Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 6

1. Are the following sets vector spaces over $\mathbb{R}$? If yes then find a basis p
   a) $V_1 = \{aX^2 | a_i \in \mathbb{R}\}$, the set of all homogeneous degree 2 polynomials.
   b) $V_2 = \{f(X) \in \mathbb{R}[X] | f(0) = 0\}$
   c) $V_3 = \{A \in M_3(\mathbb{R}) | A^T = -A$
   d) The set V_4 of all polynomials of degree ≥ 3, together with 0.
   e) $V_5 = \{A \in M_3(\mathbb{R}) | det(A) = 0$

2. The first four Hermite polynomials (which arise naturally in the study of certain differential equations in Mathematical Physics) are $1, 2t, -2 + 4t^2, -12t + 8t^3$. Do they form a basis for $\mathbb{R}_3[X]$?

3. Find a basis of $\mathbb{R}^3$ containing a vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $V = \{B \in M_2(\mathbb{R}) | BA = AB\}$. Is V a vector space over $\mathbb{R}$? If yes then give its basis and dimension.

5. Let V be the set of positive real numbers with vector addition being ordinary multiplication, and scalar multiplication being $a.v = v^a$. Show that V is a vector space.

6. Given subspaces H and K of a vector space V , the sum of H and K, written as $H + K$, is the set of all vectors in V that can be written as the sum of two vectors, one in H and the other in K; that is, $H + K = \{u + v | u \in H, v \in K\}$. Show that $H + K$ is also a subspace of V. Find $H + K$ for H, K two different lines passing through origin in $\mathbb{R}^2$.

7. Define a function $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ as $T(B) = AB$, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Is T a linear transformation? If yes then find its null space and range of T.

8. LetV denote the space of all functions $f : \mathbb{R} \to \mathbb{R}$ for which the derivatives f', f'' exist. Show that f1, f2, and f3 in V are linearly independent

provided that their wronskian $w(x)$ is nonzero for some x, where

$$w(x) = det \begin{bmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_1'(x) & f_2'(x) & f_3'(x) \\ f_1''(x) & f_2''(x) & f_3''(x) \end{bmatrix}$$