Indian Institute of Information Technology Vadodara
MA 101: Introduction to Discrete Mathematics
Tutorial 2

1. For each of the sequence of integers given below, give a simple formula or rule that gives any term of the sequence which start with the list.
   (a) $15, 8, 1, -6, -13, -20, -27, \ldots$
   (b) $3, 5, 8, 12, 17, 23, 30, \ldots$
   (c) $2, 3, 7, 25, 121, 721, 5041, 40321, \ldots$
2. Find the value of following sums
   (a) $\sum_{j=0}^{8} 3.2^j$
   (b) $\sum_{j=0}^{8} 3.(-2)^j$
   (c) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$
   (d) $\sum_{i=0}^{2} \sum_{j=0}^{3} 1$
3. Let $f, g : A \to A$ be functions. If g is surjective and $f \circ g$ is injective then does it follow that f is injective? Give justification.
4. Let A, B, C be three subsets of a universal set U. Draw a Venn diagram and shade the area representing $A \cup (B \cap C)^c$.
5. Find a bijective map $f : [a, b] \to [c, d]$ thereby proving that $|[a, b]| = |[c, d]|$ for any $a, b, c, d \in \mathbb{R}$.
6. Prove that $|\mathbb{R}| = |(0, 1)|$.
7. Determine whether each of the following sets is countable or uncountable(not countable).
   (a) $B = \{(x, y) | x \in \mathbb{N}, y \in \mathbb{Z} - \{0\}\}$.
   (b) $C = \mathbb{R} \setminus \mathbb{Q}$.
   (c) $A =$ set of all complex numbers.
8. Suppose that f is a function from A to B. We define the function S_f from $P(A)$ to $P(B)$ by the rule $S_f(X) = f(X)$ for each subset X of A. If f is injective then show that S_f is injective. Can you prove similar statement for surjectivity of S_f? What about converse?
9. Show that the set of all binary strings is countable, thereby proving set of all programs is countable.
10. A function is computable if there exists an algorithm that can do the job of the function. We can show that a problem is computable by describing a procedure and proving that the procedure always terminates and always produces the correct answer. Give an example of a computable function.
11. (HILBERT'S GRAND HOTEL) We now describe a paradox that shows that something impossible with finite sets may be possible with infinite sets. The famous mathematician David Hilbert invented the notion of the Grand Hotel, which has a countably infinite number of rooms, each occupied by a guest. When a new guest arrives at a hotel with a finite number of rooms, and all rooms are occupied, this guest cannot be accommodated without evicting a current guest. However, we can always accommodate a new guest at the Grand Hotel, even when all rooms are already occupied. How can we accommodate a new guest arriving at the fully occupied Grand Hotel without removing any of the current guests?