1. $\begin{cases} \vec{F}_1 = x^2 \hat{z} \\ \vec{F}_2 = x\hat{x} + y\hat{y} + z\hat{z} \end{cases}$

a. Calculate the divergence & curl of $\vec{F}_1$ & $\vec{F}_2$

b. Which one can be written as gradient of a scalar?

c. Find scalar potential that does the job.

d. Which can be written as the curl of a vector?

e. Find the suitable vector potential.

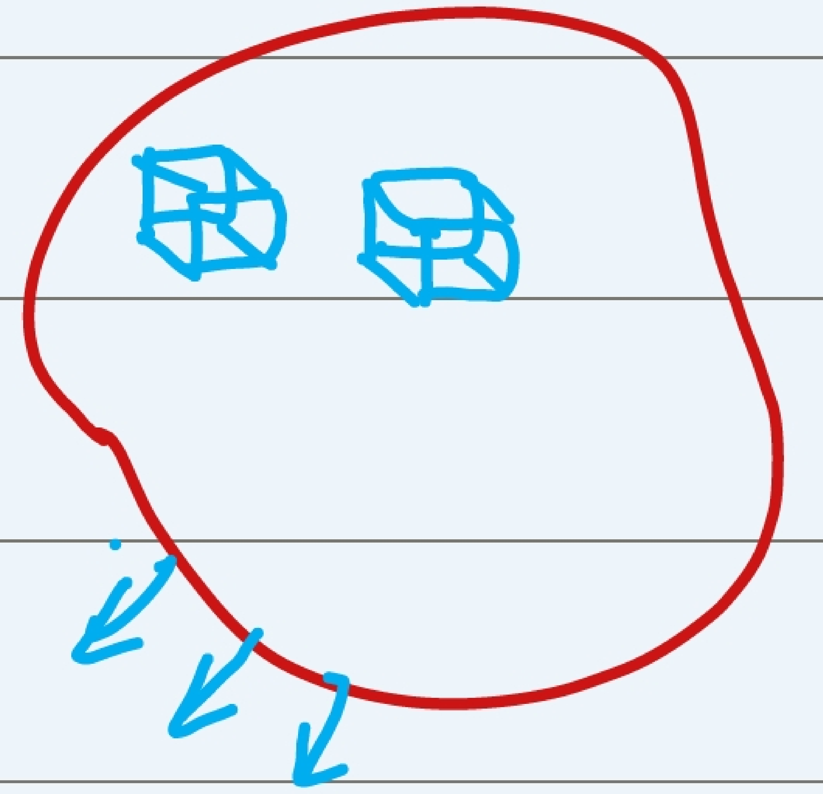
2. $\vec{V}_2 = \frac{\hat{r}}{r^2}$. Check Gauss Divergence Theorem for the function using your volume as the sphere of radius R, centered at the origin.

Divergence theorem

$\vec{F}$

$$\iint (\nabla \times \vec{F}) \cdot d\vec{a} = \oint \vec{F} \cdot d\vec{l}$$

$$\iiint (\nabla \cdot \vec{F})\, d\tau = \oiint \vec{F} \cdot d\vec{a}$$



Q Q

Q

$\vec{F} = \nabla f$

$\hookrightarrow$ Scalar potential

$\vec{F} = \nabla \times \vec{A} \rightarrow$ vector potential.

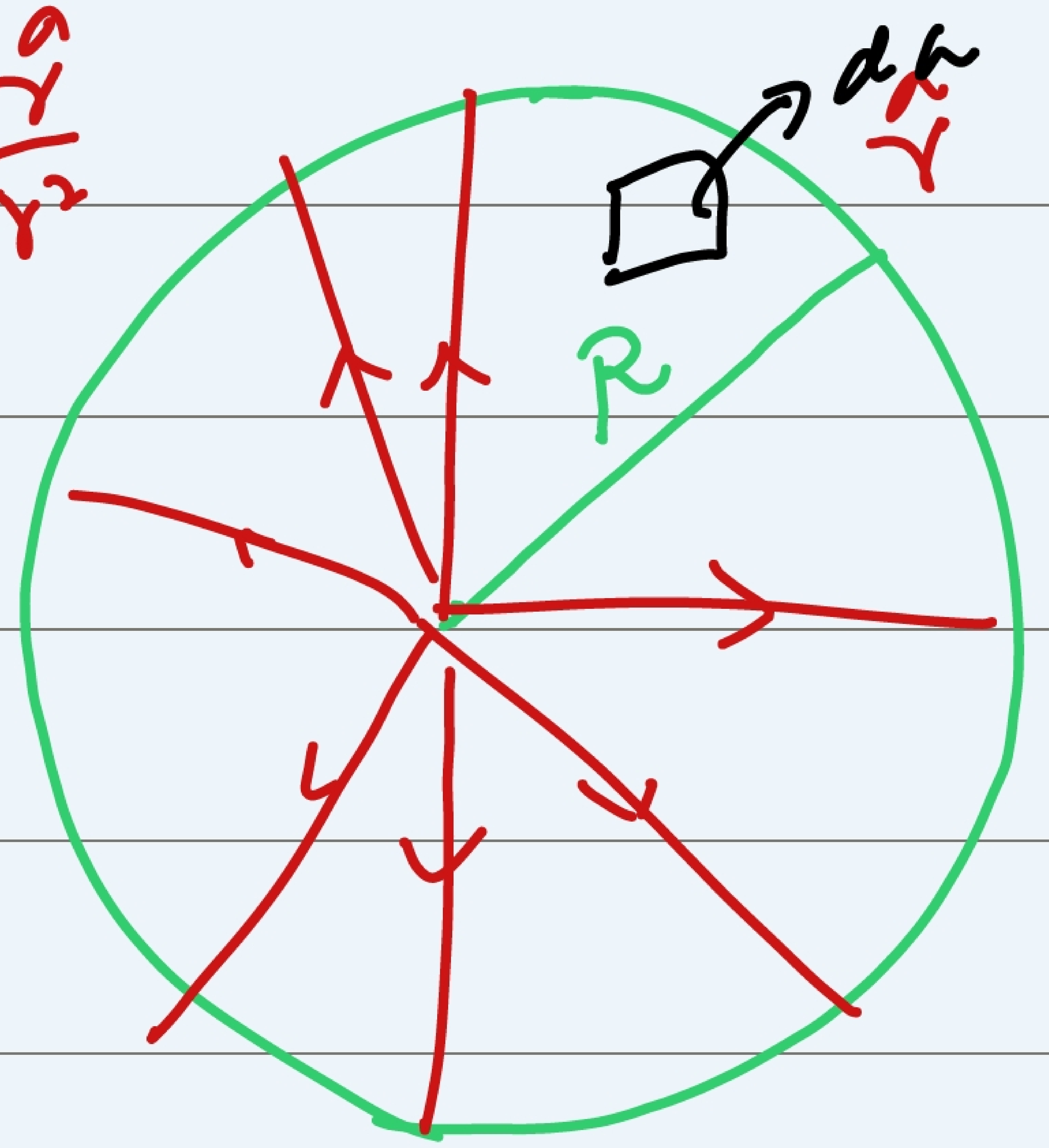
0

$$\boxed{\vec{V} = \frac{\hat{r}}{r^2}}$$

$$\vec{\nabla}\cdot\vec{V} = 0 \quad 4\pi\delta(r)$$

$$0 \leftarrow \iiint (\nabla\cdot\vec{V})\, d\tau = \int_0^{2\pi}\int_0^{\pi} \vec{V}\cdot d\vec{a} \rightarrow 4\pi$$

$$\vec{V} = \frac{\hat{r}}{r^2}$$



$r^2\hat{r}$

Spherical polar

$$\vec{\nabla}\cdot\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{1}{r^2}\right] = 0$$

$r = r'$

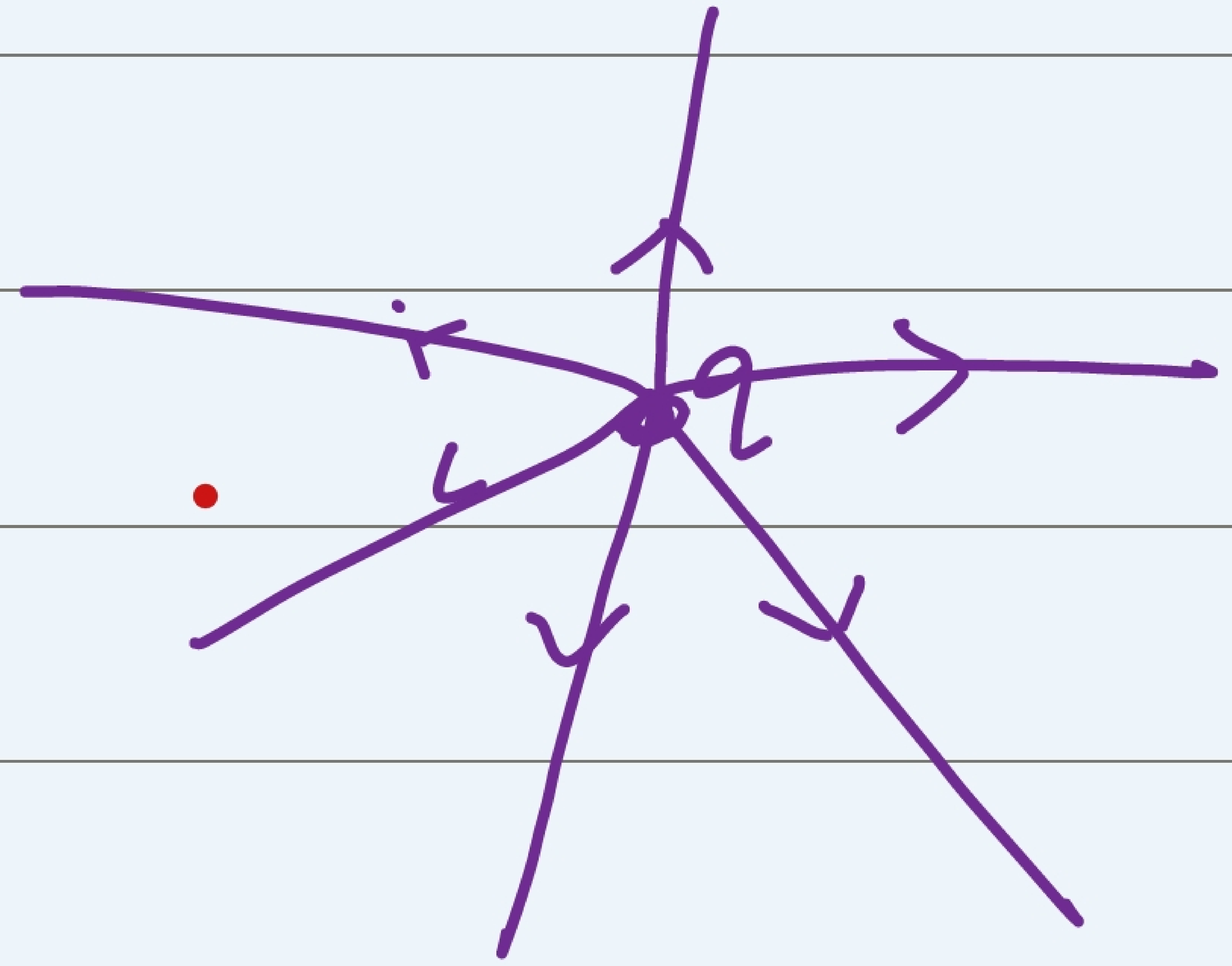
$4\pi\delta(r-r') = 4\pi$

$$\vec{V}\cdot d\vec{a} = dl_\theta\, dl_\phi\, \hat{r}$$

$$\vec{V}\cdot da = \vec{V}\cdot\left[r\,d\theta \;\; r\sin\theta\, d\phi\right]\hat{r} = \frac{\hat{r}}{r^2}\cdot r^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi = 4\pi$$

$\vec{V} = \frac{\hat{r}}{r^2}$ ✓✓



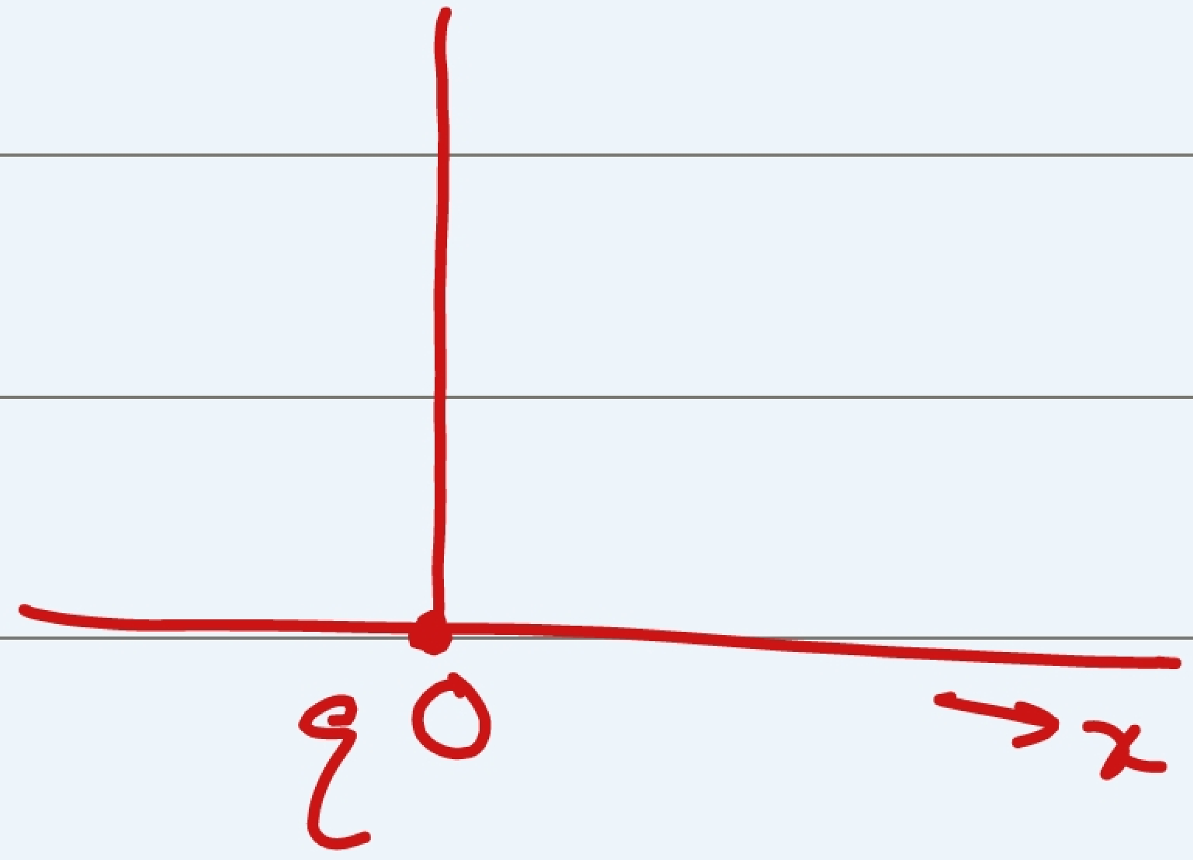
$\nabla \cdot \vec{V} = 0$

Dirac-Delta function :→

If :

$$\delta(x) = 0 \quad \text{if } x \neq 0$$
$$= \infty \quad \text{if } x = 0.$$

$$\int_{-\infty}^{+\infty} \delta(x)\,dx = 1.$$



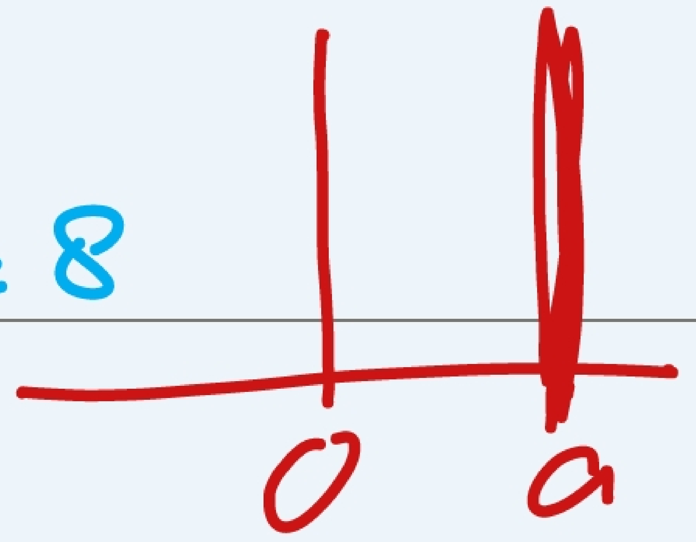
What is the charge density of a point charge q kept at origin?

$$\rho(x) = q\,\delta(x)$$

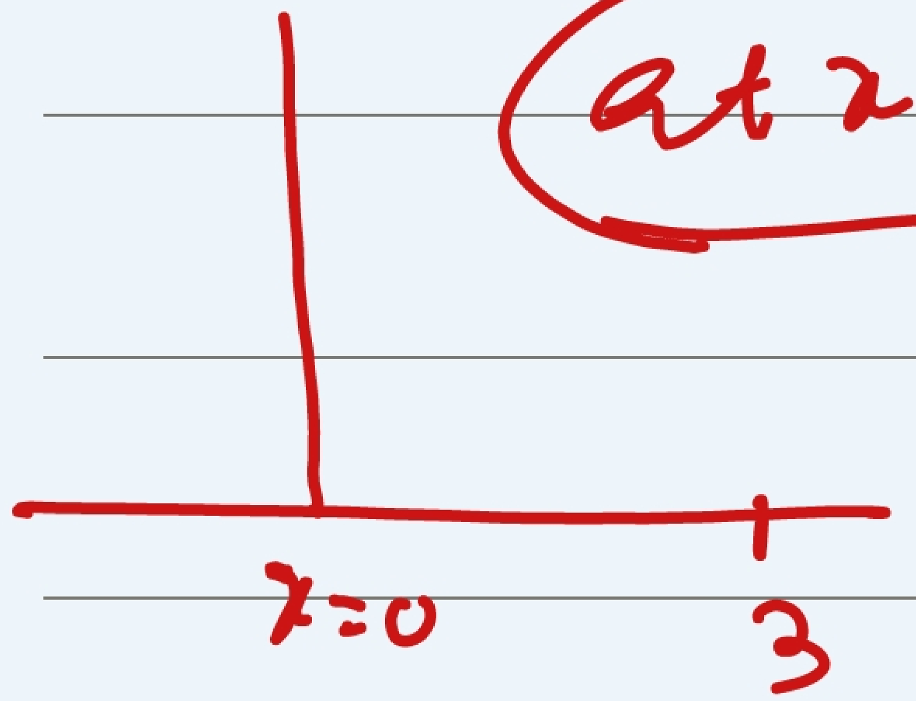
$$\int_{-\infty}^{+\infty} \rho(x)\,dx = \int_{-\infty}^{+\infty} q\,\delta(x)\,dx = q$$

$x \neq 2$

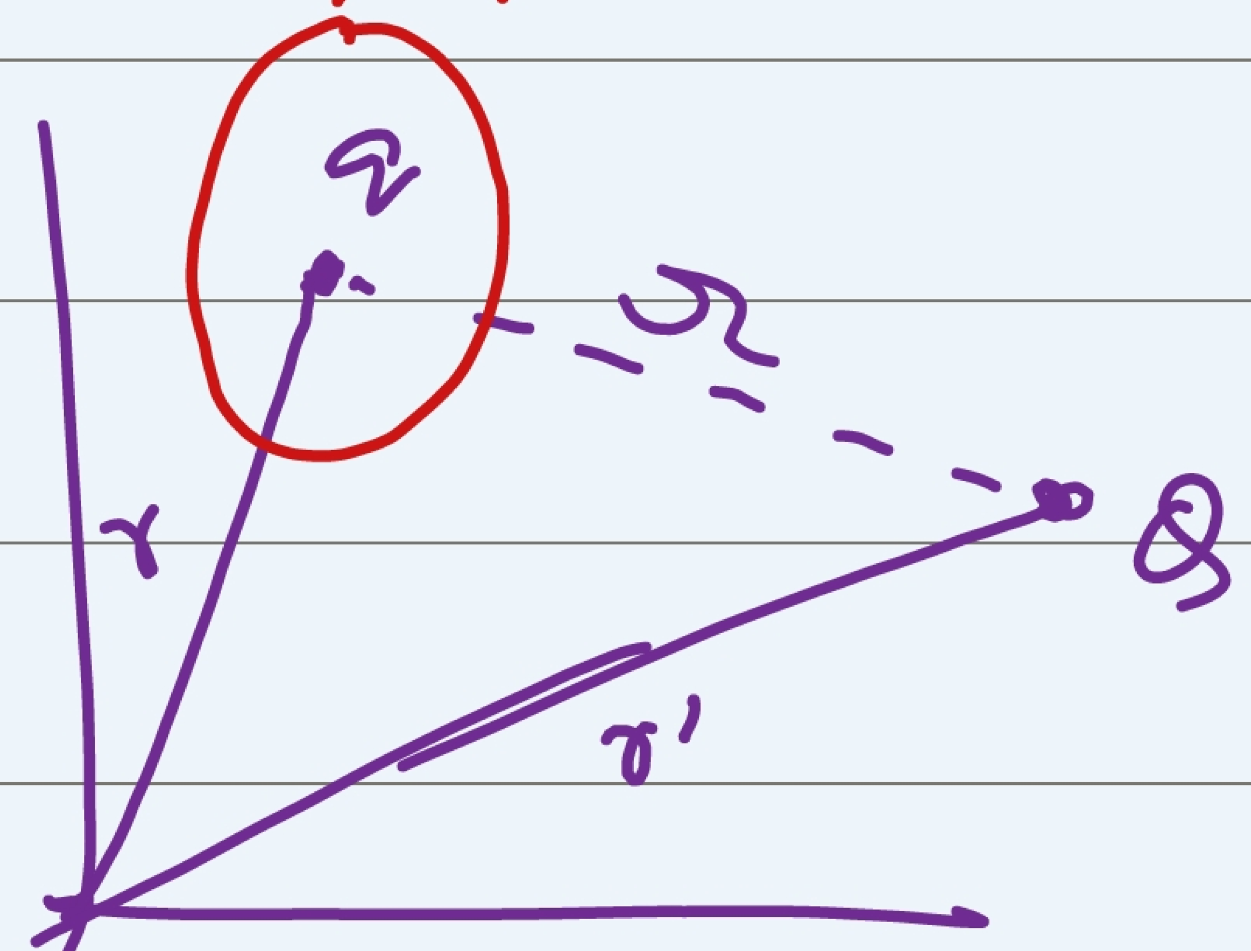
$$\int_0^3 x^3\,\delta(x-2)\,dx = 2^3 \cdot 1 = 8$$



$$\delta(x) = 0,\quad x \neq 0$$
$$= \infty,\quad x = 0$$



at $x = 2$



$$\delta(x-a) = 0,\quad x \neq a$$
$$= \infty,\quad x = a.$$

$$\int_{-\infty}^{+\infty} \delta(x-a)\,dx = 1.$$

at $x = 1$

$$\int_0^1 x^3\,\delta(x-2)\,dx = 0$$

$$\iint \nabla \cdot \rho(r)\,d\tau = \int \vec{E}(r') \cdot d\vec{a}$$

$$\delta(\vec{r}) = \delta(x)\,\delta(y)\,\delta(z)$$

$$\underline{\iiint \delta(\vec{r})\,d\vec{r}} = \iiint \delta(x)\,dx\,\delta(y)\,dy\,\delta(z)\,dz = 1$$