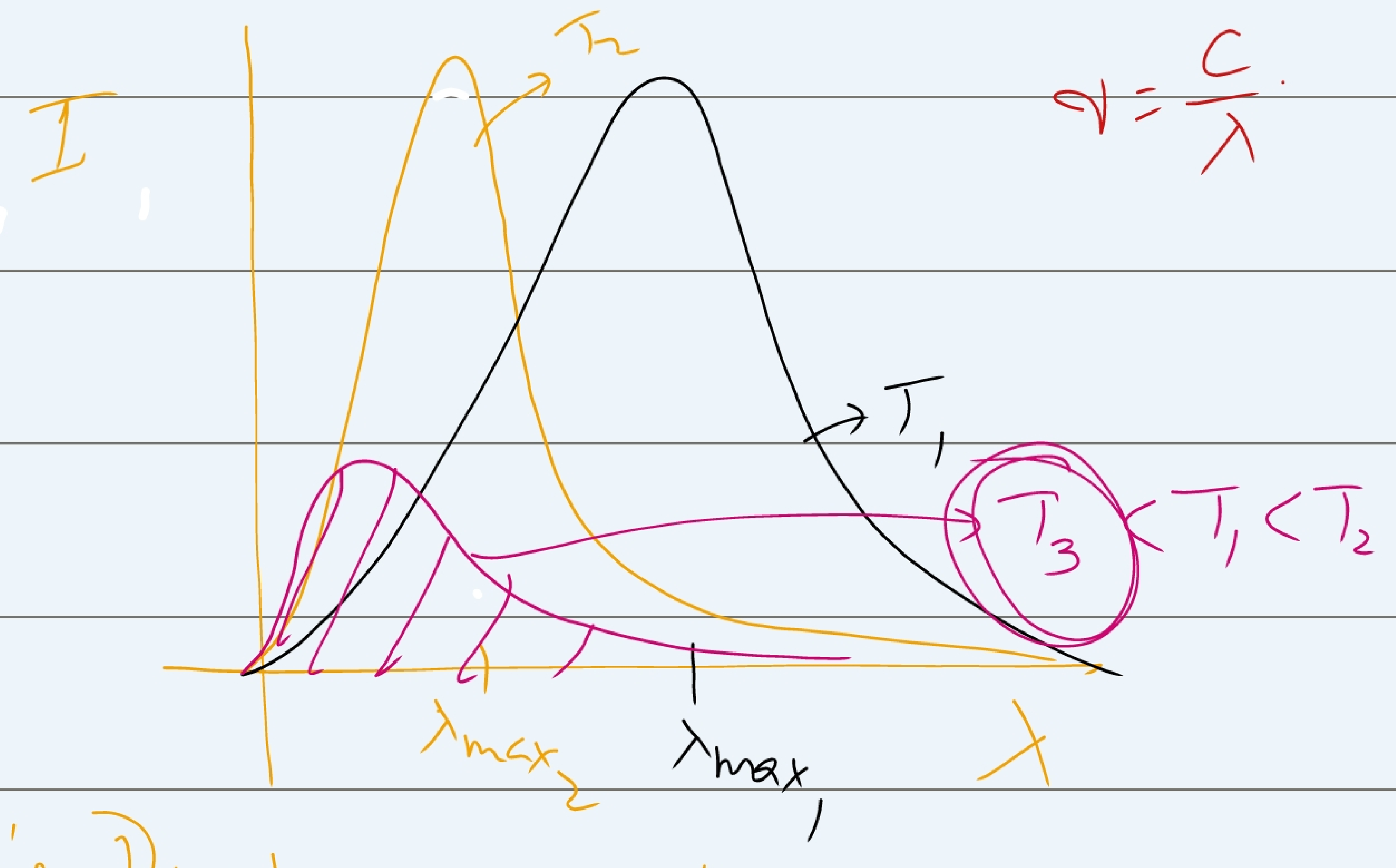
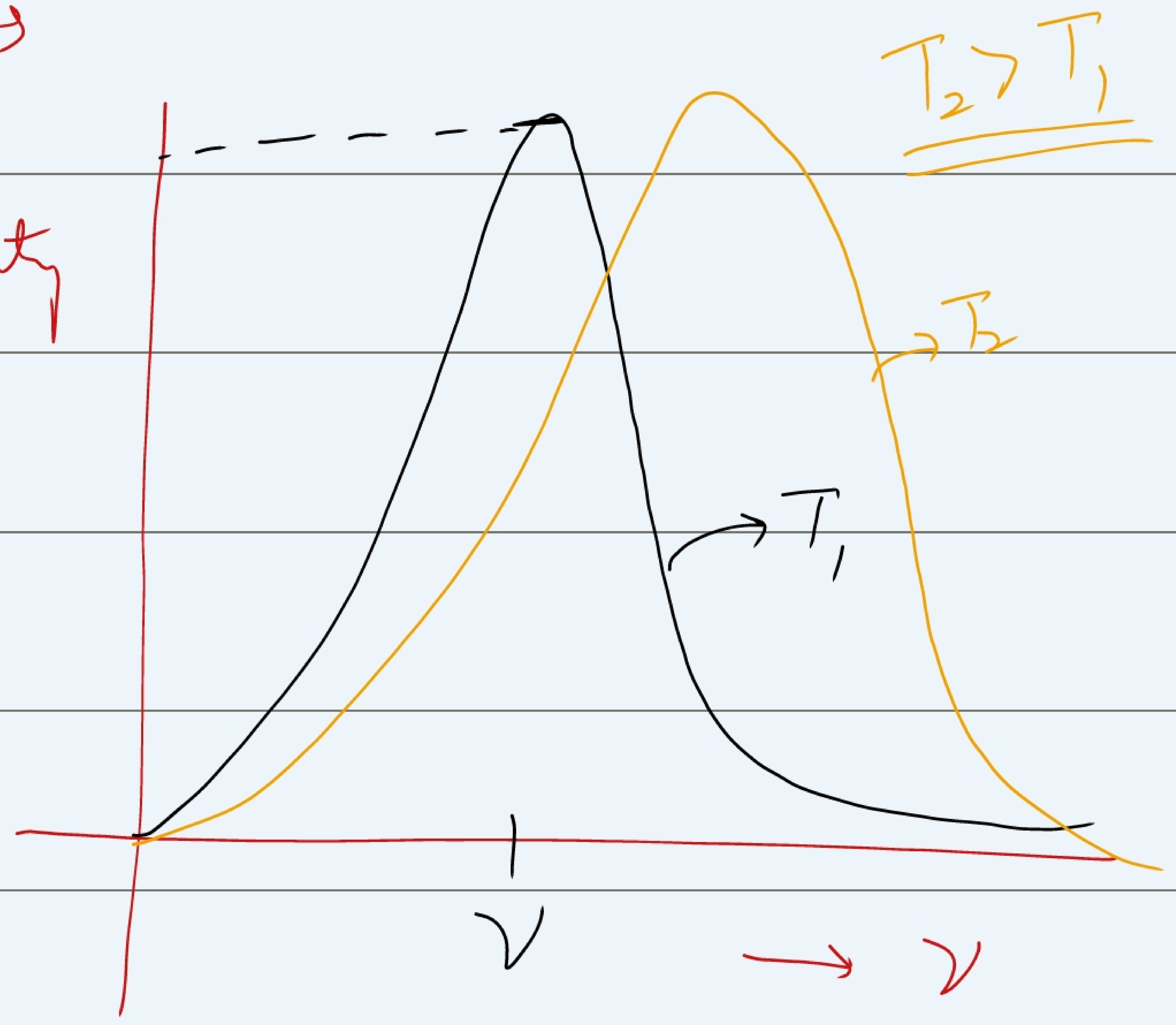
01/11/2023

Failures of Classical Mechanics :→



$\nu = \frac{c}{\lambda}$

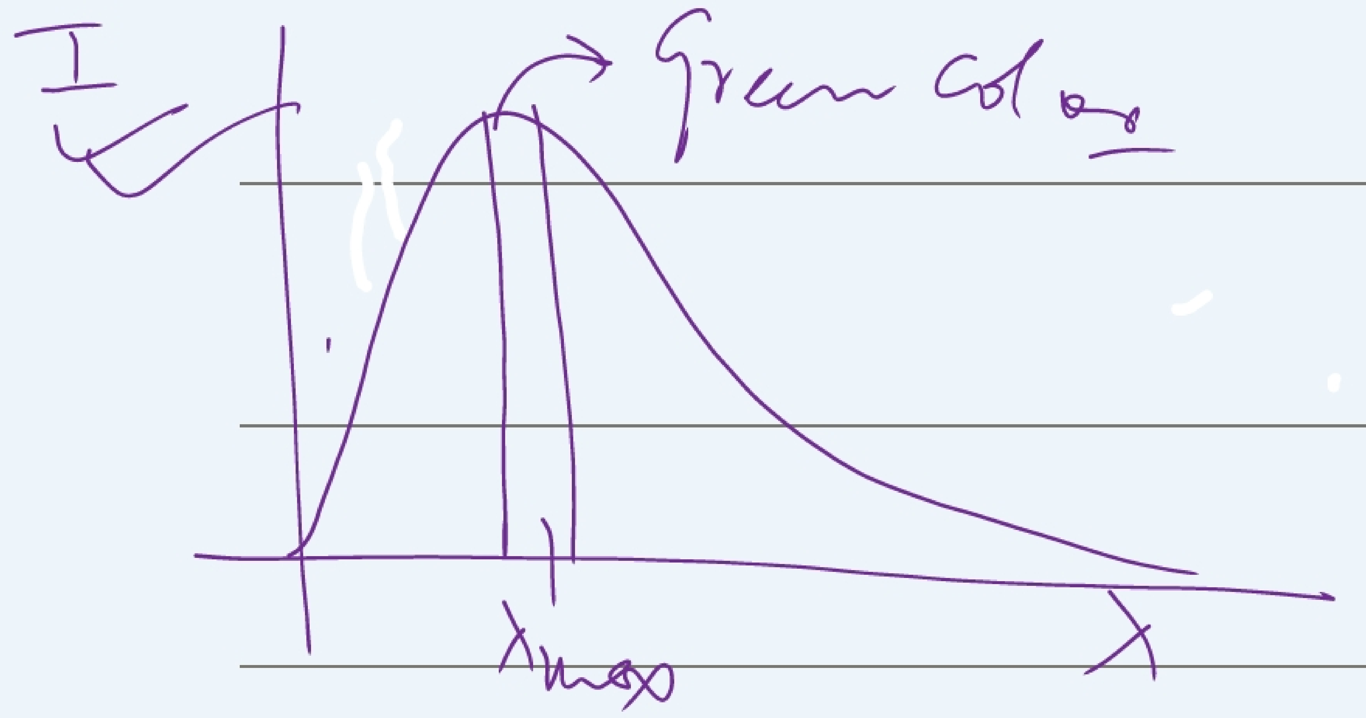


Wien's Displacement

$$\lambda_{max} T = 2.898 \times 10^{-3}\ mK$$

Stefan Boltzmann Law

$$R(T) = \epsilon \sigma T^4$$



$\lambda_{max} \cdot 6000K = 2.898 \times 10^{-3}\ mK$

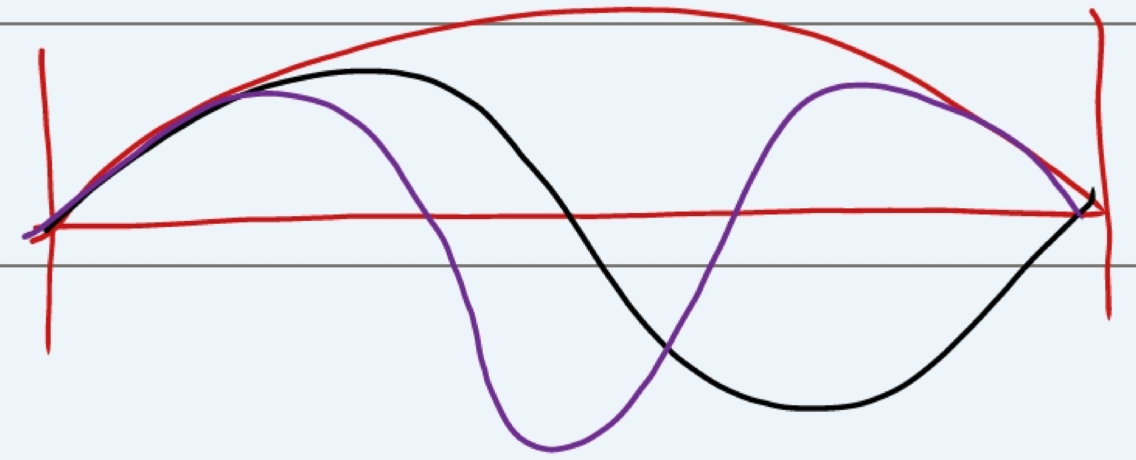
$\lambda_{max} \simeq 480 \times 10^{-9}\ m$

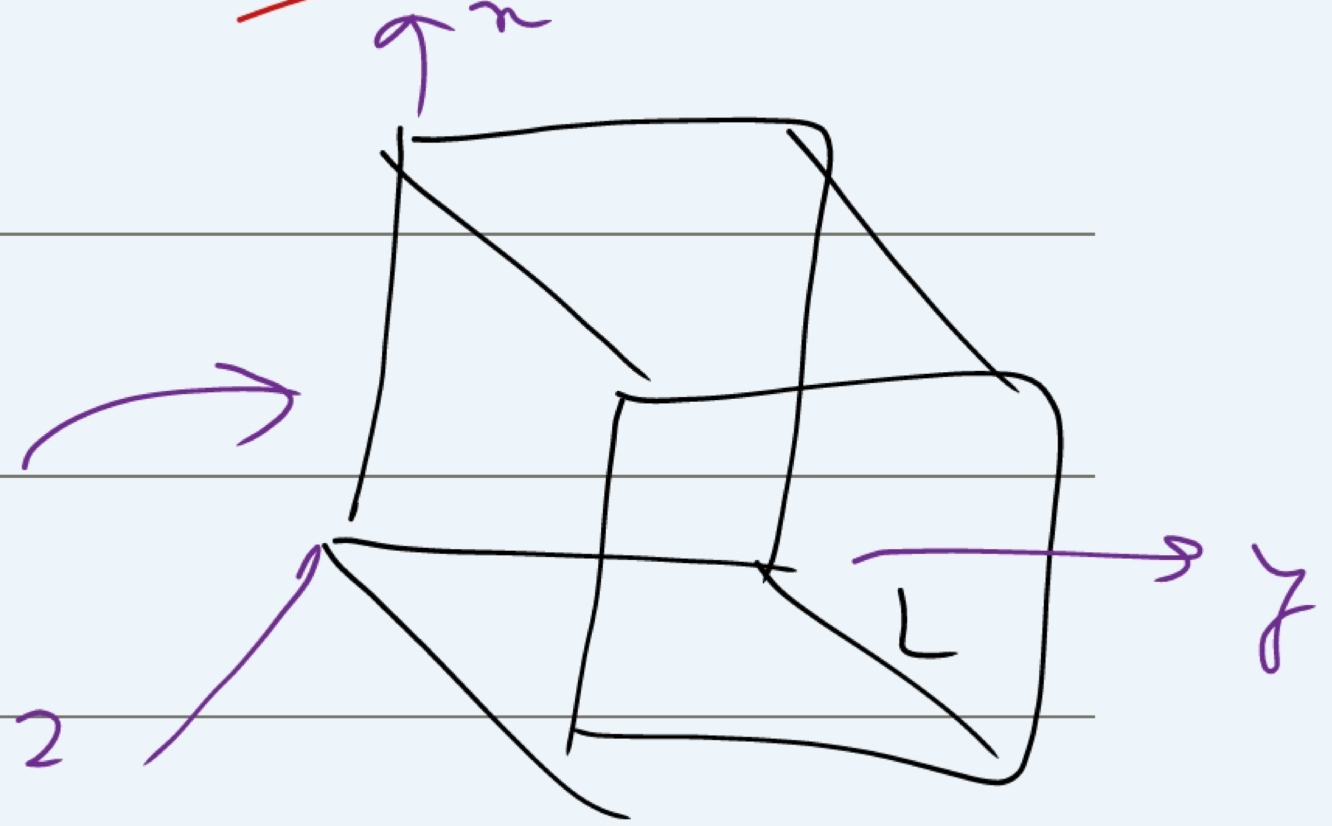
Rayleigh-Jeans Law →

$\frac{c}{\lambda_n} = \lambda_n = \frac{2L}{j}$

$\sqrt{j_x^2 + j_y^2 + j_z^2}^{\,2}$

$\epsilon = 1$ $\quad L^3$





1 mode $\quad \frac{\lambda}{2} = L$

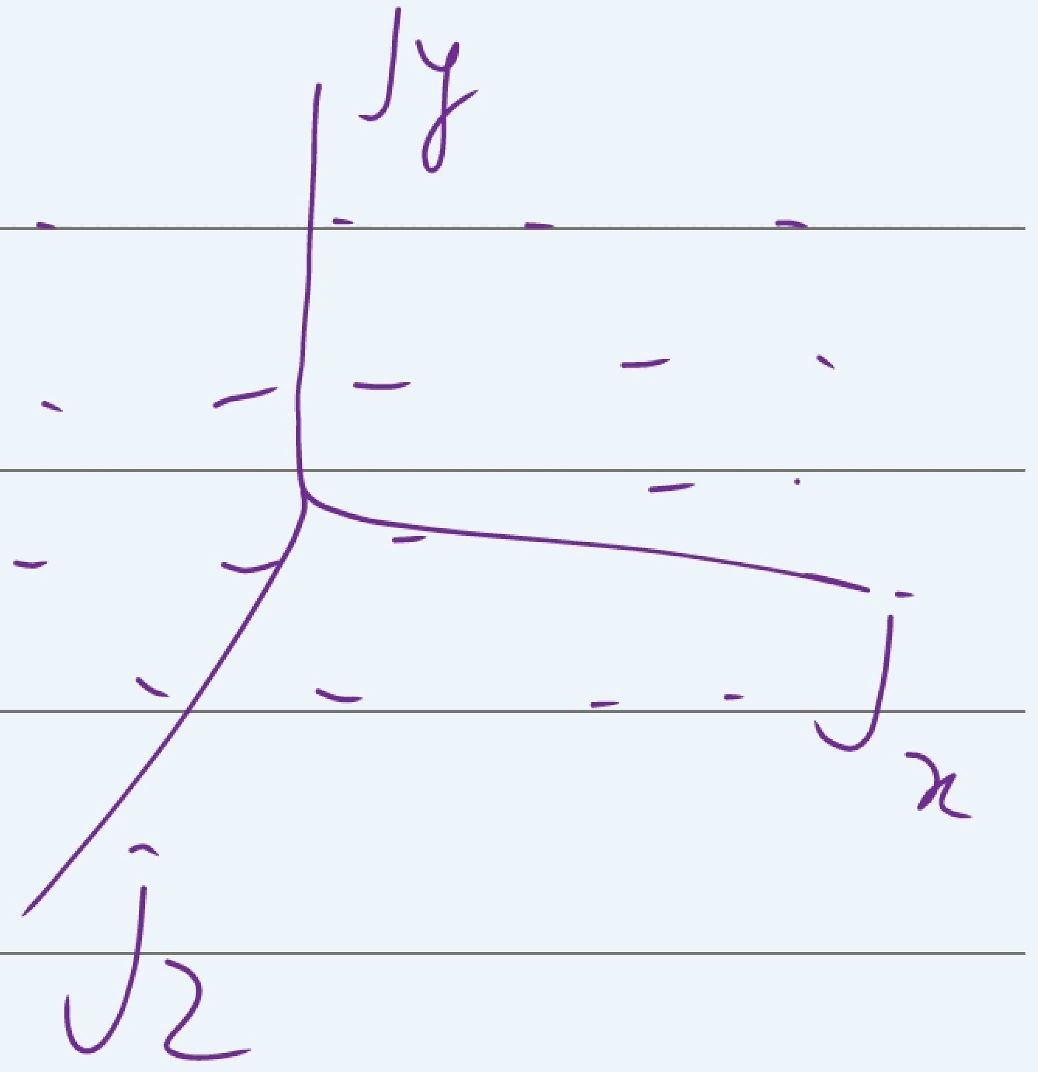
$\lambda = L$

$\frac{3}{2}\lambda = L$

$\Rightarrow \boxed{j_x \lambda = 2L}, \quad j = 1, 2, 3, \ldots$

$j_y \lambda = 2L$

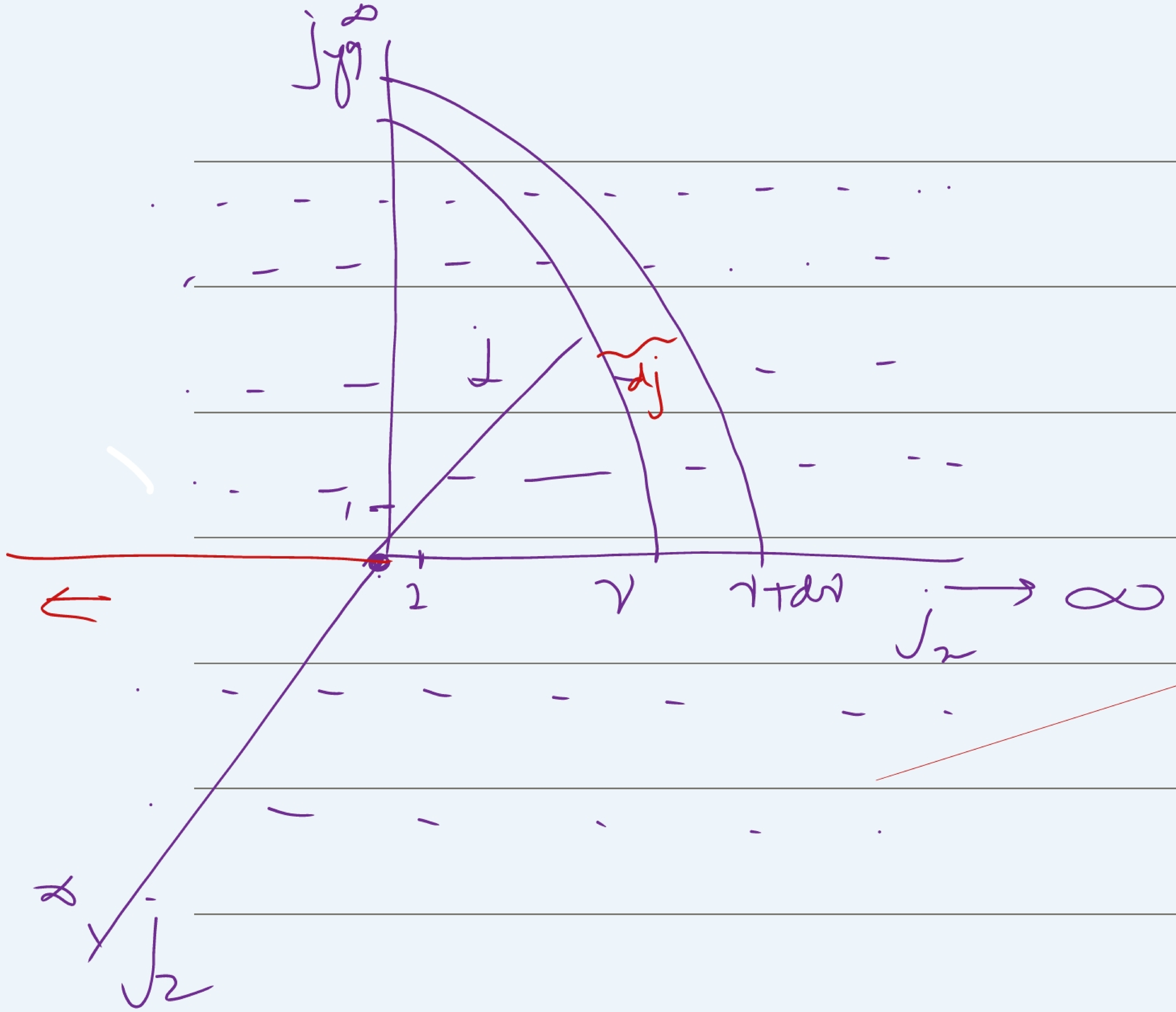
$j_z \lambda = 2L$



For any arbitrary Direction, Standing wave

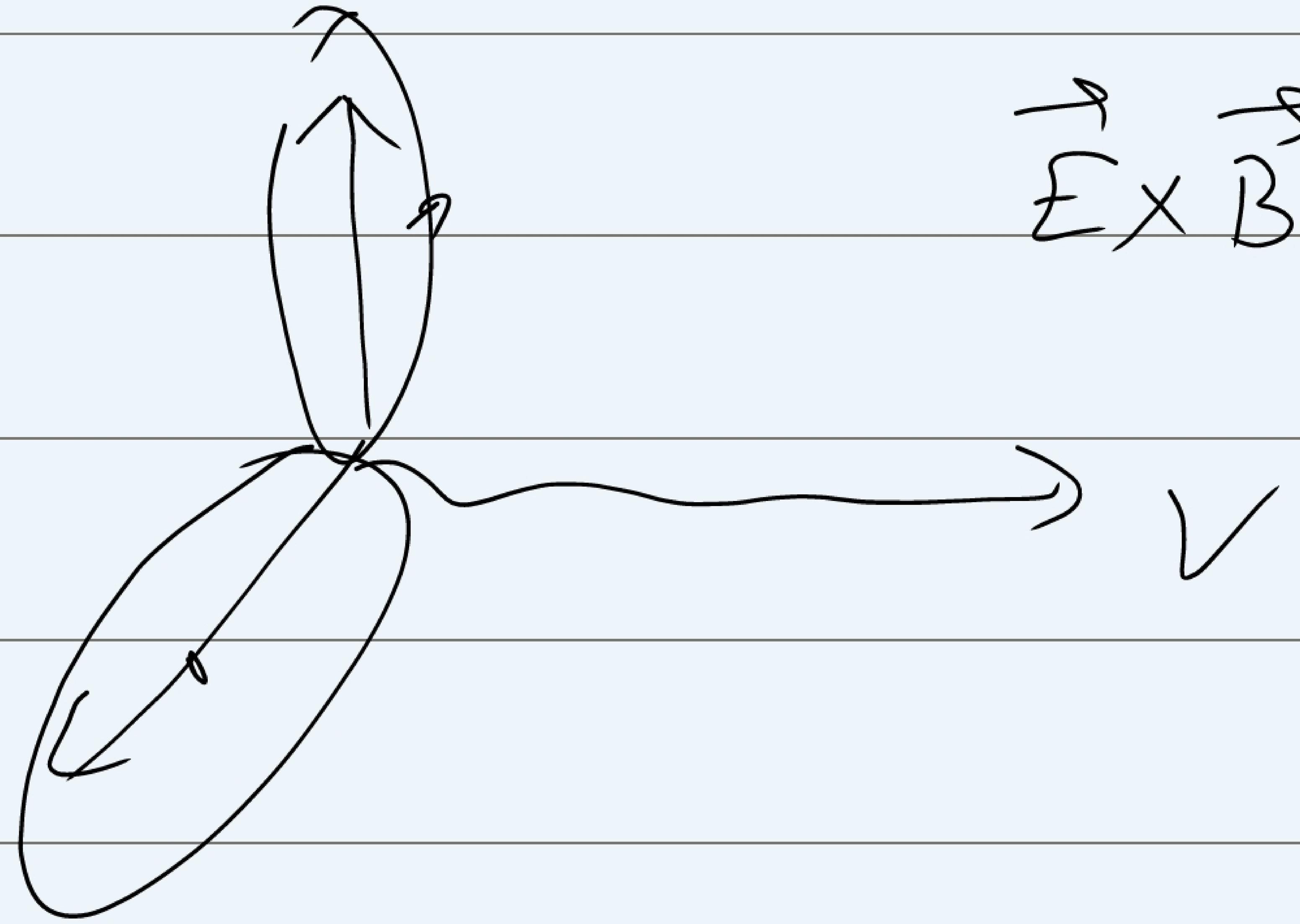
$= \sqrt{j_x^2 + j_y^2 + j_z^2}$

$j_x = \frac{2L}{\lambda}, j_y = \frac{2L}{\lambda}, j_z = \frac{2L}{\lambda}$



Volume of standing (No. of standing) wave $= \frac{4\pi j^2 dj \times \frac{1}{8} \times 2}{} = \pi j^2 dj$

$g(\nu)d\nu$



$\vec{E} \times \vec{B} =$

$$g(\nu)\,d\nu = \pi j^2\,dj \qquad j = \frac{2L}{\lambda} = \frac{2L\nu}{c}$$

$$= \pi\left(\frac{2L\nu}{c}\right)^2 \cdot \left(\frac{2L}{c}\right) d\nu \qquad dj = \frac{2L}{c}\,d\nu$$

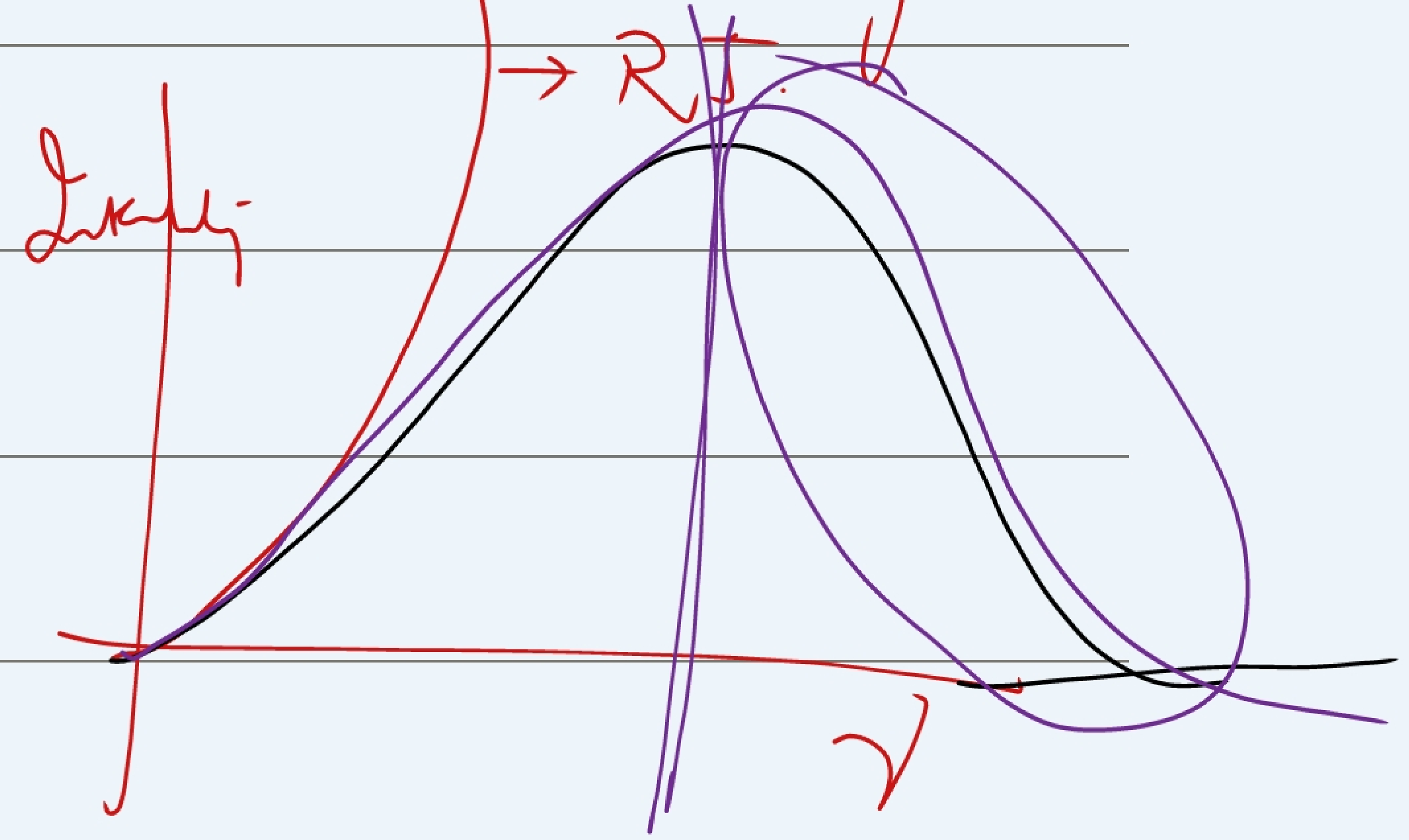
$$g(\nu)\,d\nu = \frac{8\pi \cancel{L^3}}{c^3\,\cancel{L^3}}\,\nu^2\,d\nu = \frac{8\pi}{c^3}\nu^2\,d\nu = \text{Density of standing wave formed in the cavity.}$$

$$\frac{\text{No. of standing wave}}{\text{Volume}}$$

$$\frac{h\nu}{e^{h\nu/KT}-1} = \bar{E} = K_B T \qquad \left(\begin{array}{ll} 1/2 & 1\ \text{degree} \\ 3/2 & 3\ \text{degree} \end{array}\right)$$

R.J. Radiation Law.

$$\text{Intensity} = \frac{8\pi\nu^2\,d\nu}{c^3} \cdot K_B T$$



Planck's Radiation Law:-

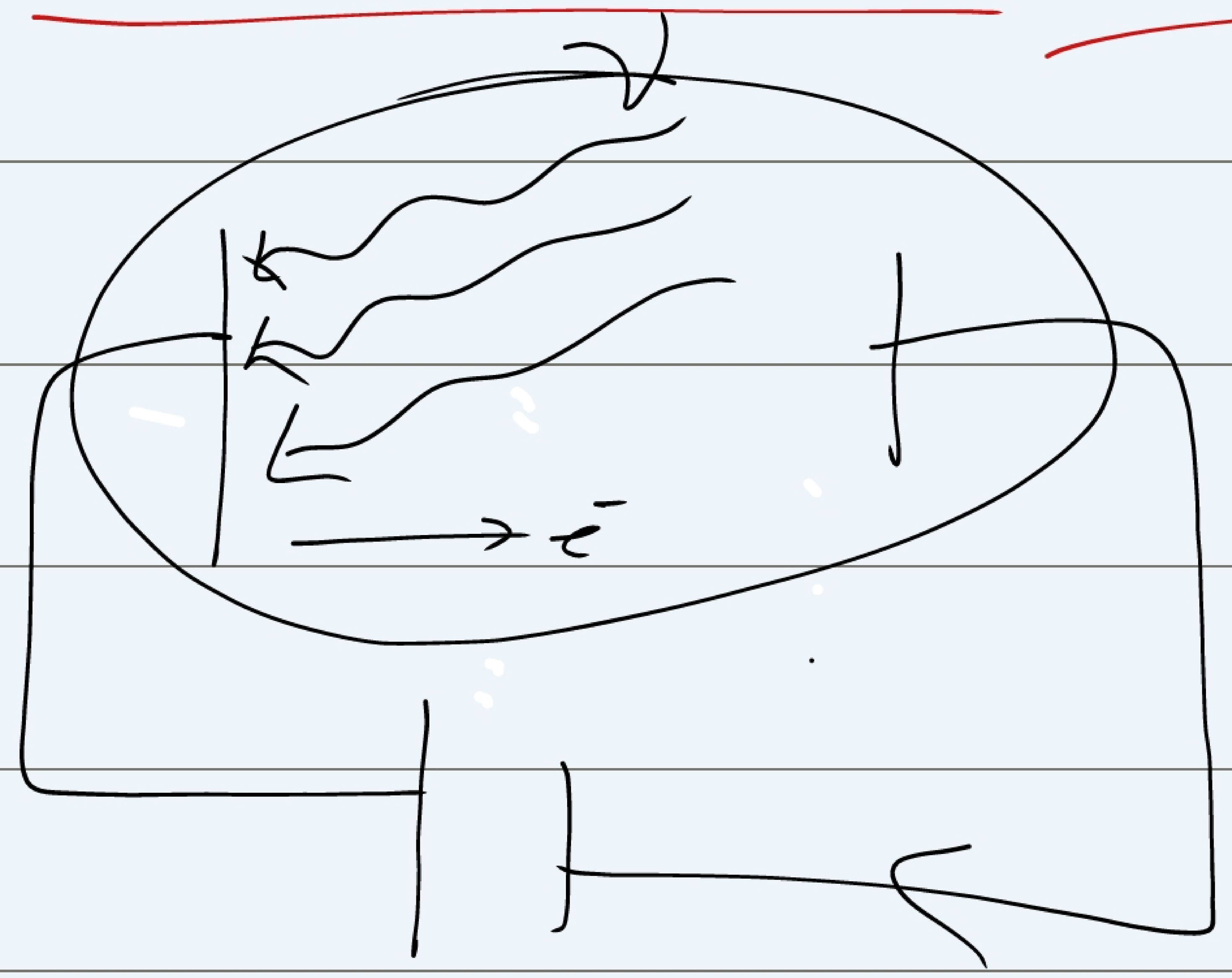
$$I = \left(\frac{8\pi\nu^2 d\nu}{c^3}\right) \cdot \frac{\cancel{h\nu}}{e^{h\nu/KT} - 1} \quad \frac{\cancel{h\nu}}{KT}$$

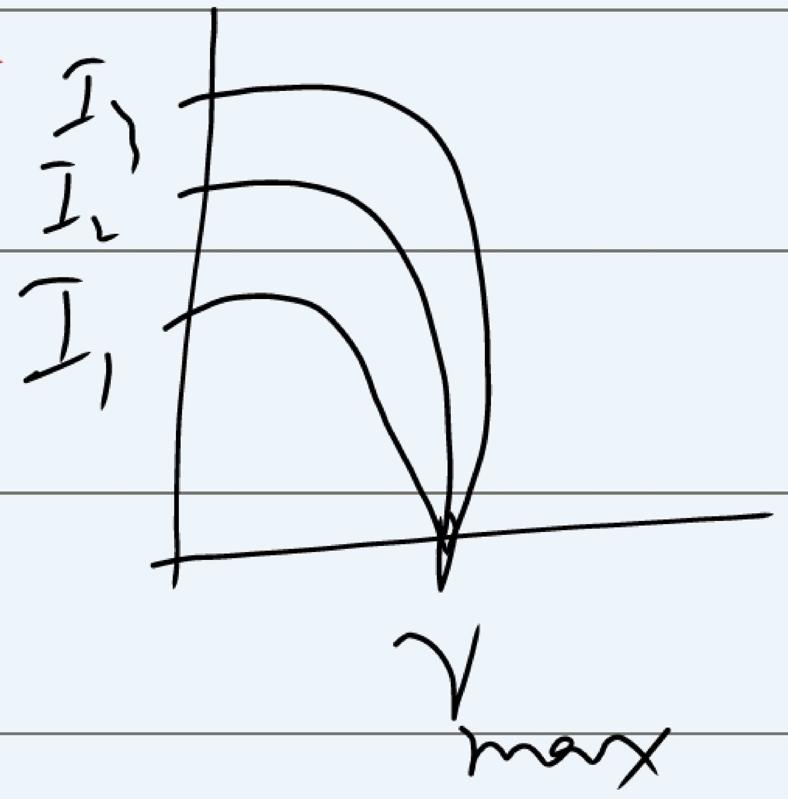
① For large ν. $\frac{h\nu}{KT} >> 1$, $h\nu >> KT$, $e^{h\nu/KT} \to \infty$

② For small ν, $\frac{h\nu}{KT} << 1$, $h\nu << KT$, $e^{h\nu/KT} \approx 1 + \frac{h\nu}{KT}$

$e^x = 1 + x + \frac{x^2}{2!} + \cdots$ $\quad e^{h\nu/KT} - 1 \approx \frac{h\nu}{KT}$, $I =$

Photoelectric Effect :-



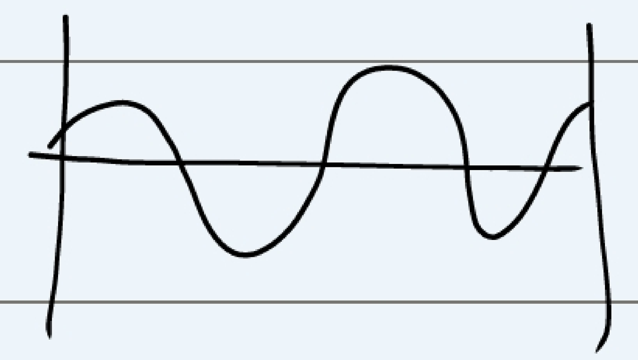


$\varepsilon = K_B T$

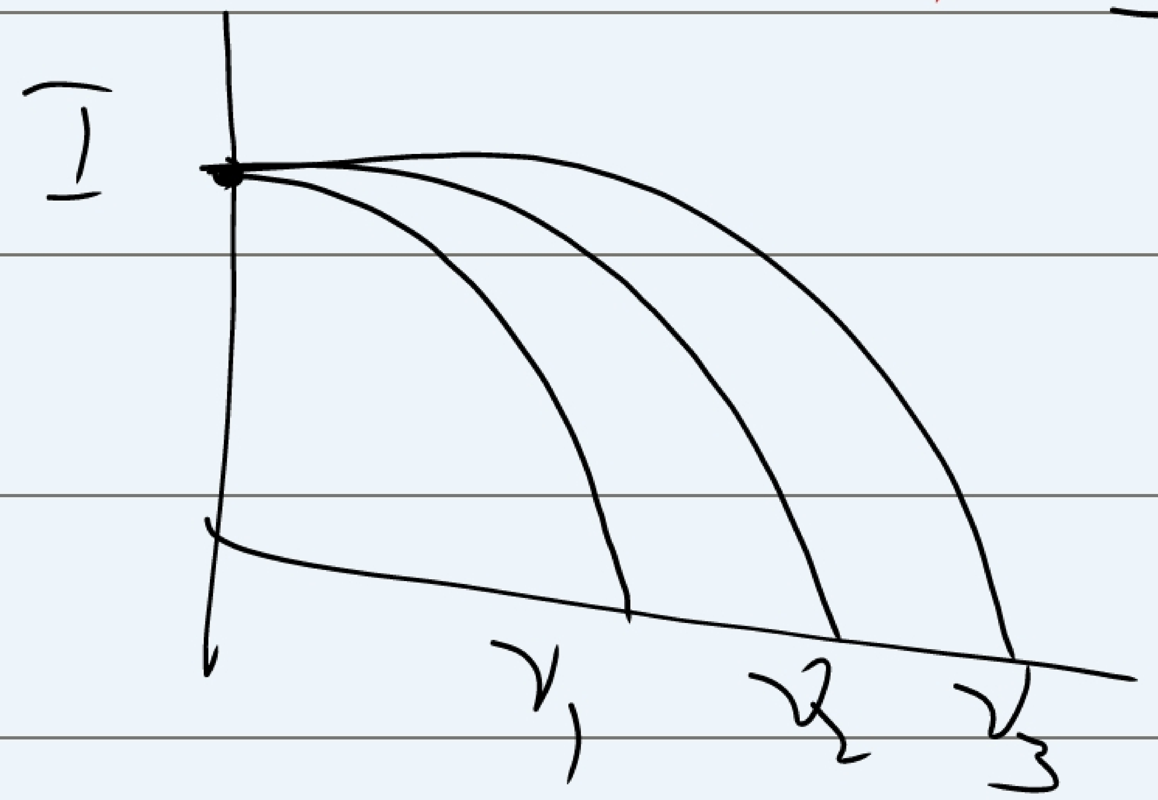
$\boxed{\varepsilon = nh\nu}$

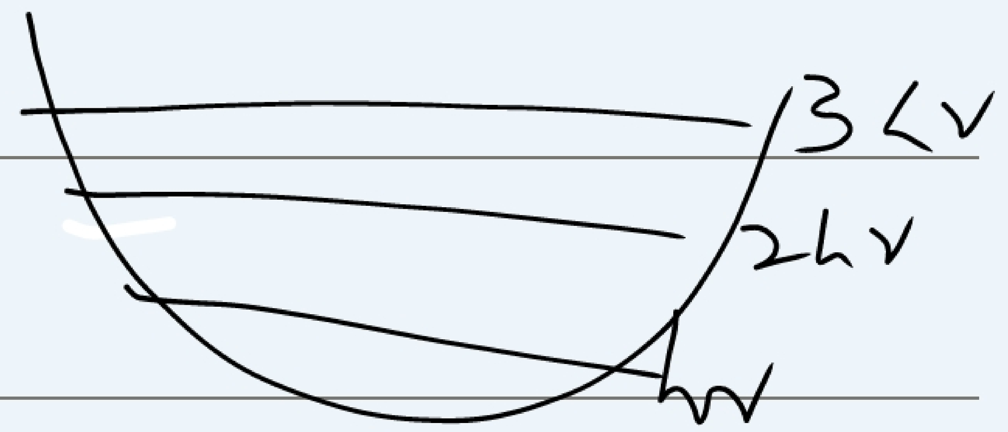
$\varepsilon_1 = h\nu$

$\varepsilon_2 = 2h\nu$



$eV_{max} = h\nu_0 + h\nu$





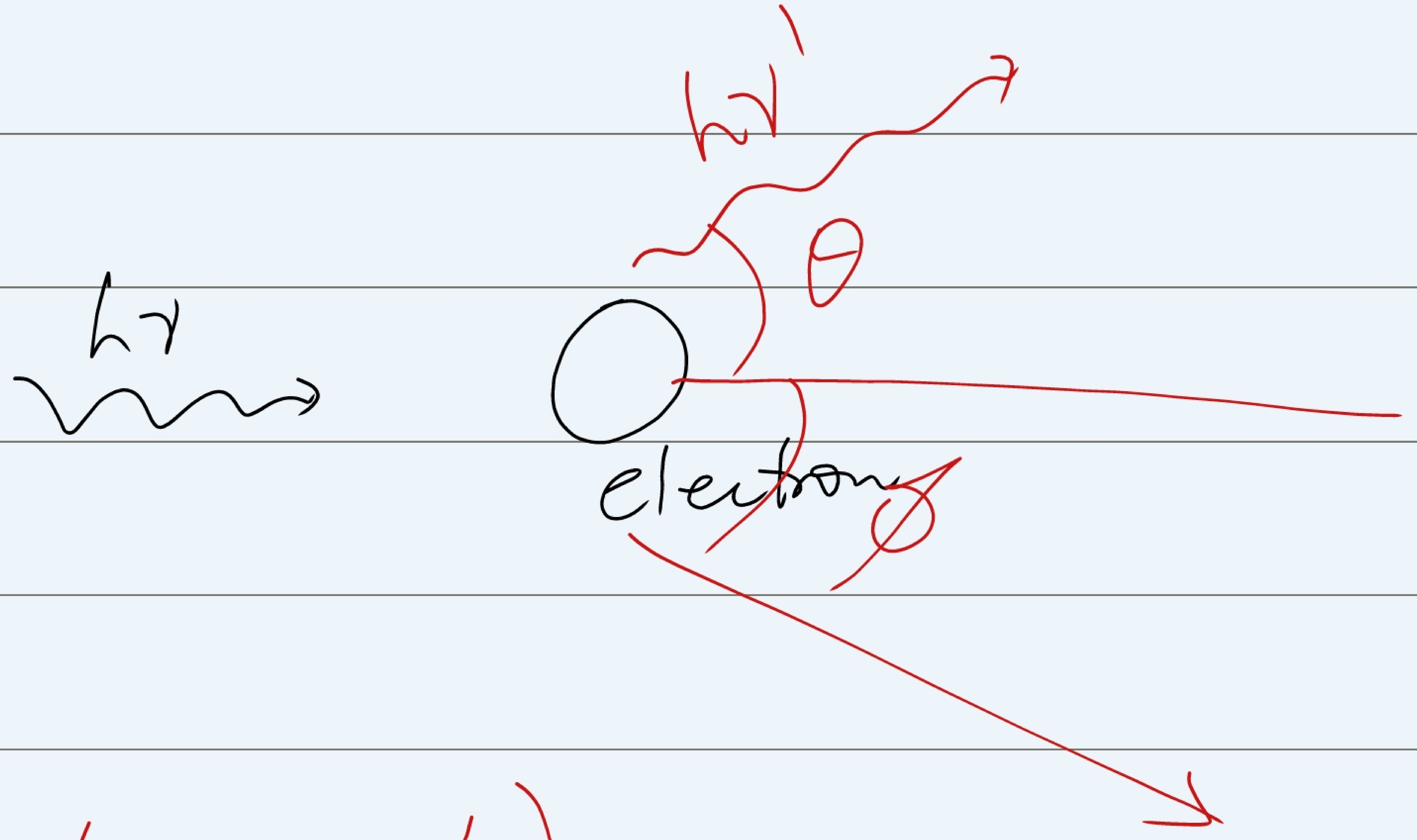
$E = \frac{1}{2} KA^2$

② Compton Effect :→

Planck's

$E = nh\nu$

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

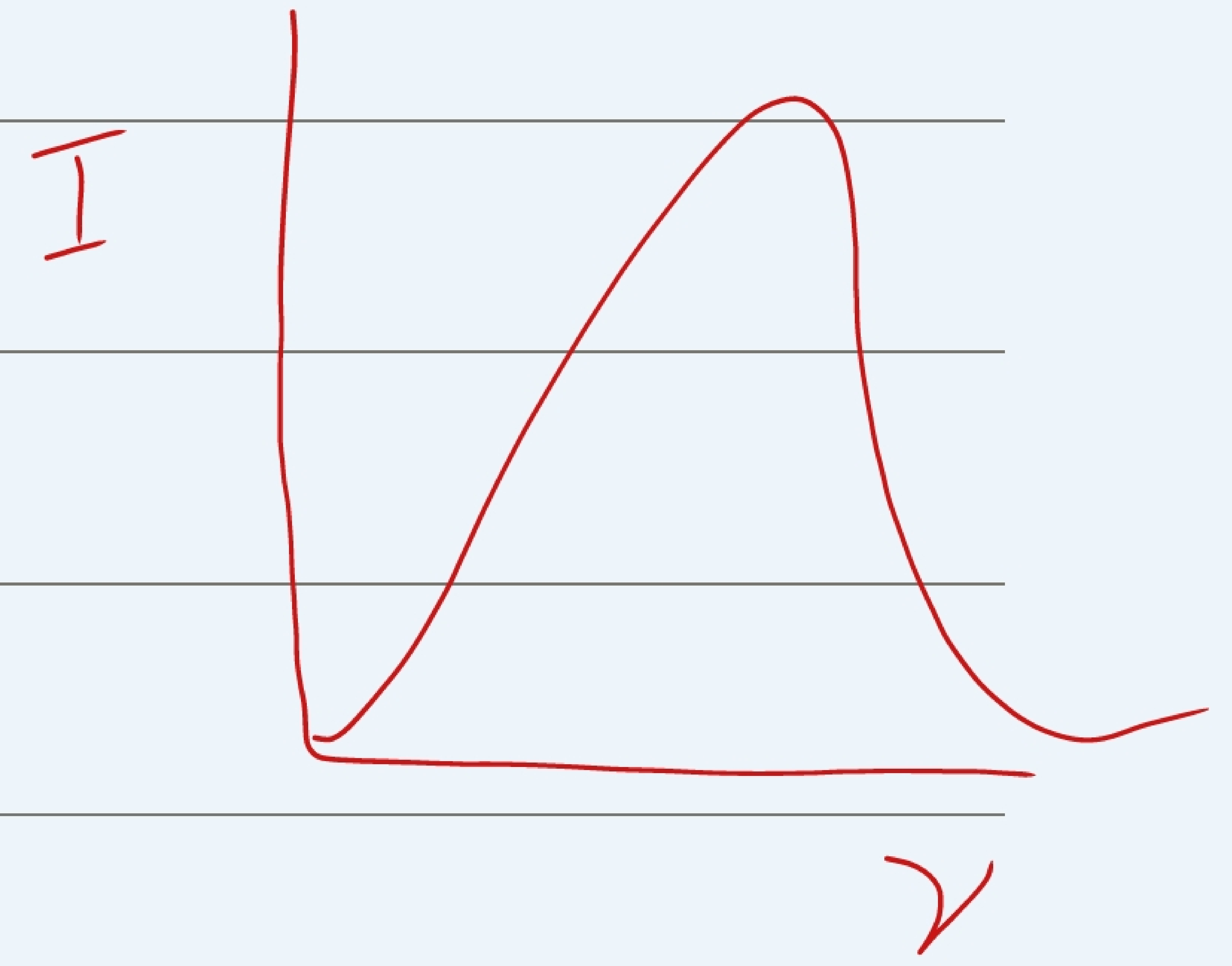


$h(\nu - \nu') \rightarrow$ Quantized.

$$\frac{h}{mc}\left[1 - \cos\theta\right]$$

$$\boxed{\frac{8\pi\nu^2}{c^3}}$$

Av. Eng $= K_B T$ $\leftarrow$ RJ : $I \propto \nu^2$



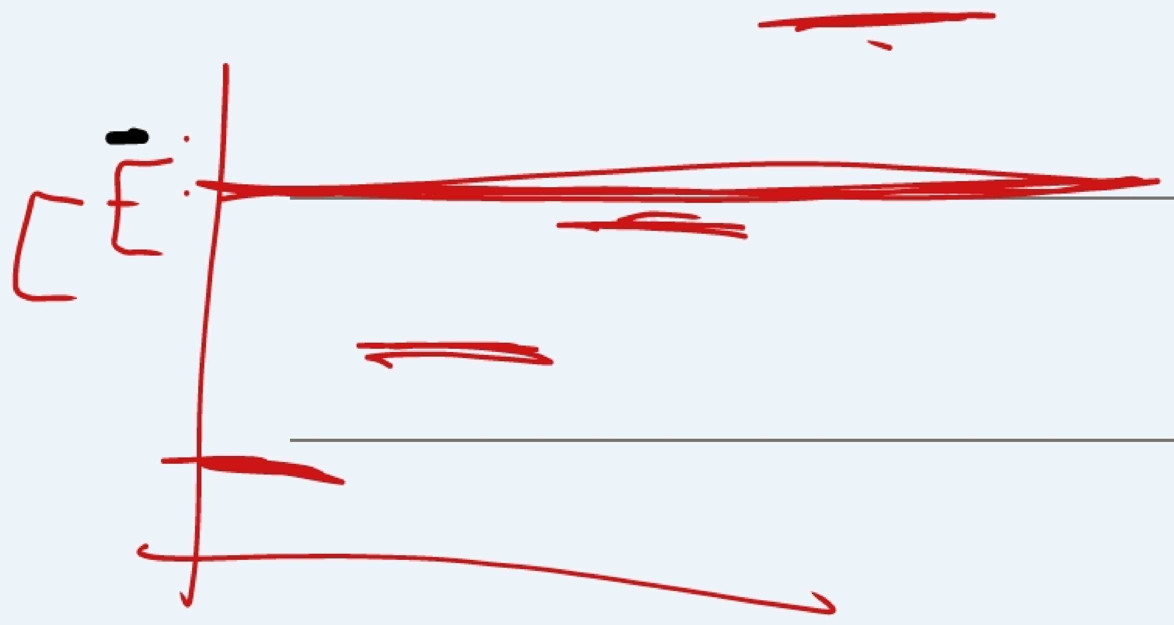
Classical System:

HO:

$\bar{E} = \frac{1}{4} K A^2$

$$\frac{n h\nu}{e^{h\nu/KT} - 1}$$

Planck:



$E_n = nh\nu$.

Compton Effect :→ $E = h\nu$

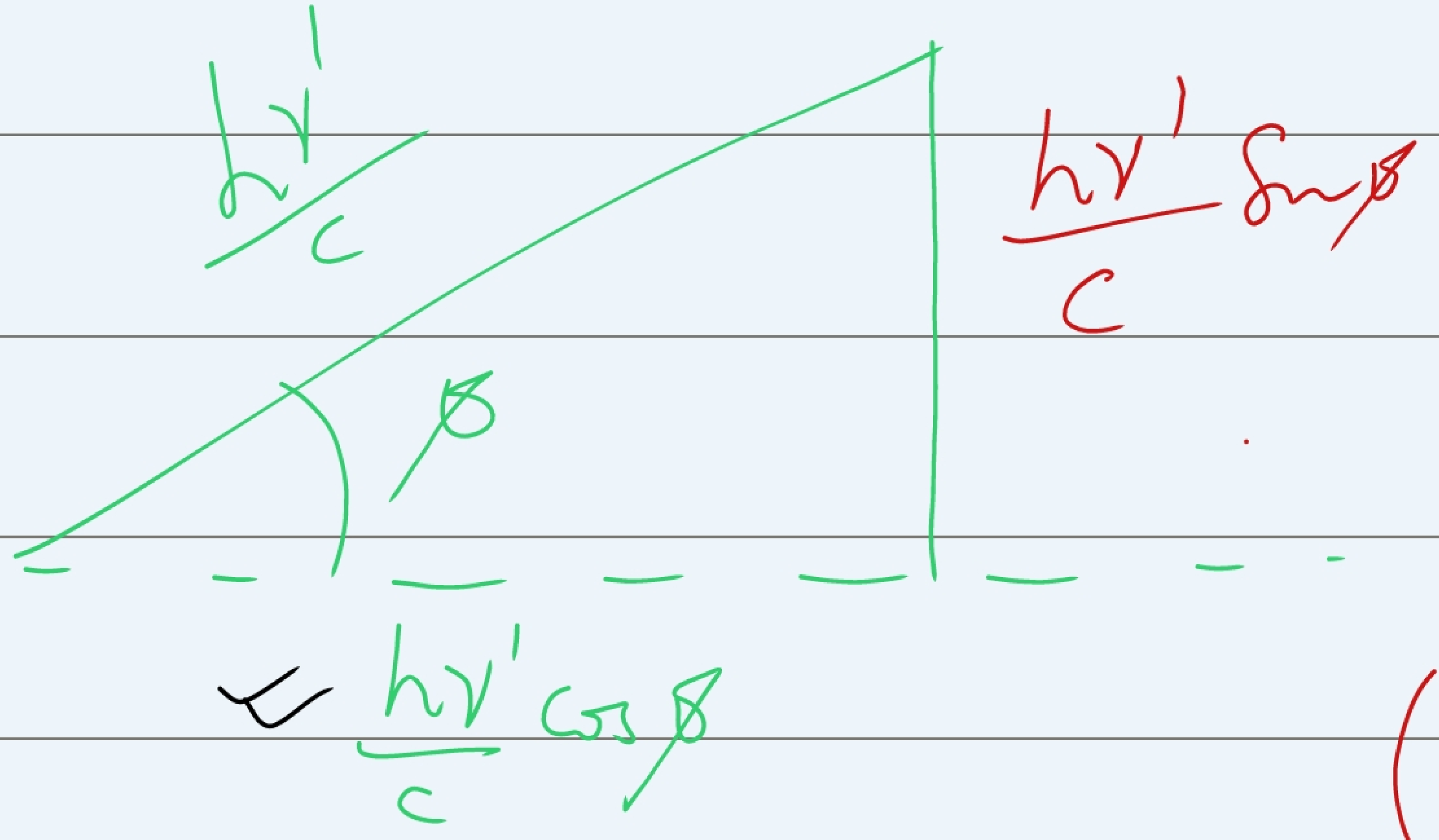
A. Beiser

(Particle nature of wave):

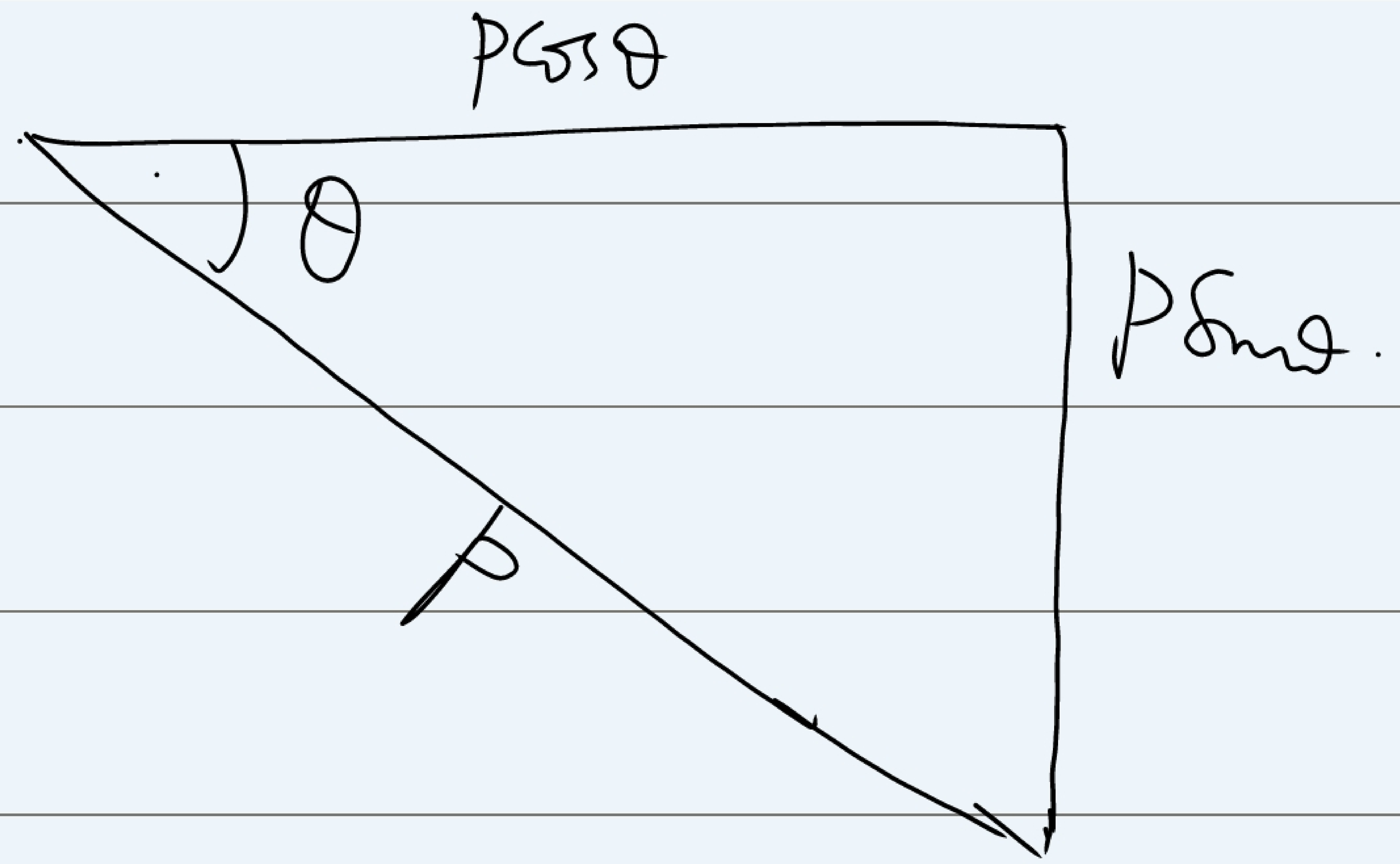
$p = \frac{E}{c} = \frac{h\nu}{c}$

STR:

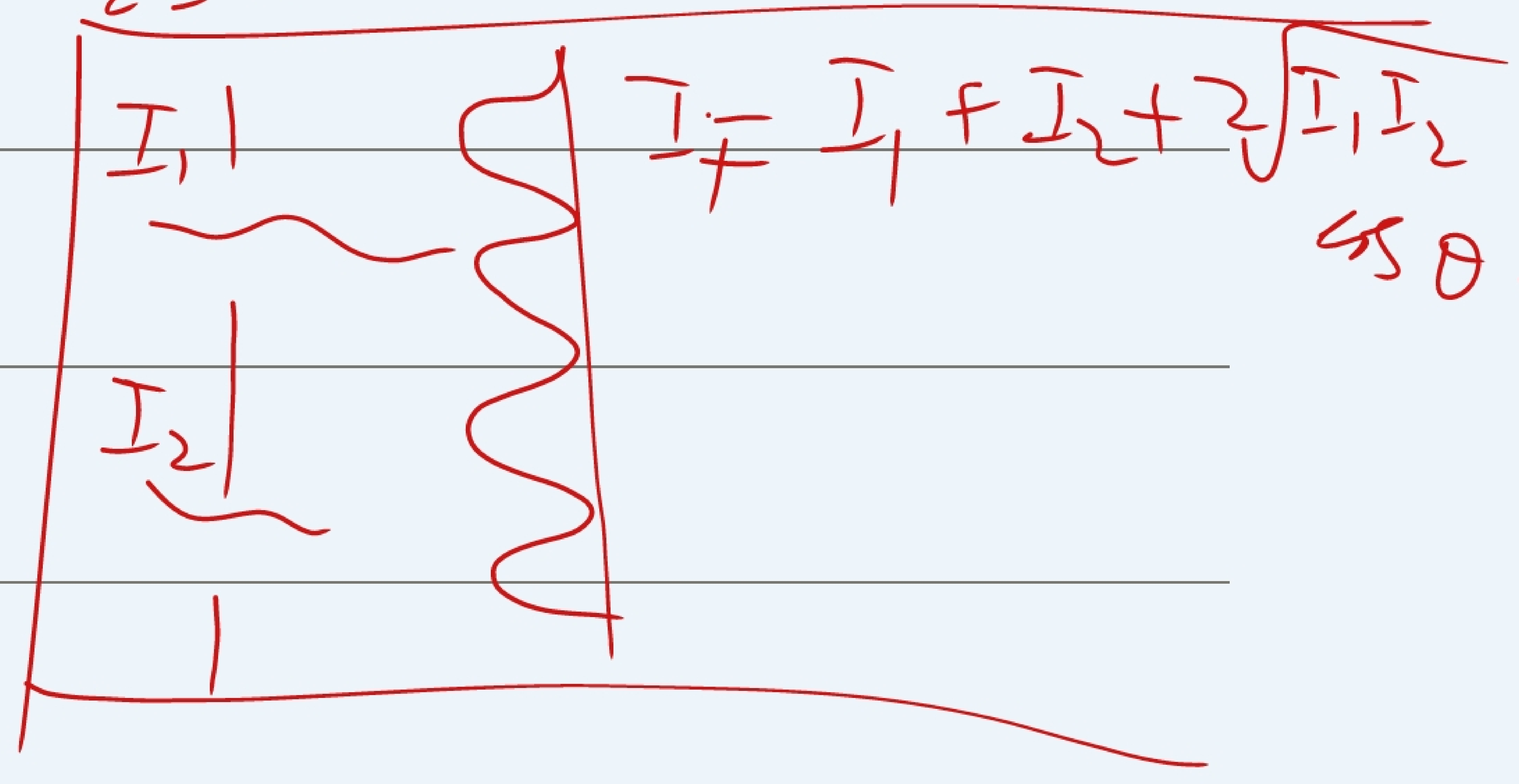
$E = \sqrt{p^2c^2 + m^2c^4}$



$(\lambda' - \lambda) = \text{Constant}\,[1 - \cos\phi]$



$E = h\nu$



$I_T = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$

Horizontal:

$$\frac{h\nu}{c} + 0 = \frac{h\nu'\cos\phi}{c} + p\cos\theta \quad — (1)$$

$$\hookrightarrow h\nu = h\nu'\cos\phi + pc\cos\theta \Rightarrow pc\cos\theta = h\nu - h\nu'\cos\phi$$

Vertical:

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \quad — (2)$$

$c \times$ (1) & (2)

$$0 = h\nu'\sin\phi - pc\sin\theta \Rightarrow pc\sin\theta = h\nu'\sin\phi$$

$$\text{L.H.S} \Rightarrow \boxed{p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\phi}$$

$$E = K.E + mc^2 = \sqrt{m^2c^4 + \underline{p^2c^2}}$$

$K.E = h(\nu - \nu')$

$\nu = \frac{c}{\lambda}$

$$\Rightarrow (K.E + mc^2)^2 = m^2c^4 + p^2c^2$$

$$\Rightarrow K.E^2 + \cancel{m^2c^4} + 2(K.E)(mc^2) = \cancel{m^2c^4} + p^2c^2$$

$$\Rightarrow \underline{p^2c^2} = K.E^2 + 2(K.E)(mc^2)$$

$$\cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu')\cos\phi = \cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu') + 2mc^2(h\nu - h\nu')$$

$$\boxed{(\lambda' - \lambda) = \left(\frac{h}{mc}\right)[1 - \cos\phi]}$$

Divide by $2h^2c^2$

$$\Rightarrow \frac{\cancel{2}mc^{\cancel{2}}}{\cancel{2}h^2\cancel{c^2}}(h\nu - h\nu') = \frac{\cancel{2}(\cancel{h}\nu)(\cancel{h}\nu')}{\cancel{2}\cancel{h^2}c^2}[1 - \cos\phi]$$

$$\Rightarrow \frac{mh}{h^2}\left[\frac{c}{\lambda} - \frac{c}{\lambda'}\right] = \frac{1}{\cancel{c^2}} \cdot \frac{\cancel{c}}{\lambda} \cdot \frac{\cancel{c}}{\lambda'}[1 - \cos\phi] \Rightarrow \frac{mc}{h}[\lambda' - \lambda] = (1 - \cos\phi)$$

Wave properties of particles :→

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p}$$

De broglie :- material : $\vec{p} = \gamma m \vec{v}$, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$\lambda_{dB} = \frac{h}{\gamma m v}$$

$$V_{group} \simeq \frac{\Delta\omega}{\Delta K} = \frac{d\omega}{dK}$$

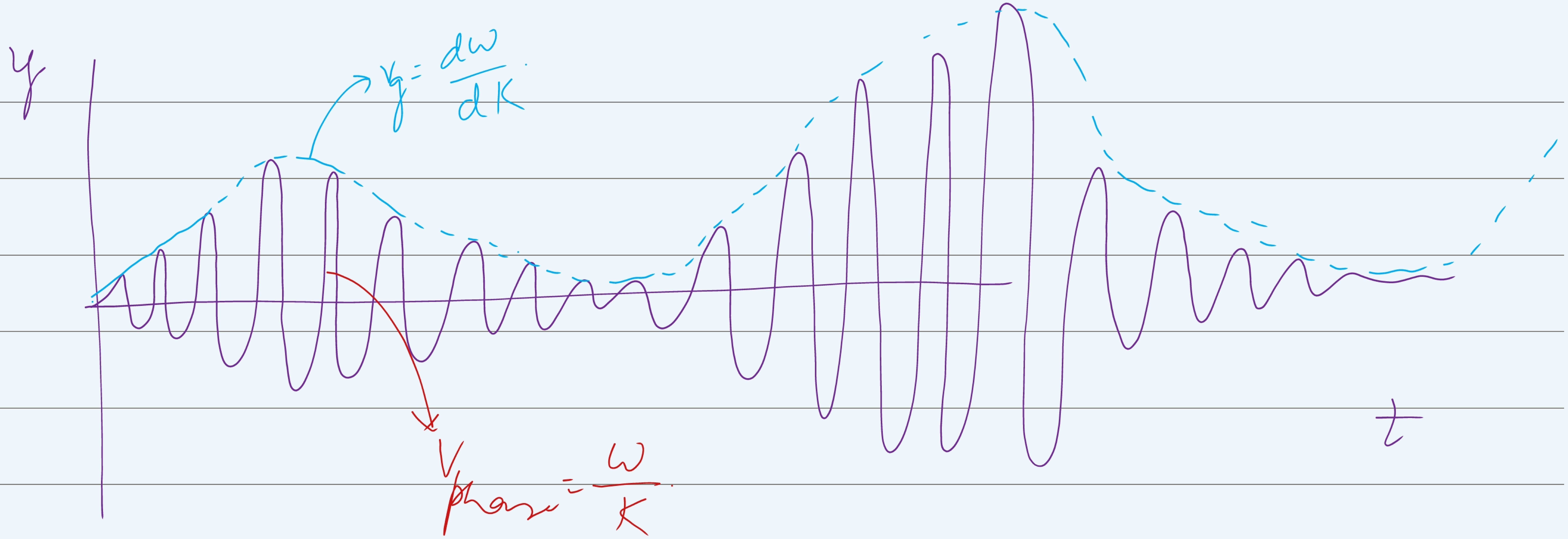
$$y_1 = A \cos(\omega t - Kx)$$

$$+$$

$$y = y_1 + y_2 = y_2 = A \cos[(\omega + \Delta\omega)t - (K + \delta K)x]$$

$$V_{phase} \approx \frac{2\omega}{2K} = \frac{\omega}{K}$$

$$= \cancel{2A} \cos\frac{1}{2}\left[\left(\frac{2\omega + \delta\omega}{2}\right)t - \left(\frac{2K + \delta K}{2}\right)x\right] \cos\frac{1}{2}\left[\frac{\delta\omega}{2}t - \frac{\delta K}{2}x\right]$$



y
$v_g = \frac{d\omega}{dK}$
$V_{phase} = \frac{\omega}{K}$
t

Davisson & Germer / Particle Diffraction:

electron

$\downarrow$

Nickel crystal

$$K.E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$