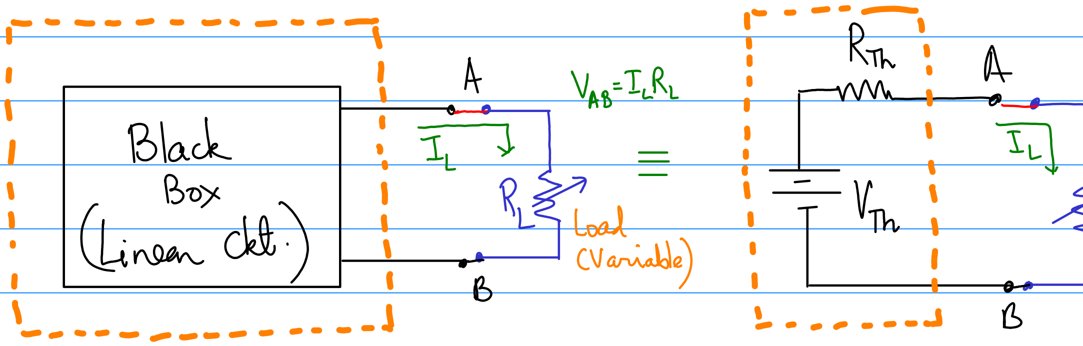
19.09.2023

Circuit Theorems

Recap:

1) Superposition Theorem (Response of each source should be looked at separately) ⇒ Add all effects.

2) Thevenin Theorem

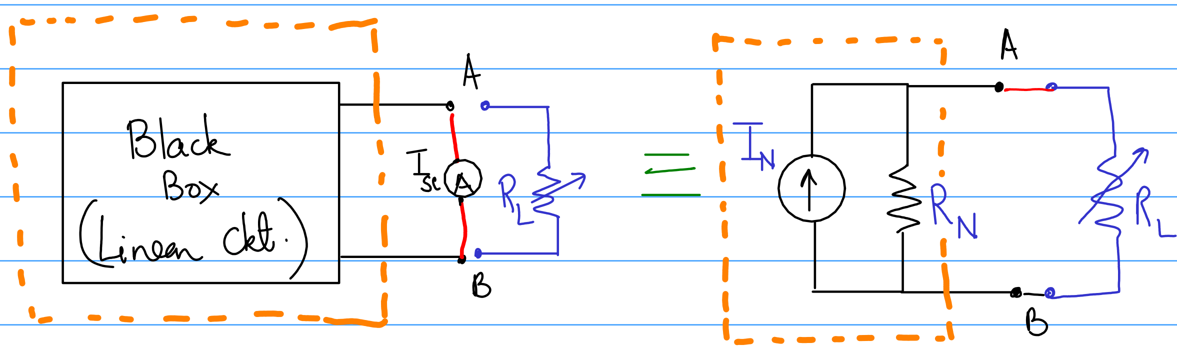


$V_{R_L} = \left(\frac{R_L}{R_L + R_{Th}}\right) V_{Th}$

$I_L = \frac{V_{Th}}{R_{Th} + R_L}$

Black Box ≡ replaced with a single voltage source V_{Th} & a series resistance R_{Th}.

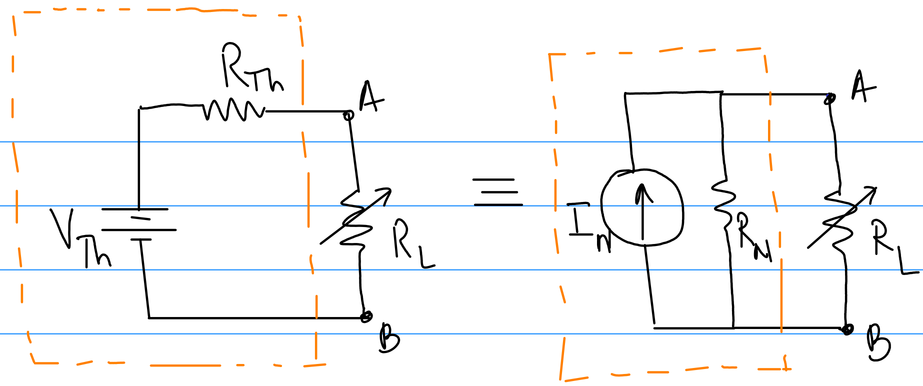
3) Norton Theorem:



here, $I_N = I_{sc}$ in Thevenin's equivalent ckt.

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

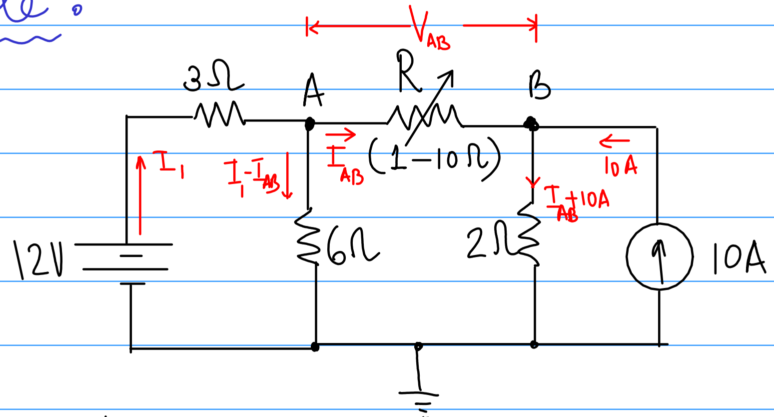
$$\boxed{R_N = R_{Th}}$$



Thevenin's voltage $V_{Th} = V_{oc}$ (open ckt. voltage)

Norton's Current $I_N = I_{sc}$ (short ckt. current)

Example :



Fill this table :

$V_{AB} = \left(\frac{R}{R+4\Omega}\right) 12V$

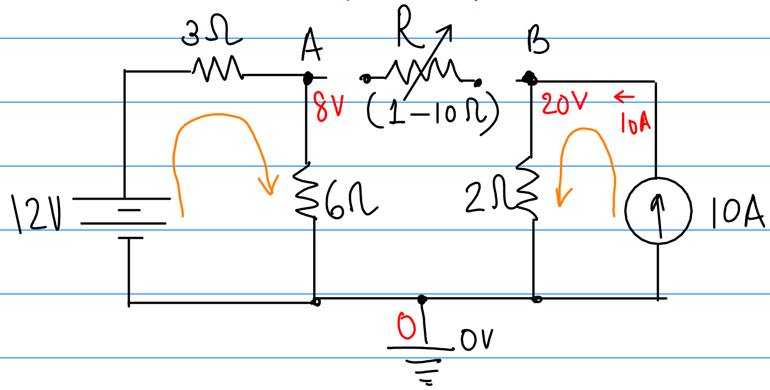
$I_{AB} = \frac{12V}{R+4\Omega}$

R (in Ω)	V_{AB} (in V)	I_{AB} (in A)
1Ω	2.4V	2.4 A
2Ω	4.0V	2.0 A
3Ω	⋮	⋮
4Ω		
⋮		
10Ω		

Here we make use of Thevenin's theorem :

ie Thevenize the ckt. across the terminals A & B

Open the resistance R across 'A' & 'B'



$$V_{AB}\,(\text{open ckt.}) = V_A - V_B$$

$$V_B = 2\Omega \cdot 10A = 20V$$

$$V_A = \frac{6\Omega}{3\Omega + 6\Omega} \cdot 12V = 8V$$

$$V_{AB} = V_A - V_B = 8V - 20V = -12V$$

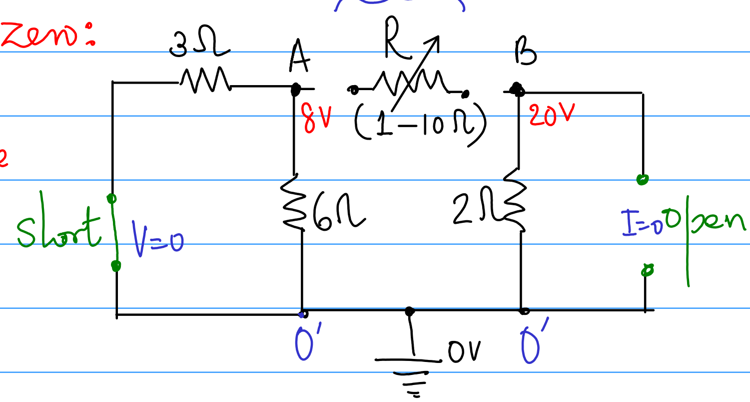
$$\boxed{V_{Th} = V_{AB} = -12V}$$

Determination of R_{Th}:

Open the resistance R across 'A' & 'B'

To make the source zero:

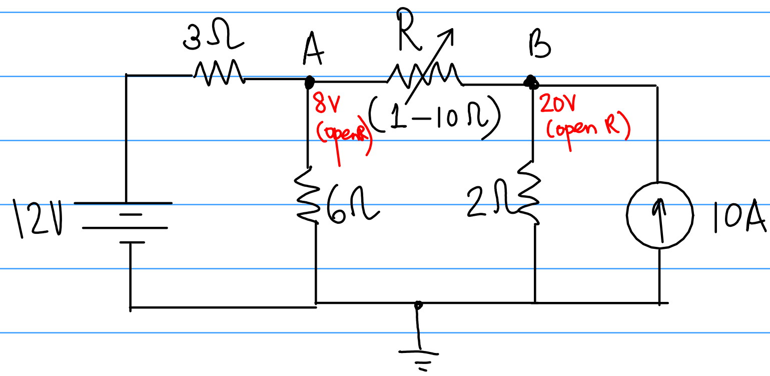
- short voltage source
- open current source



$$R_{eq(AB)} = R_{Th}$$

$$R_{Th} = (3\Omega \| 6\Omega) + 2\Omega$$

$$R_{Th} = 4\Omega$$



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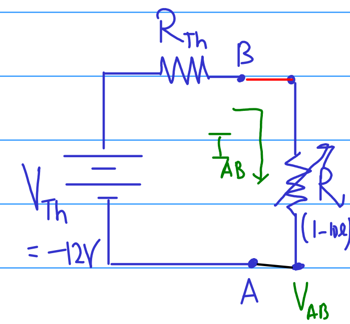
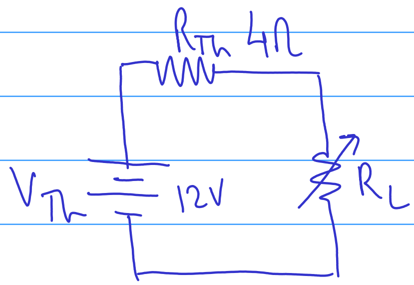


$$I_{AB} = \frac{V_{Th}}{R_{Th} + R} = \frac{12V}{4\Omega + R}$$

$$V_{AB} = I_{AB} \cdot R = \left[\frac{12V}{4\Omega + R}\right] \cdot R$$



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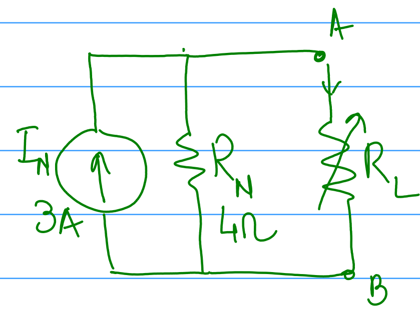
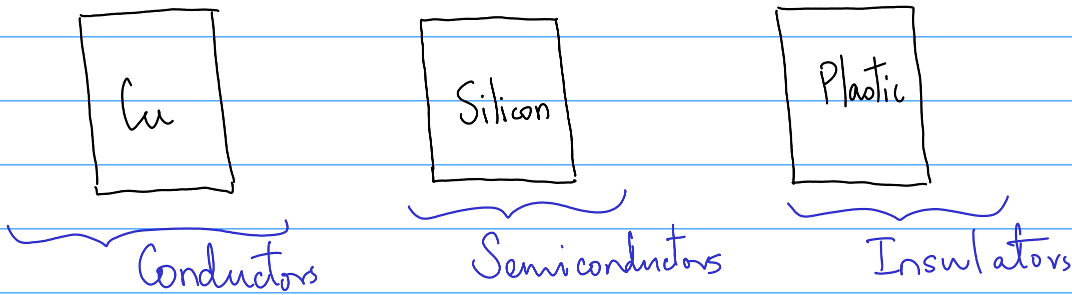


$I_{AB} =$

$V_{AB} =$

$$I_N = I_{sc} = \frac{V_{Th}}{R_{Th}} = \frac{12V}{4\Omega} = 3A$$

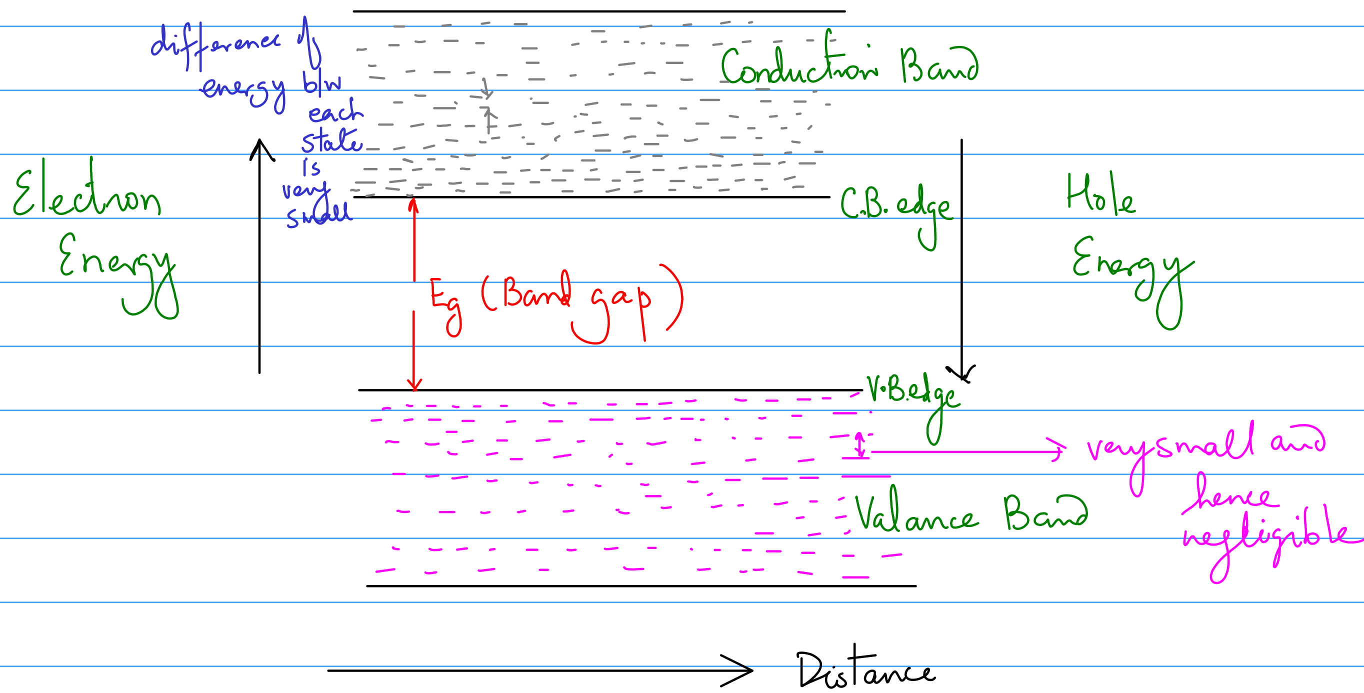
Semiconductors



In terms of Energy gap : E_g (semiconductors) $\sim$ 1-3 eVs (typical)

E_g (Insulators) $\sim$ > 3eV (typical)

Simplified Band Diagram of a Semiconductor :



In each energy band, we have discrete energy states, however, the energy separation b/w them is very small and hence assumed negligible. Therefore, We visualize each band is having continuum energy states.

Typically, at room temperature (300K)

$$E_g(\text{Silicon}) = 1.12\,eV$$

$$E_g(Ge) = 0.66\,eV$$

$$E_g(GaAs) = 1.42\,eV$$

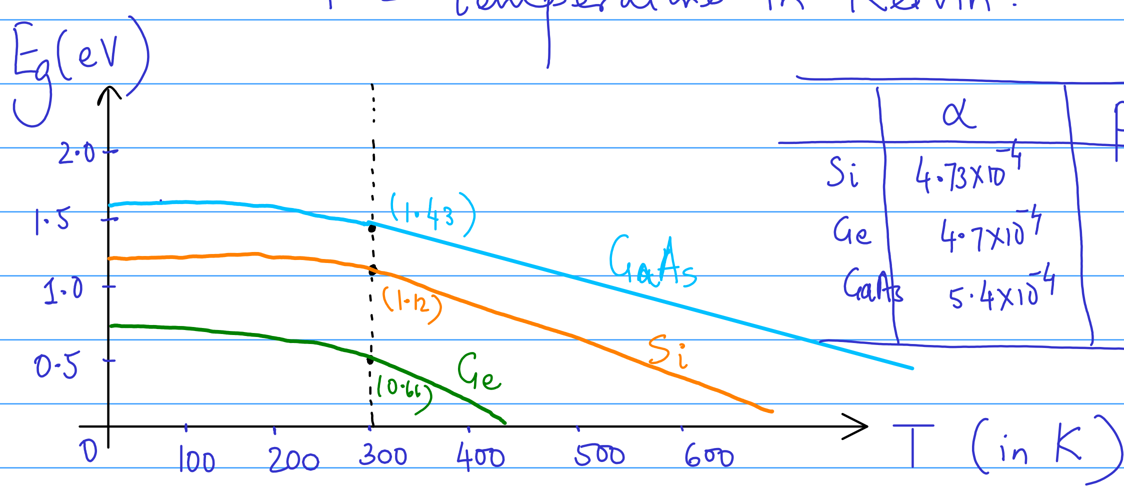
Band gap depend on temperature

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{(T+\beta)}$$

where $E_g(0)$ = Band gap at $T = 0K$

α, β are the parameters

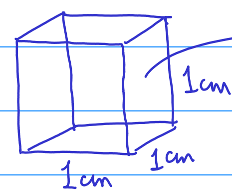
T = temperature in Kelvin.



	α	β
Si	4.73×10^{-4}	636
Ge	4.7×10^{-4}	235
GaAs	5.4×10^{-4}	204

Charge-carrier density in Conductors, Semiconductors & Insulators.

Charge-carrier density $\equiv$ No. of electrons / holes per unit volume (cm^{-3})

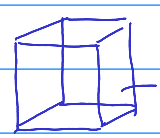


Volume = $1\,cm^3$

In any solid, atomic density $\sim 10^{22}\ cm^{-3}$

Let take specific example:

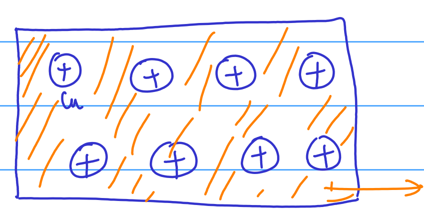
Copper : $Cu \rightleftharpoons Cu^{2+} + 2e^-$ "FREE"



$1\,cm^3$ of Copper $\equiv 10^{22}$ atoms $= \underbrace{2\times10^{22}\ \text{electrons}}_{\text{"FREE"}}$

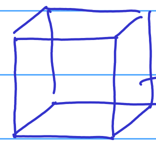
In metals (conductors), we have $\sim 10^{22}$ "FREE" electrons cm^{-3} which are available for the conduction of electrical energy.

Drude's Model



Sea of "FREE" charge carriers

What about semiconductors? (say silicon)



$1\ cm^3$

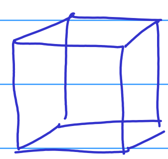
$\#$ atoms $= 10^{22}$

at $T = 0K$; "FREE" electrons/holes $\approx 0\ cm^{-3}$

at $T = 300K$ (RT) "FREE" electron/hole $\sim 10^{10}\ cm^{-3}$

Thermal Energy at RT $= k_B T \approx 25\,meV$

<u>Insulators</u> :



$1\ cm^3$ atom $\sim 10^{22}$

at RT (300K); "FREE" electron $\lesssim 10^{5}\ cm^{-3}$