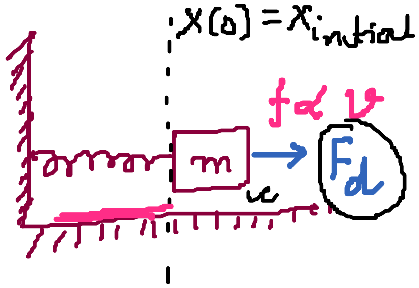
Lecture 12: Section 2

Thursday, January 27, 2022 8:51 AM

Oscillations →

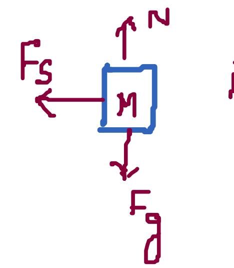


K = Spring constant

Spring is extensible

Undamped HO

Free body diagram



$\vec{N} = -\vec{F_g}$

Horizontal direction dynamics.

- Undamped oscillation
- Damped oscillation
- Forced → driven oscillation
- Forced → driven damped oscillation

Goal: $x(t) = ?$

How it oscillate

$T = ?$

Spring constant $K = ?$

Amplitude of oscillation

P.E, γ

$$\vec{F} = \vec{F_s} + \vec{F_g} + N = \vec{F_s} \Rightarrow m\ddot{x}(t) = -Kx(t)\,\hat{x}$$

$$\Rightarrow \boxed{\ddot{x}(t) + \frac{K}{m}x(t) = 0}$$

$$x(t) = A\cos(\omega_0 t) \Rightarrow -\omega_0^2\underbrace{(A\cos\omega_0 t)}_{x(t)} + \frac{K}{m}x(t) = 0$$

$$\boxed{\omega_0^2 = \frac{K}{m}}$$

$x(t) = B\sin(\omega_0 t)$

Displacement Undamped HO

$$\boxed{x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)}$$

A, B arbitrary Constant

Initial condⁿ → 1. $x(0) = x_{initial}$, 2. $\dot{x}(t) = 0$

$t=0$, $x(0) = A = x_{initial}$,

$\dot{x}(t) = -\omega_0 A\sin(\omega_0 t) + \omega_0 B\cos(\omega_0 t)$ ($t=0$)

$\dot{x}(0) = \omega_0 B = 0$

$\dot{x}(0) = \omega_0 B = 0$

$\Rightarrow B = 0$

$$\boxed{x(t) = x_{\text{initial}} \cos(\omega_0 t)} \qquad , \; \omega_0 = \sqrt{\frac{k}{m}}$$

$$\rightarrow x(t) = A \cos\omega_0 t + B \sin\omega_0 t$$

Underdamped HO

$$\boxed{x(t) = C \cos(\omega_0 t + \phi)}$$

ϕ = Initial phase !

~~$\cos(A+B)$~~

$$= C\left[\cos\omega_0 t \cos\phi - \sin\omega_0 t \sin\phi\right]$$

$$\boxed{\begin{aligned} A &= C\cos\phi \\ B &= -C\sin\phi \end{aligned}}$$

$$\rightarrow x(t) = A\cos\omega_0 t + B \sin\omega_0 t$$

$$C = \sqrt{A^2+B^2}, \quad \boxed{\tan\phi = -\frac{B}{A}}$$

Displacement

$$x(t) = C\left[\cos(\omega_0 t + \phi)\right]$$

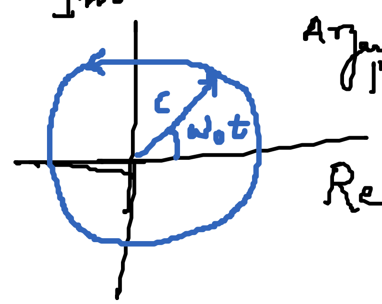
$$= \mathrm{Re}\left[C \cos(\omega_0 t + \phi) + i \underbrace{f(t)}_{C \sin(\omega_0 t)}\right]$$

$f(t)$ = Arbitrary function

$$\Rightarrow \quad \mathrm{Re}[K + iG] = K$$

Arganda plane

$$= \mathrm{Re}\left[C\cos(\omega_0 t) + i\, C \sin\omega_0 t\right]$$



$$\boxed{x(t) = C e^{i\omega_0 t}}$$

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

Time translation over rotation

Phoenix function

$\rightarrow x(t)$ | $x =$ $\dot{x}$

- Phoenix function

- Cannot be killed Differentiation $\rightarrow x(t)$

$x =$
$\rightarrow v = \dot{x}$
$a = \ddot{x}$
$PE = \frac{1}{2}kx^2$, $KE = \frac{1}{2}mv^2$

- Nice property

$$e^{i\theta_1} \cdot e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$$

$$c e^{i(\omega_0 t+\phi)} = c\, e^{i\omega_0 t}\, \boxed{e^{i\phi}}$$

x

Un-Damped HO: $\Rightarrow$

- $\dfrac{x(t)}{x(t+a)}$] Physics of the System will remain the same.

$t = 5$ sec
$t_1 = 7$ sec
$a = 2$ sec

Chain rule: $t_1 = t + a$

$$\frac{df(t')}{dt} = \frac{dx(t')}{dt} = \frac{dx(t')}{dt'}\frac{dt'}{dt} = \frac{dx(t')}{dt'}$$

$v = \dot{x}$
$a = \ddot{x}$

Energy $\Rightarrow$

$$x(t) = c\cos(\omega_0 t+\phi) = Re\left[c e^{i(\omega_0 t+\phi)}\right]$$

$E = K.E + P.E$

$\omega_0 = \sqrt{\frac{k}{m}}$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

K.E P.E $\omega_0 = \sqrt{\frac{k}{m}}$

$$= \frac{1}{2}m\left[-c\omega_0 \sin(\omega_0 t+\phi)\right]^2 + \frac{1}{2}kc^2\cos^2(\omega_0 t+\phi)$$

$\rightarrow$ average

$$= \frac{1}{2}mc^2\omega_0^2 \sin^2(\quad) + \frac{1}{2}m\omega_0^2 c^2 \cos^2(\omega_0 t+\phi)$$

$\frac{1}{2}$ $\frac{1}{2}$

$\nu = 400\,nm$ ~~Femto~~

$= \frac{1}{2} \ldots$

$= \frac{1}{2} mc^2 \omega_0^2 \sin^2(\ldots) + \frac{1}{2} \ldots$

$\nu = 400\,nm$

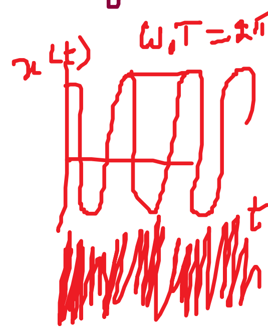
$= \frac{1}{4} mc^2 \omega_0^2$

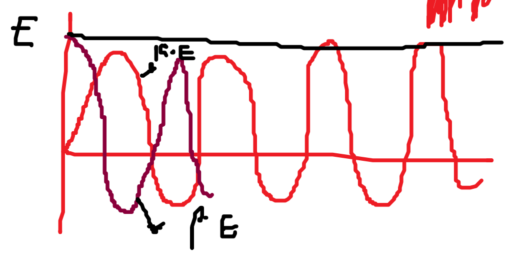
$\omega_0 = 2\pi \nu_0$

$\lambda \nu = c \Rightarrow \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{400 \times 10^{-9}}$ $400 \times 10^{-9}\,m$

$\boxed{\nu = 0.75 \times 10^{15}\,Hz} \rightarrow \omega_0 = 2\pi \times \nu$

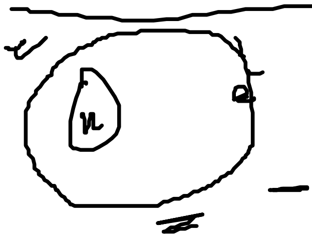
$\boxed{\langle E \rangle = \frac{1}{4} kc^2}$



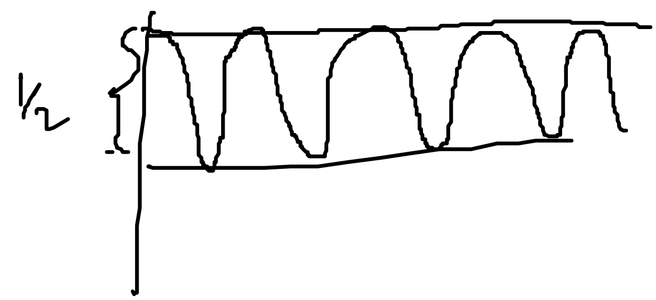


friction

Q.M



I → colour → visible



$\cos^2(x) = \frac{1}{2}$

$\sin^2(x) = \frac{1}{2}$