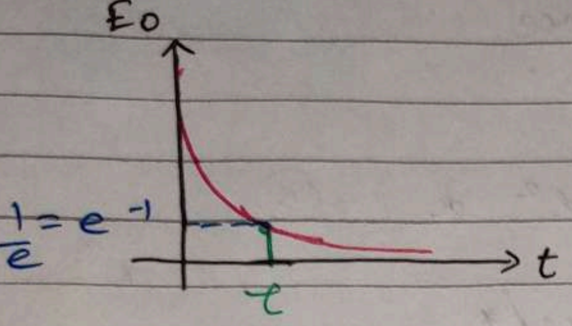


$$\gamma\tau = 1 \Rightarrow \tau = \frac{1}{\gamma} = \frac{m}{b}$$

$$\gamma t = 1 \Rightarrow t = \frac{1}{\gamma} = \frac{m}{b}$$

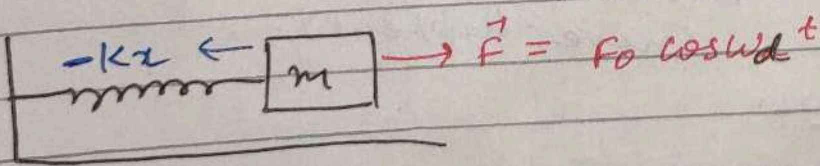
Q-factor = $\dfrac{\text{Energy stored}}{\text{Energy dissipiated per radian}}$

$$\Delta E = \left|\frac{dE}{dt}\right| \cdot \Delta t = \gamma E \Delta t = \frac{\gamma E}{\omega} \quad (\omega t = 2\pi)$$

$$\frac{\gamma E}{\omega}$$

$$\text{Q-factor} = \frac{\gamma E \Delta t}{E / 2\pi} \quad \frac{\gamma E}{\omega \cdot E} = \frac{\gamma}{\omega}$$

Forced (Driven) Harmonic oscillation:-



$$F_{net} = -kx + F_0 \cos \omega_d t$$

$$m\ddot{x} + kx = F_0 \cos \omega_d t$$

$$\ddot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \omega_d t \quad -(A)$$

for hit and trial option, we choose

- $x(t) = A\cos\omega_d t$

$\ddot{x}(t) = -\omega_d^2 A\cos\omega_d t = -\omega_d^2 x$ — (B)

$$(\omega_0^2 - \omega_d^2) A \cancel{\cos\omega_d t} = \frac{F_0}{m} \cos\cancel{\omega_d t}$$

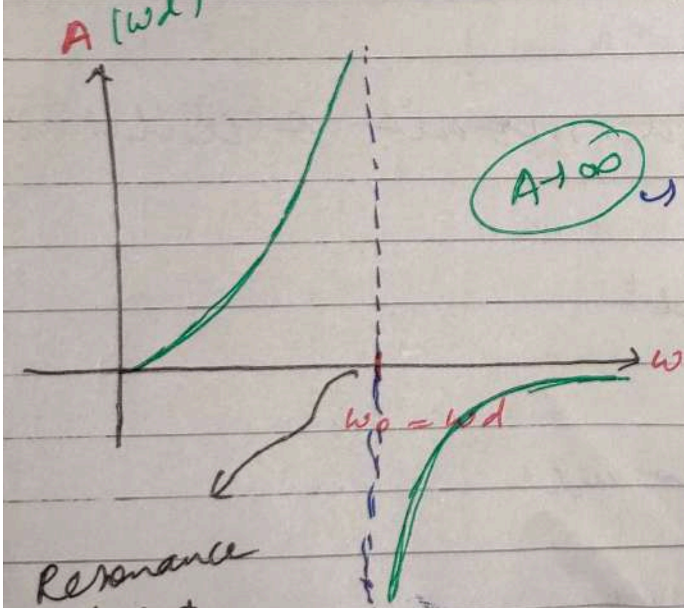
(we have choose $F = F_0 \cos\omega t$, because it will cancel out here)

$$A = \frac{F_0}{m} \frac{1}{(\omega_0^2 - \omega_d^2)}$$

$$\boxed{x(t) = \frac{F_0}{m(\omega_0^2 - \omega_d^2)} \cdot \cos\omega_d t}$$

$\omega_d \to$ driving frequency

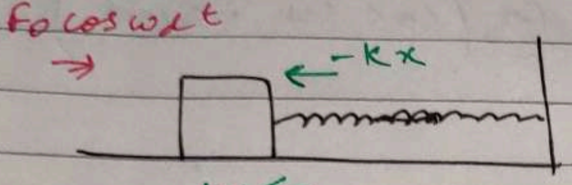
$\omega_0^2 = \frac{K}{m}$



$A \to \infty$ → physical problem (never happens)

Resonance point ($\omega_0 = \omega_d$)

Forced damped Harmonic oscillation: (Damped & driven) :-



kx (spring force)
bv (viscous force)
$F_0 \cdot \cos \omega_d t$ (driven force)
$\omega_d \rightarrow$ driven frequency

$$\vec{F}_{net} = -kx - b\dot{x} + F_0 \cos \omega_d t \qquad (bv = b\dot{x})$$

$$m\ddot{x} + b\dot{x} + kx = F_0 \cdot \cos \omega_d t$$

$$\Downarrow$$

$$\boxed{\ddot{z} + \gamma \dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i\omega_d t}}$$ → complex domain → (A)

$z = x + iy$ $\qquad F_{0y} = i \sin \omega_d t \, F_{0y}$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

~~Hit~~ trial
$z(t) = Ae^{i(\omega_d t - \delta)}$ → (B)

→ There will be a phase lack between displacement and applied force, which is acting over the system
→ It take some time for the system to 'feel' the force (phase lag can be zero or non-zero)

$$\dot{z}(t) = i\omega_d z$$
$$\ddot{z}(t) = -\omega_d^2 z$$

$$[-w_d^2 + i\gamma w_d + w_0^2]\, A e^{i(w_d t - \delta)} = \frac{F_0}{m} e^{i w_d t}$$

$$\Rightarrow \{[w_0^2 - w_d^2) + i\gamma w_d\}\, A = \frac{F_0}{m} \cdot e^{i\delta}$$

$$= \frac{F_0}{m}\left(\cos\delta + i\sin\delta\right)$$

- Real value = Real value
- Imaginary value = Imaginary value

$$A(w_0^2 - w_d^2) = \frac{F_0}{m}\cos\delta \quad -(3)\ (\text{Real})$$

$$\gamma w_d \cdot A = \frac{F_0}{m}\sin\delta \quad -(4)\ (\text{Imaginary})$$

$(3)^2 + (4)^2$

$$A^2\{\gamma^2 w^2 d^2 + (w_0^2 - w_d^2)^2\} = \frac{F_0^2}{m^2}$$

$$A^2 = \frac{F_0^2}{m^2} \cdot \frac{1}{(\gamma^2 w^2 d^2 + (w_0^2 - w_d^2)^2}$$

$$\boxed{A = \frac{F_0}{m}\sqrt{\frac{1}{(\gamma w_d)^2 + (w_0^2 - w_d^2)^2}}}$$

$(4)/(3)$

$$\tan\delta = \frac{\gamma w_d}{(w_0^2 - w_d^2)} \quad \Rightarrow \delta \rightarrow \text{can be found there.}$$

$$\cancel{x}\, z(t) = \frac{F_0}{m}\sqrt{\frac{1}{(\gamma w_d)^2 + (w_0^2 - w_d^2)^2}}\; e^{i(w_d t - \delta)}$$

$$x(t) = \mathrm{Re}[z(t)] = A w_d \cos(w_d t - \delta)$$

$\omega_0^2 = \frac{K}{m}$ ⇒ Natural frequency of system

$\omega = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$

$\omega_d \rightarrow$ driven frequency

Case-I $\omega_d \rightarrow 0$

$$A(\omega_d) \approx \frac{F_0}{m} \cdot \frac{1}{\omega_0^2}$$

$$\tan\delta = \frac{0}{\omega_0^2} \Rightarrow \tan\delta = 0$$
$$\delta \rightarrow 0$$

Case-II $\omega = \omega_d$ $A(\omega_d)$ ⇒ Amplitude is function of ω_d

$$A(\omega_d) = \frac{F_0}{m} \cdot \frac{1}{\gamma\omega_d}$$

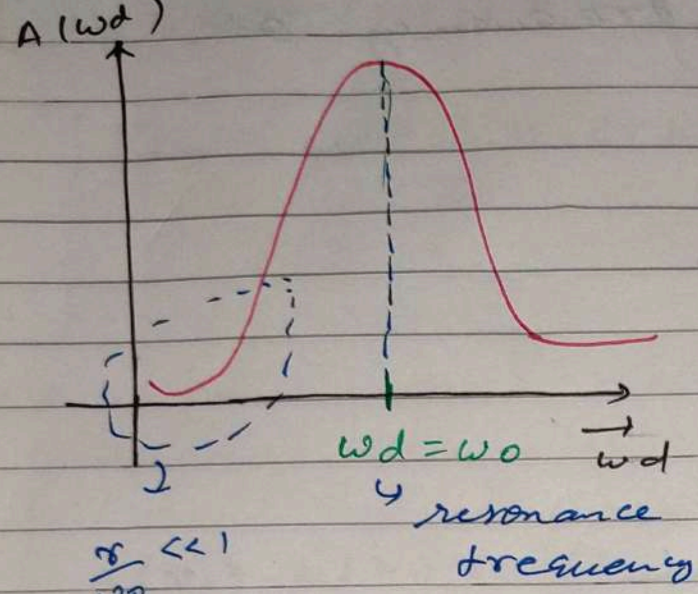
$$\tan\delta = \frac{\gamma\omega_d}{0} \rightarrow \infty \qquad \delta \rightarrow \pi/2$$

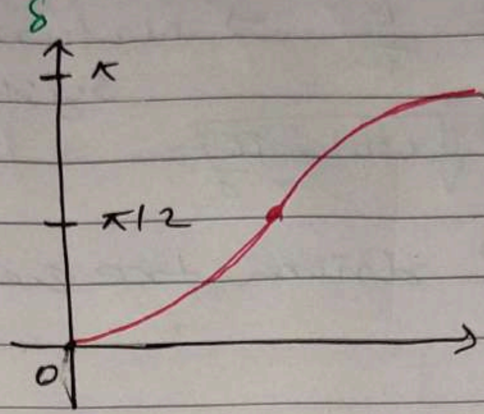
Case-III $\omega_d \rightarrow \infty$

$$A(\omega_d) = \frac{F_0}{m} \cdot \frac{1}{\rightarrow\infty} \rightarrow 0$$

$(\omega_d >>> \omega_0)$ $(\omega_0^2 - \omega_d^2 \approx \omega_d^2)$

$$\tan\delta = \frac{\gamma\omega_d}{\omega_0^2 - \omega_d^2} \rightarrow 0 \rightarrow \delta = \pi$$





$$\boxed{\frac{\gamma}{\omega_0} << 1}$$

$\simeq 1$

daming $E = E_0 \cdot e^{-\gamma t}$ $\quad \gamma t = 1$ $\quad$ } Lightly damped

$\underset{\text{Energy}}{E}$ $\quad Q = \frac{\omega_0}{\gamma}$

Energy in case of Forced damped Oscillation

$x(t) = A\cos(\omega_d t - \delta)$

$v(t) = -\omega_d A \sin(\omega_d t - \delta)$

$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m \omega_d^2 A^2 \sin^2(\omega_d t - \delta)$

$P.E = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega_d t - \delta)$

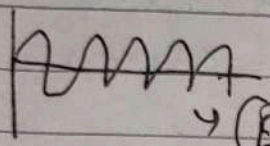
$\langle K.E \rangle = \frac{1}{4} m \omega_d^2 A^2$ $\quad \left\{ \langle \cos^2\theta \rangle = \frac{1}{2} \right.$

$\langle P.E \rangle = \frac{1}{4} k A^2$ $\quad \left. \langle \sin^2\theta \rangle = \frac{1}{2} \right\}$

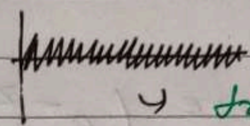
$$\langle E\rangle = \frac{1}{4} m \omega_d^2 A^2 + \frac{1}{4} K A^2$$

wavelength of visible light $\Rightarrow (380 - 760)\ nm$

$$c = \nu\lambda \Rightarrow \nu = \frac{c}{\lambda} = \frac{3\times 10^8}{380\times 10^{-9}} = 0.77\times 10^{15}\ Hz$$



y $\textcircled{E}$, don't take average



y frequency very high, so we take average of energy.

$$\langle E\rangle = \frac{1}{4} m A^2 (\omega_d^2 + \omega_0^2)$$

$$= \frac{1}{4} m \cdot \frac{F_0^2}{m^2} \cdot \frac{1}{(\gamma^2\omega_d^2 + (\omega_0^2 - \omega_d^2)^2} \times (\omega_d^2 + \omega_0^2)$$

$$\boxed{\langle E\rangle = \frac{1}{4} \cdot \underbrace{\frac{F_0^2}{m}}_{\text{constant value}} \cdot \frac{(\omega_d^2 + \omega_0^2)}{(\omega_0^2 - \omega_d^2)^2 + (\gamma\omega_d)^2}}$$

Focus on $\dfrac{(\omega_d^2 + \omega_0^2)}{(\omega_0^2 - \omega_d^2)^2 + (\gamma\omega_d)^2}$

- $\gamma << \omega_0$ (Lightly damped system)

$\omega_d \approx \omega_0$ $(\omega_d = 1.12\ Hz, \omega_0 = 1.125\ Hz)$

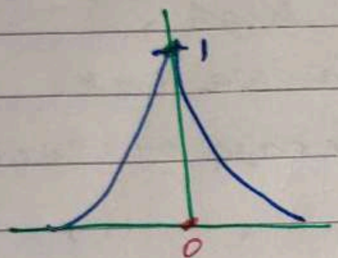
$$(\omega_0^2 - \omega_d^2)^2 = (\omega_0 + \omega_d)(\omega_0 - \omega_d)$$
$$\cong 2\omega_0(\omega_0 - \omega_d)$$

$$\langle E\rangle = \frac{1}{4}\,\frac{F_0^2}{m}\;\frac{\cancel{2}\,\omega_0^2}{\frac{\cancel{4}\,\omega_0^2}{2}(\omega_0-\omega_d)^2+\frac{\gamma^2\omega_0^2}{2}}$$

$$\boxed{\langle E\rangle = \underbrace{\frac{1}{8}\,\frac{F_0^2}{m}}_{\text{constant}}\cdot\underbrace{\frac{1}{(\omega_0-\omega_d)^2+\left(\frac{\gamma}{2}\right)^2}}_{\text{Lorentzian curve (Resonance curve)}}}$$

Gaussian curve $\left(e^{-z^2}\right)$

↳ this type of function very useful.

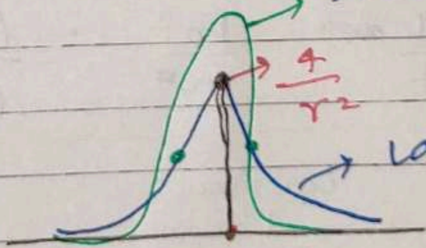


↳ class distribution marks.

↳ In nature most of things lie in average Range. → for that ~~th~~ ~~by~~ type of Nature, Gaussian curve is Best.

Let $\frac{F_0^2}{8m} = 1$ ↳ constant , at $(\omega_0=\omega_d)$

$$\langle E\rangle_{max} = \frac{4}{\gamma^2}$$



Half-maximum of this particular average energy (E_{max})

$$\langle E_{max}\rangle = \frac{4}{\gamma^2}$$

$$\langle \text{Energy}\rangle_{\text{Half-maximum}} = \frac{1}{2}\langle E_{max}\rangle = \frac{2}{\gamma^2}$$

$\langle E_{half\ max} \rangle = \dfrac{2}{\gamma^2} \Rightarrow \dfrac{1}{\frac{\gamma^2}{2}}$

$$\frac{1}{\underbrace{(\omega_0 - \omega_d)^2}_{\frac{\gamma^2}{4}} + \underbrace{\left(\frac{\gamma}{2}\right)^2}_{\frac{\gamma^2}{4}}} = \frac{1}{\frac{\gamma^2}{2}}$$

$\dfrac{\cancel{2}\gamma^2}{\cancel{4}} \Rightarrow \dfrac{\gamma^2}{2}$

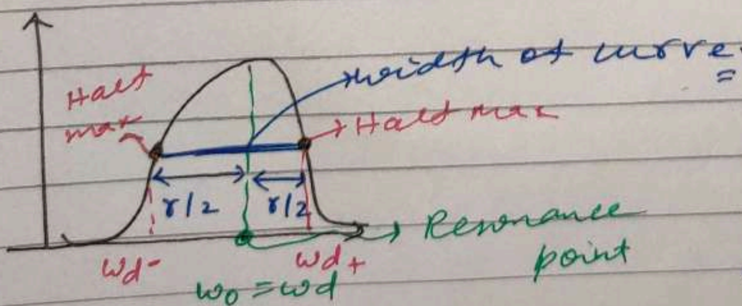
- $(\omega_0 - \omega_d)^2 = \dfrac{\gamma^2}{4} \Rightarrow$ for half maximum of $\langle E \rangle$

$\Downarrow$

$(\omega_0 - \omega_d) = \pm \dfrac{\gamma}{2}$

$\omega_{d-} = \omega_0 - \dfrac{\gamma}{2}$ $\qquad$ $\omega_{d+} = \omega_0 + \dfrac{\gamma}{2}$

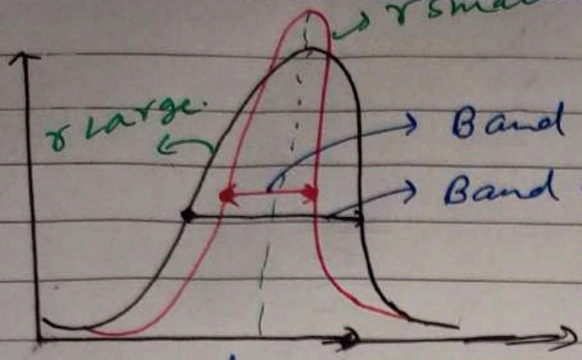
↳ Half-maximum



for Large width, Large γ

\# width / Resonance width $= \omega_{d+} - \omega_{d-} = \gamma$

for small width, Small γ



↳ selective value respones
(In Radio)

$Q = \frac{\omega_0}{\gamma} = \frac{\omega_0}{\Delta\omega}$ $\Delta\omega \to$ Resonance width
$\omega_0 \to$ Resonance frequency

$\Delta\omega \to$ Small
$Q \to$ Large } → your system will be respond for a particular value.

↳ high Q means System is very selective.