# Indian Institute of Information Technology Vadodara

## End-semester Examination

## MA 101 (Mathematics I: Linear Algebra and Matrices)

**Total Marks: 50**

1. Find the parabola $At^2 + Bt + C$ that is closest to the values $b = (0, 0, 1, 0, 0)$ at the times $t = -2, -1, 0, 1, 2$, respectively. **5 Marks**

2. Let the matrix $A$ be diagonalizable such that $A = S\Lambda S^{-1}$, where $\Lambda$ is a diagonal matrix. Then diagonalize the block matrix $B = \begin{pmatrix} A & 0 \\ 0 & 2A \end{pmatrix}$. Find its eigenvalues and eigenvectors (block) matrices. **5 Marks**

3. (a) Let $A = \begin{pmatrix} 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$. Find a basis for $Col(A)$ and find a basis for $Null(A)$.
(b) If $B$ is a $7 \times 9$ matrix with $rank(B) = 5$. Then $dim(ColB) =?$, $dim(NullB) =?$ , $dim(RowB) =?$ , $dim(NullB^T) =?$ , where $dim$ stands for dimension. **5 Marks**

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that takes $T\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Then
(i) Find $T\begin{pmatrix} 5 \\ 6 \end{pmatrix}$.
(ii) Find standard matrix representation of $T$. **5 Marks**

5. Let $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \\ 2 & 6 \end{bmatrix}$. Find dimensions of Row space of A, Column space of A and Null space of A. Is $AX = b$ always consistent for any choice of the vector $b$? If yes then give reason else give $b$ such that $AX = b$ is not consistent. **5 Marks**

6. Find an orthonormal basis for the column space of the matrix $A = \begin{pmatrix} 1 & -2*j \\ 1 & 0 \\ 1 & j \\ 1 & 3*j \end{pmatrix}$, where $j$ is 1 + the last digit of your student id.

Let $b = \begin{pmatrix} -4 \\ -3 \\ 3 \\ 0 \end{pmatrix}$. Find the projection of $b$ onto that column space of $A$.

**5 Marks**

7. Let $B = \{1, 1+X, X+X^2, 1+X^3\}$. Show that $B$ is a basis of the vector space of all polynomials of degree less than or equal 3=$\mathbb{R}_3[X]$. Consider $T : \mathbb{R}_3[X] \to \mathbb{R}_3[X]$ defined as $T(f(X)) = 2X\frac{df}{dX}$. If $T$ is a linear transformation then find a matrix representation of $T$ with respect to a basis $B$ else give rason. **5 Marks**

8. Is every matrix $A \in M_n(\mathbb{R})$ with $A^3 = A$ diagonalisable over $\mathbb{R}$? Give justication. **3 Marks**

9. Find the SVD of the matrix $A = \begin{bmatrix} 1 & 0 & 1 & j \\ 0 & 1 & 0 & j \end{bmatrix}$, where $j$ is 1 + the last digit of your student id. Find an orthonormal basis for Column space of A and Null space of A. **7 Marks**

10. Let $A = \begin{bmatrix} 0 & 2 & 4 & 6 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$. Find the determinant of $A$. What can you say about the linear transformation associated with $A$ and $A^2$? **5 Marks**