28.07.2021

The Equivalent ckt. of an Induction Motor

Recap:

- Relative speed of rot. mag. field & rotor bar.
- slip speed ; $n_{slip} = n_{sync.} - n_m$
- slip $\underline{s} = \frac{n_{slip}}{n_{sync.}} \times 100\%$
- $f_r = s f_e$

$$\boxed{\text{I.M.} \Rightarrow \text{equ. ckt. Model.}}$$

- Since the "Induction" of emf / current in the rotor bars (winding) of an I.M is in principle a "TRANSFORMER ACTION".

  Therefore, the equivalent ckt. model of I.M. corresponds to equivalent ckt. model of a Transformer.

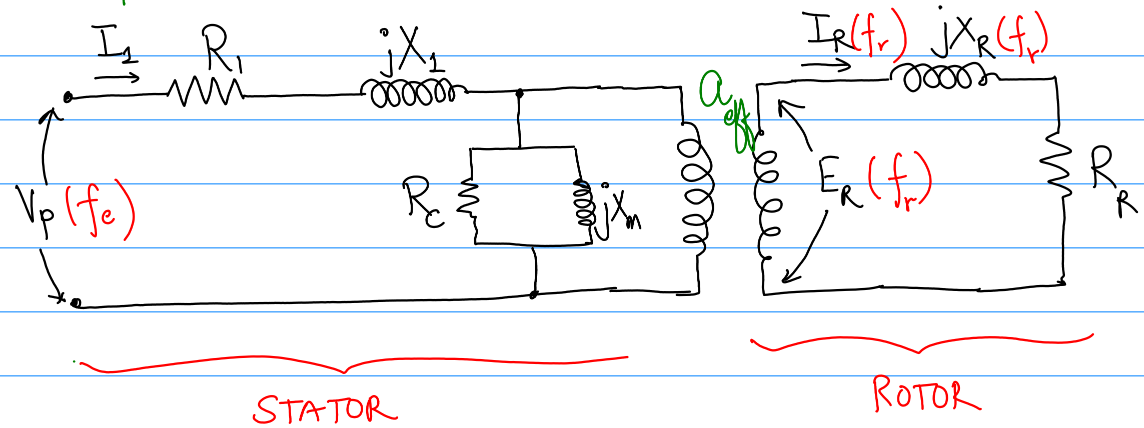
- Also, it is noted that the I.M. is a "Singly excited" machine. That is, there is NO FIELD CURRENT the electrical ckt. model does not have an internal voltage source such as E_A in case sync. machine.

- Thus the equivalent ckt. model of the I.M. will be developed considering:

  (i) the equivalent ckt model of the transformer.

  (ii) the variation in the rotor's electrical frequency with the relative mechanical speed (ie, slip)

Per-phase equivalent ckt. model.



$f_r = s f_e$

a_{eff} = Effective turn-ratio

- An I.M. equivalent ckt. differs from the transformer equivalent ckt. model primarily in effects of varying rotor electrical frequency on the rotor voltage E_R and the rotor impedance X_R

- Hence modeling the rotor ckt. is important.

Rotor Circuit Model :

In general, the greater the relative motion between the stator magnetic field ($\vec{B_s}$) and rotor magnetic field ($\vec{B_R}$), the greater the resulting rotor voltage & frequency.

Case (i) Largest relative motion

- when the rotor is stationary ie, the rotor is mechanically blocked (or locked).

$$S = 1$$

Case (ii) Smallest relative motion (No relative motion)

- when the rotor is moving with sync. speed.

$$S = 0$$

Hence, if the magnitude of the induced rotor voltage in case (i) is E_{RO}, then the magnitude of induced rotor voltage at any slip value is given as

$$\boxed{E_R = s E_{RO}}$$

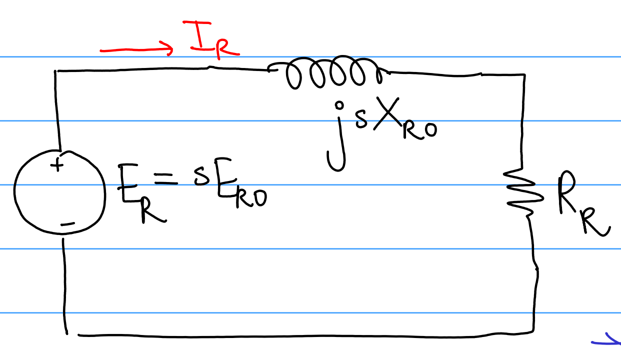
Further, we also know that

$$f_r = s f_e$$

and the reactance $X_R = \underbrace{2\pi f_R}_{\omega} L$

Therefore, $\boxed{X_R = s X_{RO}}$ where X_{RO} is the reactance of the rotor when it is mechanically blocked.

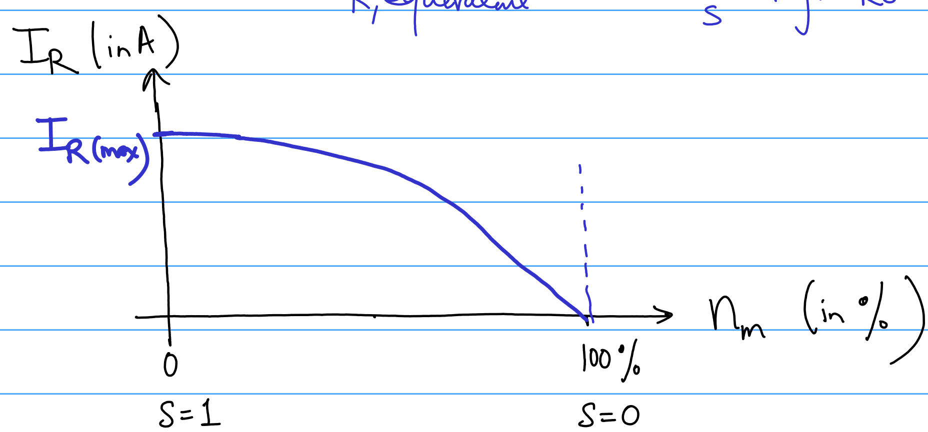
The rotor equivalent ckt. will be



$$I_R = \frac{E_R}{R_R + jX_R}$$

$$\Rightarrow I_R = \frac{sE_{RO}}{R_R + j s X_{RO}}$$

$$Z_{R,equivalent} = \frac{R_R}{s} + jX_{RO}$$

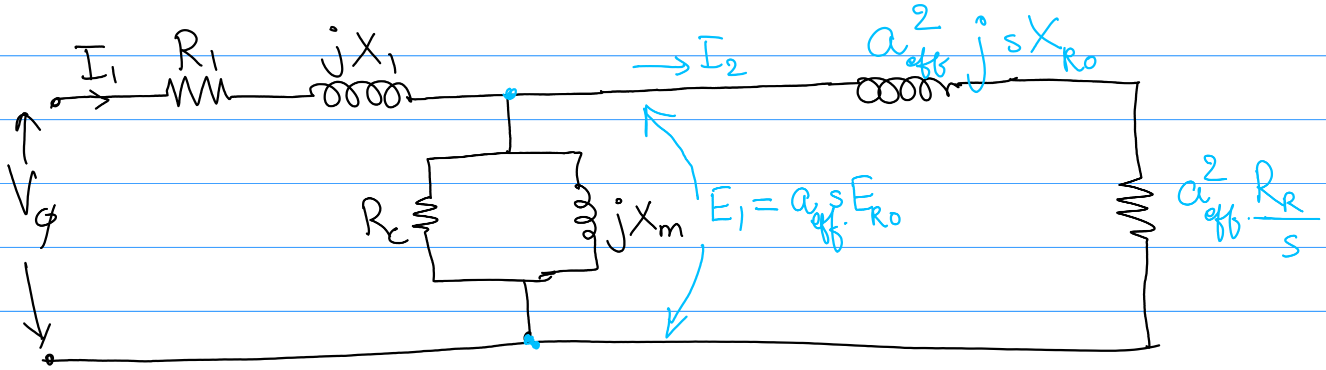


From the discussion of transformer, we know that

$$\frac{V_p}{V_s} = a \quad ; \quad \frac{I_p}{I_s} = \frac{1}{a} \quad ; \quad Z_s' = a^2 Z_s$$

Referred impedance to the primary

Per-phase equivalent ckt. model of an I.M. referred to the stator (primary)



30.07.2021

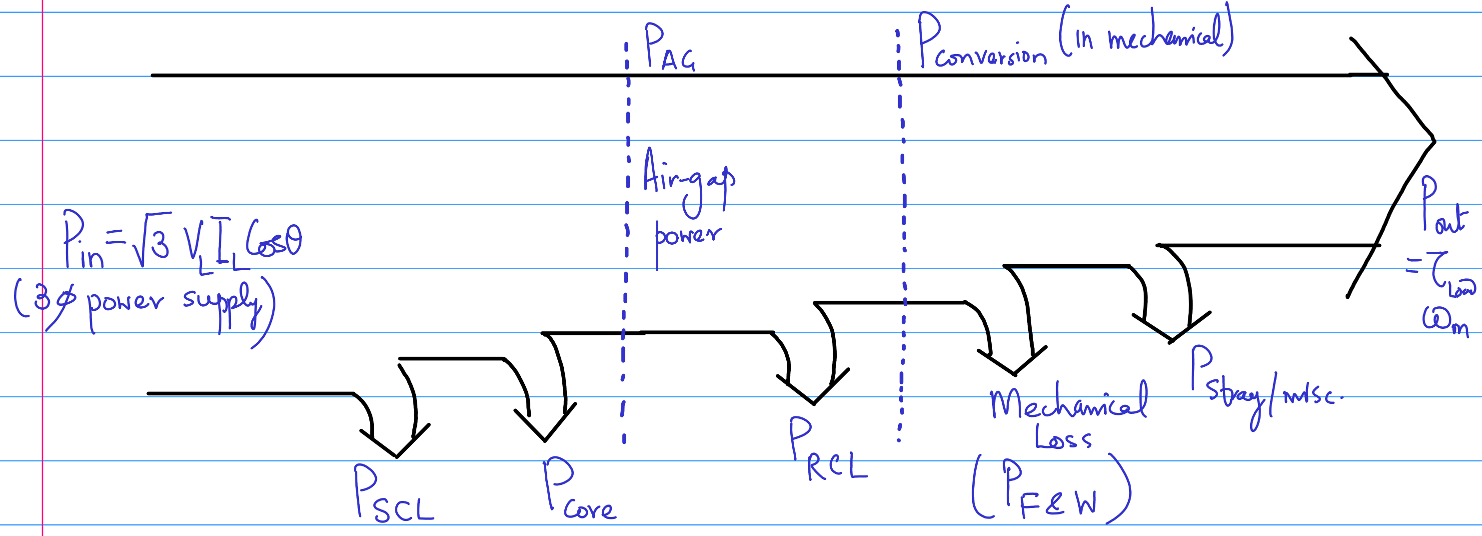
Power and Torque in Induction Motor

Recap: (i) Equivalent ckt. model of I.M. corresponds to the equivalent ckt. model of a transformer.

(ii) Stator ckt. (primary ckt.) remains the same.

(iii) Rotor ckt. (secondary ckt.) is 'SLIP' dependent. (ie, relative motion)

Power flow diagram of I.M. :



- Input electrical power, P_{in} = 3ϕ power supply $= \sqrt{3}\, V_L I_L \cos\theta$
- This input power is given to the stator winding, therefore, there is a copper loss in stator windings (P_{SCL})
- Since the stator winding is done over the laminated core, hence there are some amount of power loss due to hysteresis & eddy current. (P_{core})

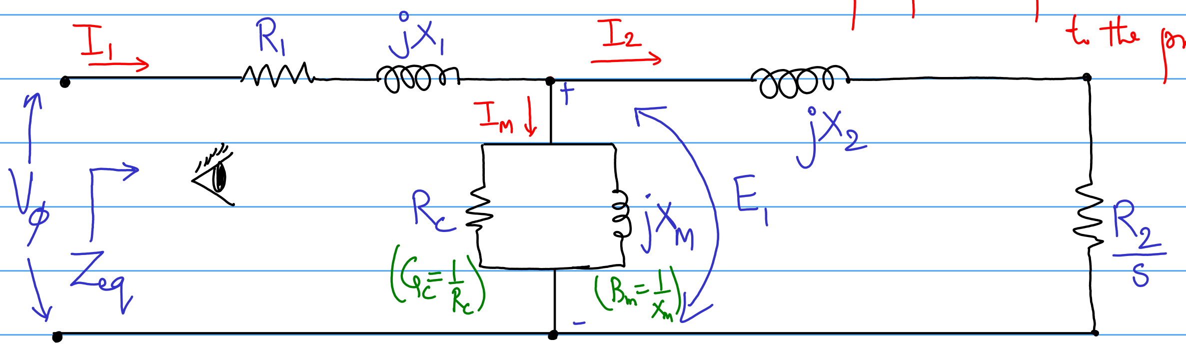
- The electrical power remaining after copper & core losses is transfered to the rotor through the air-gap (P_{AG}).
- Since the power is transfered inform of induced emf in the rotor, and since the rotor is electrically shorted, there is some amount power lost as I^2R_R in the rotor (P_{RCL}, rotor copper loss).
- After P_{RCL}, there is no electrical power loss. Therefore, the remaining power is converted into mechanical power ($P_{conv.}$).
- As a mechanical power loss, we have to account any loss due to friction or windage ($P_{F\&W}$).
- Any other losses incorported as $P_{stray/misc.}$
- The remaining power is available to the load.

$$P_{out} = \tau_{Load} \cdot \omega_m$$

$$\boxed{\underset{(mech.)}{P_{out}} = \underset{(elect.)}{P_{in}} - P_{SCL} - P_{core} - P_{RCL} - P_{F\&W} - P_{stray/misc.}}$$

Relate power & torque :

Consider per-phase equ. ckt. referred to the primary.



here,

$$X_2 = a_{eff}^2 X_{RO}$$

$$R_2 = a_{eff}^2 R_R$$

$$E_1 = a_{eff} E_{RO}$$

$$I_R = \frac{E_{RO}}{\frac{R_R}{s} + jX_{RO}}$$

- From the equivalent ckt., one can determine the input current (per phase) in the motor.

$$I_1 = \frac{V_\phi}{Z_{eq}}$$

here,

$$Z_{eq} = R_1 + jX_1 + \left[\frac{1}{G_c - jB_m + \left\{ \frac{1}{\frac{R_2}{s} + jX_2} \right\}} \right]$$

Therefore, knowing the value of I_1, we can determine, the stator copper loss & core loss.

$$P_{SCL} = 3 I_1^2 R_1$$

$$P_{core} = \frac{3 E_1^2}{R_c} = 3 G_c E_1^2$$

So, from this we get the power available in air-gap

$$P_{AG} = \underbrace{P_{in}}_{\text{input}} - \underbrace{P_{SCL}}_{3I_1^2R_1} - \underbrace{P_{core}}_{3G_cE_1^2}.$$

Now, out of available air-gap power (P_{AG}) the only power consumed is in the resistor R_2/s.

So, one can say. $P_{AG} = 3 I_2^2 \frac{R_2'}{s}$

however, the rotor copper loss is expressed as

$$P_{RCL} = 3 I_2^2 R_2$$

Comparing P_{AG} & P_{RCL} we get

$$\boxed{P_{RCL} = s P_{AG}}$$

$$P_{conv.} = P_{AG} - P_{RCL} = (1-s) P_{AG}$$

$$\boxed{P_{conv.} = (1-s) P_{AG}}$$

Finally, $P_{out} = P_{conv.} - \underbrace{P_{F\&W}}_{\text{if given}} - \underbrace{P_{stray/misc.}}_{\text{if given}}$

What is Induced Torque (or Developed Torque) in the I.M. ?

- Basically it is the torque generated by the internal electric to mechanical power conversion.

- This internal induced torque is higher than the output torque at the shaft of the rotor.

$$\Rightarrow \quad \tau_{ind} = \frac{P_{conv.}}{\omega_m} = \frac{\cancel{(1-s)}\,P_{AG}}{\cancel{(1-s)}\,\omega_{sync}}$$

$$\Rightarrow \quad \boxed{\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}}$$

Note: P_{AG} & ω_{sync} does not vary once the input to the stator is fixed.