Indian Institute of Information Technology Vadodara
MA 101: Introduction to Discrete Mathematics
Tutorial 4

1. Show that negation of $\forall x(P(x) \Rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.
2. Express the statement "If a person is female and is a parent, then this person is someone's mother" as a logical expression involving predicates, quantifiers with a domain consisting of all people, and logical connectives.
3. Express the negation of the statement $\forall x \exists y(xy = 1)$ so that no negation precedes a quantifier.
4. Use rules of inference to show that if $\forall x(P(x) \Rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true then $\forall x(R(x) \wedge S(x))$ is true.
5. For following set of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises
   i) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.
   ii) All foods that are healthy to eat do not taste good.
   Tofu is healthy to eat.
   You only eat what tastes good.
   You do not eat tofu.
   Cheeseburgers are not healthy to eat.
6. What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."
7. Use resolution to show the hypotheses "Allen is a bad boy or Hillary is a good girl" and "Allen is a good boy or David is happy" imply the conclusion "Hillary is a good girl or David is happy."
8. Use resolution to show that the hypotheses "It is not raining or Geeta has her umbrella," "Geeta does not have her umbrella or she does not get wet," and "It is raining or Geeta does not get wet" imply that "Geeta does not get wet."
9. What is wrong with this argument? Let $H(x)$ be "x is happy." Given the premise $\exists x H(x)$, we conclude that $H(Lola)$. Therefore, Lola is happy.
10. Determine whether this argument is valid.
    If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.
11. Find DNF, CNF, PDNF, PCNF of the following formulae: $P \Rightarrow (P \wedge (Q \Rightarrow P))$
12. Prove that if x is irrational then $1/x$ is also irrational. Identify the method of your proof.
13. Prove that if n is an integer and $3n + 2$ is even, then n is even using
    a) a proof by contraposition.
    b) a proof by contradiction.
14. Prove that if x and y are real numbers, then $\max(x, y) + \min(x, y) = x + y$. Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x < y$, respectively.
15. Give a constructive proof to show that there is a positive integer that equals the sum of the positive integers not exceeding it.
16. Prove that either $2 * 10^{500} + 15$ or $2 * 10^{500} + 16$ is not a perfect square.

17. Prove or disprove that there is a rational number x and an irrational number y such that x^y is irrational.

18. Prove that there is no positive integer n such that $n^2 + n^3 = 100$. Which method did you use?