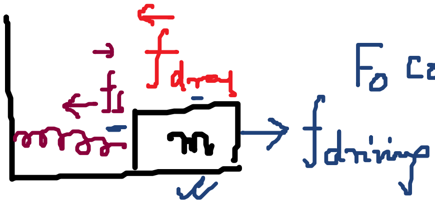
Lecture 16: Section 1

Friday, February 4, 2022 10:09 AM

Forced HO :→ Undamped – $f_{drag} = 0$
→ Damped $f_{drag} \neq 0$



$F_0 \cos(\omega_d t)$

$$\vec{F} = -\vec{f}_s - \vec{f}_{drag} + \vec{f}_{driving}$$

$$m\ddot{x} = -kx - b\dot{x} + F_0 \cos(\omega_d t)$$

$$\Rightarrow \boxed{\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos(\omega_d t)}$$

$$x(t) = A \cos(\omega_d t - \delta), \quad A = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + \gamma^2 \omega_d^2}}$$

m = mass of object.

$\omega_0^2 = \frac{k}{m}$.

ω_d = Driving frequency

$\gamma = b/m$

F_0 = Amplitude of applied force.

$$\tan\delta = \frac{\gamma \omega_d}{\omega_0^2 - \omega_d^2}$$

$$x(t) = A \cos(\omega_d t - \delta)$$

forced damped HO :→ $E(t) = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

$$= \frac{1}{2} m \left[-\omega_d A \sin(\omega_d t - \delta) \right]^2 + \frac{1}{2} k A^2 \cos^2(\omega_d t - \delta)$$

$$E(t) = \frac{1}{2} m \omega_d^2 A^2 \underbrace{\sin^2(\omega_d t - \delta)}_{1/2} + \frac{1}{2} k A^2 \underbrace{\cos^2(\omega_d t - \delta)}_{1/2}$$

$$\overline{E(t)} = \frac{1}{4} m \omega_d^2 A^2 + \frac{1}{2} k A^2$$

$k = m\omega_0^2$

$$\overline{E(t)} = \frac{1}{4} m A^2 \left[\omega_d^2 + \omega_0^2 \right]$$

$$\overline{E}(t) = \frac{1}{4} \cdot \cancel{m} \cdot \frac{F_0^2/m^{\cancel{2}} \cdot (\omega_d^2 + \omega_0^2)}{(\omega_0^2 - \omega_d^2)^2 + \gamma^2 \omega_d^2}$$

Damped HO

Lightly $\omega_0^2 > \frac{\gamma^2}{4}$

$$\bar{E}(t) = \frac{1}{4}\cdot \cancel{m} \cdot \frac{F_0^2/m^2 \cdot (\ldots)}{(\omega_0^2-\omega_d^2)^2+\gamma^2\omega_d^2}$$

Lightly ω_0, $\frac{\gamma}{4}$

Physical Approximation: Lightly damped, $\omega_0 \approx \omega_d$

$$\omega_0^2 - \omega_d^2 = (\omega_0+\omega_d)(\omega_0-\omega_d) \approx 2\omega_0(\omega_0-\omega_d)$$

$$\bar{E}(t) = \frac{1}{4}\cdot\frac{F_0^2}{m}\cdot\frac{2\omega_0^2}{4\omega_0^2\cdot(\omega_0-\omega_d)^2+\gamma^2\omega_d^2}$$

Q-factor

Dividing N^r & D^r by $2\omega_0^2$.

$$\bar{E}(t) = \frac{1}{4}\cdot\frac{F_0^2}{m}\cdot\frac{1}{2(\omega_0-\omega_d)^2+\frac{\gamma^2}{2}}$$

$\omega_d = \omega_0 - \frac{\gamma}{2}$

$$\boxed{\bar{E}(t) = \underbrace{\left(\frac{1}{8}\frac{F_0^2}{m}\cdot\right)}_{K}\frac{1}{(\omega_0-\omega_d)^2+\left(\frac{\gamma}{2}\right)^2}}$$

$(\gamma/2)^2$

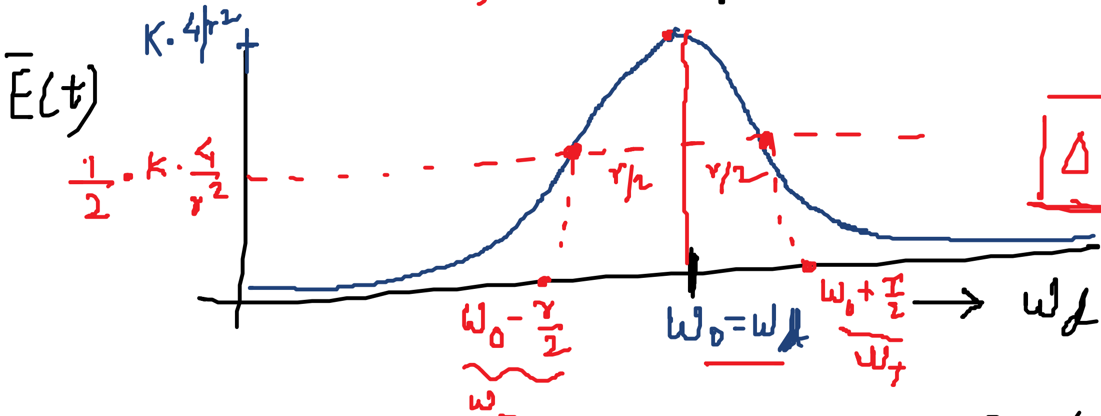
Resonance curve or Lorentzian curve.

→ fdn

m — m — sdhing

• $\omega_0 = \omega_d$, $\bar{E}_{max}(t) = K\frac{4}{\gamma^2}$ ←

, $\bar{E}_{1/2\,max}(t)$



$$\boxed{\Delta\omega = \gamma}$$

$\ldots(\omega_+ - \omega_0)^2 = (\gamma\ )^2$

ω_-

$$\bar{E}(t) = K \cdot \frac{1}{(\omega_0-\omega_d)^2+\left(\frac{\gamma}{2}\right)^2}, \quad \text{if } (\omega_0-\omega_d)^2 = \left(\frac{\gamma}{2}\right)^2$$

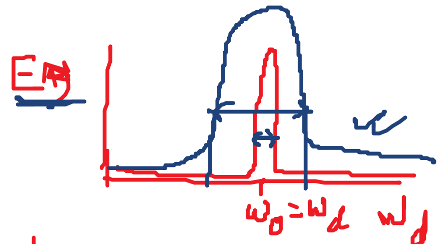
$$= K \cdot \frac{1}{2\left(\frac{\gamma}{2}\right)^2}$$

$$\omega_0 - \omega_d = \pm \frac{\gamma}{2}$$

$$\boxed{\omega_d = \omega_0 \pm \frac{\gamma}{2}}$$

$$\bar{E}(t) = \frac{1}{2}\bar{E}_{max}(t)$$

Q-factor: $Q = \dfrac{\omega_0}{\gamma} = \dfrac{\omega_0}{\boxed{\Delta\omega}}$

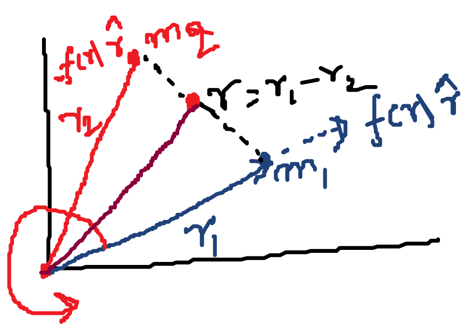


1. When $\Delta\omega$ is very small: Q very large, System is highly selective only for few frequency
2. When $\Delta\omega$ is large: Q is small

Central Force Motion:

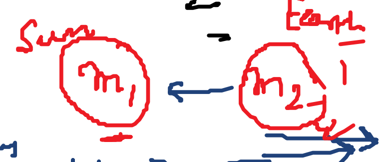
Gravitational force
Electrostatic force

$f(r)\,\hat{r}$

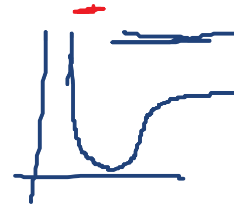


- bound
- unbound

m_1
m_2



molecules
H_2
H O H



C
N

Equation of Motion:

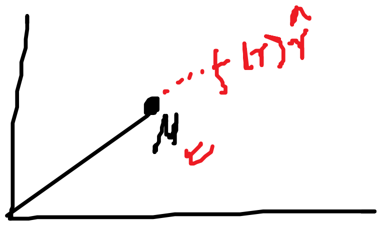
$$m_1 \ddot{r}_1 = f(r)\,\hat{r} \quad \text{— (1)}$$
$$m_2 \ddot{r}_2 = -f(r)\,\hat{r} \quad \text{— (2)}$$

$$\ddot{r}_1 - \ddot{r}_2 = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) f(r)\,\hat{r}$$

$$\Rightarrow \underbrace{\left(\frac{m_1 m_2}{m_1 + m_2}\right)}_{\text{Reduced mass } (\mu)} (\ddot{r}_1 - \ddot{r}_2) = f(r)\,\hat{r} \Rightarrow \boxed{\mu \ddot{\underline{r}} = f(r)\,\hat{r}}$$

$$r = r_1 - r_2$$

1-body problem



$$\boxed{\frac{d\vec{r}(t)}{dt},\ \frac{d\theta(t)}{dt},\ \frac{d\theta(}{d\underline{r}}}$$

C.M. $\Rightarrow$ $\underline{R} = \dfrac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

$F_{ext} = 0$, $\quad \ddot{\underline{R}} = 0$, $\quad \boxed{\underline{R} = R_0 + Vt}$

$$r_1 = R + \left(\frac{m_2}{m_1 + m_2}\right) r$$

$$r_2 = R - \left(\frac{m_1}{m_1 + m_2}\right) r$$

$$\underline{r} = r_1 - r_2$$

$$R = \frac{m_1 r_1 + m_2 (r_1 - r)}{m_1 + m_2}$$

$$= \frac{(m_1 + m_2) r_1}{m_1 + m_2} - \frac{m_2 r}{m_1 + m_2}$$

$$\boxed{\underline{r_1} = R + \frac{m_2 r}{m_1 + m_2}}$$