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Hence total no. of 2 letter passwords=$62 * 62$.

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Product rule: Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.

Your friend's mobile no starts with 98 and ends with 3, but middle digits are not known. How many trials (at max) you need to find out his/her mobile no?

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What is the value of k after the following code, where n_1, n_2, n_3 are positive integers, has been executed?

$k := 0$

for $i_1 := 1$ to n_1

for $i_2 := 1$ to n_2

for $i_3 := 1$ to n_3

$k := k + 1$

end

end

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Sum rule: If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

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A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

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Total no. of the task can be completed$=|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

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Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

Generalized Product rule: Let a task T can be broken into r no. of tasks $T_1, T_2, \ldots, T_r$ and T_1 can be completed in n_1 ways and for each way of T_1, T_2 can be completed in n_2 ways and each of these $n_1 * n_2$ ways, T_3 can be completed in n_3 ways and so on.

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   $$S = \{a_1, a_2, \cdots, a_n\}$$
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   We give a bijection $f : A \to P(S)$ as follows:

   $$f(c_1c_2\cdots c_n) = \{a_i | \text{ if } c_i = 1\}$$

   Hence $|P(S)| = |A| = 2^n$.

1. Let $A = \{1, 2, 3, \ldots, m\}$ and $B = \{1, 2, 3, \ldots, n\}$. How many functions are there from A to B? n^m

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How many bit strings of length 6 contain either three consecutive 0s or three consecutive 1s?38

It says that if there n pigeons and $n+1$ nests, then there exists at least one nest with two pigeons.

Theorem (The Pigeonhole Principle)

If $k+1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

Proof.

Suppose theorem is not true, i.e., each of k boxes contains at most one of the objects.

Then total no. of objects in the box is at most k, which is a contradiction. $\square$

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   6 sets and 7 numbers are selected.
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No. of b_i are $n + 1$.
By Pigeonhole Principle, at least two b_i are same, say b_1 and b_2.
Then $a_1 - a_2$ is a multiple of n and is of 0s and 1s.

Generalized Pigeonhole Principle

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If N objects are placed into k boxes, then there is at least one box containing at least $\lceil \frac{N}{k} \rceil$ of the objects.

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2. What is the least number of area codes needed to guarantee that the 10 crores phones in India can be assigned distinct 10-digit telephone numbers? (Assume that telephone numbers are of the form NXX-NXX-XXXX, where the first three digits form the area code, N represents a digit from 2 to 9, and X represents any digit.)

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No. of choices for
$NXX - XXXX = 8 * 10 * 10 * 10 * 10 * 10 * 10 = 80,00,000.$
By Generalized Pigeonhole Principle, there are at least $\lceil \frac{10,00,00,000}{80,00,000} \rceil = 13$ no. of times NXX-XXXX repeated.

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Question: If 9 people are seated in a row of 12 chairs, then some consecutive set of 3 chairs are filled with people.

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Let $A = \{a_1, a_2, \cdots, a_{n^2+1}\}$ be a sequence.

$$a_i \leftrightarrow (b_i, c_i)$$

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Then the number of all possible pairs (b_i, c_i) with $1 \leq b_i, c_i \leq n$ is n^2.

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Which is a contradiction.

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In a sequence of $n^2 + 1$ distinct integers, there is either an increasing subsequence of length $n + 1$ or a decreasing subsequence of length $n + 1$.

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How many different 5 members committees can be formed among 100 students?
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Ans: ^nC_i

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Above Identity is useful in defining Binomial coefficients recursively. For each $n \in \mathbb{N}$, define

$${}^nC_0 =^n C_n = 1$$

$${}^nC_i = {}^{n-1}C_{i-1} + {}^{n-1}C_i$$

Pascal's Triangle

$$\binom{0}{0}$$

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$$\cdots$$

By Pascal's identity:

$$\binom{6}{4} + \binom{6}{5} = \binom{7}{5}$$

1

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LHS is choosing r elements from a set of $m + n$ elements.
It is same as
Choosing $r - i$ elements out of first m elements and i elements out of next n elements, for some i between 0 and r. $\square$

Vandermonde's Identity

Theorem (Vandermonde's Identity)

Let $m, n, r \in \mathbb{N}$ with $r \leq m$ or $r \leq n$. Then

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Corollary

$$^{2n}C_n = \sum_{i=0}^{n} (^{n}C_i)^2$$

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Let $r, n \in \mathbb{N}$ *with* $r \leq n$*. Then*

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No. of zeros before i^{th} one and after $(i-1)^{th}$ one will give no. of i^{th} objects to be selected.

Suppose that a cookie shop has four different kinds of cookies. How many different ways can six cookies be chosen? Assume that only the type of cookie, and not the individual cookies or the order in which they are chosen, matters.

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No. of different permutations of n objects, where n_1 are of same type, say type 1, n_2 are of type 2,...,n_k objects are same of type k is

$$\frac{n!}{n_1! * n_2! * \cdots * n_k!}$$

Generating Permutations

Let $A = \{1, 2, \cdots, n\}$.
Consider lexicographic ordering on the set of permutations of A using the natural ordering of A.
$a_1 a_2 \cdots a_n < b_1 b_2 \cdots b_n$ iff there exists i such that $a_j = b_j$ for $1 \leq j < i$ and $a_i < b_i$

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Repeat above procedure on new permutation to get next permutation.

Algorithm to generate new permutation

Input$(a_1 a_2 \cdots a_n)$
$\{$ If $a_1 a_2 \cdots a_n = nn-1 \cdots 21$ then exit
Else
$j := n-1$
while$(a_j > a_{j+1})$
$\{j = j-1\}$we come out of loop when $a_j < a_{j+1}$
$k := n$
while $(a_j > a_k)$
$\{k = k-1\}$
Interchange a_j and a_k
$r := n$; $s := j+1$
while$(r > s)$
$\{$ interchange a_r and a_s
$r := r-1, s := s+1$ $\}\}$

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Let $A = \{a_1, a_2, \cdots, a_n\}$. We have

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In this lexicographic ordering, the first r-combination is $\{1, 2, \cdots, r-1, r\}$ and the last r-combination is $\{n-r+1, n-r+2, \cdots, n-1, n\}$.

An r-combination of $A = \{1, 2, \ldots, n\}$ can be represented by a sequence containing the elements in the subset in increasing order (e.g., 35n for $\{3, 5, n\}$).

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First, locate the last element a_i in the sequence such that $a_i \neq n - r + i$.

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Then, replace a_i with $a_i + 1$ and a_j with $a_i + j - i + 1$, for $j = i+1, i+2, \ldots, r$.

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What is next larger 4-combination of $\{1, 2, \dots, 6\}$ after $\{1, 2, 5, 6\}$? Find $n - r + i$ for $i = 1, 2, 3, 4$. (i.e., 3,4,5,6)

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Replace a_2 by $a_2 + 1$ and next a_j with $a_2 + j - 2 + 1$ to get

$$\{1, 3, 4, 5\}$$