Academic Year 2022-23

Tutorial #04

PH100: Mechanics and Thermodynamics

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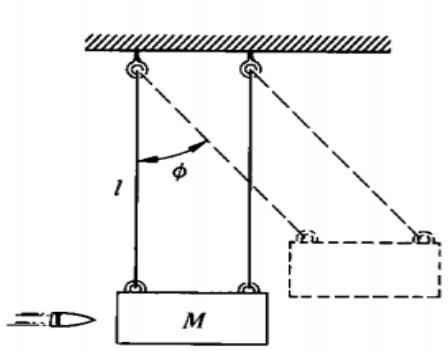
1. A simple way to measure the speed of a bullet is with a ballistic pendulum. As illustrated, this consists of a wooden block of mass *M* into which the bullet is shot. The block is suspended from cables of length *I*, and the impact of the bullet causes it to swing through a maximum angle Φ, as shown. The initial speed of the bullet is *v*, and its mass is *m*.
   a. How fast is the block moving immediately after the bullet comes to rest? (Assume that this happens quickly.)
   b. Show how to find the velocity of the bullet by measuring *m*, *M*, *I*, and Φ.



2. Mass *m* whirls on a frictionless table, held to circular motion by a string which passes through a hole in the table. The string is slowly pulled through the hole so that the radius of the circle changes from l_1 to l_2. Show that the work done in pulling the string equals the increase in kinetic energy of the mass.

3. A commonly used potential energy function to describe the interaction between two atoms is the Lennard-Jones potential:

$$U = \epsilon \left[\left(\frac{r_0}{r} \right)^{12} - 2 \left(\frac{r_0}{r} \right)^{6} \right]$$

   a. Show that the radius at the potential minimum is r_0, and that the depth of the potential well is \epsilon.

   b. Find the frequency of small oscillations about equilibrium for 2 identical atoms of mass *m* bound to each other by the Lennard-Jones interaction.

4. A particle of mass *m* moves in one dimension along the positive x axis. It is acted on by a constant force directed toward the origin with magnitude *B*, and an inverse square law repulsive force with magnitude A/x^2.
   a. Find the potential energy function *U(x)*.

b. Sketch the energy diagram for the system when the maximum kinetic energy is $K_0 = (1/2)\, m\, v_0^2$.
c. Find the equilibrium position, x_0.
d. What is the frequency of small oscillations about x_0?

5. A particle of mass *m* and velocity v_0 collides elastically with a particle of mass *M* initially at rest and is scattered through angle θ in the center of the mass system.
   a. Find the final velocity of *m* in the laboratory system.
   b. Find the fractional loss of kinetic energy of *m*.

6. A ball drops to the floor and bounces, eventually coming to rest. Collisions between the ball and floor are inelastic; the speed after each collision is *e* times the speed before the collision where $e < 1$, (*e* is called the coefficient of restitution.) If the speed just before the first bounce is v_0, find the time to come to rest.

7. A proton makes a head-on collision with an unknown particle at rest. The proton rebounds straight back with *4/9* of its initial kinetic energy. Find the ratio of the mass of the unknown particle to the mass of the proton, assuming that the collision is elastic.