Dynamical Systems and Spotted Owls

Northern spotted owls at old Growth forest in the Pacific Northwest, USA.
Population dynamics is to model the population at yearly intervals, $k = 0, 1, 2 \ldots$. The population at year k can be described by a vector $x_k = (j_k, s_k, a_k)$, where j_k, s_k, a_k are the numbers of females in the juvenile, subadult, and adult stages, respectively.

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Stage-matrix model (R. Lamberson, R. McKelvey, B. R. Noon, and C. Voss, 1992)

$$\begin{bmatrix} j_{k+1} \\ s_{k+1} \\ a_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.33 \\ 0.18 & 0 & 0 \\ 0 & 0.71 & 0.94 \end{bmatrix} \begin{bmatrix} j_k \\ s_k \\ a_k \end{bmatrix}$$

If 50% of the juveniles who survive to leave the nest also find new home ranges, then the owl population will thrive.

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What do you observe with $\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix} \}$ as a subset of $\mathbb{R}^2$?

Eigenvalues and Eigenvectors

An **eigenvector** of an $n \times n$ matrix A is a nonzero vector $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ. A scalar λ is called an **eigenvalue** of A if there is a nontrivial solution $\mathbf{x}$ of $A\mathbf{x} = \lambda\mathbf{x}$; such an $\mathbf{x}$ is called an *eigenvector corresponding to* λ.[1]

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Hence for $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$, the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue 2 and $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ is an eigenvector with eigenvalue -1.

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Given a matrix A, the polynomial $det(A - \lambda.I) = 0$ is called characteristic polynomial of A(here λ is treated as a variable). Its roots are the eigenvalues of A.

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Theorem

The eigenvalues of a triangular matrix are the entries on its main diagonal.

Some results

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If $v_1, \ldots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \ldots, \lambda_r$ of an $n \times n$ matrix A , then the set $\{v_1, \ldots, v_r\}$ is linearly independent.

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Note: every $n \times n$ matrix with real entries need not have real eigenvalues/eigenvectors.

Eigenvectors and Difference Equations

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If x_0 is an eigenvector A then $x_k = \lambda^k x_0$.