# PH100: Mechanics and Thermodynamics (3-1-0:4)

## Lecture 4

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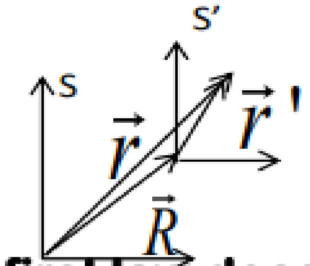
# Newton's first law of motion

1. **Gives a definition of (zero) force**
2. **Defines an *inertial frame*.**

**Zero Force: When a body moves with *constant velocity in a straight line,* either there are no forces present or the net force acting on the body is zero** $\sum_i \vec{F}_i = 0$ **If the body changes its velocity, then there must be an acceleration, and hence a total non-zero force must be present. Velocity can change due to change in its magnitude or due to change in its direction or change in both.**

**Inertial frame: If the relative velocity between the two reference frames is constant, then the relative acceleration between the two reference frames is zero,** $\vec{A} = \frac{d\vec{V}}{dt} = \vec{0}$ **and the reference frames are considered to be *inertial reference frames*. The inertial frame is then simply a frame of reference in which the first law holds.**




$$\vec{r}\,' = \vec{r} - \vec{v}t, \quad \vec{v} = \frac{d\vec{R}}{dt}$$

Galilean transformation

**Is Earth an inertial frame?**

**The first law does *not hold in an arbitrary frame. For example, it fails in the frame of a rotating* turntable.**

# Newton’s Second law of motion:

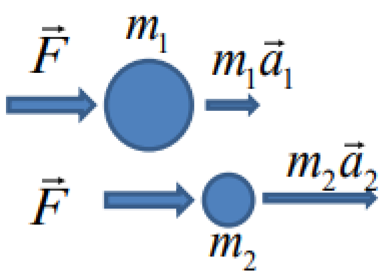
**If any force generates a change in motion, a double force will generate double change in the motion, a triple force will correspond to triple change in the motion, whether that force is impressed altogether and at once or gradually or successively.**

Change of motion is described by the change in momentum of body. For a point mass particle, the momentum is defined as $\vec{p} = m\vec{v}$

Suppose that a force is applied to a body for a time interval $\Delta t$ . *The impressed force or impulse produces a change in the momentum of the body,*

$$\bar{\vec{\mathbf{I}}} = \bar{\vec{\mathbf{F}}}\,\Delta t = \Delta\vec{\mathbf{p}}$$

**The instantaneous action of the total force acting on a body at a time *t is defined by taking the mathematical limit as the time interval Δt becomes smaller and smaller,***




$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

**Inertial mass**

$$\vec{\mathbf{F}}^{\text{total}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{p}}}{\Delta t} \equiv \frac{d\vec{\mathbf{p}}}{dt}$$

$$\vec{F}^{\text{total}} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt}$$

$$\vec{\mathbf{F}}^{\text{total}} = m\vec{\mathbf{a}}$$

Inertial mass $\equiv$ Gravitational mass

# Newton's third law of motion:

**Consider two bodies engaged in a mutual interaction. Label the bodies 1 and 2 respectively. Let $\vec{F}_{1,2}$ be the force on body 1 due to the interaction with body 2, and $\vec{F}_{2,1}$ be the force on body 2 due to the interaction with body 1.**


$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

$$\text{Gravitational force: } \vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12} \qquad \hat{r}_{12} = -\hat{r}_{21}$$

$$\text{Coulomb force: } \vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad \vec{F}_{12} = -\vec{F}_{21}$$

**All real Forces arise due to interaction!**

**If the acceleration of a body is the result of an outside force, then somewhere in the universe there must be an equal and opposite force acting on another body. The interaction may be a complicated one, but as long as the forces are equal and opposite, Newton's laws are satisfied.**

**Newton's 3rd law emphasizes Conservation of Momentum**

## Application of Newton's laws: Prescription

**Step 1**: Divide a composite system into constituent systems each of which can be treated as a point mass.

**Step 2**: Draw free body force diagrams for each point mass.

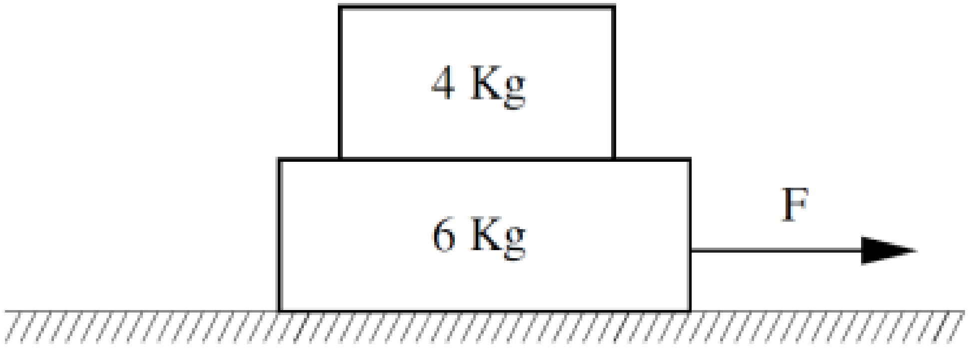
**Step 3**: Introduce a coordinate system, the inertial frame, and write the equations of motion.

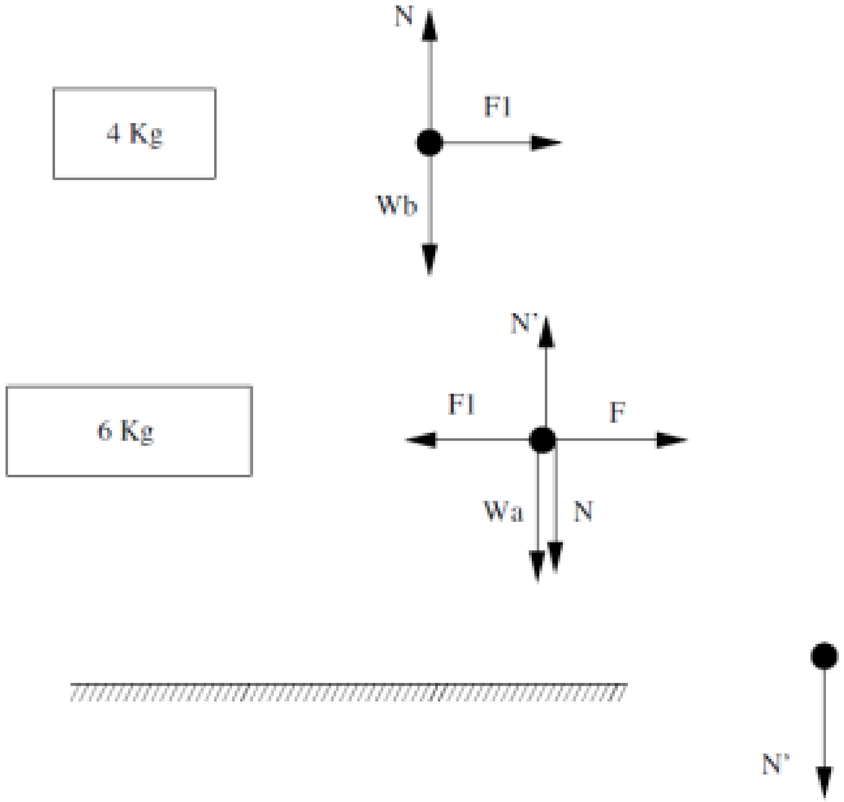
**Step 4**: Motion of a body may be constrained to move along certain path or plane. Express each constraint by an equation called constraint equation.

**Step 6**: Identify the number of unknown quantities. There must be enough number of equations ( Equations of motion + constraint equations) to solve for all the unknown quantities.

# Example 2

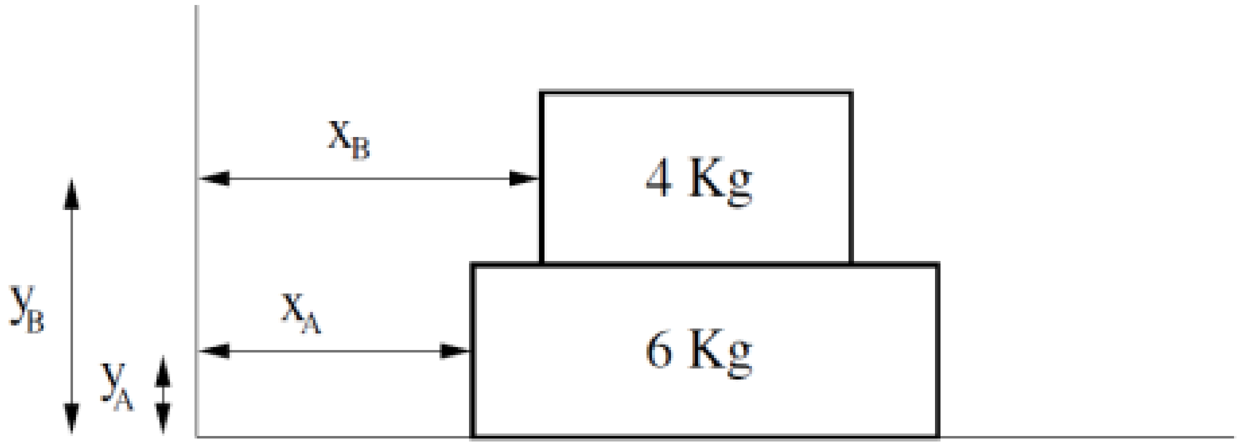
A 4 Kg block rests on top of a 6 Kg block, which rests on a frictionless table. Coefficient of friction between blocks is $0.25$. A force $F = 10N$ is applied to the lower block.



# Identify the constraints

Fix the coordinate system to the table.




$$
\begin{aligned}
y_A &= const \\
y_B &= const \\
x_A &= x_B + const
\end{aligned}
$$

# EOM in x and y-directions

Equations of Motion in Y direction.

$$
\begin{aligned}
m_A \ddot{y_A} &= N' - W_A - N \\
m_B \ddot{y_B} &= N - W_B
\end{aligned}
$$

Constraints

$$
\begin{aligned}
\ddot{y_A} &= 0 \\
\ddot{y_B} &= 0
\end{aligned}
$$

Solution

$$
\begin{aligned}
N' &= W_A + W_B \\
N &= W_B
\end{aligned}
$$

Equations of Motion in X direction.

$$
\begin{aligned}
m_A \ddot{x_A} &= F - F_1 \\
m_B \ddot{x_B} &= F_1
\end{aligned}
$$

Constraints

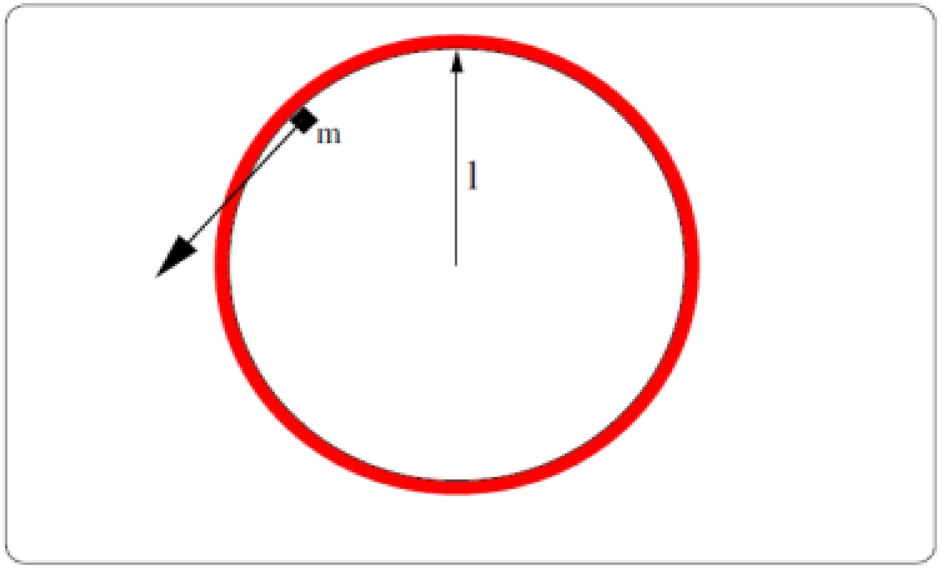
$$
\ddot{x_A} = \ddot{x_B}
$$

Solution

$$
\begin{aligned}
\ddot{x_A} = \ddot{x_B} &= \frac{F}{m_A + M_B} = 1\mathrm{m/s}^2 \\
F_1 &= m_B \ddot{x_B} = 4N
\end{aligned}
$$

The force $F_1 < \mu N = 10$ N, the maximum frictional force between the blocks. Hence the solution is consistent with assumption.

A block of mass $m$ slides on a frictionless table. It is constrained to move move inside a ring of radius $l$ fixed to the table. At $t = 0$ the block is touching the ring and has a velocity $v_0$ in tangential direction.




Find the velocity of the mass at subsequent times.

Constraint Equation is $r = l$, that is $\dot{r} = \ddot{r} = 0$.

Equations of Motion

$$\begin{aligned} m\left(\ddot{r} - r\dot{\theta}^2\right) = -ml\dot{\theta}^2 &= -N \\ m\left(r\ddot{\theta} - 2\dot{r}\dot{\theta}\right) = mr\ddot{\theta} &= -f \end{aligned}$$

Eliminating $N$, we get

$$\begin{aligned} \ddot{\theta} &= -\mu\dot{\theta}^2 \\ v(t) &= l\dot{\theta} \end{aligned}$$

# Forces of Nature

## **Fundamental Forces**

Gravitational Forces

Electromagnetic Forces

Weak Nuclear Forces

Strong Nuclear Forces

- Electrostatic force on two electrons is $10^{36}$ larger the gravitational force.
- Strong forces are nucleonic forces that is responsible for the stability of the nuclei. The magnitude is very large and does not decay as inverse square of the distance. It is a short range force.
- In large atoms weak forces play a key role in phenomenon like radioactivity. It is about $10^{25}$ stronger than the gravitation force, but $10^{11}$ weaker than the electromagnetic force.

**A unified theory for the common origin of all the forces is sought.**

# Everyday forces: Contact Forces

## Force arises from interaction between two bodies.

**By contact forces we mean the forces which are transmitted between bodies by short-range atomic or molecular interactions.**

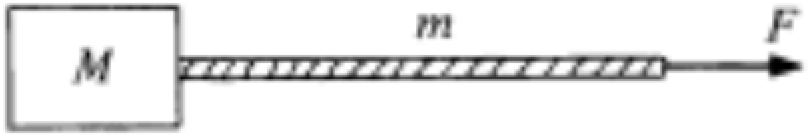
**Examples: push, pull, tension of a string, normal force, the force of friction, etc.**

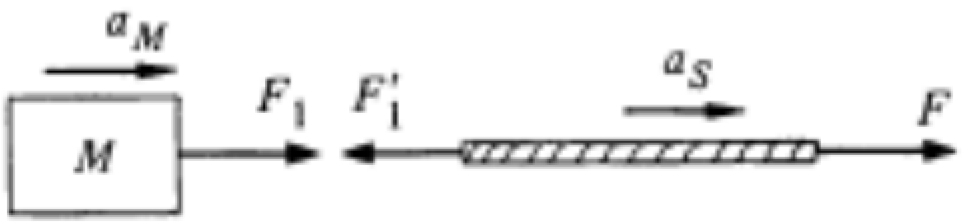
The origin of these forces can be explained in terms of the fundamental properties of matter. However, our approach will emphasize the properties of these forces and the techniques for dealing with them in physical problems, not worrying about their microscopic origins.

# Tension in a string: Most common example

A string consists of long chains of atoms. When a string is pulled, we say it is under tension. The long chains of molecules are stretched, and inter-atomic forces between atoms in the molecules prevent the molecules from breaking apart. To illustrate the behaviour of strings under tension:

Consider a block of mass *M* pulled by a string of mass *m*. A force *F* is applied to the string. What is the force that the string “transmits” to the block?




$$F_1 = Ma_M$$

$$F - F_1' = ma_S.$$

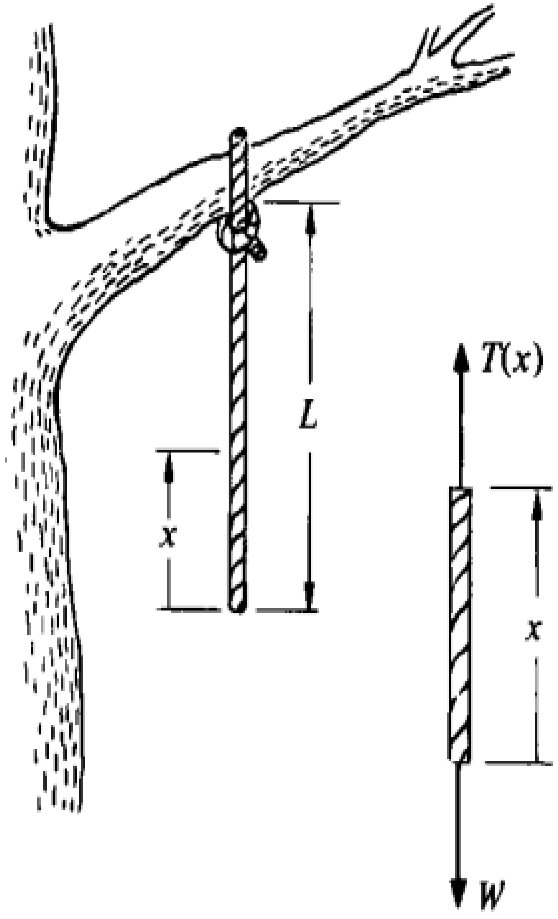
String is inextensible: $a_S = a_M$

By Newton's third law: $F_1 = F_1'$

$$a = \frac{F}{M+m},$$

$$F_1 = F_1' = \frac{M}{M+m}F.$$

The force on the block is less than *F*. The string does not transmit the full applied force. If the mass of the string is negligible compared with the block, $F_1 = F$ to good approximation.



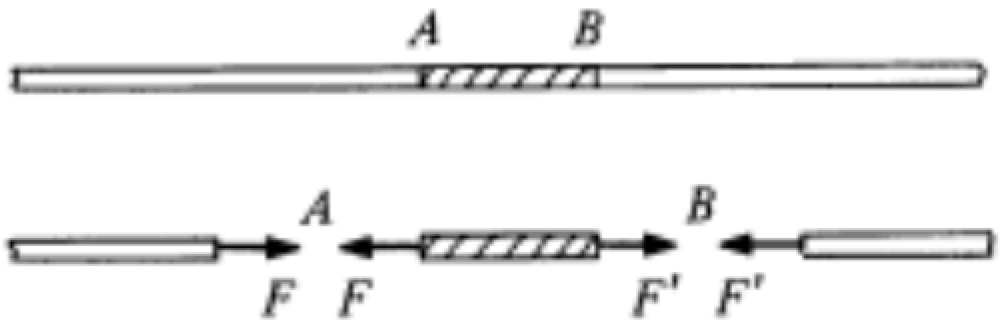

A uniform rope of mass $M$ and length $L$ hangs from the limb of a tree. Find the tension a distance $x$ from the bottom.

The force diagram for the lower section of the rope is shown in the sketch. The section is pulled up by a force of magnitude $T(x)$, where $T(x)$ is the tension at $x$. The downward force on the rope is its weight $W = Mg(x/L)$. The total force on the section is zero since it is at rest. Hence

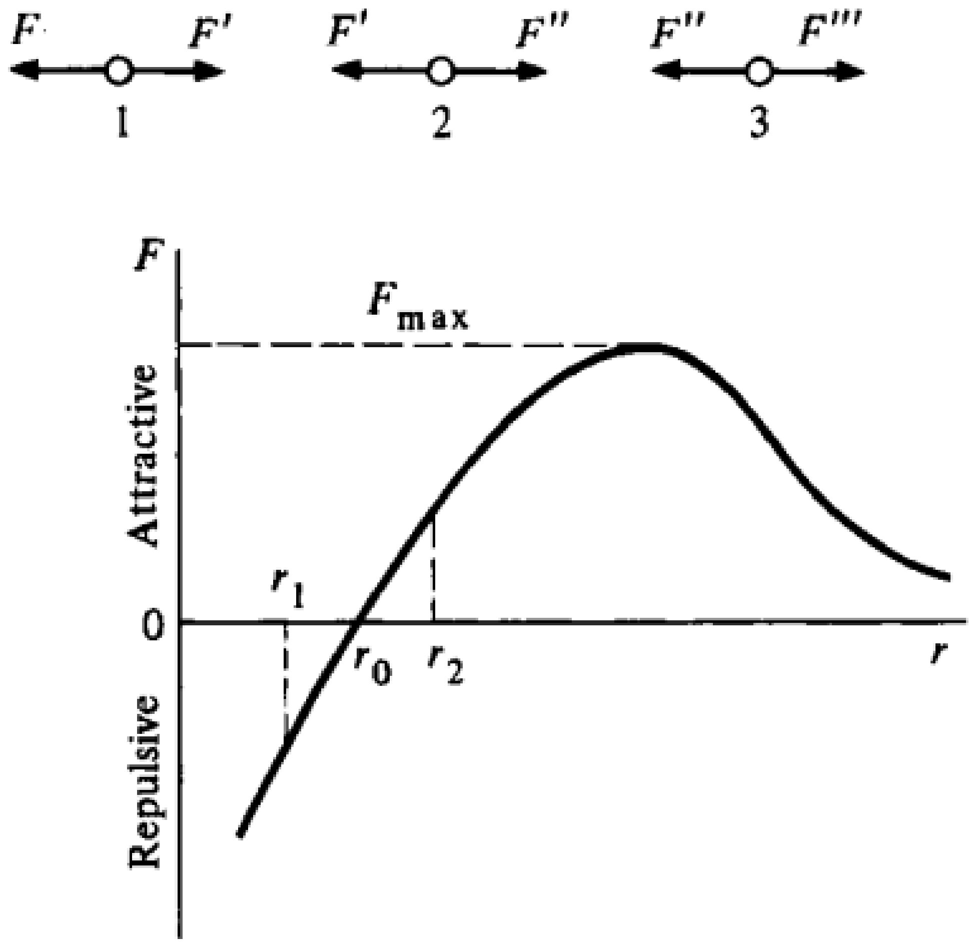
$$T(x) = \frac{Mg}{L}\,x.$$

At the bottom of the rope the tension is zero, while at the top the tension equals the total weight of the rope $Mg$.

A string is composed of short sections interacting by contact forces. Each section pulls the sections to either side of it, and by Newton's third law, it is pulled by the adjacent sections. The magnitude of the force acting between adjacent sections is called **Tension**. **There is no direction associated with tension.** In the sketch, the tension at $A$ is $F$ and the tension at $B$ is $F'$.




- Although a string may be under considerable tension, if the tension is uniform, the net string force on each small section is zero and the section remains at rest unless external forces act on it.
- If there are external forces on the section, or if the string is accelerating, the tension generally varies along the string.


F
F'
F'
F''
F''
F'''
1
2
3
F
F max
Attractive
Repulsive
0
r 1
r 0
r 2
r

# Friction

- Mostly encounter two kinds of friction:

(i) Sliding friction – Comes into play when two bodies slide on one another. All surfaces are rough at the atomic level.

(ii) Fluid/Air friction – At very small velocities, air friction is absent. Usually at moderate velocities, it is proportional to velocities (such as a ball moving through viscous fluids). At very large velocities (such as an aeroplane where the air swirls around as the plane moves), friction may be proportional to square of velocity or even higher powers.

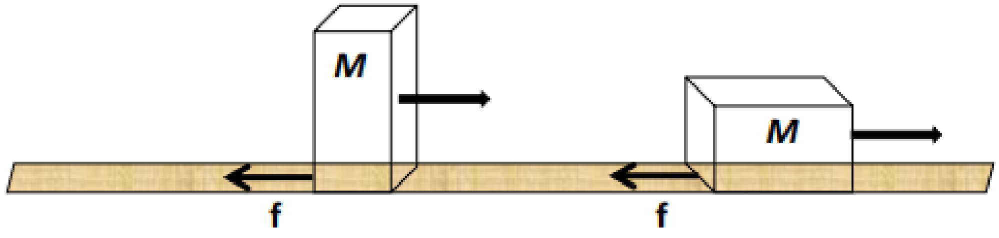
# Sliding Friction

- **Friction arises when the surface of one body moves, or tries to move, along the surface of a second body.**

- **The maximum value of the friction is $f_{friction} = \mu N$**
  **where $N$ is the normal force and $\mu$ is the coefficient of friction.**

- **When a body slides across a surface, the friction force is directed opposite to the instantaneous velocity and has magnitude $\mu N$. The force of sliding friction is slightly less than the force of static friction, but for the most part we shall neglect this effect.**

- **For two given surfaces, the force of sliding friction is independent of the area of contact.**



## Important features of Friction

- Friction is independent of the area of contact because the actual area of contact on an atomic scale is a minute fraction of the total surface area.

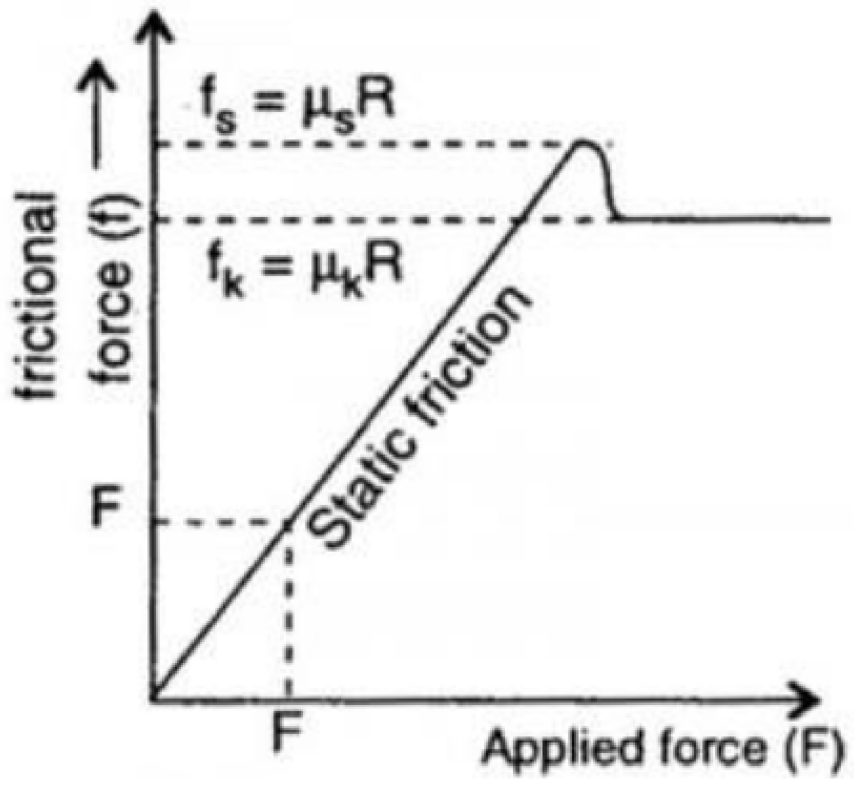
- Friction occurs because of the interatomic forces at these minute regions of atomic contact.

- Non rigid bodies, like automobile tires, are more complicated. A wide tire is generally better than a narrow one for good acceleration and braking.

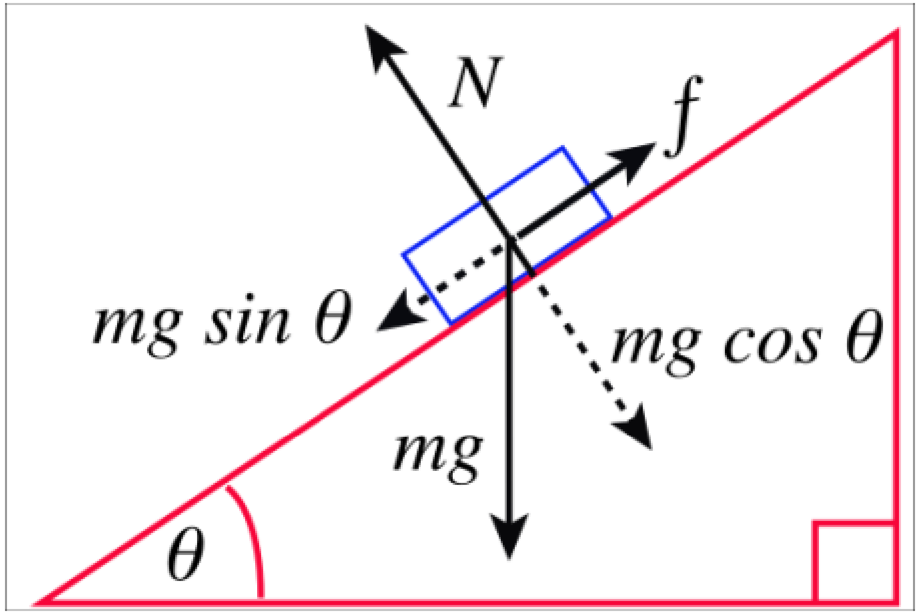
- Frictional force is also independent of relative velocity between two surfaces.

- This is approximately true for a wide range of low speeds, as the speed increases and air friction come into play, it is found that friction not only depends on the speed, but upon the square and sometimes higher powers of the speed.

# Applied force vs Frictional force



# How to experimentally determine friction or test F = μN?




At the verge of sliding

$$mg\ sin\theta = \mu N = \mu mg\ cos\theta$$

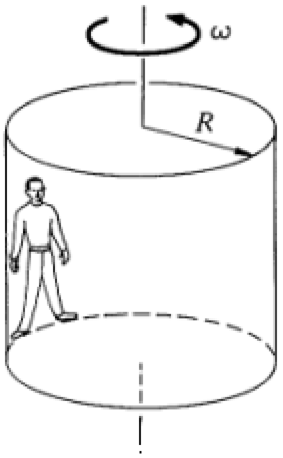
$$\mu = \tan\theta$$

An object will start to slude at a given inclination. If the same block is loaded by providing extra weight, it will still be sliding at the given angle. Coefficient of friction is constant for a given angle.

In fact if this experiment is performed by continuously varying the angle, then at the correct angle, the block begins to slide, **but not steadily.** Thus μ being constant Is only roughly true.

# The spinning terror

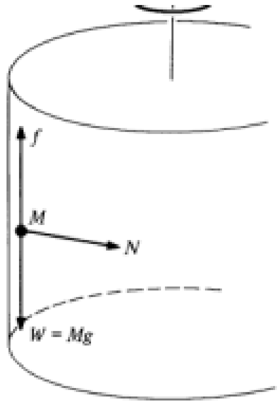
The Spinning Terror is an amusement park ride—a large vertical drum which spins so fast that everyone inside stays pinned against the wall when the floor drops away. What is the minimum steady angular velocity $\omega$ which allows the floor to be dropped away safely?




$$N = MR\omega^2.$$

By the law of static friction,

$$f \leq \mu N = \mu MR\omega^2.$$

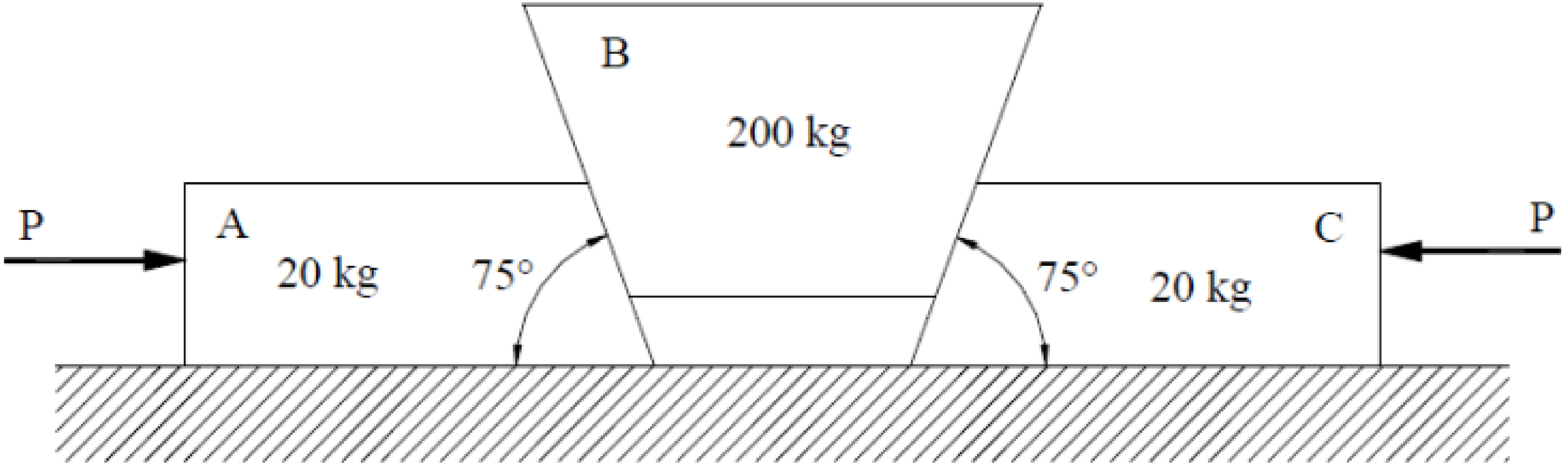


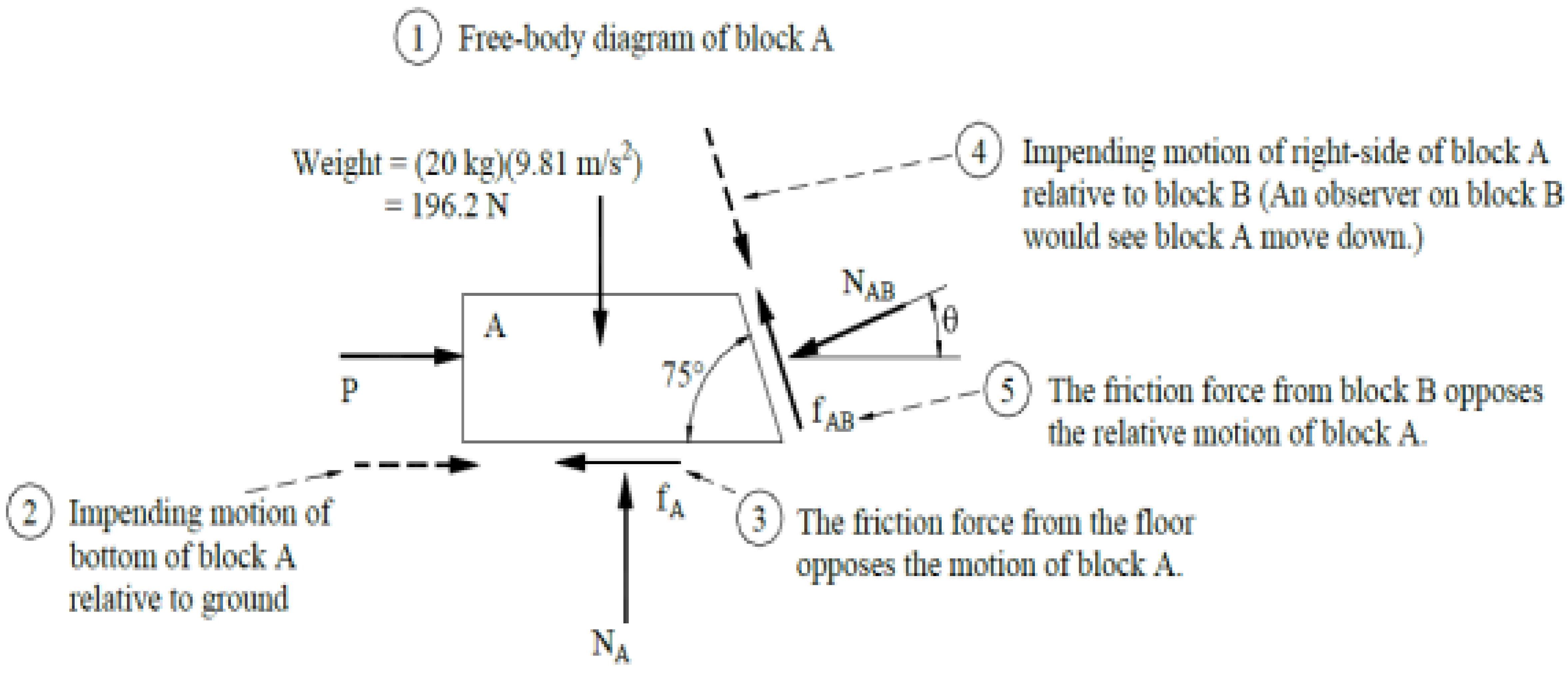

Since we require $M$ to be in vertical equilibrium, $f = Mg,$

**Therefore,** $Mg \leq \mu MR\omega^2$ or $\omega^2 \geq \frac{g}{\mu R}.$ or $\omega_{\min} = \sqrt{\frac{g}{\mu R}}.$

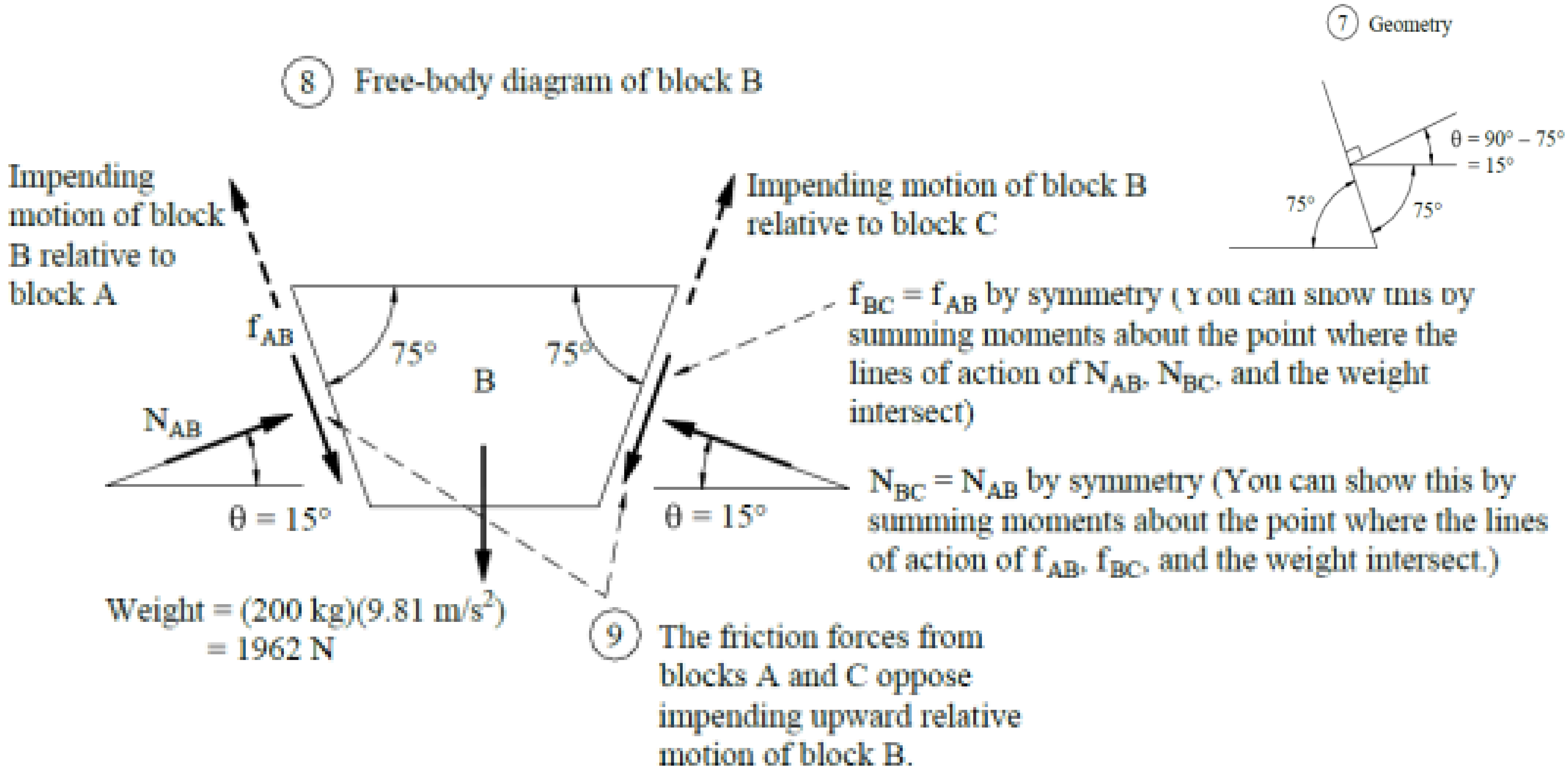
# The blocks and friction

. If the coefficient of static friction for all surfaces of contact is 0.25, determine the smallest value of the forces P that will move wedge B upward.





1 Free-body diagram of block A
Weight = (20 kg)(9.81 m/s$^2$) = 196.2 N
4 Impending motion of right-side of block A relative to block B (An observer on block B would see block A move down.)
$N_{AB}$
$\theta$
A
P
75°
$f_{AB}$
5 The friction force from block B opposes the relative motion of block A.
2 Impending motion of bottom of block A relative to ground
$f_A$
3 The friction force from the floor opposes the motion of block A.
$N_A$



7 Geometry
θ = 90° – 75°
= 15°
75°
75°
8 Free-body diagram of block B
Impending motion of block B relative to block A
Impending motion of block B relative to block C
f_AB
75°
75°
B
f_BC = f_AB by symmetry (You can show this by summing moments about the point where the lines of action of N_AB, N_BC, and the weight intersect)
N_AB
θ = 15°
θ = 15°
N_BC = N_AB by symmetry (You can show this by summing moments about the point where the lines of action of f_AB, f_BC, and the weight intersect.)
Weight = (200 kg)(9.81 m/s²)
= 1962 N
9 The friction forces from blocks A and C oppose impending upward relative motion of block B.

(6) Equations of equilibrium

$$\xrightarrow{+} \sum F_x = 0: \ P - f_A - f_{AB} \cos 75° - N_{AB} \cos \theta = 0 \qquad (1)$$

$$+\uparrow \sum F_y = 0: \ N_A - 196.2 \text{ N} + f_{AB} \sin 75° - N_{AB} \sin \theta = 0 \qquad (2)$$

Slip impends so,

$$f_A = f_{A\text{-max}} \equiv \mu N_A = 0.25 N_A \qquad (3)$$

$$f_{AB} = f_{AB\text{-max}} \equiv \mu N_{AB} = 0.25 N_{AB} \qquad (4)$$

(10) Equations of equilibrium

$$\xrightarrow{+} \sum F_x = 0: \ N_{AB} \cos 15° - N_{AB} \cos 15° + f_{AB} \cos 75° - f_{AB} \cos 75° = 0 \qquad (5)$$

(Note that this equation reduces to 0 = 0. This happens because we have assumed symmetry to conclude that $f_{BC} = f_{AB}$ and $N_{BC} = N_{AB}$.)

$$+\uparrow \sum F_y = 0: \ N_{AB} \sin 15° + N_{AB} \sin 15° - f_{AB} \sin 75° - f_{AB} \sin 75° - 1962 \text{ N} = 0 \qquad (6)$$

# Laws of Friction vs Newton's laws

- Two laws of friction:

$$F = \mu N \quad \text{(for sliding)}$$

$$F = cv^{\alpha} \quad \text{(for fluid friction); } \alpha = 1,2,\ldots$$

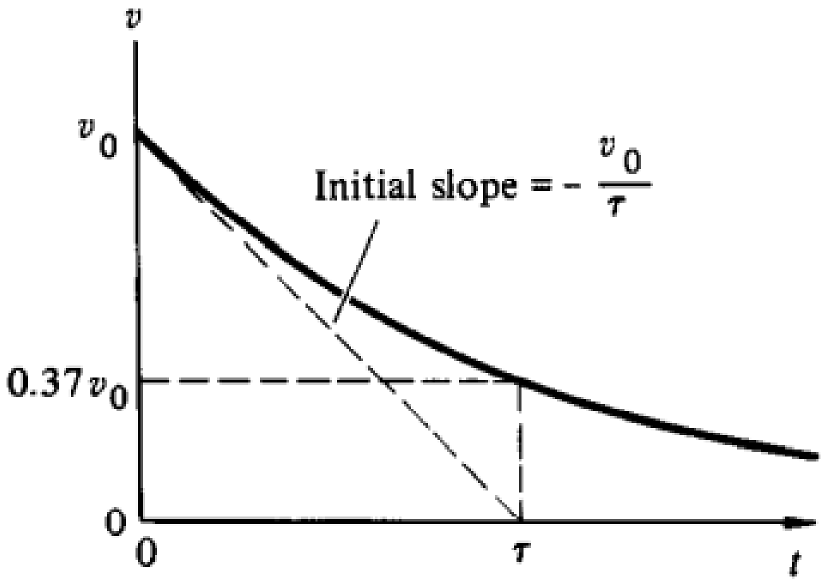
Distinguish it with $F = ma$ !!

**Newton's laws are real laws, while laws of friction are empirical laws.**

$$-Cv = m\frac{dv}{dt}$$

or

$$m\frac{dv}{dt} + Cv = 0.$$




$$\frac{dv}{dt} + \frac{C}{m}v = 0$$

$$\frac{dv}{v} = -\frac{C}{m}dt$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\int_0^t \frac{C}{m}dt$$

$$\ln\frac{v}{v_0} = -\frac{C}{m}t$$

$$\frac{v}{v_0} = e^{(-C/m)t}$$

$$v = v_0e^{-Ct/m}.$$

# Conservation of Momentum

## Momentum conservation for a system of particles

- So far we talked about point particles. There is a need to:

  (a) generalize it to extended bodies

  (b) to deal with variable mass problem

1. Momentum ($p = mv$) is a more fundamental quantity than $m$ & $v$ separately.

2. Newton's 2nd law should be written as F = $\dot{p}$ instead of '$ma$' (for variable $m$).

3. For a system of particles, **an external Force** causes change of **total momentum** of the system. The internal forces cancel each other.

**It will be useful to locate a point for a system of particles where all the mass may be concentrated at. Then the single particle EOM will continue.**

# Center of Mass

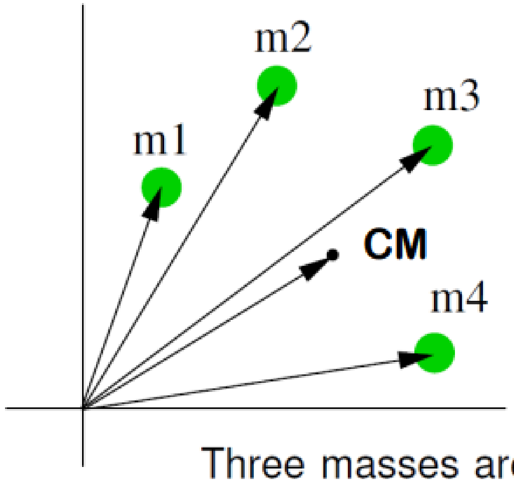
A system has $n$ particles with masses and positions given by

$$m_1,\ m_2, \ldots,\ m_n$$

$$\mathbf{r}_1,\ \mathbf{r}_2, \ldots,\ \mathbf{r}_n$$

Define a **Center of Mass** as

$$\mathbf{R}_{CM} = \frac{1}{M}\left(\sum_i m_i \mathbf{r}_i\right)$$




**Example**

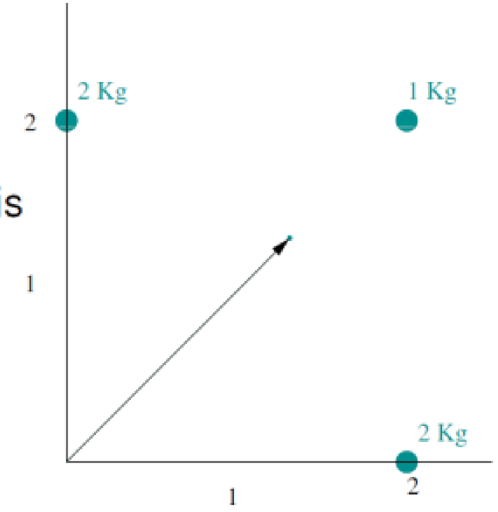
Three masses are kept in a plane as shown in the figure.

$m_1 = 2$ Kg, $m_2 = 2$ Kg and $m_3 = 1$ Kg

$r_1 = 2\mathbf{j}$, $r_2 = 2\mathbf{i}$ and $r_3 = 2\mathbf{i} + 2\mathbf{j}$

Total Mass is 5 Kg. Then Center of Mass is given by

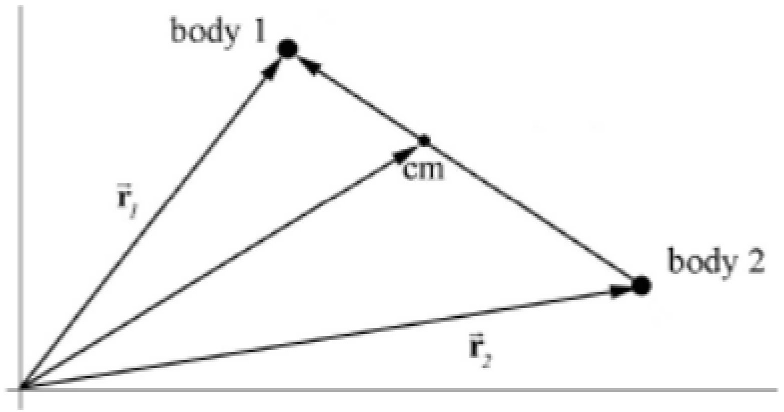
$$\begin{aligned}\mathbf{R}_{cm} &= \frac{1}{5}(2\mathbf{r_1} + 2\mathbf{r_2} + \mathbf{r_3}) \\ &= \frac{6}{5}(\mathbf{i} + \mathbf{j})\end{aligned}$$



# Center of mass

The center of mass vector, $\mathbf{R}_{cm}$, of the two-body system

$$\vec{\mathbf{R}}_{cm} = \frac{m_1 \vec{\mathbf{r}}_1 + m_2 \vec{\mathbf{r}}_2}{m_1 + m_2}.$$

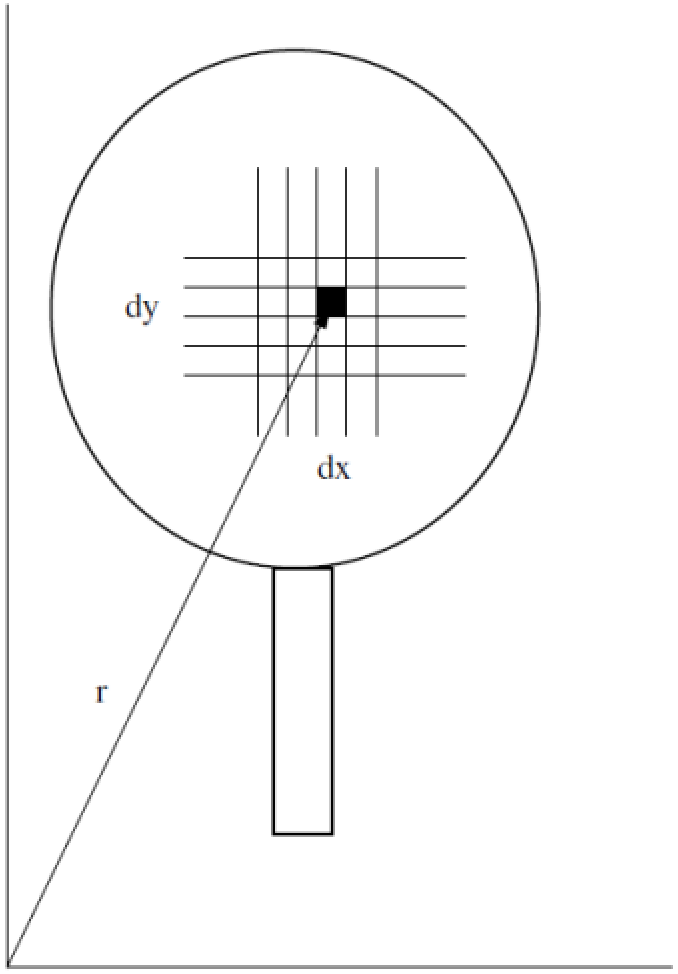



For a **continuous rigid body**, each point-like particle has mass $dm$ and is located at the position $r'$. The center of mass is then defined as an integral over the body,

$$\vec{\mathbf{R}}_{cm} = \frac{\int_{\text{body}} dm\, \vec{\mathbf{r}}'}{\int_{\text{body}} dm}.$$

# Planar Continuous Bodies

The density is given by $\rho(\mathbf{r})$. An element at $\mathbf{r}$ and of area $dxdy$ has a mass $dm = \rho(\mathbf{r})dxdy$.



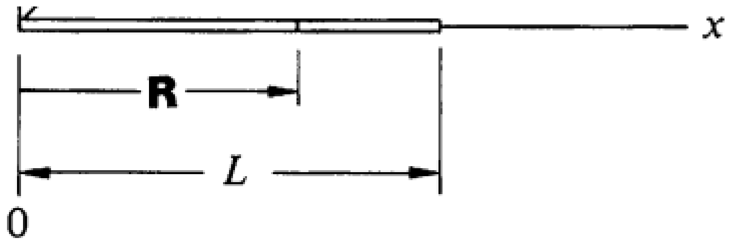

$$\begin{aligned} \mathbf{R}_{cm} &= \frac{1}{M}\sum \mathbf{r}dm \\ &= \frac{1}{M}\int \mathbf{r}\rho(\mathbf{r})dxdy \end{aligned}$$

And

(1) $$M = \int \rho(\mathbf{r})dxdy$$

# Examples

The mass per unit length $\lambda$ of a rod of length *L* varies as $\lambda = \lambda_0(x/L)$, where $\lambda_0$ is a constant and *x* is the distance from the end marked O. Find the center of mass.




$$M = \int dm = \int_0^L \frac{\lambda_0 x\, dx}{L} = \tfrac{1}{2}\lambda_0 L.$$

$$\mathbf{R}_{cm} = \frac{2}{\lambda_0 L}\int_0^L (x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}})\frac{\lambda_0 x\, dx}{L}$$

$$= \frac{2}{L^2}\frac{\hat{\mathbf{i}}}{3}x^3\Big|_0^L = \tfrac{2}{3}L\hat{\mathbf{i}}.$$

# Equations of Motion

Now, by definition,

$$\begin{aligned} M\mathbf{R}_{cm} &= \sum m_i \mathbf{r}_i \\ M\ddot{\mathbf{R}}_{cm} &= \sum m_i \ddot{\mathbf{r}}_i \end{aligned}$$

But for each particle, labled by $i$,

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i = \mathbf{F}_i^{ext} + \mathbf{F}_i^{int}$$

Hence,

$$M\ddot{\mathbf{R}}_{cm} = \sum \mathbf{F}_i^{ext} + \sum \mathbf{F}_i^{int}$$

## What are CM coordinates good for?

But by Newton's third law, all internal forces appear in pairs and are equal and opposite. Thus in the following summation all internal forces cancel each other out.

$$\sum_i \mathbf{F}_i^{int} = 0$$

Thus Equation of Motion for Center of Mass of any system

$$M\ddot{\mathbf{R}}_{CM} = \mathbf{F}^{ext} = \sum_i \mathbf{F}_i^{ext}$$

One point $\mathbf{R}_{cm}$ traces the same motion as that of a single particle of mass $M$ under the influence of a force $\mathbf{F}^{ext}$

# Translational Motion of the Center of Mass

The velocity of the center of mass is given by $\vec{\mathbf{V}}_{\mathrm{cm}} = \frac{1}{m^{\mathrm{total}}}\sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i = \frac{\vec{\mathbf{p}}^{\mathrm{total}}}{m^{\mathrm{total}}}$.

The total momentum is then expressed in terms of the velocity of the center of mass by

$$\vec{\mathbf{p}}^{\mathrm{total}} = m^{\mathrm{total}}\,\vec{\mathbf{V}}_{\mathrm{cm}}.$$

The total external force is equal to the change of the total momentum of the system,

$$\vec{\mathbf{F}}^{\mathrm{total}}_{\mathrm{ext}} = \frac{d\,\vec{\mathbf{p}}^{\mathrm{total}}}{dt} = m^{\mathrm{total}}\frac{d\,\vec{\mathbf{V}}_{\mathrm{cm}}}{dt} = m^{\mathrm{total}}\vec{\mathbf{A}}_{\mathrm{cm}}, = m^{total}\ddot{\vec{R}}_{cm}$$

where $\vec{\mathbf{A}}_{\mathrm{cm}}$, is the acceleration of the center of mass.

The system behaves as if all the mass is concentrated at the center of mass and all the external forces act at that point. This is an over simplification. The shape of the body and the point of application of force matters.

The same force on the same mass with different shape may lead to different types of motion.

$$\vec{F}^{total}_{ext} = m^{total}\ddot{\vec{R}}_{cm}$$

**Note EOM describes translational motion.**