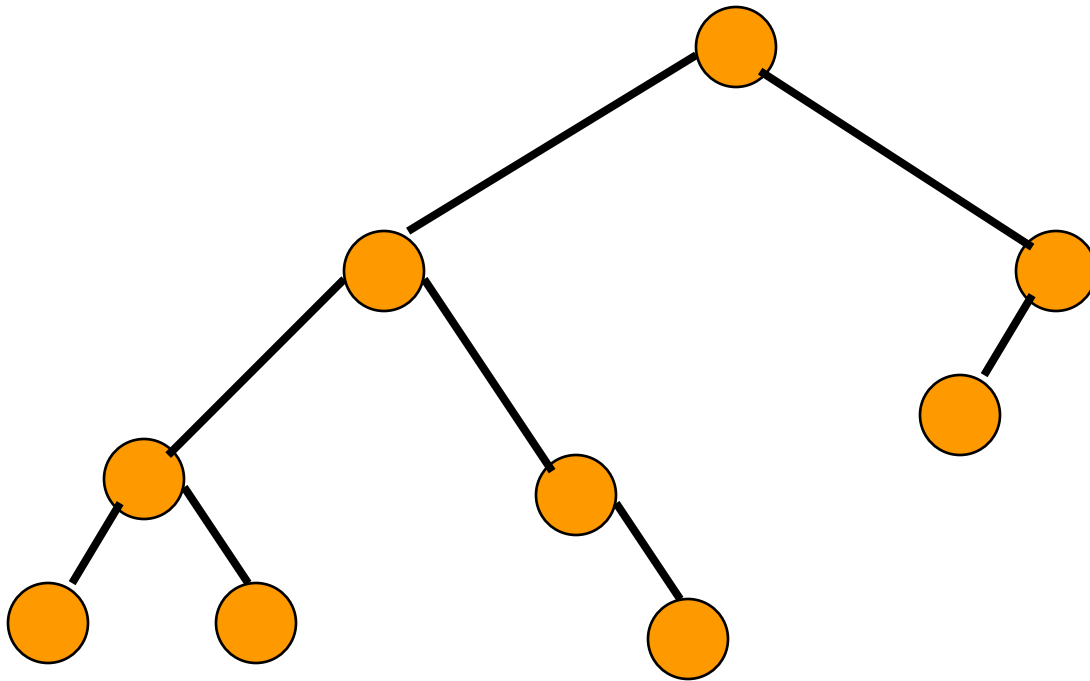
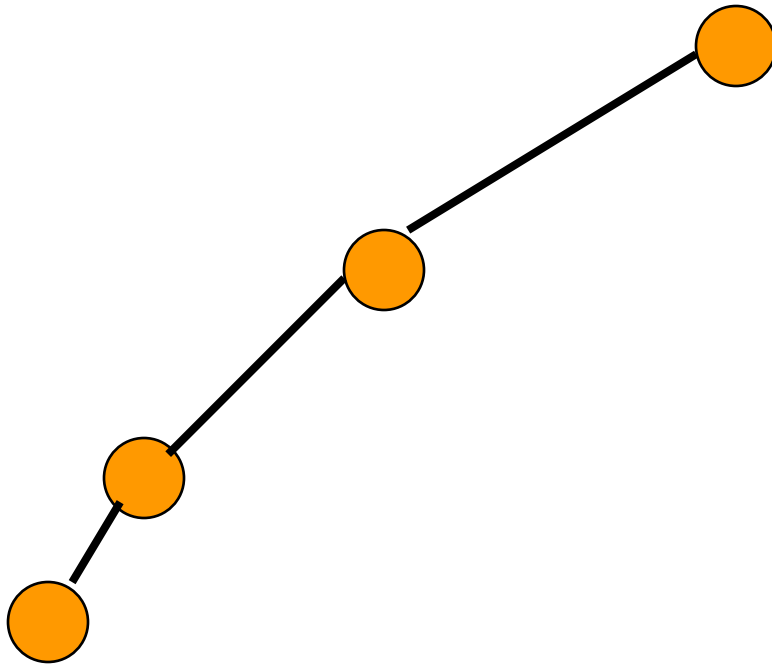


Binary Tree Properties & Representation



Minimum Number Of Nodes

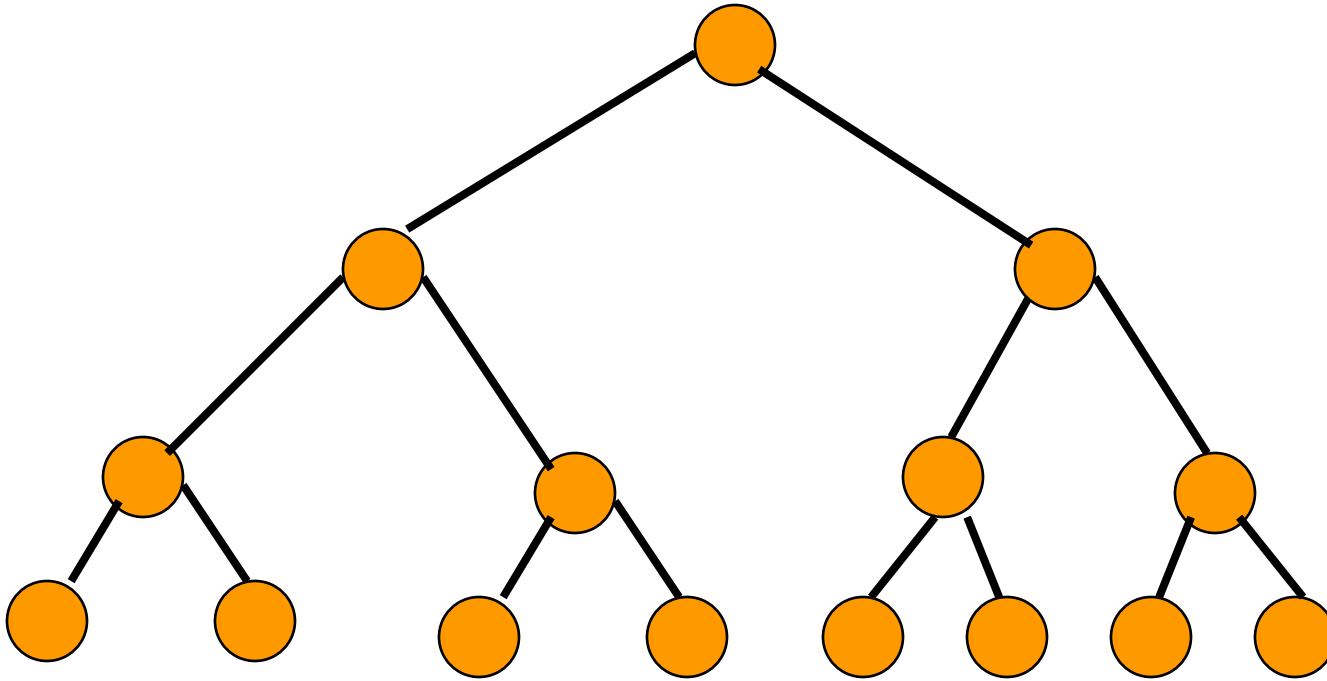
- Minimum number of nodes in a binary tree whose height is **h**.
- At least one node at each of first **h** levels.



minimum number of
nodes is **h**

Maximum Number Of Nodes

- All possible nodes at first **h** levels are present.



Maximum number of nodes

$$= 1 + 2 + 4 + 8 + \dots + 2^{h-1}$$

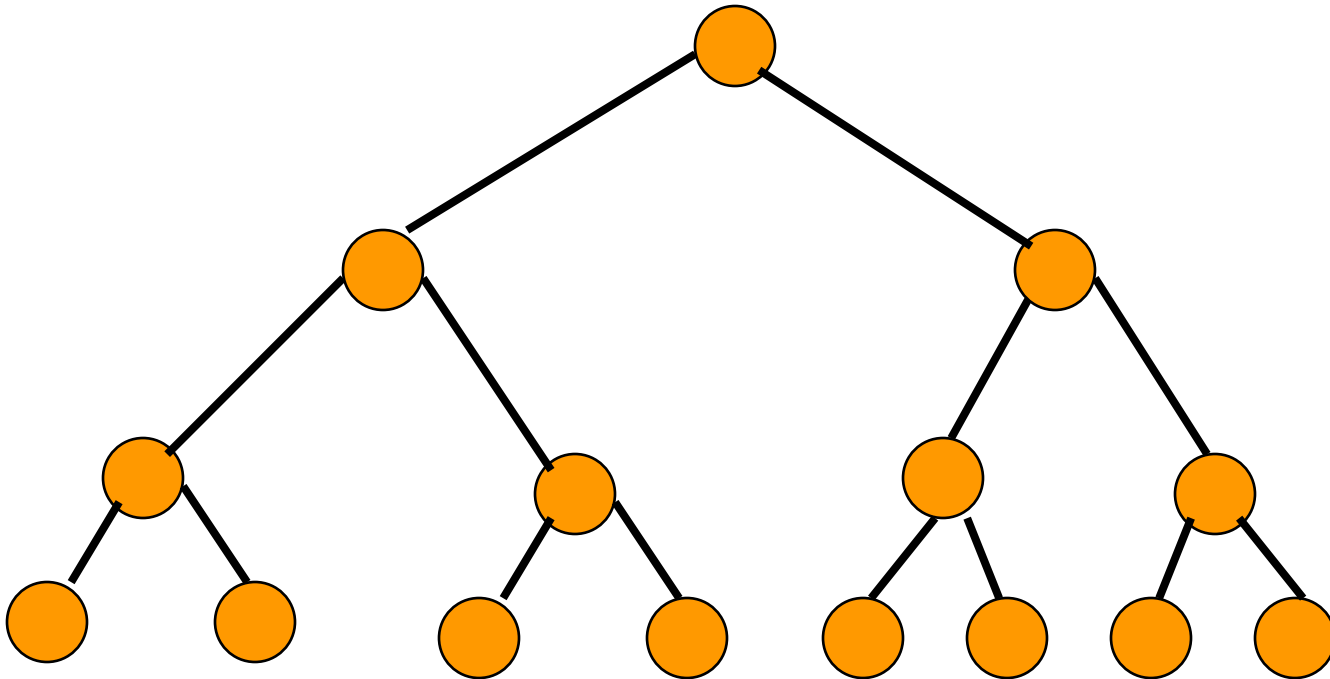
$$= 2^h - 1$$

Number Of Nodes & Height

- Let n be the number of nodes in a binary tree whose height is h .
- $h \leq n \leq 2^h - 1$
- $\log_2(n+1) \leq h \leq n$

Full Binary Tree

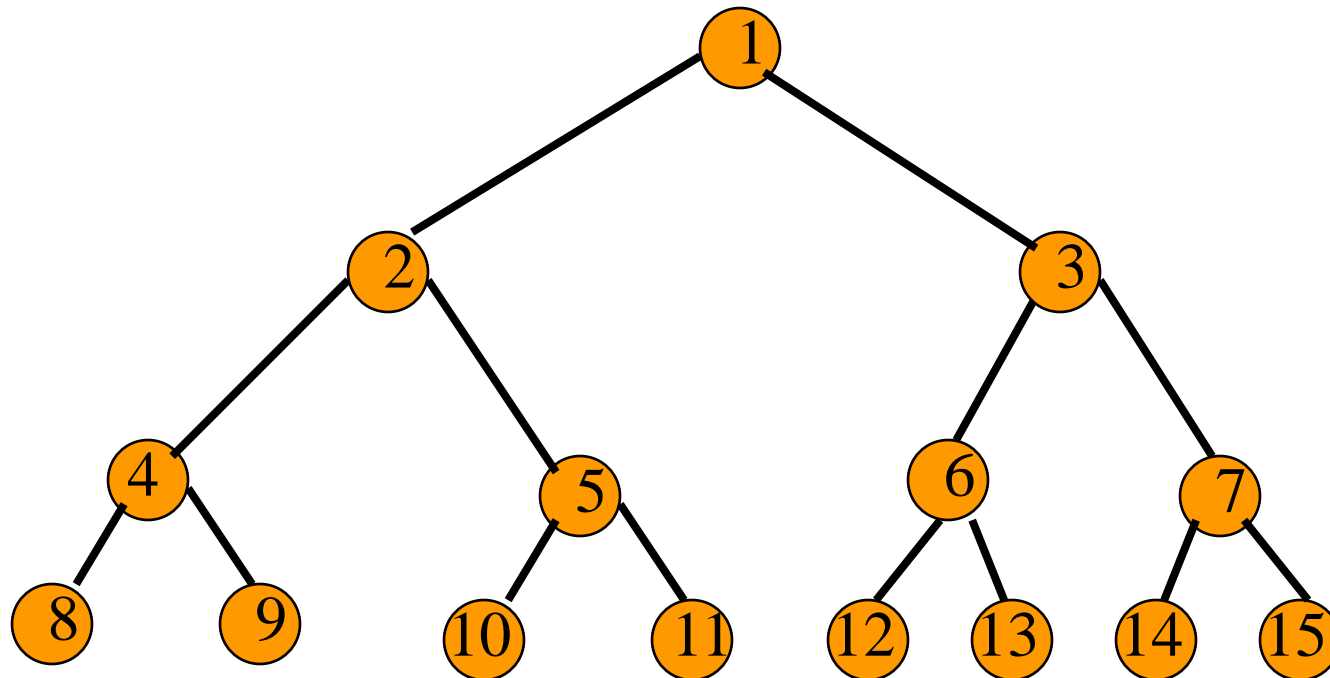
- A full binary tree of a given height h has $2^h - 1$ nodes.



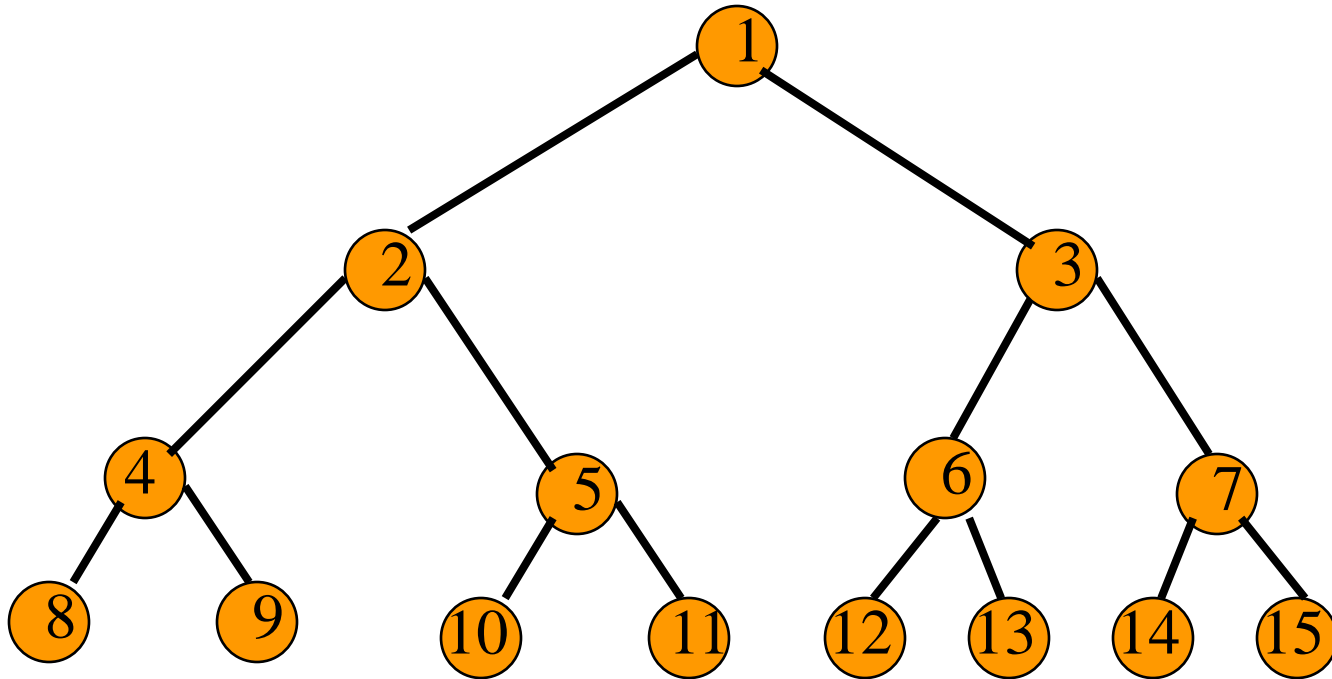
Height 4 full binary tree.

Numbering Nodes In A Full Binary Tree

- Number the nodes **1** through **$2^h - 1$** .
- Number by levels from top to bottom.
- Within a level number from left to right.

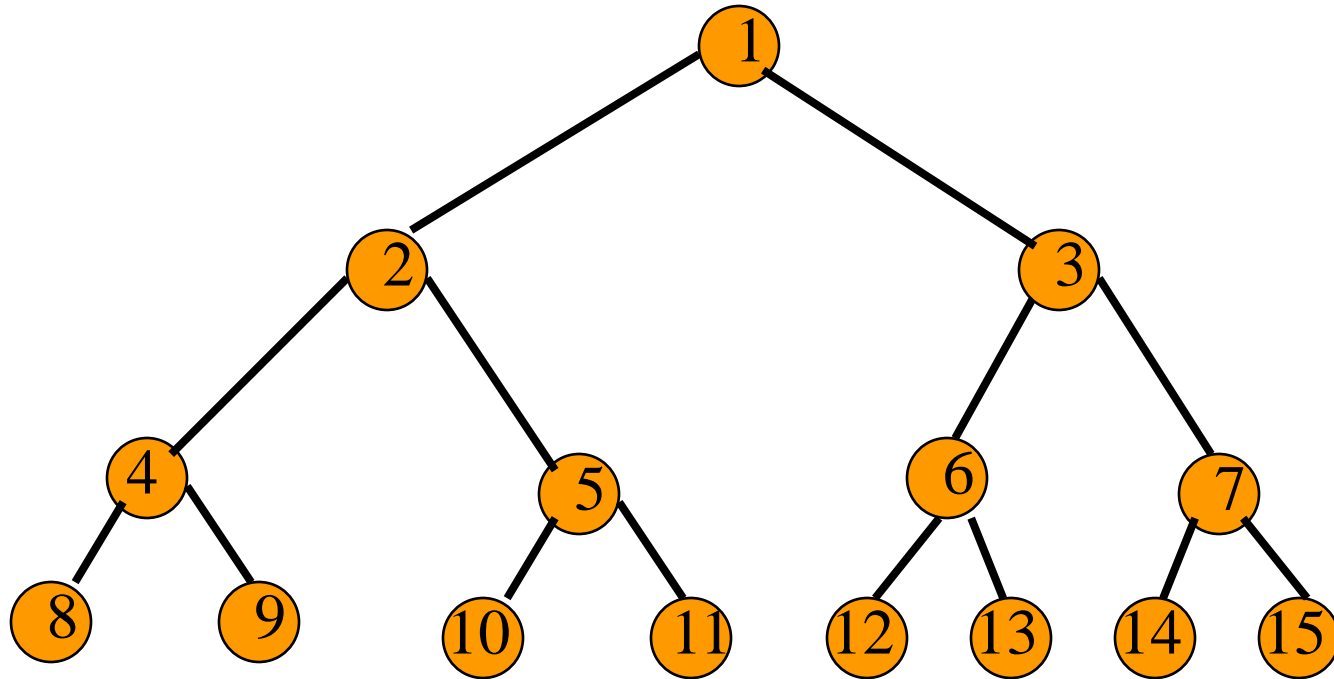


Node Number Properties



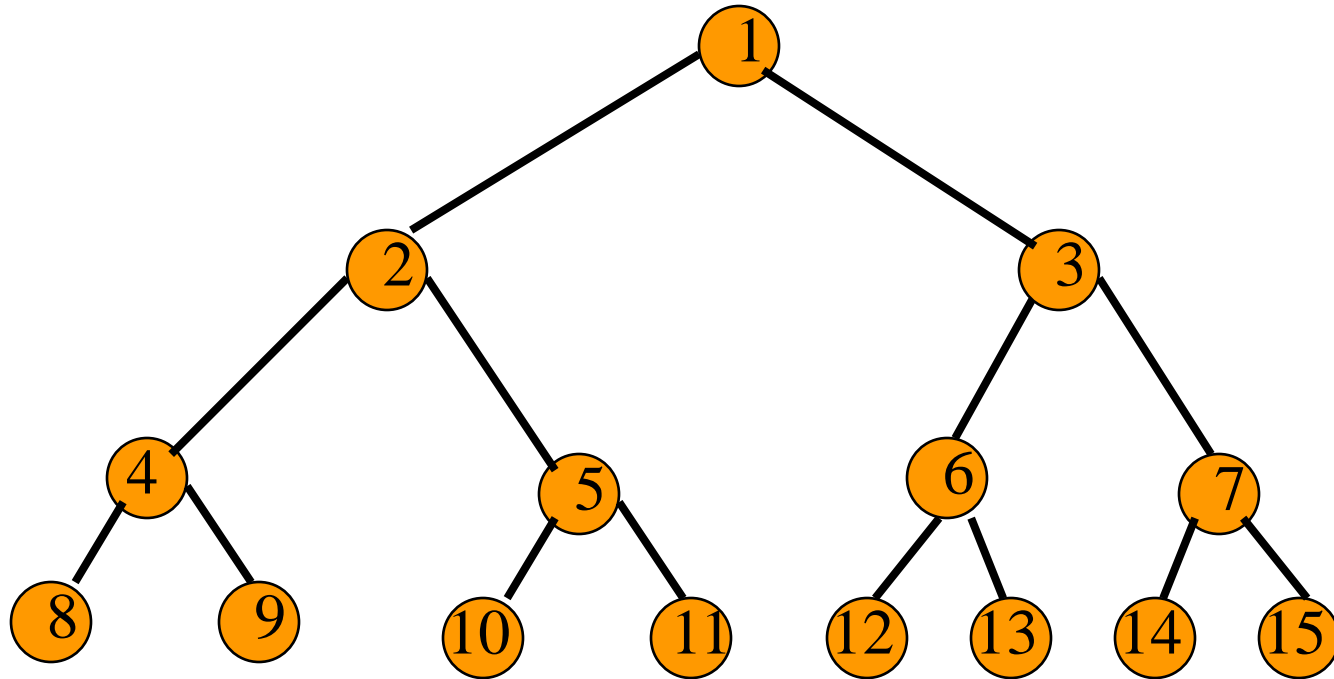
- Parent of node i is node $i / 2$, unless $i = 1$.
- Node 1 is the root and has no parent.

Node Number Properties



- Left child of node i is node $2i$, unless $2i > n$, where n is the number of nodes.
- If $2i > n$, node i has no left child.

Node Number Properties

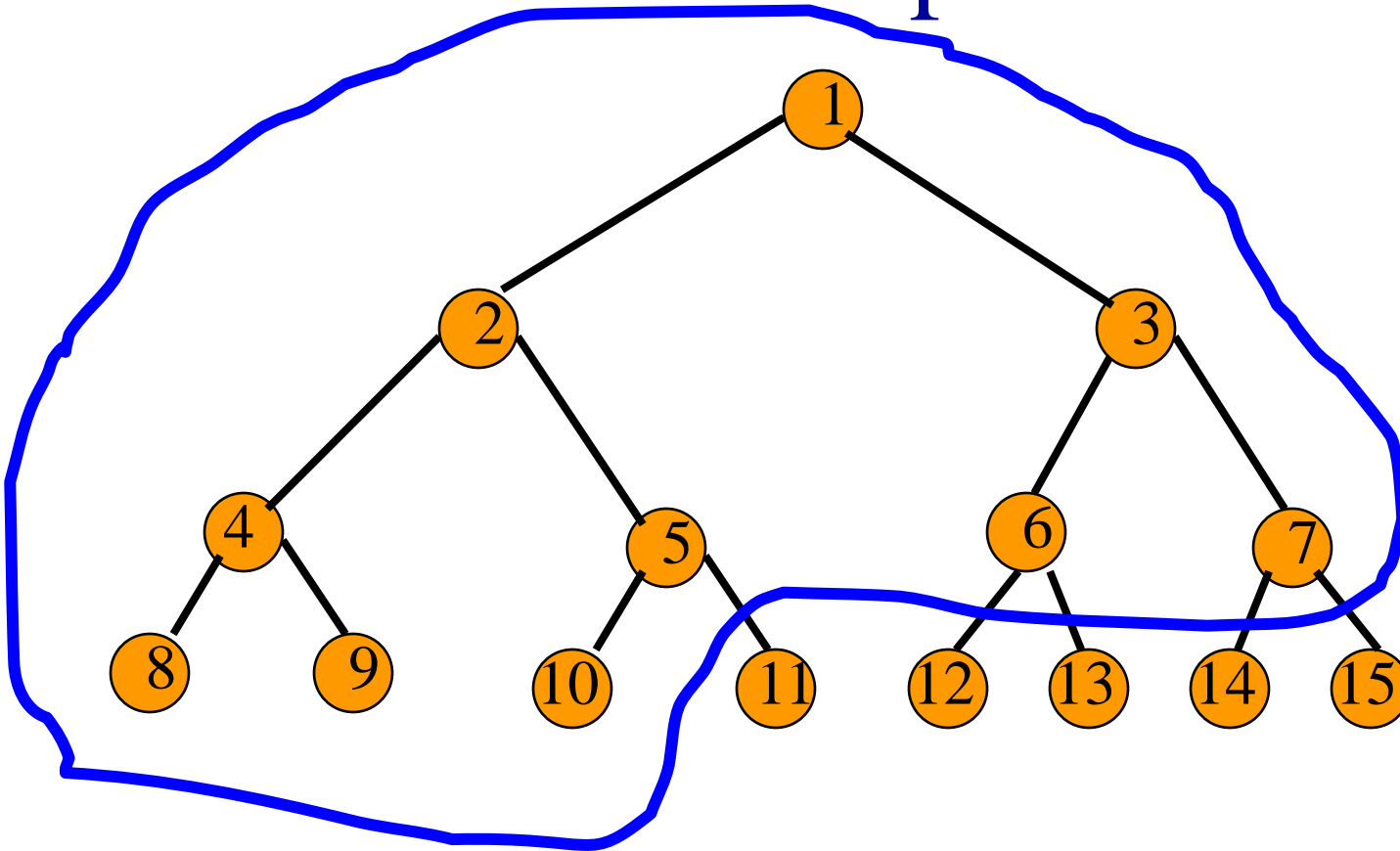


- Right child of node i is node $2i+1$, unless $2i+1 > n$, where n is the number of nodes.
- If $2i+1 > n$, node i has no right child.

Complete Binary Tree With n Nodes

- Start with a full binary tree that has at least n nodes.
- Number the nodes as described earlier.
- The binary tree defined by the nodes numbered 1 through n is the unique n node complete binary tree.

Example



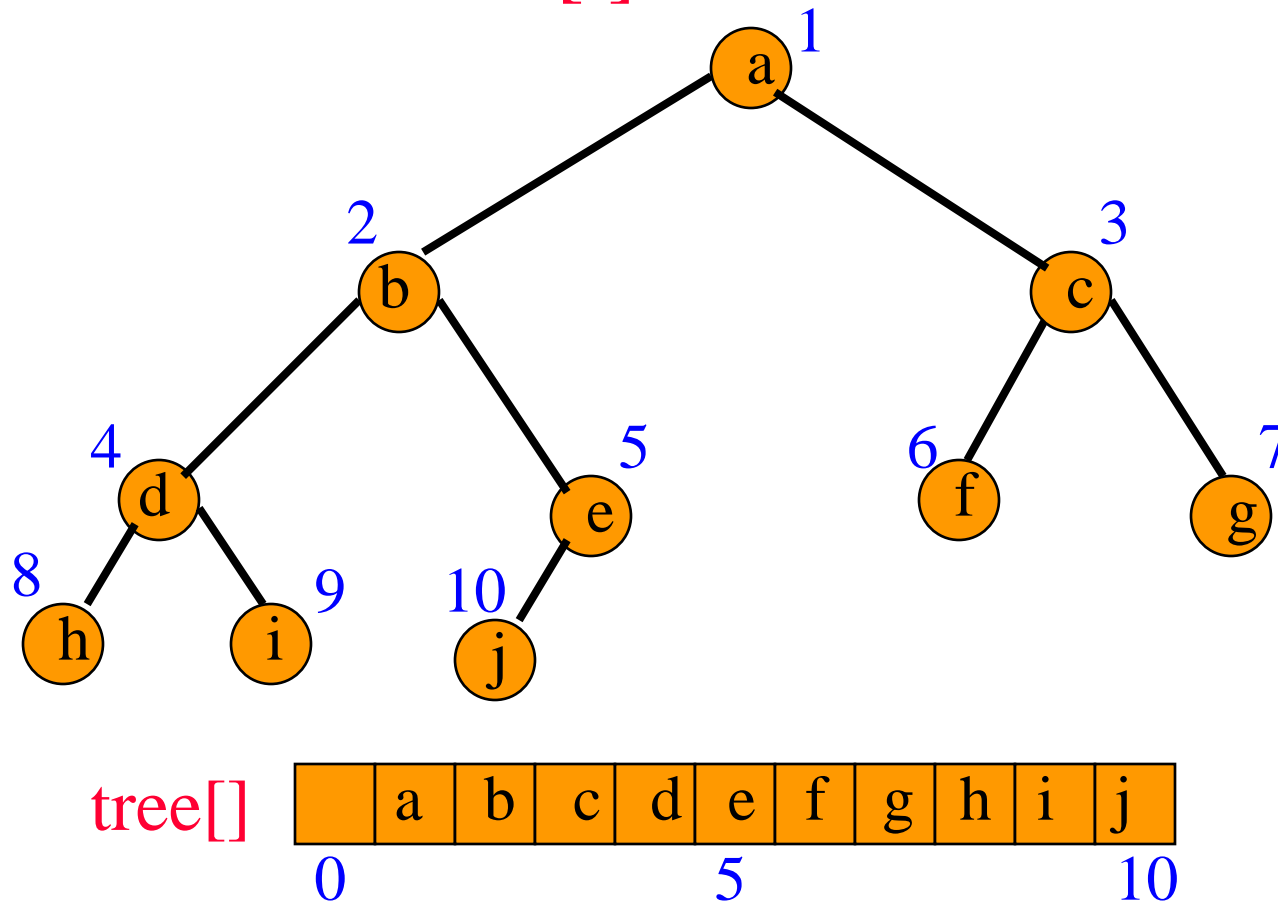
- Complete binary tree with 10 nodes.

Binary Tree Representation

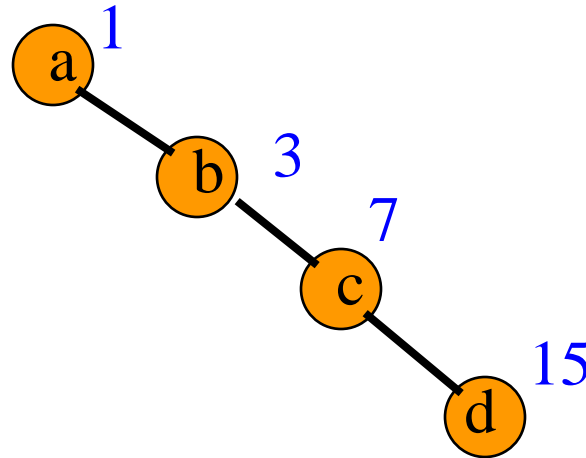
- Array representation.
- Linked representation.

Array Representation

- Number the nodes using the numbering scheme for a full binary tree. The node that is numbered i is stored in $tree[i]$.



Right-Skewed Binary Tree



tree[]

	a	-	b	-	-	-	c	-	-	-	-	-	-	-	d
0				5				10							15

- An **n** node binary tree needs an array whose length is between **n+1** and **2ⁿ**.

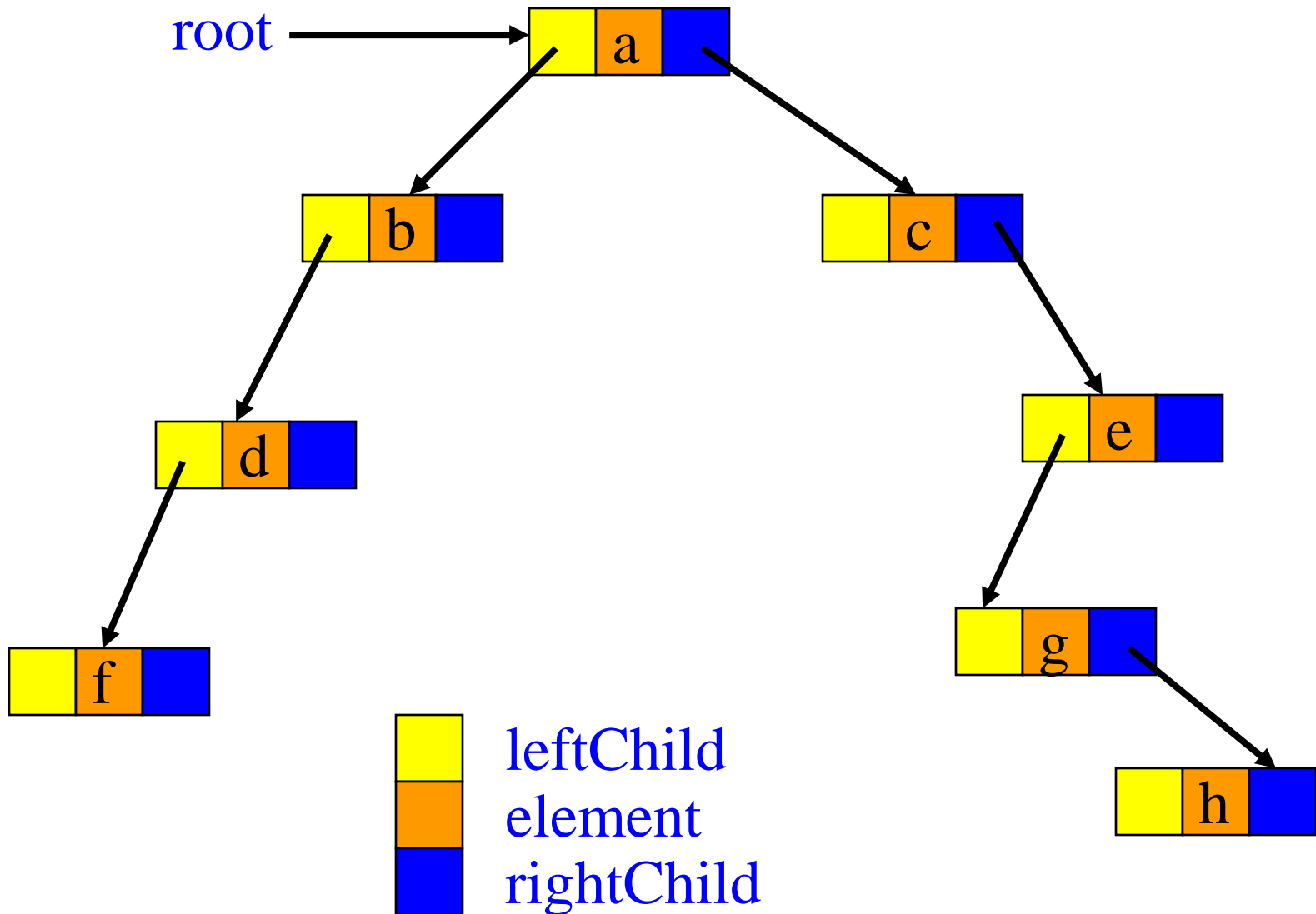
Linked Representation

- Each binary tree node is represented as an object whose data type is **BinaryTreeNode**.
- The space required by an **n** node binary tree is **$n * (\text{space required by one node})$** .

The Class BinaryTreeNode

```
public class BinaryTreeNode
{
    Object element;
    BinaryTreeNode leftChild; // left subtree
    BinaryTreeNode rightChild; // right subtree
    // constructors and any other methods come here
}
```


Linked Representation Example



Some Binary Tree Operations

- Determine the height.
- Determine the number of nodes.
- Make a clone.
- Determine if two binary trees are clones.
- Display the binary tree.
- Evaluate the arithmetic expression represented by a binary tree.
- Obtain the infix form of an expression.
- Obtain the prefix form of an expression.
- Obtain the postfix form of an expression.

Binary Tree Traversal

- Many binary tree operations are done by performing a **traversal** of the binary tree.
- In a traversal, each element of the binary tree is **visited** exactly once.
- During the **visit** of an element, all action (make a clone, display, evaluate the operator, etc.) with respect to this element is taken.

Binary Tree Traversal Methods

- Preorder
- Inorder
- Postorder
- Level order