30/10/2023

$E<0, \quad 0<\epsilon<1$, $\quad r = \dfrac{r_0}{1-\epsilon\cos\theta}$ $\quad \begin{cases} r_0 = l^2/Mc \\ \epsilon = \sqrt{1+\dfrac{2El^2}{Mc^2}} \end{cases}$

Ellipse.

① When $\theta = 0$, $\boxed{r_{max} = \dfrac{r_0}{1-\epsilon}}$

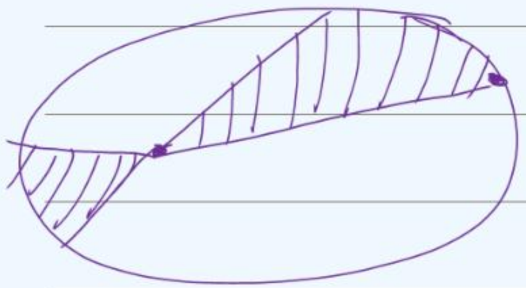
② When $\theta = \pi$, $\boxed{r_{min} = \dfrac{r_0}{1+\epsilon}}$

$\boxed{\dfrac{r_{max}}{r_{min}} = \dfrac{1+\epsilon}{1-\epsilon}, \quad \epsilon=0, \text{ circle}, \quad \epsilon \simeq 1}$

Length of major axis $= A = r_{max} + r_{min} = \dfrac{2r_0}{1-\epsilon^2} = \dfrac{2l^2/Mc}{1-\left[1+\dfrac{2El^2}{Mc^2}\right]}$

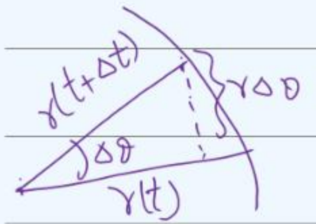
$= -\dfrac{C}{E}$

Kepler's Laws of Planetary Motion →



1. Fixed elliptical orbits.
2. Sweeps equal area in equal time.
3. $T^2 \propto A^3$

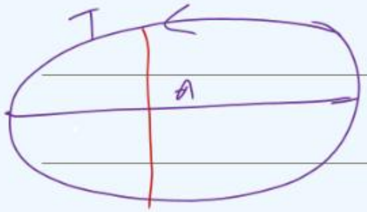
$\vec{F} = f(r)\hat{r}$, $\tau = 0$, $\vec{L}$ = constant. $|L| = mr^2\dot{\theta}$



$$\Delta A \simeq \frac{1}{2}(r+\Delta r)\, r\Delta\theta$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t} + \frac{1}{2}\frac{r\,\Delta r\,\Delta\theta}{\Delta t} \to 0 \simeq \frac{1}{2} r^2 \dot{\theta} \simeq \frac{L_z}{2m}$$

$$\boxed{\Delta A \simeq \frac{1}{2} r^2 d\theta}$$



$$L = \mu r^2 \frac{d\theta}{dt} \Rightarrow \frac{L}{2\mu} dt = \frac{\cancel{\mu} r^2}{2\cancel{\mu}} \frac{d\theta}{dt} dt$$

$$= \frac{1}{2} r^2 d\theta$$ = Area swept in the $\Delta\theta$ radians

$$\longrightarrow \frac{L}{2\mu} \cdot T = \pi a b$$

$$a = \frac{A}{2} = r_{max} + r_{min} = -\frac{C}{2E} \, , \quad b = \frac{l}{\sqrt{-2\mu E}}$$

$$\Rightarrow T^2 \propto A^3$$

Failure of classical Mechanics