

- Wave shows particle nature
- $E = hf$
- Photoelectric effect
  - Compton effect
  - Black body radiation

De Broglie waves  $\Rightarrow$  particles also show wave characteristics

$$E^2 = \frac{m^2 c^4}{0} + p^2 c^2$$

$$hf = E = pc \Rightarrow h = \frac{pc}{f} \Rightarrow \lambda = \frac{h}{p}$$

$$p = \gamma m v$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\lambda_{db} = \frac{h}{\gamma m v}$$

particle

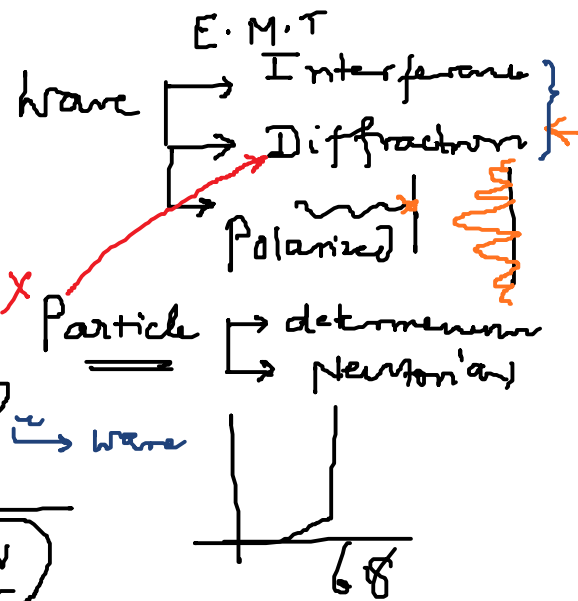
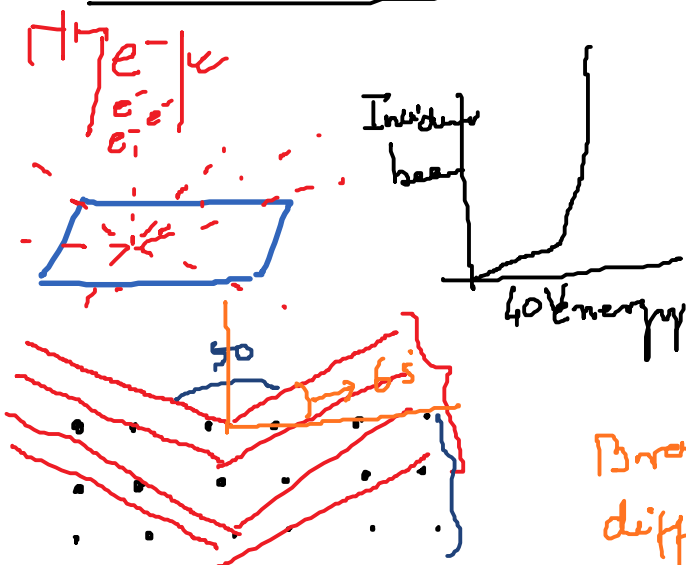
$e^- \rightarrow \sim 10^{-34}$

neutrons

Protons

Matter waves  $\Rightarrow$  ?

Davisson-Germer Experiment  $\Rightarrow$



Bragg's diffraction

$$n\lambda = 2d \sin \theta$$

$n=1$ ,  $\lambda = 2 \cdot 0.091 \times 10^{-9} \text{ m}$

$d = 0.091 \text{ nm}$ ,  $\sin 65$

$\lambda \approx 0.165 \text{ nm}$

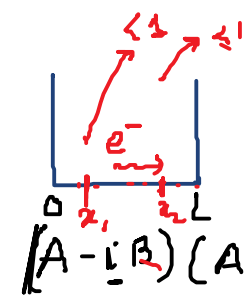
$$\lambda_{db} = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

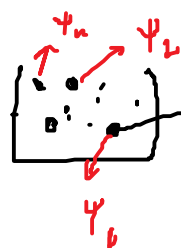
$$E = 9 \text{ V}$$

$p = \sqrt{2mE}$   $E = 2V$   
 $p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \text{ kg} \times 54 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV}}$   
 $\approx 4.0 \times 10^{-24} \text{ kg m/s}$   
 $\lambda = \frac{6.63 \times 10^{-34}}{p} \approx 0.166 \text{ nm}$

Matter waves  $\Rightarrow \Psi(x, y, z, t) : A + iB$

Probability density  $= \Psi^* \Psi = |\Psi|^2 = A^2 + B^2$   
 Head  $\frac{1}{2}$  Tail  $\frac{1}{2} = 1$

  
 $\int_0^L |\Psi|^2 dx = 1$



$\Psi = \sum \Psi_i$

Wave packet



$\Psi_1 = A \cos(\omega t - kx)$

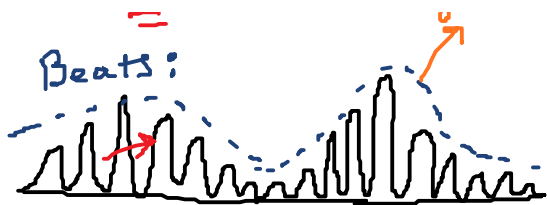
$v_1 = \frac{\omega}{k}$

$\Psi_2 = A \cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$ ,  $v_2 = \frac{\omega + \Delta\omega}{k + \Delta k}$

$A = \Psi_1 + \Psi_2 = 2A \cdot \cos\left[\left(\frac{2\omega + \Delta\omega}{2}\right)t - \left(\frac{2k + \Delta k}{2}\right)x\right]$

$\cos A + \cos B$   
 Beats:

$\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$   
 $v_g = v$   
 $\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$



$$\frac{640 \text{ Hz}, 640}{10000, 10001}$$

$$\psi = \frac{2A \cos(\omega t - kx) \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)}{\left. \begin{array}{l} 2\omega + \Delta\omega \approx 2\omega \\ 2k + \Delta k \approx 2k \end{array} \right\}}$$

$$V_{\text{phase}} = \frac{\omega}{k}$$

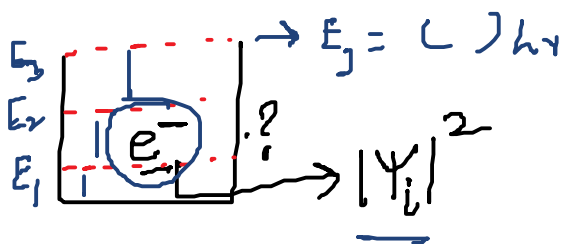
$$V_{\text{group}} = \frac{\Delta\omega}{\Delta k} \approx \frac{d\omega}{dk}$$

$$V_{\text{phase}} = \frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \nu\lambda, \quad \left| \begin{array}{l} \lambda_{db} = \frac{h}{\gamma m v} \\ \gamma = \frac{\gamma m c^2}{h} \rightarrow E = h\nu \end{array} \right.$$

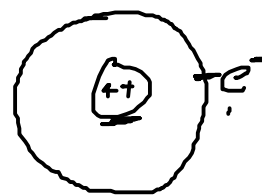
$$V_{\text{phase}} \approx \frac{c^2}{v} \cdot v$$

$$V_{\text{group}} = \frac{d\omega(\nu)}{dk(\nu)} = \frac{d\omega/d\nu}{dk/d\nu} = v$$

$$\lambda \rightarrow \frac{1}{\gamma}, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



X ————— Atom



Bohr  
atom