Academic Year 2023-24
Tutorial #05
PH100: Mechanics and Thermodynamics

1. A 0.3-kg mass is attached to a spring and oscillates at 2 Hz with a *Q* of 60. Find the spring constant and damping constant.

2. In an undamped free harmonic oscillator the motion is given by x = A sin (\omega_0)t. The displacement is maximum exactly midway between the zero crossings. In a damped oscillator, the motion is no longer sinusoidal, and the maximum is advanced before the midpoint of the zero crossings. Show that the maximum is advanced by a phase angle Φ given approximately by Φ=1/2Q, where we assume that Q is large.

3. The logarithmic decrement \delta is defined to be the natural logarithm of the ratio of successive maximum displacements (in the same direction) of a free damped oscillator. Show that \delta = \pi /Q. Find the spring constant *k* and damping constant *b* of a damped oscillator having a mass of 5 kg, frequency of oscillation 0.5 Hz, and logarithmic decrement 0.02.

4. Two particles, each of mass M, are hung between three identical springs. Each spring is massless and has spring constant k. Neglect gravity. The masses are connected as shown to a dashpot of negligible mass. The dashpot exerts a force bv, where v is the relative velocity of its two ends. The force opposes the motion. Let x_1 and x_2 be the displacements of the two masses from equilibrium.

   a. Find the equation of motion for each mass.
   b. Show that the equations of motion can be solved in terms of the new dependent variables $y_1 = x_1 + x_2$ and $y_2 = x_1 — x_2$.
   c. Show that if the masses are initially at rest and mass 1 is given an initial velocity v_0, the motion of the masses after a sufficiently long time is $x_1 = x_2 = (v_0/2$ \omega) sin (\omega t). Evaluate \omega.

5. The motion of a damped oscillator driven by an applied force Fo cos at is given by $x_a(t)$ = A cos (cos \omega t + \phi), where A and \phi are given by Eq. (10.25). Consider an oscillator which is released from rest at t = 0. Its motion must satisfy x(0) = 0, v(0) = 0, but after a very long time, we expect that $x(t) = x_a(t)$. To satisfy these conditions we can take as the solution $x(t) = x_a(t) + X_b(t)$, where $X_b(t)$ is the solution to the equation motion of the free damped oscillator, Eq. (10.8).

   a. Show that if $x_a(t)$ satisfies the equation of motion for the forced damped oscillator, then so does $x(t) = x_a(t) + X_b(t)$, where $x_b(t)$ satisfies the equation of motion of the free damped oscillator, Eq. (10.25).

b. Choose the arbitrary constants in $X_b(t)$ so that $x(t)$ satisfies the initial conditions. [$x_b(t)$ is given by Eq. (10.9). Note that A and \phi here are arbitrary.]
c. Sketch the resulting motion for the case where the oscillator is driven at resonance.