Tut-2

Problem 1.27 Prove that the divergence of a curl is always zero. *Check* it for function $\mathbf{v}_a$ in Prob. 1.15.

Problem 1.26 Calculate the Laplacian of the following functions:

(a) $T_a = x^2 + 2xy + 3z + 4$.

(b) $T_b = \sin x \sin y \sin z$.

(c) $T_c = e^{-5x} \sin 4y \cos 3z$.

(d) $\mathbf{v} = x^2\,\hat{\mathbf{x}} + 3xz^2\,\hat{\mathbf{y}} - 2xz\,\hat{\mathbf{z}}$.

Problem 1.28 Prove that the curl of a gradient is always zero. *Check* it for function (b) in Prob. 1.11.

Problem 1.29 Calculate the line integral of the function $\mathbf{v} = x^2\,\hat{\mathbf{x}} + 2yz\,\hat{\mathbf{y}} + y^2\,\hat{\mathbf{z}}$ from the origin to the point (1,1,1) by three different routes:

(a) $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 0) \to (1, 1, 1)$.

(b) $(0, 0, 0) \to (0, 0, 1) \to (0, 1, 1) \to (1, 1, 1)$.

(c) The direct straight line.

(d) What is the line integral around the closed loop that goes *out* along path (a) and *back* along path (b)?

Problem 1.30 Calculate the surface integral of the function in Ex. 1.7, over the *bottom* of the box. For consistency, let "upward" be the positive direction. Does the surface integral depend only on the boundary line for this function? What is the total flux over the *closed* surface of the box (*including* the bottom)? [*Note:* For the *closed* surface, the positive direction is "outward," and hence "down," for the bottom face.]

Problem 1.31 Calculate the volume integral of the function $T = z^2$ over the tetrahedron with corners at (0,0,0), (1,0,0), (0,1,0), and (0,0,1).

Problem 1.33 Test the divergence theorem for the function $\mathbf{v} = (xy)\,\hat{\mathbf{x}} + (2yz)\,\hat{\mathbf{y}} + (3zx)\,\hat{\mathbf{z}}$. Take as your volume the cube shown in Fig. 1.30, with sides of length 2.

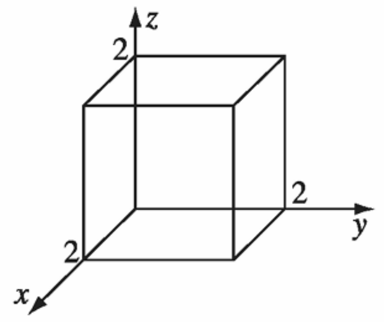

FIGURE 1.30

Problem 1.34 Test Stokes' theorem for the function $\mathbf{v} = (xy)\,\hat{\mathbf{x}} + (2yz)\,\hat{\mathbf{y}} + (3zx)\,\hat{\mathbf{z}}$, using the triangular shaded area of Fig. 1.34.

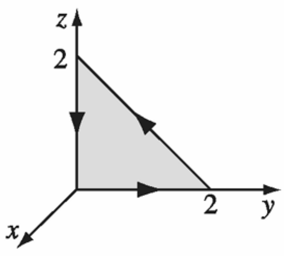

FIGURE 1.34