Academic Year 2022-23

Tutorial #01

PH100: Mechanics and Thermodynamics

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1. Consider two points located at $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$, separated by distance r = | $\mathbf{r_1}$ — $\mathbf{r_2}$|. Find a vector ***A*** from the origin to a point on the line between $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ at distance *x r* from the point at $\boldsymbol{r_1}$, where *x* is some number.

2. At $t = 0$, an elevator departs from the ground with uniform speed. At time T_1 a boy drops a marble through the floor. The marble falls with uniform acceleration $g = 9.8\ m/s^2$, and hits the ground T_2 seconds later. Find the height of the elevator at time T_1.

3. The height of a certain hill (in feet) is given by: $h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$, where *y* is the distance (in miles) north, *x* the distance east of Gandhinagar. (a) Where is the top of the hill located? (b) How high is the hill? (c) How steep is the slope (in feet per mile) at a point *1* mile north and one mile east of Gandhinagar? In what direction is the slope steepest, at that point?

4. From the given below vector field, identify any field is conservative or not.

Calculate the divergence and curl of the following vector functions:

(a) $\mathbf{v_a} = x^2\hat{\mathbf{x}} + 3xz^2\hat{\mathbf{y}} - 2xz\hat{\mathbf{z}}$,

(b) $\mathbf{v_b} = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}$,

(c) $\mathbf{v_c} = y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}$.

5. Construct a vector function that has zero divergence and zero curl everywhere.

6. By relative velocity we mean velocity with respect to a specified coordinate system. (The term velocity, alone, is understood to be relative to the observer's coordinate system.)

   (a) A point is observed to have velocity V_A relative to coordinate system *A*. What is its velocity relative to coordinate system *B*, which is displaced from system *A* by distance *R*? (*R* can change in time.)

(b) Particles *a* and *b* move in opposite directions around a circle with angular speed \omega. At $t = 0$ they are both at the point $r = l\,\mathbf{j}$, where l is the radius of the circle. Find the velocity of *a* relative to *b*.