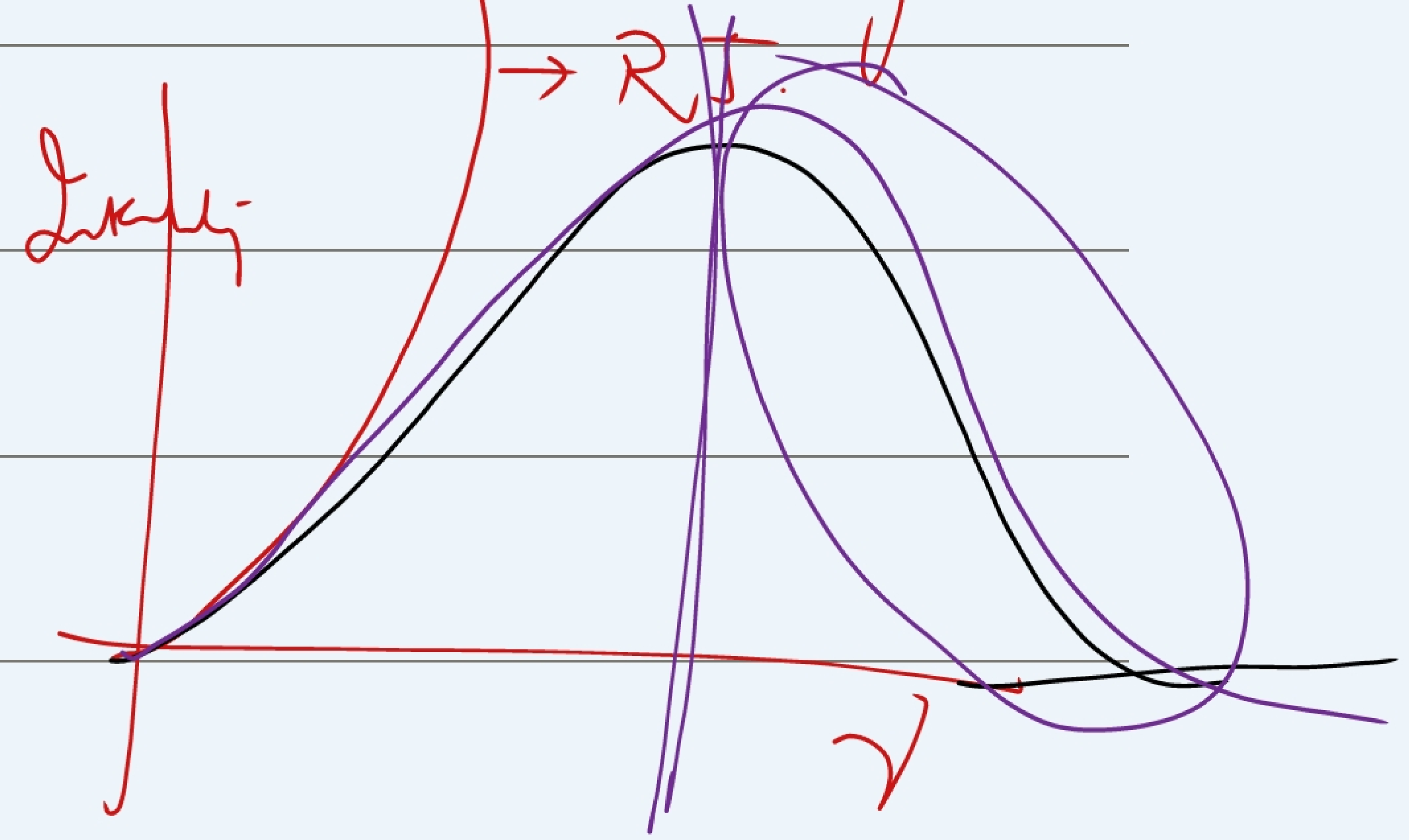
$$g(\nu)\,d\nu = \frac{8\pi \cancel{L^3}}{c^3\,\cancel{L^3}}\,\nu^2\,d\nu = \frac{8\pi}{c^3}\,\nu^2\,d\nu$$ = Density of standing wave formed in the cavity.

No. of standing waves / Volume

$$\frac{h\nu}{e^{h\nu/KT} - 1} = \bar{E} = K_B T$$ (1/2 - 1 degree, 3/2 3 degree)

R.J. Radiation Law

$$\text{Intensity} = \frac{8\pi\nu^2\,d\nu}{c^3} \cdot K_B T$$



Planck's Radiation Law:-

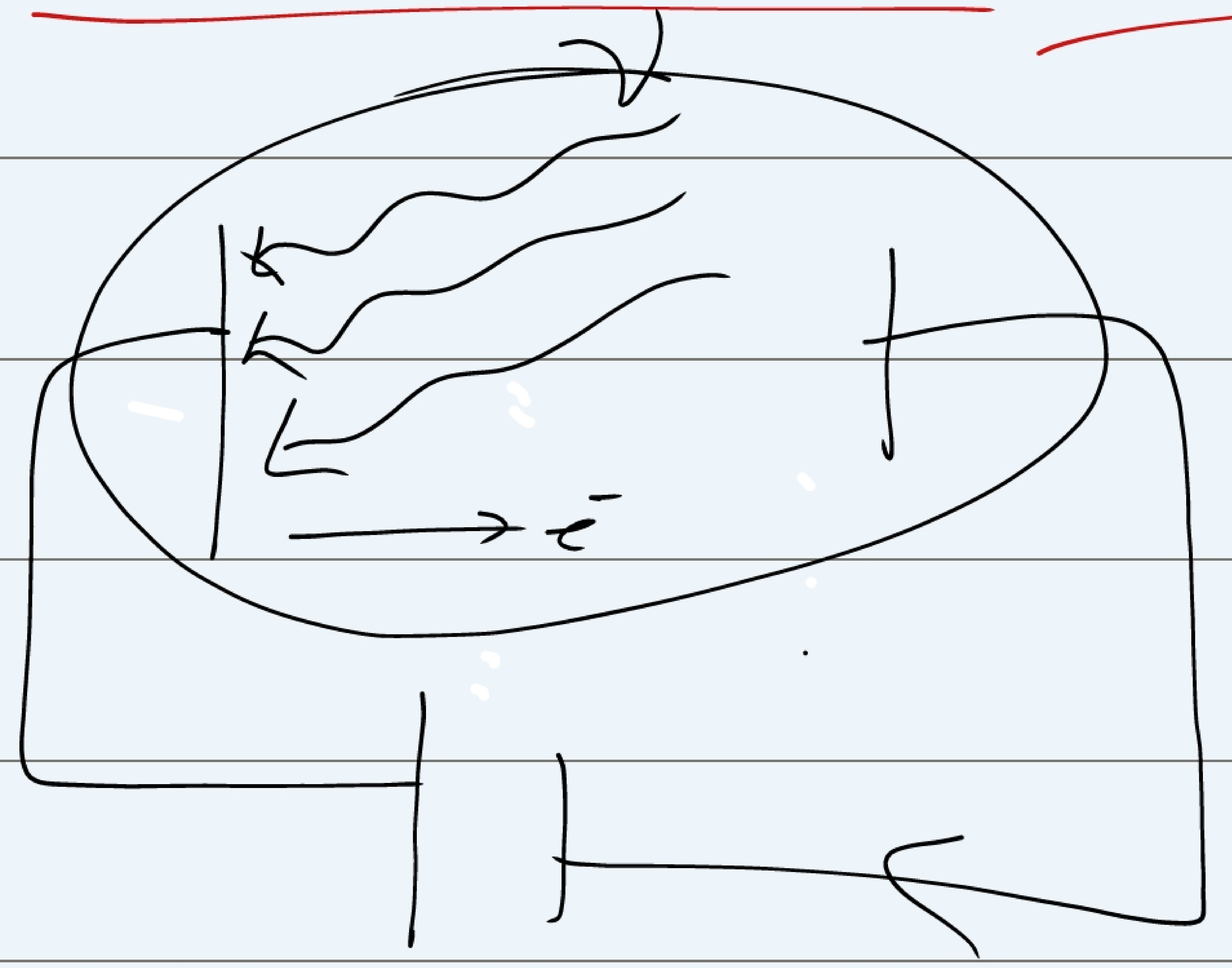
$$I = \left(\frac{8\pi\nu^2 d\nu}{c^3}\right) \cdot \frac{\cancel{h\nu}}{e^{h\nu/KT} - 1} \quad \frac{\cancel{h\nu}}{KT}$$

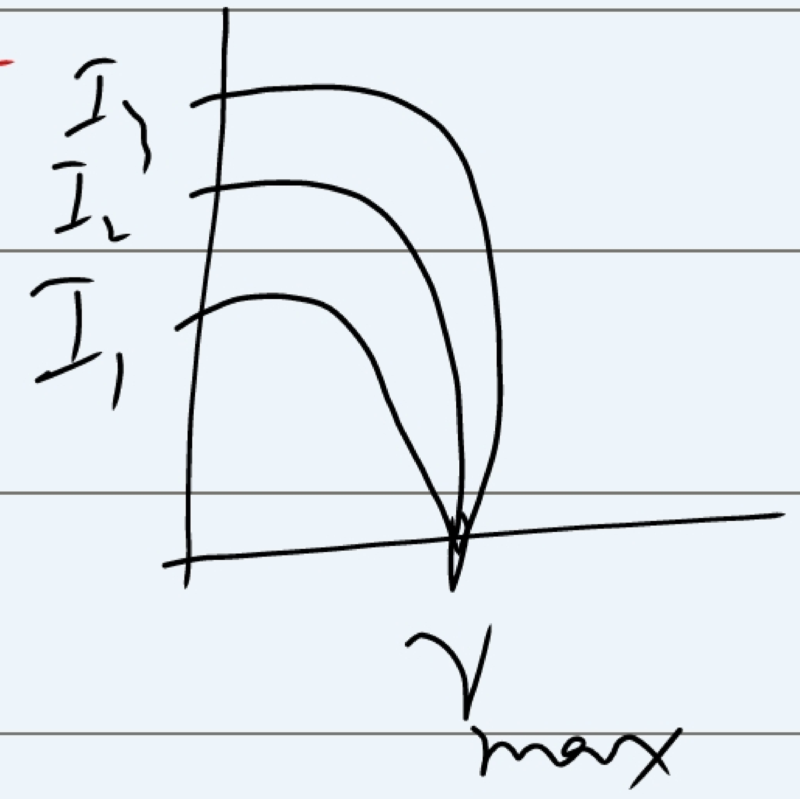
① For large ν. $\frac{h\nu}{KT} \gg 1$, $h\nu \gg KT$, $e^{h\nu/KT} \to \infty$

② For smaller ν, $\frac{h\nu}{KT} \ll 1$, $h\nu \ll KT$, $e^{h\nu/KT} \approx 1 + \frac{h\nu}{KT}$

$e^x = 1 + x + \frac{x^2}{2!} + \ldots$ $\quad e^{h\nu/KT} - 1 \approx \frac{h\nu}{KT}$, $I =$

Photoelectric Effect:-





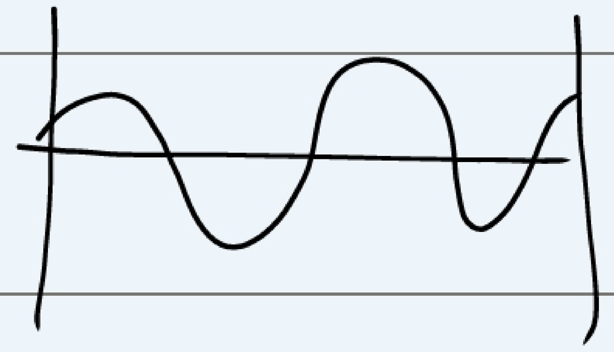
$\varepsilon = k_B T$

$$\boxed{\varepsilon = nh\nu}$$

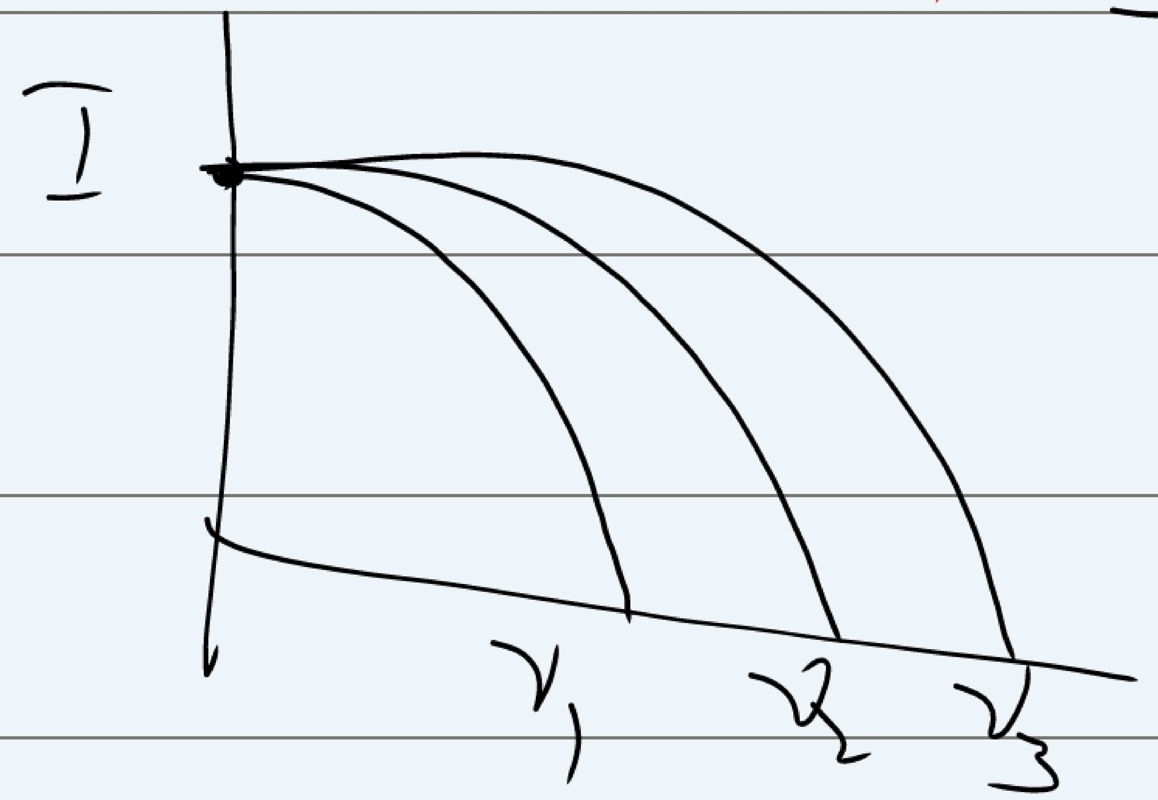
$\varepsilon_1 = h\nu$

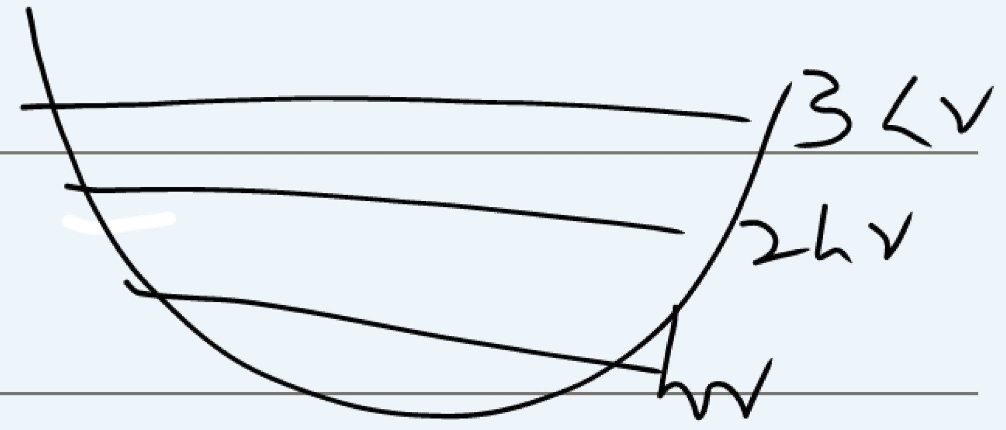
$\varepsilon_2 = 2h\nu$

$\rightarrow \nu$



$$eV_{max} = h\nu_0 + h\nu$$





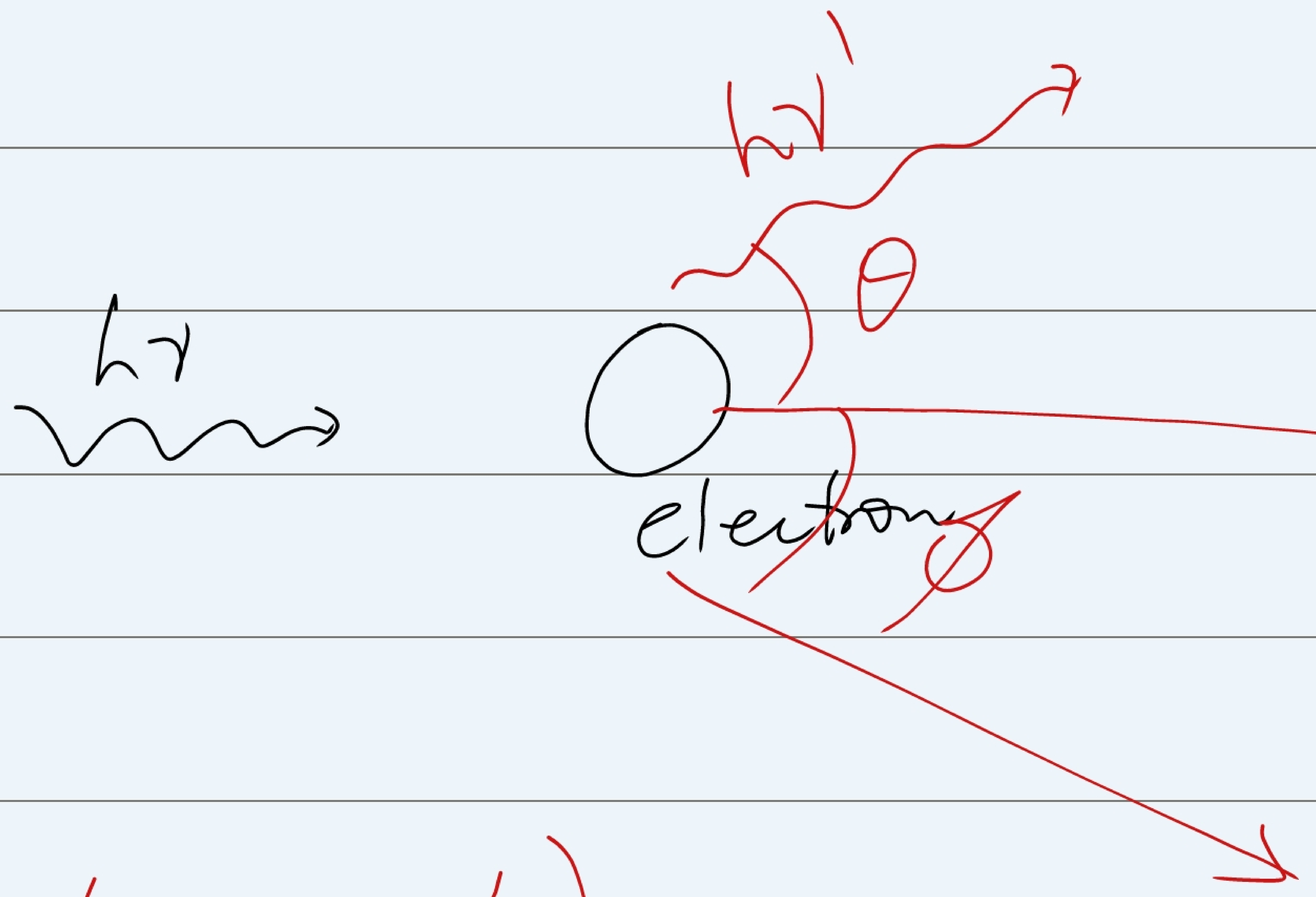
$$E = \frac{1}{2}KA^2$$

② Compton Effect:→

Planck's C

$E = nh\nu$

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

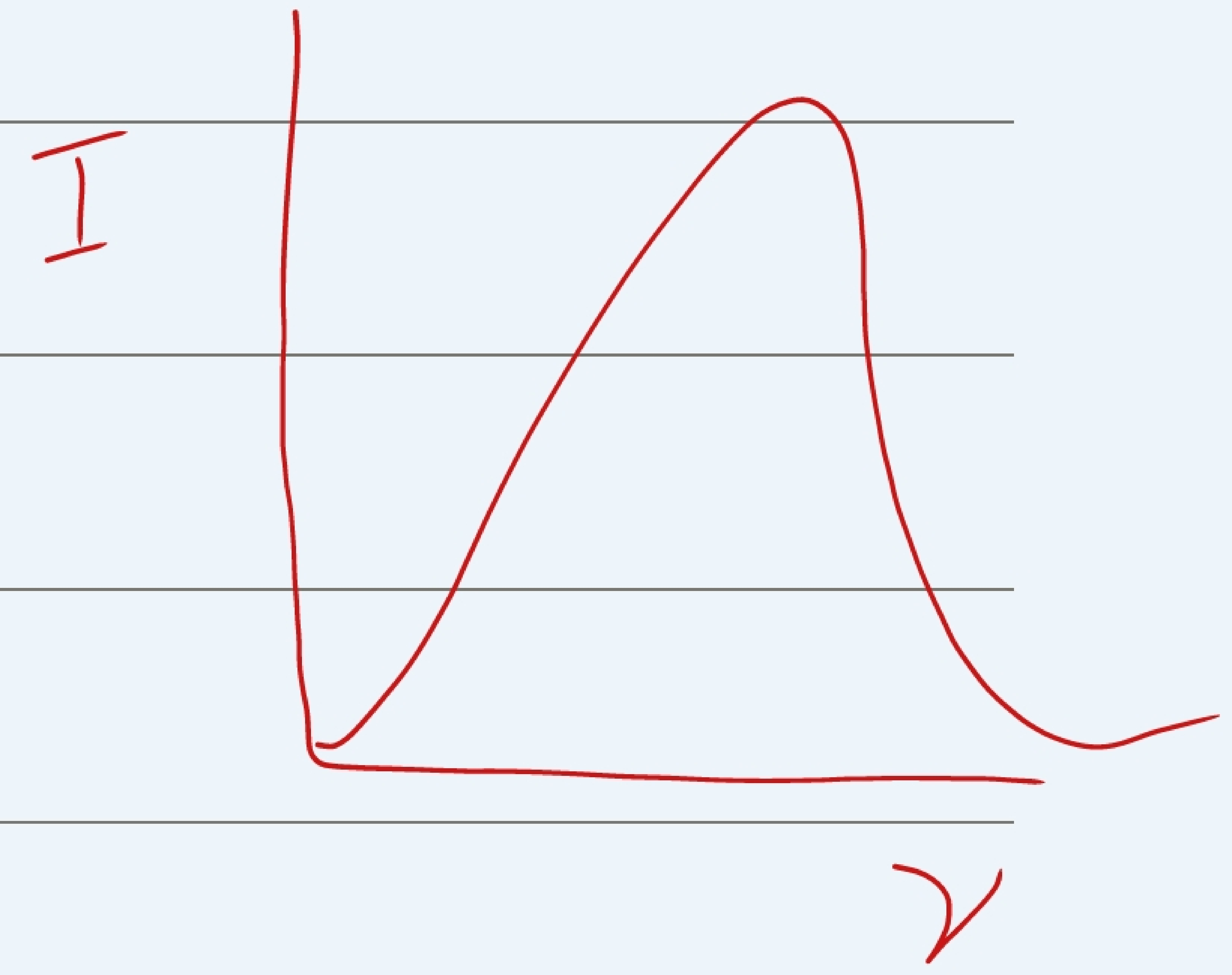


$h(\nu - \nu')$ → Quantized

$$\frac{h}{mc}\left[1 - \cos\theta\right]$$

$$\boxed{\frac{8\pi\nu^2}{c^3}}$$

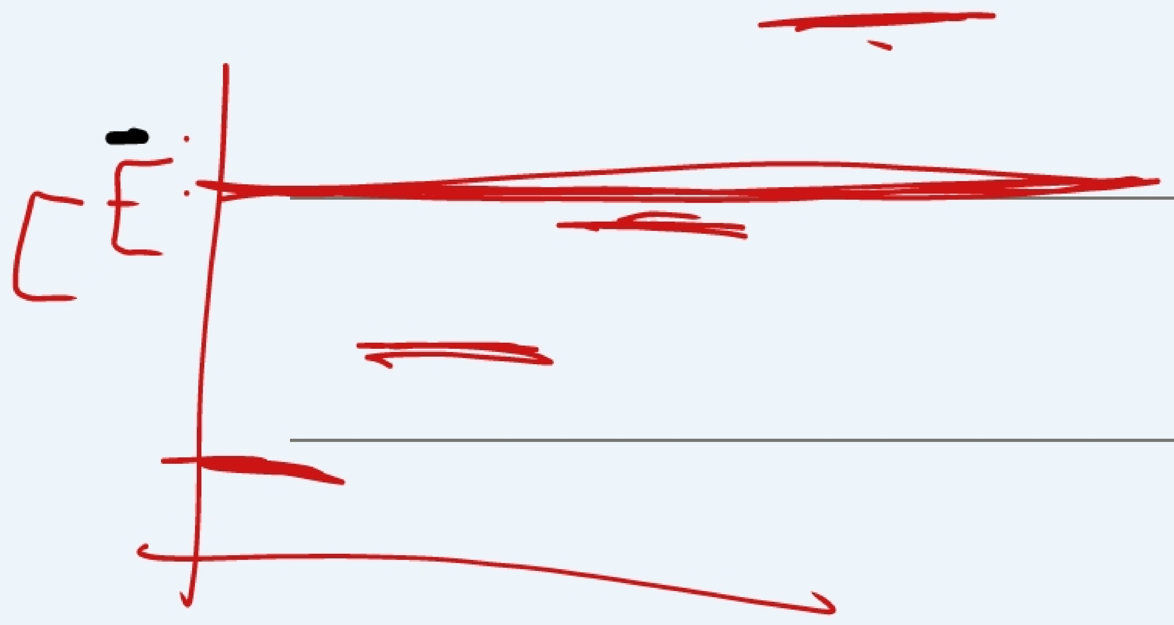
Av. Eng $= K_B T$ $\leftarrow$ RJ: $I \propto \nu^2$



Classical System:

HO:

$\bar{E} = \frac{1}{4} K A^2$



$$\frac{nh\nu}{e^{h\nu/KT} - 1}$$

Planck:

$E_n = nh\nu$

Compton Effect :→ $E = h\nu$

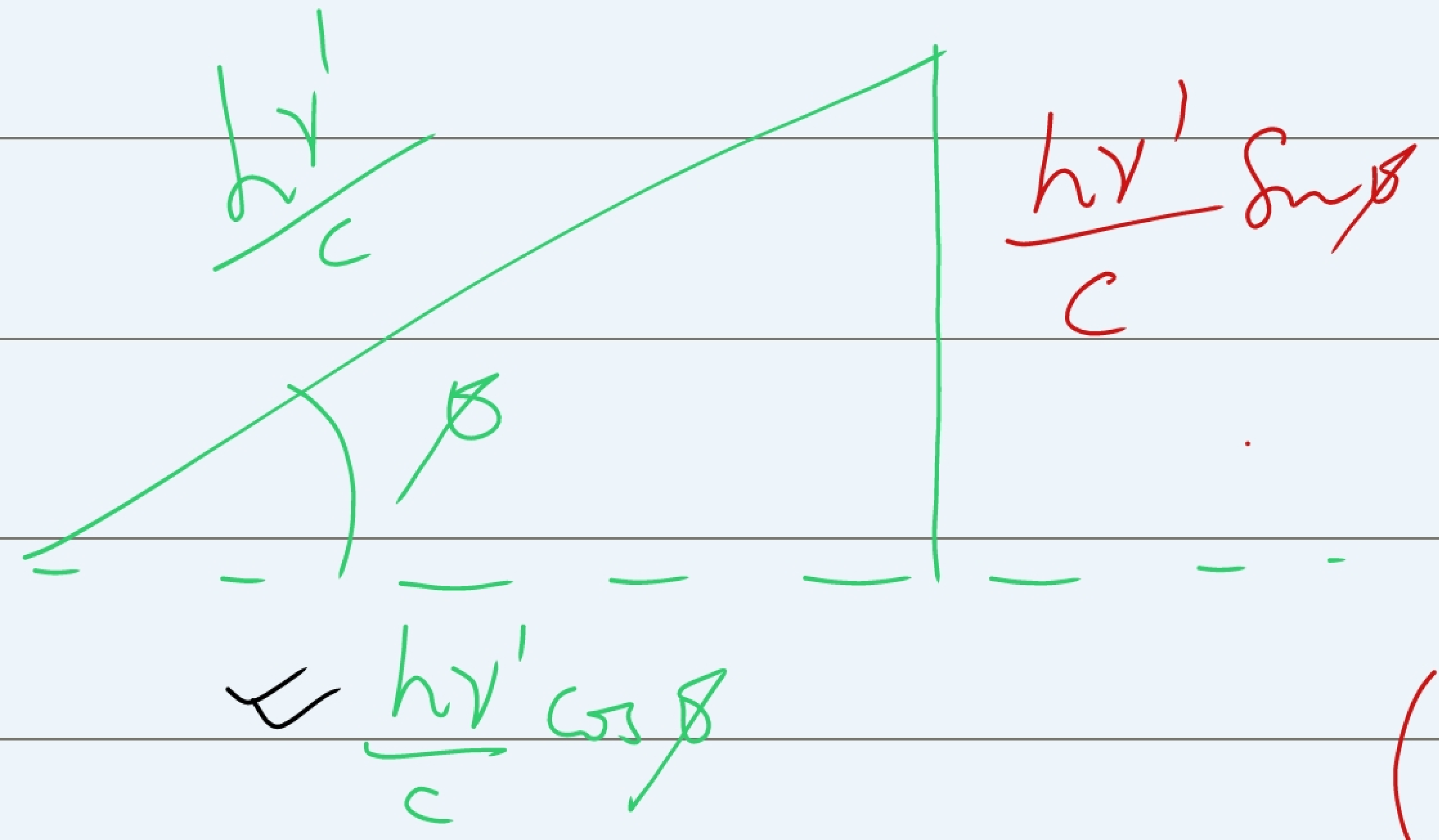
A. Beiser

(Particle nature of wave):

$p = \frac{E}{c} = \frac{h\nu}{c}$

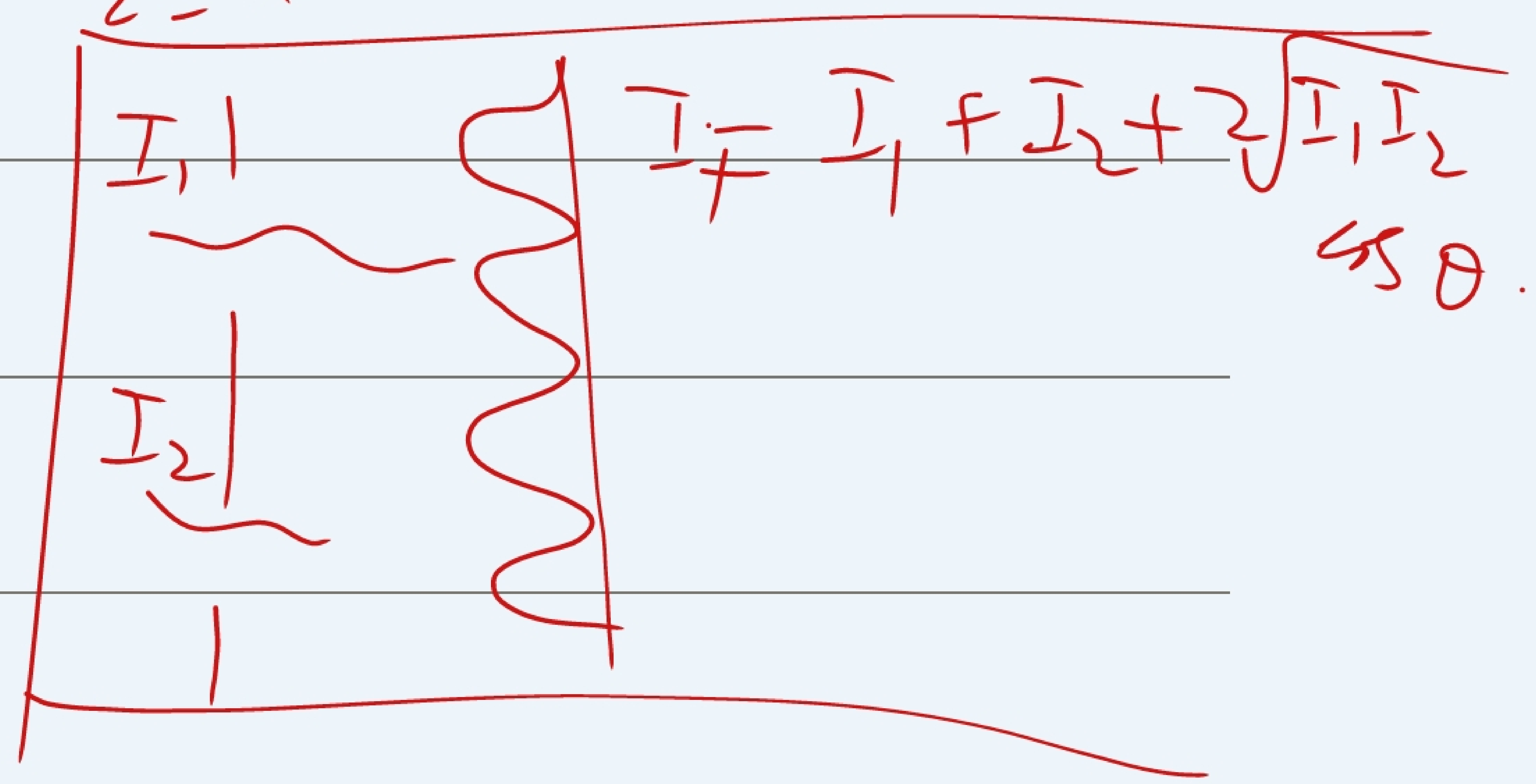
STR

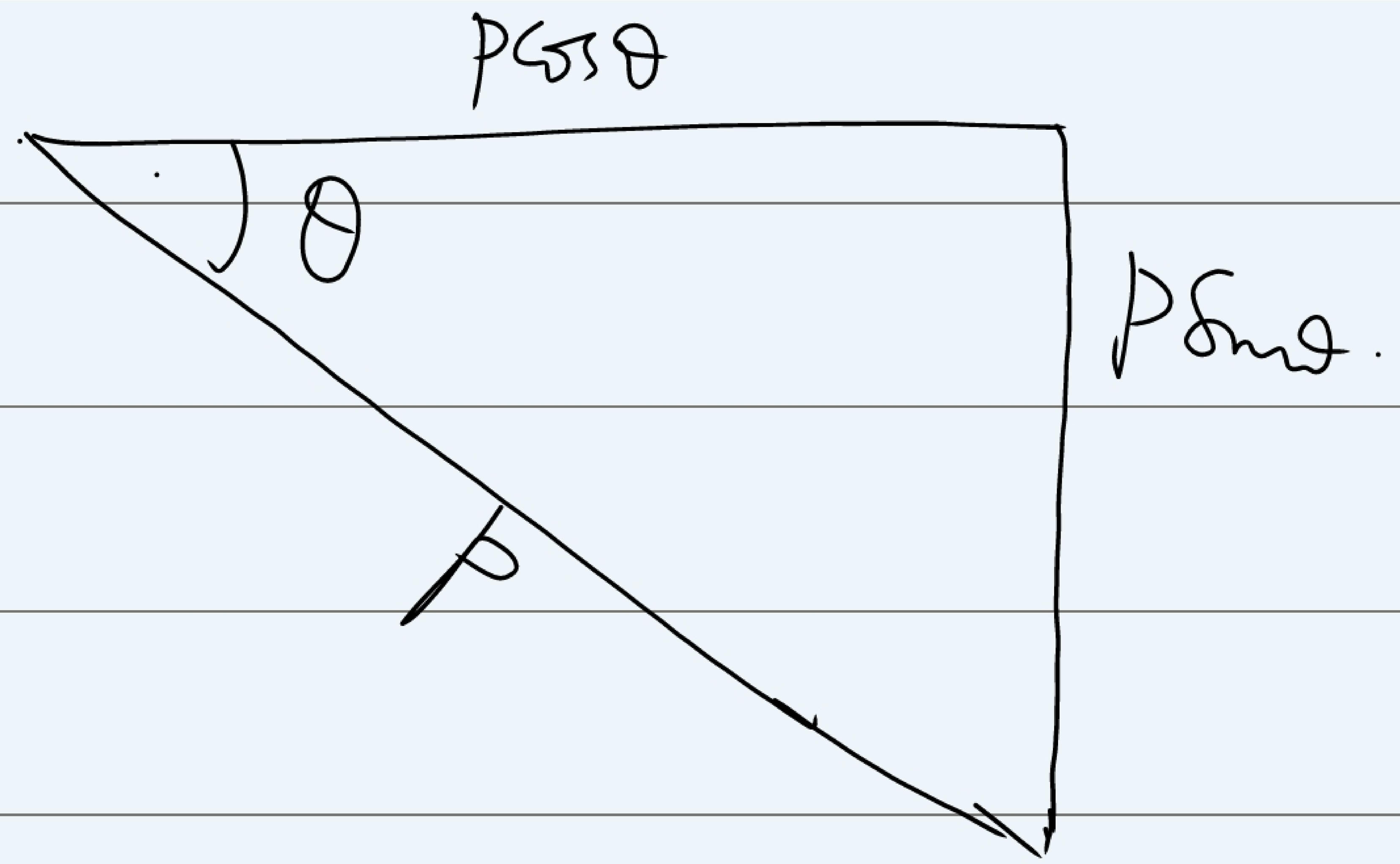
$E = \sqrt{p^2c^2 + m^2c^4}$



$(\lambda' - \lambda) = \text{Constant}\,[1 - \cos\phi]$

$E = h\nu$





Horizontal:

$$\frac{h\nu}{c} + 0 = \frac{h\nu'\cos\phi}{c} + p\cos\theta \quad — ①$$

$\hookrightarrow h\nu = h\nu'\cos\phi + pc\cos\theta \Rightarrow pc\cos\theta = h\nu - h\nu'\cos\phi$

Vertical:

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \quad — ②$$

c × ① & ②

$0 = h\nu'\sin\phi - pc\sin\theta \Rightarrow pc\sin\theta = h\nu'\sin\phi$.

L.H.S $\Rightarrow$

$$\boxed{p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\phi}$$

$$E = K.E + mc^2 = \sqrt{m^2c^4 + p^2c^2}$$

$K.E = h(\nu - \nu')$

$\nu = \frac{c}{\lambda}$

$$\Rightarrow (K.E + mc^2)^2 = m^2c^4 + p^2c^2$$

$$\Rightarrow K.E^2 + \cancel{m^2c^4} + 2(K.E)(mc^2) = \cancel{m^2c^4} + p^2c^2$$

$$\Rightarrow p^2c^2 = K.E^2 + 2(K.E)(mc^2)$$

$$\cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu')\cos\phi = \cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu') + 2mc^2 \cdot (h\nu - h\nu')$$

$$\boxed{(\lambda' - \lambda) = \left(\frac{h}{mc}\right)\left[1 - \cos\phi\right]}$$

Divide by $2h^2c^2$

$$\Rightarrow \frac{\cancel{2}mc^{\cancel{2}}}{\cancel{2}h^2\cancel{c^2}}(h\nu - h\nu') = \frac{\cancel{2}(\cancel{h}\nu)(\cancel{h}\nu')}{\cancel{2}\cancel{h^2}c^2}\left[1 - \cos\phi\right]$$

$$\Rightarrow \frac{mh}{h^2}\left[\frac{c}{\lambda} - \frac{c}{\lambda'}\right] = \frac{1}{\cancel{c^2}} \cdot \frac{c}{\lambda} \cdot \frac{c}{\lambda'}\left[1 - \cos\phi\right] \Rightarrow \frac{mc}{h}\left[\lambda' - \lambda\right] = (1 - \cos\phi)$$

Wave properties of particles:→

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p}$$

Debroglie:- material: $\vec{p} = \gamma m \vec{v}$, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$$\lambda_{dB} = \frac{h}{\gamma m v}$$

$$V_{group} \simeq \frac{\Delta\omega}{\Delta K} = \frac{d\omega}{dK}$$

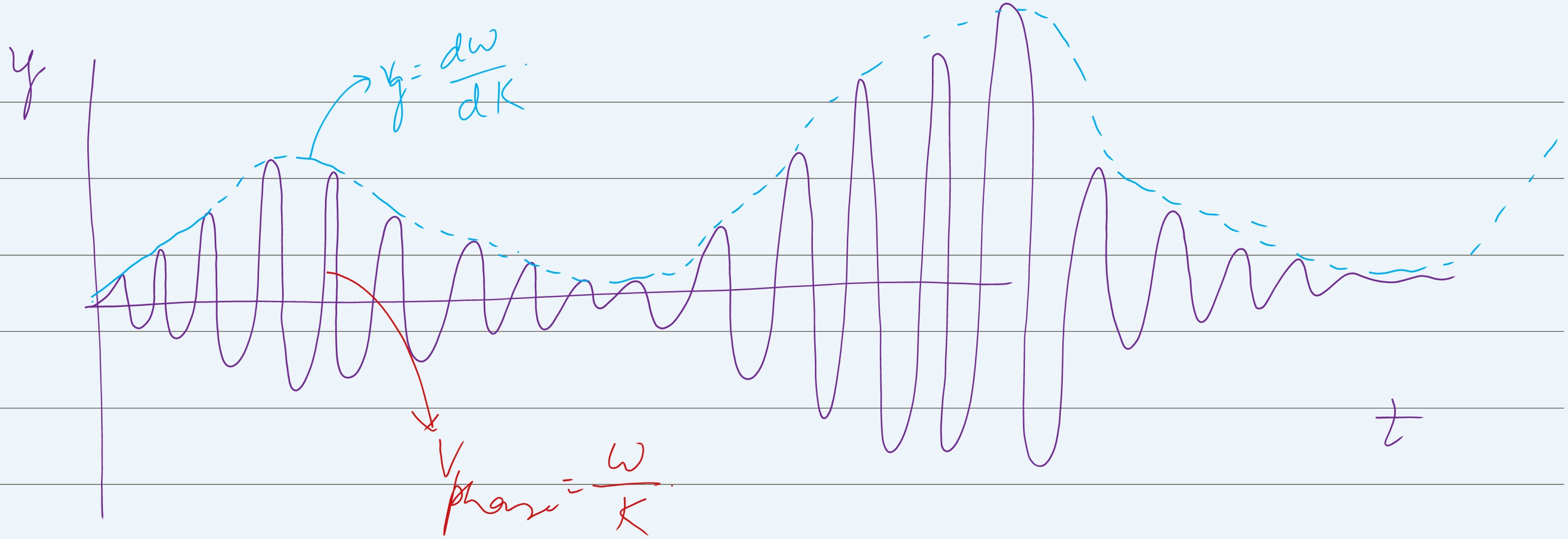
$$y_1 = \overset{A}{A\cos(\underline{\omega t - Kx})}$$

$+$ $\quad +$

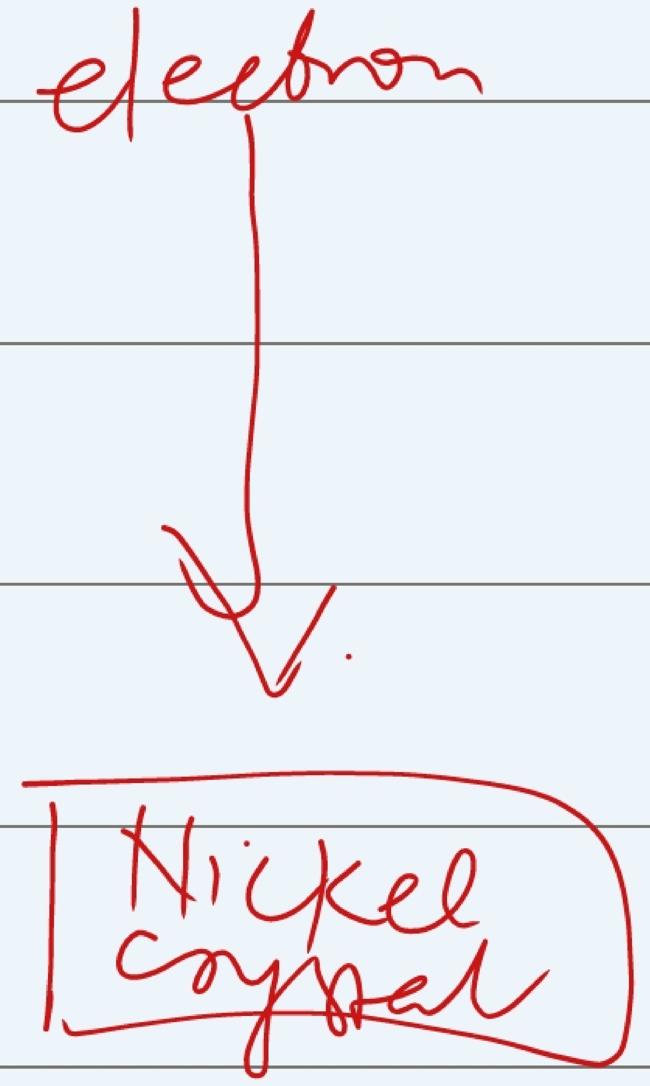
$$y = y_1 + y_2 = y_2 = A\cos\left[(\omega+\Delta\omega)t - (K+\Delta K)x\right]^{B}$$

$$V_{phase} \simeq \frac{2\omega}{2K} = \frac{\omega}{K}$$

$$= \cancel{2A} \cos\frac{1}{2}\left[\underline{\left(\frac{2\omega+\Delta\omega}{2}\right)t - \left(\frac{2K+\Delta K}{2}\right)x}\right] \cos\frac{1}{2}\left[\frac{\Delta\omega}{2}t - \frac{\Delta K}{2}x\right]$$


y
$v_g = \frac{d\omega}{dK}$
$v_{phase} = \frac{\omega}{K}$
t

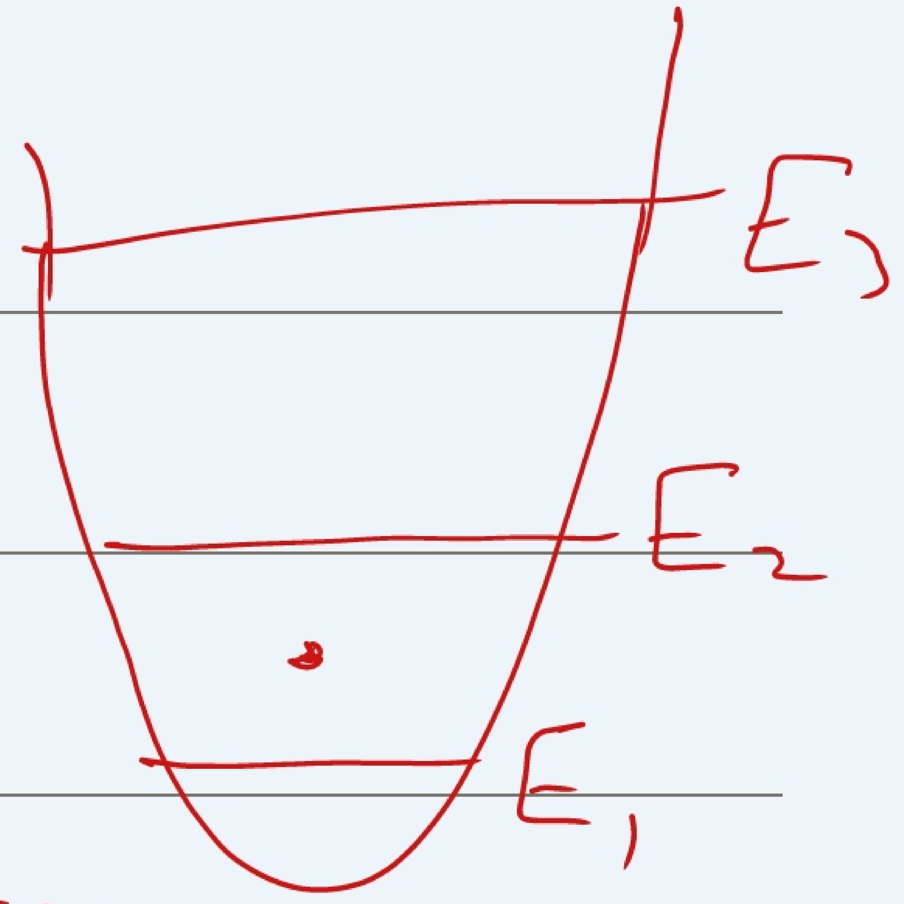
Davisson & Germer / Particle Diffraction:



$$K.E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$K.E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}, \qquad \frac{\lambda}{2} = L$$

$$\Rightarrow \lambda = \frac{2L}{n}, \quad n = 1, 2, \ldots$$



$$= \frac{n^2 h^2}{8mL^2} \qquad n = 1, 2, 3 \ldots$$

$$y_1 = A\cos(\omega t - kx)$$

$$+ \quad y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$$y \simeq 2A\cos[2\omega t - 2kt] \cos[\Delta\omega t - \Delta k x]$$

$$V_{phase} = \frac{\omega}{K} = \frac{\cancel{2\pi}\nu}{\left(\frac{\cancel{2\pi}}{\lambda}\right)} = \underline{\underline{\lambda\nu}} = \frac{h}{\gamma m v} \cdot \frac{\gamma m c^2}{h} = \frac{c^2}{v}.$$

$$E = h\nu = \gamma m c^2$$

$$\nu = \frac{\gamma m c^2}{h}$$

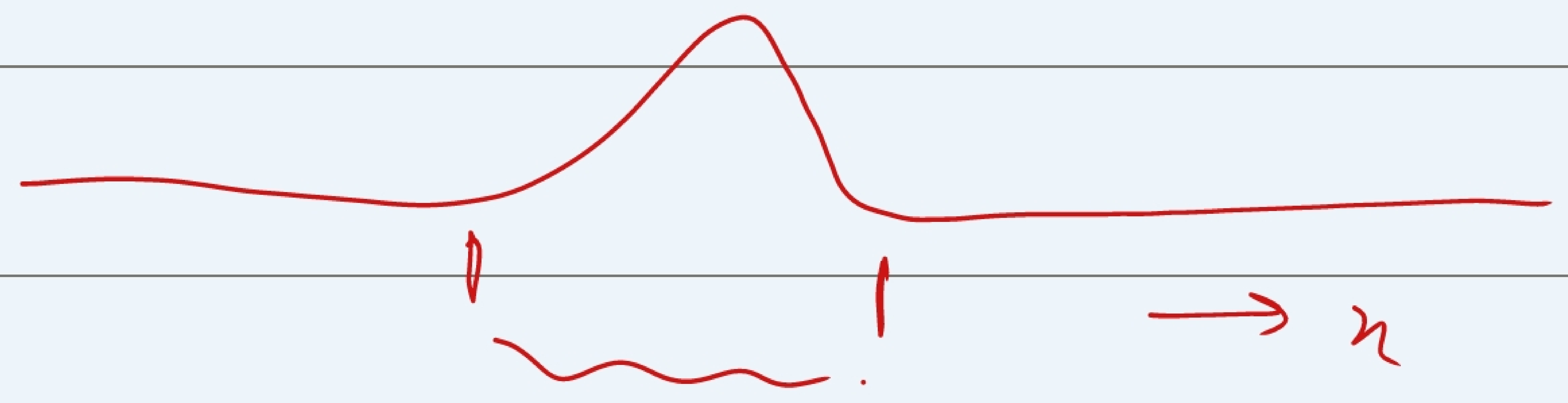
$$V_{group} = \frac{d\omega(\nu)}{dK}.$$

$$\underline{\underline{\omega}} = 2\pi\nu = \frac{2\pi \cancel{c}^{\,v}}{\lambda}$$

$$\boxed{\frac{d\omega}{dK} = \cancel{c}^{\,v}} \quad d\omega = -\frac{2\pi c}{\lambda^2} \cdot d\lambda.$$

$$K = \frac{2\pi}{\lambda}, \quad dK = -\frac{2\pi}{\lambda^2} d\lambda.$$

Heisenberg Uncertainty Principle:→



$p = \frac{h}{\lambda}$

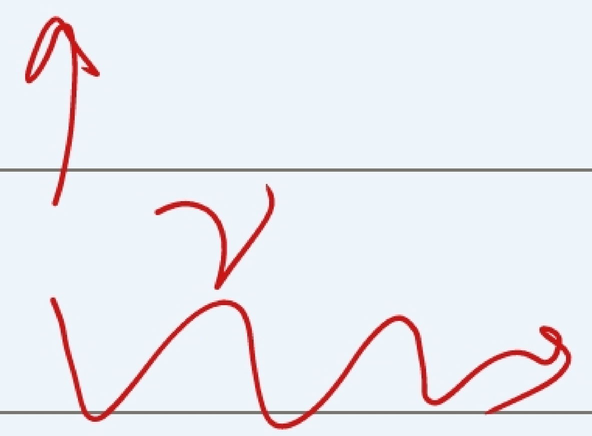
$\Delta x \to$ small

$\Delta p \to$ large.

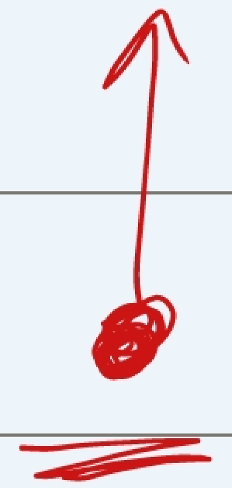
$\delta p \to$ small.

$\Delta x \to$ large.

$\Delta p \approx \frac{h}{\lambda}$



$\Delta x \sim \lambda$



$\Delta x \, \Delta p \approx h$

Postulates of Quantum Mechanics →

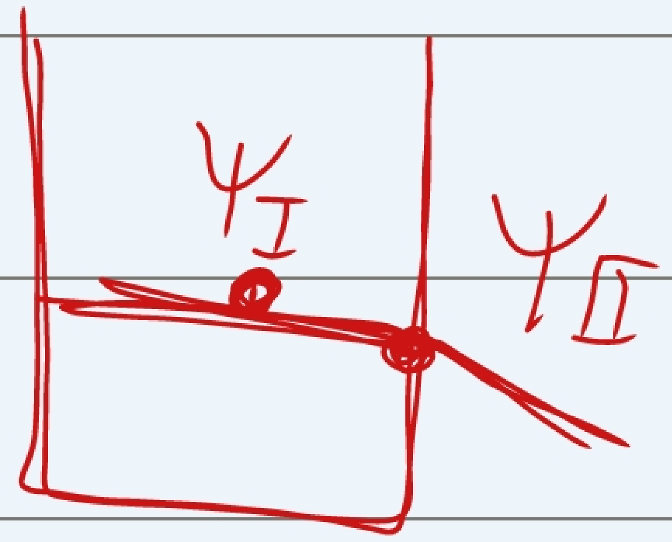
1. All physical systems have a wavefunction

$e^{i\theta} = \cos\theta + i\sin\theta$

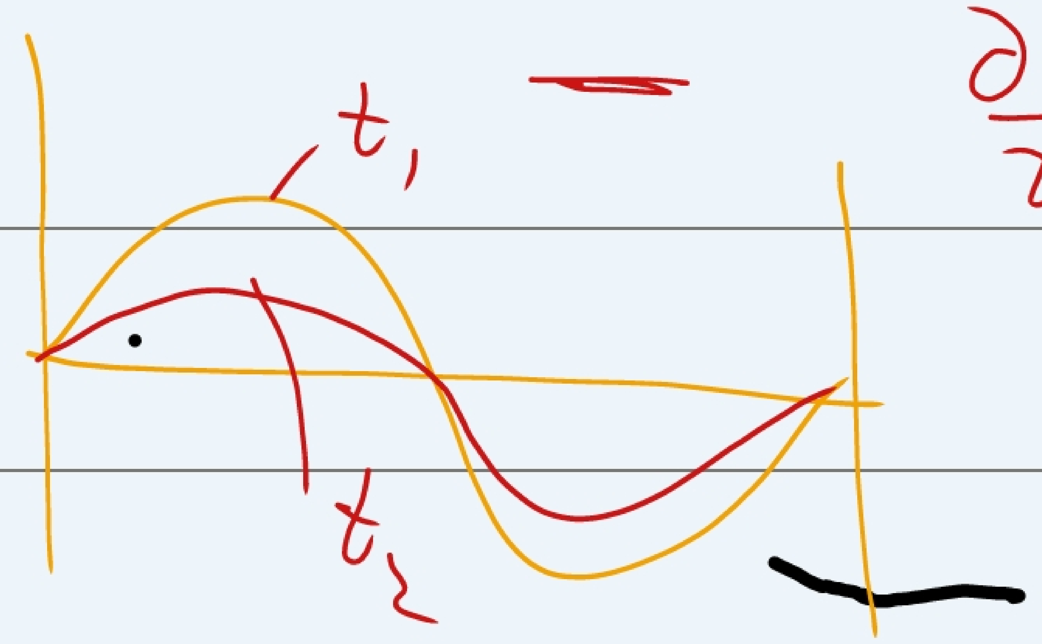
Well-behaved :- $\psi = A + iB$

— Single valued (x, t)

— Continuous and differentiable.

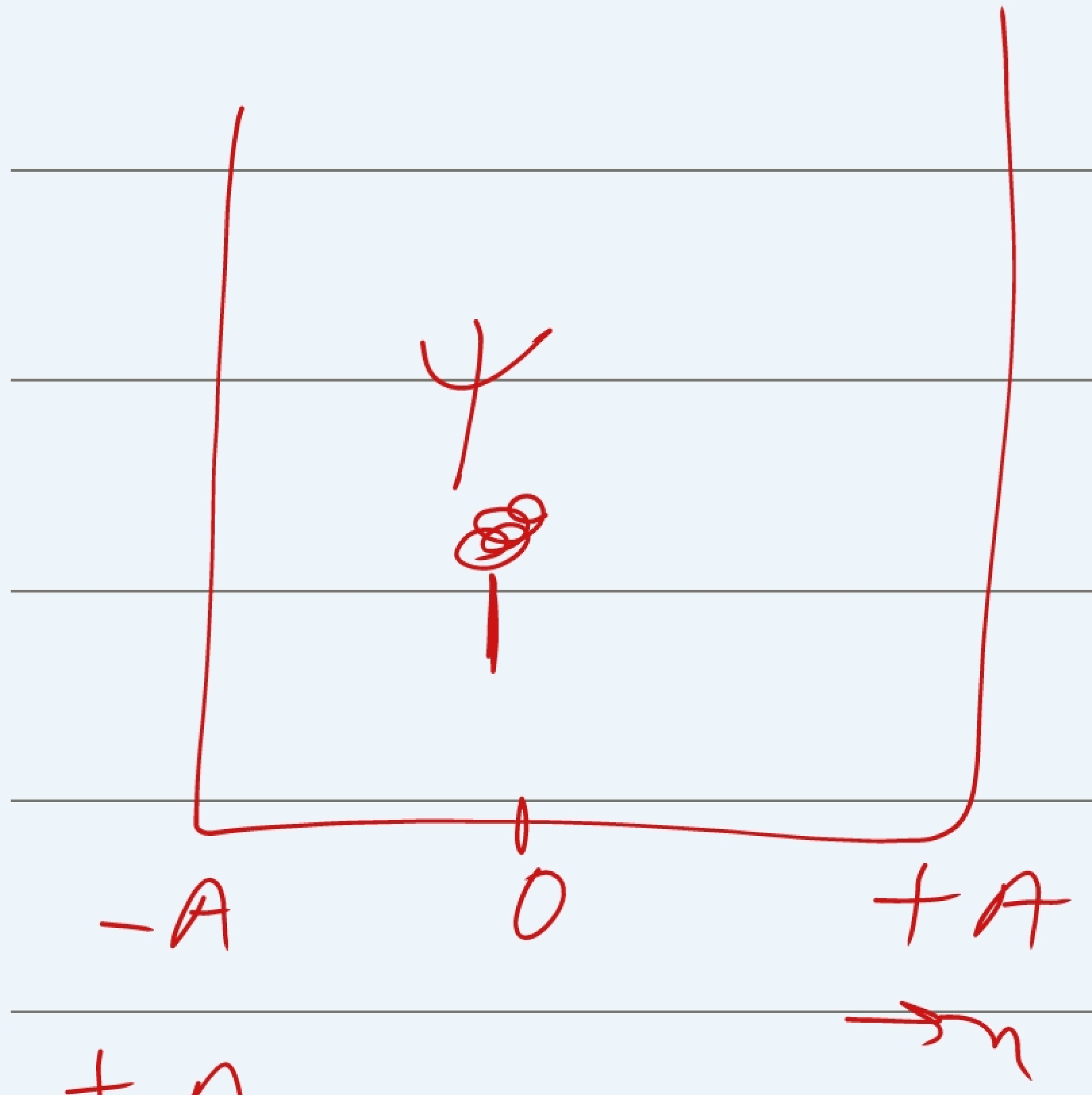


$\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z}, \frac{\partial\psi}{\partial t}$



$y(x,t)$ $\quad t = t_1, t_2$

— Square Integrable



$$\psi = A + iB$$

$$\psi^* \psi = A^2 + B^2$$

$$|\psi|^2 = A^2 + B^2$$

$$\int_{-A}^{+A} \psi^* \psi \, dx = 1$$