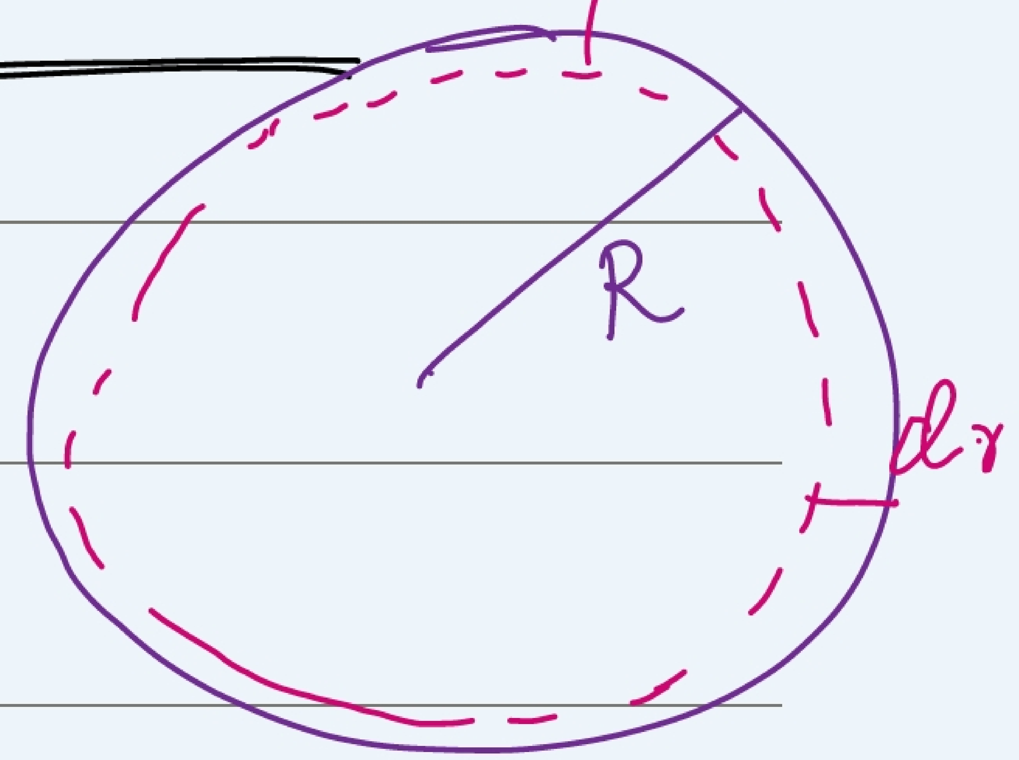
2 Gauss Law :→ $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$ → Microscopic $\Rightarrow$ $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ → Macroscopic

$\iiint (\vec{\nabla}\cdot\vec{E})\, d\tau = \frac{\int \rho\, d\tau}{\epsilon_0}$



For ex: $\vec{E} = K r^3 \hat{r}$ Spherical region : $\rho = ?$

Total charge contained in sphere of radius R. ✓

$Q_{en} = \epsilon_0 \oint \vec{E}\cdot d\vec{a}$

$= \int_0^{2\pi}\int_0^{\pi} K r^3 \hat{r}\cdot r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$

$\rho = \epsilon_0 \vec{\nabla}\cdot\vec{E} = \epsilon_0 \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 K r^3\right]$

$$\boxed{\rho = 5K\epsilon_0 r^2}$$

$Q_{en} = \int \rho\, d\tau$

$= \int_0^R 5K\epsilon_0 r^2 \cdot 4\pi r^2 dr$

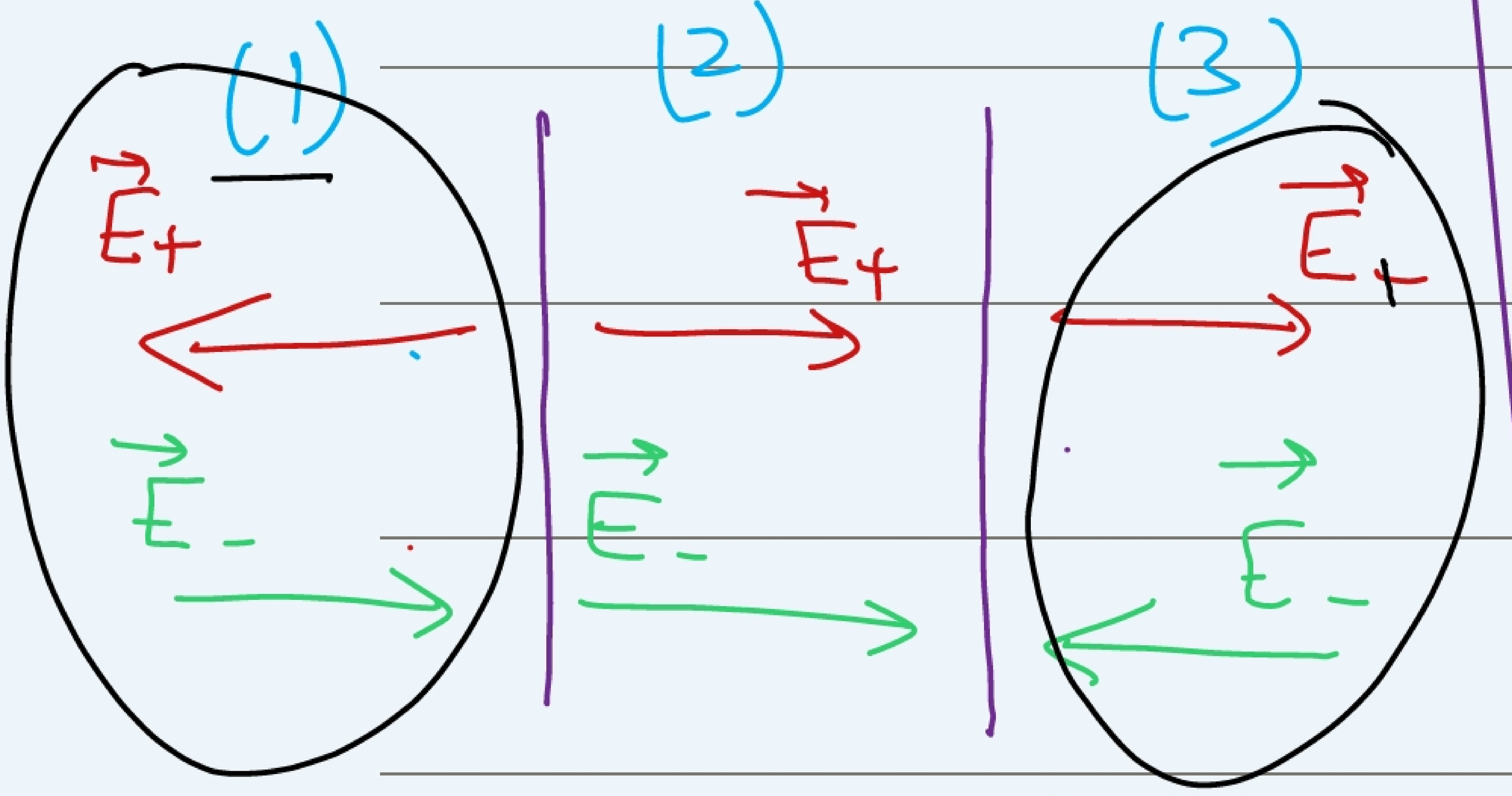
Infinite plane (σ)

$E = ?$

3D: $E \propto \frac{1}{r^2}$

2D: $E \propto \frac{1}{r}$.

1D: Distance independent field.



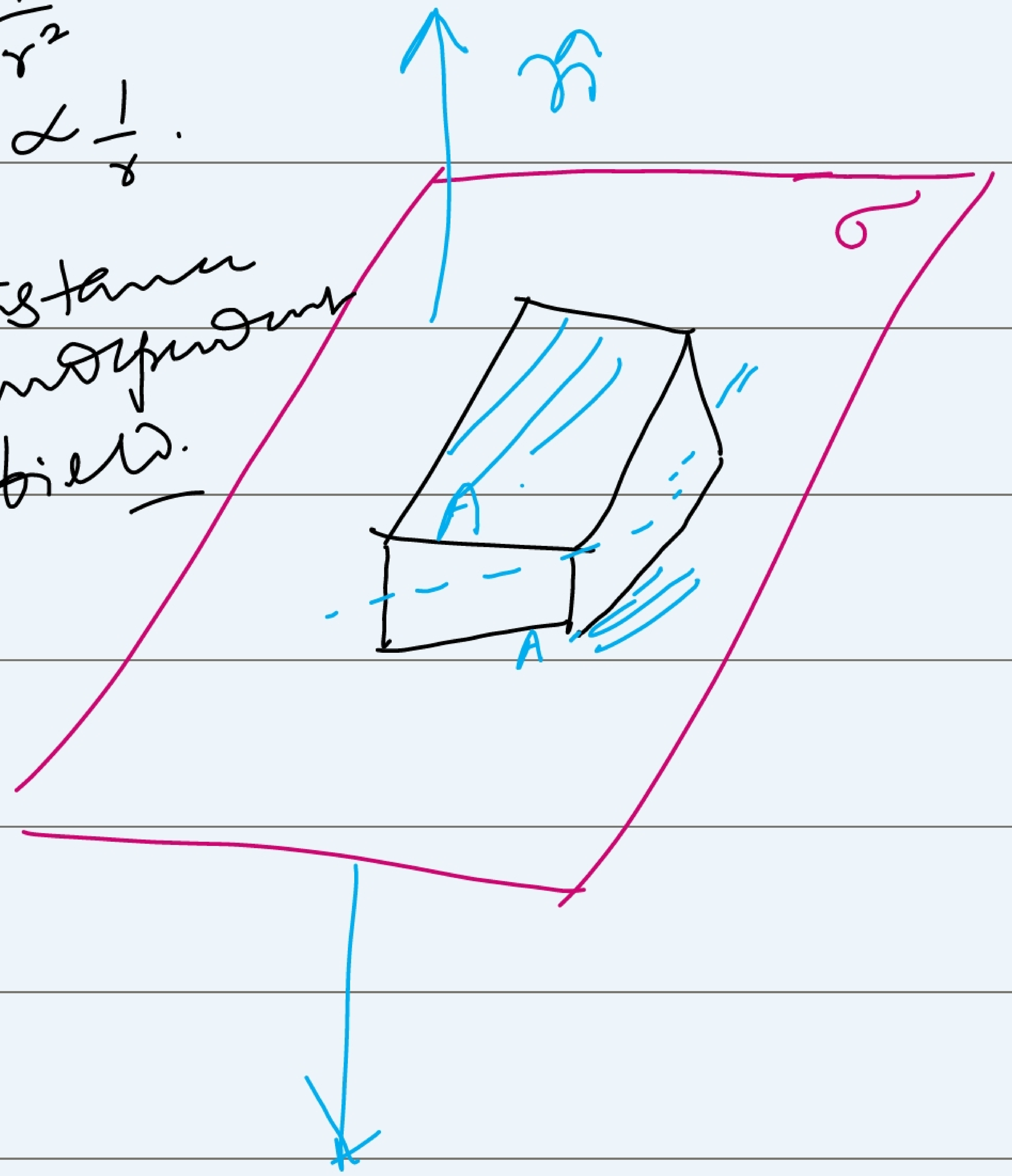
$\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$

$= \frac{\sigma}{\epsilon_0}$.

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}.$$

$$2A|E| = \frac{Q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}.$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

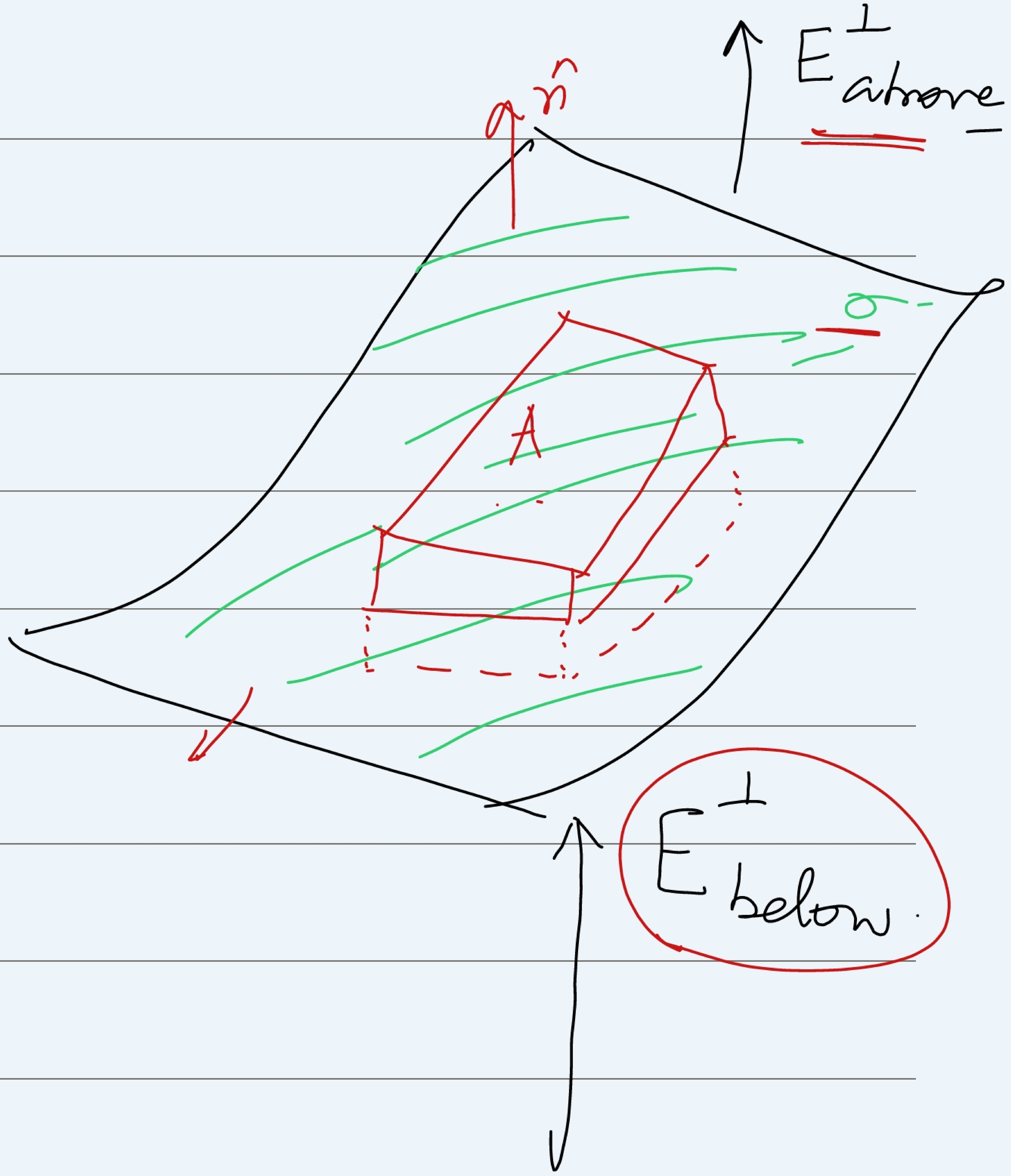


Boundary Condn:

① $\nabla \cdot E = \frac{\rho}{\epsilon_0}$, $\oint \vec{E} \cdot da = \frac{q_{en}}{\epsilon_0}$.

$$E^{\perp}_{above} A - E^{\perp}_{below} A = \frac{\sigma A}{\epsilon_0}.$$

$$\boxed{E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}.}$$



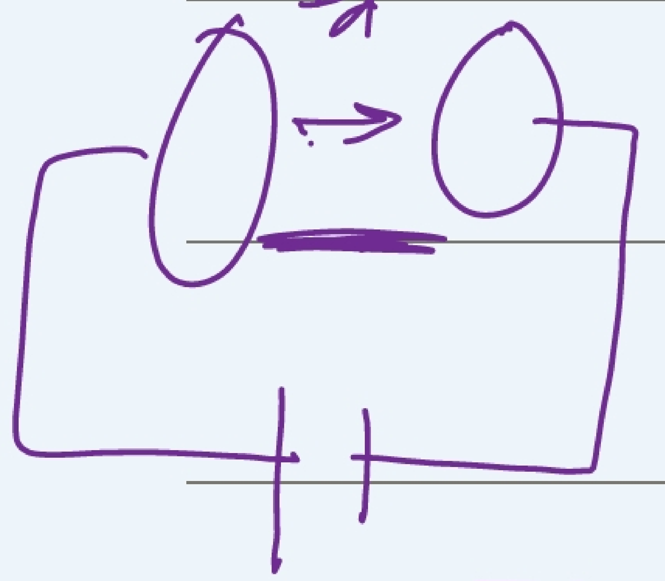
$\nabla \times \vec{E} = 0$: $\vec{E} = \nabla V$ → scalar potential

$\nabla \times B = \mu_0 J + \mu_0 J_d$

$\nabla \cdot E = \rho / \epsilon_0$ $= \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}}$$

I_d $\frac{\sigma}{\epsilon_0}$



$\frac{\partial E}{\partial t}$

$$J_d = \epsilon_0 \frac{\partial E}{\partial t}$$

$$J_d = \boxed{\epsilon_0 \frac{\partial E}{\partial t}} = \epsilon_0 \frac{\sigma}{\epsilon_0} = \epsilon_0 \frac{dQ/dt}{A \epsilon_0} = \frac{\cancel{\epsilon_0} I}{A \cancel{\epsilon_0}}$$

$$\nabla \times \vec{E} = 0 - \frac{\partial \vec{B}}{\partial t}$$

$$\oint (\nabla \times \vec{E}) \cdot da = -\frac{\partial}{\partial t} \int \vec{B} \cdot da$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 I_d$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{a} = \mu_0 \oint \vec{J} \cdot d\vec{a}$$

If. $\rho = 0$, $I = 0$.

$$\nabla \times (\nabla \times E) = \boxed{\nabla(\cancel{\nabla \cdot E}^{0}) - \nabla^2 E} = -\frac{\partial}{\partial t}(\nabla \times B)$$

$\nabla \cdot E = 0$.

$$\nabla \times \nabla \times E = -\frac{\partial (\nabla \times B)}{\partial t}.$$

$$\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}.$$

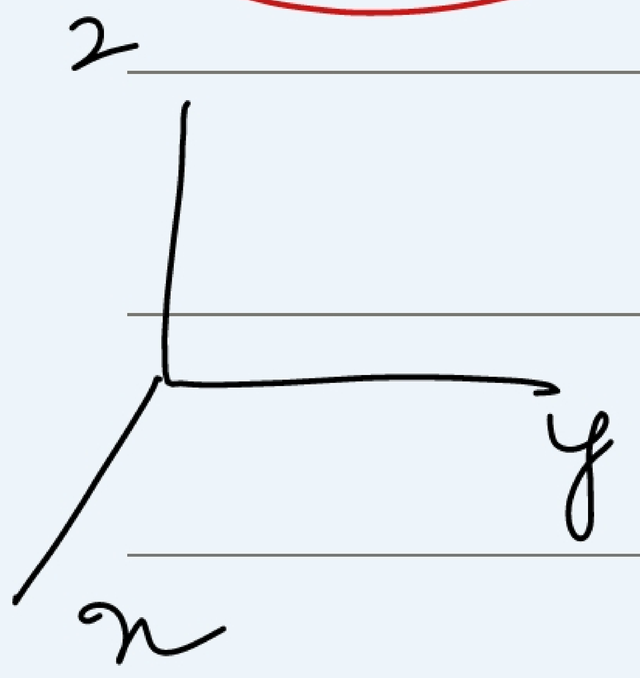
$$\boxed{\nabla^2 E = + \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

1D:
$$\frac{\partial^2 E}{\partial x^2} + \underbrace{\frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}}_{0} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

$\nabla \times E = 0$

E : Electrostatic electric field.



$$\oint (\nabla \times \vec{E}) \cdot d\vec{a} = 0.$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$\int_a^b \nabla V \cdot dl = V(b) - V(a)$$

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{vmatrix}$$

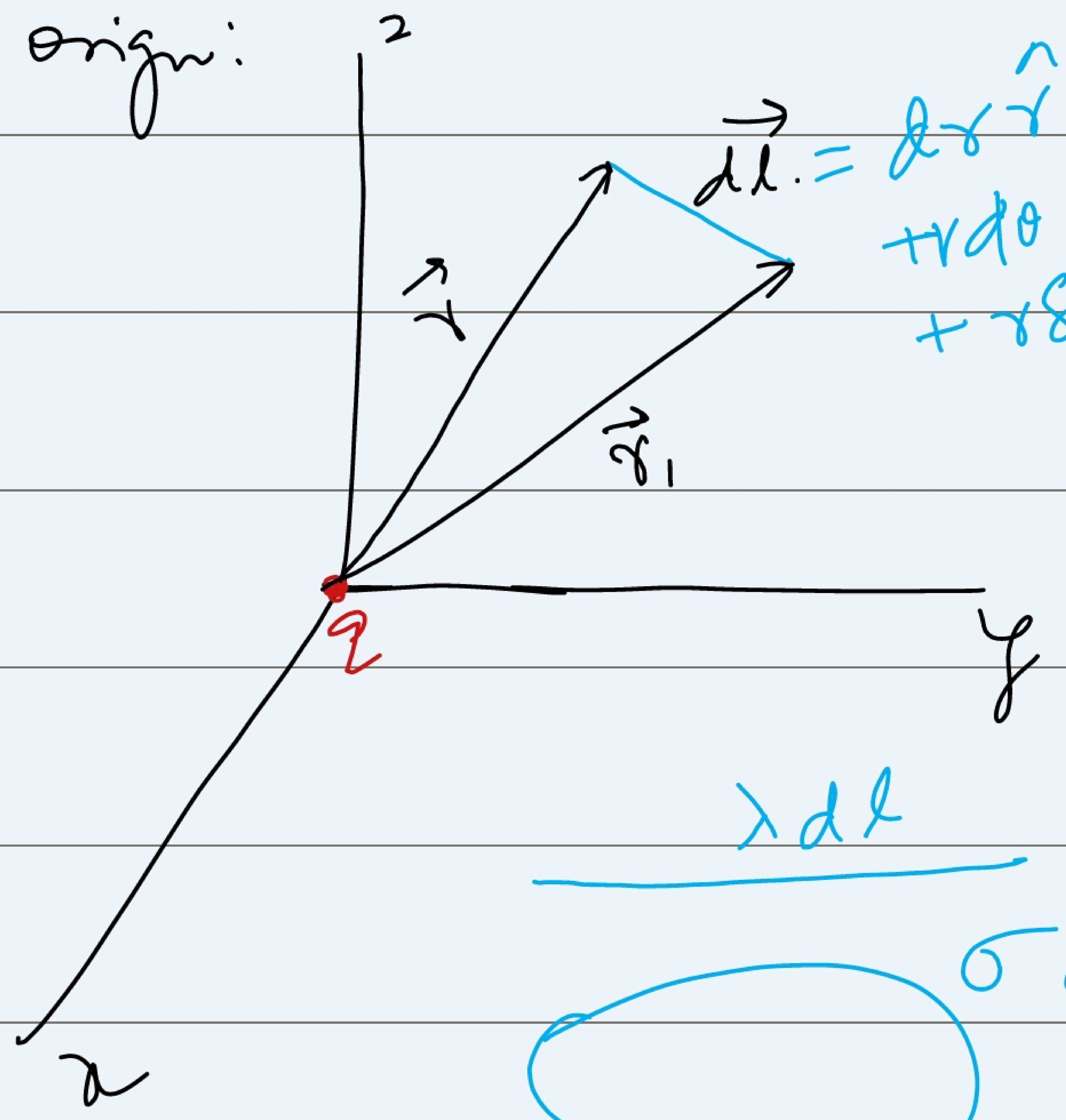
Ex: $\vec{E} = K\left[xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z}\right]$

$$\frac{\partial E_z}{\partial x} = \frac{\partial E_x}{\partial z}; \quad \frac{\partial E_z}{\partial y} = \frac{\partial E_y}{\partial z}; \quad \frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y}$$

$0 \neq 2y$

$V = ?$

Electric potential due to a point charge at origin:



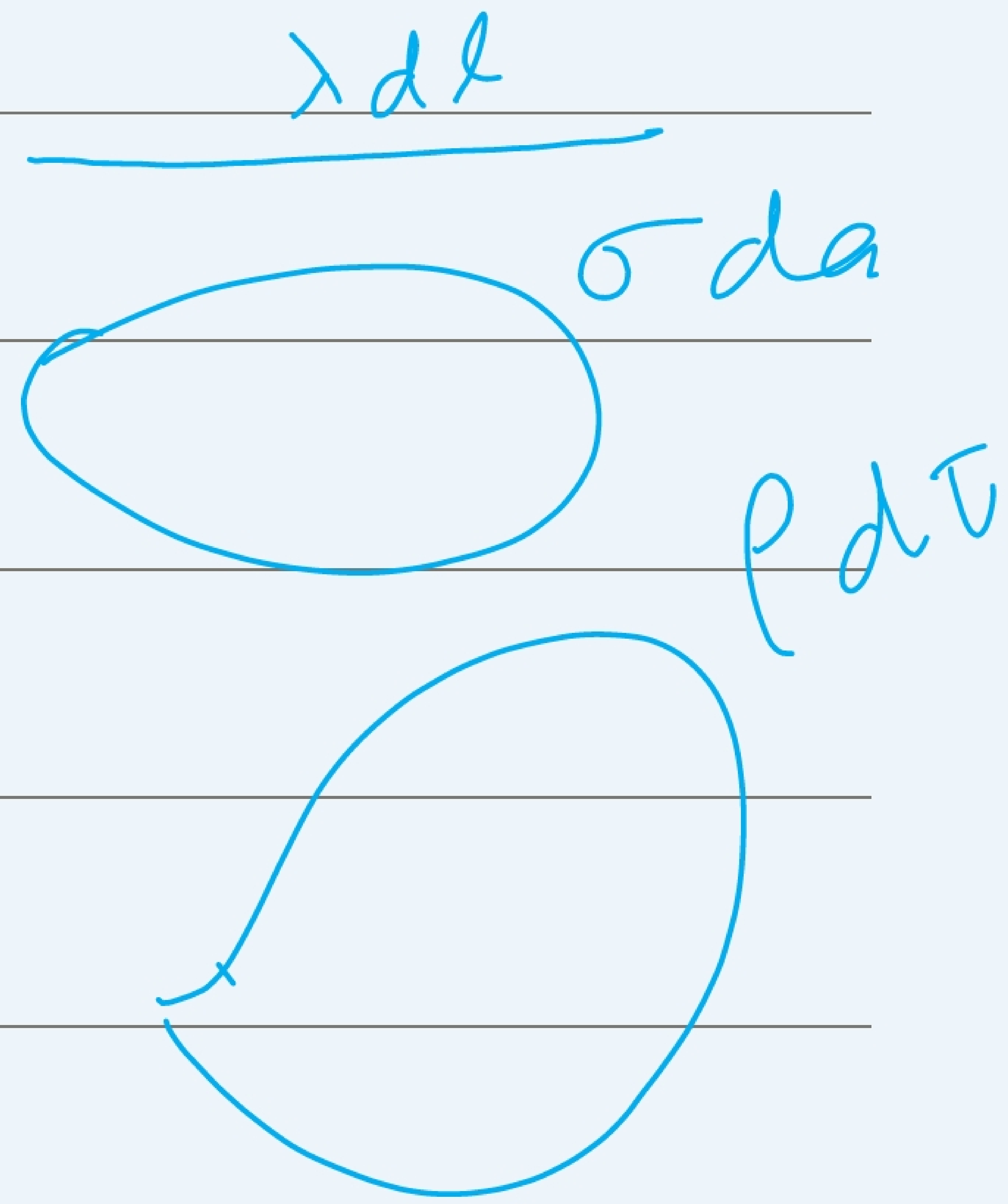
$\vec{dl} = dr\,\hat r + r\,d\theta\,\hat\theta + r\sin\theta\,d\phi\,\hat\phi$

$$\vec E(r_1) = K\frac{q}{r_1^2}\hat r_1$$

$$V(r) = -\int_\infty^r \vec E(r_1)\cdot d\vec l_1$$

$$= -K\int_\infty^r \frac{q}{r_1^2}\,dr_1$$

$$\boxed{V(r) = K\frac{q}{r}}$$



① Independent of reference frames

② $V = V_1 + V_2 + V_3 + \cdots$

③ $|k|$

n no. of charges