$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Scalar function: $T(x,y,z)$

∇T : Gradient : 3D Slope : $\underbrace{\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}}$ } vector

Vector field function :- $\vec{F}(x,y,z) = F_x(x,y,z)\,\hat{i} + F_y(x,y,z)\,\hat{j} + F_z(x,y,z)\,\hat{k}$

$\vec{\nabla}\cdot\vec{F}$: Divergence $= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$ } $\frac{\text{Flux}}{\text{Area}}$ volume } Scalar

$\vec{\nabla}\times\vec{F}$: Curl $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{vmatrix} = \frac{\text{Circulation}}{\text{Volume (Area)}}$ } vector

$$\vec{E}(\vec{r}) \propto \frac{\hat{r}}{r^2} \qquad \boxed{r=0}$$

$$\hat{r} = \frac{\vec{r}}{|r|}$$

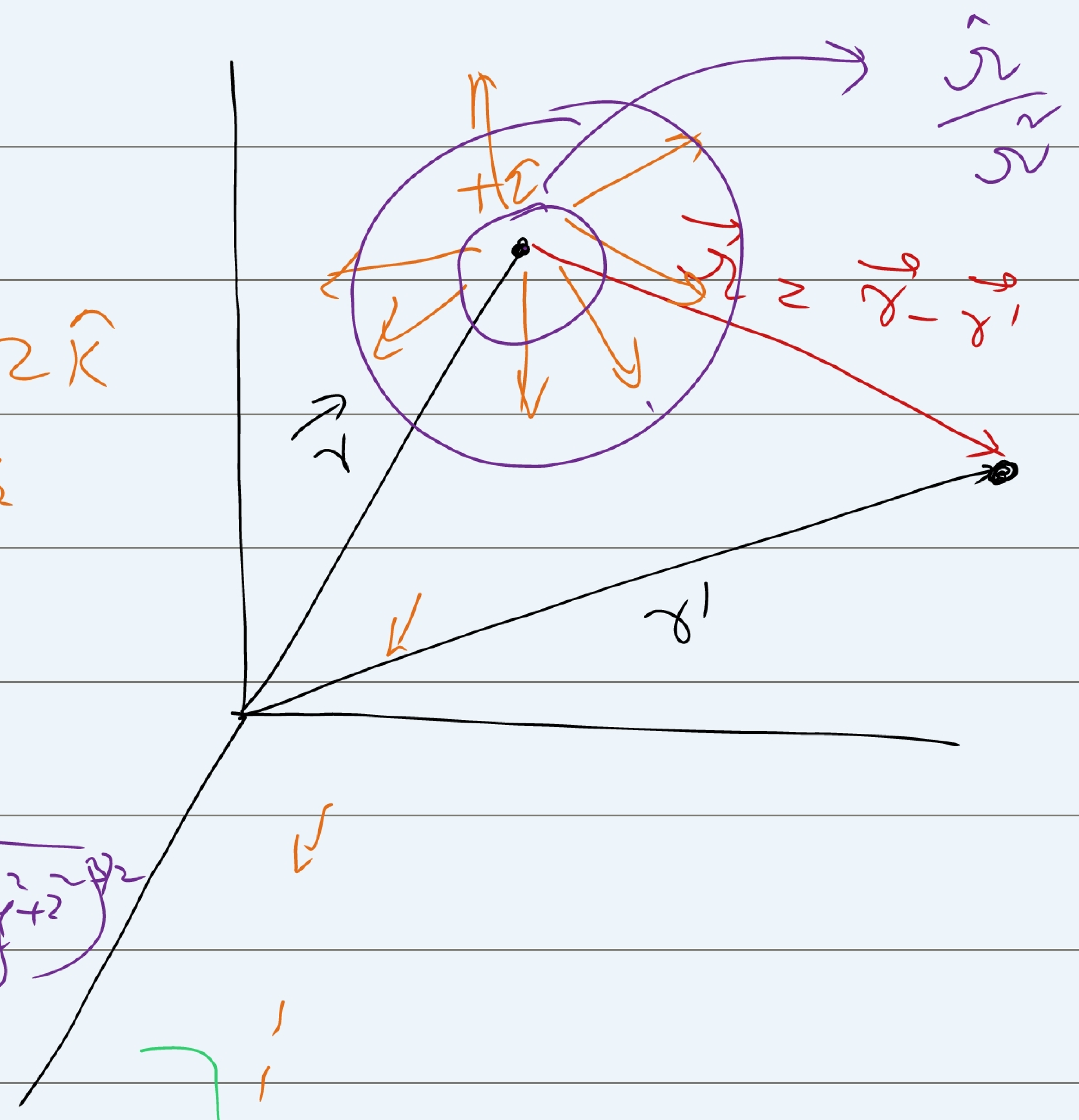
$$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = \nabla\cdot\frac{\vec{r}}{r^3}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|r| = \left[x^2 + y^2 + z^2\right]^{1/2}$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[x\hat{i} + y\hat{j} + z\hat{k}\right]\frac{1}{(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{\partial}{\partial x}\left[\frac{x}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{\partial}{\partial y}\left[\quad\right] + \frac{\partial}{\partial z}\left[\quad\right]$$



$\nabla T \to$ vector $\left[\begin{array}{l}\nabla\cdot(\nabla T) = \nabla^2 T \\ \nabla\times(\nabla T) = 0\end{array}\right.$

scalar $\leftarrow \nabla\cdot F$ — $\nabla(\nabla\cdot F) \neq \nabla^2 T$

$\nabla\times f$ $\left[\begin{array}{l}\nabla\cdot(\nabla\times F) \to \\ \nabla\times(\nabla\times f)\end{array}\right.$

Second Derivative:

$$\nabla\cdot(\nabla T) = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right]$$

Divergence of Gradient = Laplacian

$$= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T$$

∇^2 → Laplacian

$$\vec{E} = -\nabla V \qquad \nabla\cdot E = -\nabla^2 V$$

1. Few second derivative properties ∇.

Gauss's Divergence Theorem

Stokes Theorem

Spherical polar coordinate

Curl of Gradient :→

$$\nabla \times (\nabla T) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{vmatrix} = 0$$

$$\nabla T = \hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}$$

$$\hat{i}\underbrace{\left[\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial y}\right)\right]}_{0}$$

$$+\hat{j}\left[\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial z}\right)\right] + \hat{k}$$

$$\vec{F} = \nabla U$$

$$\oiint (\nabla \times \vec{F}) \cdot \vec{da} = \oint \vec{F} \cdot \vec{dl}$$

$$\nabla(\nabla\cdot F) = \hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}.$$

$$T = \nabla\cdot F = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}.$$

$$\nabla\cdot(\nabla T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$\vec{V} = \vec{\nabla}\times\vec{F}$$

$$\nabla\times F = \begin{vmatrix} \hat{i} & j & k \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{vmatrix}$$

Divergence of Curl

$$\nabla\cdot(\nabla\times\vec{F}) = \frac{\partial}{\partial x}[\nabla\times f]_x + \frac{\partial}{\partial y}[\nabla\times f]_y + \frac{\partial}{\partial z}(\nabla\times F)_z$$

$$= \frac{\partial}{\partial x}\left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]$$

$$= 0$$

$$\frac{\partial}{\partial x}\left[\frac{x}{(x^2+y^2+z^2)^{3/2}}\right] = (x^2+y^2+z^2)^{-3/2} + x\cdot\left(-\frac{3}{2}\right)\cdot\left[x^2+y^2+z^2\right]^{-5/2}\cdot 2x$$

$$= \underline{(x^2+y^2+z^2)^{-3/2}} + (-3)\,x^2\cdot\left[x^2+y^2+z^2\right]^{-5/2}$$

$$+$$

$$\frac{\partial}{\partial y}\left[\frac{y}{(x^2+y^2+z^2)^{3/2}}\right] = \underline{(x^2+y^2+z^2)^{-3/2}} + (-3)\,y^2\left[x^2+y^2+z^2\right]^{-5/2}$$

$$+$$

$$\frac{\partial}{\partial z}\left[\frac{z}{(x^2+y^2+z^2)^{3/2}}\right] = \underline{(x^2+y^2+z^2)^{-3/2}} + (-3)\,z^2\left[x^2+y^2+z^2\right]^{-5/2}$$

$$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 3r^{-3} - 3r^2 r^{-5} = 0$$

$T(x, y, z)$

$\vec{F}(x, y, z) = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

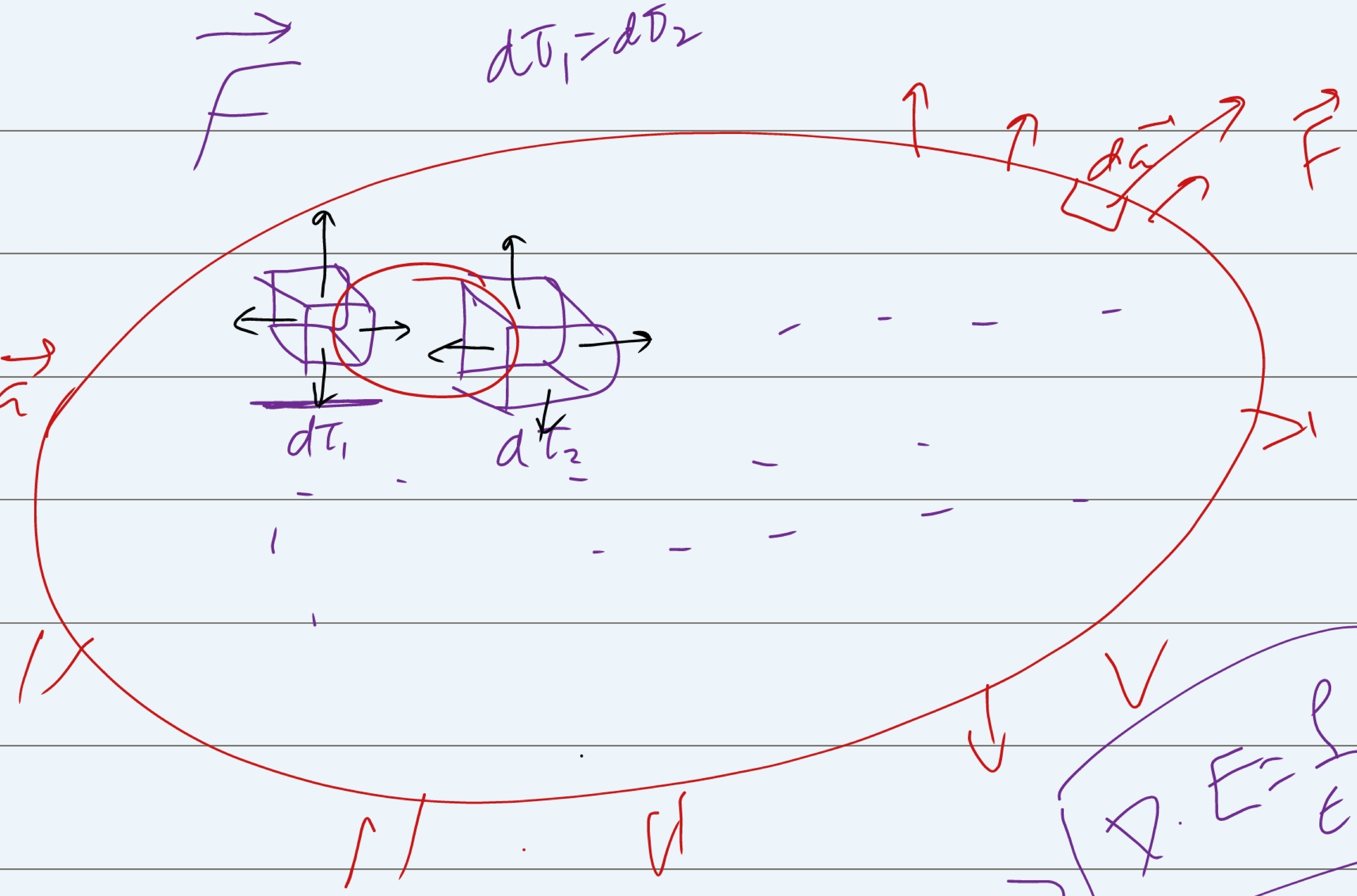
$\vec{E}(x, y, z) = x^2 y \hat{i} + z^2 y \hat{j} + k z^2 x^2 \hat{k}$, $\hat{E} = \dfrac{\vec{E}}{|E|}$

Curl of Gradient $\nabla \times \nabla T = 0$

Divergence of Curl $\nabla \cdot (\nabla \times \vec{F}) = 0$

$$\begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ F_x & F_y & F_z \end{vmatrix}$$

Gauss Divergence Theorem :→



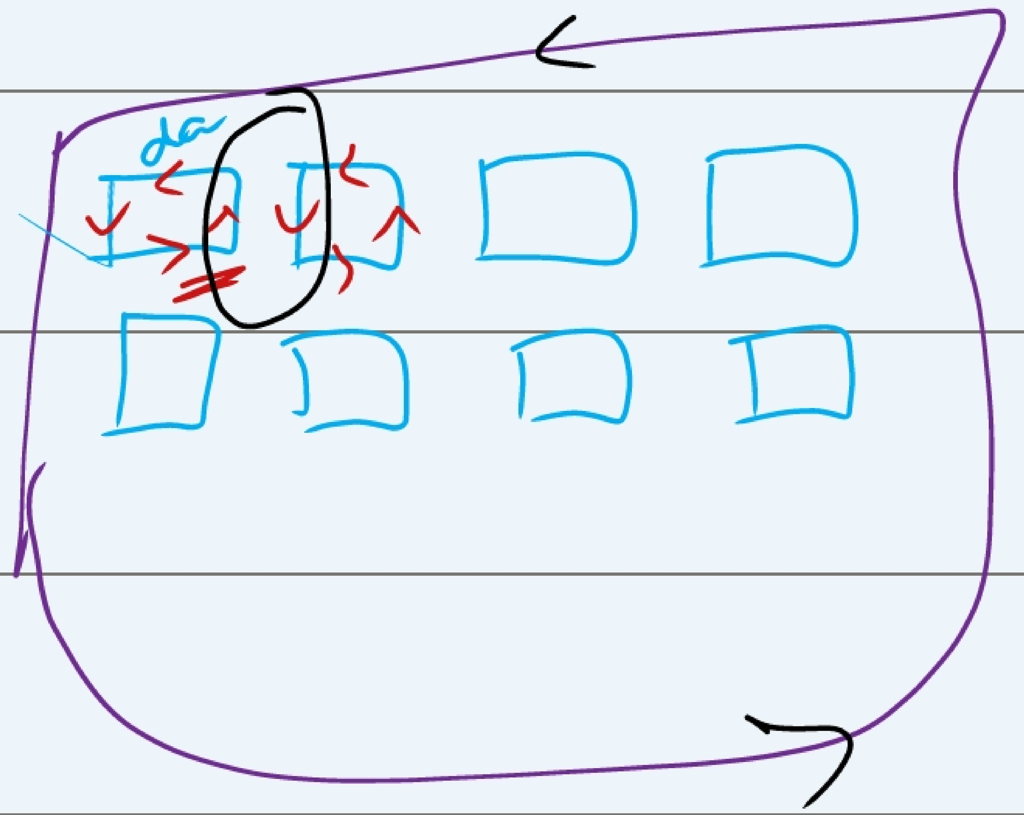
Total Flux $= \iiint_V (\nabla \cdot \vec F)\, d\tau_1 = \iint \vec F \cdot d\vec a$

$= \frac{\text{Flux}}{\text{volume}} \times \text{volume}$

$$\boxed{\frac{Q_{enc}}{\epsilon_0} = \iint \vec E \cdot d\vec a} = \iiint_V (\nabla \cdot E)\, d\tau = \frac{Q_{enc}}{\epsilon_0} = \frac{\int \rho \, d\tau}{\epsilon_0} \Rightarrow \boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$$

Stoke's Theorem: → Net Circulation $\vec{F}$

$$\text{Circulation} = \iint (\nabla \times \vec{F}) \cdot d\vec{a} = \oint \underline{\underline{\vec{F} \cdot d\vec{l}}}$$



$$\frac{\text{Circulation}}{\text{Area}} \times \text{Area}$$

$$\oint \vec{B} \cdot d\vec{l}$$