# Guided Waves :→

Inside side of Conductor : $E^{\parallel} = 0$ — (1)

$B^{\perp} = 0$ — (2)

$$\vec{E}(x,y,z,t) = \vec{E}_0(x,y)\, e^{i(kz-\omega t)} \quad \text{— (A)}$$

$$\vec{B}(x,y,z,t) = \vec{B}_0(x,y)\, e^{i(kz-\omega t)} \quad \text{— (B)}$$

— (3)

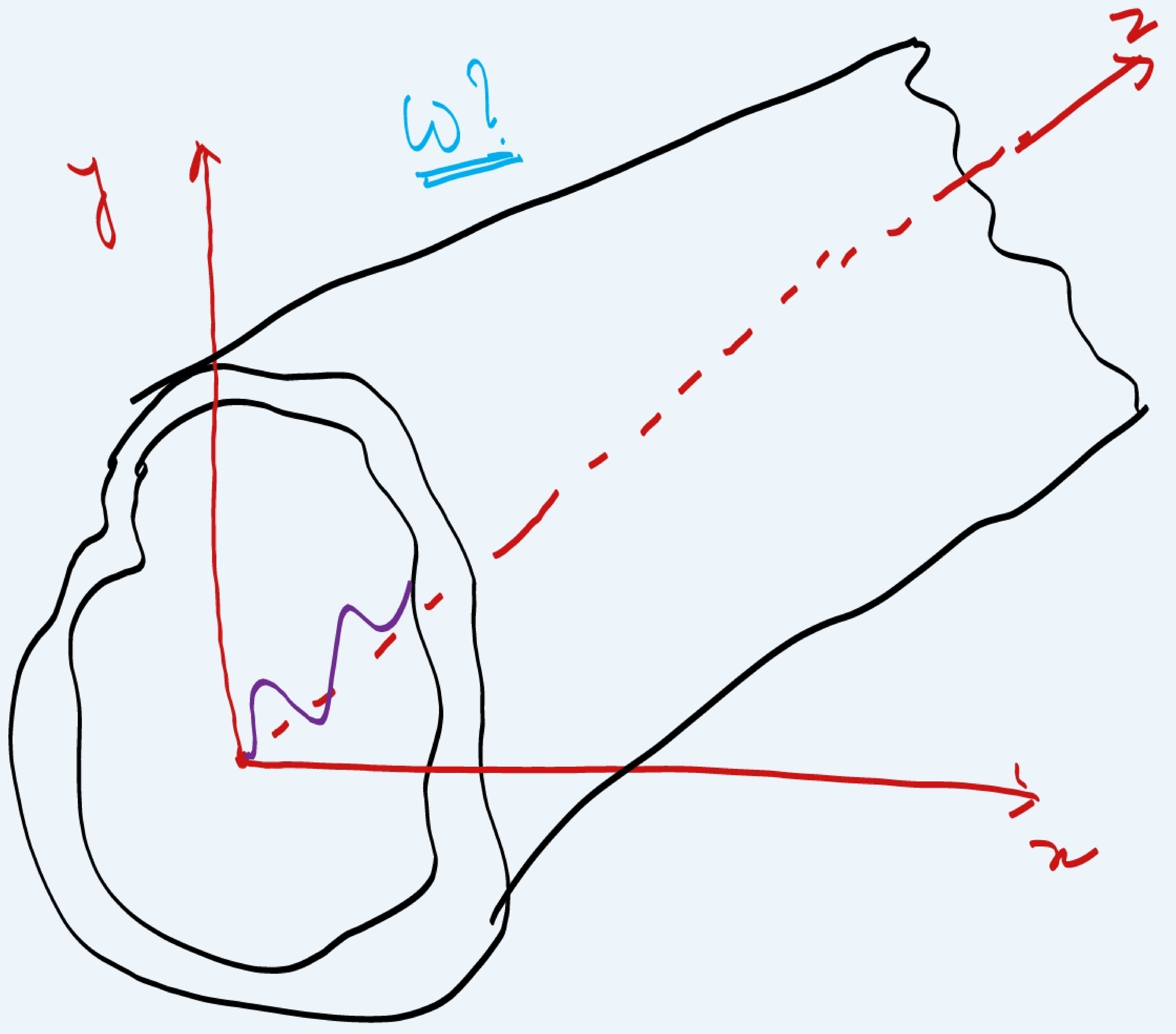
$$[B_{0x}\hat{x} + B_{0y}\hat{y} + B_{0z}\hat{z}]\, e^{i(kz-\omega t)}$$

$$\boxed{\begin{array}{l} \nabla\cdot\vec{E} = 0 \quad \text{(I)} \\ \nabla\cdot\vec{B} = 0 \quad \text{(II)} \end{array}}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{(III)}$$

$$\nabla\times\vec{B} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \quad \text{(IV)}$$

— (4)



$$\vec{E}_0 = E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z} \quad , \quad \vec{B}_0 = B_{0x}\hat{x} + B_{0y}\hat{y} + B_{0z}\hat{z}.$$

$$E_x = E_{0x} e^{i(kz-\omega t)} \hat{x}$$

$$E_y = E_{0y} e^{i(kz-\omega t)}$$

$$E_z = E_{0z} e^{i(kz-\omega t)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}\left[ B_0 e^{i(kz-\omega t)} \right] = i\omega B_0 e^{i(kz-\omega t)}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{vmatrix} = i\omega B_0 e^{i(kz-\omega t)}$$

x- Component: $\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] = i\omega B_{0x} e^{i[kz-\omega t]}$

$$\Rightarrow \left[\frac{\partial E_{0z}}{\partial y} - ik E_{0y}\right] e^{i(kz-\omega t)} = i\omega B_x \Rightarrow \boxed{\frac{\partial E_z}{\partial y} - ik E_y = i\omega B_x}$$

y-component!

$$-\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] = i\omega B_{0y}\, e^{i(kz-\omega t)}$$

$$\boxed{-\left[\frac{\partial E_z}{\partial x} - ikE_x\right] = i\omega B_y}$$

$$\Rightarrow ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y.$$

$$\begin{vmatrix} x & y & \hat{z} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

z-component:

$$\boxed{\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} = i\omega B_{0z} e^{i(kz-\omega t)} = i\omega B_z.}$$

$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t}.$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ B_x & B_y & B_z \end{vmatrix} = \frac{-1}{c^2}(i\omega) E_0 e^{i(kz-\omega t)}$$

$$= -\left(\frac{i\omega}{c^2}\right) E_x$$

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}.$$

$$\boxed{\frac{\partial B_z}{\partial y} - ikB_y = -\frac{i\omega}{c^2} E_x.}$$

(5)

(1) $$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z$$

(2) $$\frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x$$

(3) $$\frac{\omega}{c^2}\,\cancel{ik}E_x - \frac{\omega}{c^2}\frac{\partial E_z}{\partial x} = i\frac{\omega^2}{c^2} B_y$$

(4) $$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -\frac{i\omega}{c^2} E_z$$

(5) $$k\frac{\partial B_z}{\partial y} - ik^2 B_y = -\frac{i\omega \cancel{k}}{\cancel{c^2}} E_x$$

(6) $$ikB_x - \frac{\partial B_z}{\partial x} = -\frac{i\omega}{c^2} E_y$$

✓ $$E_x = \frac{i}{\left[\frac{\omega^2}{c^2} - k^2\right]}\left[k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right] \quad - \quad (6)$$

✓ $$E_y = \frac{i}{\frac{\omega^2}{c^2} - k^2}\left[k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right]$$

$$(2)\times\frac{\omega}{c^2} + k\times(6) \Rightarrow B_x = \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2}\left[k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right]$$ ✓

$$(3)\times\frac{\omega}{c^2} + (5)\times k \Rightarrow B_y = \frac{i}{(\omega/c)^2 - k^2}\left[k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right]$$ ✓

$$\underline{\nabla\cdot\vec{E}=0} \Rightarrow \frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0$$

$$\vec{E}=\vec{E}_0\, e^{i(kz-\omega t)}$$

$$\vec{E}_0=E_{0x}\hat{x}+E_{0y}\hat{y}+E_{0z}\hat{z}$$

$$\left[\frac{\partial E_{0x}}{\partial x}+\frac{\partial E_{0y}}{\partial y}+\frac{\partial E_{0z}}{\partial z}\right]e^{i[kz-\omega t]}=0$$

$$E_x=E_{0x}\,e^{i(kz-\omega t)}\,\hat{x}$$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+ikE_z=0$$

$$\frac{i}{\left(\omega/c\right)^2-k^2}\left[k\frac{\partial^2 E_z}{\partial x^2}+\cancel{\omega\frac{\partial^2 B_z}{\partial x\partial y}}+k\frac{\partial^2 E_z}{\partial y^2}-\cancel{\omega\frac{\partial^2 B_z}{\partial x\partial y}}\right]+ikE_z=0$$

$$\nabla^2 V=-\rho/\epsilon_0 \quad =0$$

$$\Rightarrow \boxed{\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\left[\frac{\omega^2}{c^2}-k^2\right]E_z=0}$$

$B_z$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \left[\left(\frac{\omega}{c}\right)^2 - K^2\right] E_z = 0$$

$$\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \left[\left(\frac{\omega}{c}\right)^2 - K^2\right] B_z = 0.$$

Hollow Conductor

$$E = \underline{\vec{E}_0}\, e^{i[kz - \omega t]}$$

$$\downarrow$$

$$\vec{E}_0 = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$$

$$\downarrow$$

$$E_x, E_y \longrightarrow E_z$$

(1) If $E_z = 0$, $B_z \neq 0$ : TE

2. If $E_z \neq 0$, $B_z = 0$ : TM

3. If $E_z \neq 0$, $B_z \neq 0$. : TEM

Hollow Guided wave

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TEM No wave

TEM:

$E_z = 0$, $\nabla \cdot E = 0$, $\left\{ \begin{array}{l} \dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial x} = 0 \\ \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = 0 \end{array} \right\}$

$B_z = 0$, $\nabla \times E = -\dfrac{\partial B}{\partial t}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & \cancel{E_z}^{0} \end{vmatrix}$$

$\hat{x}$ $\quad -\dfrac{\partial E_y}{\partial z} = 0$

$\hat{y}$ $\quad -\dfrac{\partial E_x}{\partial z} = 0$

$\oiint \nabla \times \vec{E} \cdot da = \oint \vec{E} \cdot \vec{dl}$

$E_x, E_y$ are not function z.

$\Downarrow$

$\boxed{\nabla \times E = 0.}$

$E = \nabla \phi$

$0 =$

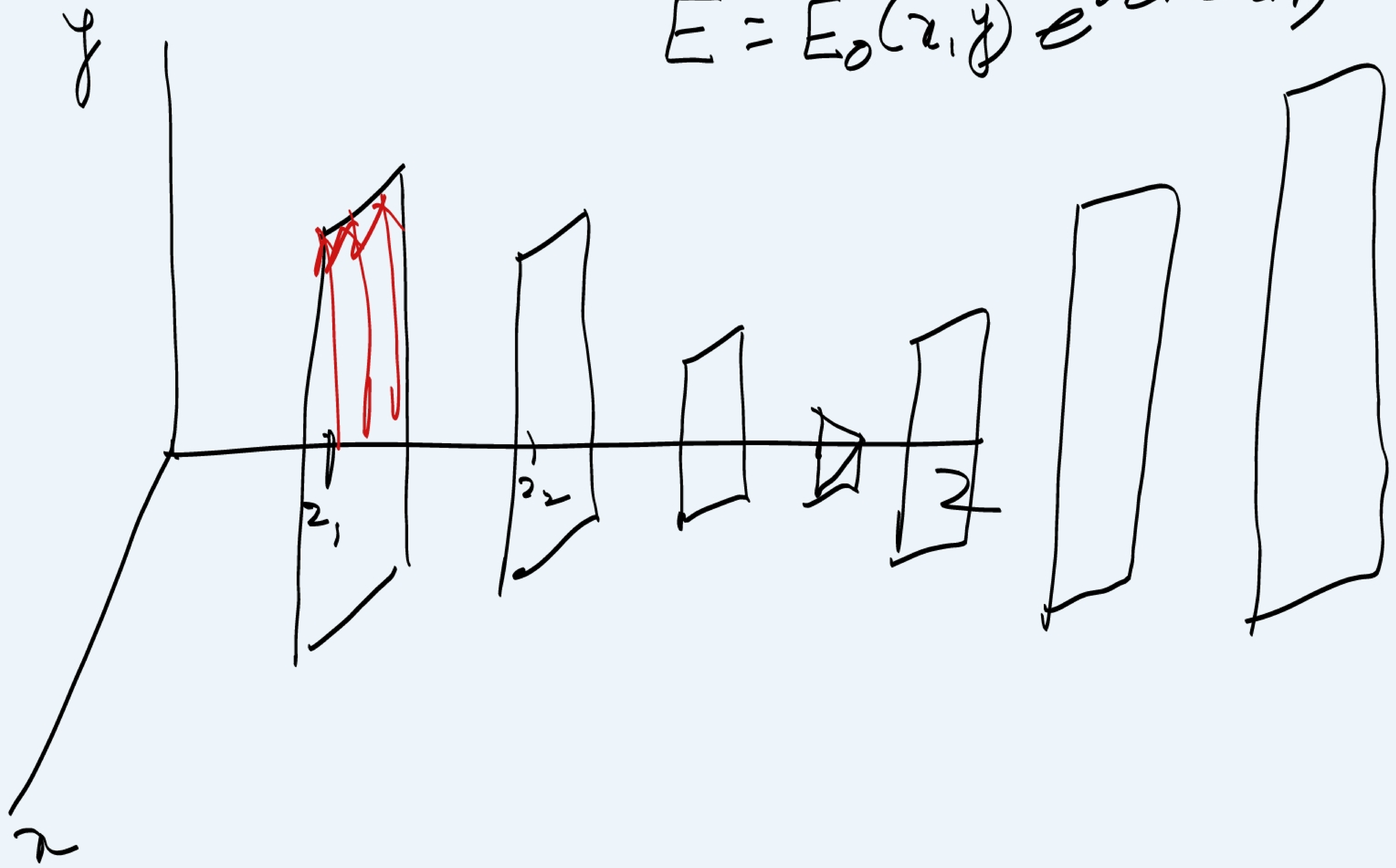
$\nabla \cdot E = 0$

$\nabla^2 \phi = 0$

Laplace

$\nabla^2 \phi = 0.$

$\phi = $ constant




$$E = E_0(x,y)\, e^{i(kz-\omega t)}$$

$$\vec{E} = E_0[x,y,z]\, e^{i(kz-\omega t)}.$$

# Rectangular Waveguide :-

TE : $E_z = 0$

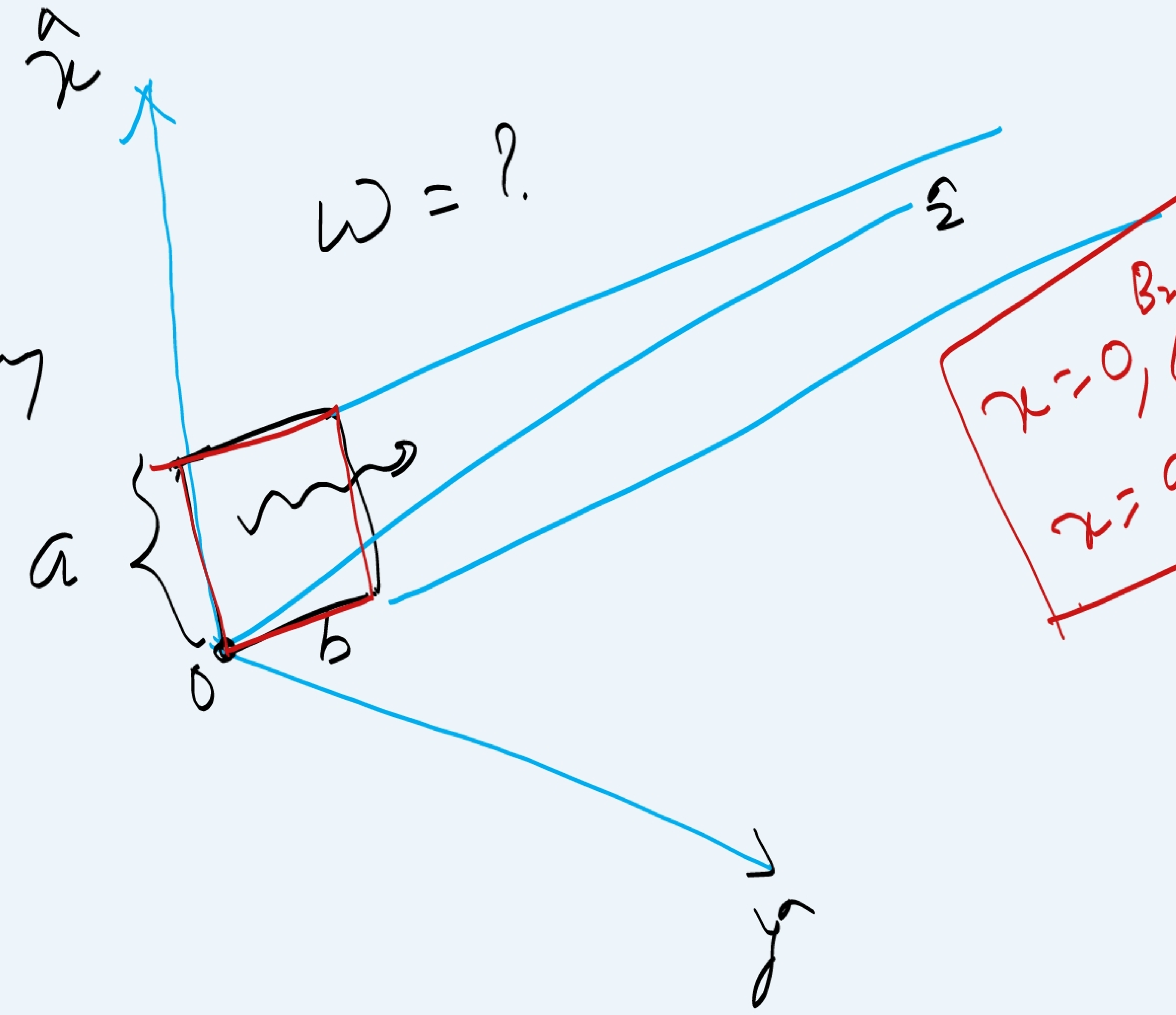
$$\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \left[\left(\frac{\omega^2}{c^2}\right) - k^2\right] B_z = 0 \quad -(1)$$

$$B_z(x,y) = X(x)\,Y(y) \quad -(2)$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + \left[\left(\frac{\omega}{c}\right)^2 - k^2\right] XY = 0$$

$$\Rightarrow \underbrace{\frac{1}{X}\ddot{X}}_{-k_x^2} + \underbrace{\frac{\ddot{Y}}{Y}}_{-k_y^2} + \left(\frac{\omega}{c}\right)^2 - k^2 = 0$$

Min. frequency




$x = 0,\ B_z = 0$

$x = a,\ B_z = 0$

$\parallel x$

$$\Rightarrow k^2 = \left(\frac{\omega}{c}\right)^2 - \left[k_x^2 + k_y^2\right]$$

$$= \left(\frac{\omega}{c}\right)^2 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\frac{\ddot{X}}{X} = -K_x^2 \Rightarrow \ddot{X} + K_x^2 X = 0$$

$$X(x) = A \sin K_x x + B \cos K_x x$$

$x = 0$, $B = 0$ $\Rightarrow X(x) = A \sin K_x x$ $\rightarrow X(x) = A \sin(K_x x)$

$= A \sin\left(\frac{m\pi}{a} x\right)$.

$x = a$, $X(a) = 0$ $\Rightarrow K_x = \frac{m\pi}{a}$, $m = 0, 1, 2, \ldots$

$\ddot{Y} + Y K_y^2 = 0$, $K_y = \frac{n\pi}{b}$, $n = 0, 1, 2, \ldots$ $\quad Y(y) =$

$Y = C \sin(\underline{K_y y}) + D \cos(K_y y) \Rightarrow Y(y) = C \sin\left(\frac{n\pi}{b} y\right)$

$$B_z = X(x) \, \& \, Y(y) = B_0 \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right)!$$

$$K^2 = \left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$E = E_0 e^{i(\underline{K}x - \omega t)}$$

$$\tilde{K} = K_1 + iK_2$$

$$K = \sqrt{\underline{\omega}^2 - \underbrace{c^2\pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]}_{\omega_{mn}}} = \frac{1}{c}\left[\sqrt{\omega^2 - \omega_{mn}^2}\right]$$

$\omega_{00}$ is not possible

$\nu_{10}, \nu_{01}$

$$\omega > \omega_{mn} \qquad \omega_{mn} = c\pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}, \quad \underline{\omega_{10}} = \frac{c\pi}{a}, \quad \omega_{01} = \frac{c\pi}{a}.$$

$$\boxed{v = \frac{\omega}{K}} = \frac{\omega c}{\sqrt{\omega^2 - \omega_{mn}^2}}, \quad \underline{v > c}$$

$$v_g = \frac{d\omega}{dK} = \frac{1}{\frac{dK}{d\omega}} < c$$

$$= c \cdot \underline{\sqrt{\omega^2 - \omega_{mn}^2}}$$