Lecture 10: Section 1

Friday, January 21, 2022 10:10 AM

Conservative force → $\boxed{\vec{F} = -\nabla U}$

$$\oint \vec{F}\cdot d\vec{l} = \iint_A (\nabla \times \vec{F})\cdot d\vec{a} \quad \Big| \quad \nabla \times (\nabla U) = 0$$

*. Equilibrium & Stability : $\vec{F} = -\nabla U$ → $\boxed{\nabla U = 0}$
↳ 0

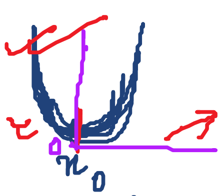
$(x, y) = (0,0)$

1. $U = x^2 + y^2$, $\nabla U = 2x\hat{i} + 2y\hat{j}$, $\vec{F} = -\nabla U$
$= \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}$ $\quad = -2[x\hat{i} + y\hat{j}]$ } Stable Equilibrium

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \nabla^2 U = (\nabla\cdot\nabla)\, U = 2+2 = 4$$

2. $U = -(x^2+y^2)$, $\nabla U = -2(x\hat{i} + y\hat{j})$, $(0,0)$
$\vec{F} = 2(x\hat{i} + y\hat{j})$ } Unstable Equilibrium

Harmonic Approximation :→



Taylor Series:

$$f(x) = f(x_0) + (x-x_0) f'(x_0) + \frac{(x-x_0)^2}{2!} f''(x_0) + \cdots$$

$$\rightarrow U(x) = U(x_0) + (x-x_0)\underbrace{U'(x_0)}_{0} + \frac{(x-x_0)^2}{2!} U''(x_0) + \cdots$$

$$\boxed{U(x) \approx \frac{1}{2}(x-x_0)^2 U''(x_0)} \Rightarrow U = \frac{1}{2}Kx^2$$

$U(x_0) = 0$

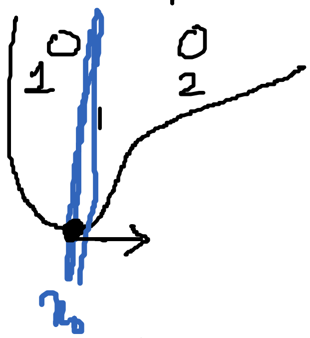
$K = U''(x_0)$

$$\omega = \sqrt{\frac{K}{m}}$$

$U(x_0) = 0$

$\omega = \sqrt{\frac{K}{m}}$ μ

Examples →



Morse potential →

$$\rightarrow V(x) = D\left[1 - e^{-\alpha(x-x_0)}\right]^2$$

$$= D\left[1 + e^{-2\alpha(x-x_0)} - 2e^{-\alpha(x-x_0)}\right]$$

-2α $\quad +2\alpha$

$U(x_0) = 0, \quad U(\infty) = D$

$$\left.\frac{dU}{dx}\right|_{x=x_0} = 2\alpha D\left[1 - e^{-\alpha(x-x_0)}\right] e^{-\alpha(x-x_0)}$$

$U'(x_0) = 0$

$$\frac{d^2U}{dx^2} = 2\alpha D\left[-\alpha e^{-\alpha(x-x_0)} + 2\alpha e^{-2\alpha(x-x_0)}\right]$$

$-\alpha + 2\alpha$

$U''(x=x_0) = 2D\alpha^2 \approx K \quad \Rightarrow \omega = \sqrt{\frac{2D\alpha^2}{\mu}}$

$m_1, m_2 \quad \mu$ = Reduced mass

$\mu = \frac{1}{m_1} + \frac{1}{m_2}$

Potentials

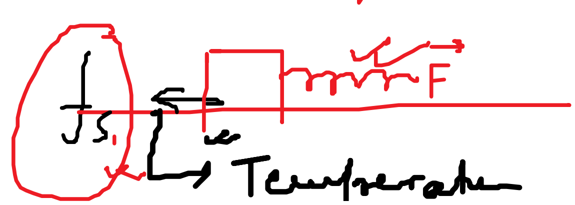
- HO
- Morse potential
- Periodic
- Leonard Jone's potential
- potch-Teller

→ Conservative forces

→ Equilibrium & Stability

$$W = \int_a^b \vec{F}\cdot d\vec{r}$$

$a \quad b$



$$W = \int_a^b \vec{F}\cdot d\vec{r}$$

$$= \int_a^b \left(\vec{F}^{C} + \vec{F}^{NC}\right)\cdot d\vec{r}$$

$$= \int_a^b F^{C}\cdot dr + \int_a^b F^{NC}\cdot dr$$

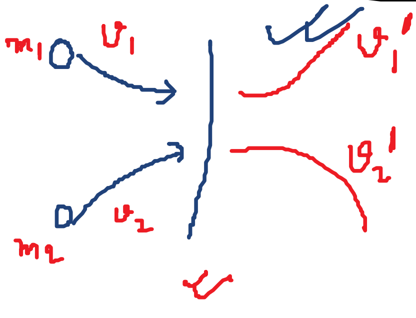
$$K.E_b - K.E_a = -U_b + U_a + W^{NC}$$

JS, Temperature

$$\Rightarrow \boxed{(K.E_b + U_b) - (K.E_a + U_a) = W^{NC}} \rightarrow 0$$

$W^{NC} = 0$, Mechanical Energy

Conservation Laws & particle collisions



Negligible external force

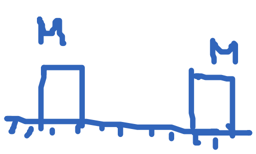
$F_{ext} = 0$

$$\frac{dp}{dt} = 0$$

$$\boxed{p_i = p_f}$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Elastic collision K.E unchanged



Initial $m_1 = M$, $v_1 = v$
$m_2 = M$, $v_2 = 0$

final
$m_1 = M$, $v_1' = 0$
$m_2 = M$, $v_2 = v$

Inelastic collision K.E changes

Initial: $v_1 = v$, $v_2 = 0$

final: $(M+M)\, v' = Mv$

$\Rightarrow v' = \frac{v}{2}$

$m_2 = M, \quad v_2 = v$

$p_i = p_f \Rightarrow Mv = Mv$

$K.E_i = K.E_f \Rightarrow \frac{1}{2}Mv^2 = \frac{1}{2}Mv^2$

$\Rightarrow v' = \frac{v}{2}$

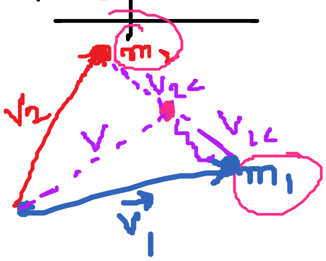
$\frac{1}{2}Mv^2 = \frac{1}{2}(2M)\left(\frac{v}{2}\right)^2 + Q$

$K.E_i = K.E_f + Q$

Collision in C.M. frame of reference →

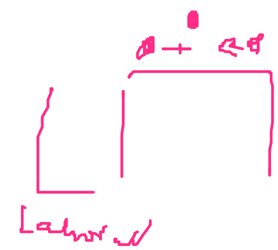
Two particles $\quad m_1 \,\|\, m_2 \quad v_1, \; v_2$

$$\Rightarrow \vec{V} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$



$$V_{1c} = V_1 - V = V_1 - \left(\frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}\right) = \frac{m_2(\vec{V}_1 - \vec{V}_2)}{m_1 + m_2}$$

$$V_{2c} = V_2 - V = -\frac{m_1}{m_1 + m_2}(\vec{V}_1 - \vec{V}_2)$$



$$p_{1c} + p_{2c} = \frac{m_1 m_2}{m_1 + m_2}(V_1 - V_2) - \frac{m_1 m_2}{m_1 + m_2}(V_1 - V_2) = 0$$

Elastic collision in CM →

$$\begin{cases} V_{1c} = V'_{1c} \\ V_{2c} = V'_{2c} \end{cases}$$

Q