

Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 3

1. Find an LU factorization of the matrices, if possible.

$$\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$$

2. Solve the equation $Ax = b$ by using the LU factorization given for A .

$$A = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

3. Use the definition of Ax to write the matrix equation as a vector equation.

$$\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

4. Determine if $Ax = b$ has a solution by checking whether b is a linear combination of a_1, a_2 , and a_3 , where i^{th} column of A is equal to a_i .

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, a_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

5. Give a geometric description of $\text{Span}\{v_1, v_2\}$ for the vectors

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Can you write down an algebraic equation for it?

6. Given $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, find one nontrivial solution of $Ax = 0$ by inspection. For which values of b , $AX = b$ is consistent.

7. Let A be the matrix given below and $v_p = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ be a particular solution to $AX = b$. Then find its all solutions.

$$\begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

8. Can every vector of \mathbb{R}^4 be written as a linear combination of columns of A ?

$$A = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

9. For any two matrices A, B of size 2×2 , what is the relation between $\text{Col}(A)$ and $\text{Col}(AB)$?