
MA102: MATHEMATICS II: INTRO. TO DISCRETE MATHEMATICS
MIDSEM (PEN-PAPER) EXAMINATION

MARKS: 40

QUESTION PAPER: Right

DATE: June 15, 2022

Instructions: Clearly write your name and roll number on the top of each page. Solutions must be written clearly.

1. What is the difference between Proof by contraposition and Proof by contradiction? Explain it with the help of example. Give an example of a proof which is Proof by cases but not Exhaustive Proof. **5 Marks**
2. What can you conclude if you meet two people on the island of knights and knaves where both say that "I am a knave"? Prove it. **3 Marks**
3. Is following function $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ bijective? If yes then prove it or give justification.

$$f(m, n) = \frac{(m + n - 1)(m + n - 2)}{2} + m$$

5 Marks

4. Let i be the last digit of your student id. Is $|(-2, i]| = |[-2, i + 1)|$? If yes then prove it else give justification. **5 Marks**

5. Prove that for any natural number n , $2(\sqrt{n+1} - 1) < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}}$ **5 Marks**

6. Is the relation $\{(x, y) | x - y \in \mathbb{R}\}$ an equivalence relation or partial order on the set of complex numbers- \mathbb{C} .
Is it possible for a relation to be equivalence as well as partial order? Give justification. **5 Marks**

7. Find PDNF of the formula $(P \vee R) \wedge (P \vee \neg Q)$. **5 Marks**

8. Five women are side by side enjoying a sunny day at the beach. Each one is using a towel, eating a sandwich and drinking a type of juice. Follow the clues to find out information about each one.

The woman drinking Grape juice is next to the woman using the Green towel. The Chef is somewhere between the Pilot and the woman drinking Orange juice, in that order. The Translator is next to the woman eating the Bacon sandwich. The 30-year-old woman is exactly to the left of the woman drinking Grape juice. Michaela is next to the woman using the Yellow towel. The 40-year-old woman is exactly to the right of the 35-year-old woman. The Chef is exactly to the right of the woman drinking Apple juice. The woman using the Yellow towel is somewhere between the 30-year-old woman and the woman eating the Pork sandwich, in that order. Michaela is drinking Mango juice. At one of the ends is the 40-year-old woman. The Engineer is exactly to the left of the 35-year-old woman. Eliana is next to the Translator. The woman eating the Ham sandwich is somewhere between Yasmin and the woman eating the Bacon sandwich, in that order. The woman using the Red towel is eating the Cheese sandwich. At the first position is the woman using the White towel. The woman using the Yellow towel is somewhere between the 30-year-old woman and the woman using the Green towel, in that order. Eliana is exactly to the right of the woman drinking Orange juice. The woman using the Yellow towel is somewhere to the left of Lydia. The oldest woman is an Engineer. At one of the ends is the woman eating the Pork sandwich. Michaela works as an Engineer.

Give justification.

7 Marks