Indian Institute of Information Technology Vadodara
MA 101: Introduction to Discrete Mathematics
Tutorial 6

1. Guess and prove the formula for sum of first n odd natural numbers, using mathematical induction.

2. Guess and prove the formula of $1.1! + 2.2! + 3.3! + ... + n.n!$

3. Show that if n is a positive integer, then ${}^{2n}C_2 = 2({}^{n}C_2) + n^2$ where ${}^{n}C_i = \frac{n!}{i!(n-i)!}$.

4. Suppose that $a_{m,n}$ is defined recursively for $(m, n) \in \mathbb{N} \times \mathbb{N}$ by $a_{0,0} = 0$ and

$$a_{m,n} = \begin{cases} a_{m-1,n} + 1 & \text{if } n = 0 \text{ and } m > 0 \\ a_{m,n-1} + n & \text{if } n > 0. \end{cases}$$

   Show that $a_{m,n} = m + n(n + 1)/2$ for all $(m, n) \in \mathbb{N} \times \mathbb{N}$.

5. Give a recursive definition of the sequence $a_n, n = 1, 2, 3, ...$ if $a_n = n(n + 1)$

6. Give a recursive definition of the set of positive integers not divisible by 5.

7. Let f_n be the nth Fibonacci number. Prove that

$$f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1},$$

   where n is a positive integer.

8. Give a recursive definition of the functions max and min so that $\max(a_1, a_2, ..., a_n)$ and $\min(a_1, a_2, ..., a_n)$ are the maximum and minimum of the n numbers $a_1, a_2, ..., a_n$ respectively.

9. A partition of a positive integer n is a way to write n as a sum of positive integers where the order of terms in the sum does not matter. For instance, $7 = 3+2+1+1$ is a partition of 7. Let P_m equal the number of different partitions of m, and let $P_{m,n}$ be the number of different ways to express m as the sum of positive integers not exceeding n. Find P_5, P_6. You may also use following recursive definition.

$$P_{m,n} = \begin{cases} 1 & \text{if } m = 1 \\ 1 & \text{if } n = 1 \\ P_{m,m} & \text{if } m < n \\ 1 + P_{m,m-1} & \text{if } m = n > 1 \\ P_{m,n-1} + P_{m-n,n} & \text{if } m > n > 1. \end{cases}$$

10. Given a positive integer n, consider a square of side n made up of n^2 1×1 squares. Prove that the total number S_n of squares present is $S_n = \frac{n(n+1)(2n+1)}{6}$