

Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 1

1. In a region around Gandhinagar, about 6% of a city's population moves to the surrounding suburbs each year, and about 4% of the suburban population moves into the city. In 2021, there were 10,000,000 residents in the city and 800,000 in the suburbs. Set up a system of equations that describes this situation, where x_0 is the initial population in 2015. Then estimate the populations in the city and in the suburbs two years later, in 2023.
2. Consider an economy with three sectors, Chemicals and Metals, Fuels and Power, and Machinery. Chemicals sells 30% of its output to Fuels and 50% to Machinery and retains the rest. Fuels sells 80% of its output to Chemicals and 10% to Machinery and retains the rest. Machinery sells 40% to Chemicals and 40% to Fuels and retains the rest. Develop a system of equations that leads to prices at which each sector's income matches its expenses.
3. Compute AB , using the indicated block partitioning.

$$A = \left[\begin{array}{cc|cc} 2 & -1 & 3 & 1 \\ 1 & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad B = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ -1 & 0 & 0 \\ \hline 0 & 5 & 1 \\ 1 & -1 & 0 \end{array} \right]$$

4. Find the inverse of $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}$
5. Find the inverse of $M = \begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$.
6. Derive a formula of inverse of $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ if $D, A - BD^{-1}C$ are non-singular.

7. Find the inverse of $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, where $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 5 \\ 0 & 6 \end{bmatrix}$, $D = \begin{bmatrix} 7 & 0 \\ 8 & 9 \end{bmatrix}$
8. Suppose A is an $m \times n$ matrix and there exist $n \times m$ matrices C and D such that $CA = I_n$ and $AD = I_m$. Prove that $m = n$, and $C = D$.
9. Compute S^5, A^{10} , where $S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Can you guess and prove the formula for A^n ?