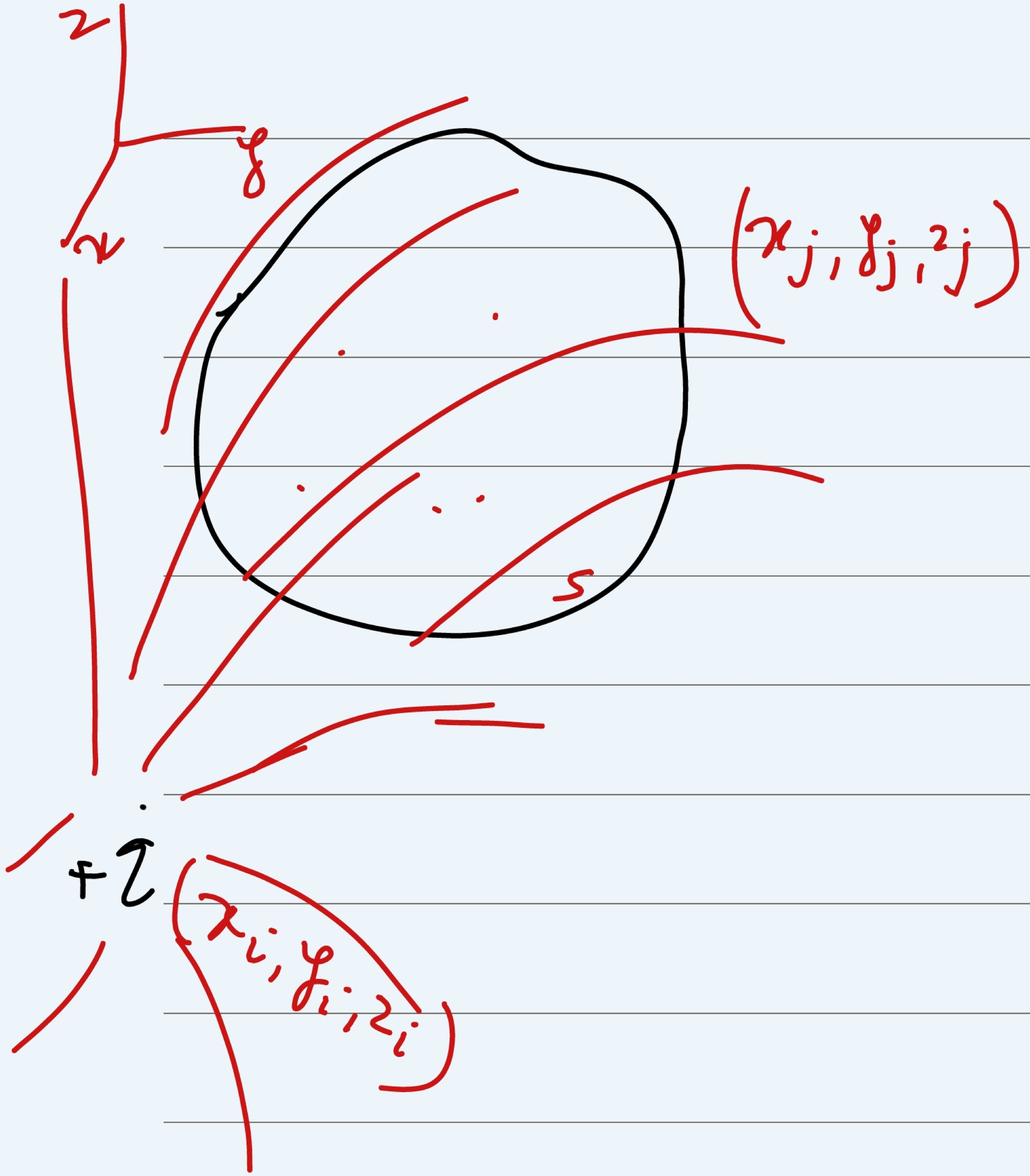
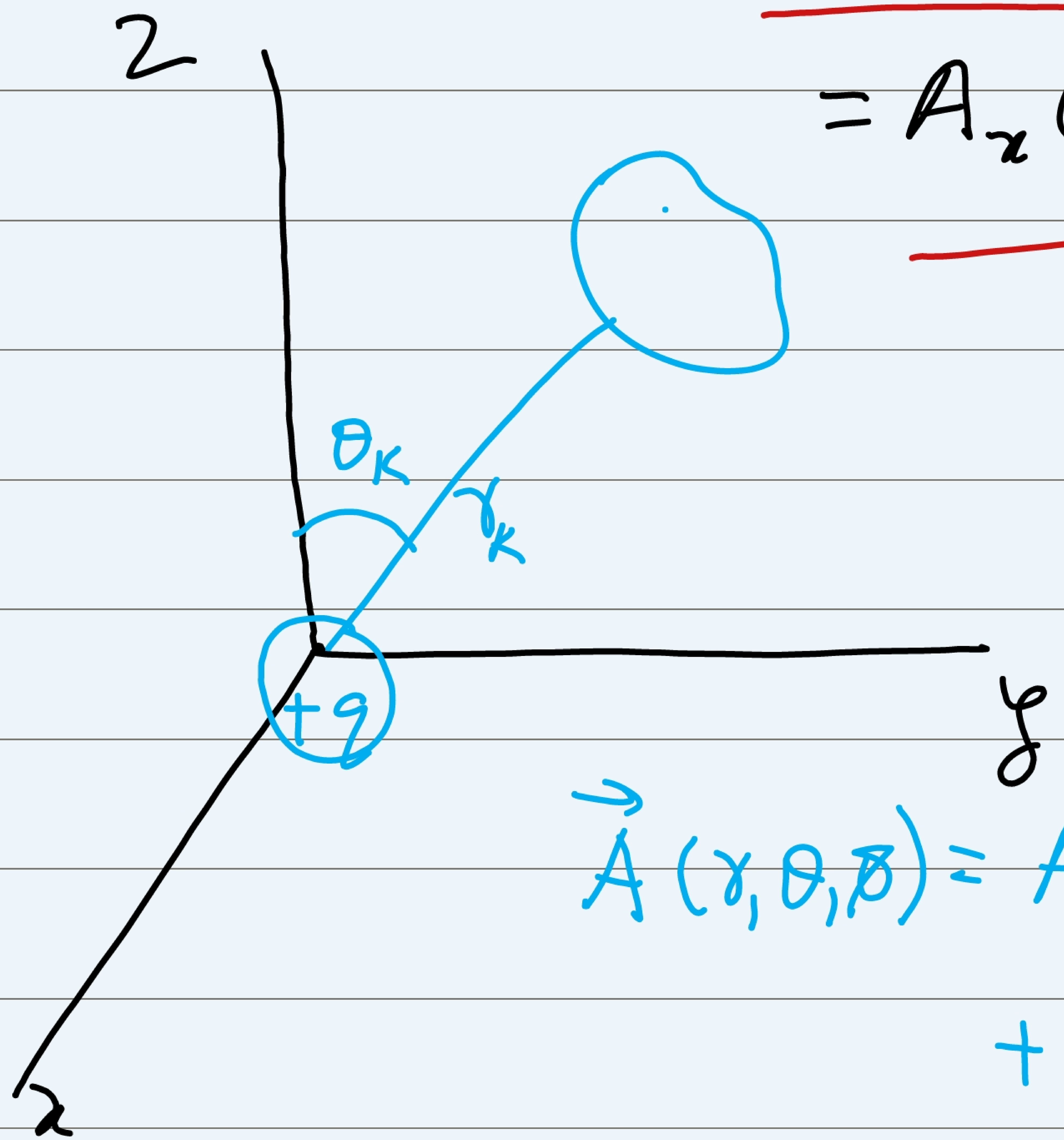
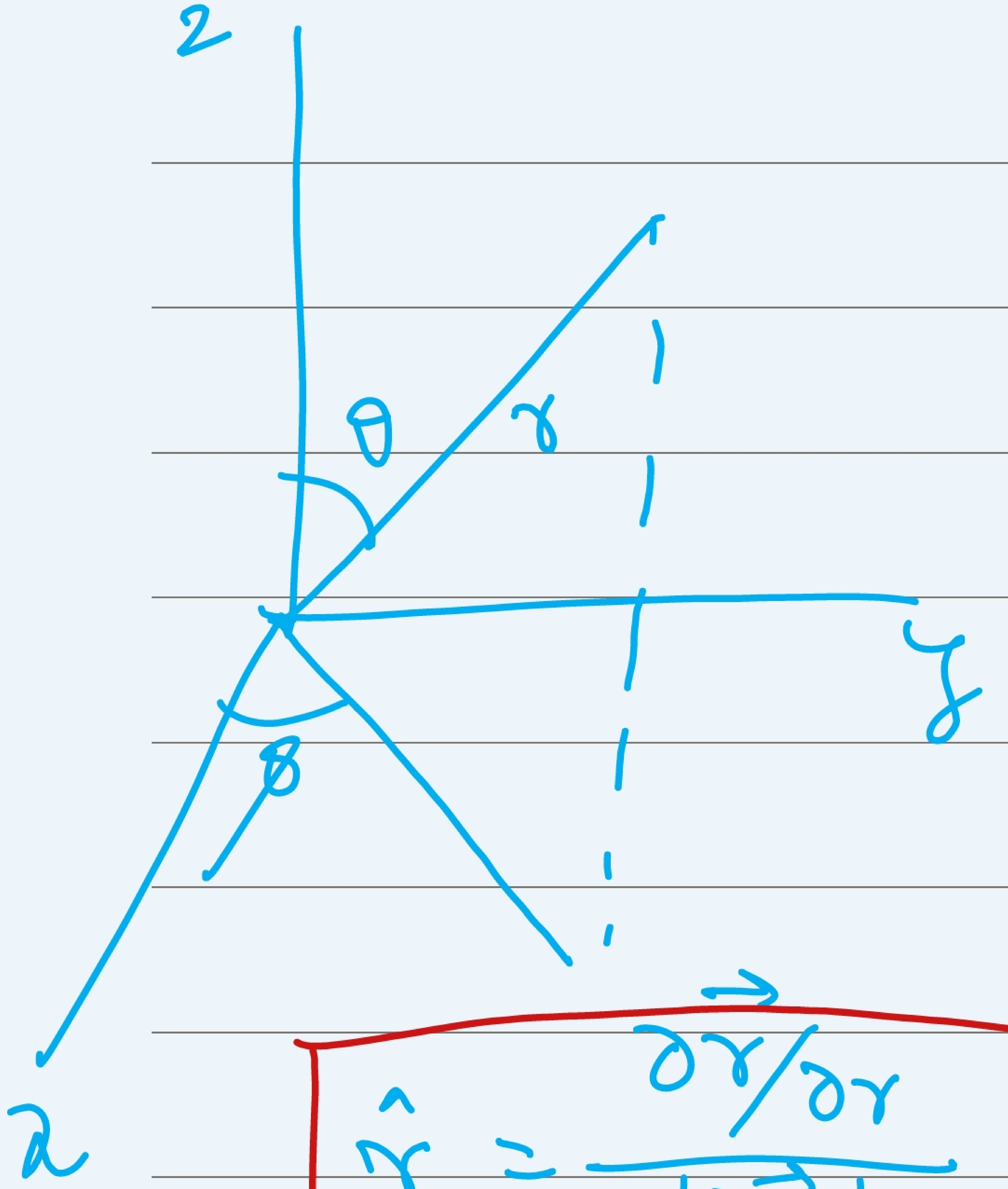




$$\vec{A}(x,y,z)$$
$$= A_x(x,y,z)\,\hat{i} + A_y(x,y,z)\,\hat{j} + A_z(x,y,z)\,\hat{k}$$

$$\vec{A}(r,\theta,\phi) = A_r(r,\theta,\phi)\,\hat{r} + A_\theta(r,\theta,\phi)\,\hat{\theta} + A_\phi(r,\theta,\phi)\,\hat{\phi}$$



$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta.$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = r\hat{r}$$

$$= r\sin\theta\cos\phi\,\hat{i} + r\sin\theta\sin\phi\,\hat{j} + r\cos\theta\,\hat{k}$$

$$\boxed{\hat{r} = \frac{\partial\vec{r}/\partial r}{\left|\frac{\partial\vec{r}}{\partial r}\right|} = \frac{\sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}}{1}}$$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}.$$

$$\vec{\nabla}\cdot\vec{F} =$$

$$\vec{\nabla}\times\vec{F} =$$

$\hat{r}$

$\hat{\theta}$

$\hat{\phi}$

$$\hat{\theta} = \frac{\frac{\partial \vec{r}}{\partial \theta}}{\left|\frac{\partial \vec{r}}{\partial \theta}\right|} = \frac{\cancel{r}\cos\theta\cos\phi\,\hat{i} + \cancel{r}\cos\theta\sin\phi\,\hat{j} - \cancel{r}\sin\theta\,\hat{k}}{\cancel{r}}$$

$$\boxed{\hat{\theta} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}}$$

$$\vec{r} = r\sin\theta\cos\phi\,\hat{i} + r\sin\theta\sin\phi\,\hat{j} + r\cos\theta\,\hat{k}$$

$$\hat{\phi} = \frac{\partial \vec{r}/\partial \phi}{\left|\partial \vec{r}/\partial \phi\right|} = \frac{-\cancel{r}\cancel{\sin\theta}\sin\phi\,\hat{i} + \cancel{r}\cancel{\sin\theta}\cos\phi\,\hat{j}}{\cancel{r}\cancel{\sin\theta}} \quad \boxed{\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}}$$

$\hat{i}, \hat{j}, \hat{k}$

$$\left\{\begin{aligned}\hat{r} &= \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}\\ \hat{\theta} &= \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}\end{aligned}\right.$$

$$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$

$\hat{r}\cdot\hat{r} = 1$

$\hat{r}\cdot\hat{\theta} = 0.$

$\underline{\nabla}$? in spherical polar coordinates

$$\boxed{\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}}$$

$$\left\{\begin{aligned}&T(x_1, y_1, z_1)\\ &\quad\downarrow\\ &T(x_2, y_2, z_2)\end{aligned}\right.$$

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$\boxed{dT = (\vec{\nabla} T)\cdot\vec{dl}}$$

$$\boxed{\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}}$$

$$du(r,\theta,\phi) = \frac{\partial u}{\partial r}\,\underline{dr} + \frac{\partial u}{\partial \theta}\,\underline{d\theta} + \frac{\partial u}{\partial \phi}\,d\phi = \nabla \vec{u}\cdot d\vec{l}$$

$$dT = \nabla T \cdot d\vec{l}$$

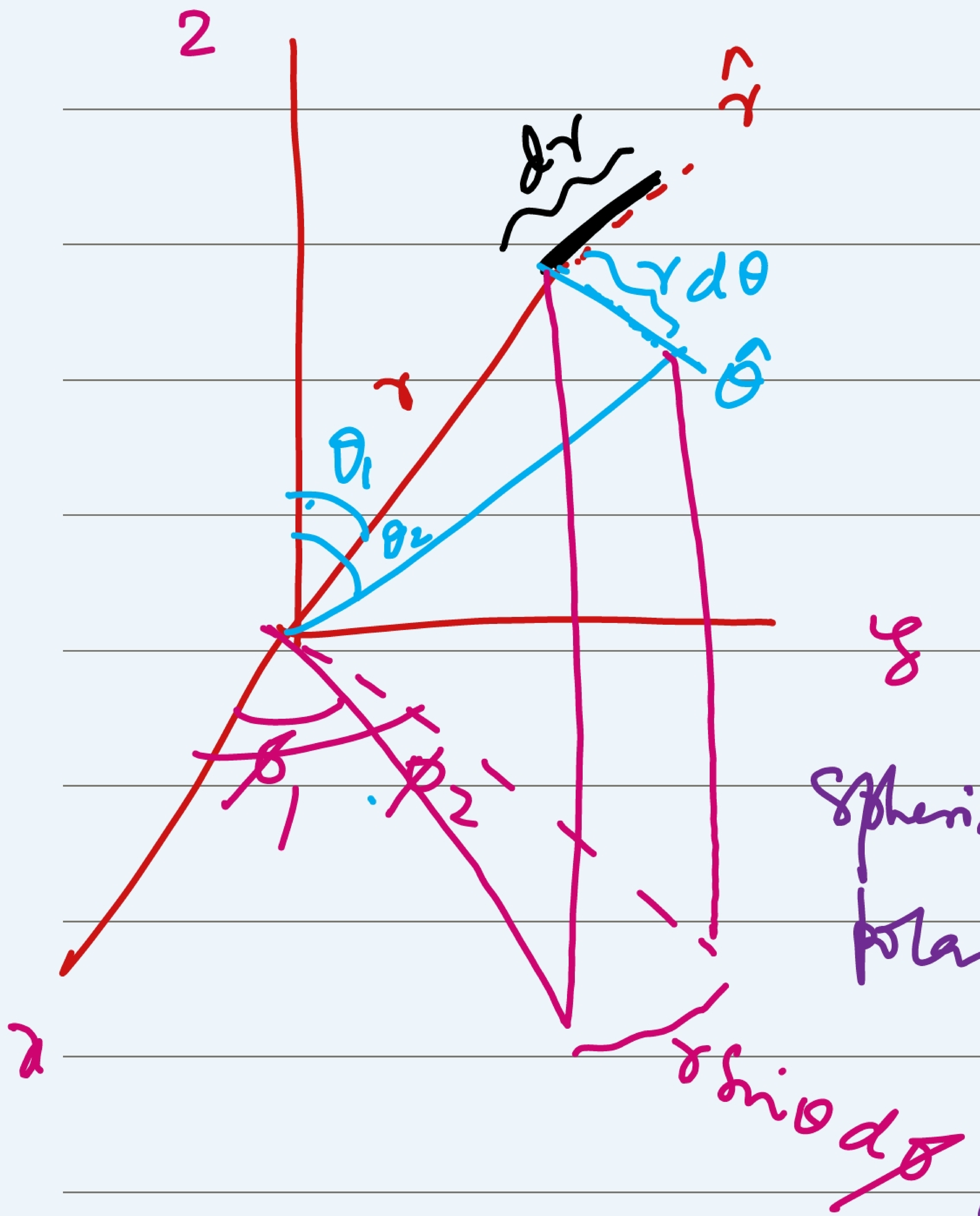
$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$= \underline{\left[(\nabla u)_r\,\hat{r} + (\nabla u)_\theta\,\hat{\theta} + (\nabla u)_\phi\,\hat{\phi}\right]}\cdot$$

$$dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$= (\nabla u)_r\,\underline{dr} + (\nabla u)_\theta\, r\,\underline{d\theta} + (\nabla u)_\phi\, r\sin\theta\, d\phi$$

$$\nabla u = \frac{\partial u}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}\,\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial u}{\partial \phi}\,\hat{\phi}$$



$$d\vec{l} = \underline{dr}\,\hat{r} + \underline{r\,d\theta}\,\hat{\theta} + \underline{r\sin\theta}\,d\phi\,\hat{\phi}$$

$$d\tau = dl_r\, dl_\theta\, dl_\phi$$

$$= r^2 \sin\theta\, dr\, d\theta\, d\phi$$

Spherical polar $\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{\underline{r}}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{\underline{r\sin\theta}}\frac{\partial}{\partial \phi}\hat{\phi}$

Cartes $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.

$\vec{\nabla} \times \vec{A}$

$$\vec{A} = A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$$

$$\vec{\nabla} \cdot \vec{A} = \left[\hat{r} \frac{\partial}{\partial r} + \frac{\hat{\theta}}{r} \frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{r \sin\theta} \frac{\partial}{\partial \phi} \right] \cdot \left[A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi} \right]$$

$\frac{\hat{r}}{r^2} \to \nabla \cdot = 0$

$$= \hat{r} \cdot \frac{\partial}{\partial r} \left[A_r \hat{r} \right]$$

$\vec{A} = \vec{F} = r^2 \hat{r}$

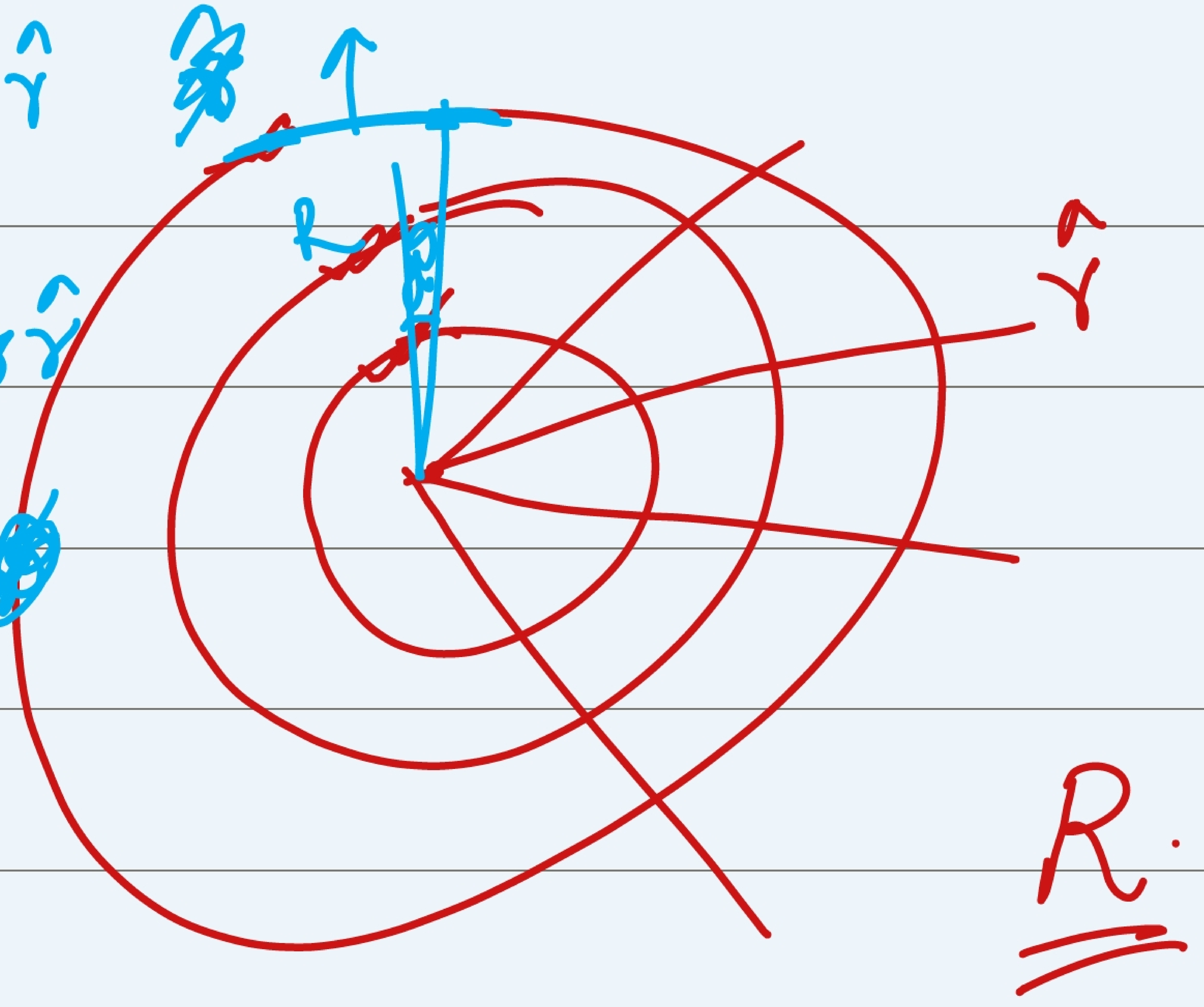
$$= \hat{r} \cdot \left[\frac{\partial A_r}{\partial r} \hat{r} + A_r \underbrace{\frac{\partial \hat{r}}{\partial r}}_{0} \right]$$

$$\boxed{\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 A_r \right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \left(A_\theta \sin\theta \right) + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}}$$

$$\iiint_V (\nabla \cdot \vec{F})\, d\tau = \iint \vec{F} \cdot d\vec{a}$$

$$\hat{r}\, da = dl_\theta\, dl_\phi\, \hat{r}$$

$$= r\, d\theta\; r \sin\theta\, d\phi\, \hat{r}$$



1. $\vec{F} = r^2 \hat{r}$.

$$\int_0^{2\pi}\int_0^{\pi} \vec{F} \cdot \hat{r}\, da = r^4 \sin\theta\, d\theta\, d\phi$$

$$= 4\pi R^4$$

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 r^2\right] = 4r$$

$$d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{R} (\nabla \cdot F)\, d\tau = 4\int_0^R r^3\, dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = 4\pi R^4$$

$$\frac{R^4}{\cancel{4}} \quad \cancel{2} \quad 2\pi$$

$$\vec{F}_2 = \frac{\hat{r}}{r^2}$$

$$\underbrace{\iiint_V (\vec{\nabla}\cdot\vec{F}_2)\, d\tau}_{\swarrow 0} = \underbrace{\iint_a \vec{F}_2 \cdot da}_{\to 4\pi}$$

$$\vec{\nabla}\cdot\vec{F}_2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{1}{r^2}\right) = 0$$

$$\vec{F}_2 \cdot da\,\hat{r} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$$

$$\int_0^{2\pi}\int_0^{\pi} \vec{F}_2 \cdot d\vec{a} = \underbrace{\sin\theta\, d\theta}_{2}\, \underbrace{d\phi}_{2\pi}$$

$$= 4\pi$$

Dirac-Delta function.

$$\left\{ \begin{array}{l} \exp\left[-\dfrac{(x-a)^2}{2s^2}\right] \\[2em] \exp\left[-x^2\right] \end{array} \right.$$