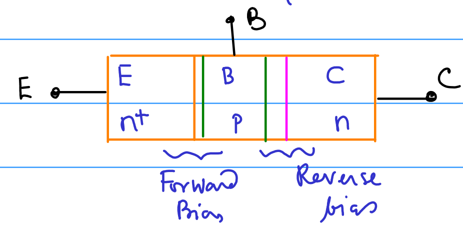
28.11.2023

Frequency Response of CE Amplifier

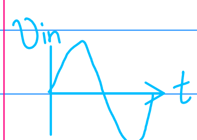
(Reference: Chapter 16, Malvino & Bates)

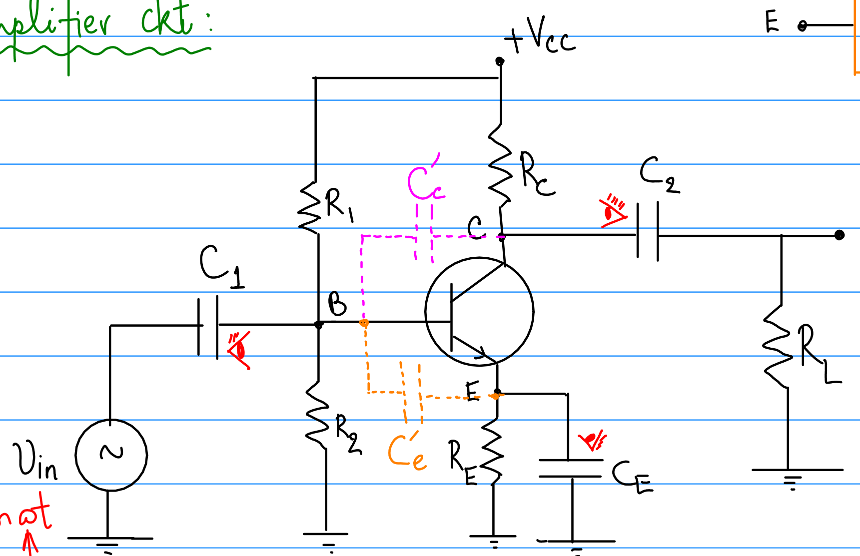
1. How voltage gain (A_v) of the CE amplifier changes with varying the frequency of the input signal.

2. Representation of gain (voltage/power) in units of decibel (dB)

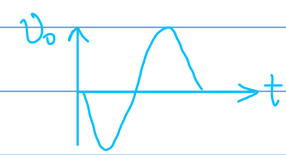


CE Amplifier ckt:





$v_o = v_{o,out} \sin(\omega t - 180°)$



$v_{in} = v_{o,in} \sin \omega t$

Ang. freq.

$\omega = 2\pi f$ where $f = \frac{1}{T}$ (in Hz)

C_c', C_e' are the junction capacitor only operating at high-freq.

- As we know, for DC analysis:
  - the capacitors are 'OPEN'.
- For ac analysis: All the capacitors are "SHORT"

$$X_C = \frac{1}{\omega C}$$

$\omega / f \rightarrow$ Mid-freq. range (1kH–50kHz)

In mid-frequency range the value of X_c is small as compared to the equivalent resistance as seen the capacitor, and hence all the ac voltage is dropped across the resistor. Therefore, the capacitors are electrically short for the ac signal.

Ques: Is this assumption valid in the low-frequencies?

— NO — ⇒ effectively the gain reduces.

Ques: Is this assumption valid in the high-frequencies?

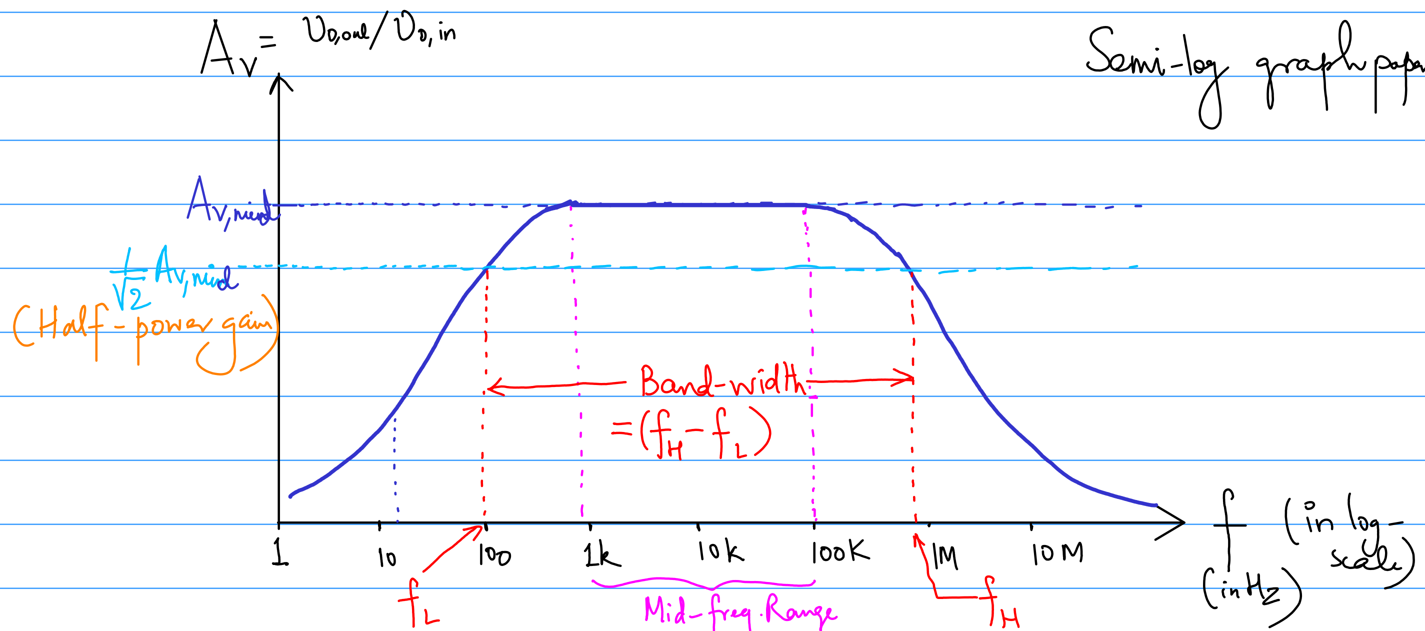
— YES — ⇒ effectively the gain reduces.

Why?

- At high-frequencies, the base-emitter junction and base-collector junction effectively behaves as a capacitors and affect the input & output ac voltages.

⇒ Effectively the gain reduces.

Semi-log graph paper



Mathematically,

$$A_v(f) = \frac{A_{v(mid)}}{\sqrt{1+(f_L/f)^2}\,\sqrt{1+(f/f_H)^2}}$$

where, f_L = Lower cut-off frequency

f_H = Higher cut-off frequency.

Case (i) $f < f_L$ (ie, the frequency is lower the low cut-off freq.)

$$\frac{f_L}{f} > 1 \quad \text{whereas} \quad \frac{f}{f_H} \approx 0$$

$$\boxed{A_v(f) \approx \frac{A_{v,mid}}{\sqrt{1+\left(\frac{f_L}{f}\right)^2}}}$$

at low-frequencies

Case (ii) $f_L < f < f_H$ (ie, mid-freq. range).

$$\frac{f_L}{f} \approx 0 \qquad \frac{f}{f_H} \approx 0$$

$$A_v(f) = A_{v\,mid}$$

Case (iii) $f > f_H$ (high freq. range)

$$\frac{f_L}{f} \approx 0 \qquad \frac{f}{f_H} > 1$$

$$\boxed{A_v(f) = \frac{A_{v\,mid}}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}}$$ at high frequency

Unit of voltage gain in Decibel (dB)

$$\boxed{A_v(dB) = \left[20 \log_{10} A_v\right] dB}$$

known $A_v = \frac{v_o}{v_{in}}$

eg: if $A_v = 100$

$$A_v(dB) = 20 \log_{10} 100 \ dB$$
$$= 40 dB$$

Unit of power gain in decibel (dB)

$$\boxed{A_p(dB) = 10 \log_{10}(A_p) \; dB}$$

where $A_p = \frac{P_o}{P_{in}} = \frac{\text{ac power output}}{\text{ac power input}}$.

We have designed a multi-stage amplifier in which each stage is having voltage gain as

$$A_{v_1}, \; A_{v_2}, \; A_{v_3} \; \cdots\cdots \; A_{v_n}$$

Overall gain is given as:

$$A_v = A_{v_1} \cdot A_{v_2} \cdot A_{v_3} \cdots\cdots \cdot A_{v_n}$$

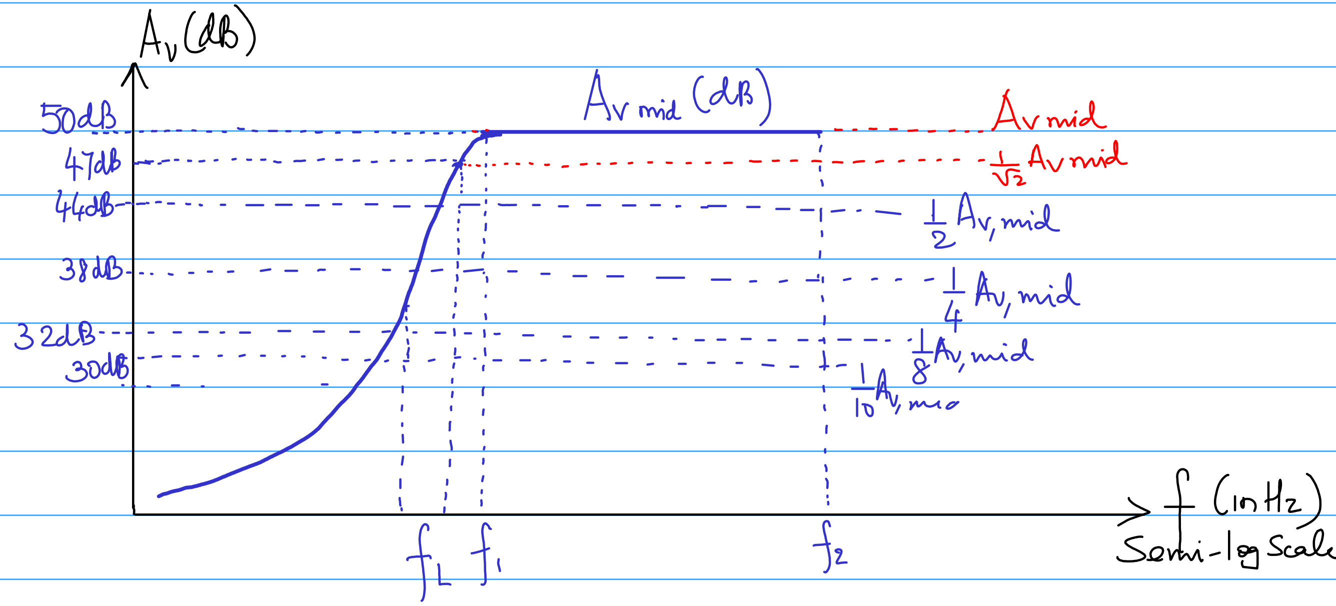
Overall gain in dB

$$A_v(dB) = 20 \log_{10} A_v$$

$$= 20 \log_{10} \left[A_{v_1} \cdot A_{v_2} \cdot A_{v_3} \cdots\cdots A_{v_n} \right]$$

$$= 20 \log_{10} A_{v_1} + 20 \log_{10} A_{v_2} + \cdots\cdots + 20 \log_{10} A_{v_n}$$

$$\boxed{A_v(dB) = A_{v_1}(dB) + A_{v_2}(dB) + \cdots\cdots + A_{v_n}(dB)}$$



At lower cut-off frequency the voltage gain is reduced as a factor $\frac{1}{\sqrt{2}}$.

What the value of this reduction in dB.

$$\text{Red}^n \text{ in gain (dB)} = 20 \log_{10}\left(\frac{1}{\sqrt{2}}\right) \text{ dB}$$

$$= \underbrace{20 \log_{10} 1}_{=0} - 20 \log_{10} 2^{1/2} \text{ dB}$$

$$= -20 \cdot \frac{1}{2} \log_{10} 2 \text{ dB}$$

$$= -3 \text{ dB}$$

$$\boxed{\text{Red}^n \text{ in gain at cut-off frequency is 3dB.}}$$

* Note : 1) Whenever the voltage gain is reduced by half ($\frac{1}{2}$) the voltage gain in dB is reduced by 6dB.

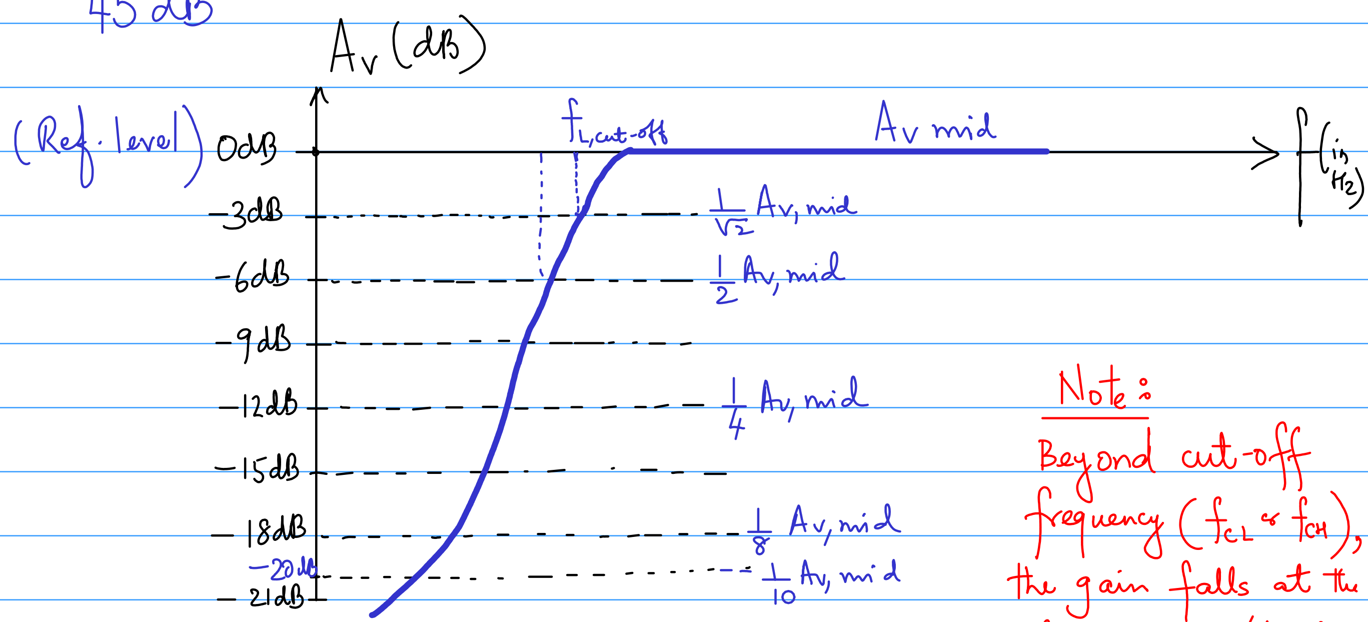
2) Whenever the voltage gain is reduced by $\frac{1}{10}$ th, the voltage gain is reduced by 20dB

Example:

Let's say mid-freq. voltage gain is 65dB

65dB → 62dB (at cut-off freq.) → 59 dB (gain red^n by half) $\xrightarrow{\text{Red}^n \text{ by } \frac{1}{4}\text{th}}$ 53dB

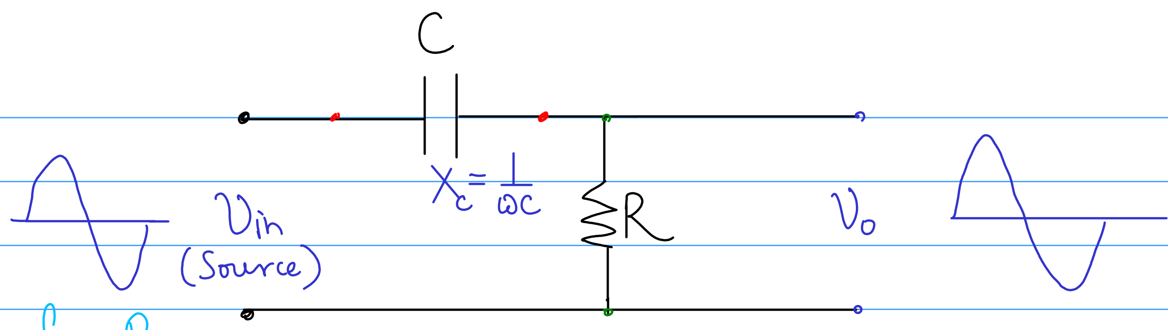
65dB $\xrightarrow{\text{Red}^n \text{ in } \frac{1}{10} \text{ of the gain}}$ 45 dB



Note: Beyond cut-off frequency (f_{cL} or f_{cH}), the gain falls at the rate of 20dB/decade.

Low frequency response of a CE Amplifier:

We know that at low frequencies ($f \to 0$), the reactance (X_C) of the coupling & by-pass capacitors are not approximated to zero. (ie, the capacitors are no more behaving as a "SHORT" to ac-signal.



High freq. Response:

$\Rightarrow$ The R-C circuit allows high-frequency signal to pass through. In other-words, it blocks the low-frequency signal.

Low freq. Response:

The capacitors are no more 'SHORT'.

$$\Rightarrow \boxed{V_o = \frac{R}{\sqrt{X_c^2 + R^2}} \cdot V_{in}}$$

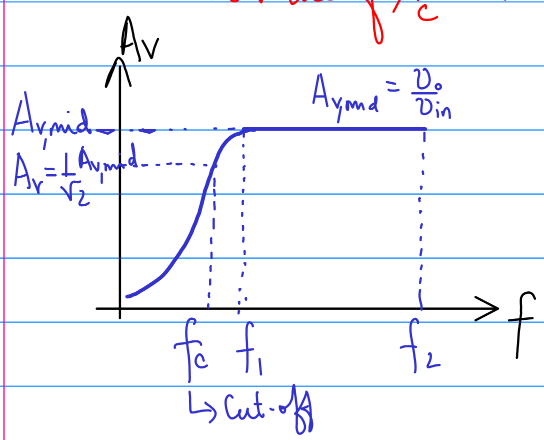
$X_c = \dfrac{1}{2\pi f C}$

Low freq.

If $\underbrace{X_c = R}_{\text{the freq 'f' at which the value of } X_c = R}$ $\Rightarrow$ $V_o = \dfrac{1}{\sqrt{2}} V_{in}$ ✓

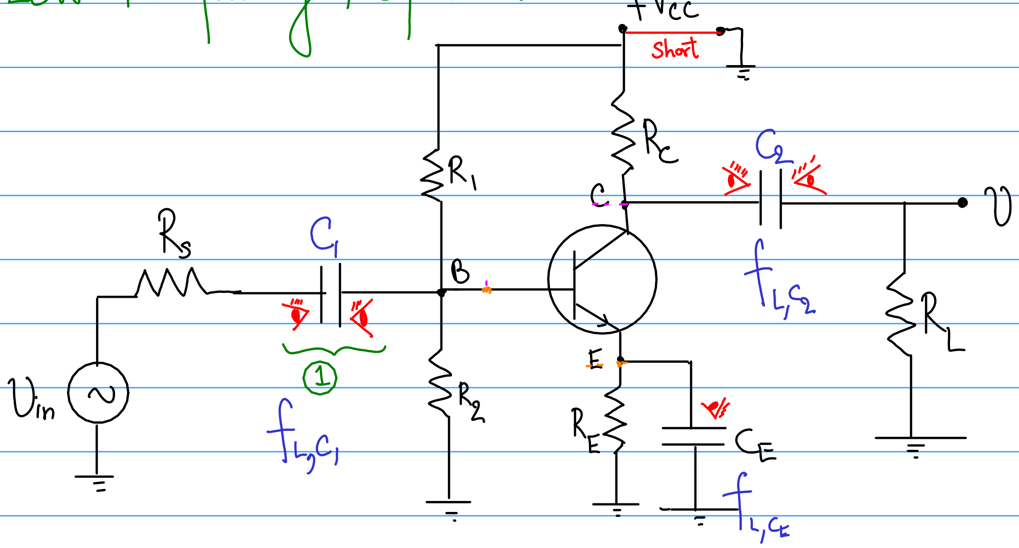
$\dfrac{1}{2\pi f_c C} = R$

$\Rightarrow f_c = \dfrac{1}{2\pi RC}$



$\Rightarrow$ The $A_v = \dfrac{V_o}{V_{in}} = \dfrac{1}{\sqrt{2}} (A_{v,mid}) \Rightarrow$ the frequency at which $X_c = R$ is the cut-off freq.

Low-Frequency Response:



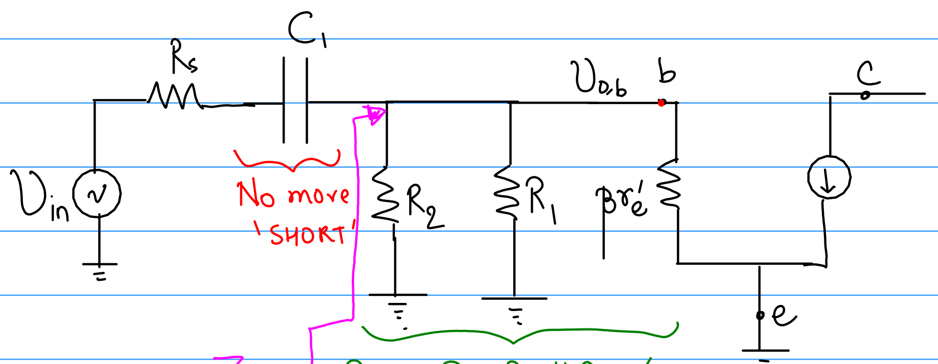
$r_e' =$ ac-emitter resistance

$$r_e' = \frac{25\,mV}{I_E}$$

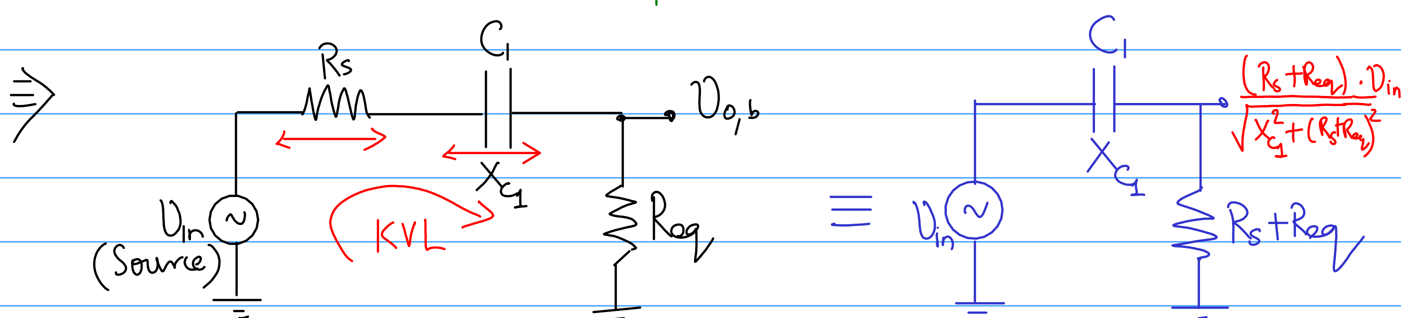
① Effect of the capacitor C_1 :

$Z = X + jY$

Resistance (X), Reactance (Y)



$$Z_{in,stage} = R_{eq} = R_1 \| R_2 \| \beta r_e'$$



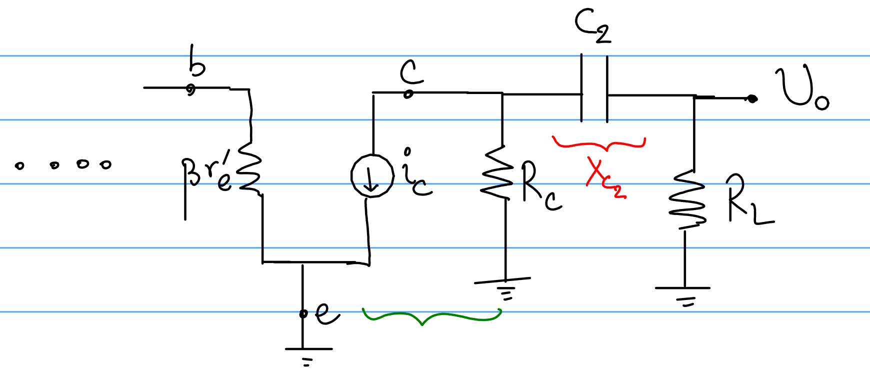
if $X_{C_1} = R_s + R_{eq}$ then we are at the lower cut-off freq.

$$\frac{1}{2\pi f_{L,C_1} C_1} = R_s + R_{eq}$$

$$\boxed{f_{L,C_1} = \frac{1}{2\pi\,(R_s + R_{eq})\,C_1}}$$

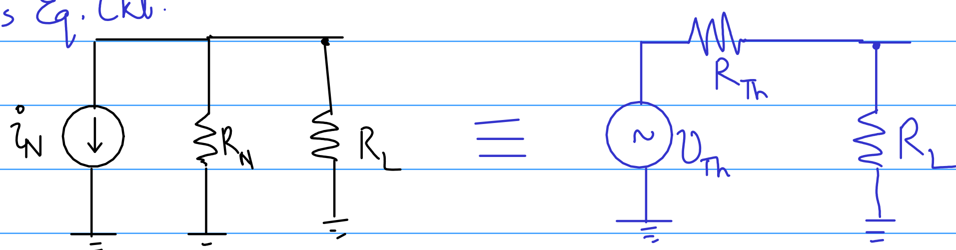
where $R_{eq} = R_1 \| R_2 \| \beta_{ac} r_e'$
& $r_e' = \dfrac{25\,mV}{I_E}$

② Effect of the Capacitor C_2 :



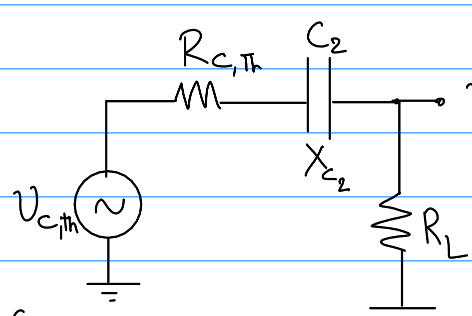
Recall 1st chapter

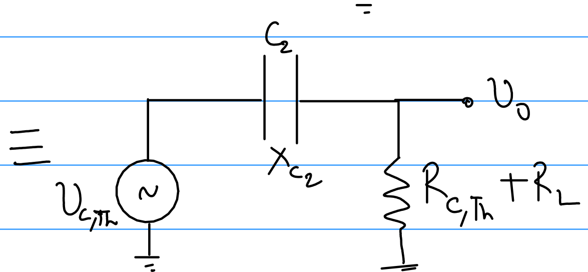
Norton's Eq. Ckt.



where, $R_{Th} = R_N$; $V_{Th} = i_N \cdot R_N$

If we apply to the collector circuit, the equivalent ckt. will be as follows:





$$v_o = \frac{(R_{C,Th} + R_L)}{\sqrt{X_{C_2}^2 + (R_{C,Th} + R_L)^2}} \cdot v_{C,Th}$$

If $X_{C_2} = R_{C,Th} + R_L$ then we have cut-off freq. due to C_2.

$$\frac{1}{2\pi f_{L,C_2} \cdot C_2} = R_{C,Th} + R_L$$

$$\boxed{f_{L,C_2} = \frac{1}{2\pi (R_{C,Th} + R_L) C_2}}$$

<u>Example:</u>

$R_1 = 10\,k\Omega$; $R_2 = 1.5\,k\Omega$,
$R_C = 1.3\,k\Omega$; $R_E = 200\,\Omega$; $R_L = 10\,k\Omega$
$R_S = 50\,\Omega$
$C_1 = 10\,\mu F = C_2$; $\beta = 100$; $V_{CC} = 12\,V$

$$r_e' = \frac{25\,mV}{I_E} = 1.98\,\Omega \approx 2\,\Omega$$

Let calculate cut-off freq. due to C_1

$$f_{L,C_1} = \frac{1}{2\pi (R_S + \underbrace{R_1 \| R_2 \| \beta r_e'}) C_1}$$

$\beta r_e' = 100 \times 2\,\Omega = 200\,\Omega$

Now, $200\,\Omega \,\|\, 1500\,\Omega = \dfrac{200\,\Omega \times 1500\,\Omega}{200\,\Omega + 1500\,\Omega}$

$=$

$$\approx 173.4\ \Omega$$

Also, $$173.4\,\Omega \parallel 10\,k\Omega = \frac{173.4\,\Omega \times 10\,k\Omega}{173.4\,\Omega + 10\,k\Omega}$$

$$\approx 170.4\ \Omega$$

$$\Rightarrow \quad R_1 \parallel R_2 \parallel \beta r_e' \approx 170.4\,\Omega$$

$$\Rightarrow \quad f_{L,C_1} = \frac{1}{2\times(3.14)\times(50\,\Omega + 170.4\,\Omega)\times 10\,F} \times 10^6\ Hz$$

$$\Rightarrow \quad f_{L,C_1} = \frac{10^{\cancel{6}\,5}}{6.28 \times 220.4 \times 1\cancel{0}} = 72.5\ Hz$$

$$\boxed{f_{L,C_1} = 72.5\ Hz}$$

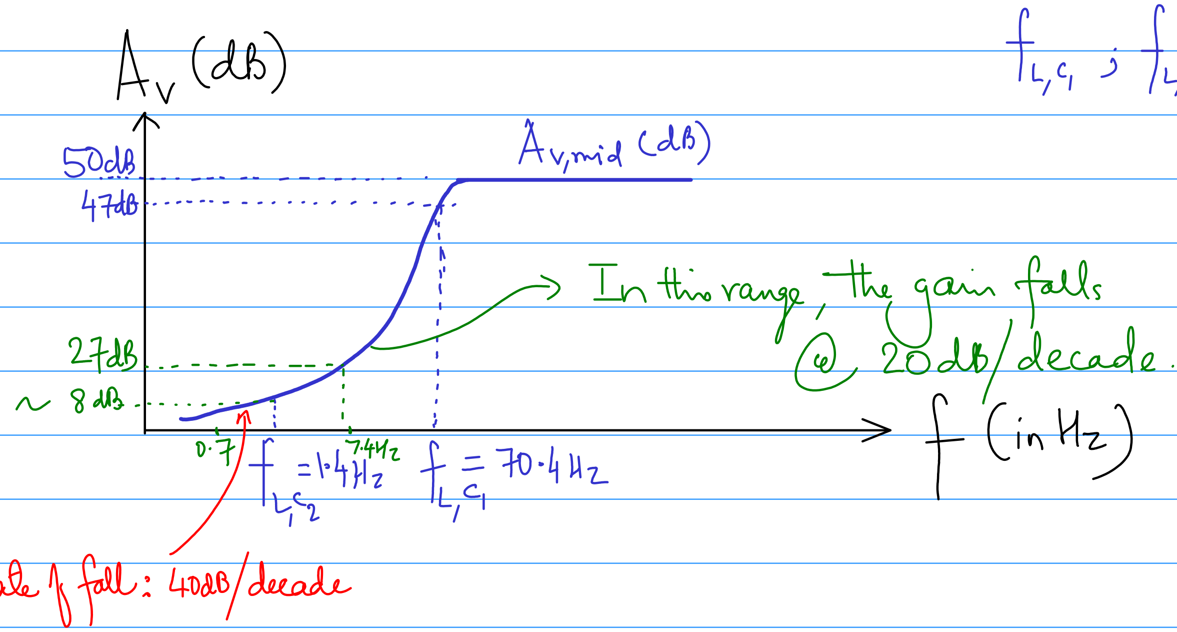
Let's calculate lower cut-off freq. due to C_2

$$f_{L,C_2} = \frac{1}{2\pi\,(R_C + R_L)\cdot C_2}$$

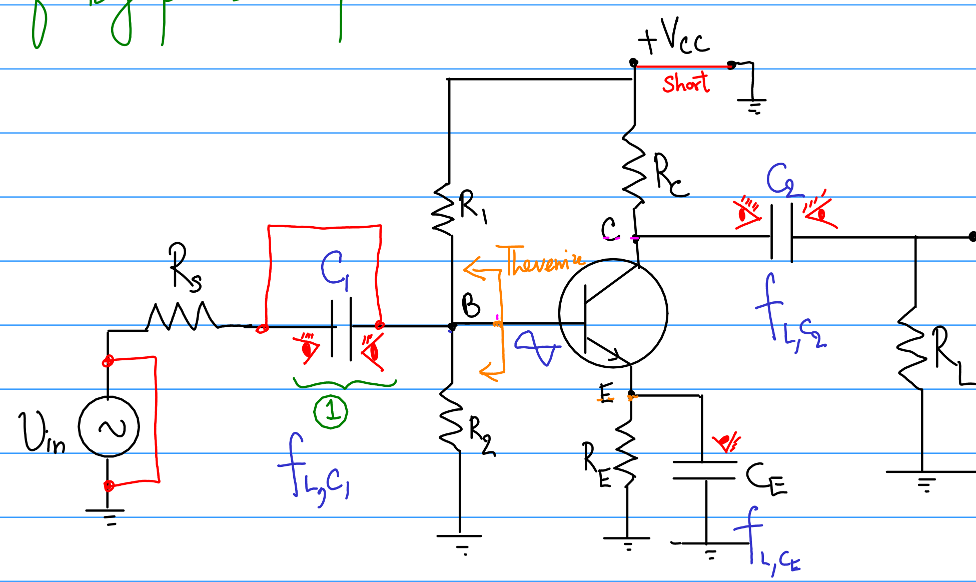
$$= \frac{1}{2\times(3.14)\times(1.3\,k\Omega + 10\,k\Omega)\times 10} 10^6\ Hz$$

$$\boxed{f_{L,C_2} = 1.4\ Hz}$$

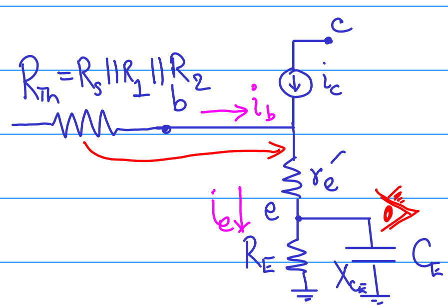
f_{L,C_1} ; f_{L,C_2}



③ Effect of By-pass Capacitor:



ac-equivalent ckt:



$i_c = \beta_{ac} i_b$

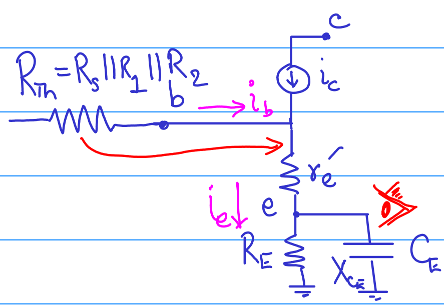
$i_c \approx i_e \Rightarrow i_e = \beta_{ac} i_b$

$\Rightarrow R_{eq:(emitt.)} = \frac{R_{Th}}{\beta_{ac}}$

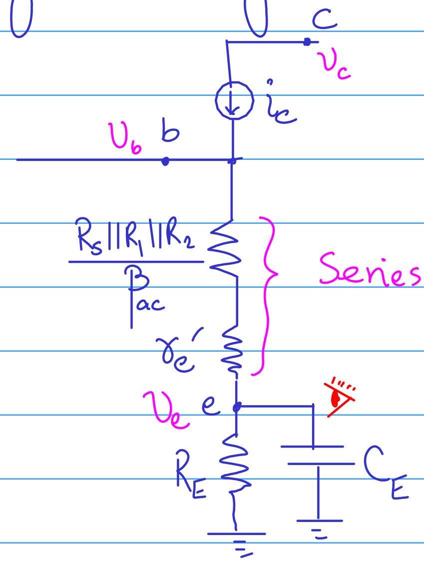
Step 1 : Thevenize from B towards input of the Base terminal.

$$R_{Th} = R_S \parallel R_1 \parallel R_2$$

Step 2 : Bring the equivalent resistance (R_{Th}) to the emitter terminal by dividing it by β_{ac}



$\equiv$



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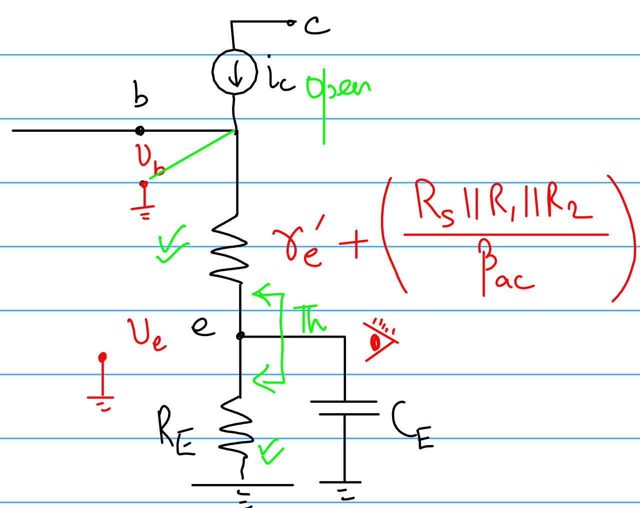
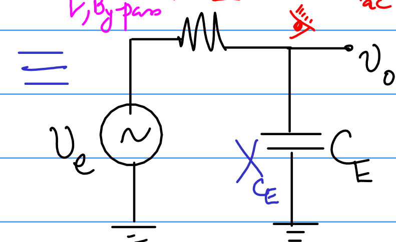


$$R_{eq_{L,By\text{-}pass}} = R_E \parallel \left[r_e' + \left(\frac{R_S \parallel R_1 \parallel R_2}{\beta_{ac}} \right) \right]$$

$\equiv$



From this equivalent ckt.

$$v_o = \frac{X_{C_E}}{\sqrt{X_{C_E}^2 + R_{eq_{L,By\text{-}pass}}^2}} v_e$$

if $X_{C_E} = R_{eq_{L,By\text{-}pass}}$.

$\Rightarrow$ cut-off frequency due to by-pass capacitor.

$$\frac{1}{2\pi f_{L,By\text{-}pass} C_E} = R_{eq, By\text{-}pass}$$

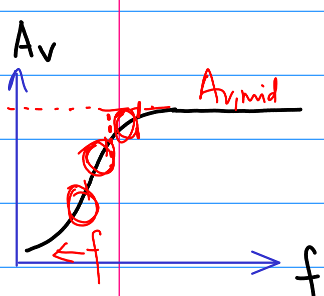
$$\Rightarrow \boxed{f_{L, By\text{-}pass} = \frac{1}{2\pi R_{eq, By\text{-}pass} C_E}}$$

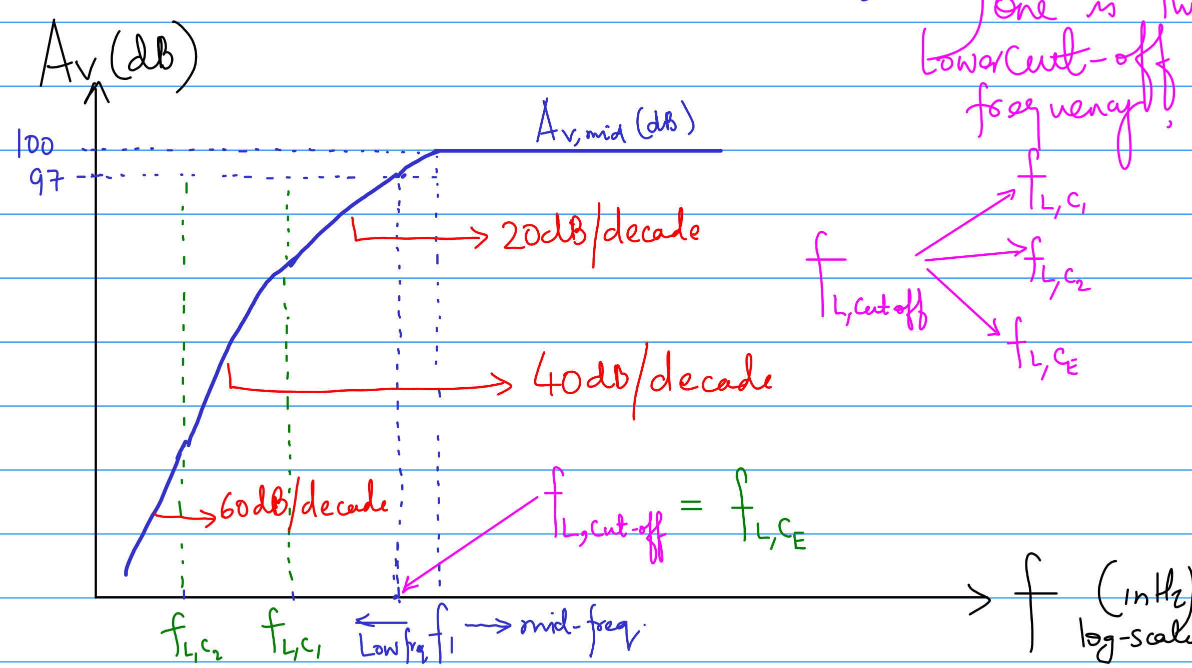
Overall low frequency response :

We have three cut-offs

$$f_{L, C_1} \; ; \; f_{L, C_2} \; ; \; f_{L, C_E}$$

Among the three the heighest one is the Lower Cut-off freqüency!





$$f_{L,cut\text{-}off} \to f_{L,C_1},\; f_{L,C_2},\; f_{L,C_E}$$