Priority Queues

Two kinds of priority queues:

- Min priority queue.
- Max priority queue.

Min Priority Queue

- Collection of elements.
- Each element has a priority or key.
- Supports following operations:
  - isEmpty
  - size
  - add/put an element into the priority queue
  - get element with min priority
  - remove element with min priority

Max Priority Queue

- Collection of elements.
- Each element has a priority or key.
- Supports following operations:
  - isEmpty
  - size
  - add/put an element into the priority queue
  - get element with max priority
  - remove element with max priority

Complexity Of Operations

good implementation is heaps

isEmpty, size, and get => O(1) time

put and remove => O(log n) time where n is the size of the priority queue

Applications

Sorting

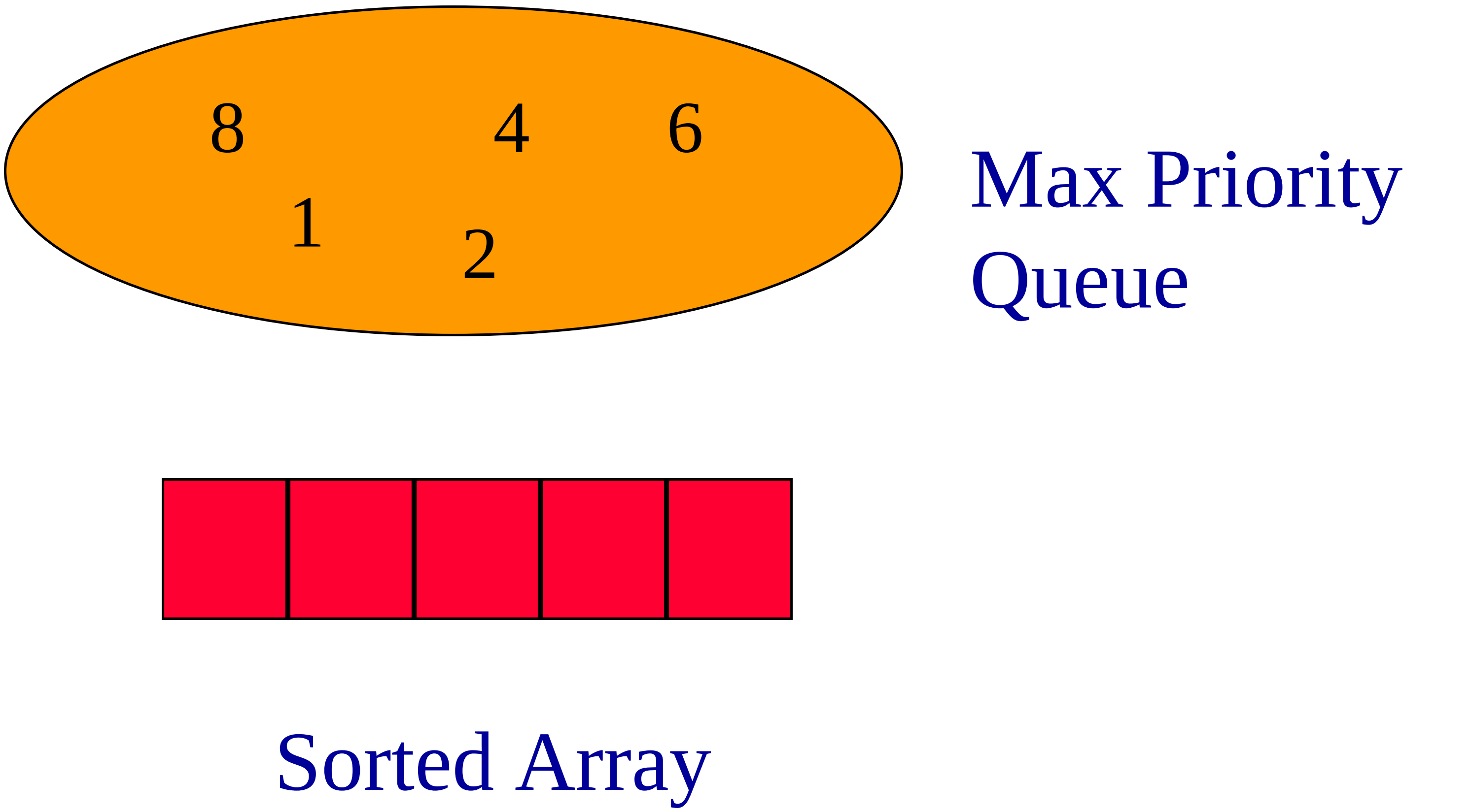
- use element key as priority
- put elements to be sorted into a priority queue
- extract elements in priority order
    - if a min priority queue is used, elements are extracted in ascending order of priority (or key)
    - if a max priority queue is used, elements are extracted in descending order of priority (or key)

Sorting Example

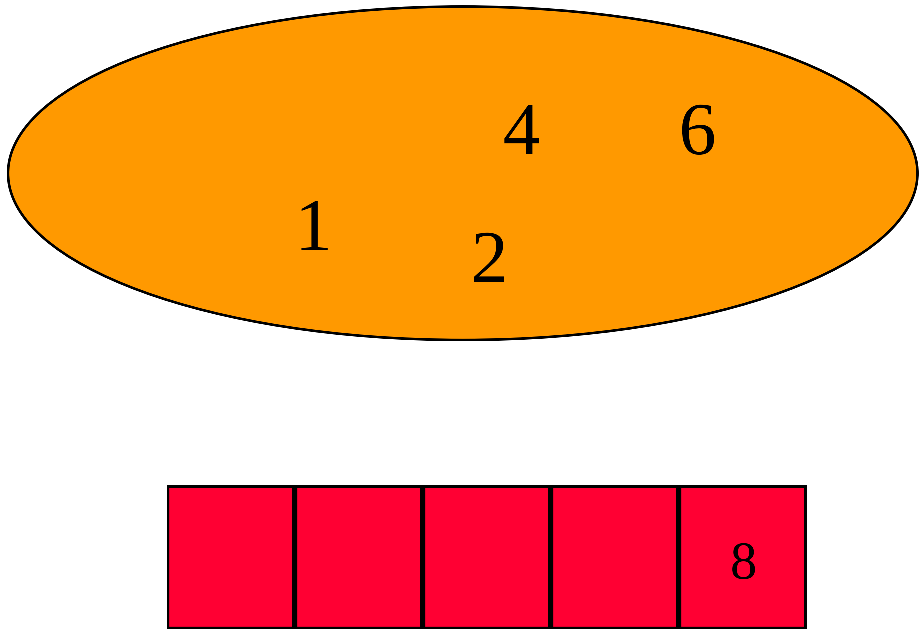
Sort five elements whose keys are 6, 8, 2, 4, 1 using a max priority queue.

- Put the five elements into a max priority queue.
- Do five remove max operations placing removed elements into the sorted array from right to left.

After Putting Into Max Priority Queue



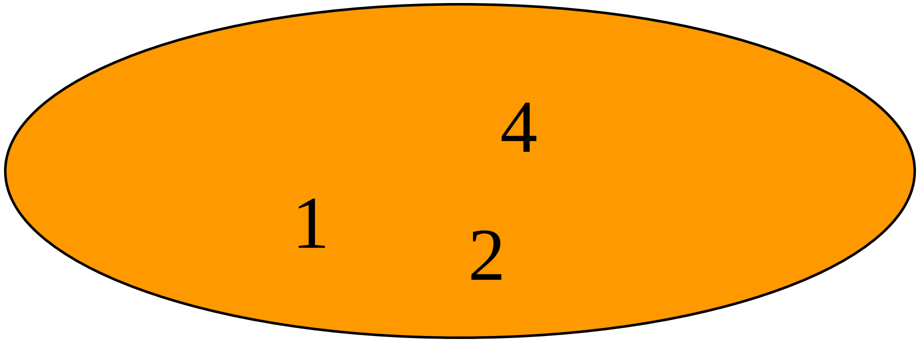
After First Remove Max Operation



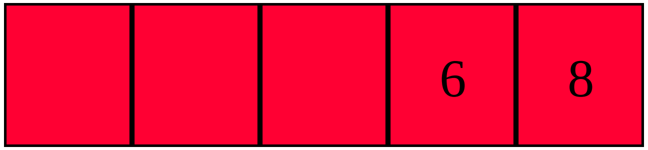
Max Priority Queue

Sorted Array

After Second Remove Max Operation

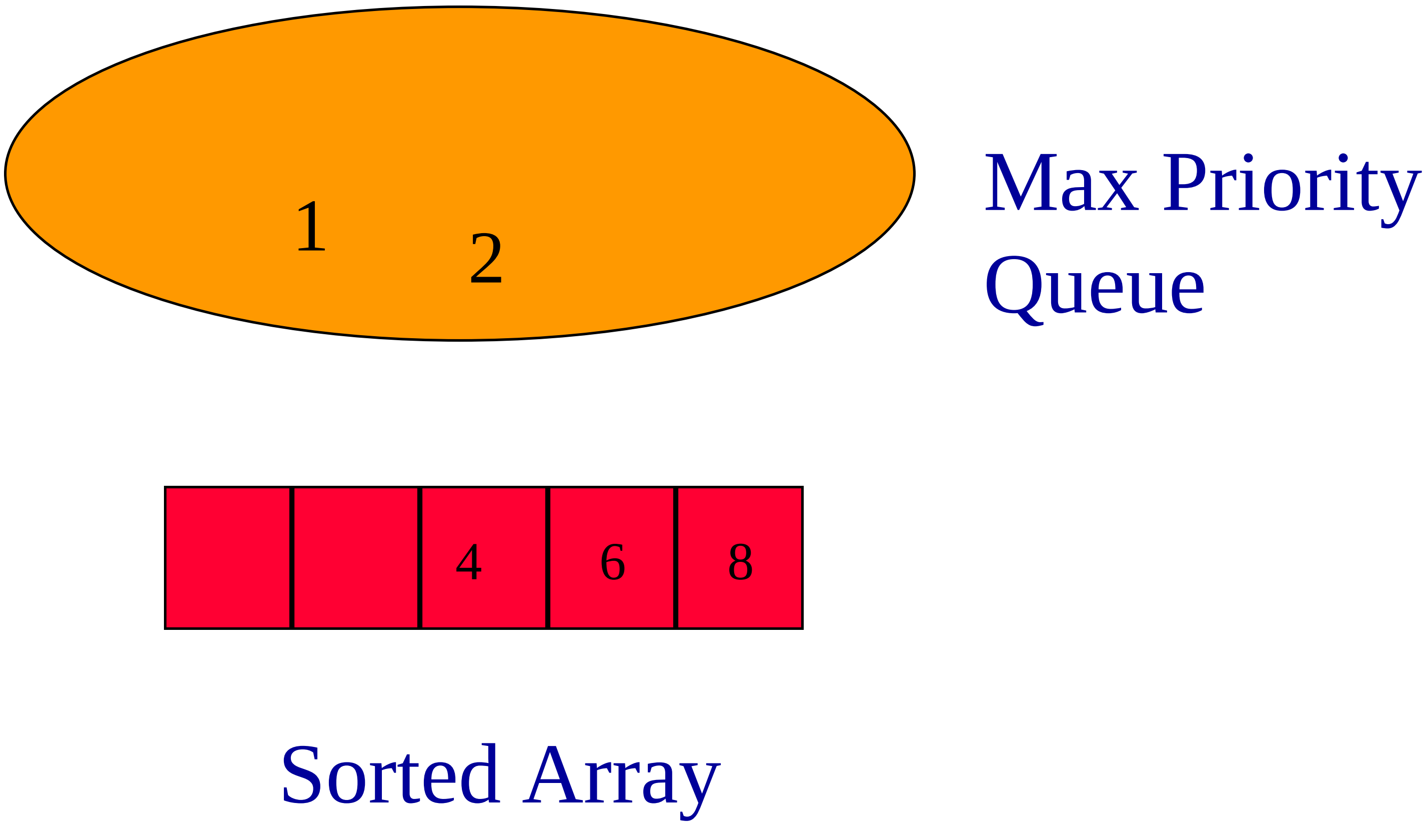


Max Priority Queue

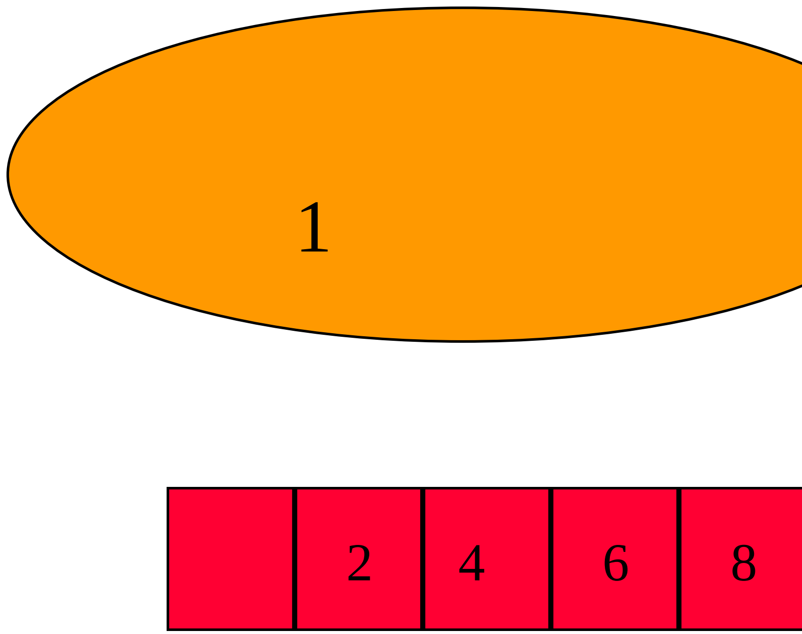


Sorted Array

After Third Remove Max Operation



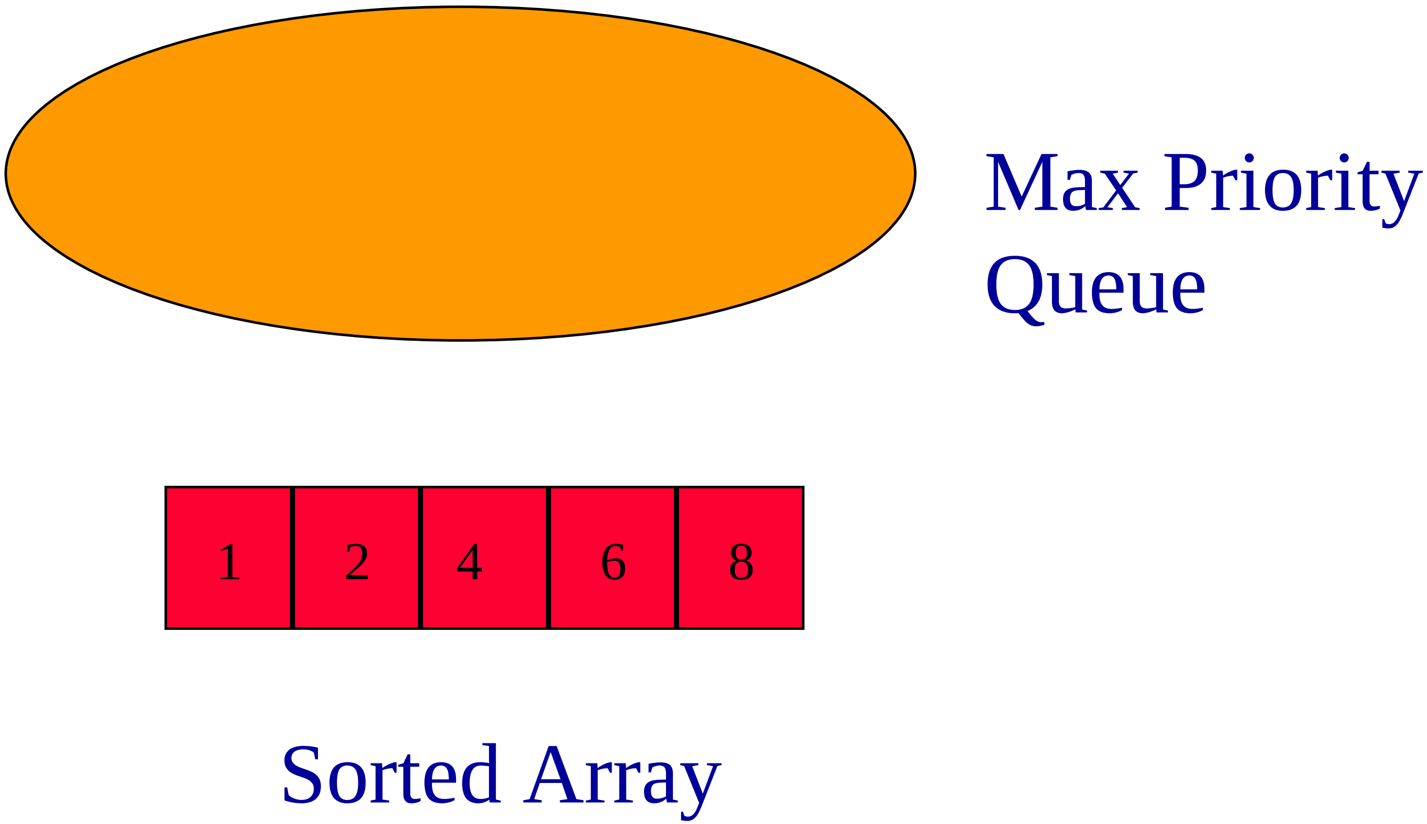
After Fourth Remove Max Operation



Max Priority Queue

Sorted Array

After Fifth Remove Max Operation



Complexity Of Sorting

Sort n elements.

- n put operations => $O(n \log n)$ time.
- n remove max operations => $O(n \log n)$ time.
- total time is $O(n \log n)$.
- compare with $O(n^2)$ for sort methods

Heap Sort

Uses a max priority queue that is implemented as a heap.

Machine Scheduling

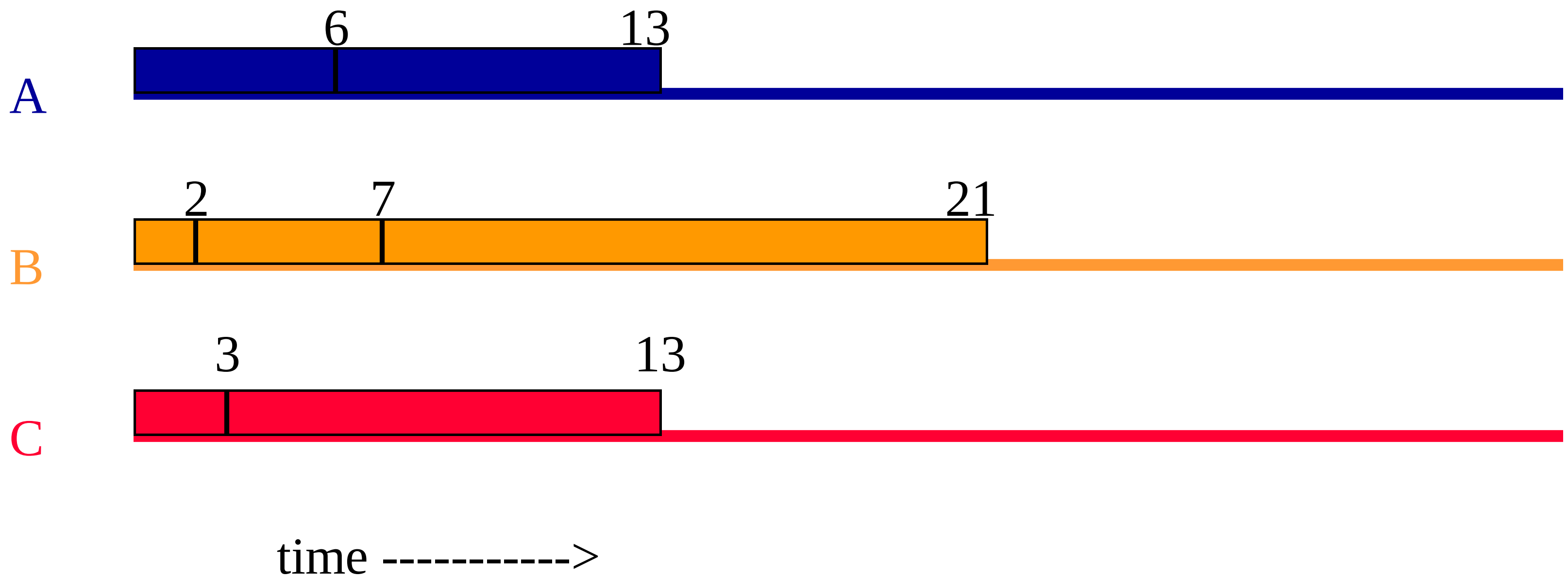
- m identical machines (drill press, cutter, sander, etc.)
- n jobs/tasks to be performed
- assign jobs to machines so that the time at which the last job completes is minimum

Machine Scheduling Example

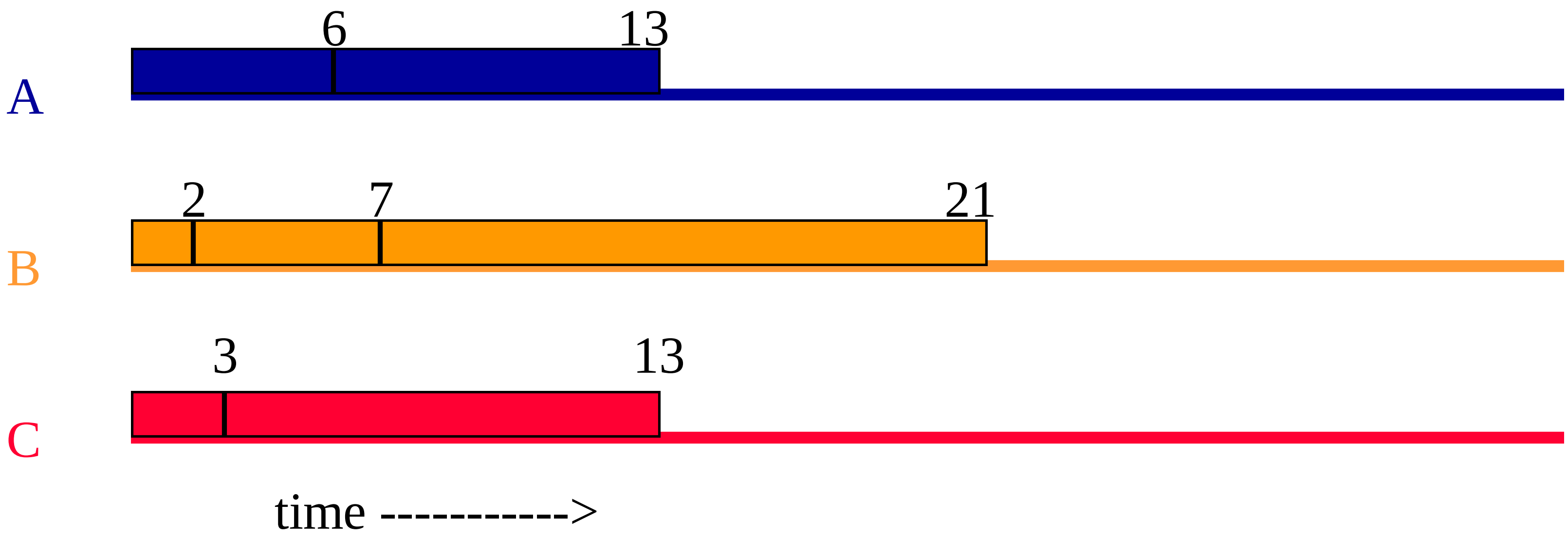
3 machines and 7 jobs

job times are [6, 2, 3, 5, 10, 7, 14]

possible schedule



Machine Scheduling Example



Finish time = 21

Objective: Find schedules with minimum finish time.

LPT Schedules

Longest Processing Time first.

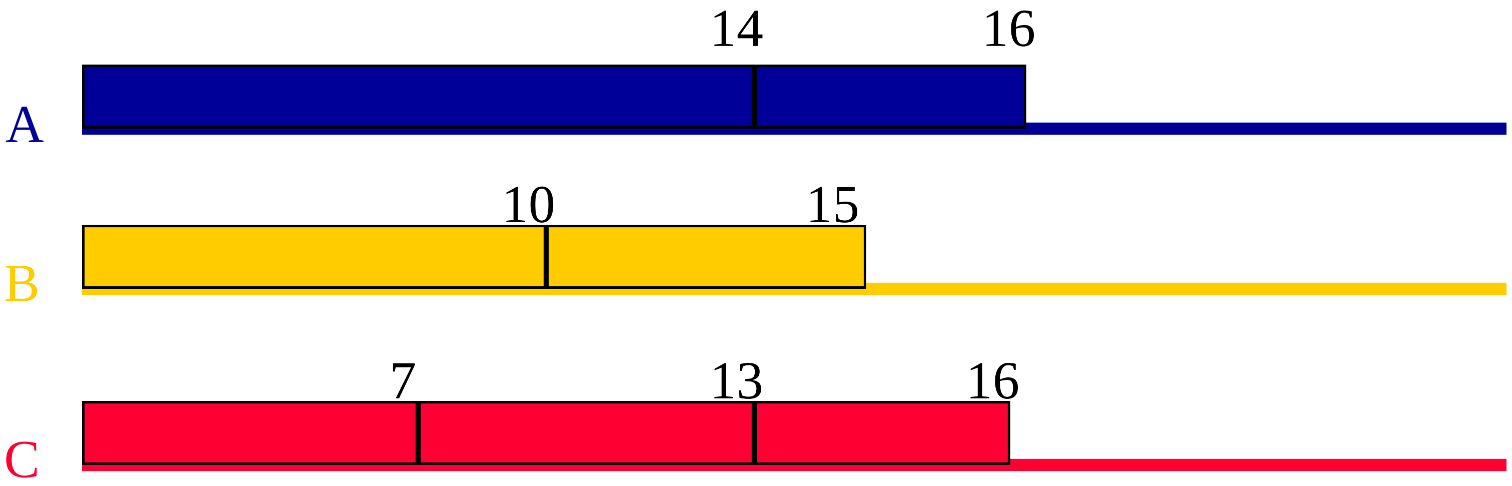
Jobs are scheduled in the order

14, 10, 7, 6, 5, 3, 2

Each job is scheduled on the machine on which it finishes earliest.

LPT Schedule

[14, 10, 7, 6, 5, 3, 2]



Finish time is 16!

LPT Schedule

- LPT rule does not guarantee minimum finish time schedules.
- Usually LPT finish time is much closer to minimum finish time.
- Minimum finish time scheduling is NP-hard.

NP-hard Problems

- Infamous class of problems for which no one has developed a polynomial time algorithm.
- That is, no algorithm whose complexity is $O(n^k)$ for any constant k is known for any NP-hard problem.
- The class includes thousands of real-world problems.
- Highly unlikely that any NP-hard problem can be solved by a polynomial time algorithm.

NP-hard Problems

- Since even polynomial time algorithms with degree $k > 3$ (say) are not practical for large n, we must change our expectations of the algorithm that is used.
- Usually develop fast heuristics for NP-hard problems.
  - Algorithm that gives a solution close to best.
  - Runs in acceptable amount of time.
- LPT rule is good heuristic for minimum finish time scheduling.

Complexity Of LPT Scheduling

- Sort jobs into decreasing order of task time.
  - O(n log n) time (n is number of jobs)
- Schedule jobs in this order.
  - assign job to machine that becomes available first
  - must find minimum of m (m is number of machines) finish times
  - takes O(m) time using simple strategy
  - so need O(mn) time to schedule all n jobs.

Can we do better than O(mn)?

Using A Min Priority Queue

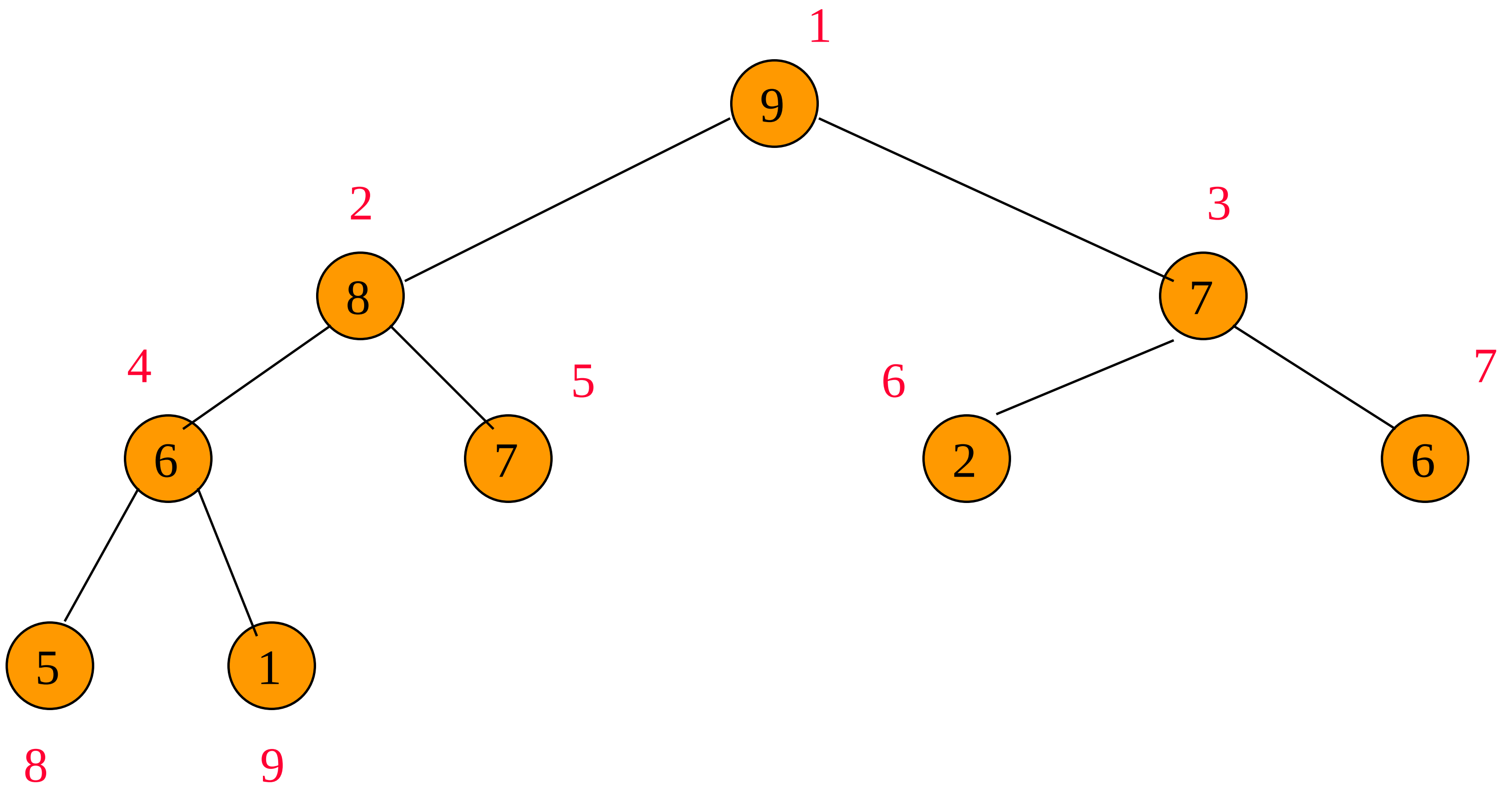
- Min priority queue has the finish times of the m machines.
- Initial finish times are all 0.
- To schedule a job remove machine with minimum finish time from the priority queue.
- Update the finish time of the selected machine and put the machine back into the priority queue.

Using A Min Priority Queue

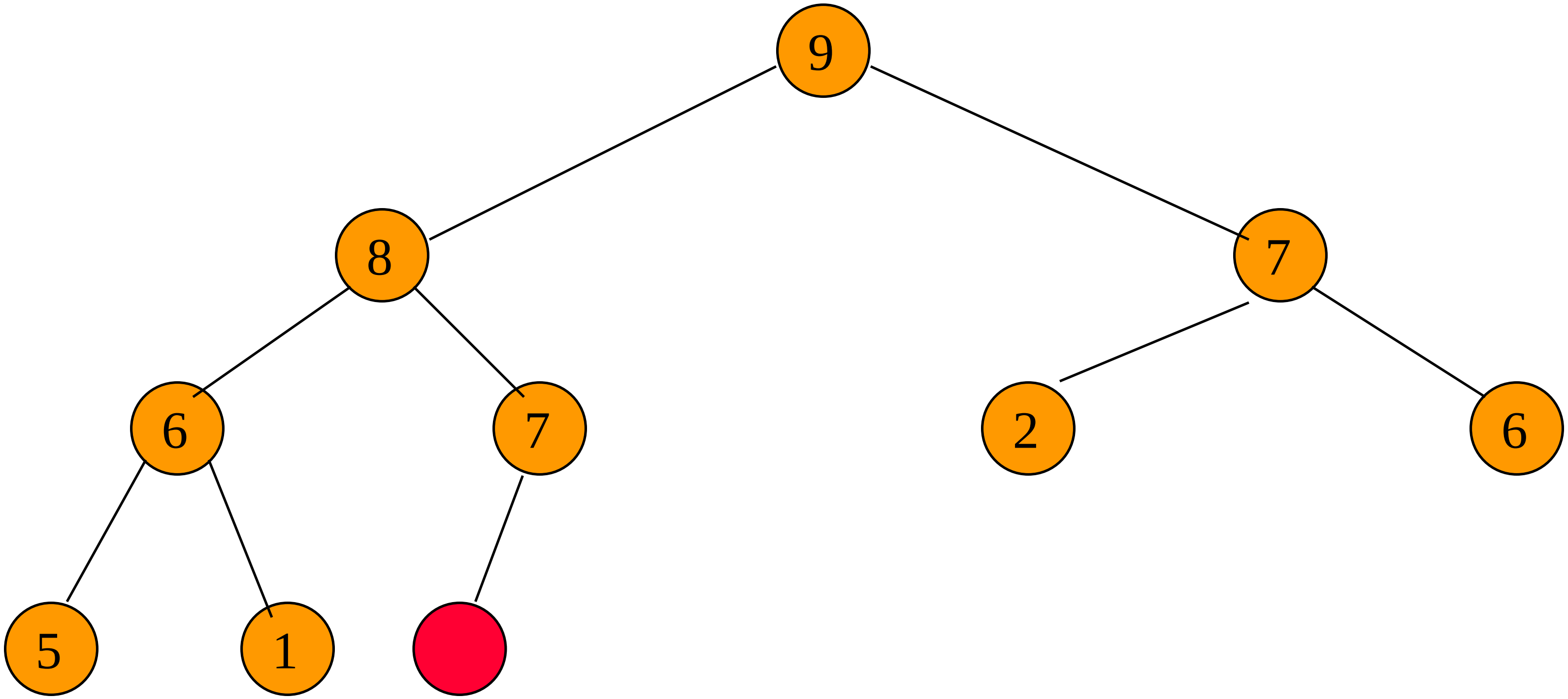
- m put operations to initialize priority queue
- 1 remove min and 1 put to schedule each job
- each put and remove min operation takes $O(\log m)$ time
- time to schedule n jobs is $O(n \log m)$
- overall time is

$$O(n \log n + n \log m) = O(n \log (mn))$$

Moving Up And Down A Heap

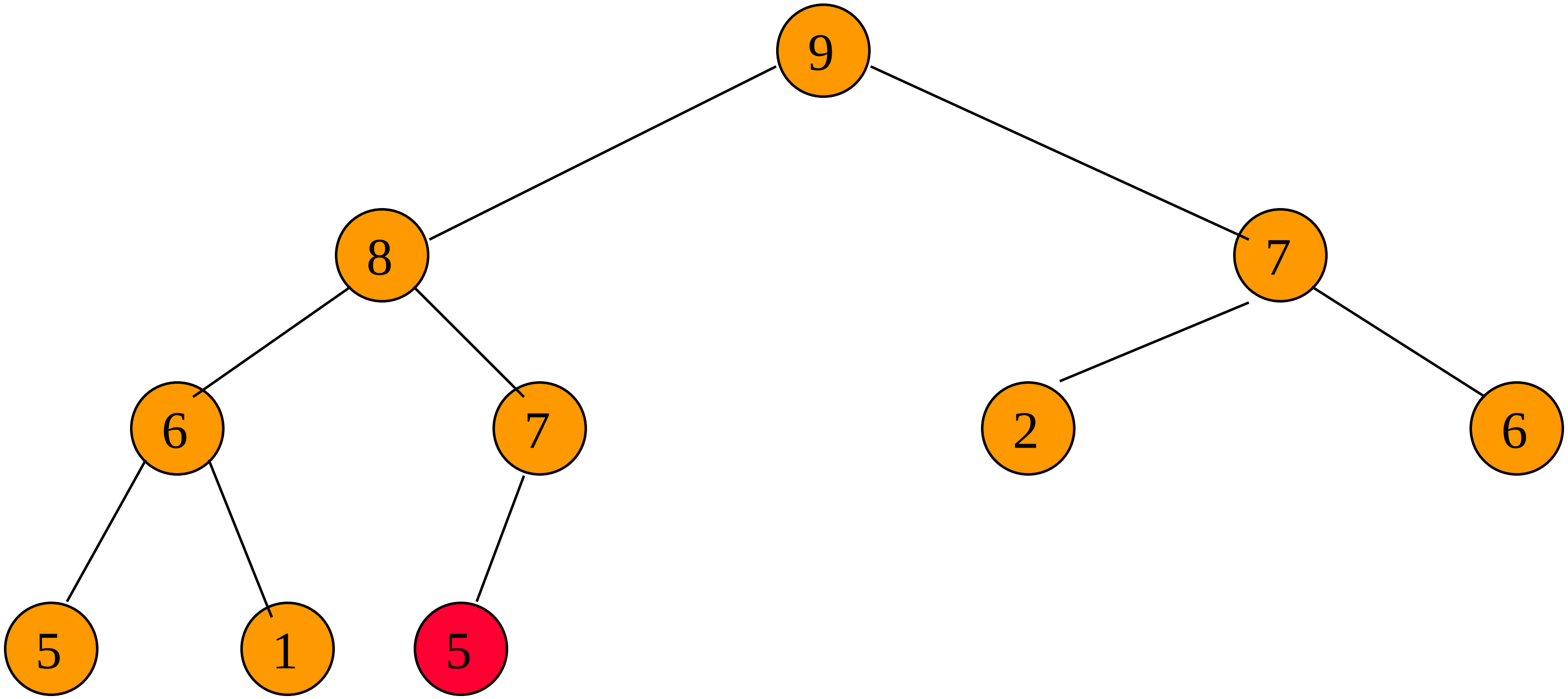


Putting An Element Into A Max Heap



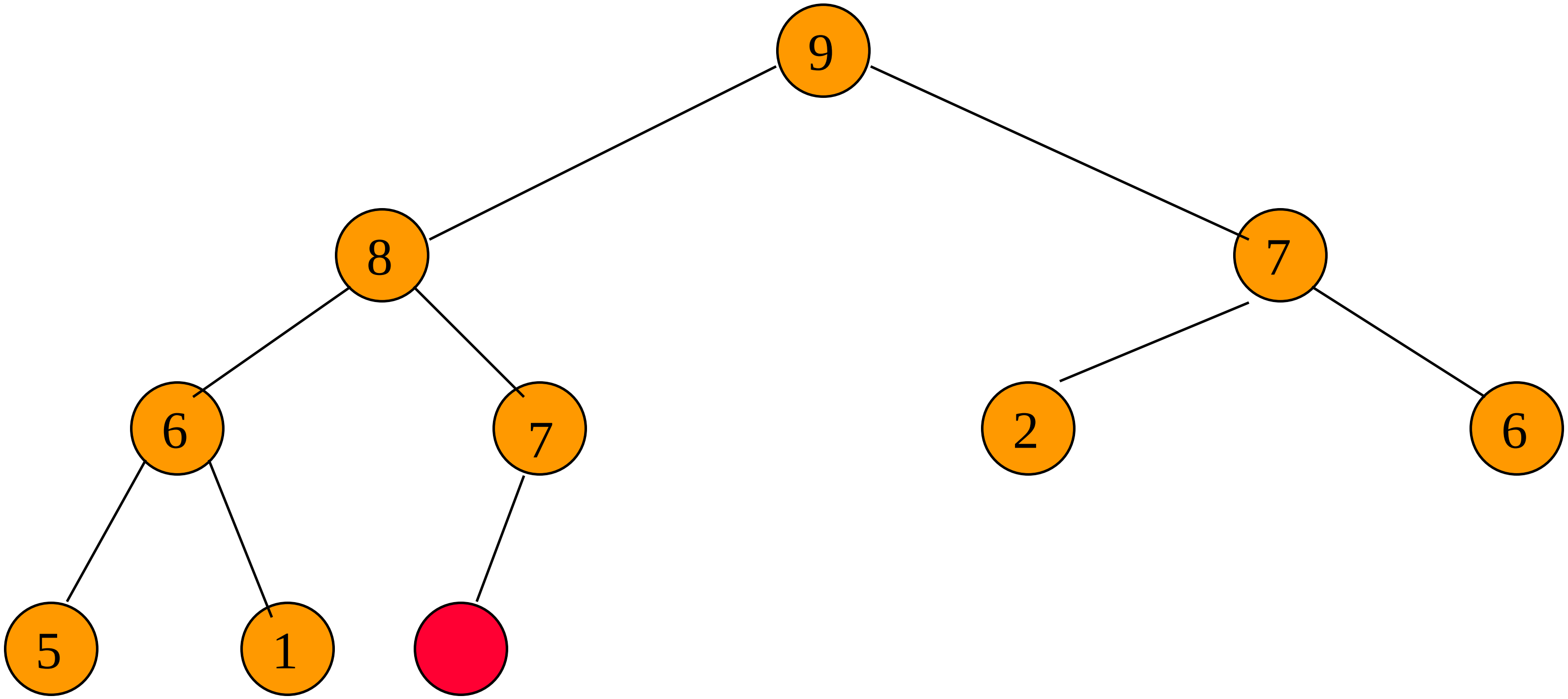
Complete binary tree with 10 nodes.

Putting An Element Into A Max Heap



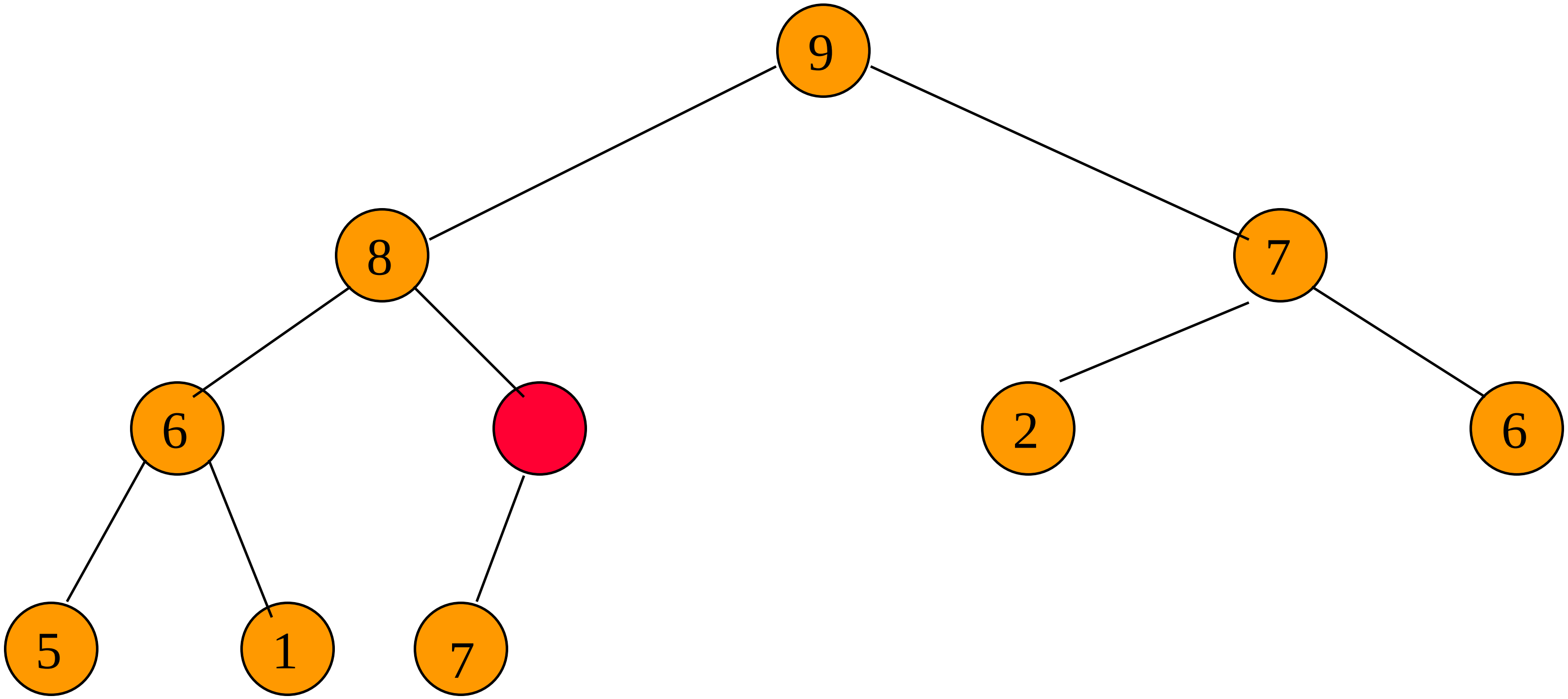
New element is 5.

Putting An Element Into A Max Heap



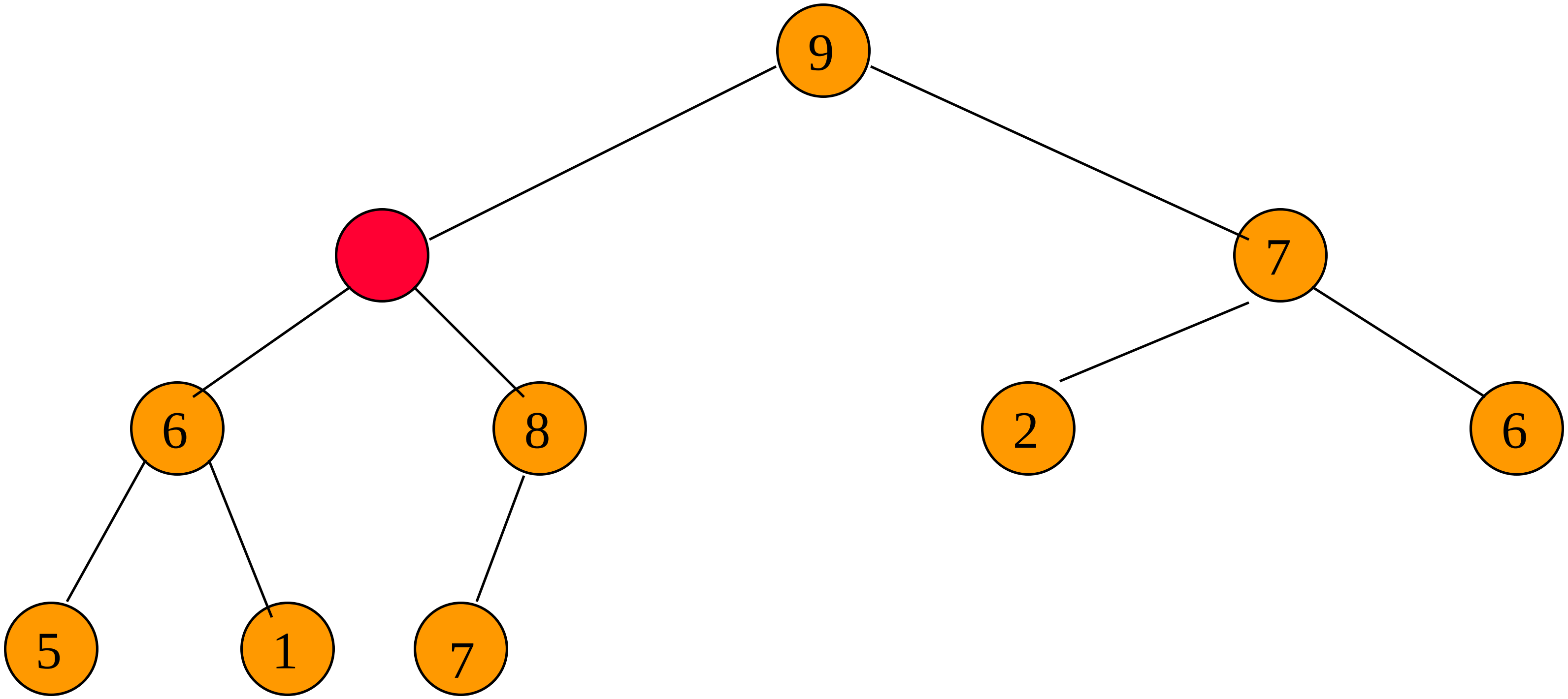
New element is 20.

Putting An Element Into A Max Heap



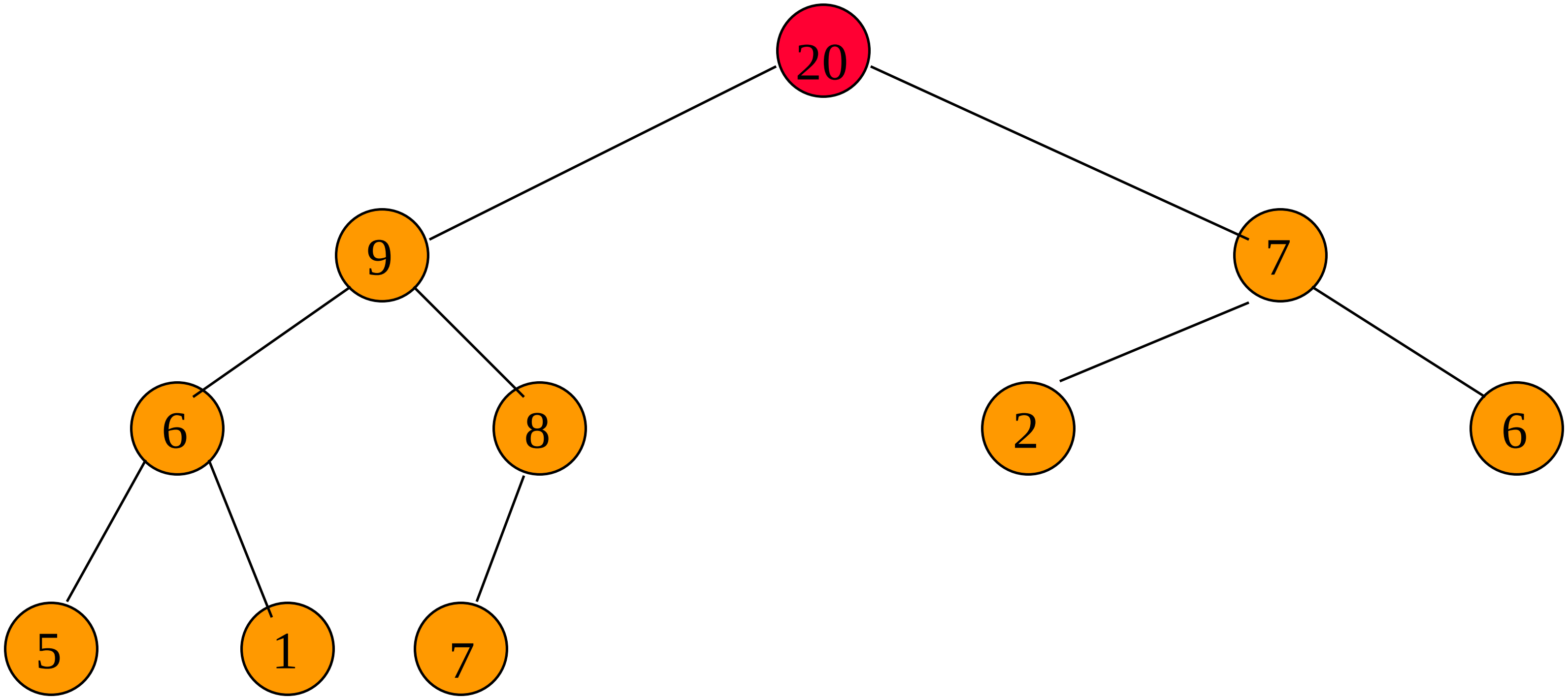
New element is 20.

Putting An Element Into A Max Heap



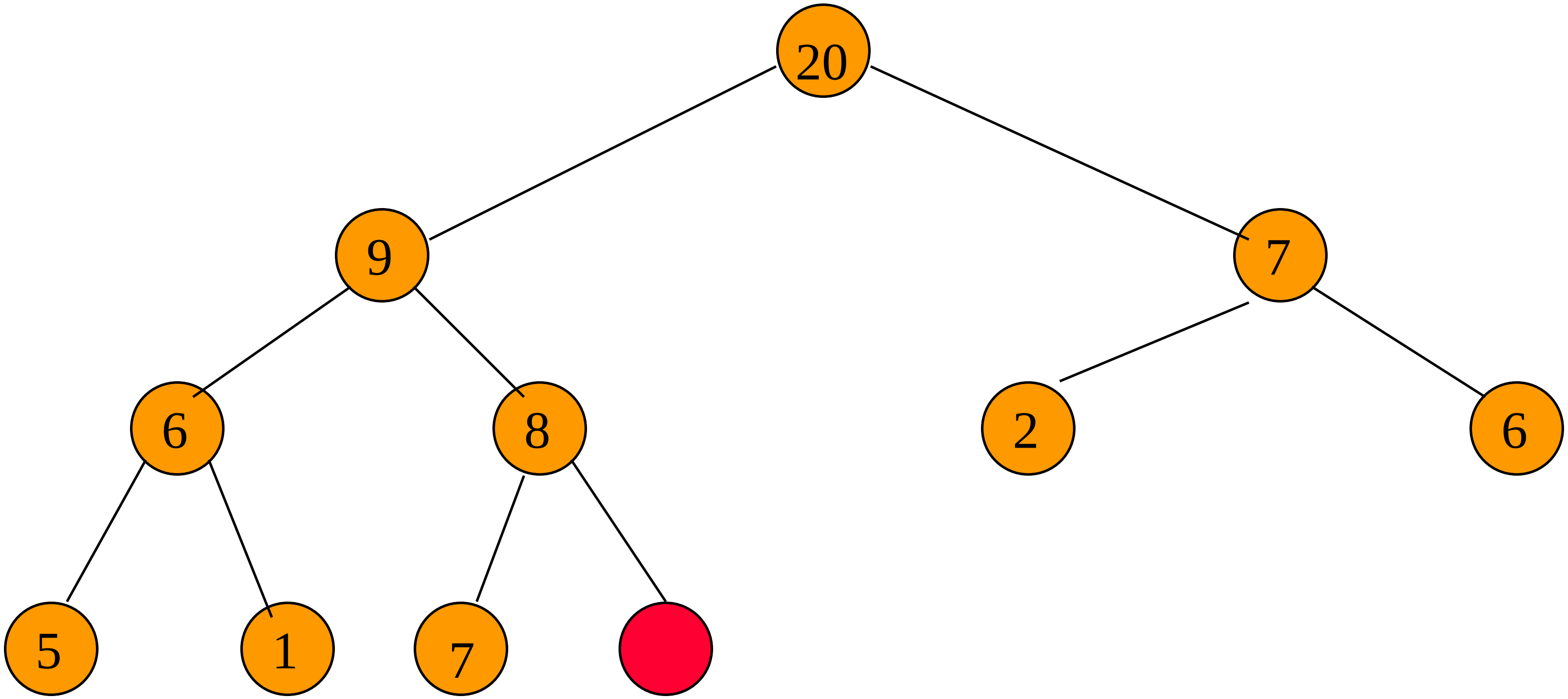
New element is 20.

Putting An Element Into A Max Heap



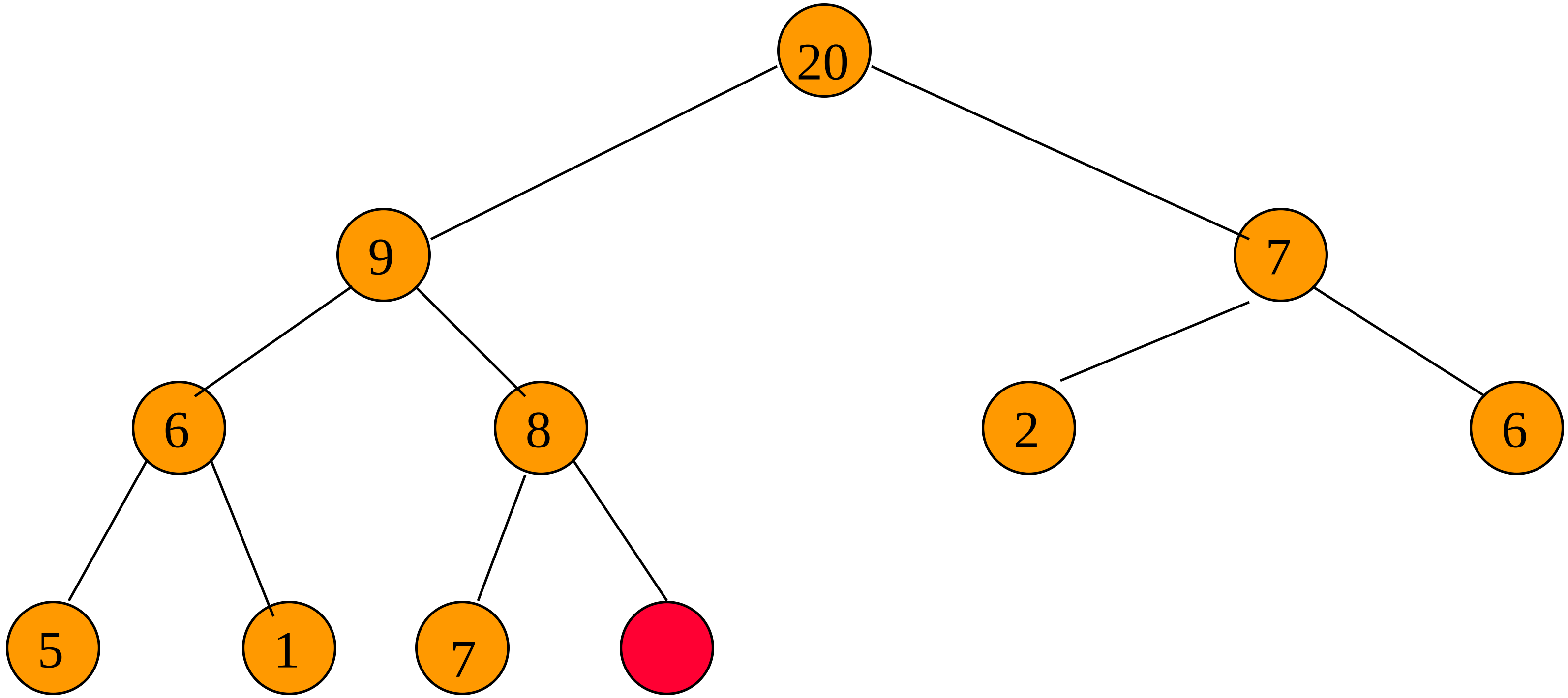
New element is 20.

Putting An Element Into A Max Heap



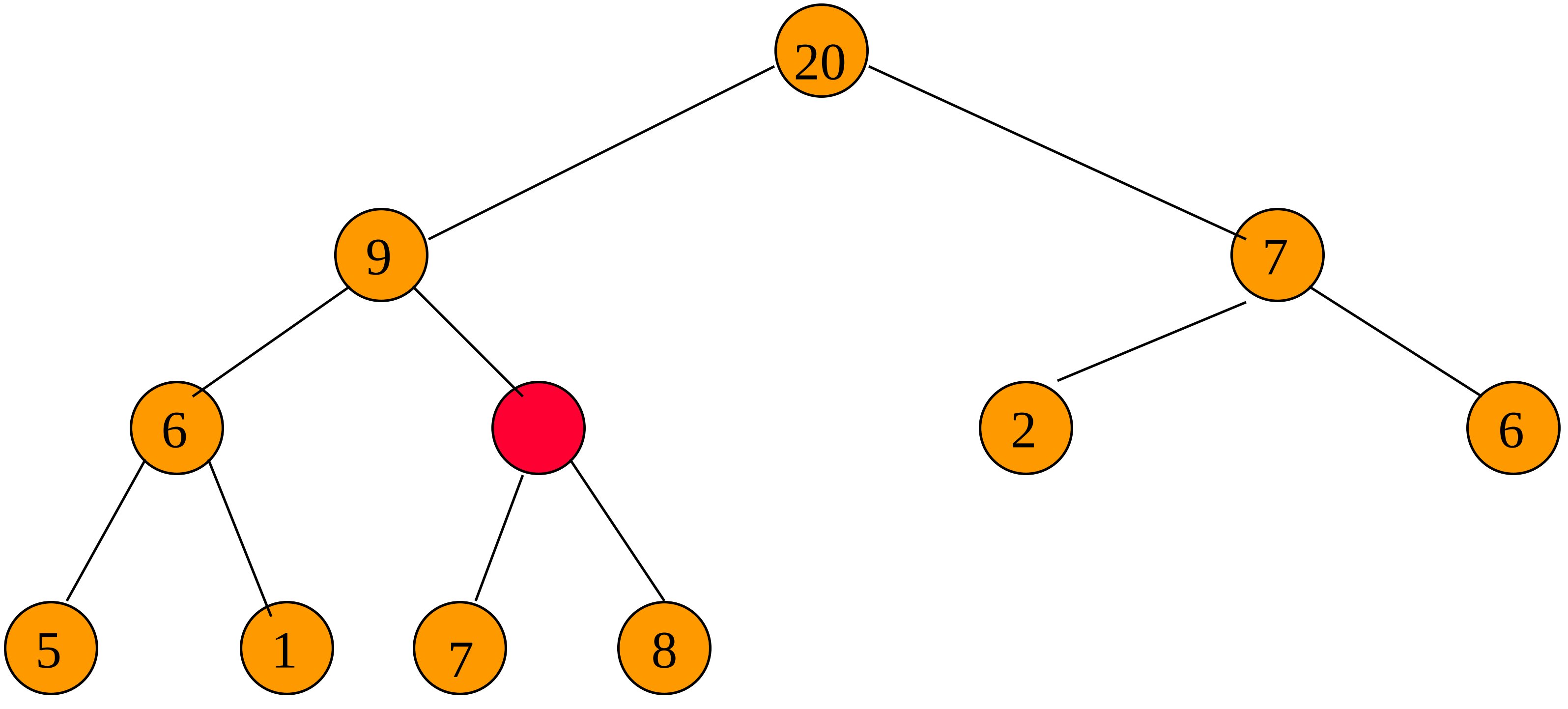
Complete binary tree with 11 nodes.

Putting An Element Into A Max Heap



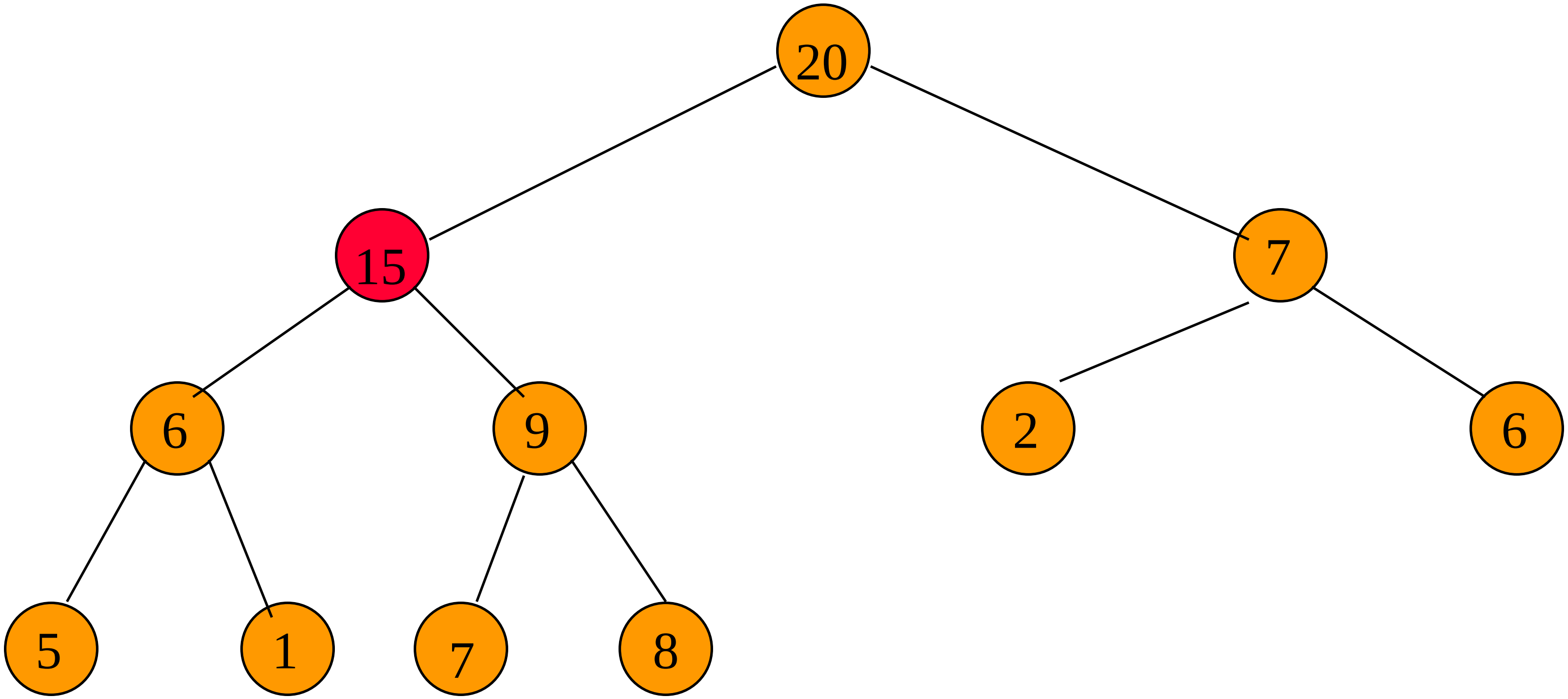
New element is 15.

Putting An Element Into A Max Heap



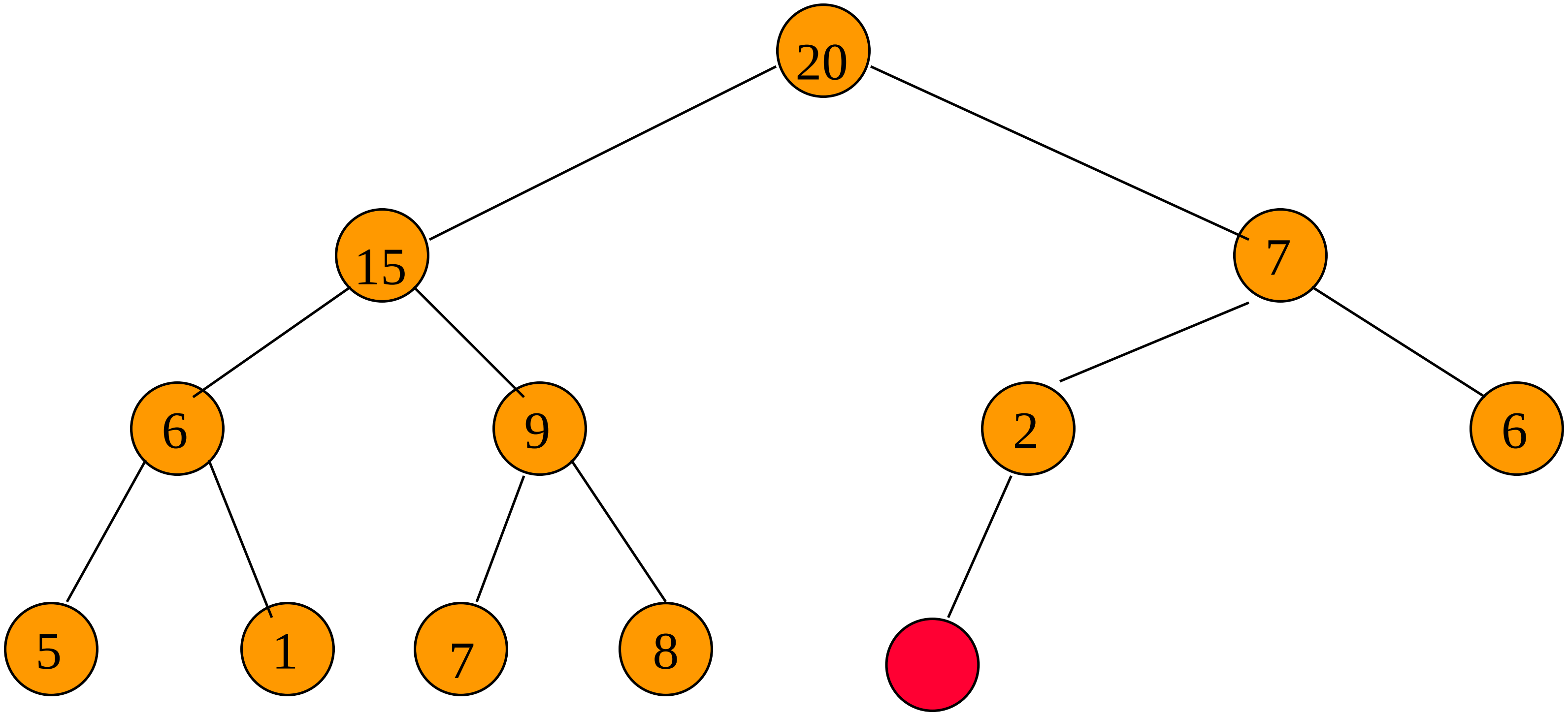
New element is 15.

Putting An Element Into A Max Heap



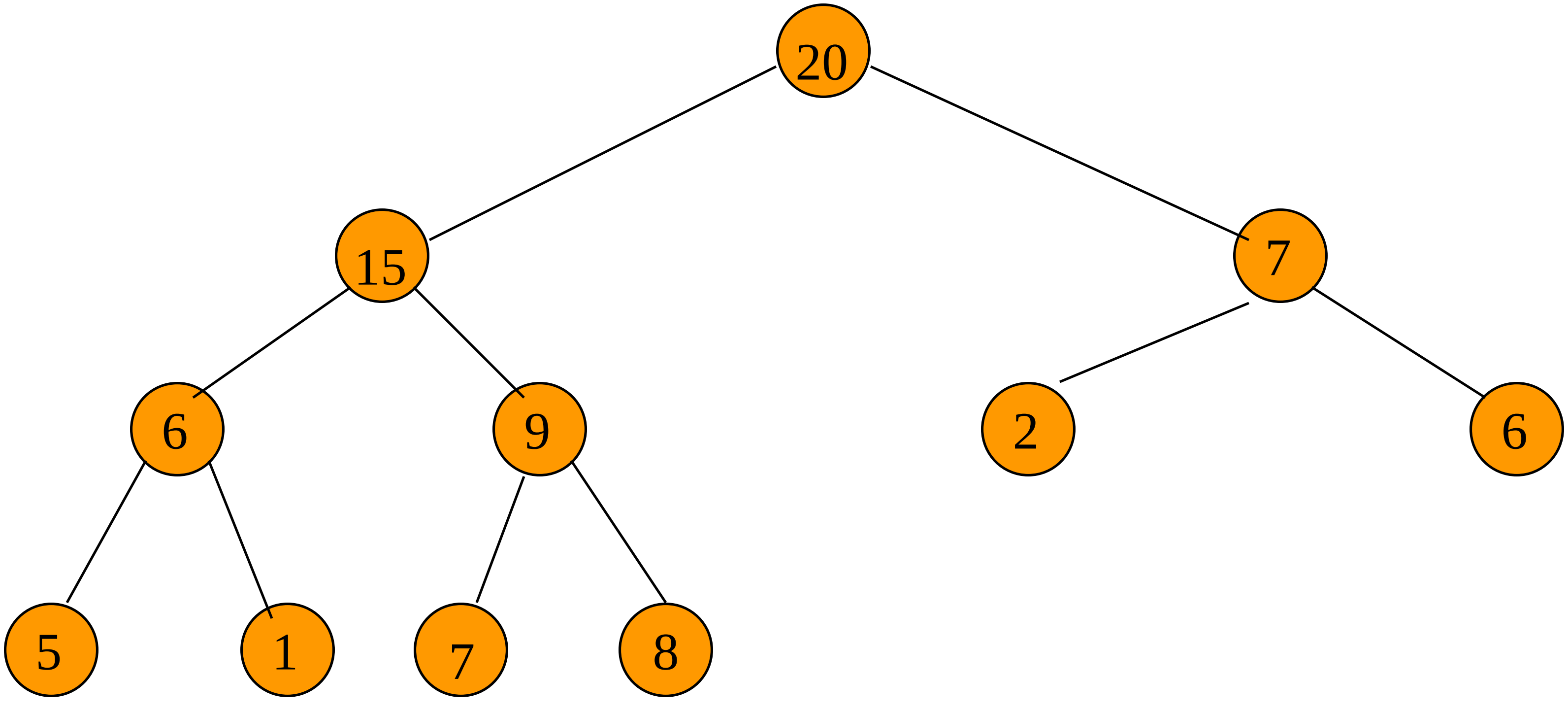
New element is 15.

Complexity Of Put



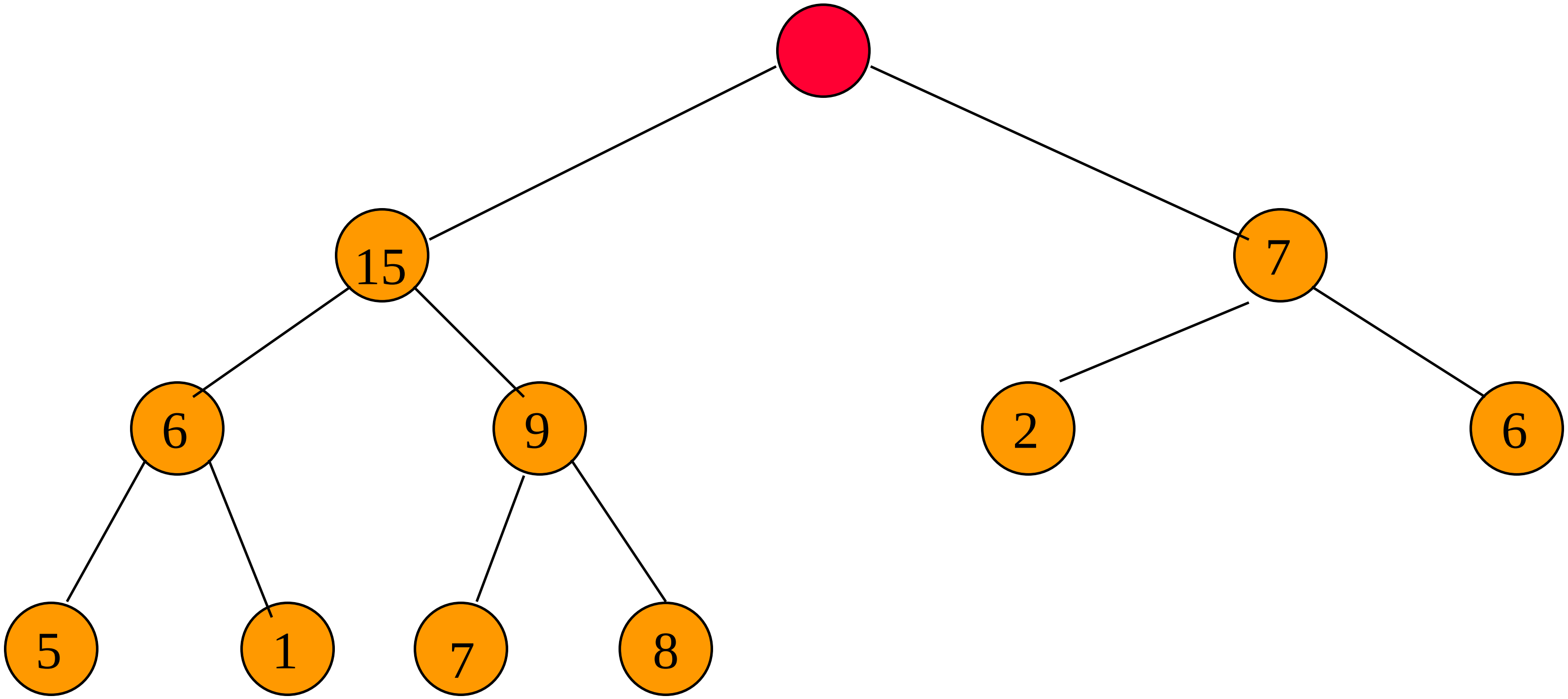
Complexity is O(log n), where n is heap size.

Removing The Max Element



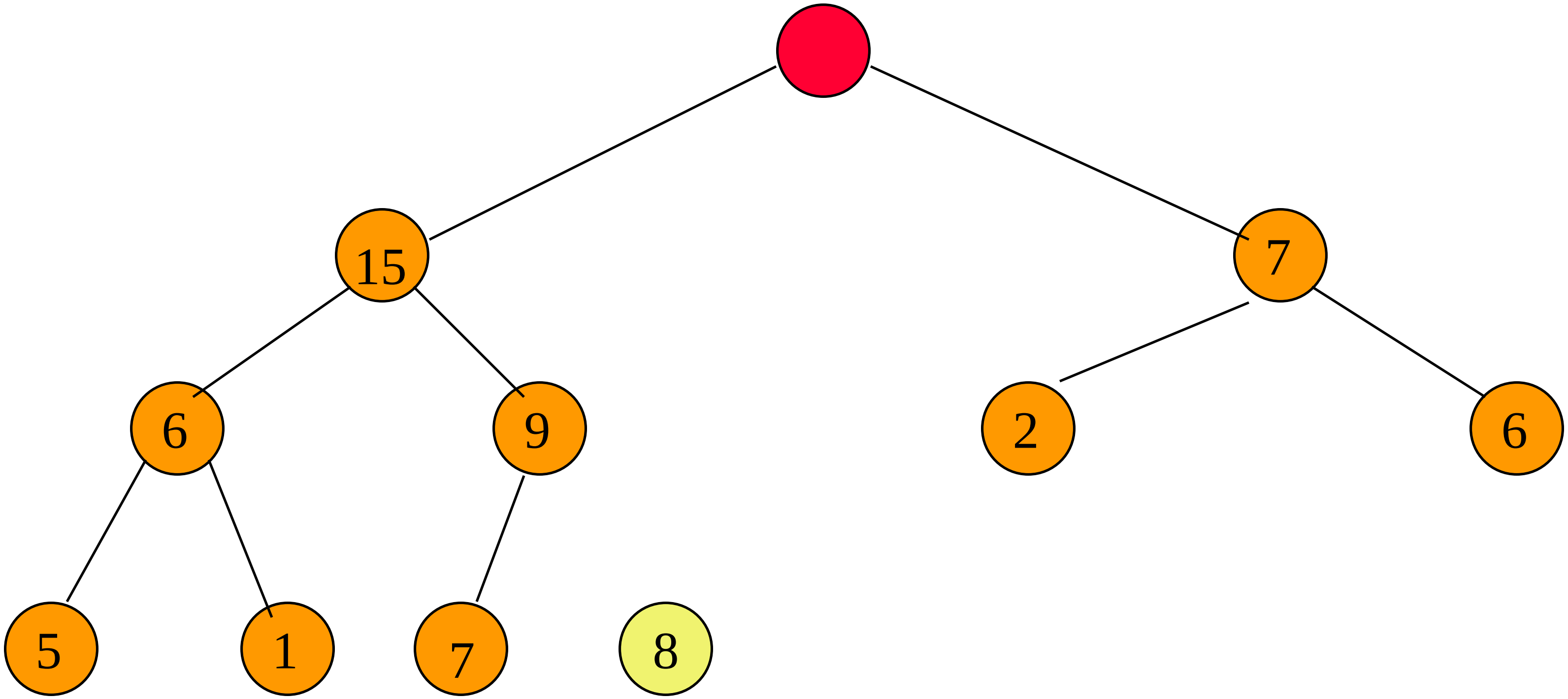
Max element is in the root.

Removing The Max Element



After max element is removed.

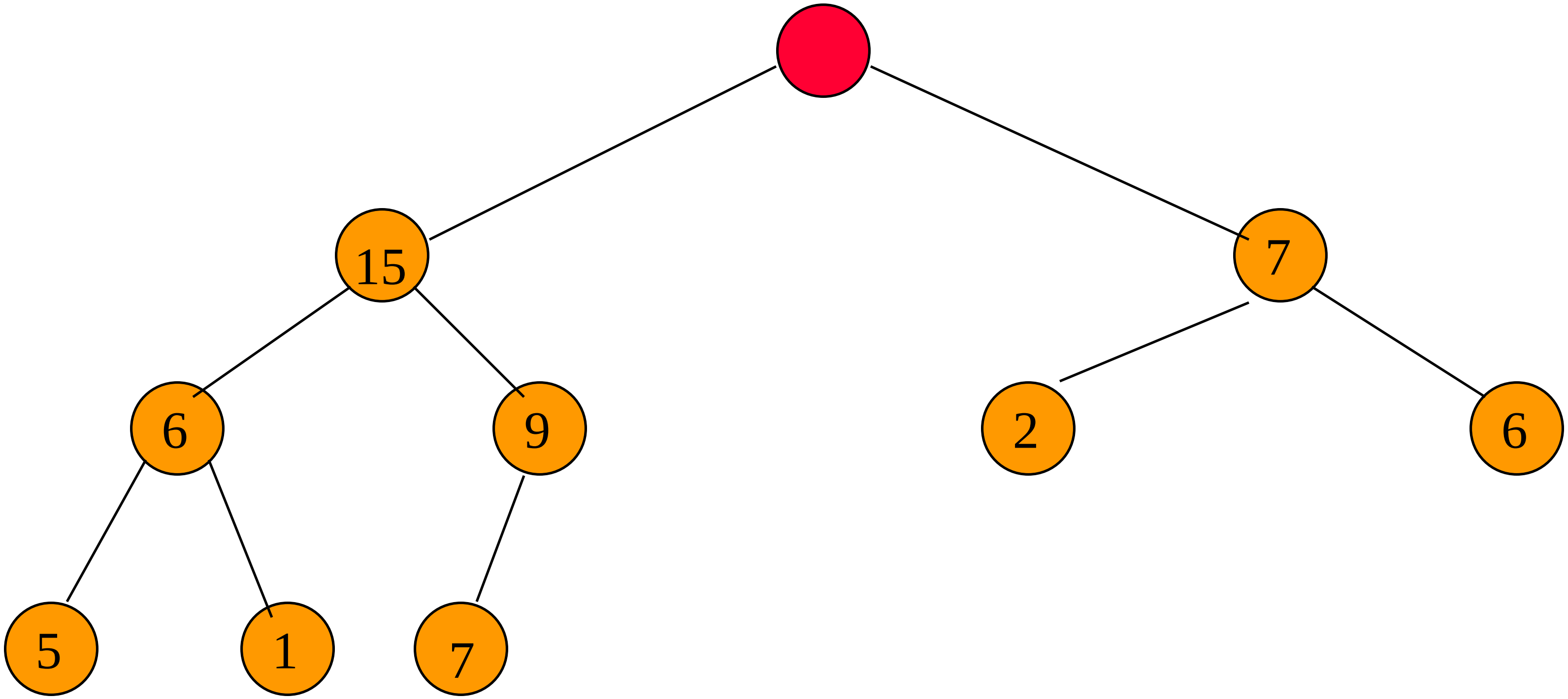
Removing The Max Element



Heap with 10 nodes.

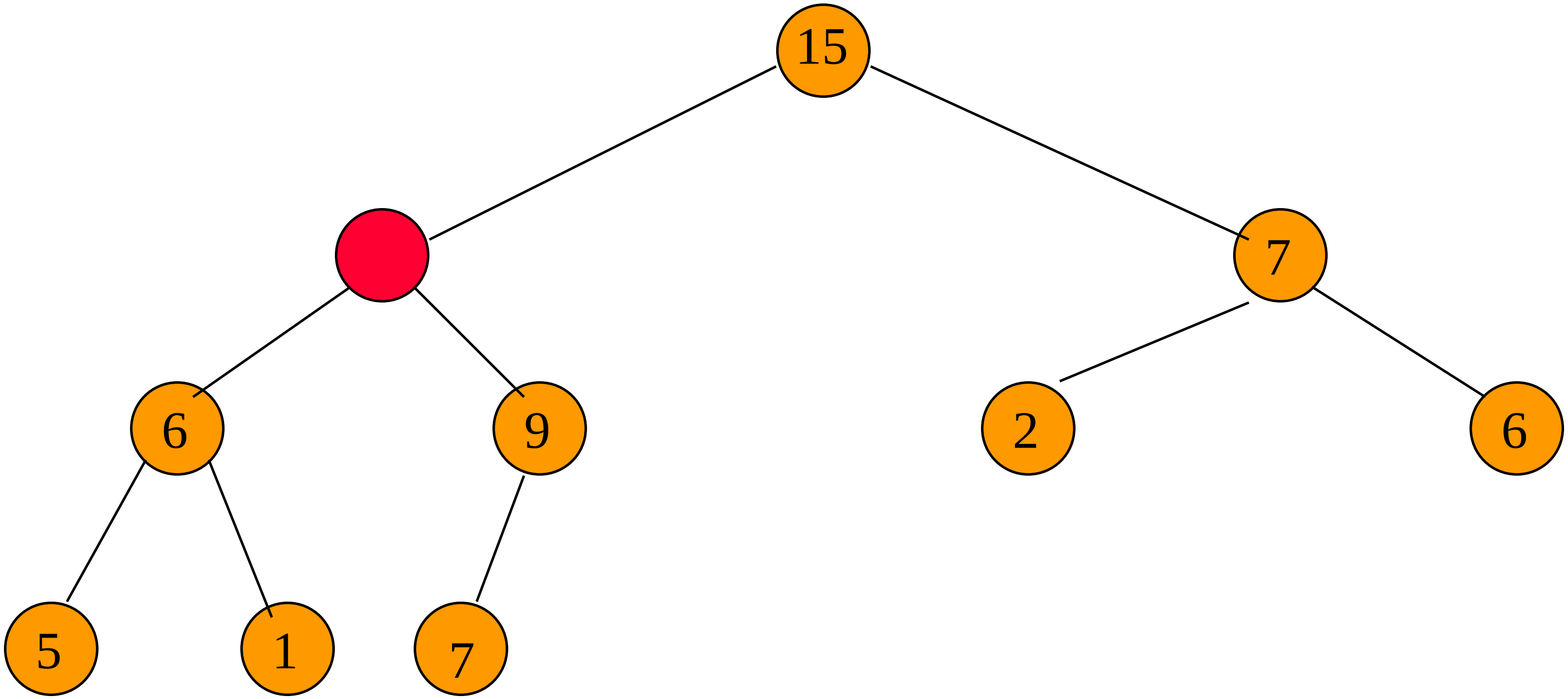
Reinsert 8 into the heap.

Removing The Max Element



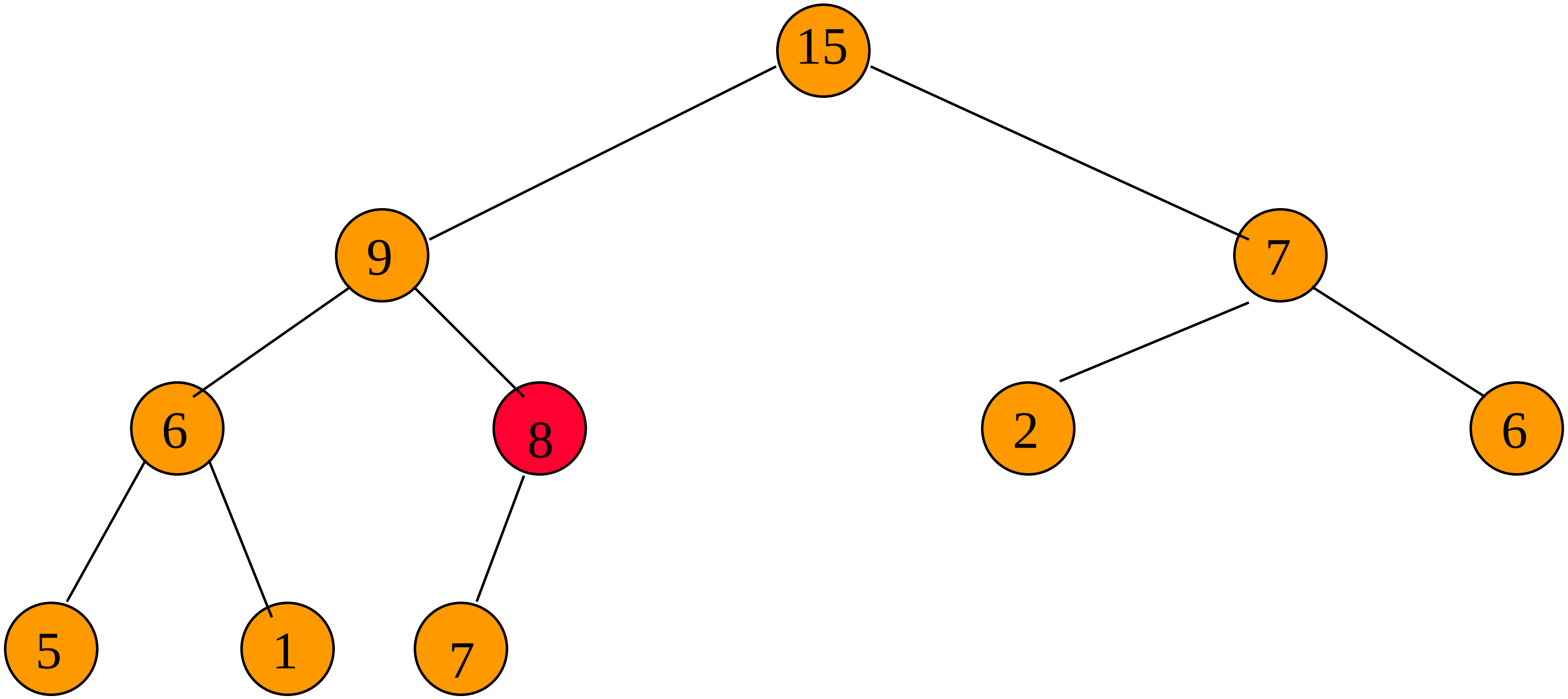
Reinsert 8 into the heap.

Removing The Max Element



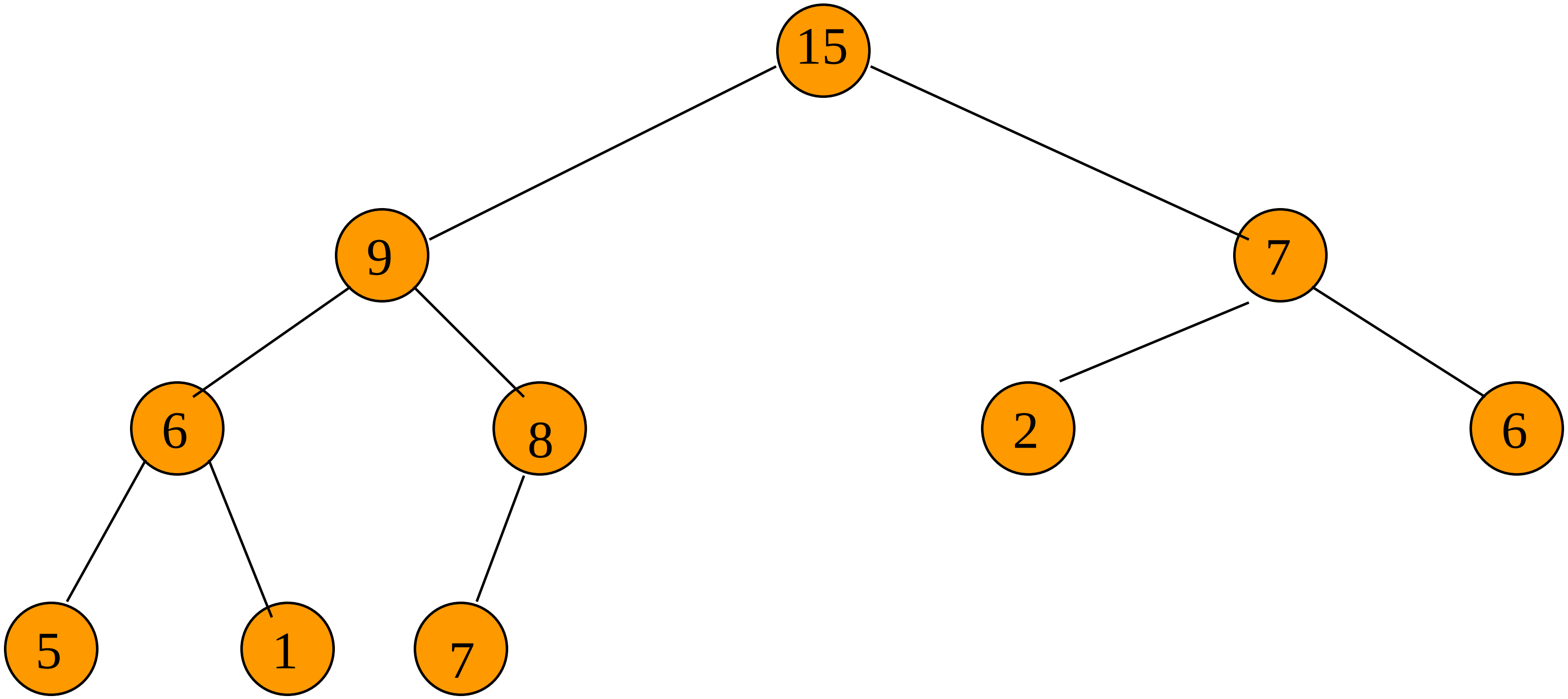
Reinsert 8 into the heap.

Removing The Max Element



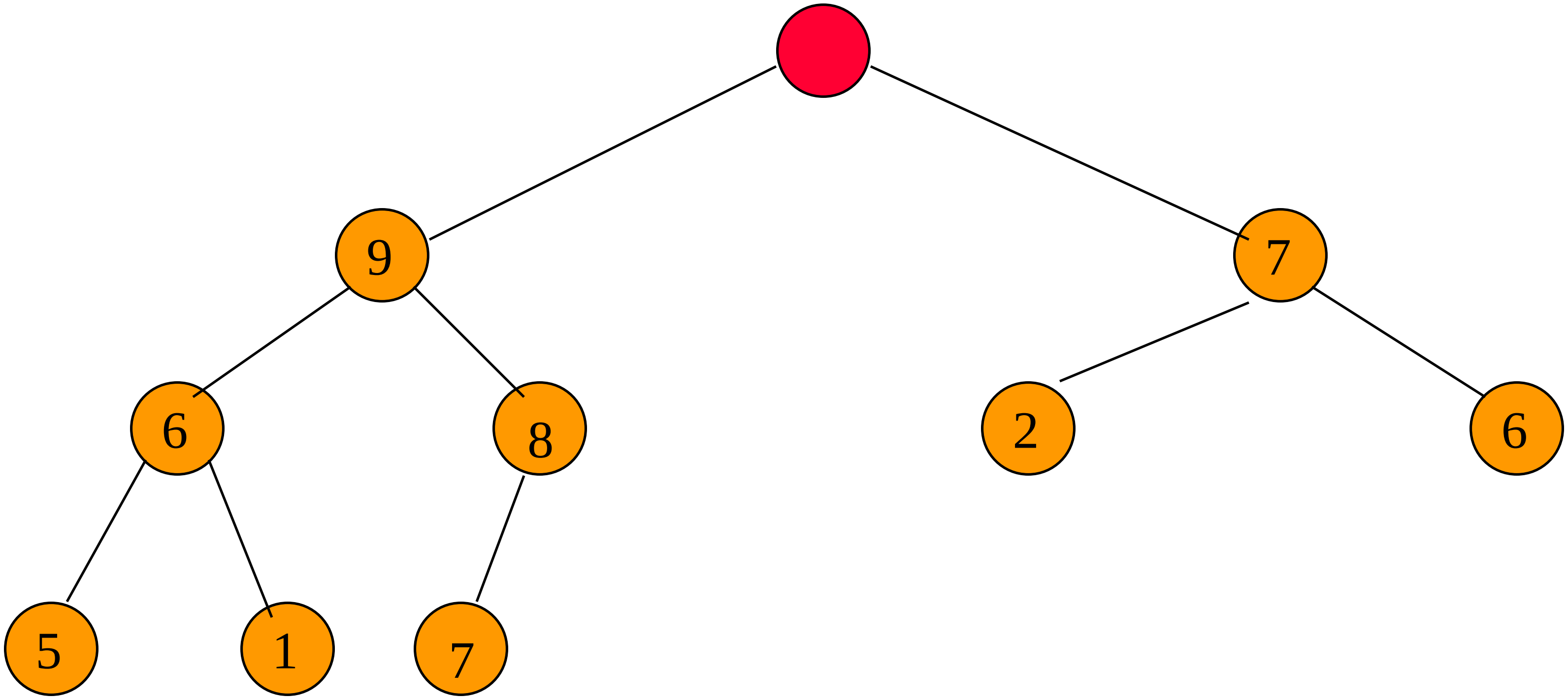
Reinsert 8 into the heap.

Removing The Max Element



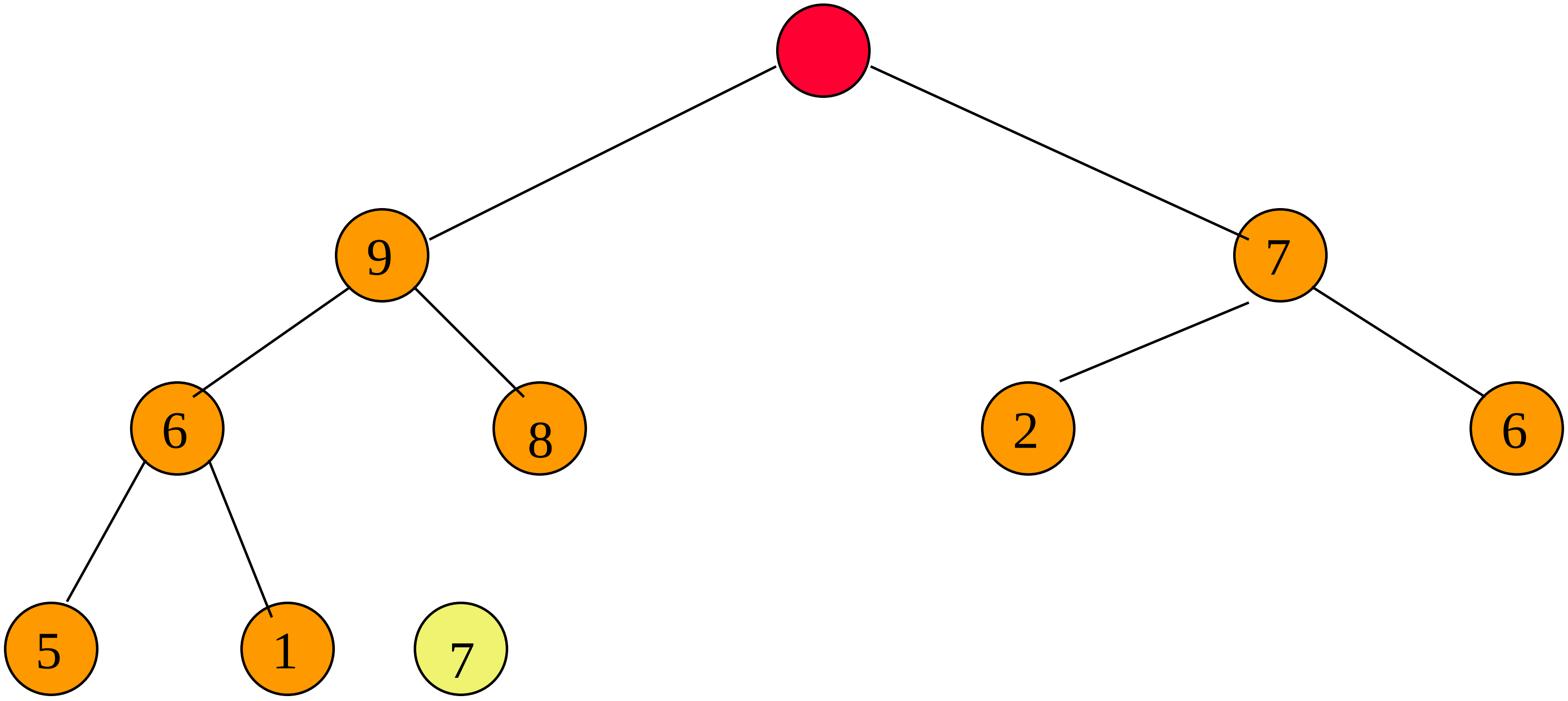
Max element is 15.

Removing The Max Element



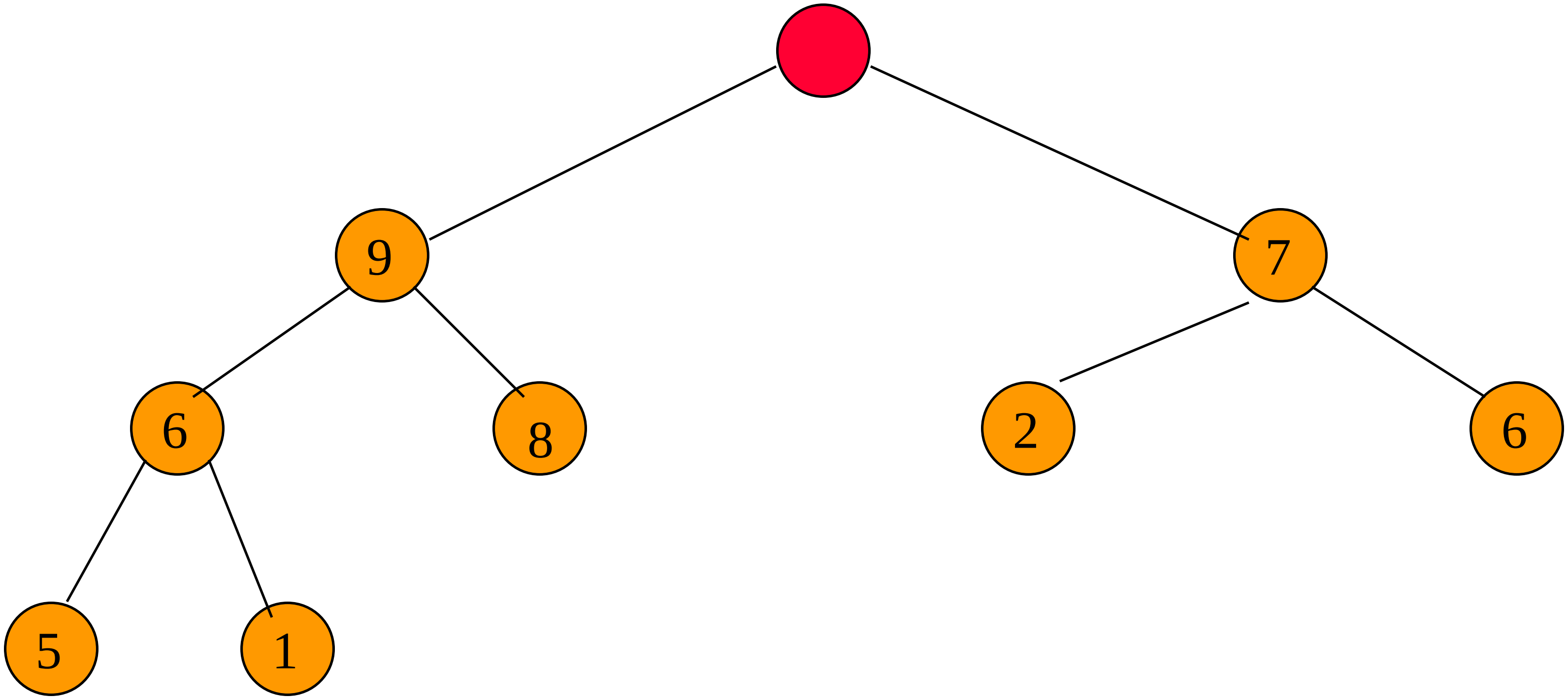
After max element is removed.

Removing The Max Element



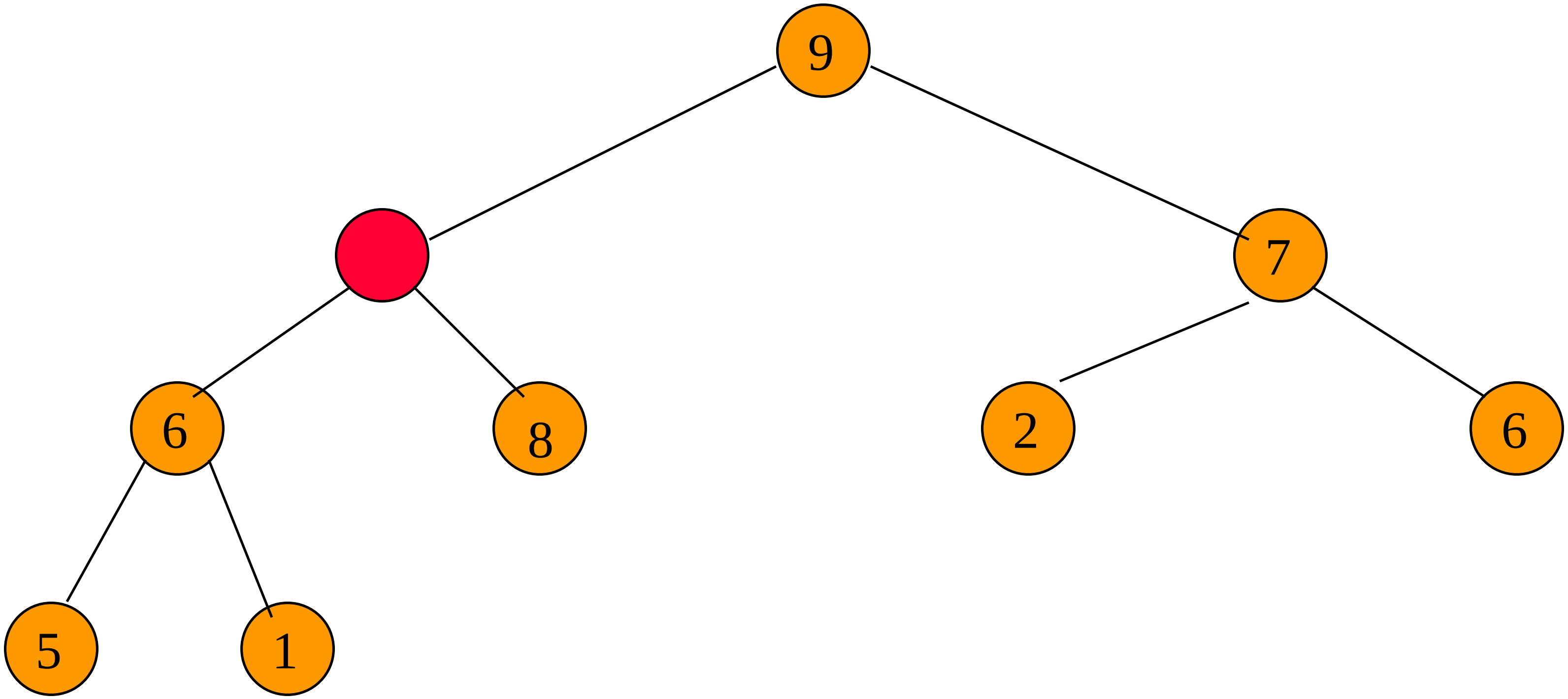
Heap with 9 nodes.

Removing The Max Element



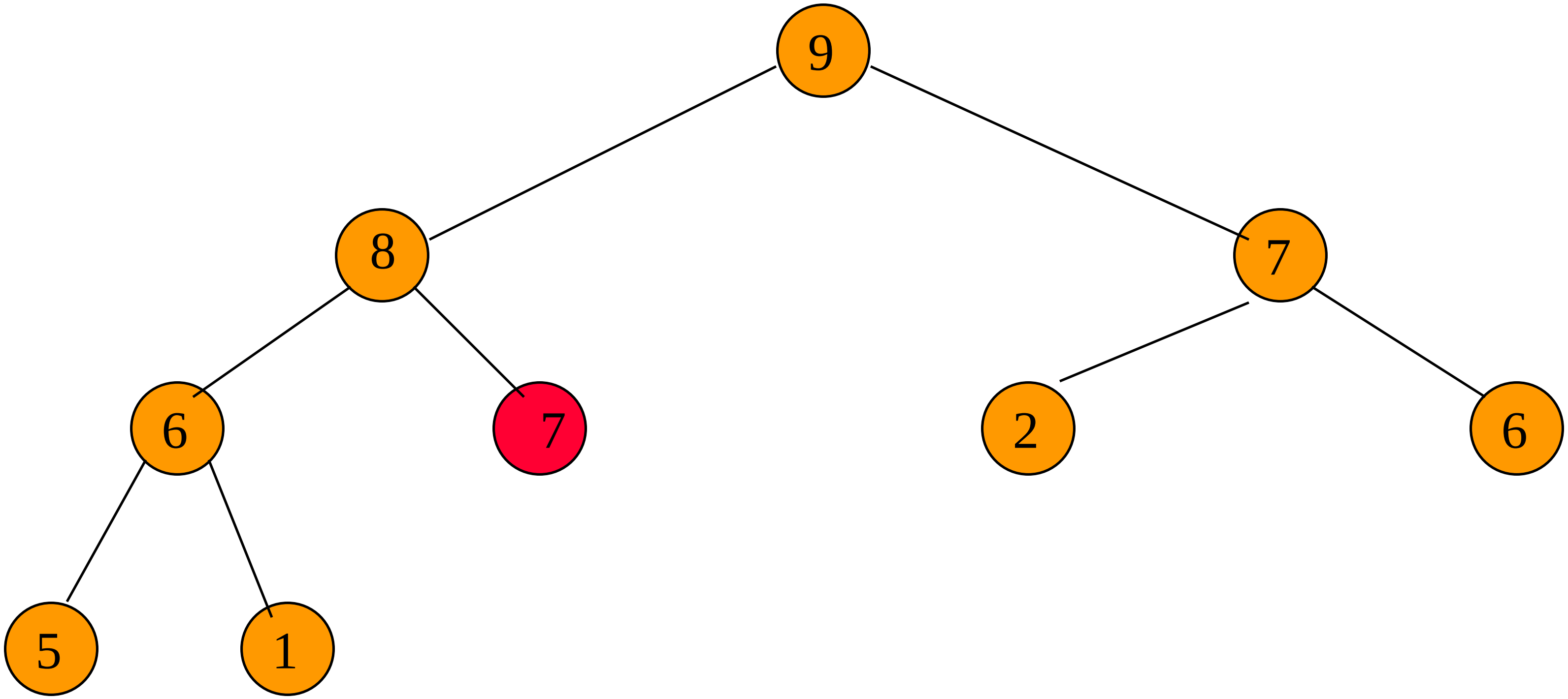
Reinsert 7.

Removing The Max Element



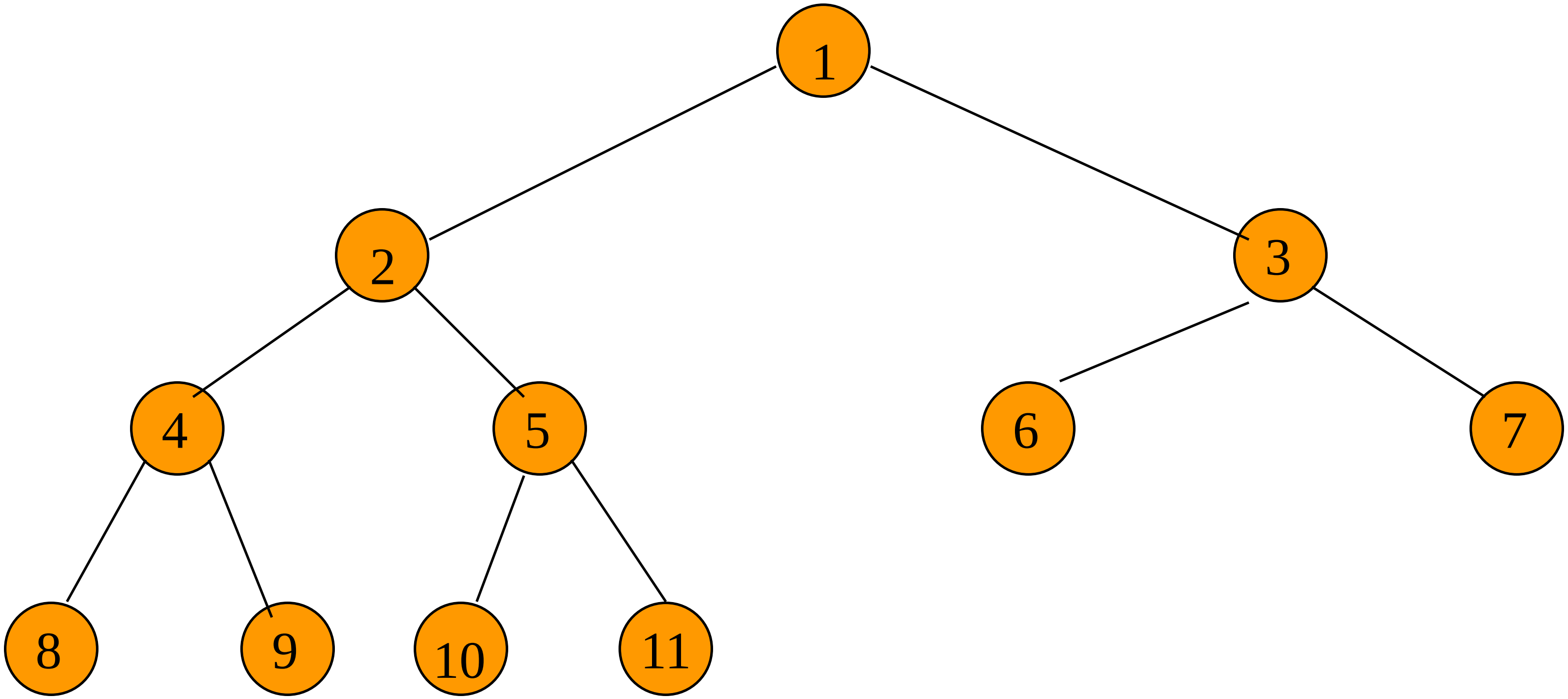
Reinsert 7.

Removing The Max Element



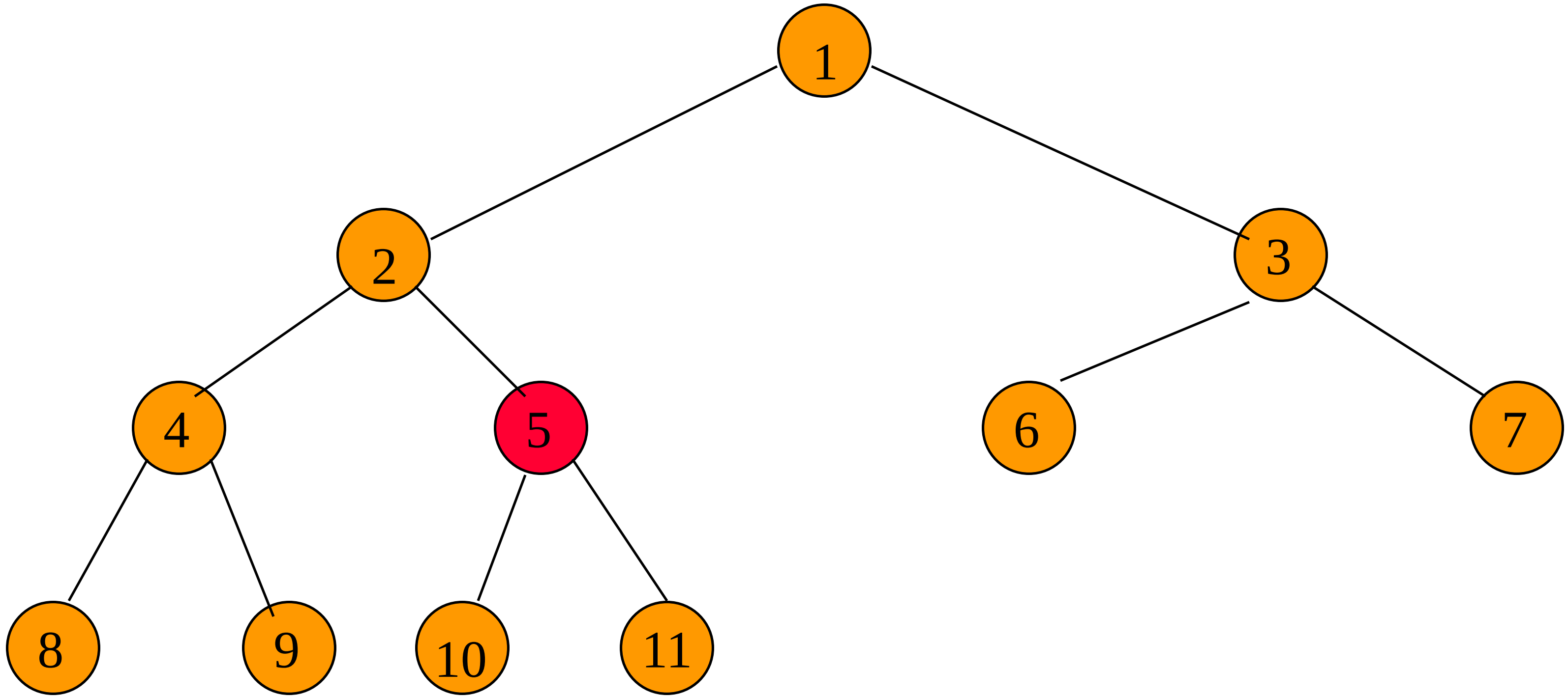
Reinsert 7.

Initializing A Max Heap



input array = [-, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

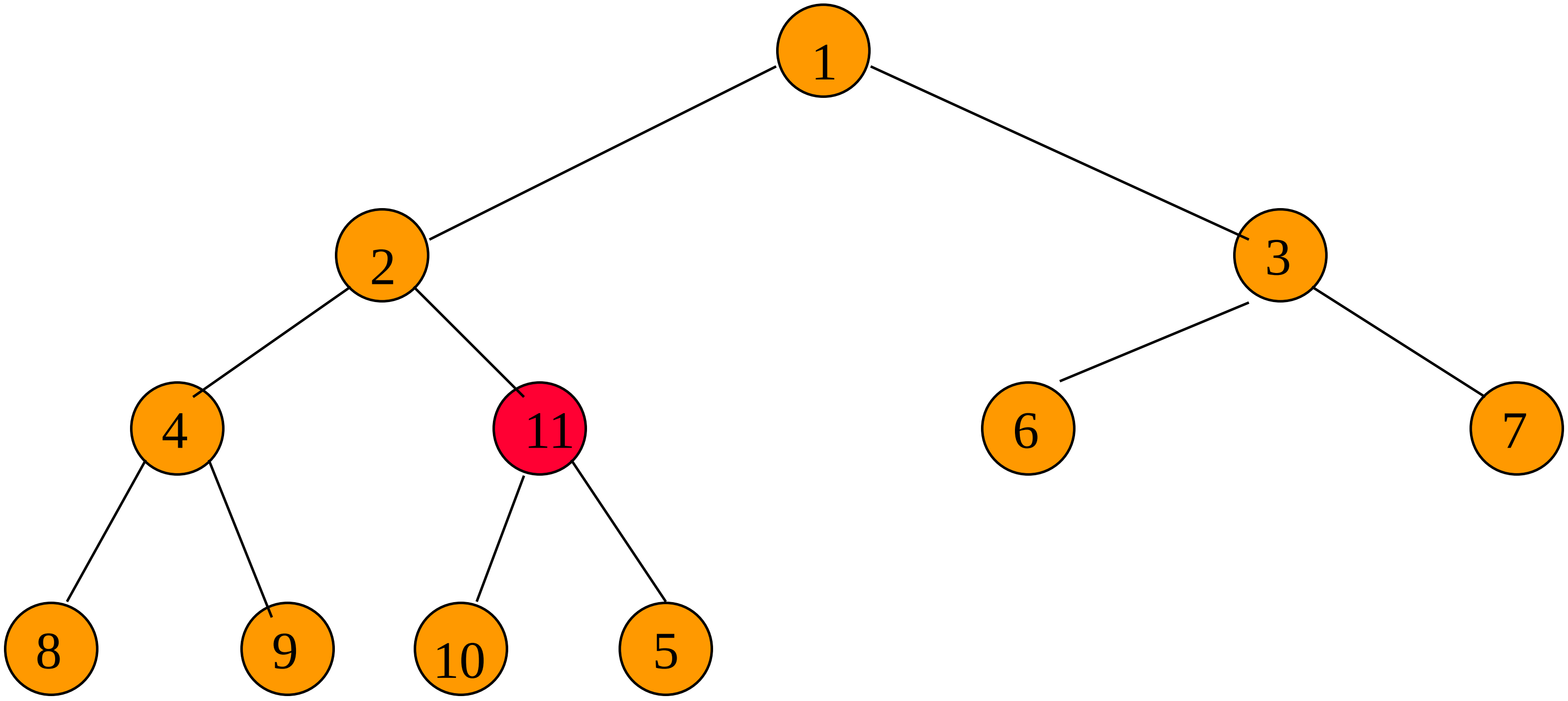
Initializing A Max Heap



Start at rightmost array position that has a child.

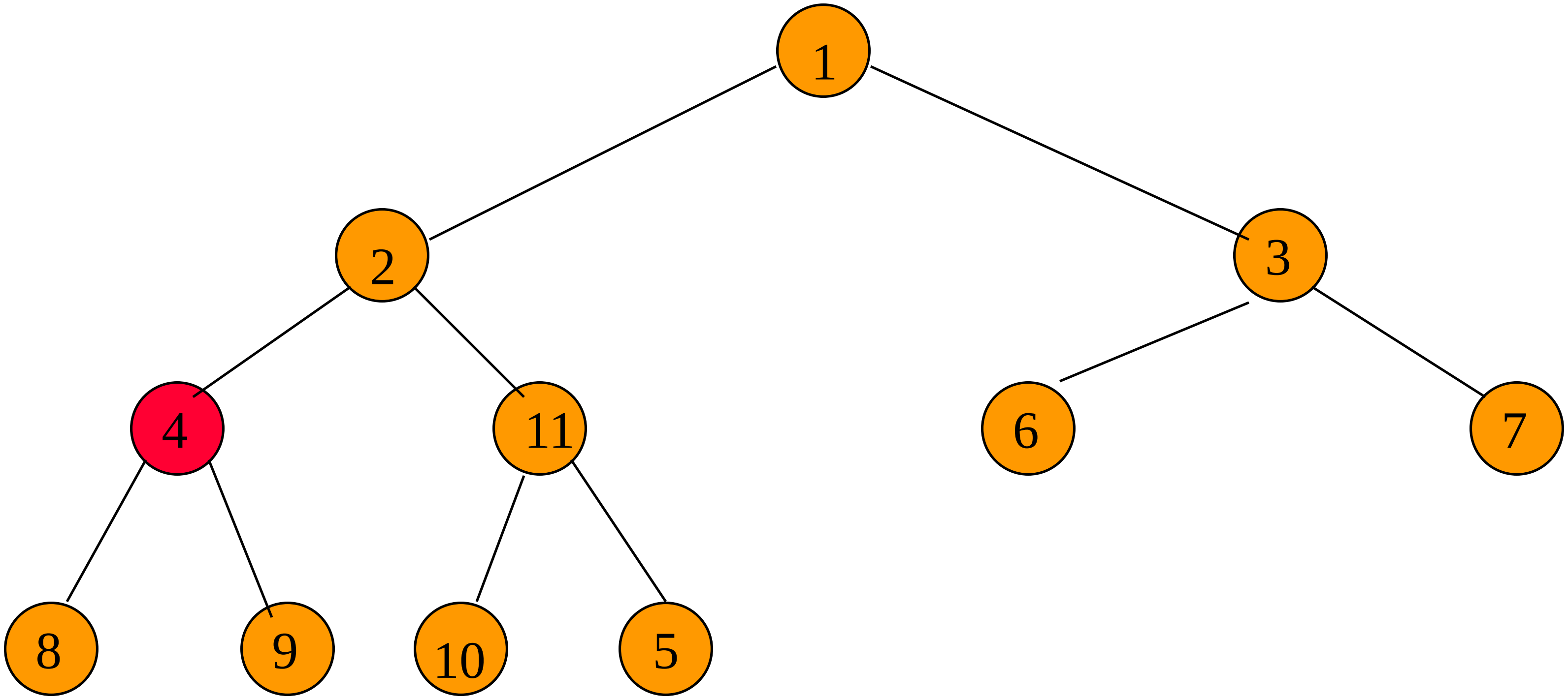
Index is n/2.

Initializing A Max Heap

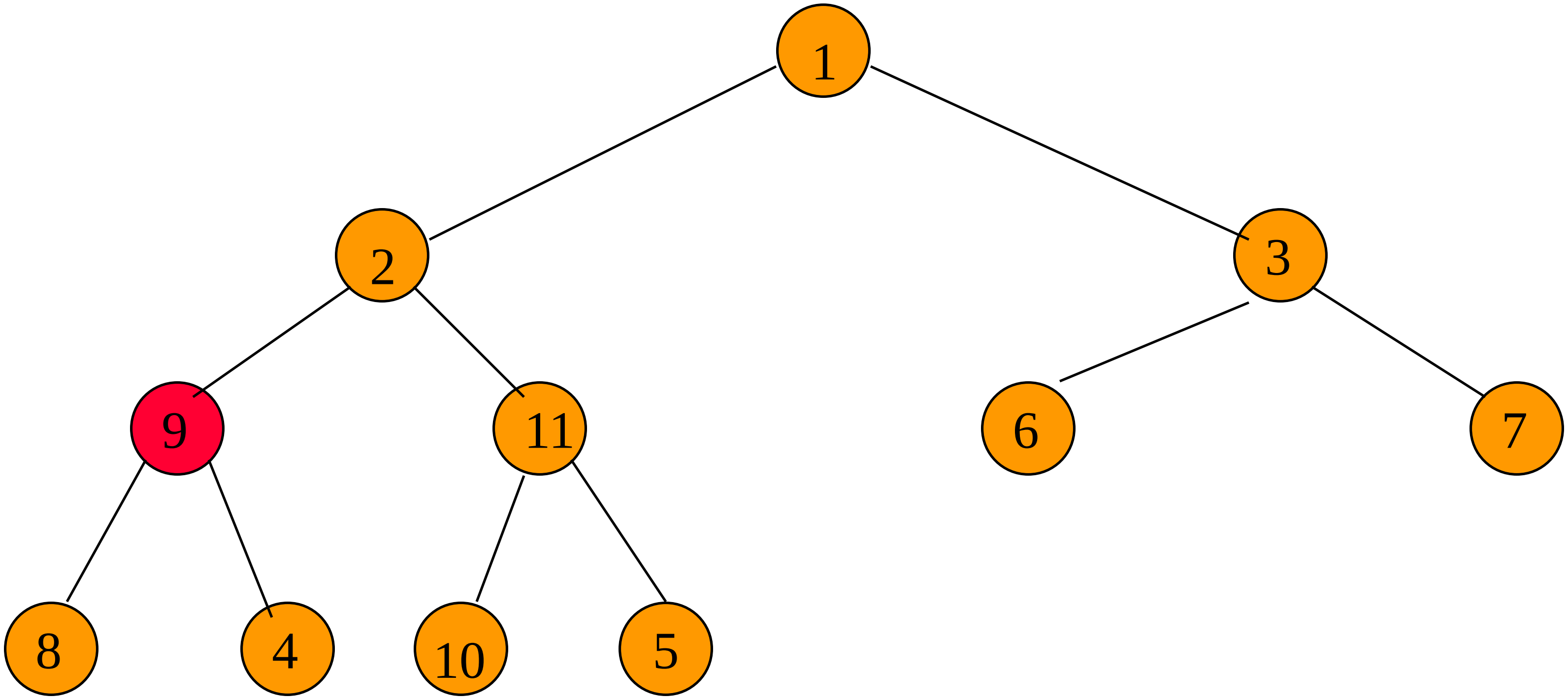


Move to next lower array position.

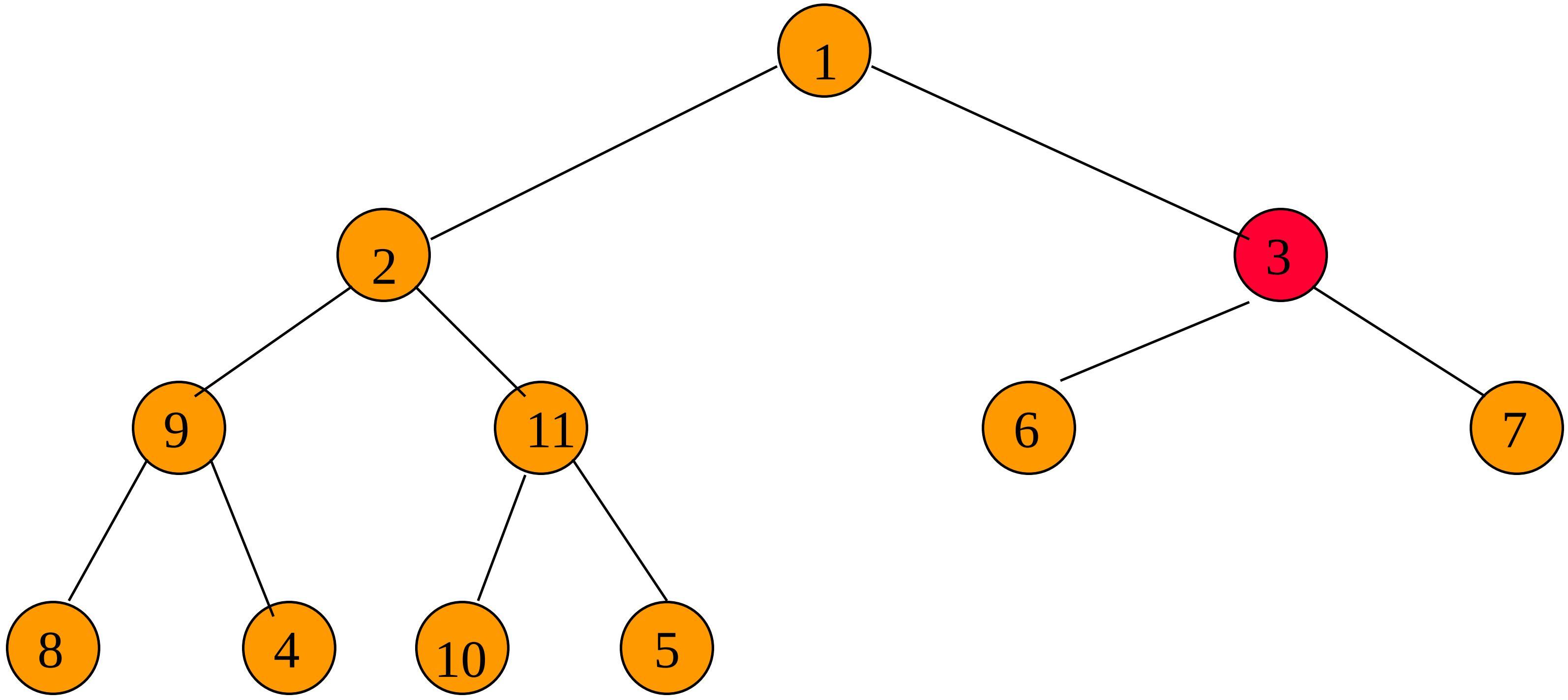
Initializing A Max Heap



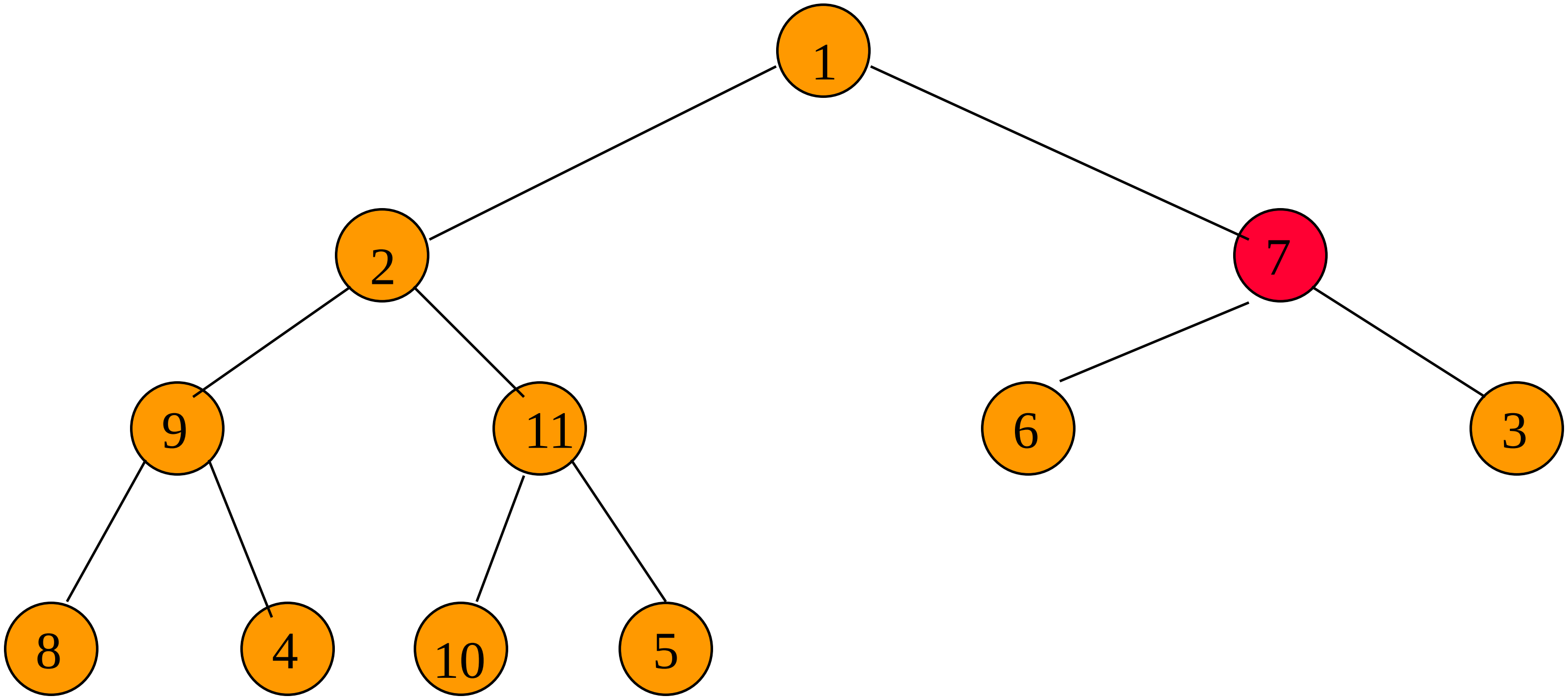
Initializing A Max Heap



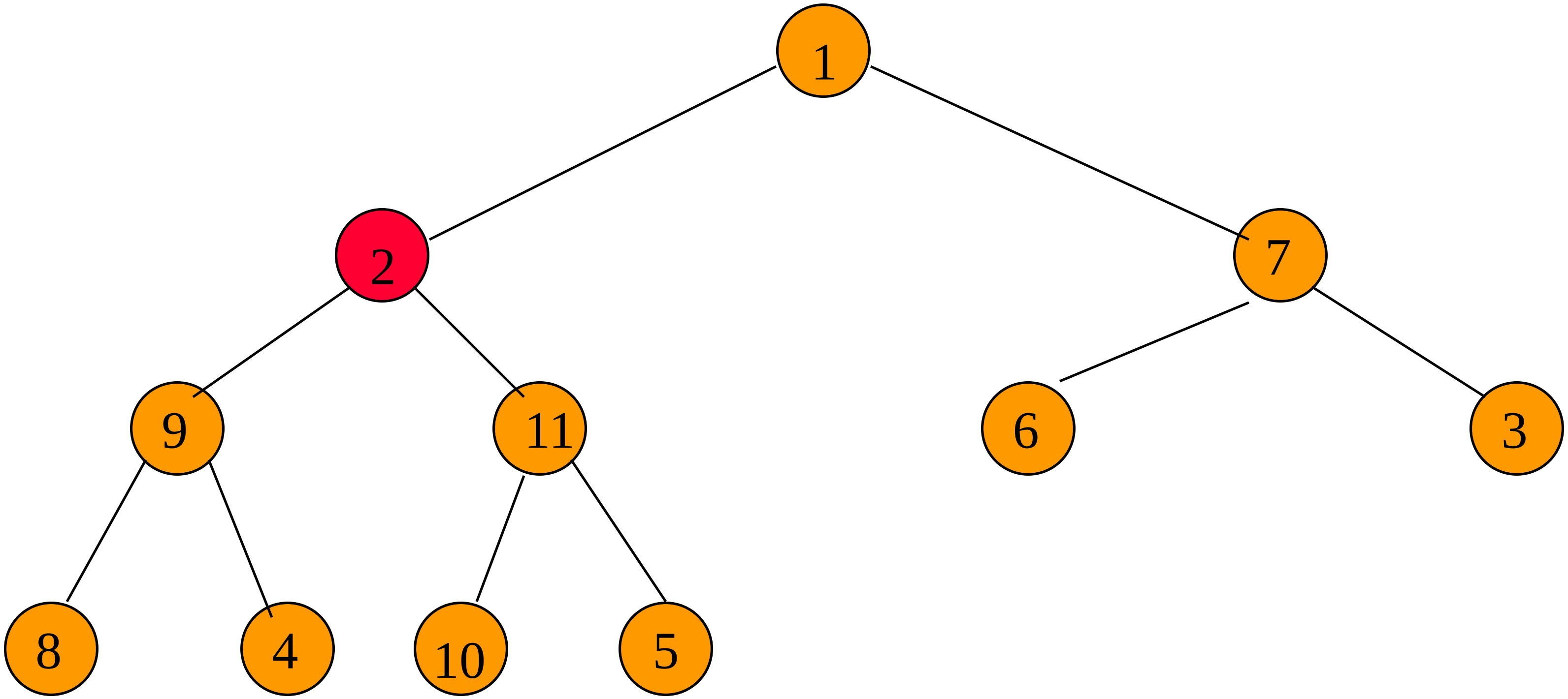
Initializing A Max Heap



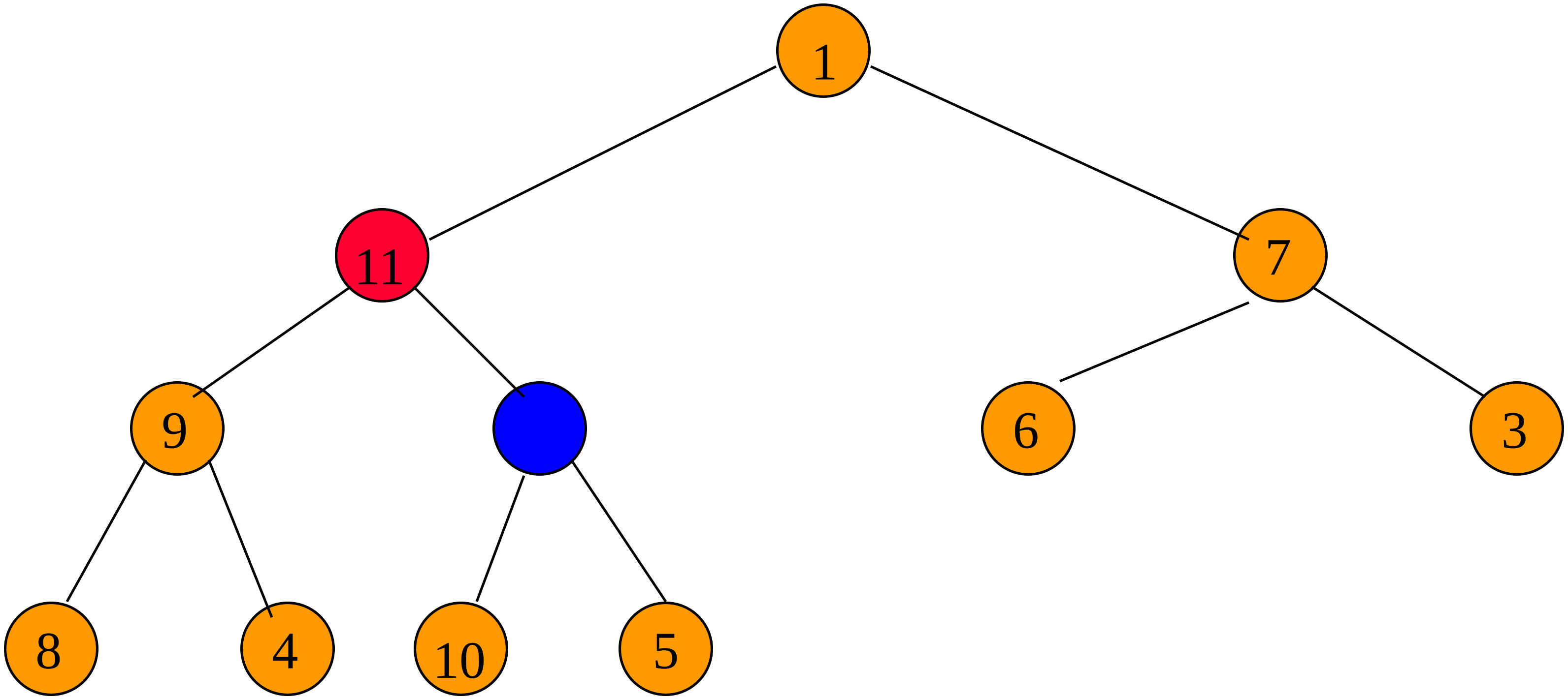
Initializing A Max Heap



Initializing A Max Heap

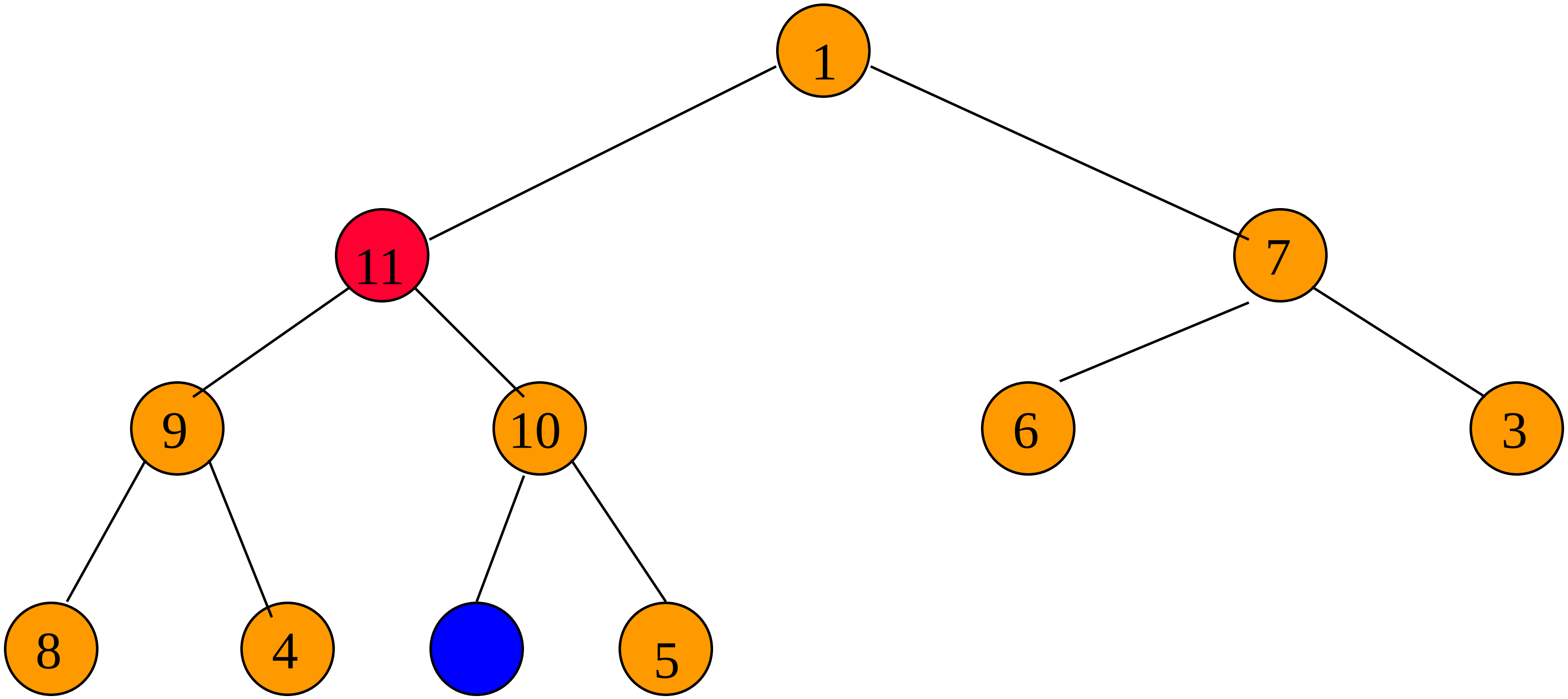


Initializing A Max Heap



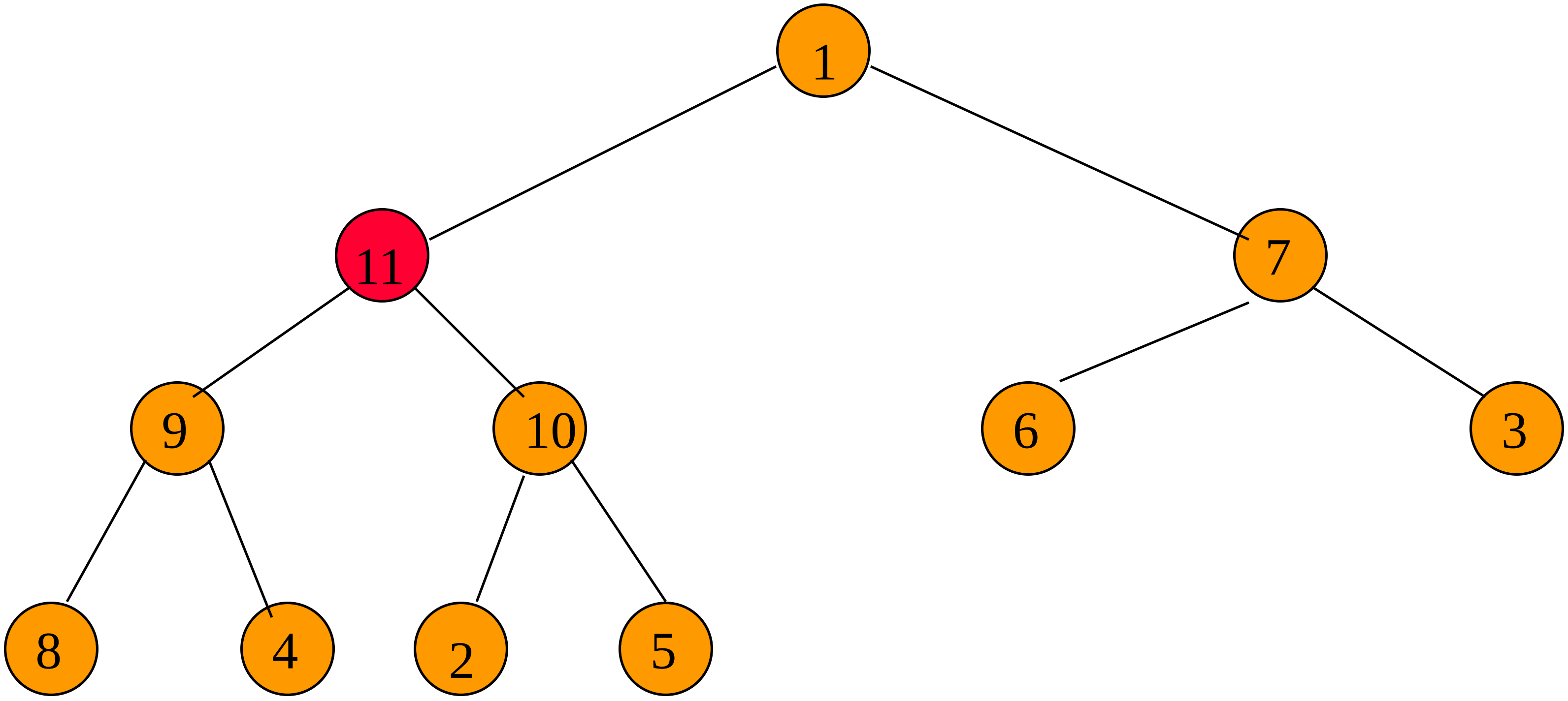
Find a home for 2.

Initializing A Max Heap



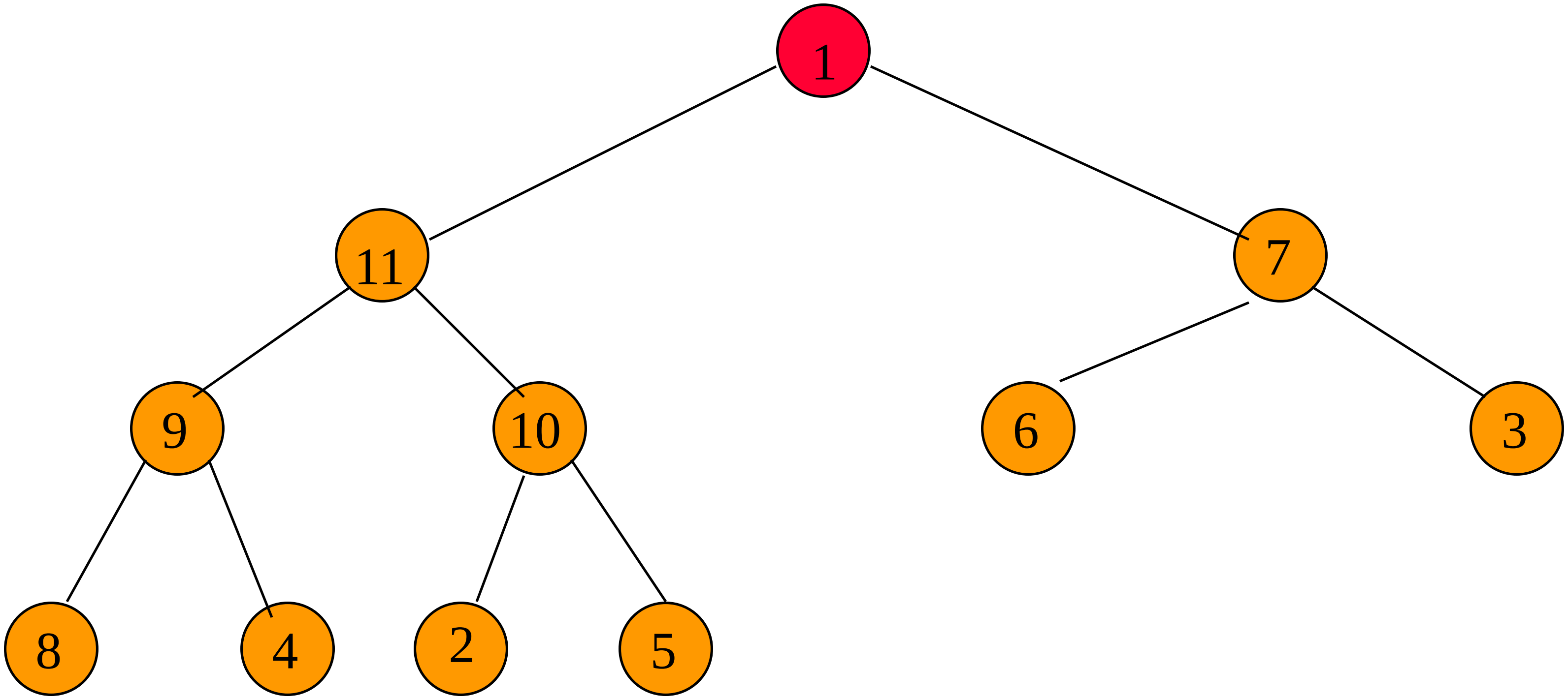
Find a home for 2.

Initializing A Max Heap



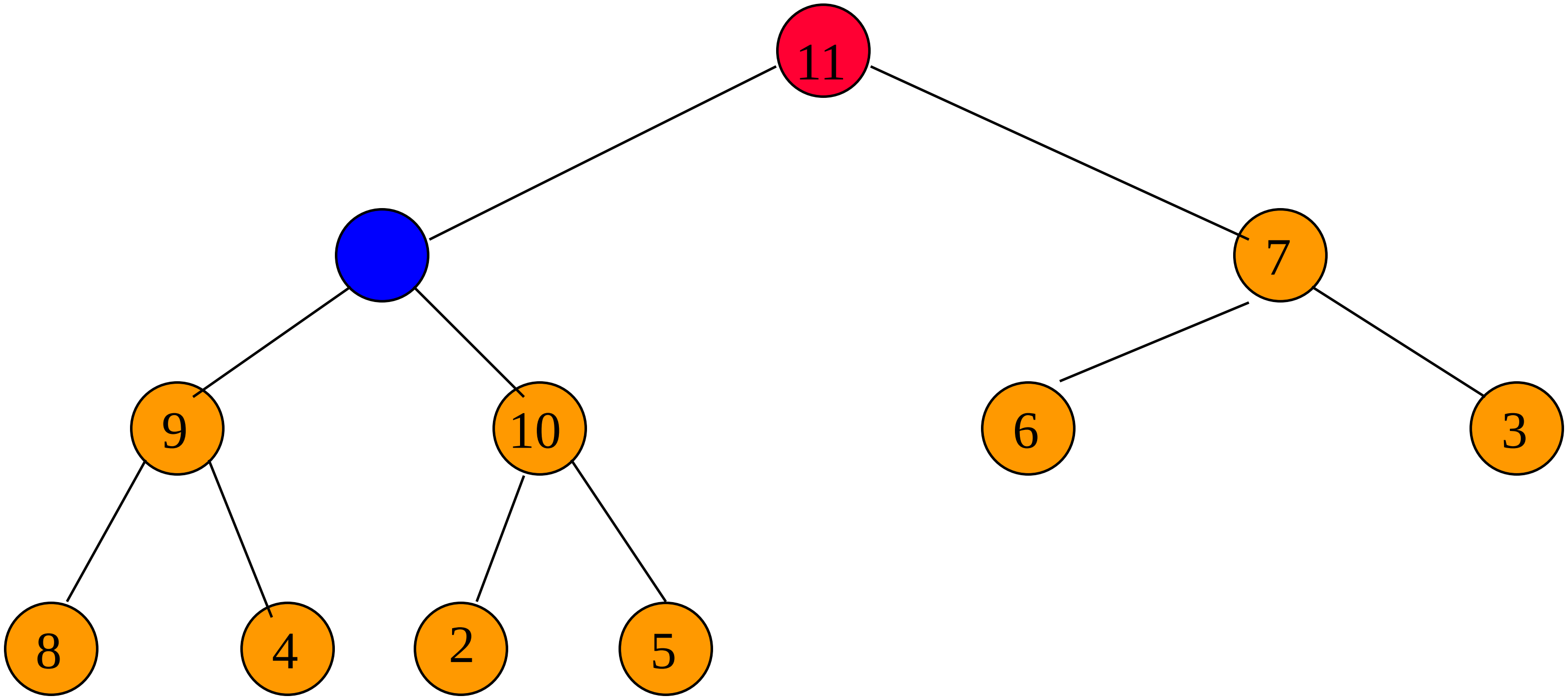
Done, move to next lower array position.

Initializing A Max Heap



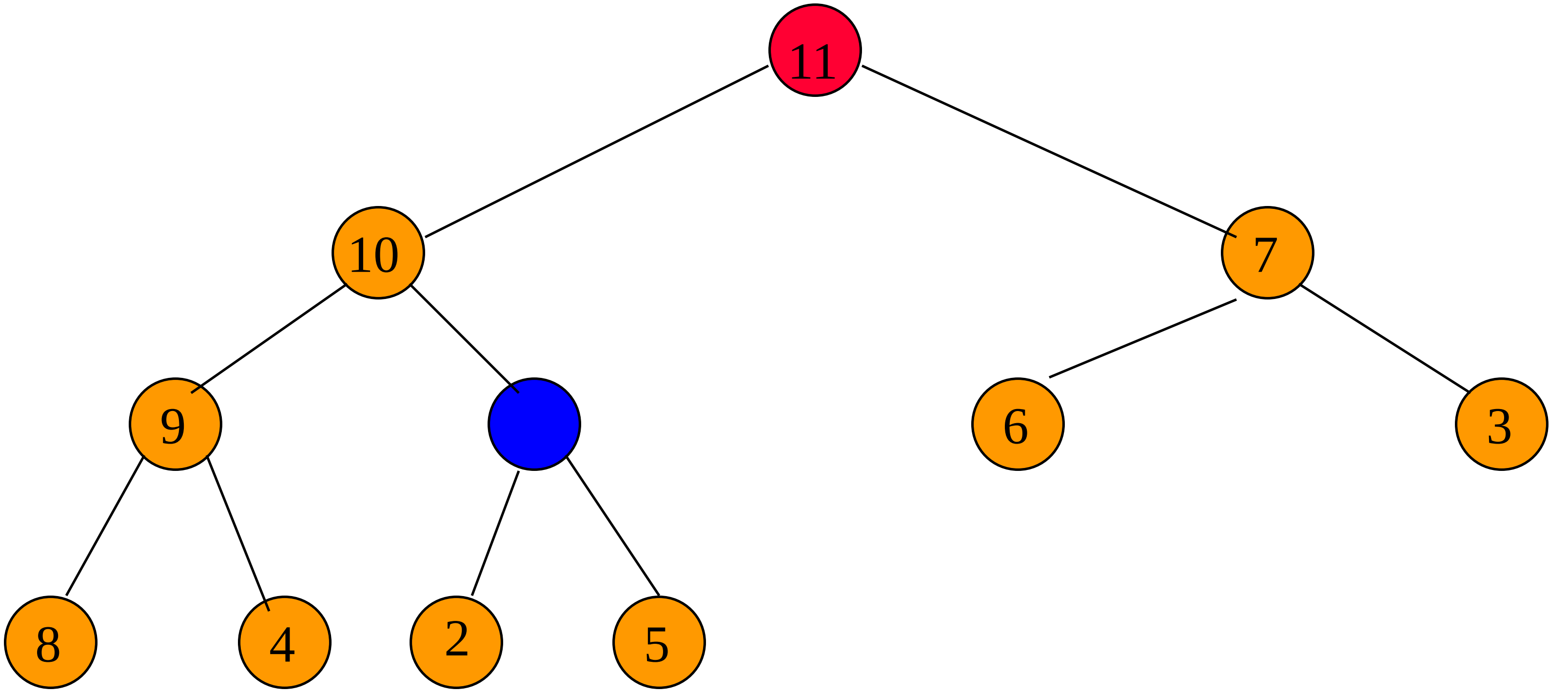
Find home for 1.

Initializing A Max Heap



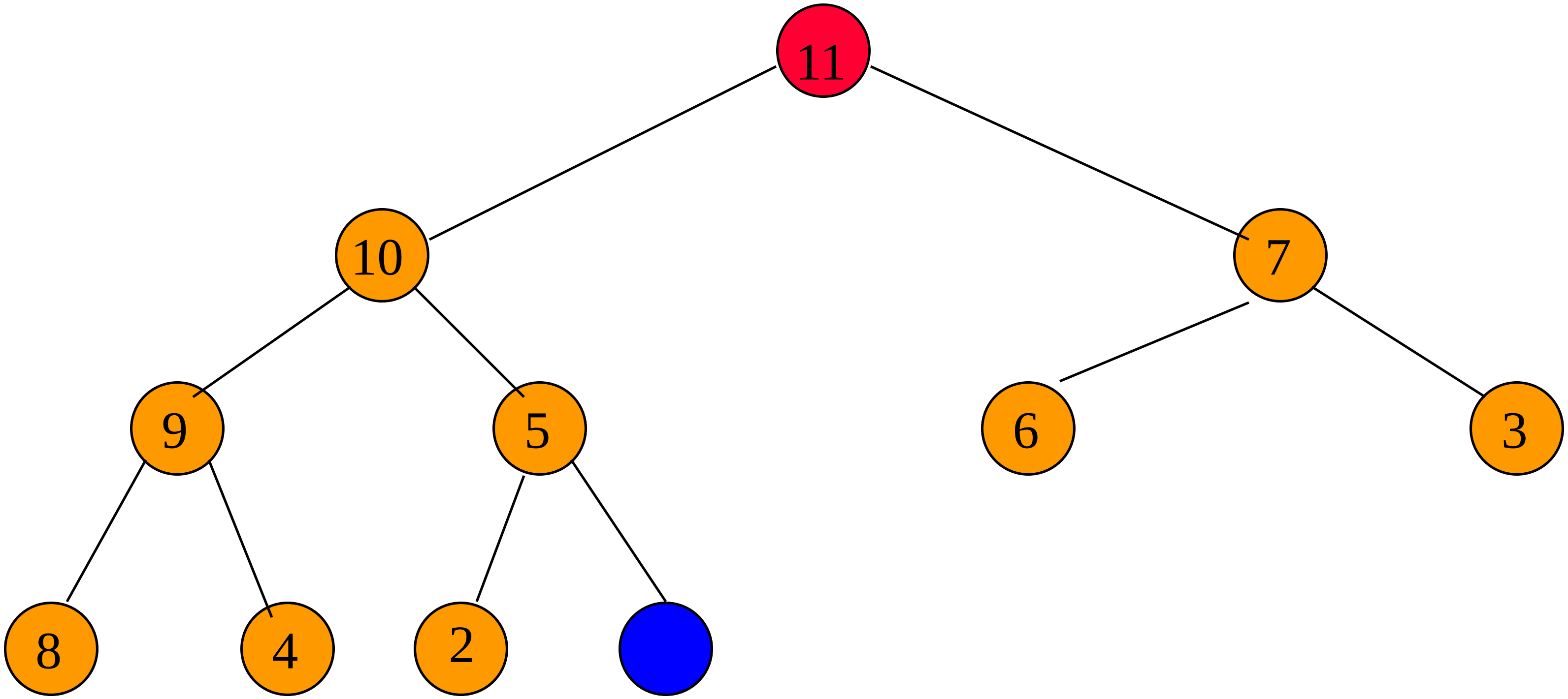
Find home for 1.

Initializing A Max Heap



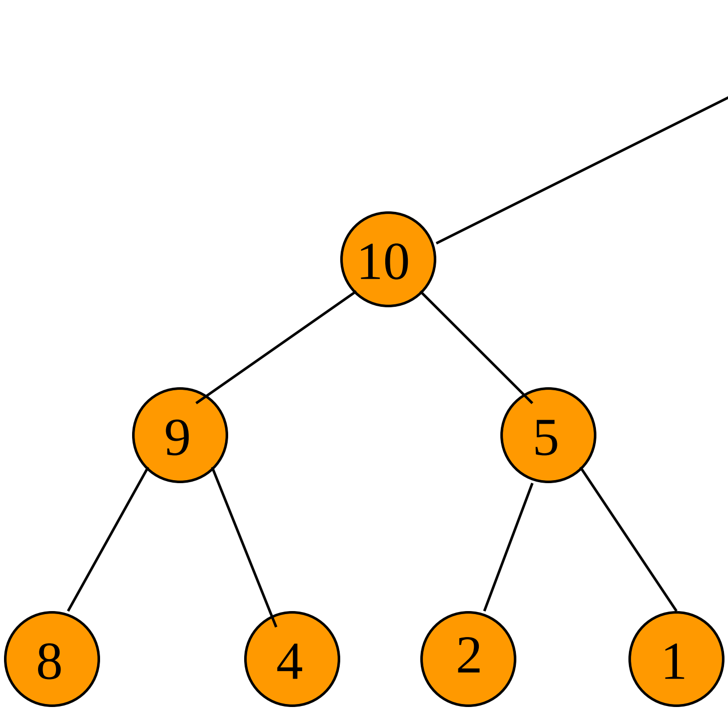
Find home for 1.

Initializing A Max Heap



Find home for 1.

Initializing A Max Heap




```
BuildMaxHeap(A, size)
{
  for(i=size/2 downto 1)
  {
     Heapify(A,size);
   }
}
```

Done.

```
HeapSort(A,size)
{
  BuildMaxHeap(A,size);
  for(i=size downto 2)
  {
     exchange(A[1] <->A[i]);
     size=size-1;
     Heapify(A,1);
   }
}
```

```
BuildMaxHeap(A, size)
{
  for(i=size/2 downto 1)
  {
     Heapify(A,size);
   }
}
```

```
Heapify(A, i)
{
  Lchild=2*i;
  Rchild=Lchild + 1;
if(Lchild≤size and a[Lchild]>A[i])
  then  largest=Lchild;
  else   largest=i;
if(Rchild≤size and a[Rchild]>A[largest])
  then  largest=Rchild;
If(largest≠i)
  then exchange(A[i] <->A[largest]);
         Heapify(A,largest)
}
```

Huffman Codes

Useful in lossless compression.

Consider a text file with 6 different characters:

	a	b	c	d	e	f
Frequency (x 1000)	45	13	12	16	9	5
Fixed length code	000	001	010	011	100	101
Variable length code	0	101	101	111	1101	1100