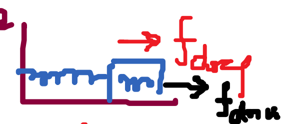
Lecture 15: Section 2

Thursday, February 3, 2022 8:57 AM

Oscillation

$E(t) = \frac{1}{2} KA^2$

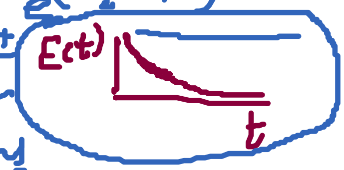


$f_{drag} = -bv$

- **Undamped:** $x(t) = A \cos(\omega_0 t + \phi)$

$E(t) = \frac{1}{2} KA^2 e^{-\gamma t}$

- **Damped:**
  - Lightly: $\omega_0^2 > \frac{\gamma^2}{4} = \frac{b^2}{4m^2}$ → $x(t) = A e^{-\frac{\gamma}{2}t} \cos(\omega t + \phi)$
  - Critically: $\omega_0^2 = \frac{\gamma^2}{4}$ → $x(t) = (A + Bt) e^{-\frac{\gamma}{2} t}$
  - Heavily: $\omega_0^2 < \frac{\gamma^2}{4}$ → $x(t) = A_+ e^{(-\frac{\gamma}{2} \pm \Gamma)t}$



Energy is continuous

Forced HO:

$-bv$ • Undamped

$\leftarrow f_{drag}$ • Damped.

m → $F_{driving}$ $F_0 \cos(\omega_d t)$

- **Undamped:** $\vec{F} = -\vec{f}_s + F_0 \cos \omega_d t$

$\Rightarrow m\ddot{x} = -Kx + F_0 \cos \omega_d t$

$\omega_0^2 = \frac{K}{m}$

homogenous ⇒ $\boxed{\ddot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \omega_d t}$

non-homogenous

$\Rightarrow \boxed{x(t) = A \cos \omega_d t} \Rightarrow \left[-\omega_d^2 + \omega_0^2\right] x = \frac{F_0}{m} \cos \omega_d t$

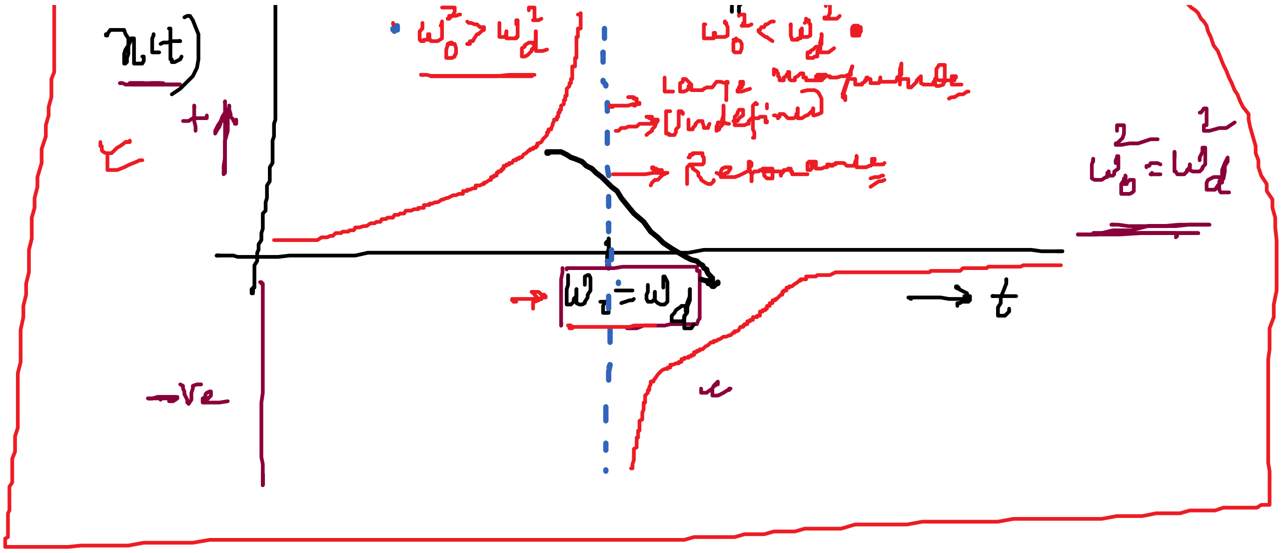
$\Rightarrow \left[\omega_0^2 - \omega_d^2\right] A \cos \omega_d t = \frac{F_0}{m} \cos \omega_d t$

~~$x(t) = A \cos \omega_d t$~~ $\Rightarrow \boxed{A = \frac{F_0}{m} \cdot \frac{1}{\left[\omega_0^2 - \omega_d^2\right]}}$

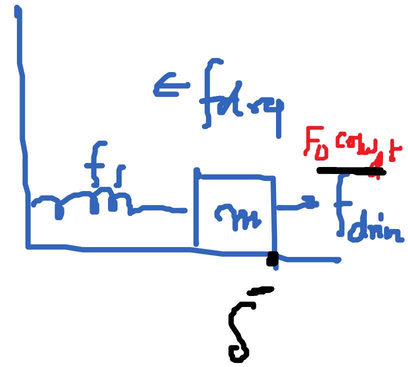
$x(t) = \text{homogenous} + \text{non-homogenous}$

$\Rightarrow x(t) = \frac{F_0}{m} \cdot \frac{1}{\left[\omega_0^2 - \omega_d^2\right]} \cos \omega_d t + A \cos(\omega_0 t + \phi)$

$x(t)$	• $\omega_0^2 > \omega_d^2$	$\omega_0^2 < \omega_d^2$ •
		Large magnitude



Forced damped Oscillator :→



$$\vec{F} = -\vec{f}_s - \vec{f}_{drag} + \vec{F}_{driving}$$

$$m\ddot{x} = -Kx - b\dot{x} + F_0 \cos\omega_d t$$

$$\Rightarrow \ddot{x} + \frac{b}{m}\dot{x} + \frac{K}{m}x = \frac{F_0}{m}\cdot\cos\omega_d t, \quad i\left[\ddot{y} + \gamma\dot{y} + \omega_0^2 y = \frac{F_0}{m}\cdot(\sin\omega_d t)\right]$$

y

$$\Rightarrow \boxed{\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i(\omega_d t)}}$$

Forced damped.

$$\gamma = \frac{b}{m}, \quad \omega_0^2 = \frac{K}{m}$$

$$z(t) = A e^{i(\omega_d t - \delta)}$$

Driving frequency.

It take some time to feel the impact of driving force.

$\delta \neq 0$

$(i\omega_d)^2 z(t)$

$$\Rightarrow \left[-\omega_d^2 + i\gamma\omega_d + \omega_0^2\right] z(t) = \frac{F_0}{m} e^{i(\omega_d t)}$$

$$\left[(\omega_0^2 - \omega_d^2) + i\gamma\omega_d\right] A e^{i(\cancel{\omega_d t} - \delta)} = F \ldots e^{i\cancel{\omega_d} t}$$

$$\Rightarrow \left[(\omega_0^2-\omega_d^2)+i\gamma\omega_d\right] A\, \cancel{e^{i(\omega_d t-\delta)}} = \frac{F_0}{m}\cancel{e^{i\omega_d t}}$$

$$\Rightarrow \left[(\omega_0^2-\omega_d^2)+i\gamma\omega_d\right] A = \frac{F_0}{m} e^{i\delta} = \frac{F_0}{m}\left[\cos\delta + i\sin\delta\right]$$

Real: $(\omega_0^2-\omega_d^2)A = \frac{F_0}{m}\cos\delta$ — (1)

Imaginary: $\gamma\omega_d A = \frac{F_0}{m}\sin\delta$ — (2)

(2)/(1)

Squaring (1) & (2) $\Rightarrow A^2\left[(\omega_0^2-\omega_d^2)^2+\gamma^2\omega_d^2\right] = \frac{F_0^2}{m^2}$

$$A = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega_d^2)^2+\gamma^2\omega_d^2}} \qquad \tan\delta = \frac{\gamma\omega_d}{\omega_0^2-\omega_d^2}$$

$$z(t) = \frac{F_0/m}{\sqrt{\{\quad\}}} e^{i(\omega_d t-\delta)}$$

$$x(t) = \mathrm{Re}[z(t)] = A(\omega_d)\cos[\omega_d t - \delta] \qquad \omega_d \gg \omega_0$$

$$A(\omega_d) = \frac{F_0/m}{\sqrt{(\underbrace{\omega_0^2-\omega_d^2}_{0})^2+\gamma^2\omega_d^2}}, \quad \tan\delta = \frac{\gamma\omega_d}{\omega_0^2-\omega_d^2} = \frac{\infty}{\infty}$$

$\omega_0^2=\omega_d^2 \Rightarrow A(\omega_d) = \frac{F_0}{m}\cdot\frac{1}{\gamma\omega_d}$. finite value

$\omega_0^2 = \omega_d^2 \Rightarrow A(\omega_d) = \frac{F_0}{m} \cdot \frac{1}{\gamma \omega_d}$. finite value

1. $\omega_d \to 0$, $A(\omega_d) = \frac{F_0}{m \omega_0^2}$ $\tan\delta = 0$, $\delta = n\pi$

2. $\omega_0 = \omega_d$, $A(\omega_d) = \frac{F_0}{m} \cdot \frac{1}{\gamma \omega_d}$ $\tan\delta = \infty$, $\delta = (2n+1)\frac{\pi}{2}$

3. $\omega_d \to \infty$, $A(\omega_d) = 0$, $\tan\delta =$ ____

phase diagram of $\left.\begin{array}{l} A(\omega_d) \sim \omega_d \\ \tan\delta \sim \omega_d. \end{array}\right\}$