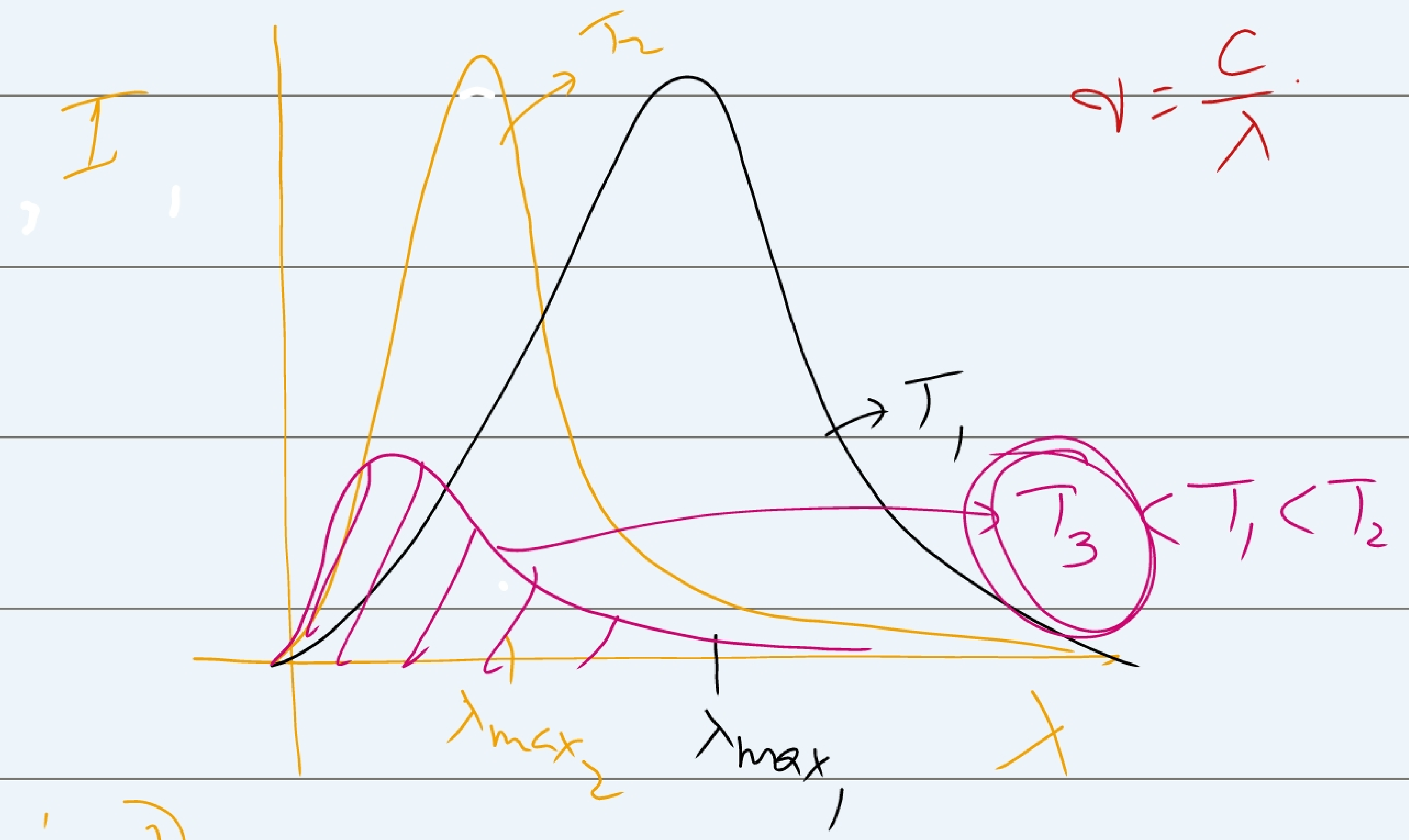
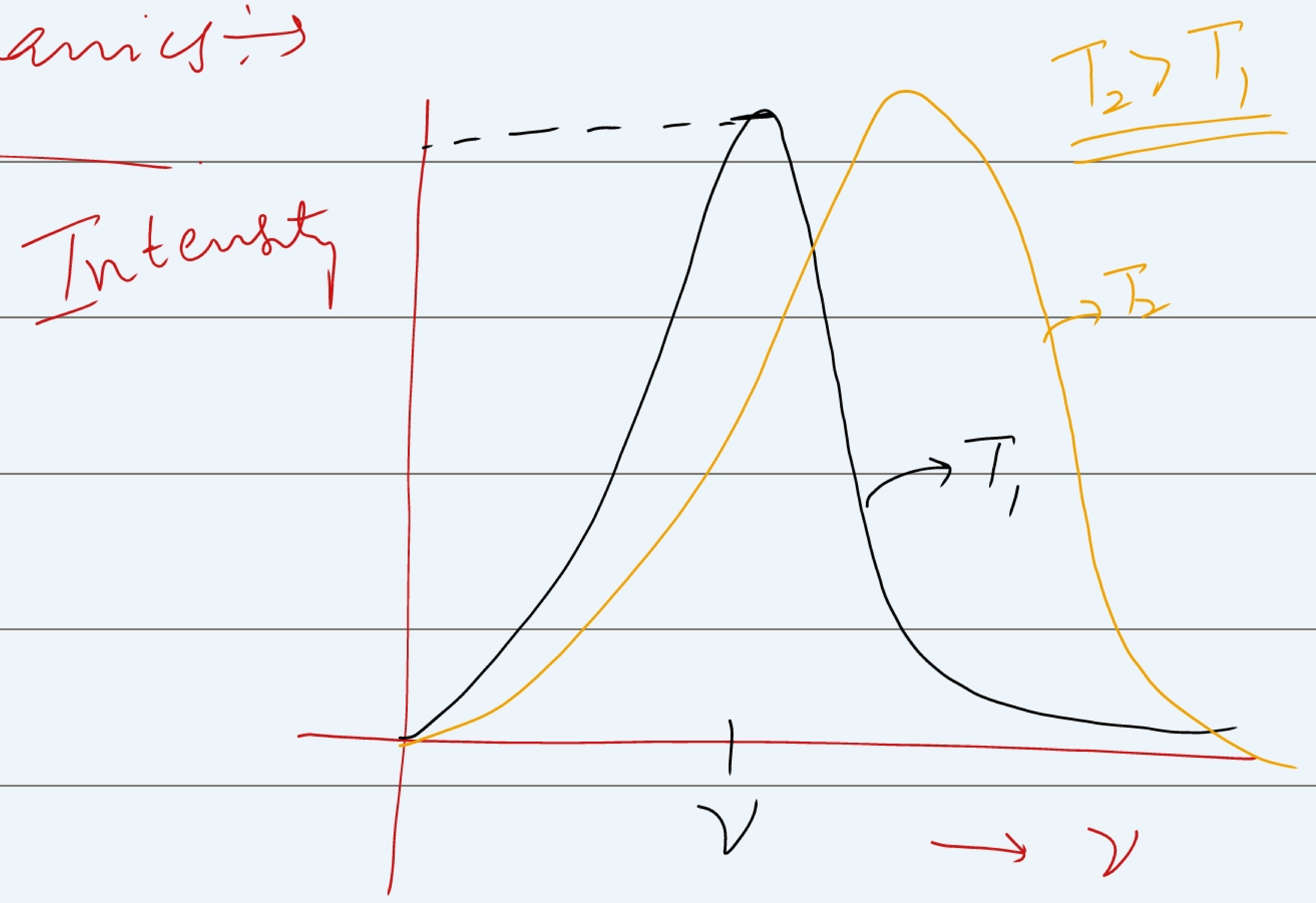
01/11/2023

Failures of classical Mechanics :→



$\nu = \frac{c}{\lambda}$

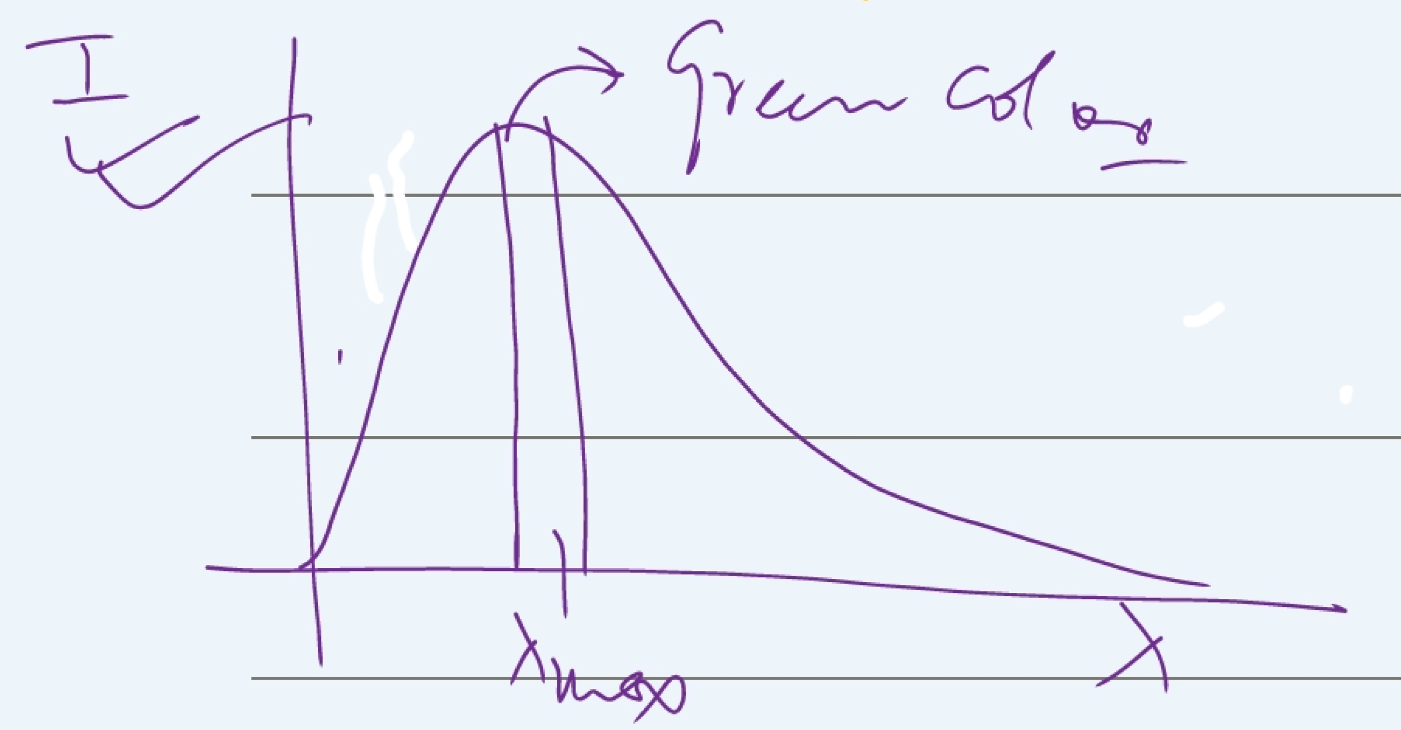


Wien's Displacement

$$\lambda_{max} T = 2.898 \times 10^{-3}\ \text{mK}$$

Stefan Boltzmann Law

$$R(T) = \epsilon \alpha T^4$$



$$\lambda_{max} \cdot 6000K = 2.898 \times 10^{-3}\ \text{mK}$$

$$\lambda_{max} \simeq 480 \times 10^{-9}\ \text{m}$$

Postulates of Q.M →

1. $\psi(x, y, z, t)$:

$= A + iB$

Born Approximation:

$|\psi|^2 = P.D = \psi^* \psi = A^2 + B^2$

Normalization:

$$\int_a^b \psi^* \psi \, \partial x = 1 = \int_{-\infty}^{+\infty} \psi^* \psi \, \partial x$$

Well-behaved:

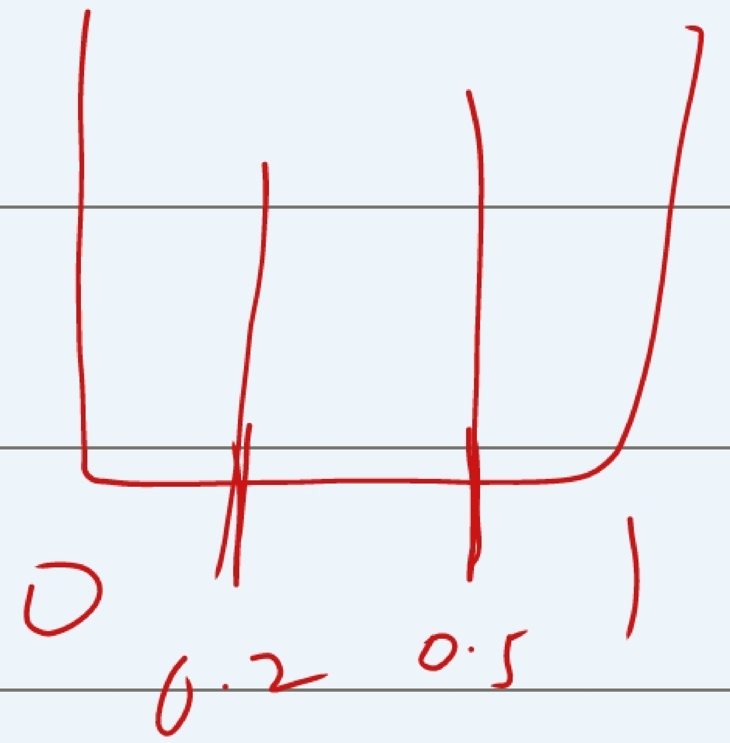
Single-valued.

Continuous & differentiable

$\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial x}$ Continuous & Differentiable.

Square integrable

$\psi(x) = ax \quad , \quad [0, 1]$



$$\int_{0.2}^{0.5} a^2 x^2 \, dx = a^2 \frac{x^3}{3} \Big|_{0.2}^{0.5}$$

$$\int_a^b \psi^* \psi \, dx$$

2. How to find wavefunction: Schrödinger Eq^n:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}, \quad \frac{A \sin(kx - \omega t)}{A \cos(kx - \omega t)}$$

$$E = K.E + P.E$$

$$= \frac{p^2}{2m} + U(x,y,z,t)$$

$$\psi(x,t) = A e^{i[kx - \omega t]}$$

$$= A e^{-i[\omega t - kx]}$$

$$= A e^{-i2\pi\left[\nu t - \frac{x}{\lambda}\right]}$$

$$\boxed{\psi(x,t) = A e^{-i\left[\frac{E}{\hbar}t - \frac{p}{\hbar}x\right]}}$$

$$\frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\omega = 2\pi\nu, \quad k = 2\pi/\lambda$$

$$E = h\nu \Rightarrow \nu = \frac{E}{h}$$

$$2\pi\nu = \frac{2E\pi}{h} = \frac{E}{\hbar}$$

$$\hbar = h/2\pi$$

$$\psi(x,t) = A e^{-\frac{i}{\hbar}\left[Et - px\right]}$$

$$\frac{\partial \psi}{\partial x} = \frac{ip}{\hbar}\psi$$

$$\Rightarrow p\psi = \frac{\hbar}{i}\frac{\partial \psi}{\partial x}, \quad \boxed{\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}}$$

$$\frac{\partial \psi}{\partial t} = -\frac{iE}{\hbar}\psi \Rightarrow E\psi = \overset{i^2}{-}\frac{\hbar}{i}\frac{\partial \psi}{\partial t}$$

$$\boxed{E\psi = i\hbar\frac{\partial \psi}{\partial t}} \quad \boxed{\hat{E} = i\hbar\frac{\partial}{\partial t}}$$

4. Measurement of physical Quantity is

$$\boxed{\begin{aligned} \hat{p}\,\psi &= a\,\psi \\ \hat{E}\,\psi &= b\,\psi \end{aligned}}$$

$$\hat{p}\,\hat{p} = \left(\frac{\hbar}{i}\right)^2 \frac{\partial}{\partial x}\frac{\partial}{\partial x}$$

Free particle: $E = \frac{p^2}{2m} + U(x,y,z,t)$

For any quantum system: $\hat{E}\Psi = \frac{\hat{p}^2}{2m}\Psi + U\Psi$

$$\boxed{i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + U\Psi(x,t)}$$

Time-dependent Schrodinger Eqn.

$$\psi(x,t) = A e^{-\frac{i}{\hbar}[Et - px]}$$

$$= A e^{-\frac{i}{\hbar}Et} e^{+\frac{i}{\hbar}px}$$

$$\boxed{\psi(x,t) = \psi(x) e^{-\frac{i}{\hbar}Et}}$$

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \, \psi(x) e^{-\frac{i}{\hbar}Et}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} e^{-\frac{i}{\hbar}Et}$$

TDSE:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U\psi(x,t)$$

$$\Rightarrow (i\cancel{\hbar})\left(\frac{-i}{\cancel{\hbar}}E\right)\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U\psi(x)$$

$$E\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U\psi(x)$$

TIDSE:

$$\boxed{\psi''(x) + \frac{2m}{\hbar^2}\left[E - U\right]\psi(x) = 0}$$

3. Linear & Superposition principle:

Intensity

e^- Ψ_1 $\quad P_1 = \Psi_1^* \Psi_1$

e^- Ψ_2 $\quad P_2 = \Psi_2^* \Psi_2$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

1. $A \sin(kx - \omega t)$
2. $A \cos(kx - \omega t)$
3. $A \sin(kx - \omega t) + B \cos(kx - \omega t)$

ψ_1

ψ_2

$$(\psi_1+\psi_2)^*(\psi_1+\psi_2)$$

$$= \underline{\psi_1^*\psi_1} + \underline{\psi_1^*\psi_2 + \psi_2^*\psi_1} + \psi_2^*\psi_2$$

$$= |\psi_1|^2 + |\psi_2|^2 + \text{Interference term}$$

5. Expectation values:-

$$\langle x \rangle = \frac{\int_{-\infty}^{+\infty} \psi^* x \psi \, \partial x}{\underbrace{\int_{-\infty}^{+\infty} \psi^* \psi \, \partial x}_{1}}$$

$$\langle p \rangle = \int_{-\infty}^{+\infty} \psi^* \hat{p} \psi \, \partial x .$$

gf

$$\int_{-\infty}^{+\infty} \psi^* \psi \left(\frac{\hbar}{i} \frac{\partial}{\partial x}\right) \partial x$$

$$= \frac{\hbar}{i} \int_{-\infty}^{+\infty} |\psi|^2 \partial$$

gf

$$\int_{-\infty}^{+\infty} \hat{p} \psi^* \psi \, \partial x$$

$$= \int_{-\infty}^{+\infty} \left(\frac{\hbar}{i} \frac{\partial}{\partial x}\right) |\psi|^2 \, \partial x$$

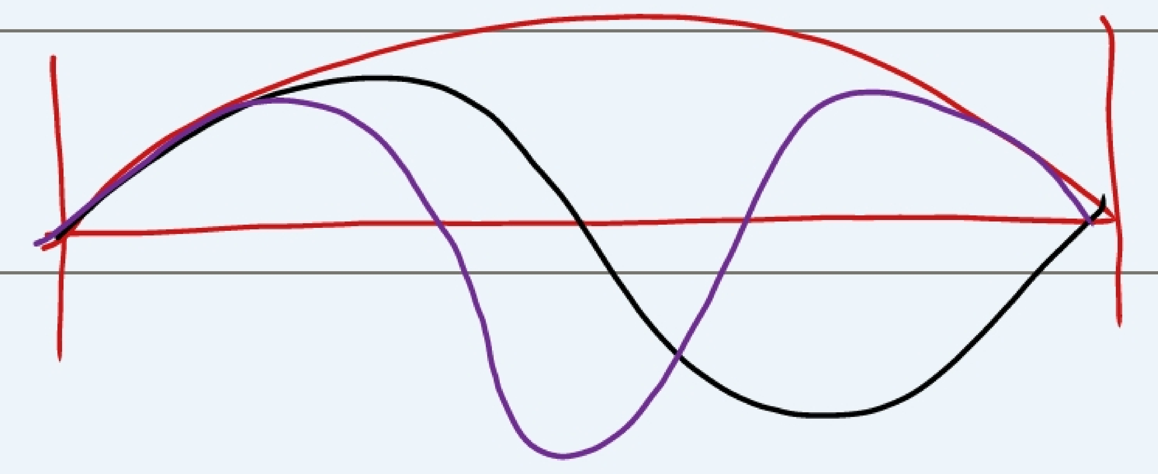
$$= \frac{\hbar}{i} \cdot |\psi|^2 \Big|_{-\infty}^{+\infty}$$

$$= 0$$

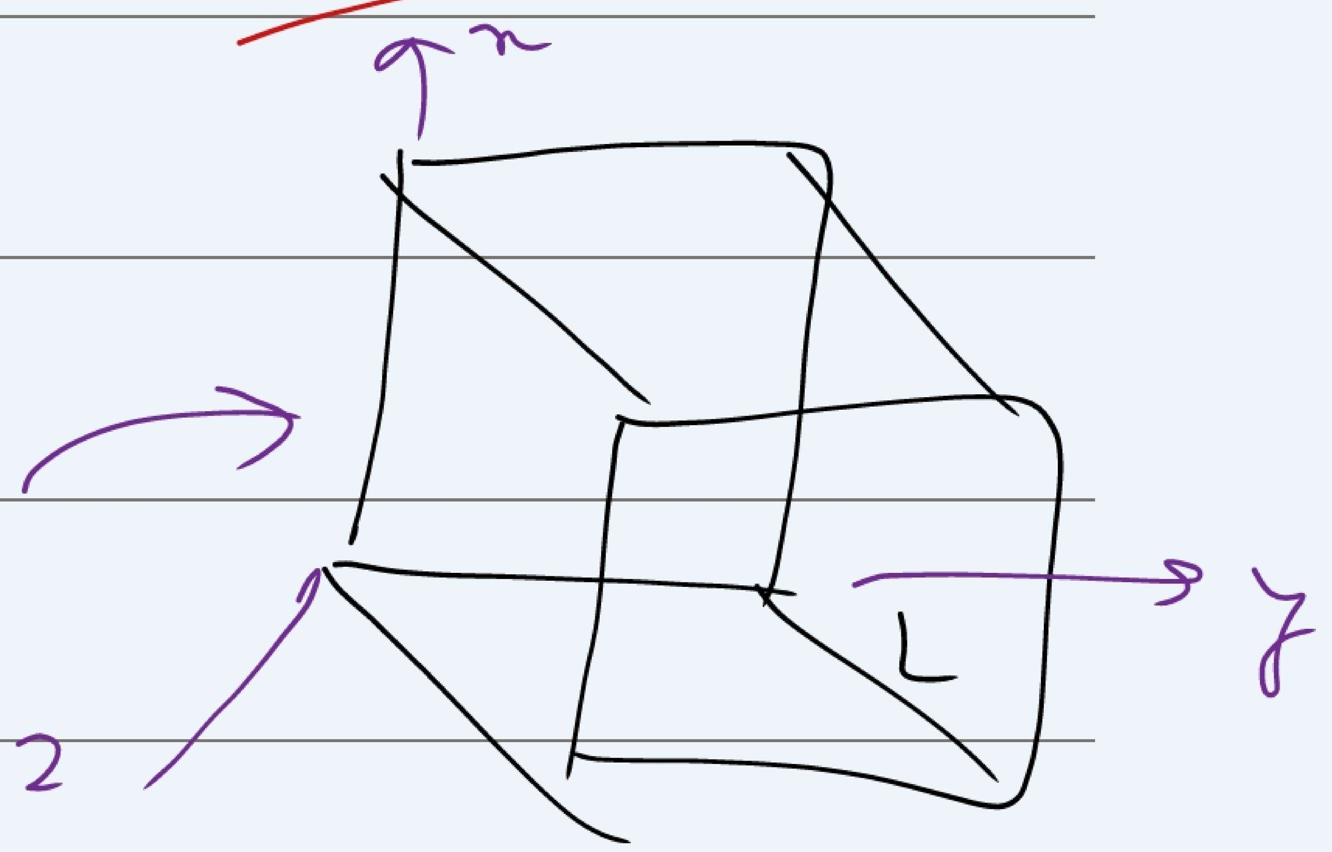
Rayleigh-Jeans Law →

$\frac{c}{\lambda_n} = \lambda_n = \frac{2L}{j}$

$\epsilon = 1$ $\quad L^3$



$\sqrt{j_x^2 + j_y^2 + j_z^2}^{\,2}$



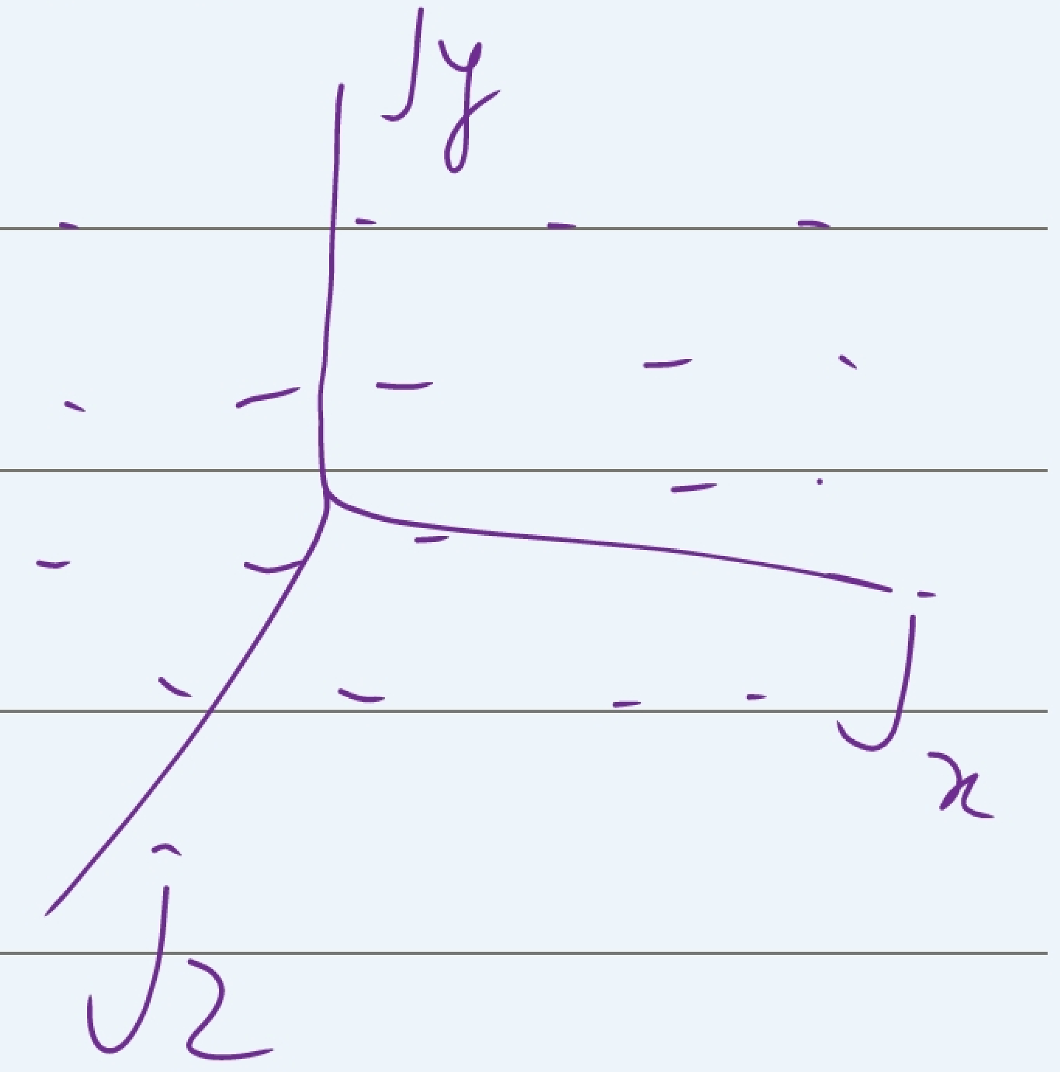
1 mode $\quad \frac{\lambda}{2} = L$ $\quad \Rightarrow \boxed{j_x \lambda = 2L}$, $\quad j = 1, 2, 3, \ldots$

$\lambda = L$

$\frac{3}{2}\lambda = L$

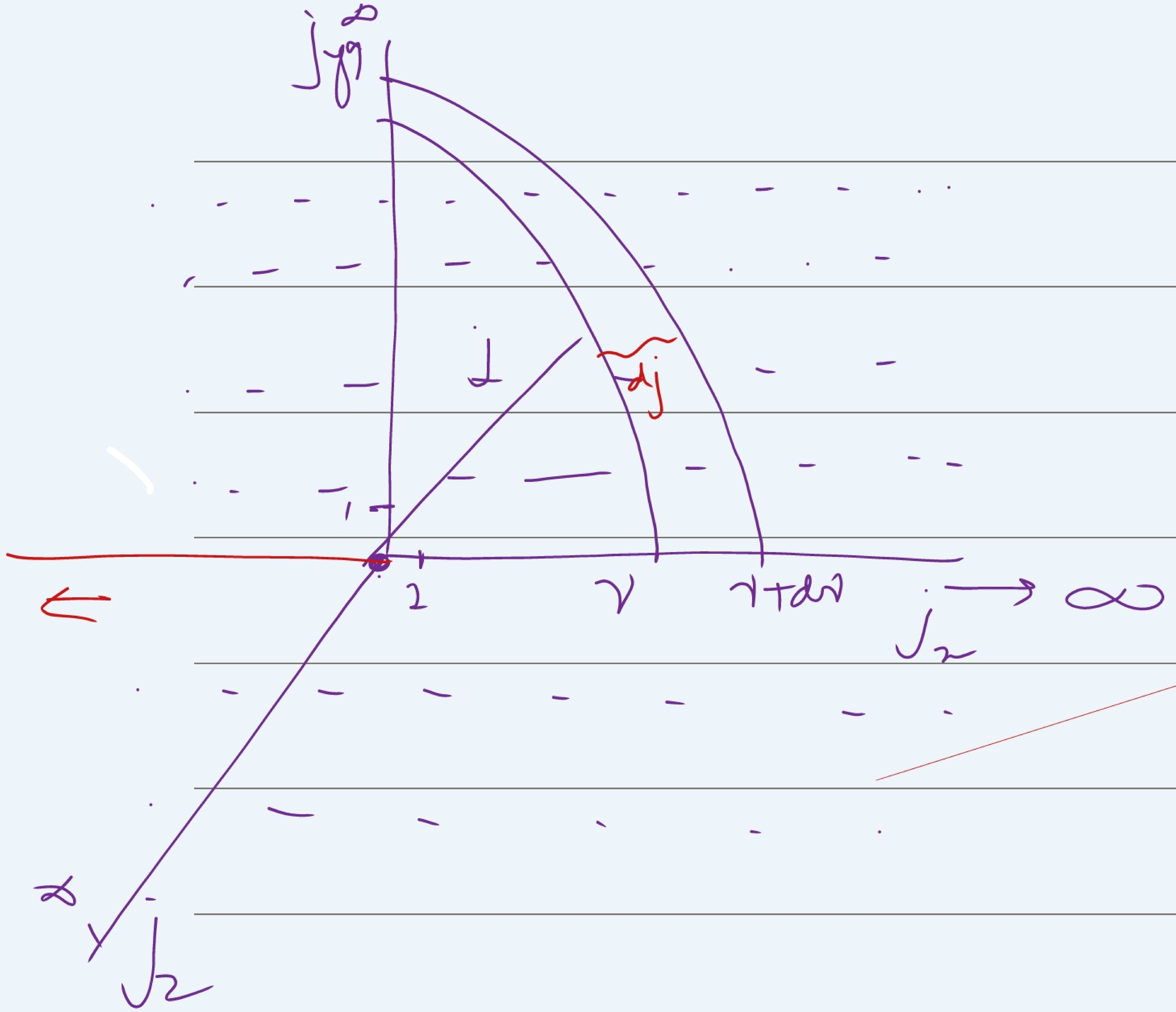
$j_y \lambda = 2L$

$j_z \lambda = 2L$



For any arbitrary Direction, Standing wave $= \sqrt{j_x^2 + j_y^2 + j_z^2}$

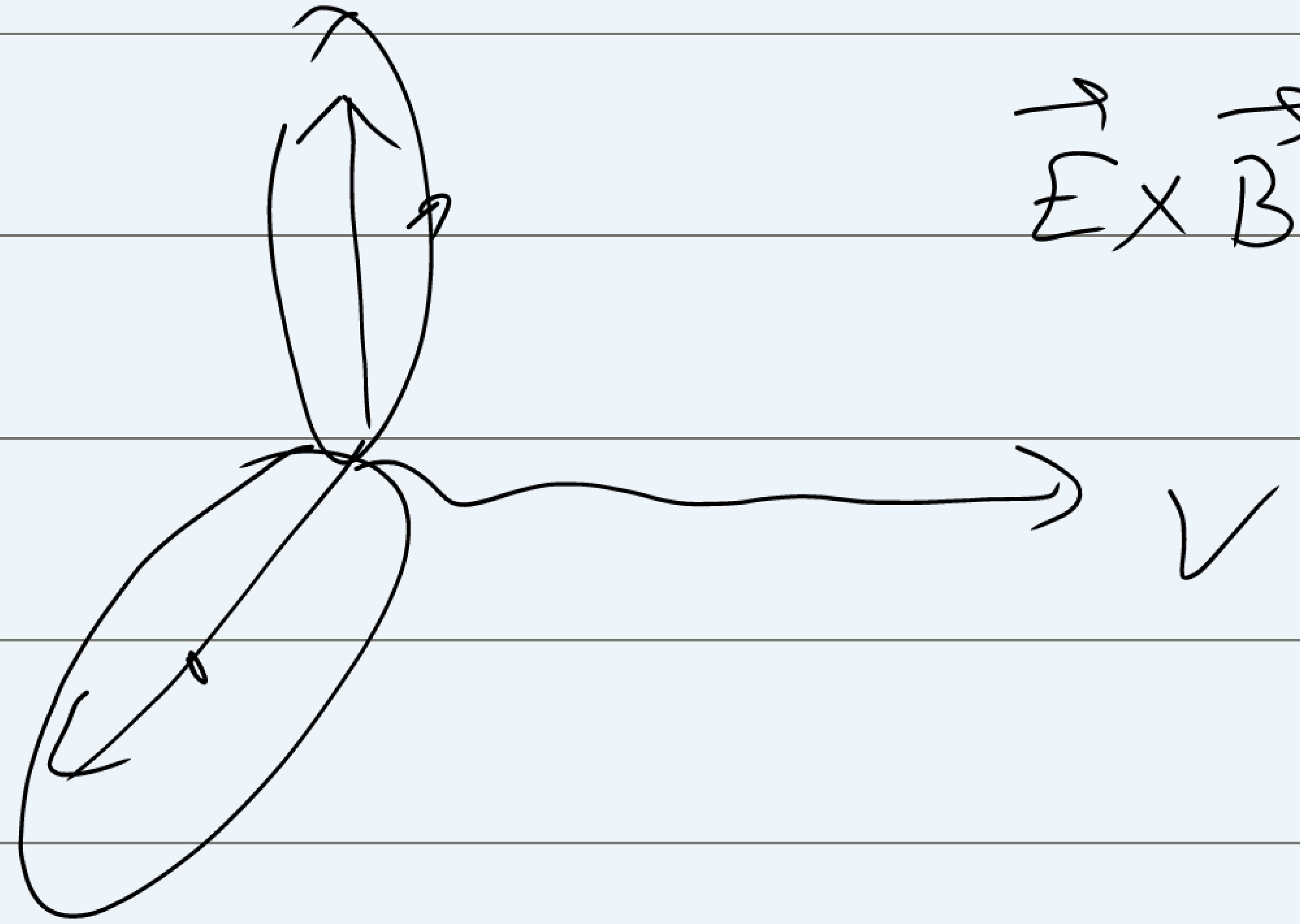
$j_x = \frac{2L}{\lambda}, \ j_y = \frac{2L}{\lambda}, \ j_z = \frac{2L}{\lambda}$



Volume of Standing (No. of Standing) Wave

$g(\nu)d\nu$

$= 4\pi j^2 dj \times \frac{1}{8} \times 2 = \pi j^2 dj$



$\vec{E} \times \vec{B} =$

$$g(\nu)\,d\nu = \pi j^2\,dj$$

$$j = \frac{2L}{\lambda} = \frac{2L\nu}{c}$$

$$= \pi\left(\frac{2L\nu}{c}\right)^2 \cdot \left(\frac{2L}{c}\right) d\nu$$

$$dj = \frac{2L}{c}\,d\nu$$

$$g(\nu)\,d\nu = \frac{8\pi \cancel{L^3}}{c^3 \cancel{L^3}}\,\nu^2\,d\nu = \frac{8\pi}{c^3}\,\nu^2\,d\nu = \text{Density of standing wave formed in the cavity.}$$

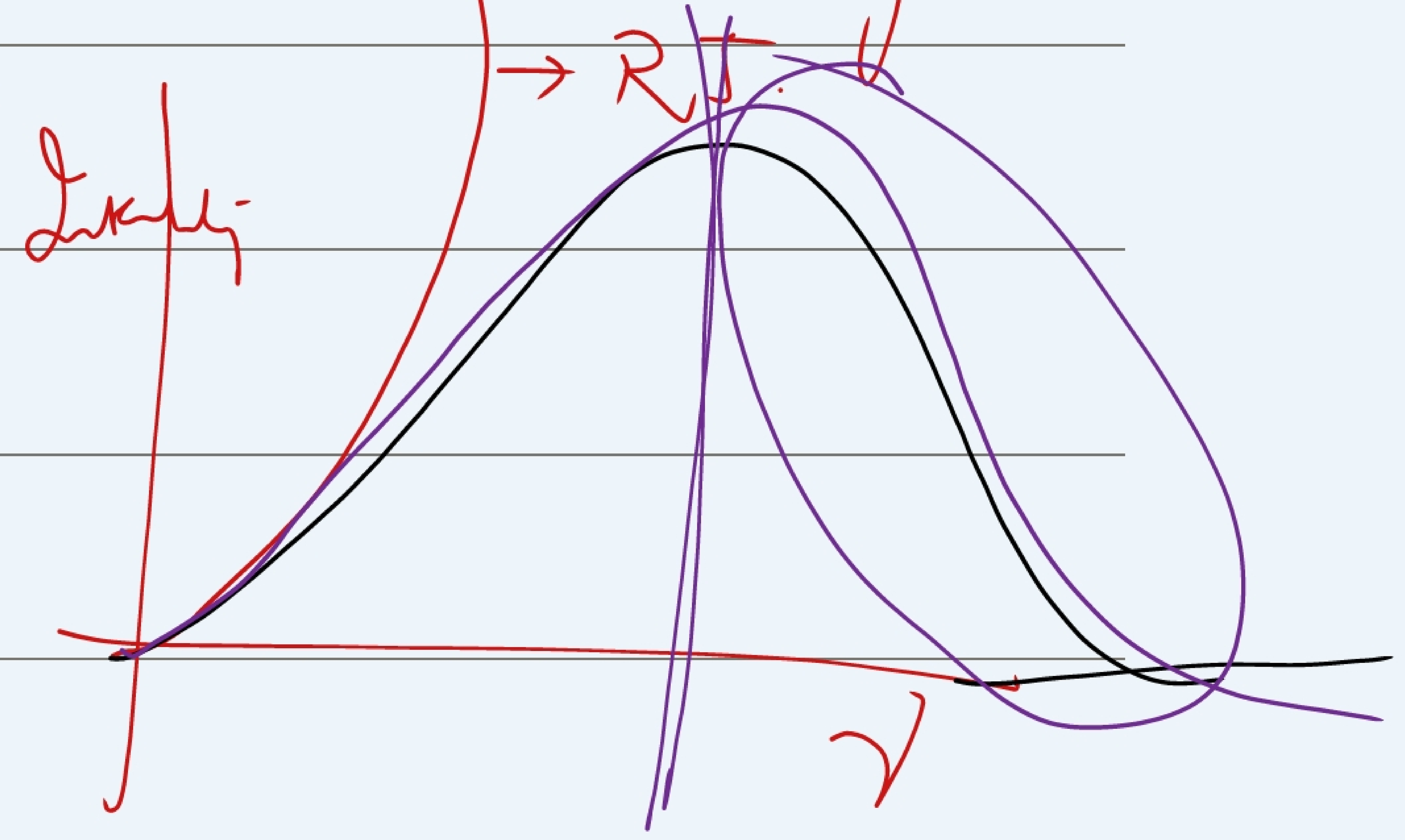
No. of standing waves / Volume

$$\frac{h\nu}{e^{h\nu/kT} - 1} = \bar{E} = K_B T$$

($\frac{1}{2}$ - 1 degree; $\frac{3}{2}$ - 3 degree)

R.J. Radiation Law

$$\text{Intensity} = \frac{8\pi\nu^2\,d\nu}{c^3}\cdot K_B T$$



Planck's Radiation Law:-

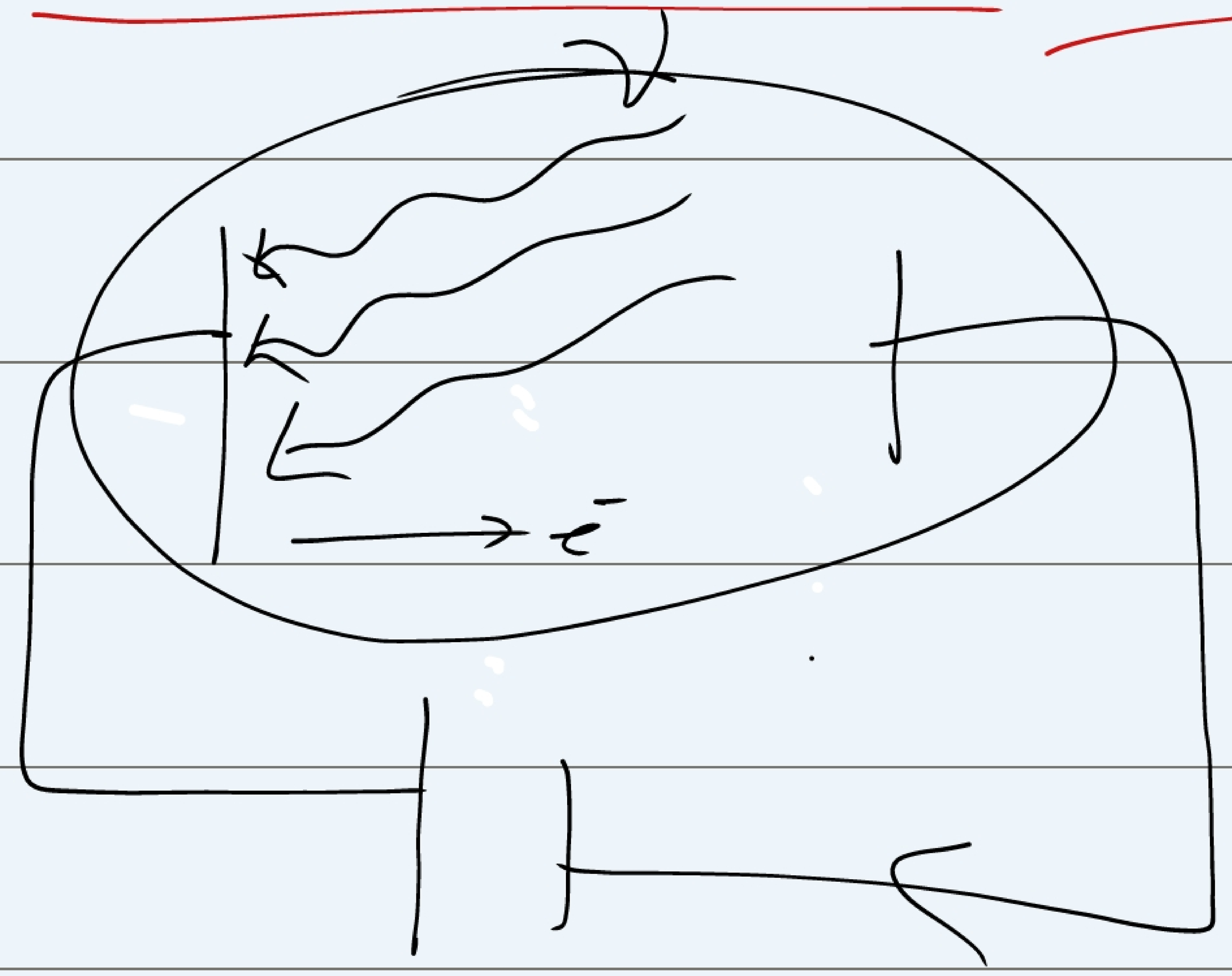
$$I = \left(\frac{8\pi\nu^2 d\nu}{c^3}\right) \cdot \frac{\cancel{h\nu}}{e^{h\nu/KT} - 1} \quad \frac{\cancel{h\nu}}{KT}$$

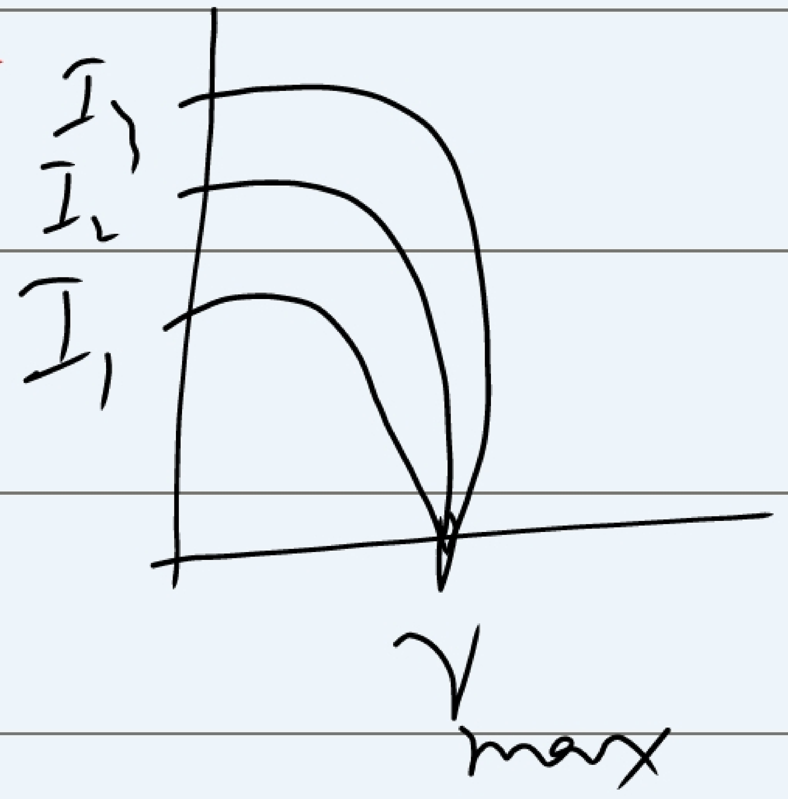
① For large ν: $\frac{h\nu}{KT} >> 1$, $h\nu >> KT$, $e^{h\nu/KT} \to \infty$

② For smaller ν, $\frac{h\nu}{KT} << 1$, $h\nu << KT$, $e^{h\nu/KT} \approx 1 + \frac{h\nu}{KT}$

$e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\quad e^{h\nu/KT} - 1 \approx \frac{h\nu}{KT}$, $I =$

Photoelectric Effect :-



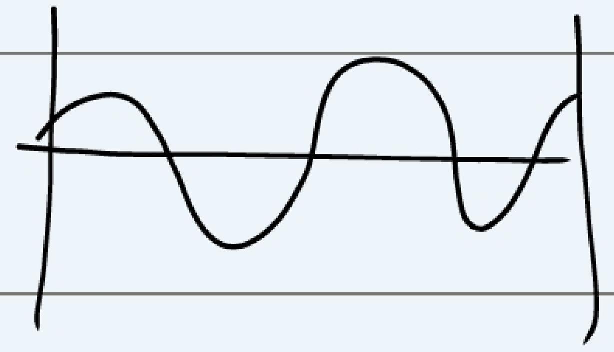


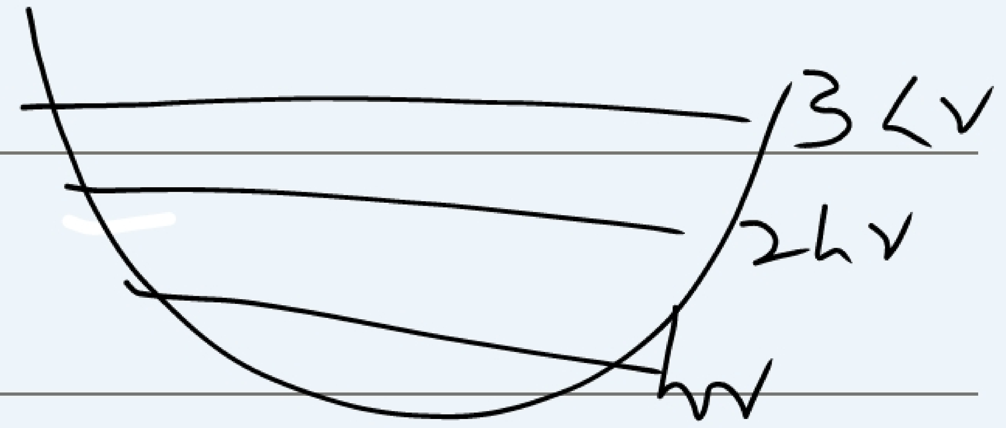
$\mathcal{E} = k_B T$

$\boxed{\mathcal{E} = nh\nu}$

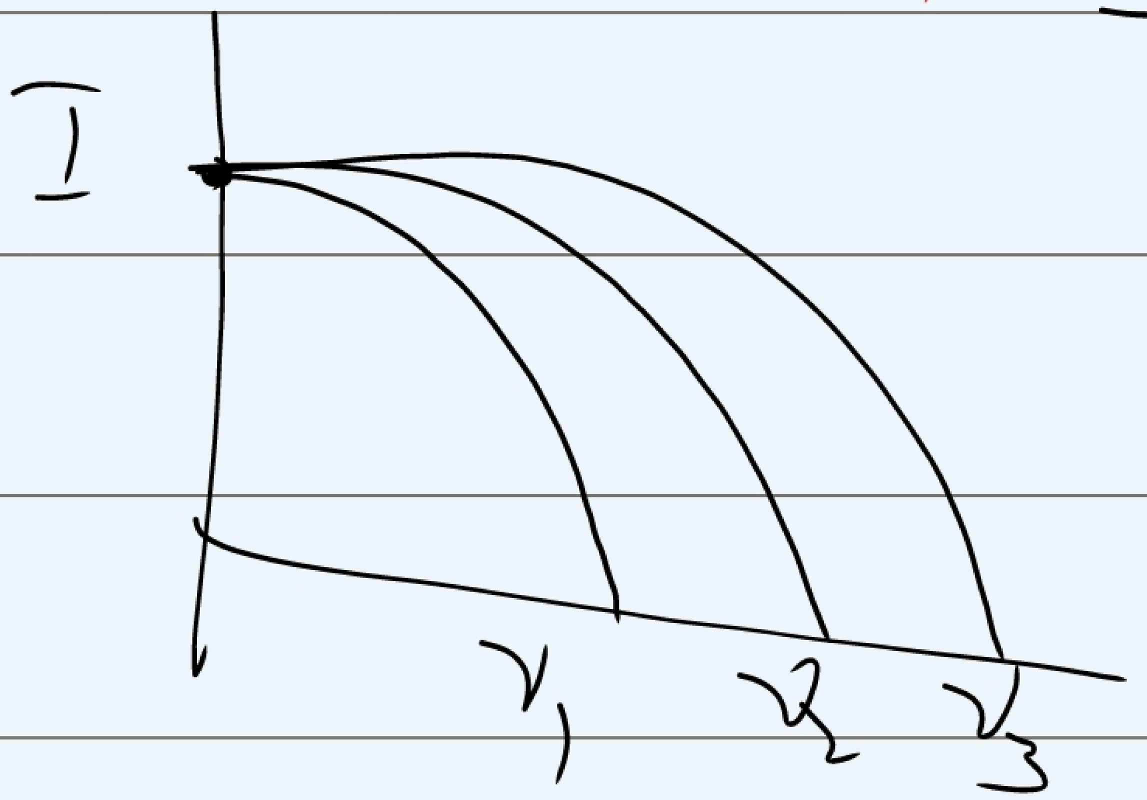
$\mathcal{E}_1 = h\nu$

$\mathcal{E}_2 = 2h\nu$





$$eV_{max} = h\nu_0 + h\nu$$



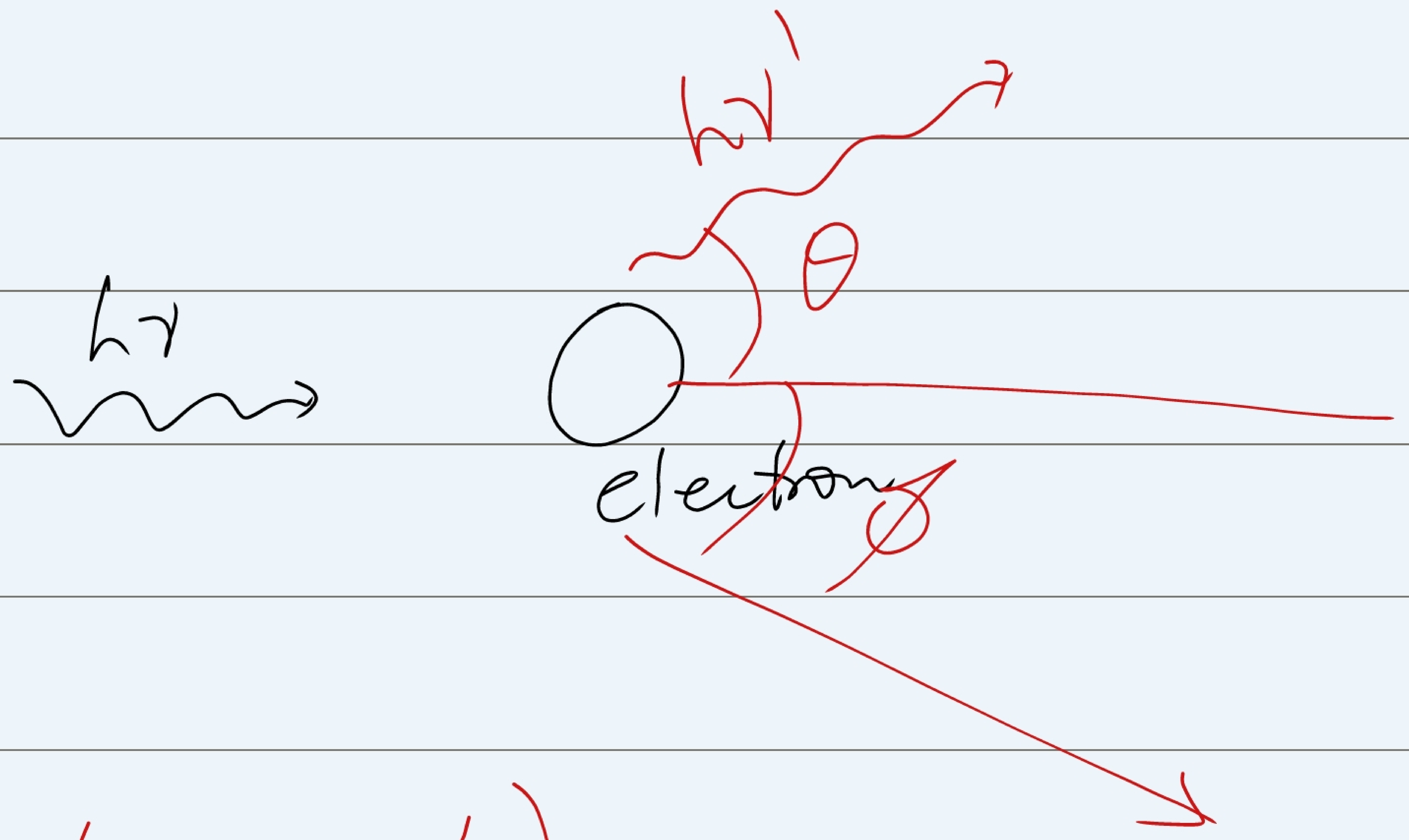
$$E = \frac{1}{2}KA^2$$

② Compton Effect :→

Planck's C

$E = nh\nu$

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

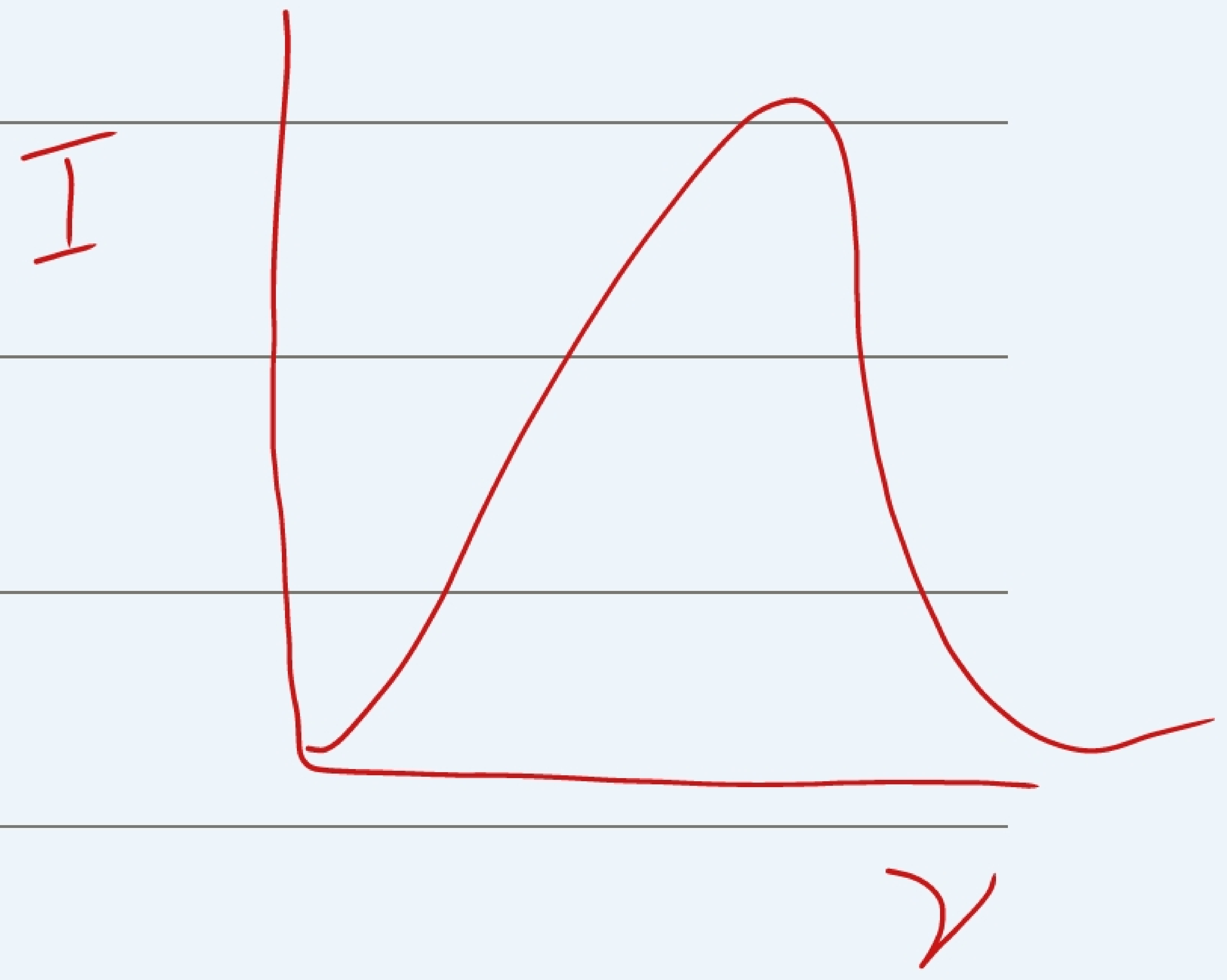


$$h(\nu - \nu') \rightarrow \text{Quantized}$$

$$\frac{h}{mc}\left[1 - \cos\theta\right]$$

$$\boxed{\frac{8\pi\nu^2}{c^3}}$$

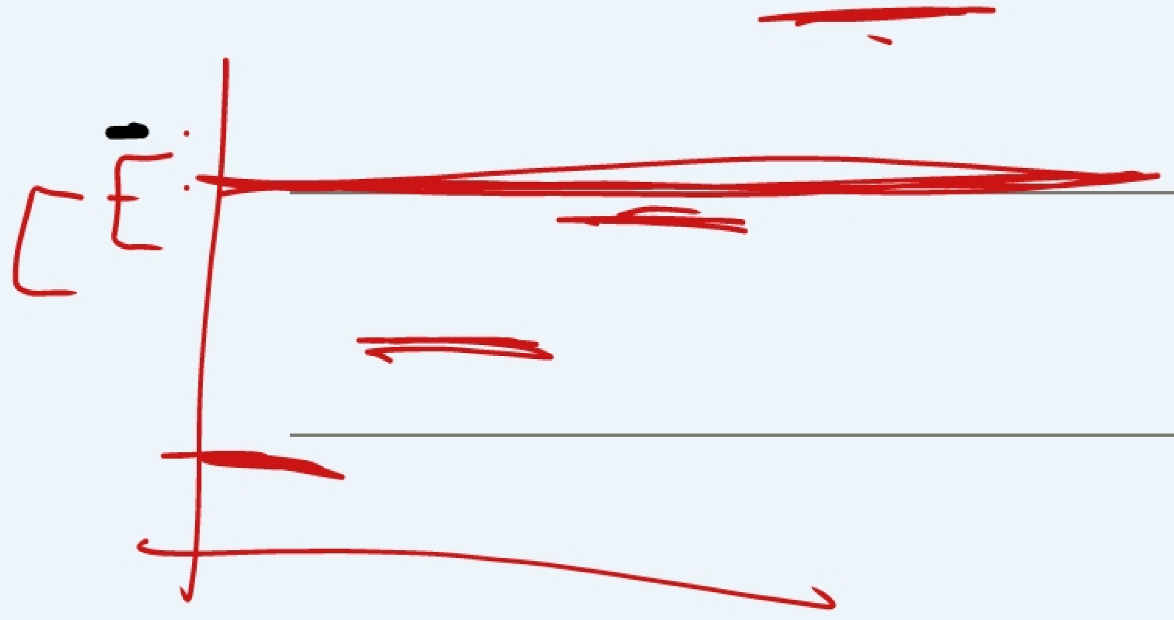
Av. Eng $= k_B T$ $\leftarrow$ RJ: $I \propto \nu^2$



Classical System:

HO:

$\bar{E} = \frac{1}{4} K A^2$



$$\frac{nh\nu}{e^{h\nu/kT} - 1}$$

Planck:

$E_n = nh\nu$

Compton Effect :→ $E = h\nu$

A. Beiser

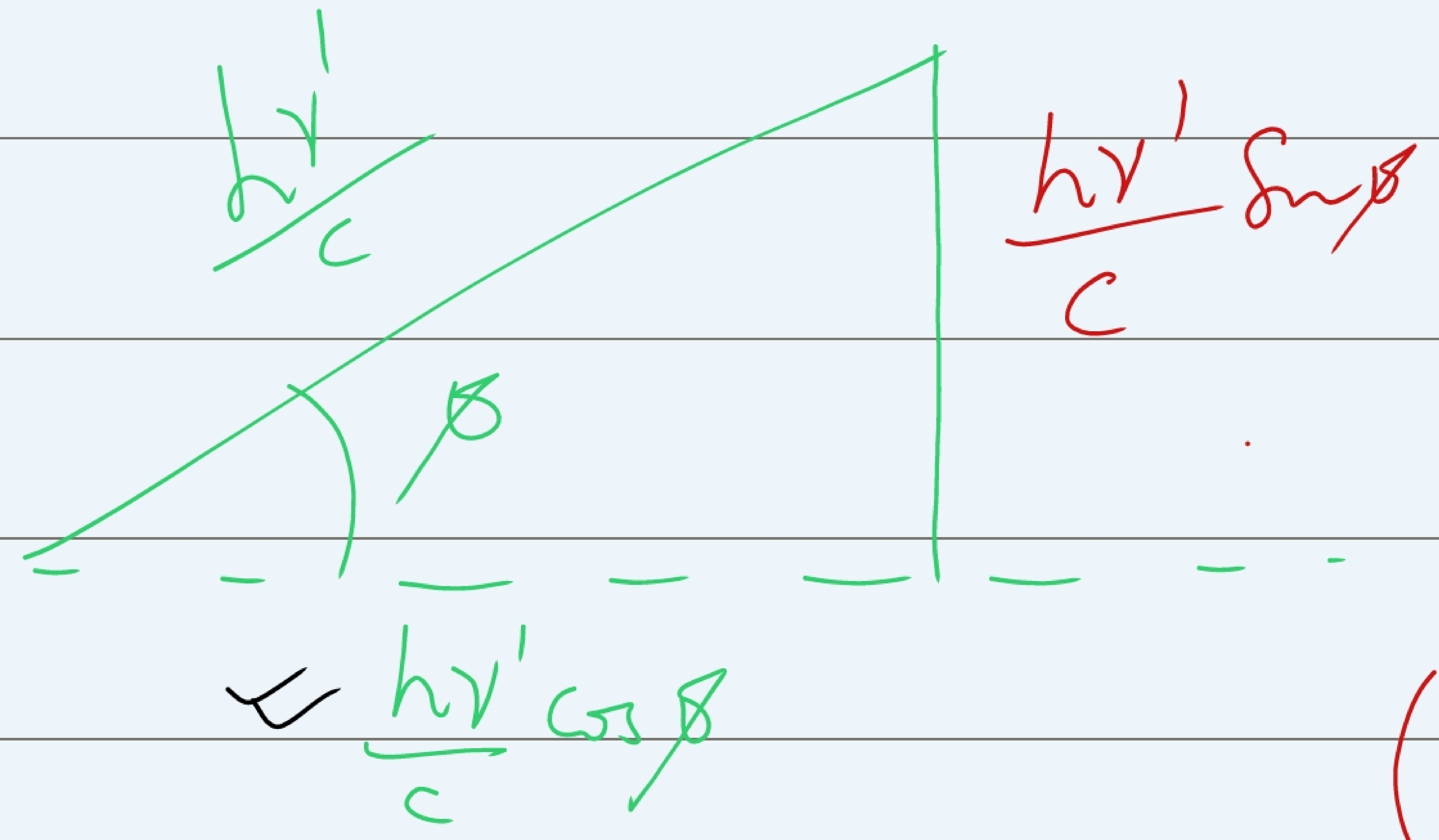
(Particle nature of wave):

$p = \frac{E}{c} = \frac{h\nu}{c}$

STR:

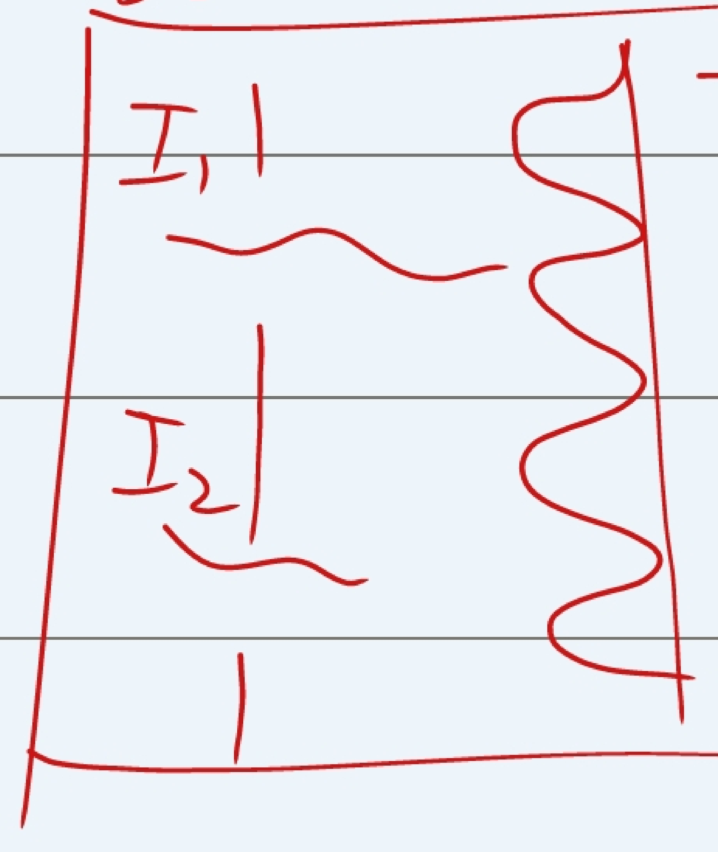
Scattered electron

$E = \sqrt{p^2c^2 + m^2c^4}$

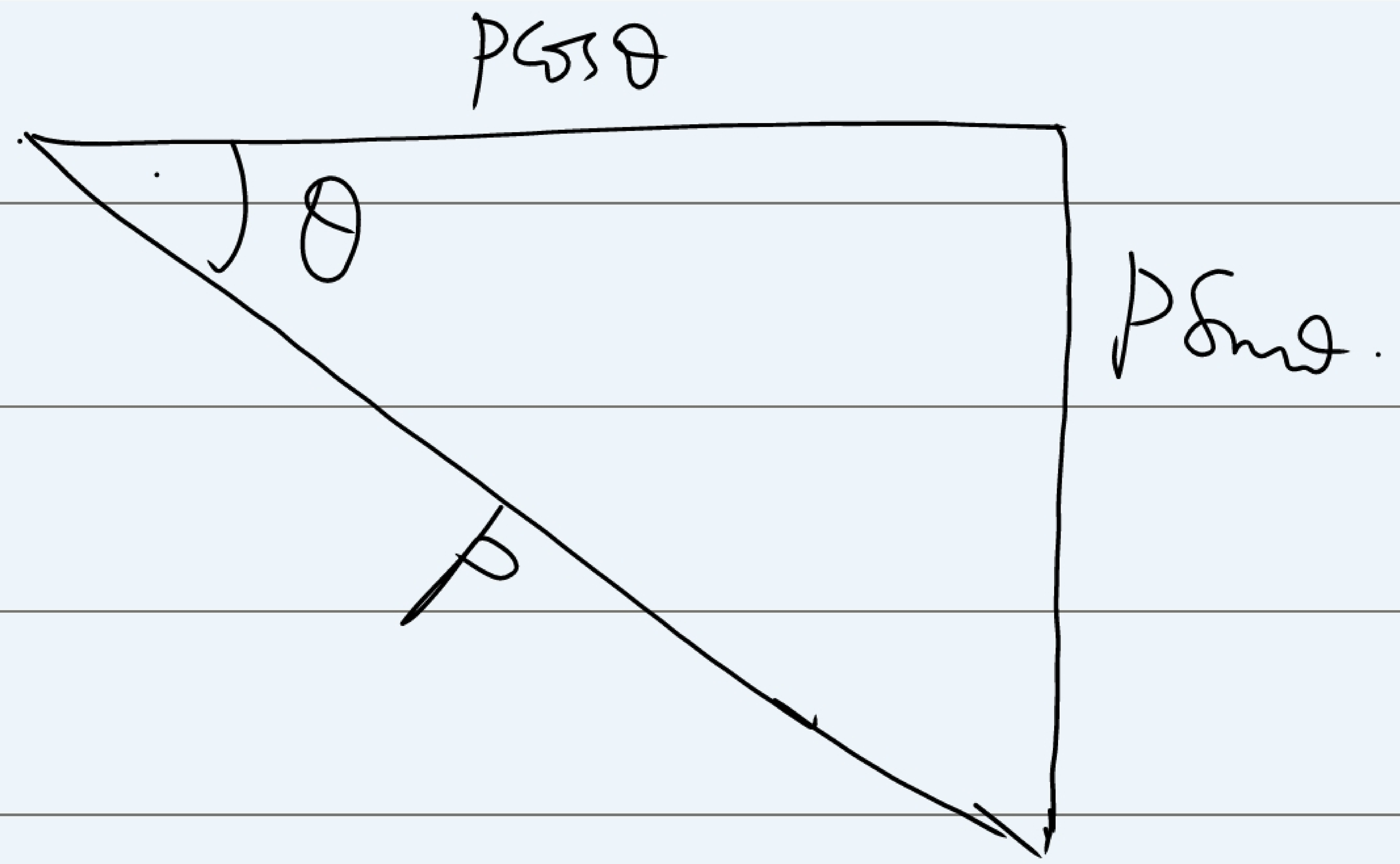


$(\lambda' - \lambda) = \text{Constant}\,[1 - \cos\phi]$

$E = h\nu$



$I_T = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$.



Horizontal: $\dfrac{h\nu}{c} + 0 = \dfrac{h\nu'}{c}\cos\phi + p\cos\theta$ — (1)

$\hookrightarrow h\nu = h\nu'\cos\phi + pc\cos\theta \Rightarrow pc\cos\theta = h\nu - h\nu'\cos\phi$

Vertical: $0 = \dfrac{h\nu'}{c}\sin\phi - p\sin\theta$ — (2)

$c \times$ (1) & (2)

$0 = h\nu'\sin\phi - pc\sin\theta \Rightarrow pc\sin\theta = h\nu'\sin\phi$

L.H.S $\Rightarrow$ $\boxed{p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\phi}$

$$E = K.E + mc^2 = \sqrt{m^2c^4 + p^2c^2}$$

$$K.E = h(\nu - \nu')$$

$$\Rightarrow (K.E + mc^2)^2 = m^2c^4 + p^2c^2$$

$$\nu = \frac{c}{\lambda}$$

$$\Rightarrow K.E^2 + \cancel{m^2c^4} + 2(K.E)(mc^2) = \cancel{m^2c^4} + p^2c^2$$

$$\Rightarrow p^2c^2 = K.E^2 + 2(K.E)(mc^2)$$

$$\cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu')\cos\phi = \cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu') + 2mc^2(h\nu - h\nu')$$

$$\boxed{(\lambda' - \lambda) = \left(\frac{h}{mc}\right)[1 - \cos\phi]}$$

Divide by $2h^2c^2$

$$\Rightarrow \frac{\cancel{2}mc^{\cancel{2}}}{\cancel{2}h^2\cancel{c^2}}(h\nu - h\nu') = \frac{\cancel{2}(\cancel{h}\nu)(\cancel{h}\nu')}{\cancel{2}\cancel{h^2}c^2}[1 - \cos\phi]$$

$$\Rightarrow \frac{mh}{h^2}\left[\frac{c}{\lambda} - \frac{c}{\lambda'}\right] = \frac{1}{\cancel{c^2}}\cdot\frac{\cancel{c}}{\lambda}\cdot\frac{\cancel{c}}{\lambda'}[1 - \cos\phi] \Rightarrow \frac{mc}{h}[\lambda' - \lambda] = (1 - \cos\phi)$$

Wave properties of particles :→

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p}.$$

Debroglie :- material : $\vec{p} = \gamma m \vec{v}$, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$\lambda_{dB} = \frac{h}{\gamma m v}.$$

$$V_{group} \simeq \frac{\Delta \omega}{\Delta K} = \frac{d\omega}{dK}.$$

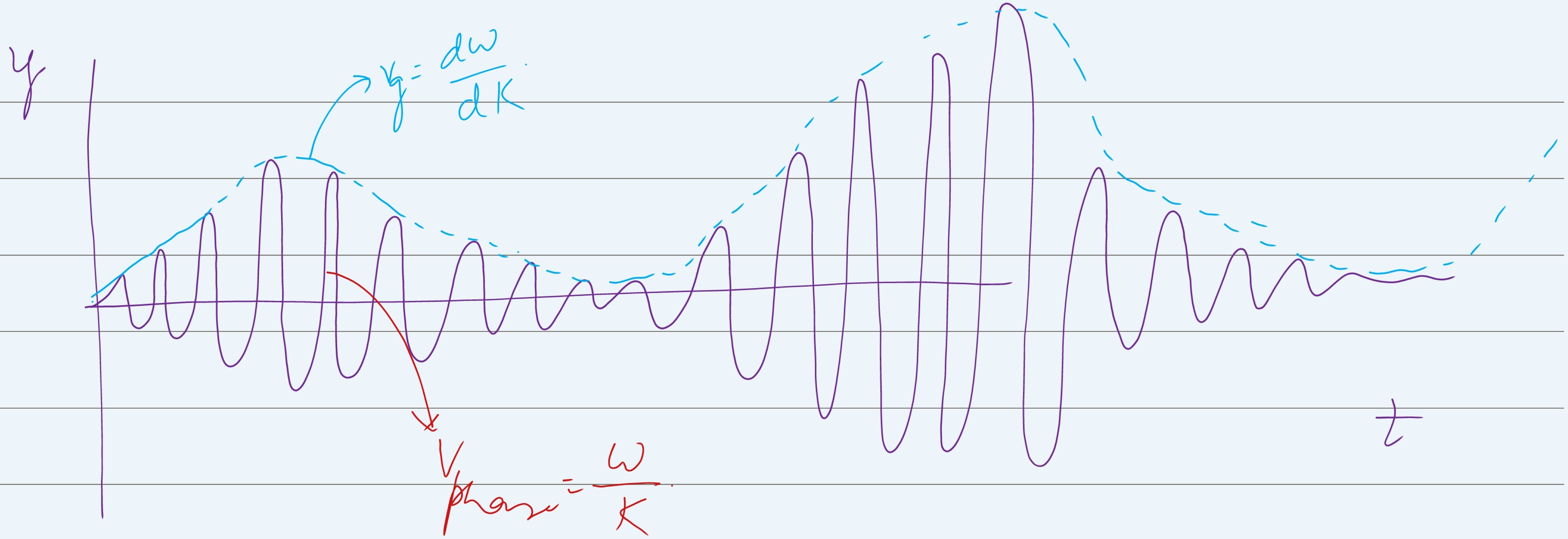
$$y_1 = A\cos(\underline{\omega t - Kx})$$

$$+ \qquad +$$

$$y = y_1 + y_2 = y_2 = A\cos\left[(\omega + \Delta\omega)t - (K + \Delta K)x\right]$$

$$V_{phase} \approx \frac{2\omega}{2K} = \frac{\omega}{K}.$$

$$= \cancel{2A} \cos\frac{1}{2}\left[\underline{\left(\frac{2\omega + \Delta\omega}{2}\right)t - \left(\frac{2K + \Delta K}{2}\right)x}\right] \cos\frac{1}{2}\left[\frac{\Delta\omega}{2}t - \frac{\Delta K}{2}x\right]$$



y
$v_g = \frac{d\omega}{dk}$
$v_{phase} = \frac{\omega}{k}$
t

Davisson & Germer / Particle Diffraction:

electron

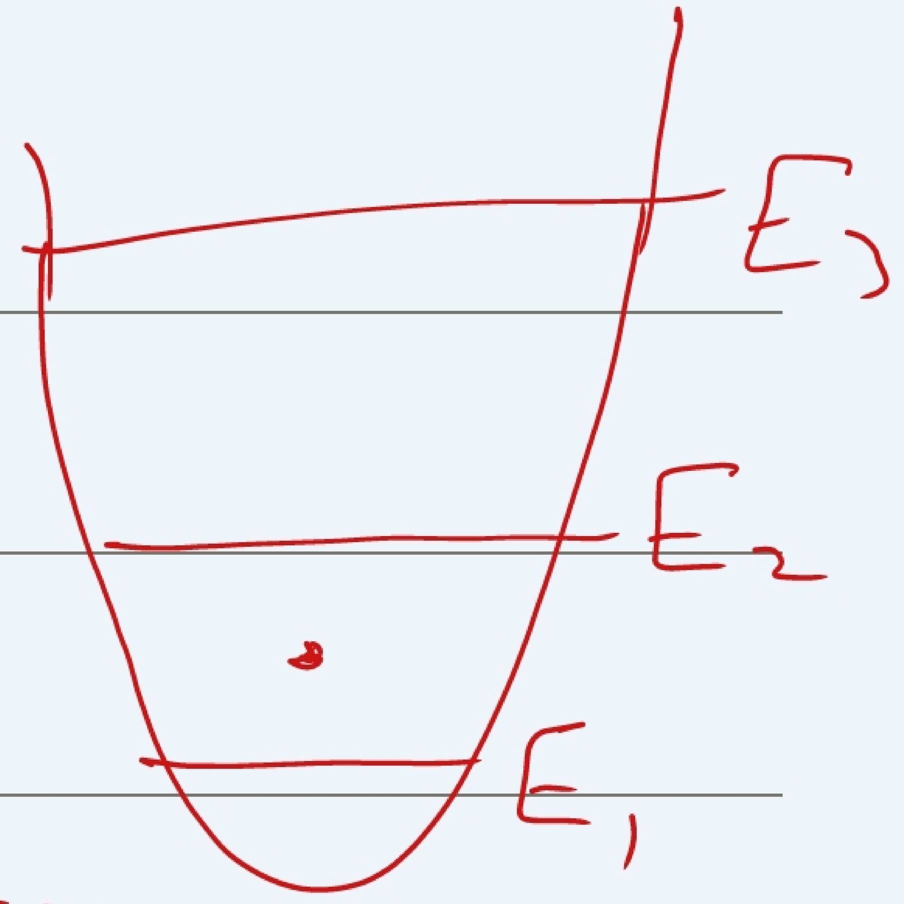
↓

Nickel crystal

$$K.E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$K.E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}, \qquad \frac{\lambda}{2} = L$$

$$\Rightarrow \lambda = \frac{2L}{n}, \quad n = 1, 2, \ldots$$



$$= \frac{n^2 h^2}{8mL^2}$$

$$n = 1, 2, 3 \ldots$$

$$y_1 = A\cos(\omega t - kx)$$

$$+ \quad y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$$y \simeq 2A\cos[2\omega t - 2kt]\cos[\Delta\omega t - \Delta k x]$$

$$V_{phase} = \frac{\omega}{K} = \frac{2\pi\nu}{\left(\frac{2\pi}{\lambda}\right)} = \underline{\lambda\nu} = \frac{h}{\gamma m v} \cdot \frac{\gamma m c^2}{h} = \frac{c^2}{v}$$

$$E = h\nu = \gamma m c^2$$

$$\nu = \frac{\gamma m c^2}{h}$$

$$V_{group} = \frac{d\omega(\nu)}{dK}$$

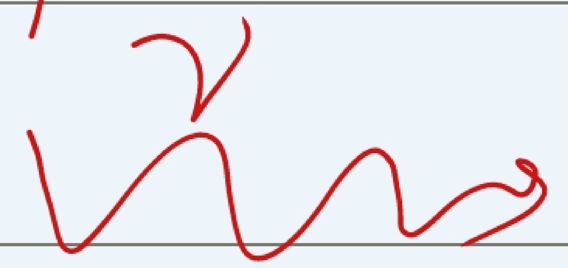
$$\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$$

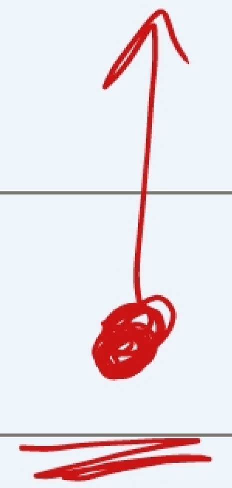
$$\boxed{\frac{d\omega}{dK} = c} \qquad d\omega = -\frac{2\pi c}{\lambda^2} \cdot d\lambda$$

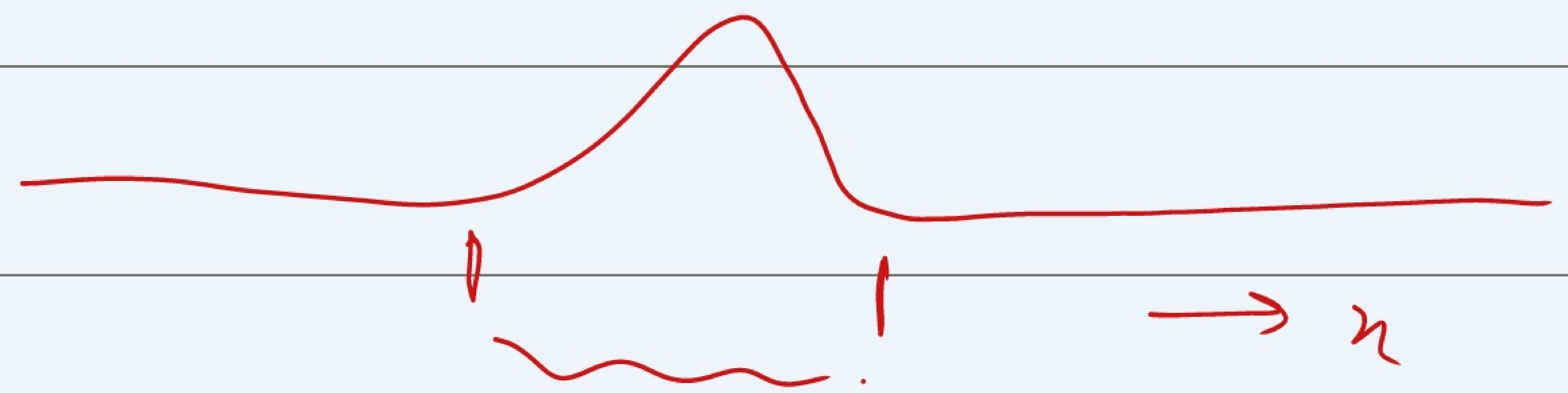
$$K = \frac{2\pi}{\lambda}, \quad dK = -\frac{2\pi}{\lambda^2} d\lambda$$

Heisenberg Uncertainty Principle:→

$\Delta p \approx \frac{h}{\lambda}$ $\Delta x \sim \lambda$







$p = \frac{h}{\lambda}$.

$\Delta x \to$ small

$\Delta p \to$ large.

$\delta p \to$ small.

$\Delta x \to$ large.

$\Delta x \, \Delta p \approx h$

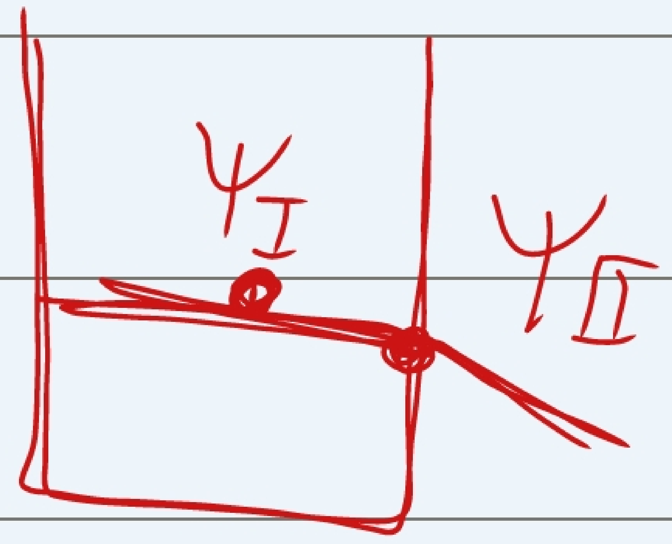
Postulates of Quantum Mechanics →

1. All physical systems have a wavefunction

$e^{i\theta} = \cos\theta + i\sin\theta$

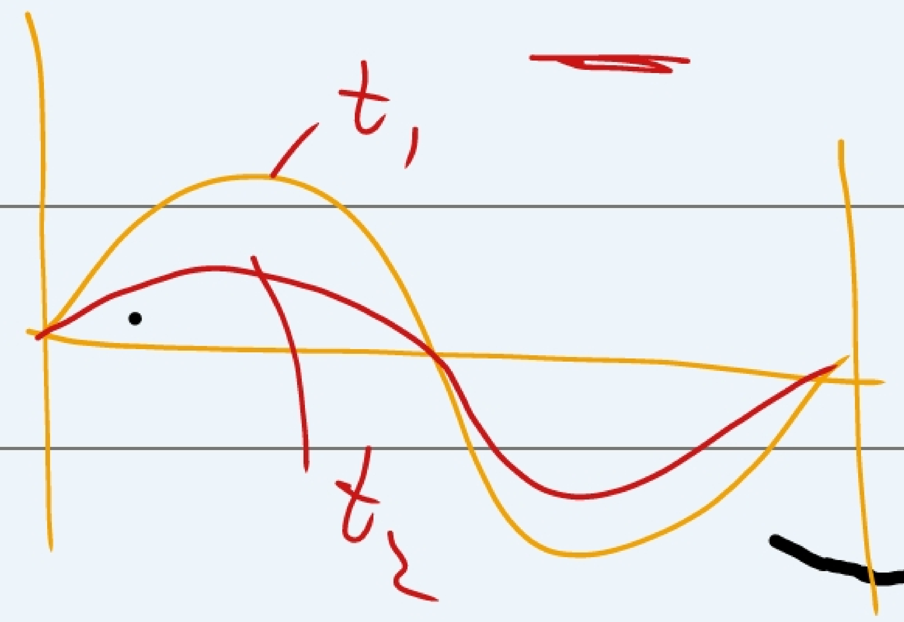
Well-behaved :- $\Psi = A + iB$

— Single valued (x, t)



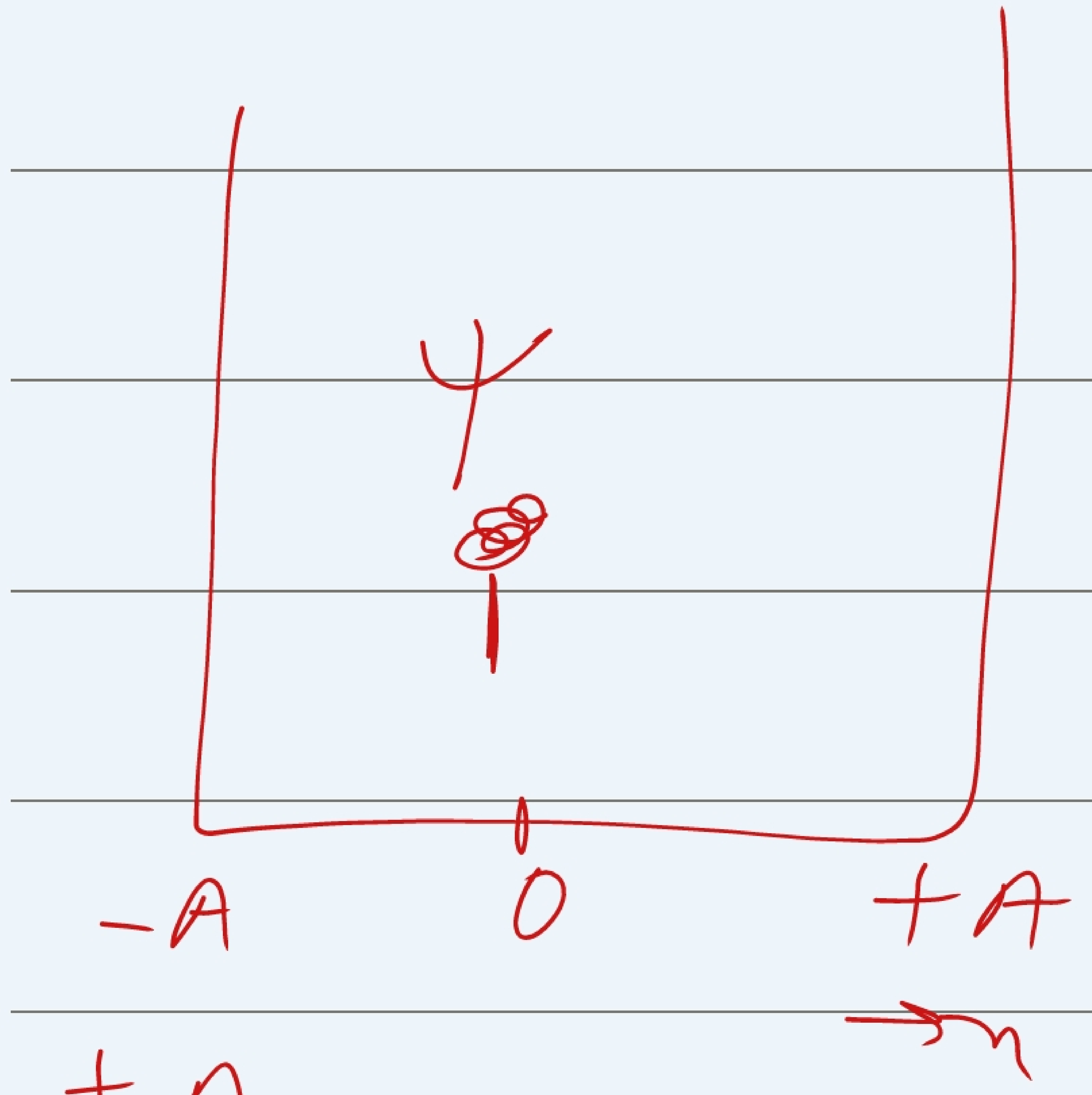
— Continuous and differentiable

$\frac{\partial\Psi}{\partial x}, \frac{\partial\Psi}{\partial y}, \frac{\partial\Psi}{\partial z}, \frac{\partial\Psi}{\partial t}$



$t = t_1, t_2$

— Square Integrable



$$\psi = A + iB$$

$$\psi^* \psi = A^2 + B^2$$

$$|\psi|^2 = A^2 + B^2$$

$$\int_{-A}^{+A} \psi^* \psi \, dx = 1$$