Academic Year 2023-24

Tutorial #10

PH100: Mechanics and Thermodynamics

1. Which of the following wave functions cannot be solutions of Schrödinger's equation for all values of x? Why not? (a) y =A sec x; (b) y = A tan x; (c) y = A exp(x^2); (d) y = A exp($-x^2$).

2. The wave function of a certain particle is y = A $\cos^2 x$ for -pi/2 < x < pi/2. (a) Find the value of A. (b) Find the probability that the particle be found between x = 0 and x = pi/4.

3. As mentioned in Sec. 5.1, in order to give physically meaningful results in calculations a wave function and its partial derivatives must be finite, continuous, and single-valued, and in addition must be normalizable. Equation (5.9) gives the wave function of a particle moving freely (that is, with no forces acting on it) in the +x direction as

$$\Psi = Ae^{-(i/\hbar)(Et-pc)}$$

   where *E* is the particle's total energy and *p* is its momentum. Does this wave function meet all the above requirements? If not, could a linear superposition of such wave functions meet these requirements? What is the significance of such a superposition of wave functions?

4. Show that the expectation values <px> and <xp> are related by

$$\langle px\rangle - \langle xp\rangle = \frac{\hbar}{i}$$

   This result is described by saying that p and x do not commute, and it is intimately related to the uncertainty principle.

5. Obtain Schrödinger's steady-state equation from Eq.(3.5) with the help of de Broglie's relationship λ= h/mv by letting y = Ψ and finding $\partial^2\Psi/\partial x^2$.

6. One of the possible wave functions of a particle in the potential well of Fig. 5.17 is sketched there. Explain why the wavelength and amplitude of momentum vary as they do.

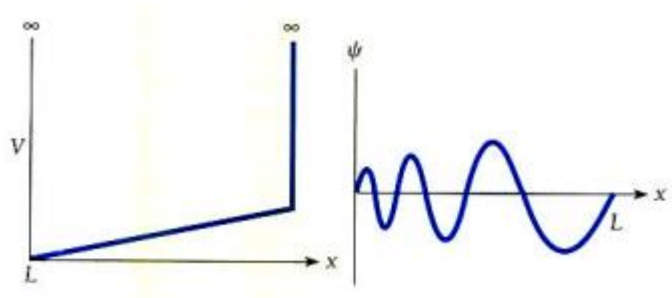

Figure 5.17

7. An eigenfunction of the operator d^2/dx^2 is e^{2x}. Find the corresponding eigenvalue.