Tut-07

Problem 3.33 A "pure" dipole p is situated at the origin, pointing in the z direction.

(a) What is the force on a point charge q at $(a, 0, 0)$ (Cartesian coordinates)?

(b) What is the force on q at $(0, 0, a)$?

(c) How much work does it take to move q from $(a, 0, 0)$ to $(0, 0, a)$?

Problem 3.36 Show that the electric field of a (perfect) dipole (Eq. 3.103) can be written in the coordinate-free form

$$\boxed{\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}].} \qquad (3.104)$$

Problem 4.5 In Fig. 4.6, $\mathbf{p}_1$ and $\mathbf{p}_2$ are (perfect) dipoles a distance r apart. What is the torque on $\mathbf{p}_1$ due to $\mathbf{p}_2$? What is the torque on $\mathbf{p}_2$ due to $\mathbf{p}_1$? [In each case, I want the torque on the dipole *about its own center*. If it bothers you that the answers are not equal and opposite, see Prob. 4.29.]

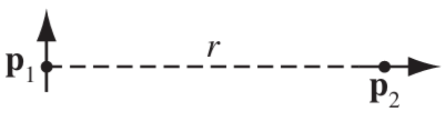

FIGURE 4.6

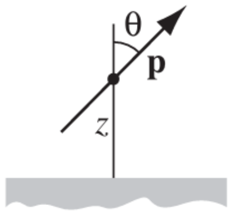

FIGURE 4.7

Problem 4.7 Show that the energy of an ideal dipole $\mathbf{p}$ in an electric field $\mathbf{E}$ is given by

$$\boxed{U = -\mathbf{p}\cdot\mathbf{E}.} \qquad (4.6)$$

Problem 4.8 Show that the interaction energy of two dipoles separated by a displacement $\mathbf{r}$ is

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[\mathbf{p}_1\cdot\mathbf{p}_2 - 3(\mathbf{p}_1\cdot\hat{\mathbf{r}})(\mathbf{p}_2\cdot\hat{\mathbf{r}})]. \qquad (4.7)$$

[*Hint:* Use Prob. 4.7 and Eq. 3.104.]

Problem 4.14 When you polarize a neutral dielectric, the charge moves a bit, but the *total* remains zero. This fact should be reflected in the bound charges σ_b and ρ_b. Prove from Eqs. 4.11 and 4.12 that the total bound charge vanishes.

Problem 4.16 Suppose the field inside a large piece of dielectric is $\mathbf{E}_0$, so that the electric displacement is $\mathbf{D}_0 = \epsilon_0\mathbf{E}_0 + \mathbf{P}$.

(a) Now a small spherical cavity (Fig. 4.19a) is hollowed out of the material. Find the field at the center of the cavity in terms of $\mathbf{E}_0$ and $\mathbf{P}$. Also find the displacement at the center of the cavity in terms of $\mathbf{D}_0$ and $\mathbf{P}$. Assume the polarization is "frozen in," so it doesn't change when the cavity is excavated.

(b) Do the same for a long needle-shaped cavity running parallel to $\mathbf{P}$ (Fig. 4.19b).

(c) Do the same for a thin wafer-shaped cavity perpendicular to $\mathbf{P}$ (Fig. 4.19c).

Assume the cavities are small enough that $\mathbf{P}$, $\mathbf{E}_0$, and $\mathbf{D}_0$ are essentially uniform. [*Hint:* Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.]
