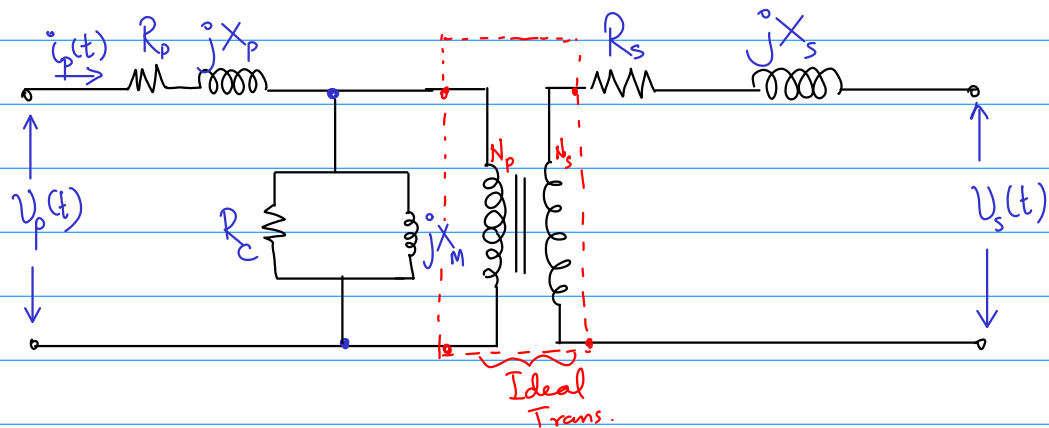


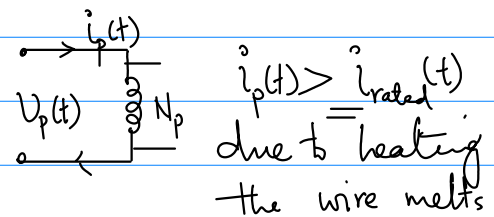
## Equivalent ckt. model of a real transformer



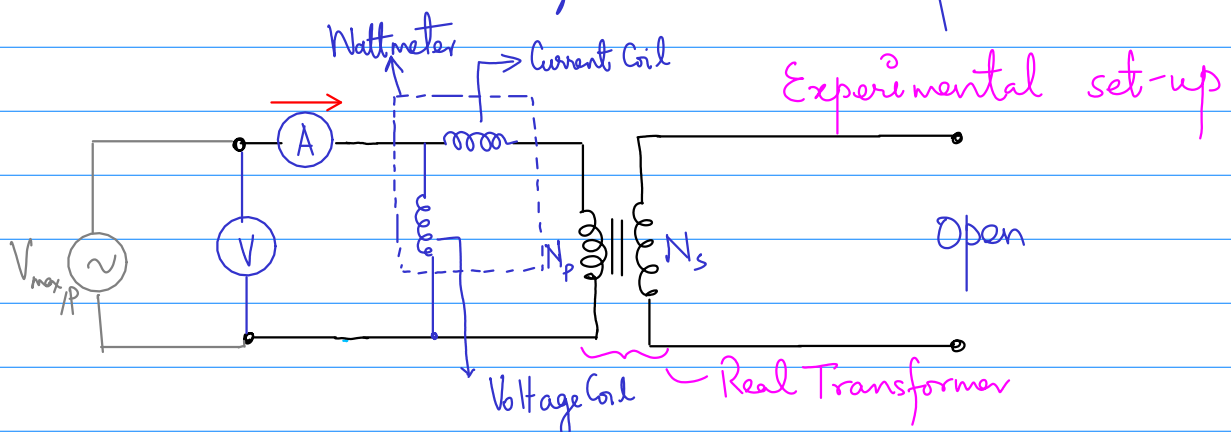
Experimental determination of ckt. elements of the equivalent ckt. model of a real transformer.

### 1. Open-circuit Test :

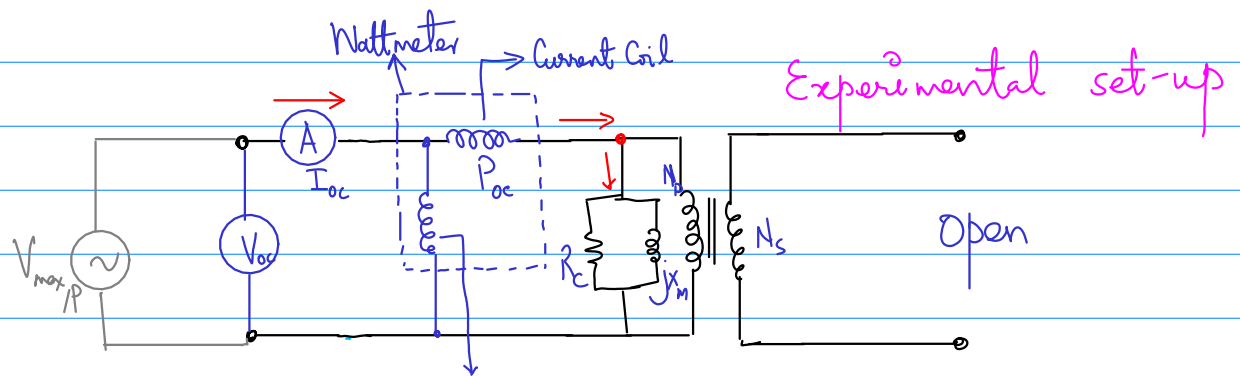
- The secondary terminals of transformer is OPEN.
- The primary terminals are connected to a FULL-RATED line voltage.



- You need voltmeter, ammeter and a wattmeter to measure voltage, current & power.

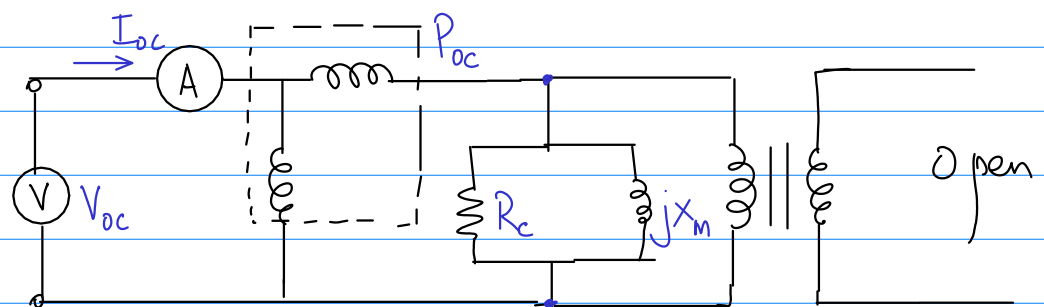


Under this experimental condition, all the input current must be flowing through the excitation branch.

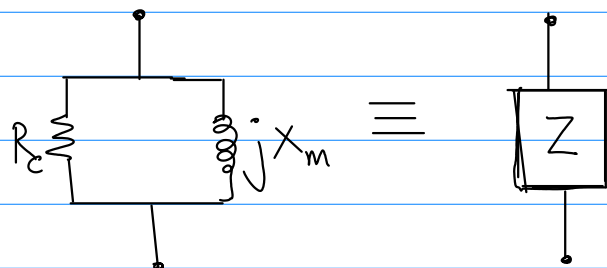


That is, the series elements  $R_p$  &  $X_p$  are assumed to be too small in comparison to the elements in the excitation branch.

Essentially all the input voltage is dropped across the excitation branch.



Excitation branch



For  $R_c$  (Core-Resistance)  
we define.

$$G_c = \frac{1}{R_c} \quad (\text{Core-Conductance})$$

For  $X_m$  we define

$$B_m = \frac{1}{X_m} \quad (\text{Core-Susceptance})$$

Therefore, the admittance of the excitation branch

$$Y_E = G_c - jB_m = \frac{1}{R_c} - j \frac{1}{X_m}$$

hence,

$$Z_E = \frac{1}{Y_E} \quad \text{or,} \quad Y_E = \frac{1}{Z_E}$$

here, the measurable quantities are:  $V_{oc}$ ,  $I_{oc}$ , &  $P_{oc}$

$$Z_E = \frac{V_{oc}}{I_{oc}}$$

we also know that

$$P_{oc} = V_{oc} I_{oc} \underbrace{\cos \theta}_{\text{power factor}}$$

$$\Rightarrow \cos \theta = \frac{P_{oc}}{V_{oc} \cdot I_{oc}}$$

$$\Rightarrow \theta = \left[ \cos^{-1} \frac{P_{oc}}{V_{oc} I_{oc}} \right] = \text{Impedance angle}$$

determined

$\Rightarrow$  We have, both the magnitude & angle of the impedance offered by the excitation branch.

We can also write,

$$Y_{IE} = \frac{I_{oc}}{V_{oc}} \angle -\theta$$

$$\Rightarrow \left[ \frac{1}{R_c} - j \frac{1}{X_m} \right] = \frac{I_{oc}}{V_{oc}} \angle -\theta$$

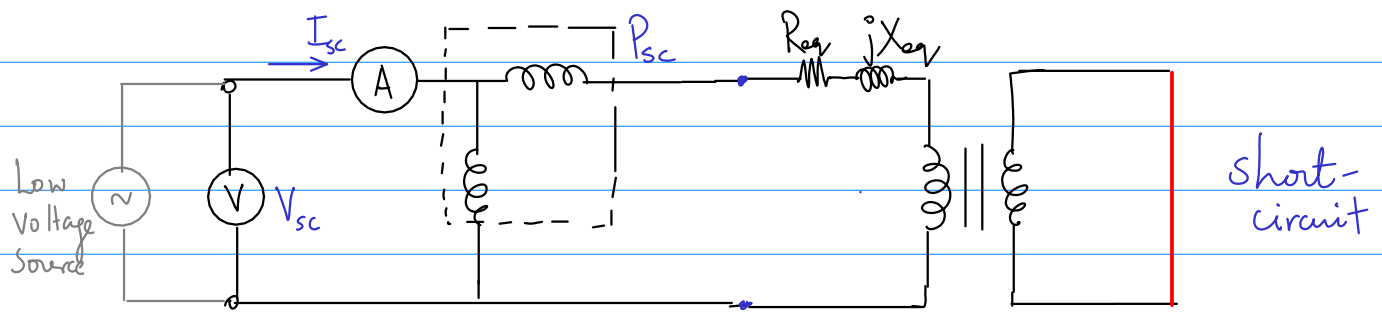
Convert into Rectangular form.  
 $(R + jX)$

$$\Rightarrow \boxed{\begin{aligned} R_c &= \frac{1}{\text{Real Part}} \\ X_m &= \frac{1}{\text{Imaginary Part}} \end{aligned}}$$

2

## SHORT-CIRCUIT TEST :

- The secondary-terminals are short-circuited.
- The primary terminals are connected to low-voltage source.
- We need voltmeter, ammeter & wattmeter.



In this experimental condition since the input voltage is so low that we assume there will be very low (or negligible) current flows through the excitation branch. Therefore, we can safely ignore the excitation branch and hence all voltage is dropped across the series elements.

Since we are measuring  $V_{sc}$ ,  $I_{sc}$ ,  $P_{sc}$

$$|Z_{sc}| = \frac{V_{sc}}{I_{sc}}$$

$$\text{Impedance Angle } \theta = \cos^{-1} \left( \frac{P_{sc}}{V_{sc} \cdot I_{sc}} \right)$$

$$R_{eq} + jX_{eq} = \underbrace{Z_{sc}}_{\text{Convert into rectangular form. \& equate the real \& Imag part to determine } R_{eq} \& X_{eq}} \angle \theta$$

Convert into rectangular form. & equate the real & Imag part to determine  $R_{eq}$  &  $X_{eq}$

$$R_{eq} = \underline{R_p} + a^2 R_s = \text{Real part of } Z_{sc}$$

$$X_{eq} = \underline{X_p} + a^2 X_s = \text{Imaginary part of } Z_{sc}$$

In short circuit test, it is possible to determine the total series impedance referred to the primary voltage level.

Limitation: There is no direct way to split the series impedances into primary & secondary components.

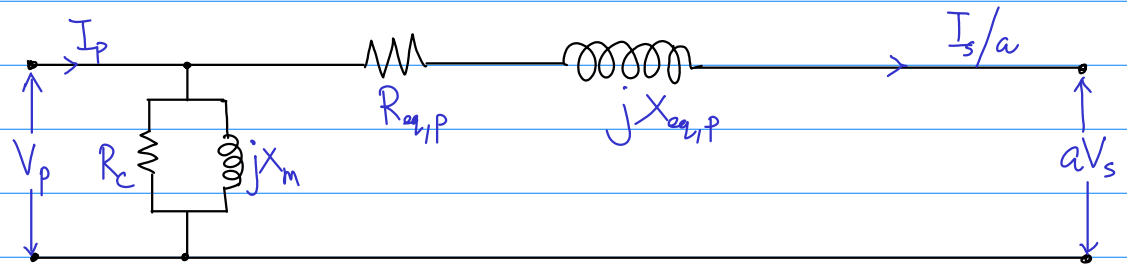
Ques: How do we split?

— Only way out is through estimation.

# Work out example problem 2.2 given at page no. 92.

Problem : The equivalent ckt. impedances of a given 20 kVA, 8000/240-V, 60-Hz transformer are to be determined. The open-ckt. test and short-ckt. test were performed on the primary side of the transformer, and the following data were taken :

Open-ckt. Test	Short-ckt. Test
$V_{oc} = 8000 \text{ V}$	$V_{sc} = 489 \text{ V}$
$I_{oc} = 0.214 \text{ A}$	$I_{sc} = 2.5 \text{ A}$
$P_{oc} = 400 \text{ W}$	$P_{sc} = 240 \text{ W}$



$$R_{eq,p} = R_p + a^2 R_s$$

$$X_{eq,p} = X_p + a^2 X_s$$

Open-Ckt. test : Analysis of the data .

$$V_{oc} = 8000 \text{ V}$$

$$I_{oc} = 0.214 \text{ A}$$

$$P_{oc} = 400 \text{ W}$$

$$\text{p.f.} = \cos \theta = \frac{P_{oc}}{V_{oc} \times I_{oc}} = \frac{400 \text{ W}}{8000 \times 0.214}$$

$$p.f = 0.233 \text{ (lagging)}$$

The admittance of the excitation branch

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1}(0.233) = \frac{0.214 A}{8000 V} \angle -\cos^{-1}(0.233)$$

$$Y_E = 0.0000267 \angle -76.5^\circ$$

Convert  $Y_E$  into rectangular form:

$$Y_E = 2.67 \times 10^{-5} \cdot \cos(-76.5^\circ) + j 2.67 \times 10^{-5} \cdot \sin(-76.5^\circ)$$

$$Y_E = 6.2 \times 10^{-6} - j 2.5 \times 10^{-5}$$

We also know that

$$Y_E = \frac{1}{R_c} - j \frac{1}{X_m}$$

$$\Rightarrow 6.2 \times 10^{-6} - j 2.5 \times 10^{-5} = \frac{1}{R_c} - j \frac{1}{X_m}$$

Now, we equate the real & imaginary terms in LHS with RHS.

$$\Rightarrow R_c = \frac{1}{6.2 \times 10^{-6}} = 160 k\Omega \Rightarrow \text{Core-Resistance}$$

$$\Rightarrow X_m = \frac{1}{2.5 \times 10^{-5}} = 40 k\Omega \Rightarrow \text{Core-Magnetization}$$



Short-ckt Test : Analysis of the data :

$$V_{sc} = 489 \text{ V}$$

$$I_{sc} = 2.5 \text{ A}$$

$$P_{sc} = 240 \text{ W}$$

$$\text{p.f.} = \cos \theta = \frac{P_{sc}}{I_{sc} \cdot V_{sc}} = \frac{240 \text{ W}}{489 \text{ V} \times 2.5 \text{ A}}$$

$$\cos \theta = 0.204 \quad (\text{lagging})$$

$$Z = \frac{V_{sc}}{I_{sc}} \angle \cos^{-1}(\text{p.f.}) = \frac{489 \text{ V}}{2.5 \text{ A}} \angle \cos^{-1}(0.204)$$

$$Z = 195.6 \angle 78.2^\circ \Omega$$

Convert into rectangular form:

$$Z = 195.6 \cos(78.2^\circ) + j 195.6 \sin(78.2^\circ)$$

$$Z = 39.9 + j 191.4 \Omega$$

We know that  $Z = R_{eq,p} + j X_{eq,p}$

$$\Rightarrow 39.9 + j 191.4 \Omega = R_{eq,p} + j X_{eq,p}$$

Equating real & Imaginary terms in LHS with RHS.

$$R_{eq,p} = 39.9 \Omega$$

$$X_{eq,p} = 191.4 \Omega$$

$$R_p + a^2 R_s = 39.9 \Omega$$

$$X_p + a^2 X_s = 191.4 \Omega$$

$$\text{here } a = \frac{8000V}{240V}$$

$$a = 33.3$$

$$R_p + (33.3)^2 R_s = 39.9 \Omega$$

$$X_p + (33.3)^2 X_s = 191.4 \Omega$$

Let's consider  $R_p \sim 5 \Omega$

$$5 \Omega + (33.3)^2 R_s = 39.9 \Omega$$

$$\Rightarrow R_s = \frac{34.9}{(33.3)^2} \Omega$$

$$\Rightarrow R_s = 0.031 \Omega$$

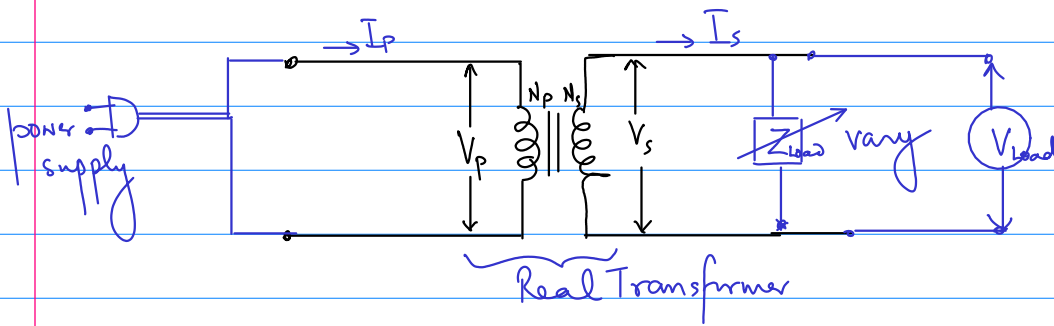
Similarly, we can consider  $X_p = 10 \Omega$

$$10 \Omega + (33.3)^2 X_s = 191.4 \Omega$$

$$X_s = \frac{181.4 \Omega}{(33.3)^2} = 0.16 \Omega$$

$R_c, X_m, R_p, R_s, X_p, X_s$  - Determined.

## Voltage Regulation and Efficiency of Real Transformer



We define voltage regulation of a real transformer as.

$$V.R. = \left[ \frac{V_{s,NL} - V_{s,FL}}{V_{s,FL}} \right] \times 100 \%$$

$V_{s,NL}$  = Output voltage when No-Load is connected to the secondary.

$V_{s,FL}$  = Output voltage when full-load is connected to the secondary.

$$V.R. = \frac{\frac{V_p}{a} - V_{s,FL}}{V_{s,FL}} \times 100$$

In case of ideal transformer,

$$V.R. = 0\%$$

Typically, we design a transformer whose V.R. is min.

(i) If the load connected to the secondary is INDUCTIVE in nature:

$$V.R. = +ve$$

(ii) If the load connected to the secondary is Resistive then  $V.R. = +ve$

(iii) If the load connected to the secondary is Capacitive in nature then

$$V.R. = -ve.$$

Efficiency of a Real Transformer:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

Ideal transformer  $\eta = 100\% \quad (P_{out} = P_{in})$

$$\eta_{Real} = \left[ \frac{V_s I_s \cos \theta}{P_{Core-loss} + P_{Copper-loss} + V_s I_s \cos \theta} \right] \times 100\%$$