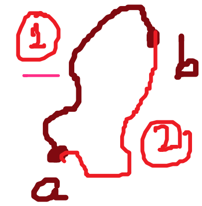
Lecture 8: Section 1

Tuesday, January 18, 2022 8:51 AM

$$\vec{F}^{(n)}_{ext} = \frac{d\vec{P}_c}{dt_c} \qquad \underline{\vec{F}(t)}$$

$$\Downarrow$$

$$0$$



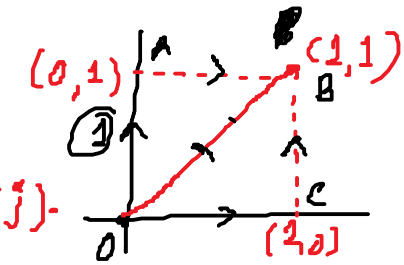
Work done by a force →

$$W = \int_a^b \vec{F}\cdot d\vec{r} \qquad W^{(1)}_{ba} \neq W^{(2)}_{ba}$$

Ex 1: $\vec{F} = xy\,\hat{i} + y^2\,\hat{j}$, $W = \int \vec{F}\cdot d\vec{r}$

$$= \int (xy\,\hat{i} + y^2\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$$

$$= \int_A^B xy\,dx + \int_A^B y^2\,dy$$



(1) $OA \to AB$: Portion of hwy

(2) $OC \to CB$: P "

(3) OB : P..

(1) OA : $x = 0$

AB : $y = 1$, $dy = 0$

$$W^{(1)} = \int_0^1 y^2\,dy + \int_A x\,dx$$

$$= \int_0^1 y^2\,dy + \int_0^1 x\,dx = 5/6.$$

(2) $OC \to CB$.

OC : $y = 0$ ⟶ $W = 0$

CB : $x = 1$, $dx = 0$ ⟶ $W^{(2)} = \int_0^1 y^2\,dy = 1/3$

$$\Rightarrow W^{(1)} \neq W^{(2)} \neq W^{(3)}$$

(3) $y = x$, $dy = dx$

$$W^{(3)} = \int xy\,dx + \int y^2\,dy$$

$$= \int y^2 dy + \int y^2 dy = 2/3$$

Path matters

$\vec{F}_{ext}$

$$W_{ba} = \oint_a^b \vec{F} \cdot d\vec{r} = \int_a^b m \left(\frac{d\vec{v}}{dt} \cdot \vec{v} \right) dt$$

$$\frac{d}{dt} v^2 = \frac{d}{dt}(\vec{v} \cdot \vec{v}) = \frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt} = 2 \frac{d\vec{v}}{dt} \cdot \vec{v}$$

$$W_{ba} = \int_a^b \frac{m}{2} \cdot \frac{d}{dt}(v^2)\, dt = \frac{1}{2} m \left[V_b^2 - V_a^2 \right].$$

Work energy Theorem

$$\boxed{W_{ba} = \frac{1}{2} m \left[V_b^2 - V_a^2 \right]}$$

$$\boxed{W_{ba} = \frac{1}{2} m \left[\dot{X}_{(b)}^2 - \dot{X}_{(a)}^2 \right]}$$

$X = C M.$

$$W_{ba} = \int_a^b \vec{F} \cdot d\vec{r}$$

Path-dependent.

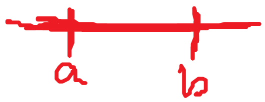
$$d\vec{r} = dx\, \hat{i} + dy\, \hat{j} + dz\, \hat{k}$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

Path-Independent: (I) 1D case: $\vec{F} = F_x \hat{i}$, $dx\, \hat{i} = d\vec{r}$

Path-Independent:

(1) 1D case: $\vec{F} = F_x \hat{i}$, $dx\,\hat{i} = \dots$



$$\int_a^b \vec{F}\cdot d\vec{r} = \int_a^b F_x\,dx$$

$$U_{ba} = I(b) - I(a)$$

(2) Central forces: $\vec{F} = f(r)\,\hat{r}$

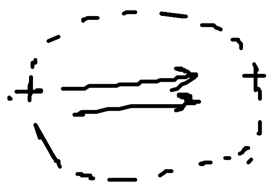
$$\vec{F} = K\frac{q_1 q_2}{r^2}\,\hat{r}$$

$$\vec{F} = -G\frac{Mm}{r^2}\,\hat{r}$$

$$W = \int_a^b \vec{F}\cdot d\vec{r} = \int_a^b f(r)\,dr$$

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

$\hat{r}\cdot\hat{r} = 1$, $\hat{r}\cdot\hat{\theta} = 0$



$$W = \oint \vec{F}\cdot d\vec{r} = 0 \Rightarrow \text{Conservative force}$$

Potential energy: $W_{ba} = \int_a^b \vec{F}\cdot d\vec{r} = -U_b + U_a$

$= K.E_b - K.E_a$ $\Rightarrow K.E_b + U_b = K.E_a + U_a$

$$U_b - U_a = -\int_a^b \vec{F}\cdot d\vec{r}$$

$$-\frac{\partial U}{\partial r} = \vec{F}$$

$a = \infty$

$$\vec{F} = -\frac{GMm}{r^2}\,\hat{r}, \quad U(b) - \underbrace{U(a)}_{0} = +\int_a^b \frac{GMm}{r^2}\,dr$$

$$= -GMm\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$U(b) = -\frac{GMm}{r}$$

$$U(b) = -\frac{GMm}{r}$$

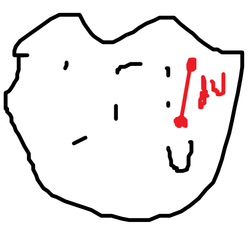
$$U_b - U_a = -\int_a^b \vec{F}\cdot d\vec{r} \quad \Rightarrow \quad \boxed{dU = -\vec{F}\cdot d\vec{r}} \leftarrow$$

$$\left.\begin{aligned} \vec{F} &= F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \\ d\vec{r} &= dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k} \end{aligned}\right]$$

1D case $\quad \vec{F} = -\frac{dU}{dx}$

3D case $\quad \vec{F} = \cancel{-\frac{dU}{d\vec{r}}}$

Let note that $U(x, y, z)$



$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz \leftarrow$$

$$= \left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$$\boxed{\vec{F} = -\nabla U}$$

$$\boxed{dU = (\underbrace{\vec{\nabla} U}_{\vec{F}})\cdot d\vec{r}}$$

$$dU = -\vec{F}\cdot d\vec{r}$$

Ex: $\quad U = \frac{1}{2}K\left[x^2 + y^2 + z^2\right]$

$$\boxed{\vec{F} = -\nabla U} = -\left[\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right]$$

$$\vec{F} = -K\vec{r}$$

$$= -k\left[x\hat{i} + y\hat{j} + z\hat{k}\right] = -K\vec{r}$$

$\vec{r}$

$F = -K\vec{r} \qquad = -k\left[x\hat{i} + y\hat{j} + z\hat{k}\right] = -K\vec{r}$

$\vec{F} = -\nabla\vec{V}$ ⟶ Stokes Theorem's $\qquad F = -\nabla U$

$\longrightarrow \oint \vec{F}\cdot d\vec{r} = \iint_S (\nabla \times \vec{F})\cdot da$