18.06.2021

AC Machinery Fundamentals

(Reference: Chapter 4)

Two major classes — (i) Synchronous Machine

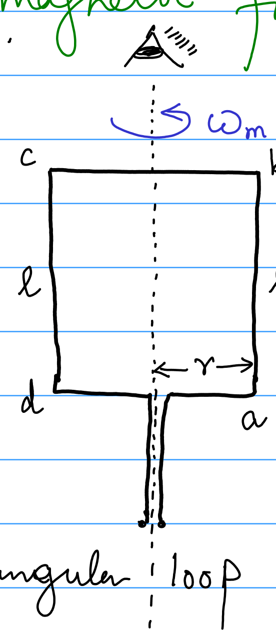
(ii) Induction Machine

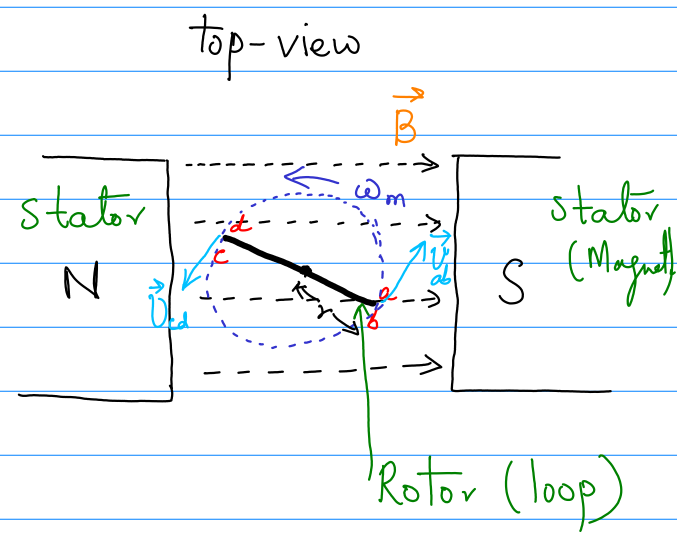
In any electrical machine (generator / motor), we have two major parts :

(i) Stator : Non-movable

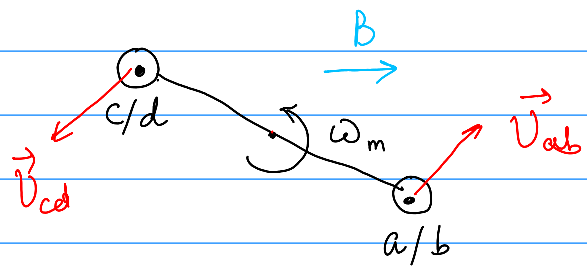
(ii) Rotor : Movable (Rotating)

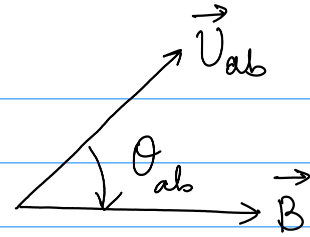
I : A simple rotating loop placed in a uniform magnetic field :

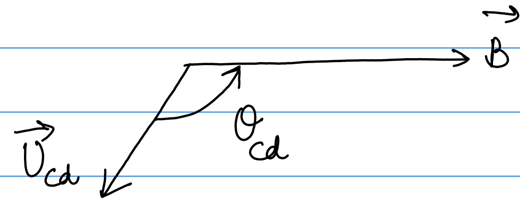




In this simple machine, where the conducting wire loop (in rectangular geometry) is rotating in a uniform magnetic field created by the bar magnets.







We know that the motional emf is given as:

$$e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

(i) Induced emf in the segment ab of the moving loop.

$$e_{ab} = (\vec{v}_{ab} \times \vec{B}) \cdot \vec{l} = v_{ab} \cdot B \sin\theta_{ab} \cdot l$$

(ii) Induced emf in the segment bc

In the 1st half of the segment bc, $(\vec{v} \times \vec{B})$ points into the page of the figure, whereas the 2nd half points out of the page of the figure. Since the length "bc" $(2r)$ is in the plane of the figure,

$$(\vec{v} \times \vec{B}) \cdot \vec{l} = 0$$

$$\Rightarrow \quad e_{bc} = 0$$

(iii) Induced emf in segment "cd".

$$e_{cd} = (\vec{v}_{cd} \times \vec{B}) \cdot \vec{l} = vBl \sin\theta_{cd}$$

(iv) Induced emf in segment 'da'.

$$e_{da} = 0$$

Therefore, Total induced emf :

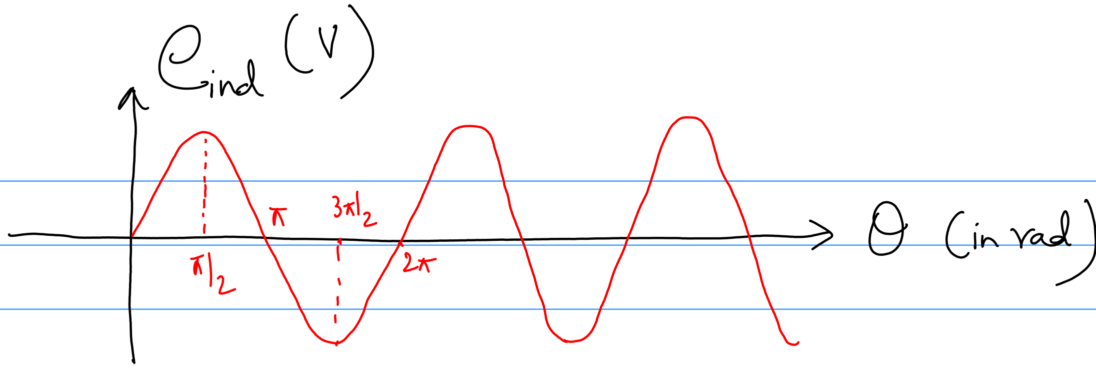
$$e_{ind} = e_{ab} + \cancel{e_{bc}}^{\,0} + e_{cd} + \cancel{e_{da}}^{\,0}$$

$$e_{ind} = vBl\left[\sin\theta_{ab} + \sin\theta_{cd}\right]$$

Since, $\theta_{ab} = 180^\circ - \theta_{cd}$ $\left[\sin 180 - \theta = \sin\theta\right]$

$$\boxed{e_{ind} = 2vB \cdot l \underset{\sim\sim}{\sin\theta}}$$

$\Rightarrow$ Induced emf is θ dependent, meaning e_{ind} is sinusoidal in nature.



⇒ We have a generator which generates ac voltage.

ω_m = uniform angular speed of the rotating loop.

$$\theta = \omega_m t$$

$$v = \omega_m r$$

$$e_{ind} = 2 \cdot (\omega_m r) \cdot B \cdot l \sin \omega_m t$$

$$\text{Area of loop} = l \times 2r = A$$

$$e_{ind} = \underbrace{A \cdot B}_{\phi} \omega_m \sin \omega_m t$$

$$\boxed{e_{ind}(t) = \phi\, \omega_m \sin \omega_m t}$$

In general, the voltage induced in real machine depends upon:

(i) The magnetic flux density in the machine.

(ii) The mechanical speed of the rotor (loop).

(iii) A constant representing the contruction of the machine.

II Effect on the current carrying loop placed within a uniform magnetic field.

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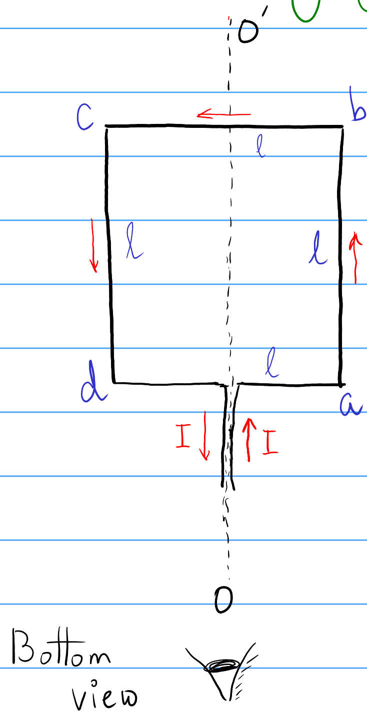
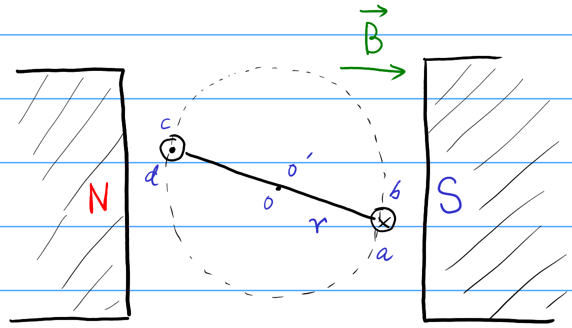
Current-carrying loop in uniform magnetic field



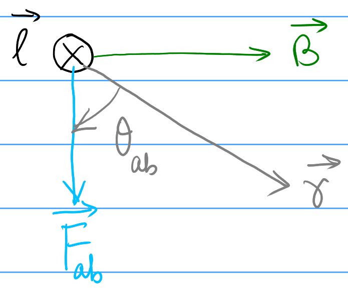


Bottom view

$$\vec{F}_{ind.} = I\,(\vec{\ell} \times \vec{B})$$

'ℓ' = length of the wire & its direction is the direction of current I.

(i) The segment 'ab':

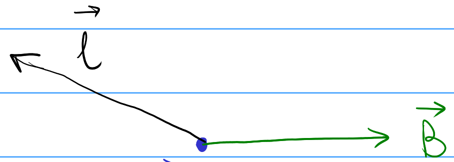


$$\vec{F}_{ab} = I\,(\vec{\ell} \times \vec{B})$$

$\vec{F}_{ab} = I\ell B$ downward

$\vec{\tau}_{ab} = \vec{r} \times \vec{F}_{ab} = rI\ell B \sin\theta_{ab}$ into the plane of the figure

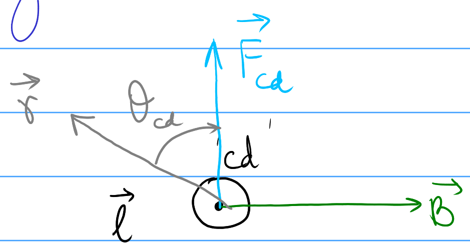
(ii) The segment 'bc'



$\vec{F}_{bc}$, $\vec{r}$ are into the plane of the page,

$\vec{\tau}_{bc} = 0$

(iii) The segment 'cd'



$\vec{F}_{cd} = I l B$ upward

$\vec{\tau}_{cd} = r I l B \sin\theta_{cd}$ into the plane of the page/figure

(iv) The segment 'da'

$\vec{r}$, $\vec{F}_{da}$ out of the page/figure.

$\vec{\tau}_{da} = 0$

For any arbitrary angle 'θ' w.r.t magnetic field direction the loop experience net torque on it,

$$\vec{\tau}_{tot} = \vec{\tau}_{ab} + \underbrace{\vec{\tau}_{bc}}_{=0} + \vec{\tau}_{cd} + \underbrace{\vec{\tau}_{da}}_{=0}$$

$$\vec{\tau}_{tot} = r \cdot I \cdot l \cdot B \sin\theta_{ab} + r \cdot I \cdot l \cdot B \sin\theta_{cd}$$

here, $\theta_{ab} = \theta_{cd} = \theta$ (vertically opp. angles)

$$\boxed{\tau_{tot} = 2 r I l B \sin\theta}$$

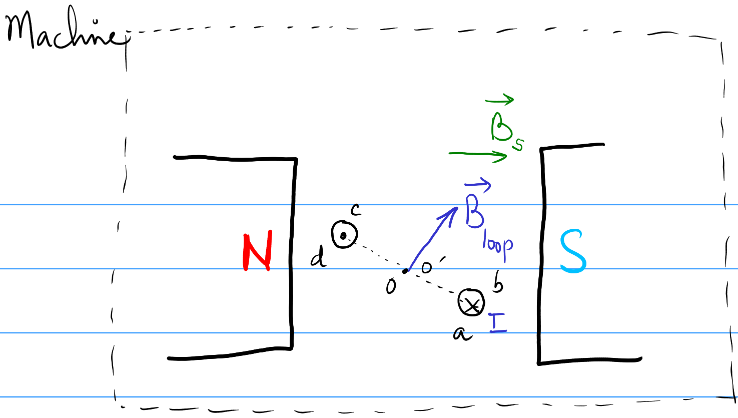
Conclusion: 1) The total torque experienced by the current carrying loop is sinusoidal in nature.

2) The magnitude of the torque depends on the angle ie,

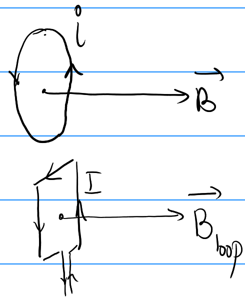
a) if the plane of the loop is parallel to the direction of magnetic flux density, then the

τ_{tot} = maximum

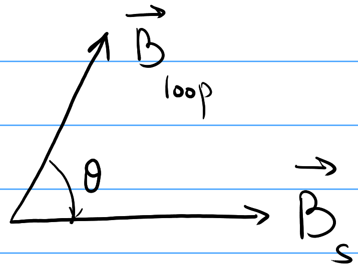
b) if the plane of the loop is perpendicular to the direction of $\vec{B}$, then the $\tau_{tot} = 0$



$\vec{B}_s$ = stator's magnetic flux density.
= uniform in magnitude & directⁿ.



$\vec{B}_{loop}$ = Magnetic flux density produced by the current flowing the loop.



$$B_{loop} = \frac{\mu I}{G}$$

'G' is the geometric parameter

Substituting 'I' and $A = 2rl$ is the expression of τ_{tot}

$$\tau_{tot} = \frac{AG}{\mu} \cdot B_{loop} \cdot B_s \sin\theta$$

$$\tau_{tot} = k \cdot B_{loop} \cdot B_s \sin\theta$$

where $k = \frac{AG}{\mu}$

$$\Rightarrow \boxed{\vec{\tau}_{tot} = k \cdot \vec{B}_{loop} \times \vec{B}_s}$$

Here, one point is clear that the magnetic flux density $\vec{B}_{loop}$ tries to allign itself toward the $\vec{B}_s$.

Suppose "somehow" we creat $\vec{B}_s$ which is rotating in the space, then, as a nature of $\vec{B}_{loop}$, trying to allign towards $\vec{B}_s$, the $\vec{B}_{loop}$ will chase $\vec{B}_s$.

This continues till the current 'I' is flowing in the loop.

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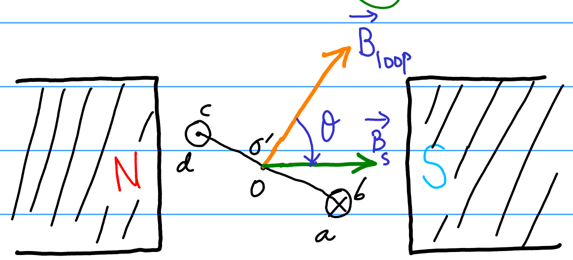
The Rotating Magnetic Field

Recap:

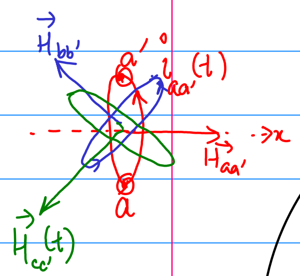


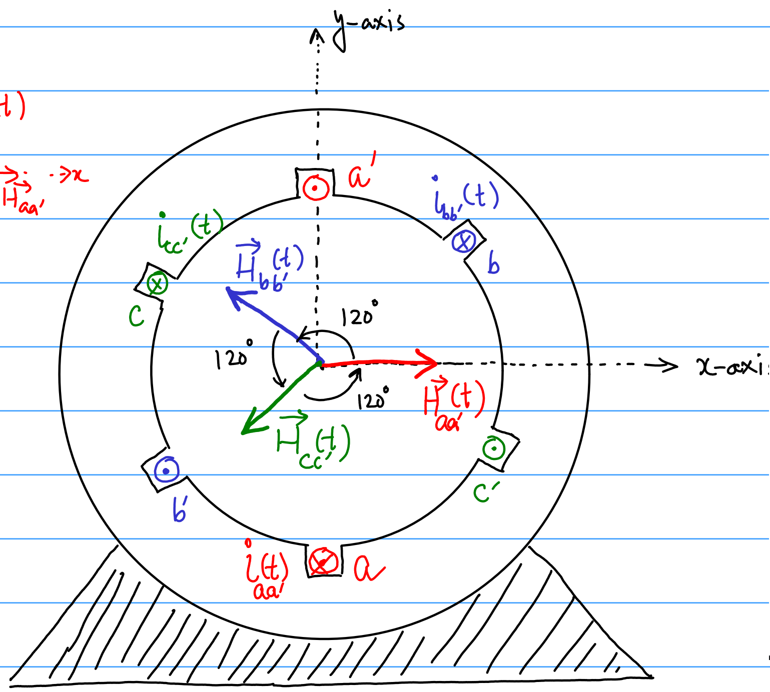
$$\boxed{\vec{\tau}_{ind} = k\,\vec{B}_{loop} \times \vec{B}_s}$$

where $k = \dfrac{AG}{\mu}$ = Depends on the construction of the machine

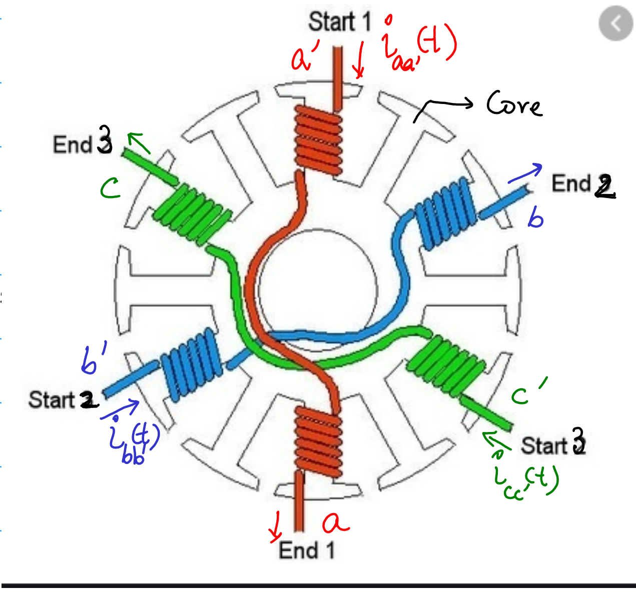
- $\vec{B}_s$ is the uniform magnetic flux density due to magnets (here it is a 'STATOR')
- $\vec{B}_{loop}$ is the uniform magnetic flux density produced by the current carrying loop. (here it is a 'ROTOR').
- A torque ($\vec{\tau}_{ind}$) will be induced on the 'ROTOR' which will cause the 'ROTOR' to turn and allign itself such that the two fields become parallel ($\theta = 0°$).
- However, if there is some way to make the 'STATOR' magnetic field ($\vec{B}_s$) to "ROTATE", then the induced torque acting on the 'ROTOR' will cause it to constantly "CHASE" the $\vec{B}_s$ around a circle.
- This is the basic principle of operation of all ac-motors.

How can the stator's magnetic field be made to rotate?





3-φ stator



3-φ stator windings

Assume that the currents in the three coils aa', bb', cc' are given as:

$$i_{aa'}(t) = I_m \sin \omega t \quad A \quad \text{--- connected to phase-I}$$

$$i_{bb'}(t) = I_m \sin(\omega t - 120^\circ) \quad A \quad \text{--- connected to phase-II}$$

$$i_{cc'}(t) = I_m \sin(\omega t - 240^\circ) \quad A \quad \text{--- connected to phase-III}$$

Due to these currents, the magnetic field intensity in the space of the 'STATOR'.

$$\vec{H}_{aa'}(t) = H_m \sin \omega t \quad \angle 0^\circ \quad \text{(ie, along x-axis)}$$

Similarly

$$\vec{H}_{bb'}(t) = H_m \sin(\omega t - 120^\circ) \quad \angle 120^\circ \text{ w.r.t x-axis}$$

$$H_{cc'}(t) = H_m \sin(\omega t - 240^\circ) \quad \angle 240^\circ \text{ w.r.t x-axis}$$

Since $\vec{B} = \mu \vec{H}$

Therefore,

$$\vec{B}_{aa'}(t) = B_m \sin(\omega t) \quad \angle 0^\circ \text{ w.r.t x-axis}$$

$$\vec{B}_{bb'}(t) = B_m \sin(\omega t - 120^\circ) \quad \angle 120^\circ \text{ w.r.t x-axis}$$

$$\vec{B}_{cc'}(t) = B_m \sin(\omega t - 240^\circ) \quad \angle 240^\circ \text{ w.r.t x-axis}$$

Let us determine resultant magnetic flux density in the space of the STATOR as a function of Time.

Let find resultant flux density at two instants of time.

(i) at an instant when $\omega t = 0$

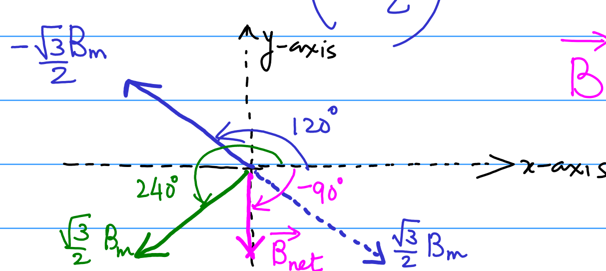
$$\vec{B}_{aa'} = B_m \sin 0 = 0$$

$$\vec{B}_{bb'} = B_m \sin(0^\circ - 120^\circ) \quad \angle 120^\circ \text{ w.r.t x-axis}$$

$$\vec{B}_{cc'} = B_m \sin(0^\circ - 240^\circ) \quad \angle 240^\circ \text{ w.r.t x-axis}$$

$$\vec{B}_{net} = \vec{B}_{aa'} + \vec{B}_{bb'} + \vec{B}_{cc'}$$

$$\vec{B}_{net} = 0 + B_m\left(-\frac{\sqrt{3}}{2}\right) \angle 120^\circ \text{ w.r.t x-axis} + B_m\left(\frac{\sqrt{3}}{2}\right) \angle 240^\circ \text{ w.r.t x-axis}$$



$$\vec{B}_{net} = \frac{3}{2}B_m \angle -90^\circ \text{ w.r.t x-axis}$$

(ii) at an instant $\omega t = 90°$

$$\vec{B}_{aa'} = B_m \sin 90° = B_m \; x\text{-axis}$$

$$\vec{B}_{bb'} = B_m \sin(90° - 120°) \quad \angle 120° \text{ w.r.t } x\text{-axis}$$

$$\vec{B}_{cc'} = B_m \sin(90° - 240°) \quad \angle 240° \text{ w.r.t } x\text{-axis}$$

$$\boxed{\vec{B}_{net} = \frac{3}{2} B_m \angle 0° \text{ w.r.t } x\text{-axis}}$$

Thus, we observe that the resultant magnetic flux density $\vec{B}_{net}$ is changed its direction from $-ve$ y-axis to x-axis.

Conclusion:

At any instant of time 't', the resultant magnetic flux density have the constant magnetude ($1.5 B_m$). It rotates in the space of the stator.

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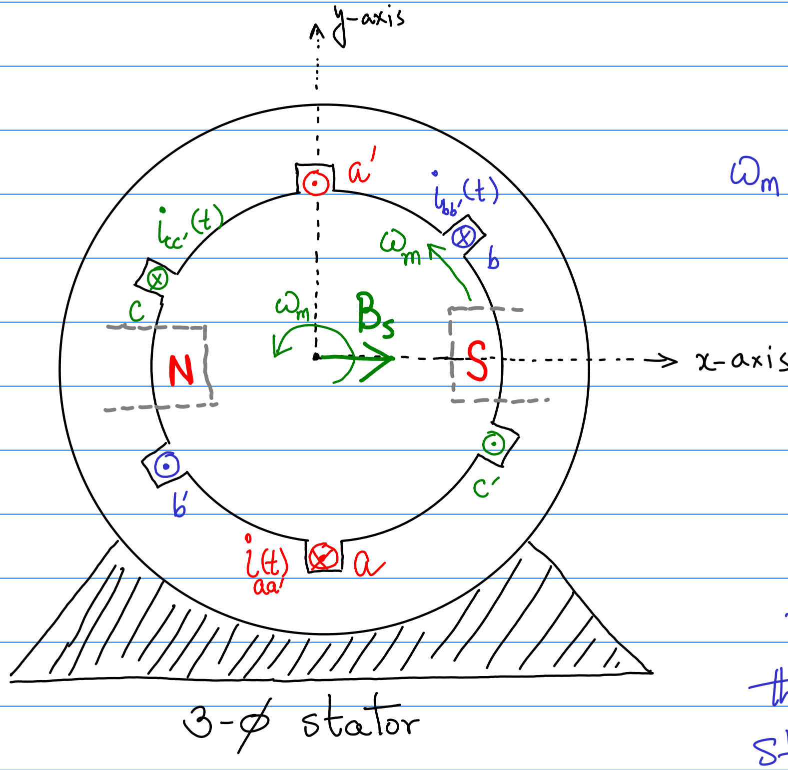
..... Continued

One Set of coils as :
a - c' - b - a' - c - b'

⇓

feature of a two poles bar-magnet



ω_m = mechanical angular speed.
It is the angular speed at which the resultant magnetic flux density ($\vec{B}_s$) is rotating in the space of the stator coils.

How to relate this mechanical rotation with the electrical frequency?

⇒ The effective magnetic poles (N & S) (here two-poles) complete one mechanical rotation around the stator surface for each electrical cycle of the applied current.

Lets consider electrical freq. = 50 Hz

ie, # electrical cycles per second = 50

Therefore, # mechanical rotations per second = 50 ie, $f_e = f_m$ where, f_e = electrical frequency (oscillations per second)

f_m = mechanical speed (revolution per second)

ie, $\omega_e = \omega_m$

How would happen when the set of coils (a-c′-b-a′-c-b′) in the stator is repeated ?

Example: repeated twice in the stator

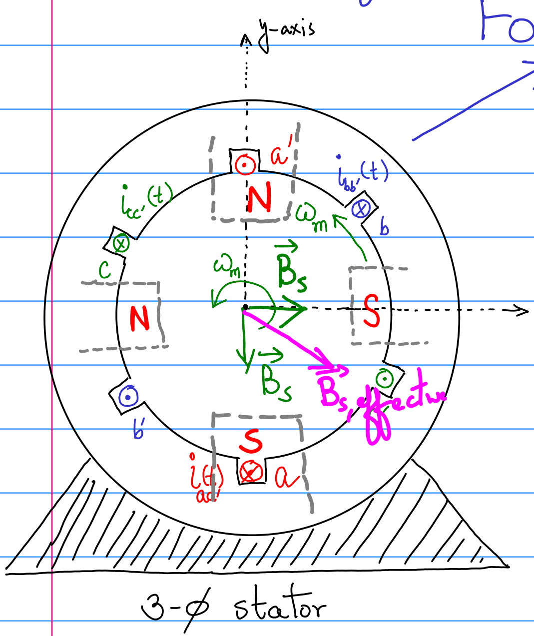
ie, $\underbrace{a-c'-b-a'-c-b'}_{\text{Set-I}} - \underbrace{a-c'-b-a'-c-b'}_{\text{Set-II}}$

Set-I ⇓ Two-poles bar magnet

Set-II ⇓ Two-poles bar magnet

Effectively, in the space of the stator, we have

Four-poles —— two poles → N
two poles → S



⇒ In this design, we observe that one set of magnetic poles moves halfway (half cycle) around the stator surface in one complete electrical cycle.

$f_e = 2 f_m$ or, $\omega_e = 2\omega_m$

In general, we can have 'n' number of repeatation of the set of coils ie,

$$\underbrace{a-c'-b-a'-c-b'}_{\text{Set-I}}-\underbrace{a-c'-b-a'-c-b'}_{\text{Set-II}}-\underbrace{\cdots\cdots}_{\text{Set-n}}$$

Set-I $\Downarrow$ Two-poles

Set-II $\Downarrow$ Two-poles

Set-n $\Downarrow$ Two-poles

$\Rightarrow$ We have a set of stator coils giving 'P' number of magnetic poles.

then, we can say that there will be $P/2$ number of sets of coils.

$$\theta_e = \frac{P}{2}\theta_m$$

$$\Rightarrow \quad f_e = \frac{P}{2} f_m$$

$$\Rightarrow \quad \omega_e = \frac{P}{2}\omega_m$$

We know that f_m = revolution/sec

n_m = revolution per minute (rpm)

$$\frac{n_m}{60} = f_m \qquad \text{Since } f_m = \frac{2 f_e}{P}$$

$$\boxed{n_m(\text{rpm}) = \frac{120\, f_e\,(\text{Hz})}{P}}$$

where P = # of poles in the stator.

Example:

$f_e = 50\,\text{Hz}$; $P = 8$

$$n_m(\text{rpm}) = \frac{\overset{15}{\cancel{120}} \times 50}{\cancel{8}} = 750 \text{ rpm}$$