

Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 2

1. Consider an economy with three sectors, Chemicals and Metals, Fuels and Power, and Machinery. Chemicals sells 30% of its output to Fuels and 50% to Machinery and retains the rest. Fuels sells 80% of its output to Chemicals and 10% to Machinery and retains the rest. Machinery sells 40% to Chemicals and 40% to Fuels and retains the rest. Develop a system of equations that leads to prices at which each sector's income matches its expenses. Then write the augmented matrix that can be row reduced to find these prices. Find a set of equilibrium prices when the price for the Machinery output is 100 units.
2. Balance the chemical equation:
$$NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2.$$
3. The augmented matrix of two linear systems have been reduced by row operations to the forms given below. Are they in Row echelon form or Reduced row echelon form? Describe the solution set of the original system, for each matrix.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$
$$\begin{bmatrix} 1 & -2 & 0 & 3 & -2 \\ 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

4. Find the intersection of the planes given below. Write it down in equation form as well as parametric form.

$$5x_1 - x_2 + 2x_3 = 7, -2x_1 + 6x_2 + 9x_3 = 0$$

5. For the following system of linear equations, choose h and k such that the system has: (a) no solution, (b) unique solution, (c) infinitely many solutions.

$$x_1 + hx_2 = 2$$

$$4x_1 + 8x_2 = k$$

6. Suppose the system below is consistent for all possible values of f and g . What can you say about the coefficients c and d ? Justify your answer.

$$x_1 + 3x_2 = f$$

$$cx_1 + dx_2 = g.$$

7. Two matrices are said to be row equivalent if one can be obtained from another by performing finitely many elementary row operations. Is following matrix row equivalent to identity matrix?

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Can you generalize it?

8. Give an example of an inconsistent underdetermined system (A system of linear equations with fewer equations than unknowns) of two equations in three unknowns.
9. A system of linear equations with more equations than unknowns is sometimes called an overdetermined system. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.
10. Find a polynomial whose graph pass through $(1, 12), (2, 15), (3, 16)$. What is its degree? Given n points in \mathbb{R}^2 , will there always exists a polynomial passing through them?