14.05.2022

Magnetic Fields in Electrical Machines: Transformer, Motor, Generator

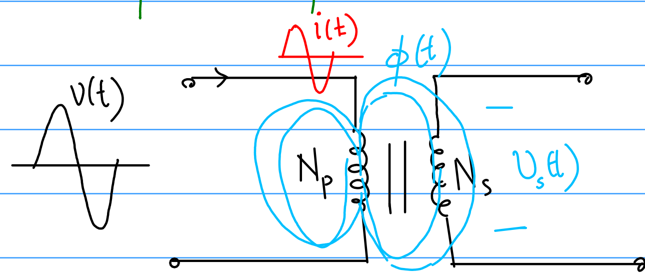
(Reference: Chapter-1, Sec 1.4 onward)

There are four ways the magnetic field used in electrical machines:

1. A current-carrying wire produces a magnetic-field in the area around it. — Biot-Savart's law / Ampere's Law

2. A time-varying magnetic field induces a voltage (emf) in a coil of wire if it passes through that coil.

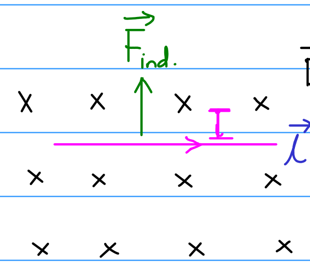
— Basis of transformer action



3. A current carrying wire in presence of magnetic field has a force induced on it.

— Basis of the Motor action

$$\vec{F}_{ind} = I\,(\vec{l} \times \vec{B})$$

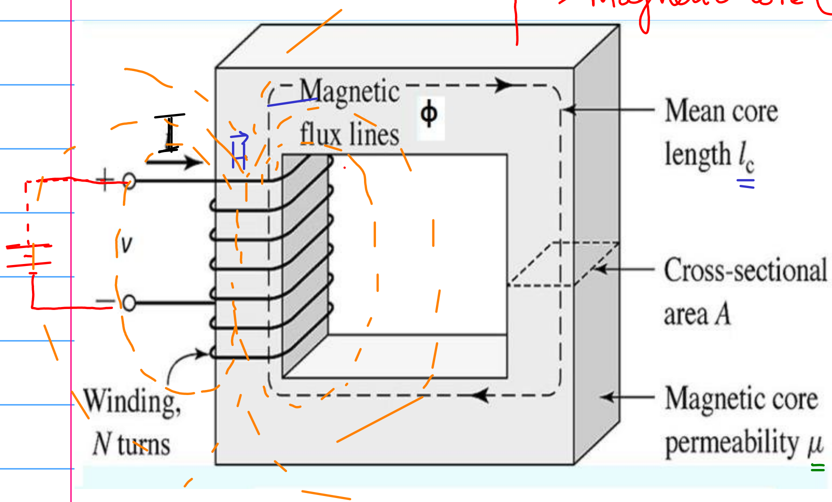


4. A moving wire in the pressence of a magnetic field has a voltage induced on it (motional emf).

— Basis of the Generator Action.

Production of Magnetic field :

Magnetic Core (Ferromagnetic Material)



Ampere's law to determine the magnetic field intensity due to current I_{net}.

$$\oint \vec{H} \cdot \vec{dl} = I_{net} = NI$$

$\vec{H}$ = Magnetic field intensity produced by the total current $I_{net} = NI$

$\vec{dl}$ = Elemental (differential) length along the path of the integral.

Assuming that the magnetic field produced by the current I remain inside the core (ideal case).

With this assumption, we can determine the path integral in Ampere's law :

$$I_{net} = NI$$

$$\oint \vec{H} \cdot \vec{dl} = H l_c$$

Therefore, we have $\quad H l_c = NI$

$$\boxed{H = \frac{NI}{l_c}}$$

$H =$ Magnetic field Intensity

Physically it is a measure of the "Effort" that the current I_{net} (=NI) is putting into the establishment of magnetic field.

We also define $\vec{B}$ which is "magnetic flux density":

$$\vec{B} = \mu \vec{H}$$

Magnetic flux density

Magnetic Permeability

— It is an extent to which magnetic field lines can enter into a substance.

Magnetic field Intensity

$\vec{H}$ = Magnetic field intensity : Unit – Ampere-turn/meter

$\vec{B}$ = Magnetic flux density : Unit – T or (Weber/m^2)

μ = Magnetic permeability : Unit – Henry/m

ϕ = Magnetic flux : Unit – Weber

$$H l_c = NI \qquad — (1)$$

$$B = \mu H \qquad — (2)$$

$$\Rightarrow \quad B = \mu \frac{NI}{l_c} \qquad — (3)$$

$$\phi = \int \vec{B} \cdot d\vec{A} = B \cdot A \qquad — (4)$$

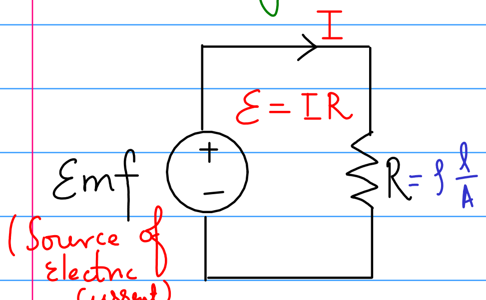
$$\boxed{\phi = \frac{\mu N \cdot I A}{l_c}}$$

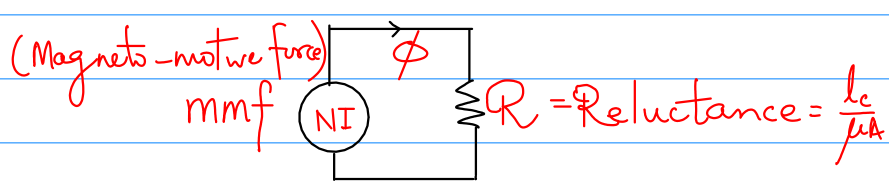
$$\underbrace{\phi}_{\substack{\text{Effect} \\ \text{(Magnetic flux)}}} = \underbrace{\left(\frac{\mu A}{l_c}\right)}_{\substack{\text{Parameter of the core material} \\ \text{in which magnetic field is} \\ \text{produced.}}} \cdot \underbrace{(NI)}_{\text{Source of magnetic field}}$$

Analogy with the Electric Ckt :

Can we have a magnetic ckt ?

Yes, we have.





$$NI = \mathcal{R} \cdot \phi$$

where, $\boxed{\mathcal{R} = \dfrac{l_c}{\mu A}}$

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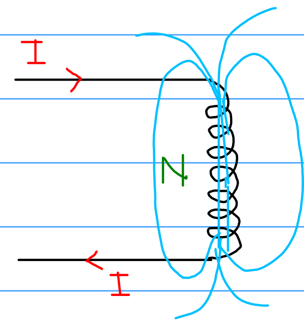
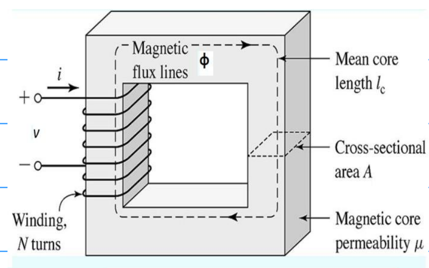
(Sec-2)

Magnetic Circuit



$$I \longrightarrow \underbrace{H}_{\text{Magnetic Intensity}} \xrightarrow{\mu} \underbrace{B}_{\text{Magnetic flux density}} \xrightarrow[(l_c,\, A)]{\text{Geometry}} \phi$$

Total Magnetic flux.



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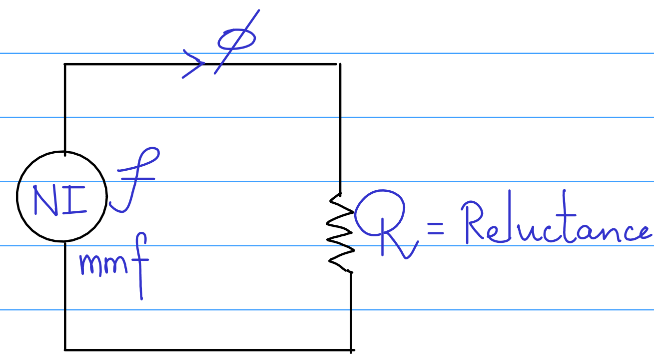


Magneto-motive force (mmf) : $NI = \mathcal{R}\phi$ { $\mathcal{E} = RI$ Electric ckt }

where, $\mathcal{R} = \dfrac{l_c}{\mu A}$ [Unit: A-turn/Wb] Also in electric ckt; we define $\frac{1}{R}$ as conductance

Also, in magnetic ckt; we define

$\dfrac{1}{\mathcal{R}}$ as Permeance ($\mathcal{P}$) [Unit: Wb/A-turn]

The reluctance obeys the same rules as resistance in electric ckt. That ie, if there are two or more reluctances connected in the magnetic ckt. the the equivalent reluctance follows the same rule of series & parallel connections of resistances in electric ckt.

Let say, in magnetic ckt, we have $R_1, R_2, \ldots R_n$ reluctances.

(i) Series connection : $R_{eq} = \sum_{i=1}^{n} R_i$

(ii) Parallel connection : $\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$

Example Problem :

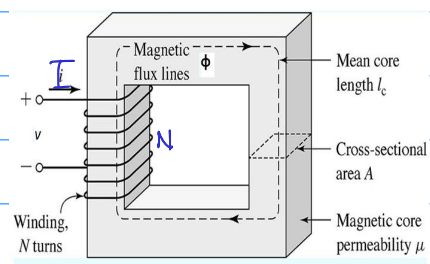
$\mu = \mu_0 \mu_r$

$I = 1A$

$\mu_r = 2500$

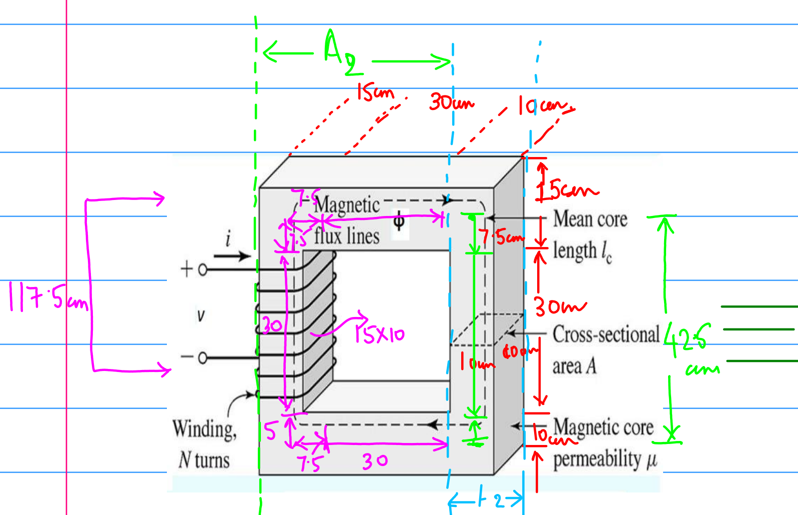
$\mu_0 = 4\pi \times 10^{-7}$

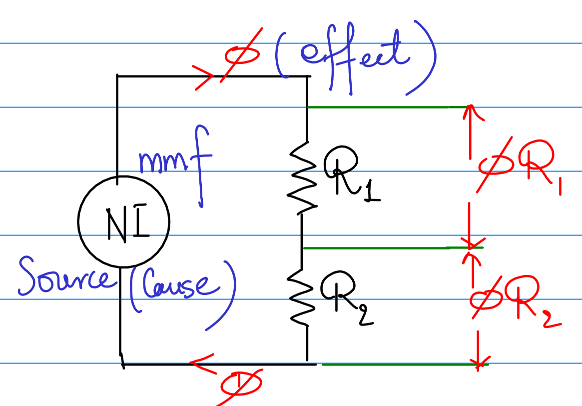
$N = 200$



Arm-I ($A_1 = 15 \times 10\ cm^2$) ($l_1 = 117.5\ cm$) $R_1 = \frac{l_1}{\mu A_1}$

Arm-II ($A_2 = 10 \times 10\ cm^2$) ($l_2 = 42.5\ cm$) $R_2 = \frac{l_2}{\mu A_2}$





$$R_1 = \frac{117.5\ cm}{4\pi\times10^{-7}\times 2500\times 15\times10\ cm^2} = (\quad)$$

$$R_2 = \frac{42.5\ cm}{4\pi\times10^{-7}\times 2500\times 10\ cm\times 10\ cm} = (\quad)$$

$$R_{eq} = R_1 + R_2$$

therefore, $\phi = \dfrac{NI}{R_{eq}}$ Wb

$$\phi = \frac{200\,\text{turns}\times 1\,A}{(\ ?\ )\ \text{A-turns/Wb}}$$

Similarly:

Magnetic Core & Current Coil arrangement $\equiv$