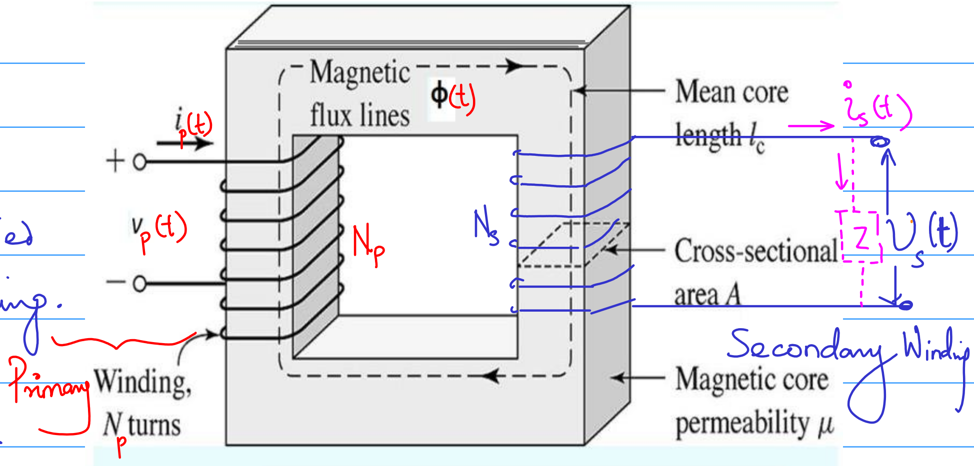
24.05.2022

The Ideal Transformer

(Lossless Transformer)

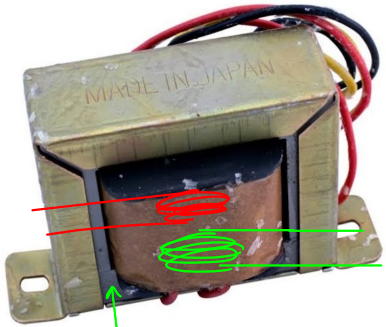
Construction:

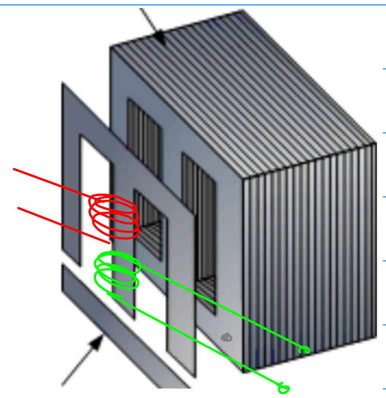


Here, $V_p(t)$ is the source voltage connected to the primary winding.

eg: $V_p = 220V \angle 0°, 50Hz$

Due to this, there is time varying current $i(t)$ in the primary winding.



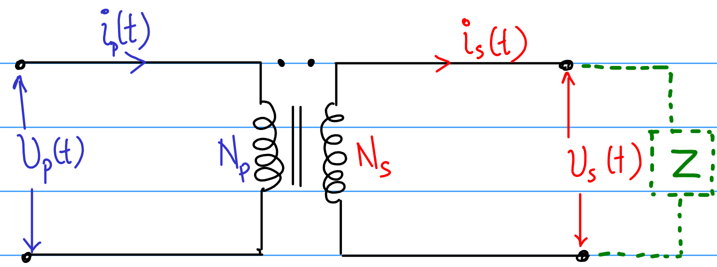


As a result, the time varying magnetic flux $\phi(t)$ is created within the magnetic core.

Since, the secondary winding is experiencing the time-varying magnetic flux $\phi(t)$, there will induced voltage across the terminals of the secondary winding.

Remember: The primary & the secondary windings are not connected electrically. They are connected magnetically.

Ckt. Symbol of the transformer:



1. Turn-Ratio:

We define turn-ratio of a transformer as the ratio of the number of turns in the primary winding to the number of turns in the secondary winding.

Turn-ratio $\boxed{a = \dfrac{N_p}{N_s}}$

In terms of voltages;

$$\boxed{a = \frac{v_p(t)}{v_s(t)}}$$

example: if $N_p = 100$ and $N_s = 300$

then, $a = \dfrac{1}{3}$ (or, $1:3$)

$\Rightarrow$ for one turn in the primary winding, we have 3-turns in the secondary winding.

the amplitude of

if $= 1:3$ and if $v_p(t) = 220V$

then the amplitude of the secondary voltage will be

$$\frac{v_p(t)}{v_s(t)} = a = \frac{1}{3}$$

$$\Rightarrow \quad v_s(t) = 3v_p(t) = 3 \times 220V = 660V$$

$\Rightarrow$ This design is for the step-up transformer

In case the $a = 3:1$ then, we have step-down transformer

Turn-ratio in terms of Currents:

$$\boxed{\frac{i_p(t)}{i_s(t)} = \frac{1}{a} = \frac{N_s}{N_p}}$$

In terms of phasor quantities:

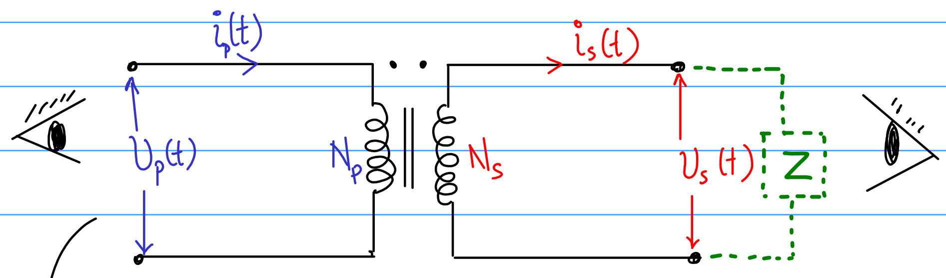
$V_p \angle \theta_p$; $V_s \angle \theta_s$

$I_p \angle \phi_p$; $I_s \angle \phi_s$

$$\boxed{a = \frac{V_p}{V_s}} \quad ; \quad \boxed{\frac{1}{a} = \frac{I_p}{I_s}}$$

⇒ The turn-ratio (a) is only responsible for the changes in voltages & currents in the two windings.

Impedances in the Ideal Transformer :



The apparent impedance as seen from the primary terminals :

$$Z_p = \frac{V_p}{I_p}$$

Similary, the apparent impedance as seen from the secondary terminals.

$$Z_s = \frac{V_s}{I_s}$$

Now, $Z_p = \dfrac{V_p}{I_p} = \dfrac{a V_s}{\left(\frac{1}{a}\right) I_s} = a^2 \dfrac{V_s}{I_s} = a^2 Z_s$

$$\boxed{Z_p = a^2 Z_s}$$

Power in the ideal transformer :

(Transformer is loss-less)

Let, the real power supplied from the source to the primary winding.

$$P_p = V_p I_p \underbrace{\cos\theta_p}_{\text{p.f. of primary}}$$

Real power taken out from the secondary winding.

$$P_s = V_s I_s \underbrace{\cos\theta_s}_{\text{p.f. of secondary}}$$

Since, we are dealing with the loss-less transformer,

$$P_p = P_s \quad \text{satisfy,}$$

In order to satisfy this, we have

$$\cos\theta_p = \cos\theta_s \Rightarrow \theta_p = \theta_s = \theta$$

Similarly;
Reactive Power,

$$Q_p = Q_s$$

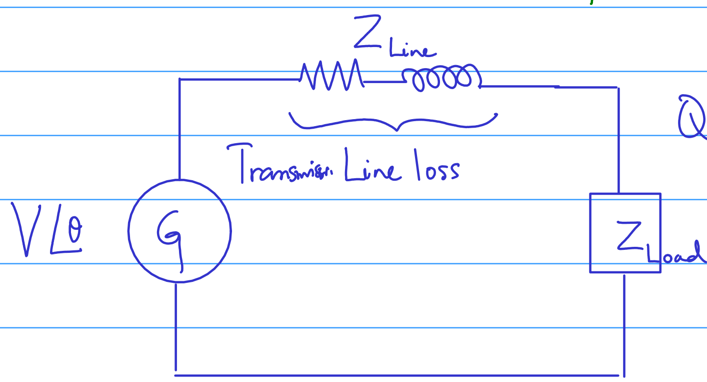
Apparent Power, $S_p = S_s$

Reference : Chapter - 2

Class Assignment :

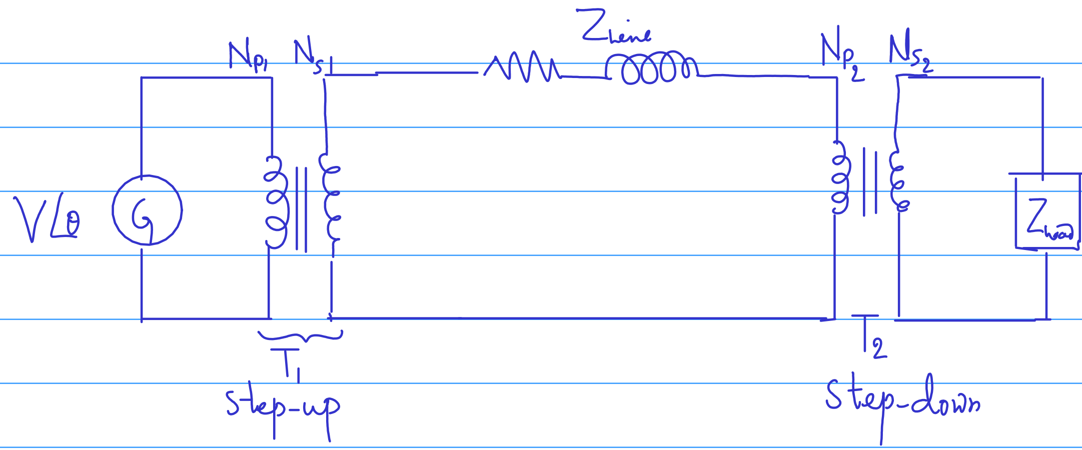
Work-out example problem 2.1 given in the chapter.

a)



Ques : Determine power loss in the transmission line.

b)



Ques : Determine the power-loss in the transmission line.

$\Rightarrow$ The

26.05.2022 (Sec-2) & 27.05.2022 (Sec-1)

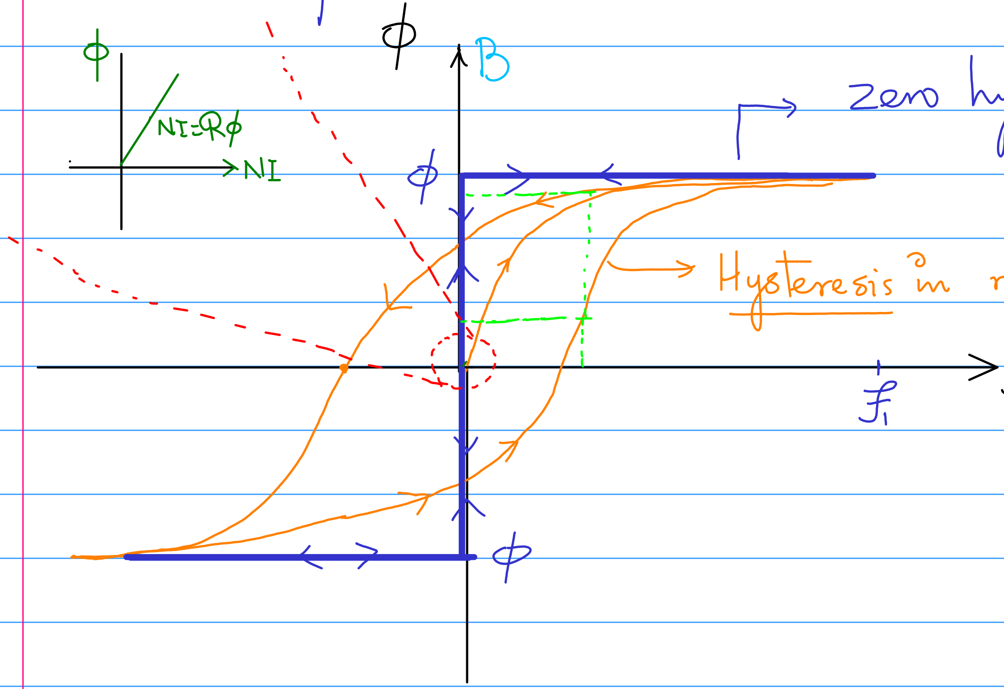
Single-Phase Real Transformer

In discussing the lossless transformer (ideal), followings are assumed.

(i) The resistance of both windings (ie, primary & secondary) is zero.
- Energy loss due to resistive heating is zero. (Joule heating, i^2Rt)

(ii) The leakage of magnetic flux from the core is zero.
- The flux in the core couple both the windings, ie, primary & secondary.

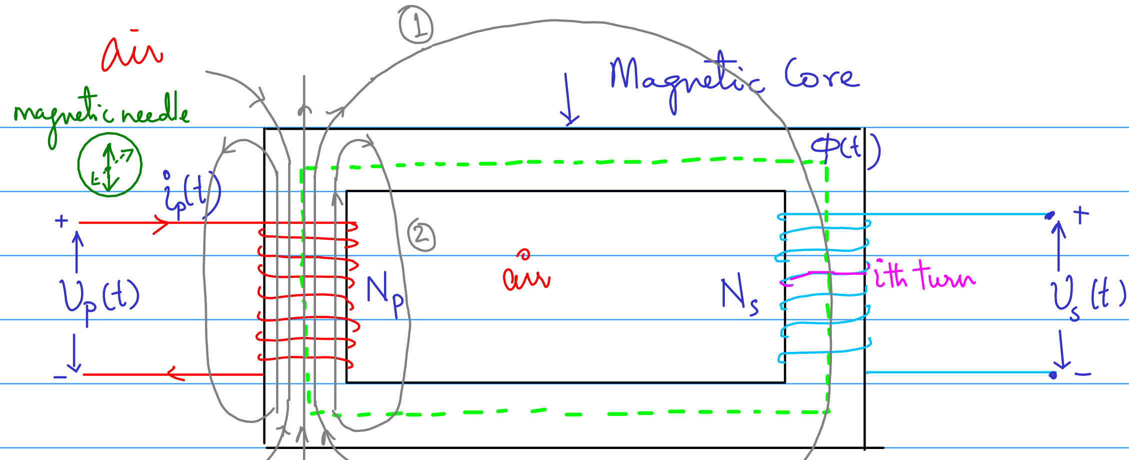
(iii) The magnetization in the core of the transformer follows the curve shown below:



$$\boxed{NI = \mathcal{R}\phi}$$

.... Analogous to Ohm's law.

Real Transformer:



A transformer without load connected to the secondary

Induced emf in the secondary winding is given as

$$\boxed{v_s(t) = N_s \frac{d\phi(t)}{dt}}$$

What happens in real transformer in terms of flux leakage?

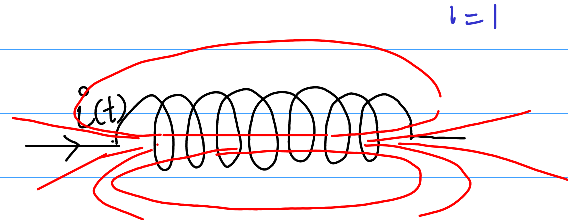
In case there is leakage of magnetic flux through each turn of the windings, the magnitude of the voltage in the ith turn of the winding is given as

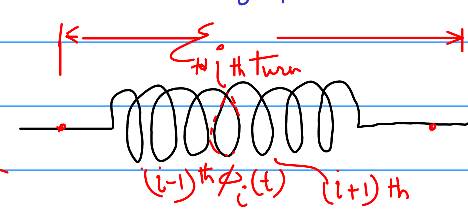
$$\mathcal{E}_i(t) = 1 \cdot \frac{d\phi_i(t)}{dt}$$

Therefore,

The total emf induced in the windings is

$$\mathcal{E}_{tot}(t) = \sum_{i=1}^{N_s} \mathcal{E}_i(t) = \sum_{i=1}^{N_s} \frac{d\phi_i(t)}{dt}$$





$$\mathcal{E}_{tot}(t) = \frac{d}{dt} \underbrace{\sum_{i=1}^{N_s} \phi_i(t)}$$

$\lambda(t)$ = Flux linkage of the winding (Wb-turn)

$$\mathcal{E}_{tot}(t) = \underbrace{\frac{d\lambda(t)}{dt}} \quad \text{where } \lambda(t) = \sum_{i=1}^{N_s} \phi_i(t)$$

how flux linkage of the winding is changing with time 't'.

Since the flux linkage $\lambda(t)$ through the winding is not just a product of number of turns to that of magnetic flux (ie, $N\phi$). $\lambda(t) \neq N\phi$ in reality

Because the flux passing through each turn of the winding is slightly different in magnitude.

But, we also know that in real transformer, the number of turns (N) is very large (say ~1000)

In practice, we define an average flux per turn

as,

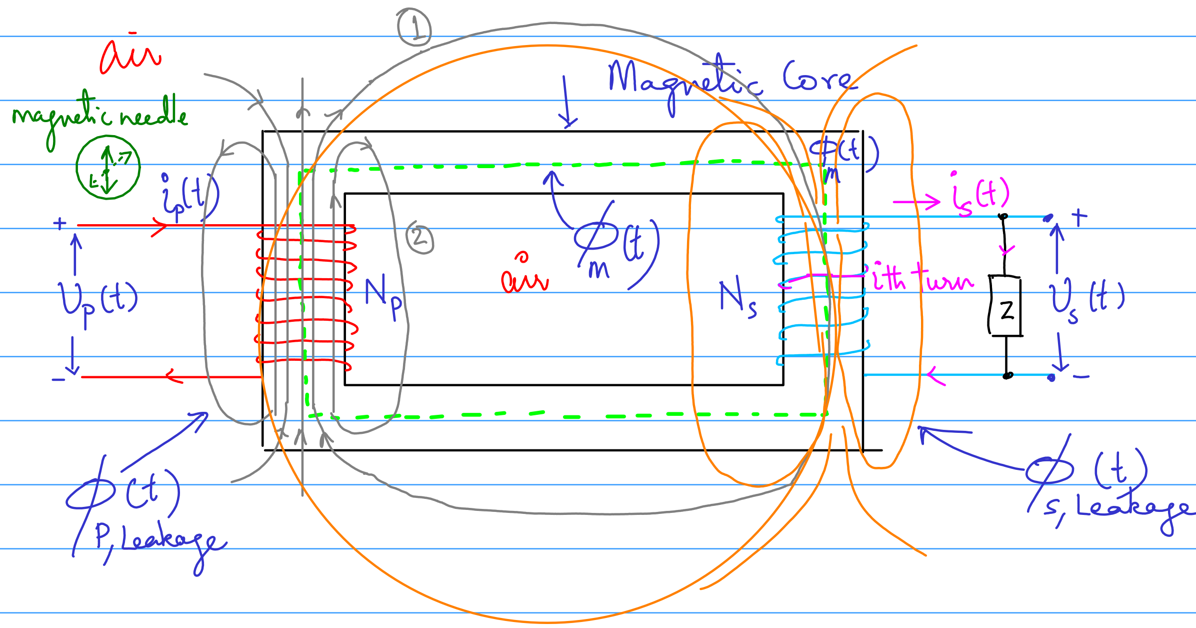
$$\phi_{ave.} = \frac{\lambda}{N} \qquad \boxed{\lambda \neq N\phi}$$

Therefore,

$$\mathcal{E}_{ind.}(t) = \frac{d\lambda(t)}{dt} = \underbrace{N\frac{d\phi_{ave}(t)}{dt}}$$

how the average flux changing with time.

Real Transformer in operation (connected to a load impedance Z)



Primary winding, we have $i_p(t)$

$$\phi_p(t) = \phi_m(t) + \phi_{p,Leakage}(t)$$

Similarly, in secondary winding, we have $i_s(t)$

$$\phi_s(t) = \phi_m(t) + \phi_{s,\,leakage}$$

where, $\phi_m(t)$ is the mutual flux experienced by both the windings simultaneously.

Therefore,

$$v_p(t) = N_s \frac{d\phi_p(t)}{dt}$$

$$v_p(t) = N_s \frac{d}{dt}\left(\phi_m(t) + \phi_{p,\text{leakage}}(t) \right)$$

$$v_p(t) = N_s \frac{d\phi_m(t)}{dt} + N_s \frac{d\phi_{p,\text{leakage}}(t)}{dt}$$

$$v_p(t) = \mathcal{E}_{p,m}(t) + \underbrace{\mathcal{E}_{p,\text{Leakage}}(t)}_{\substack{\text{Loss of voltage} \\ \text{(due to leakage} \\ \text{flux)}}}$$

Similarly

$$v_s(t) = \mathcal{E}_{s,m}(t) + \mathcal{E}_{s,\text{Leakage}}(t)$$

here, we observe that

$$\frac{v_p(t)}{v_s(t)} \neq \frac{N_p}{N_s} \text{ (turn-ratio)}$$

Rather,

$$\frac{\mathcal{E}_{p,m}(t)}{\mathcal{E}_{s,m}(t)} = \frac{N_p \frac{d\phi_m(t)}{dt}}{N_s \frac{d\phi_m(t)}{dt}} = \frac{N_p}{N_s} = \text{turn-ratio}$$

$$\boxed{\frac{\mathcal{E}_{p,m}(t)}{\mathcal{E}_{s,m}(t)} = \frac{N_p}{N_s} = a = \text{turn-ratio}}$$

While designing a transformer we try to minimize the leakage flux through each of the windings.

Therefore, in case

$$\phi_m(t) \gg \phi_{p,\,leakage} \text{ or } \phi_{s,\,leakage}$$

$\Rightarrow$ we can ignore the leakage flux as compared to the mutual flux.

$$v_p(t) = \mathcal{E}_{p,m}(t) + \underbrace{\mathcal{E}_{p,\,Leakage}}$$

$$v_s(t) = \mathcal{E}_{s,m}(t) + \underbrace{\mathcal{E}_{s,\,Leakage}}$$

Ignored.

$$\boxed{\frac{v_p(t)}{v_s(t)} = \frac{\mathcal{E}_{p,m}(t)}{\mathcal{E}_{s,m}(t)} = \frac{N_p}{N_s} = \text{Turn ratio } (a)}$$