$$\oint \vec{E}\cdot d\vec{a} = \frac{q}{\epsilon_0} \quad \text{Gauss's Law}$$

$$\nabla \times E = 0, \quad \oint \vec{E}\cdot d\vec{l} = 0$$

$$\oint \vec{B}\cdot d\vec{a} = 0, \quad \vec{\nabla}\cdot\vec{B} = 0$$

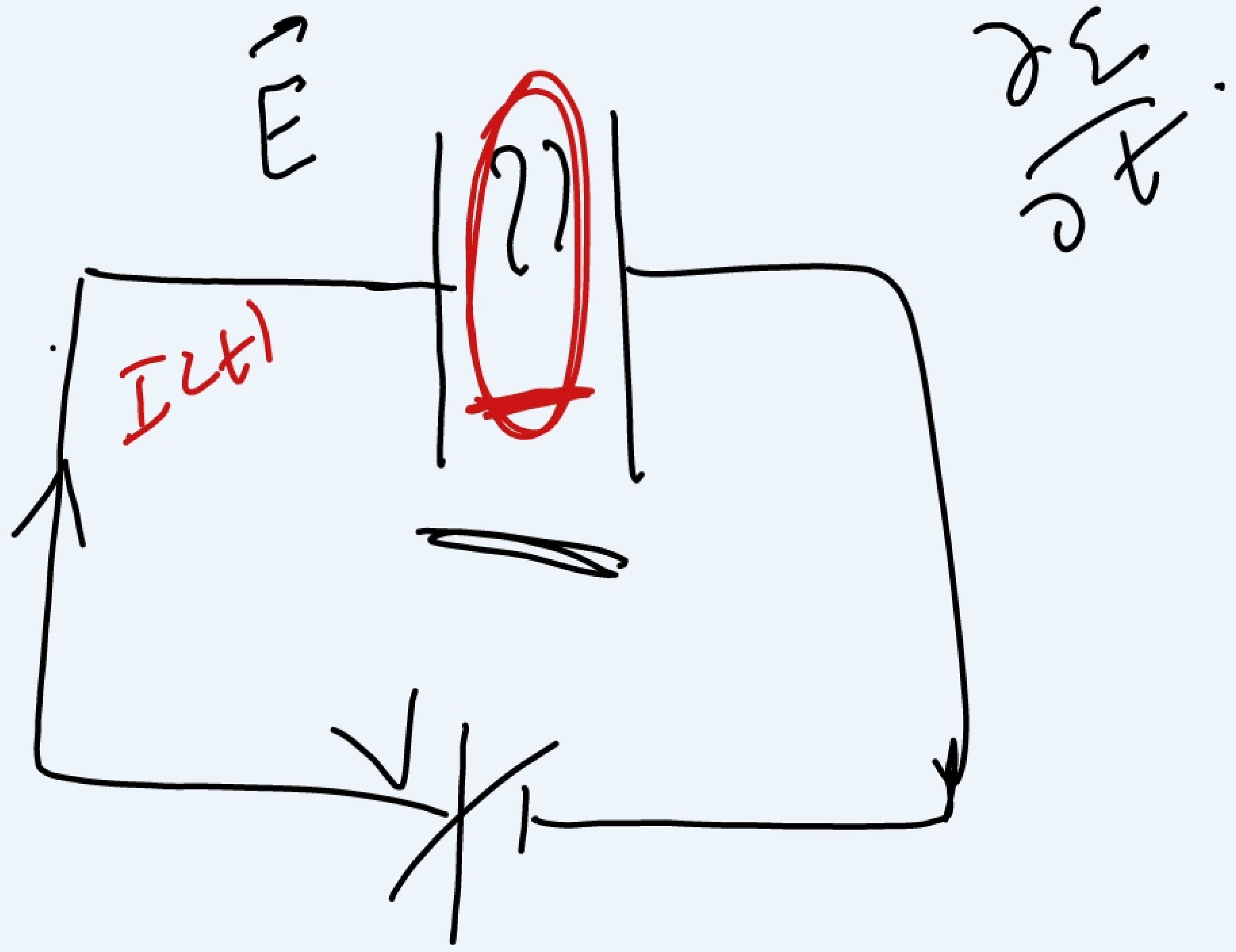
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I, \quad \nabla \times B = \mu_0 J$$

$$\boxed{\mathcal{E}_{\text{induced}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}}$$

$$\oint (\nabla \times E)\cdot da$$

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial B}{\partial t}\cdot d\vec{A}$$

$$\boxed{\begin{array}{c|c} \nabla\cdot B = 0 & \nabla\cdot\vec{E} = \rho/\epsilon_0 \\ \nabla\times B = \mu_0 I & \nabla\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \end{array}}$$



$$\underbrace{\nabla\cdot(\vec{\nabla}\times\vec{E})}_{0} = \underbrace{-\nabla\cdot\frac{\partial\vec{B}}{\partial t}}_{0}$$

$$\mathcal{E} = -\frac{d\phi_B}{\partial t}$$

$$\boxed{\nabla\cdot J + \frac{\partial\rho}{\partial t} = 0}$$

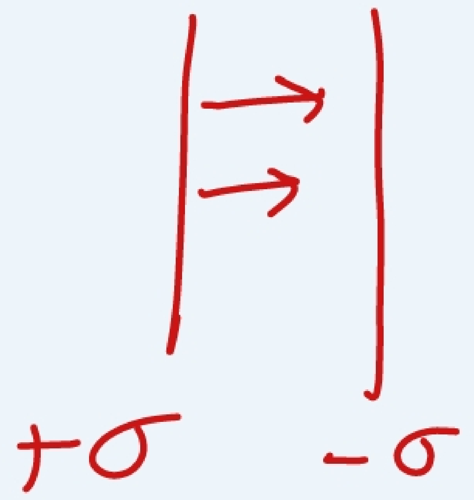
$$\underbrace{\nabla\cdot\nabla\times\vec{B}}_{0} = \mu_0\nabla\cdot J + \underline{??}\ \underbrace{\mu_0\epsilon_0\frac{\partial E}{\partial t}}$$

→ for steady currents

$$\nabla\cdot(\nabla\times B) = \mu_0\left[\nabla\cdot J + \cancel{\epsilon_0}\frac{\partial}{\partial t}\underbrace{\nabla\cdot E}_{\rho/\cancel{\epsilon_0}}\right]$$

$$0$$

$$\nabla\times B = \mu_0 J + \mu_0 \underline{J_d}.$$



$$\vec{E} = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

$$\frac{\partial E}{\partial t} = \frac{I}{A\epsilon_0}$$

$$\Rightarrow \epsilon_0 \frac{\partial E}{\partial t} = \frac{I}{A}$$

$$J_d$$

$$\oint \vec{B}\cdot da \neq 0 \quad q_m$$

Maxwell's Eqⁿ of E-M:

$$\boxed{\begin{aligned} &\nabla\cdot E = \rho/\epsilon_0 && \nabla\times E = -\frac{\partial B}{\partial t} \\ &\nabla\cdot B = 0 && \nabla\times B = \mu_0 J + \mu_0\epsilon_0\frac{\partial E}{\partial t} \end{aligned}}$$

Free space: $\rho = 0,\ J = 0$

$$\nabla\cdot E = 0 \;\frac{\rho_e}{\epsilon_0}, \qquad \nabla\times E = -\frac{\partial B}{\partial t}$$

$$\nabla\cdot B = 0\;\mu_0 J_m \qquad \nabla\times B = \mu_0\epsilon_0\frac{\partial E}{\partial t}$$

Poynting vectors:-

$$W = \vec{F}\cdot d\vec{l}$$

$$= q\left[\vec{E} + \vec{v}\times\vec{B}\right]\cdot\vec{v}\,dt$$

$$= q\,\vec{E}\cdot\vec{v}\,dt$$

$$\boxed{\frac{dW}{dt} = \vec{E}\cdot\vec{J}\,d\tau}$$

$$\nabla\times B = \mu_0 J + \mu_0\epsilon_0\frac{\partial E}{\partial t}.$$

$$\vec{E}\cdot J = \frac{1}{\mu_0}\vec{E}\cdot(\nabla\times B) - \epsilon_0\vec{E}\cdot\frac{\partial E}{\partial t}.$$

$$\frac{1}{2}\epsilon_0 E^2$$

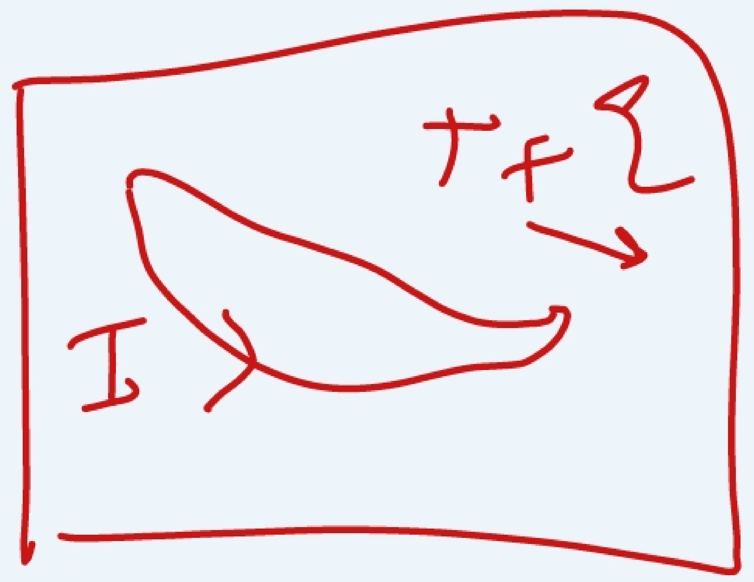
$$\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}\left[E^2\right]$$

$$\frac{B^2}{2\mu_0}.$$

$\vec{E}, \vec{B}$

t

dt



$$q = \rho\,d\tau$$

$$\rho v = \frac{q}{\text{volume}}\times\frac{L}{T}$$

$$= \frac{I}{A} = J$$

$$\vec E\cdot\vec J = \frac{1}{\mu_0}\vec E\cdot(\vec\nabla\times\vec B) - \epsilon_0\vec E\cdot\frac{\partial\vec E}{\partial t}$$

$$\nabla\cdot(\vec E\times\vec B) = \vec B\cdot(\nabla\times\vec E) - \vec E\cdot(\nabla\times\vec B)$$

$$\Rightarrow \vec E\cdot(\vec\nabla\times\vec B) = \vec B\cdot\left[-\frac{\partial\vec B}{\partial t}\right] - \nabla\cdot(\vec E\times\vec B)$$

$$\vec E\cdot\vec J = \frac{1}{\mu_0}\left[-\frac{1}{2}\frac{\partial}{\partial t}B^2 - \nabla\cdot(\vec E\times\vec B)\right] - \frac{\epsilon_0}{2}\frac{\partial}{\partial t}E^2$$

$$\frac{dW}{dt} = \int(\vec E\cdot\vec J)\,d\tau = -\frac{\partial}{\partial t}\int\left[\frac{1}{2\mu_0}B^2 - \frac{\epsilon_0}{2}E^2\right]d\tau - \frac{1}{\mu_0}\int\nabla\cdot(\vec E\times\vec B)\,d\tau = -\int\frac{\partial u}{\partial t}\,d\tau - \int(\nabla\cdot\vec S)\,d\tau$$

$$\nabla\cdot(\vec A\times\vec B)$$

$$u = \frac{\epsilon_0}{2}E^2 + \frac{B^2}{2\mu_0}$$

poynting vector

Power / Area

$$\boxed{\vec S = \frac{\vec E\times\vec B}{\mu_0}}$$

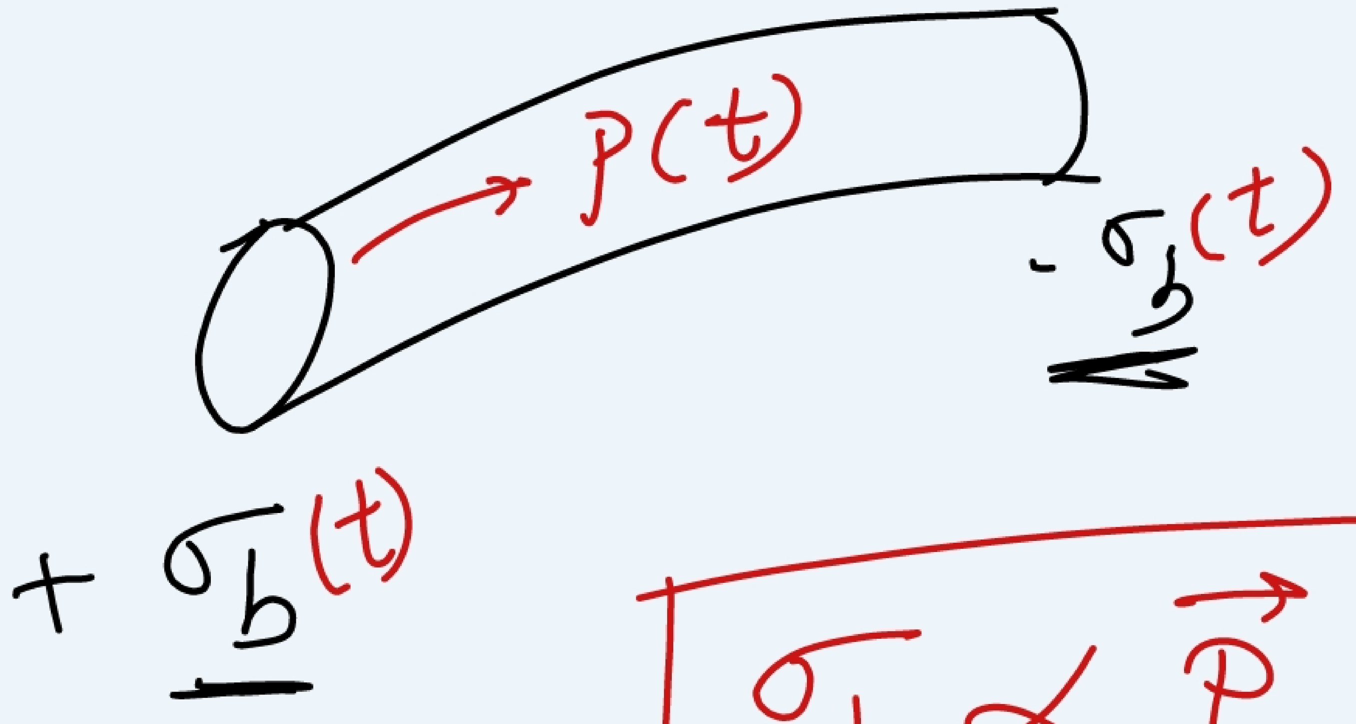
$$\oint\vec S\cdot d\vec a$$

$$\frac{dW}{dt} = 0 \; ; \qquad -\frac{\partial u}{\partial t} = \nabla \cdot \vec{S} \Rightarrow \boxed{\nabla \cdot \vec{S} + \frac{\partial u}{\partial t} = 0}$$ → Conservation of energy

$$\boxed{\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0}$$ → Conservation charge.

$$\frac{\text{Power}}{\text{Area}} = \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Maxwell's Eqn in Matter:



$$\boxed{\sigma_b \propto \vec{P}}$$

$$dI = \frac{d\sigma_b}{dt}\, da_\perp = \frac{\partial P}{\partial t}\, da_\perp$$

$$\boxed{J_p = \frac{dI}{da_\perp} = \frac{\partial P}{\partial t}.}$$

$$D = \epsilon_0 E + \vec{P}$$

$$\rho_b = -\nabla \cdot \vec{P} \qquad \vec{J}_b = \nabla \times \vec{M}$$

$$\sigma_b = \vec{P} \cdot \hat{n} \qquad \vec{K}_b = \vec{M} \times \hat{n}$$

$$\nabla \times \vec{B} = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}.$$

$+q \; (-q \;\; +q) \; (-q \;\; +q) \; -q$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \qquad \epsilon_0 \nabla \cdot E = \rho_f + \rho_b$$

$$\nabla \cdot (\epsilon_0 E + P) = \rho_f$$

$$\boxed{\nabla \cdot D = \rho_f}$$

$$\vec{J} = J_f + J_b + J_p$$

$$\rho = \rho_b + \rho_f$$

$$\nabla \times B = \mu_0 \left[J_f + J_b + \frac{\partial P}{\partial t} \right] + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}.$$

$\nabla \times M$

$$\nabla \times \left[\frac{B}{\mu_0} - M \right] = J_f + \frac{\partial \vec{D}}{\partial t}.$$

$$\boxed{\nabla \times \vec{H} = J_f + \frac{\partial D}{\partial t}.}$$

Maxwell's Eqn in matter

$\underline{\nabla \cdot D} = \rho_f$ $\quad$ $\nabla \times E = -\frac{\partial B}{\partial t}$.

$\nabla \cdot B = 0$ $\quad$ $\underline{\nabla \times H} = J_f + \frac{\partial D}{\partial t}$.

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2}}, \quad v = \cancel{E}\frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

$\vec{E} \perp \vec{B}$