Indian Institute of Information Technology-Vadodara
MA 101: Introduction to Discrete Mathematics
Tutorial 9

1. Suppose that $A = \{1, 2, 3, 4\}$ and R be the relation on A defined as $(a, b) \in R$ iff $a < b$. Find the matrix, graph representation of R with respect to the natural ordering.

2. Suppose that the relation R on a set is represented by the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

   Is R reflexive, symmetric, antisymmetric?

3. Determine whether the relation R on the set of real numbers- $\mathbb{R}$ is reflexive, symmetric, antisymmetric, transitive, where $(x, y) \in R$ if and only if
   (i) $x + y = 0$
   (ii) $xy \geq 0$
   (iii) $x = 1$ or $y = 1$

4. Let A be the relation "to be wife of" and B "to be father of " on the set of all humans. What does the relation $A \circ B$ mean in this case?

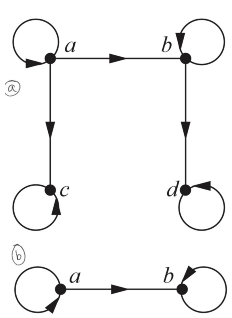
5. Let $A = \{2, 3, 4, ..., 100\}$ with partial order of divisibility.
   (a) How many maximal elements does $(A, |)$ have?
   (b) Give a subset of A that is a linear order under divisibility and is as large as possible.

6. A person's blood type is determined by the presence (T) or absence (F) of antigens A, B and Rh, as shown in the table below.

A	B	Rh	Type
F	F	F	O^-
F	F	T	O^+
F	T	F	B^-
F	T	T	B^+
T	F	F	A^-
T	F	T	A^+
T	T	F	AB^-
T	T	T	AB^+

   A person with blood type X can donate blood to a person with blood type Y, if and only if all of the antigens present in X are contained in Y. Let P be the set of the eight possible blood types, and let R be the relation on P such that XRY if and only if a person with blood type X can donate blood to a person with blood type Y. Answer the following questions.
   (a) Can a person with A^+ blood type donate to one with A^- ?
   (b) What types of blood can a person with A^+ blood type receive?
   (c) Draw a directed graph for R.
   (d) Show that R is a partial order.
   (e) Make a Hasse diagram for R.
   (f) What are the minimal (universal donor) and maximal (universal acceptor) elements of P?

7. Determine whether the relation with the directed graph shown is a partial order.



8. Display all the partial orders on a set with three elements with the help of Hasse diagram. How many of them are lattices?

9. Let R be a partial order on a finite set S. Describe how to use the matrix representation M_R to find the least and greatest element of A if they exist.
Greatest element: $y \in (S, \preceq)$ is greatest if $x \preceq y$ <u>for all</u> $x \in S$.
Least element: $z \in (S, \preceq)$ is least if $z \preceq x$ <u>for all</u> $x \in S$.

10. Give an example of an infinite lattice with neither a least element nor a greatest element.

11. Give an example of an infinite lattice with a least element and a greatest element.