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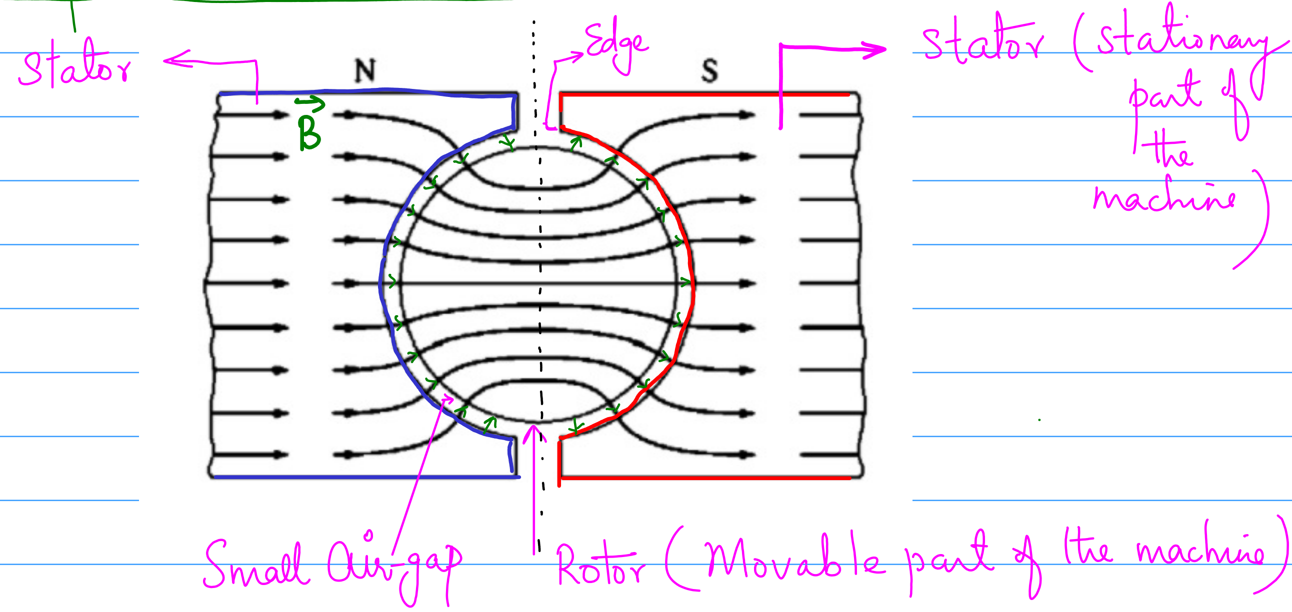
Fundamentals of DC Machinery

(Ref. : Chapter # 8)

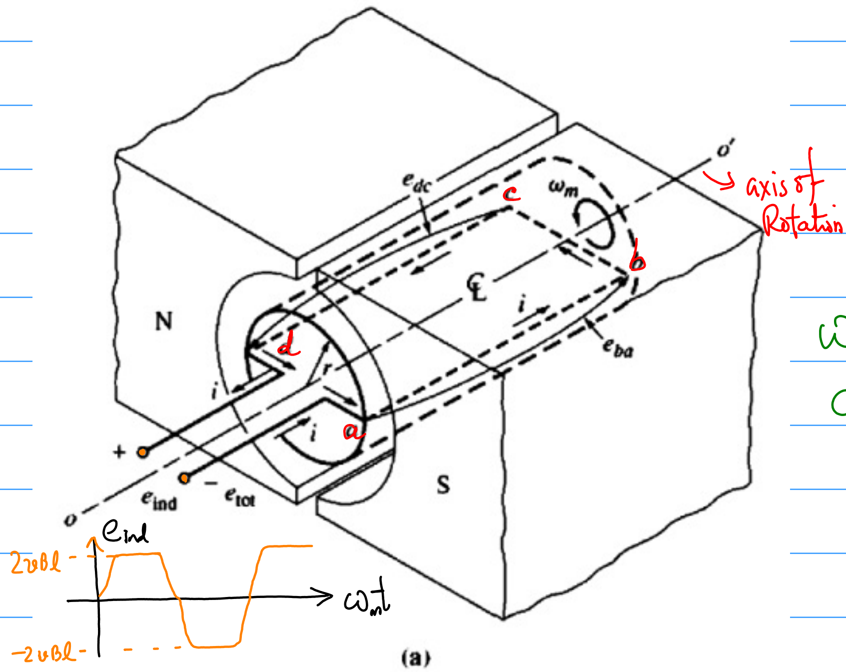
- DC Generators : Mechanical Energy → DC electrical energy
- DC Motors : DC Electrical Energy → Mechanical energy
- In DC machines, the internal voltages/currents are time-varying (in polarity), ie, ac, however, the output is only DC.
- Special arrangement : Commutator-Brush arrangement

This arrangement is being used to convert internal alternating voltages/currents to dc voltages/currents at the output terminals.

Simple DC Machine :



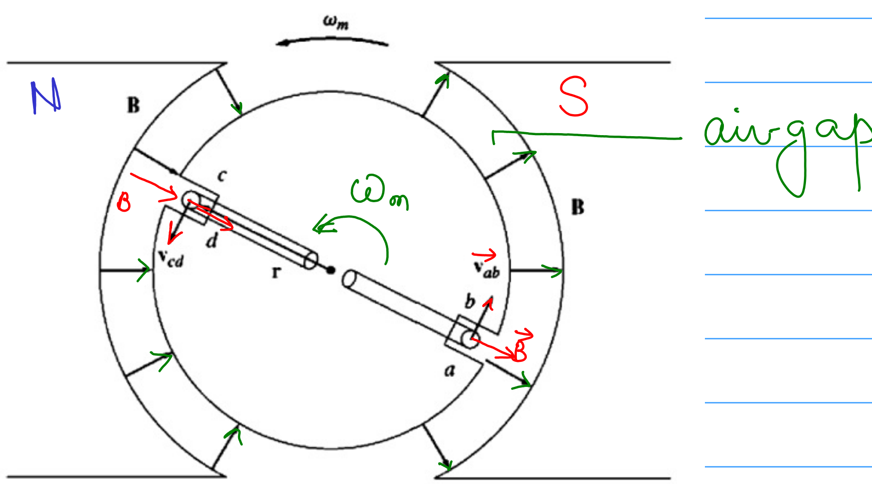
① A single loop of wire rotating about a fixed axis placed with the space of the rotor.



(a)

abcd : Rectangular single loop of wire.

ω_m = mechanical angular frequency of the rotating loop of wire.

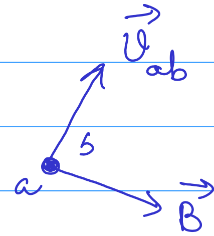


Let determine the induced voltage in a rotating loop of wire.

Since we know that $e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$

(i) For segment ab :

$$e_{ab} = (\vec{v}_{ab} \times \vec{B}) \cdot \vec{l} = vBl$$



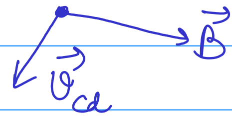
$$e_{ab} = \begin{cases} vBl & \text{into the page} \quad \text{under the pole face.} \\ 0 & \text{beyond the pole face (or within the edges of the pole)} \end{cases}$$

(ii) For segments bc/ da :

Since $(\vec{v} \times \vec{B})$ and $\vec{l}$ are $\perp$ to each other,

$$e_{bc/da} = 0$$

(iii) Segment cd :



$$e_{cd} = \begin{cases} vBl & \text{positive out of the page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole face} \end{cases}$$

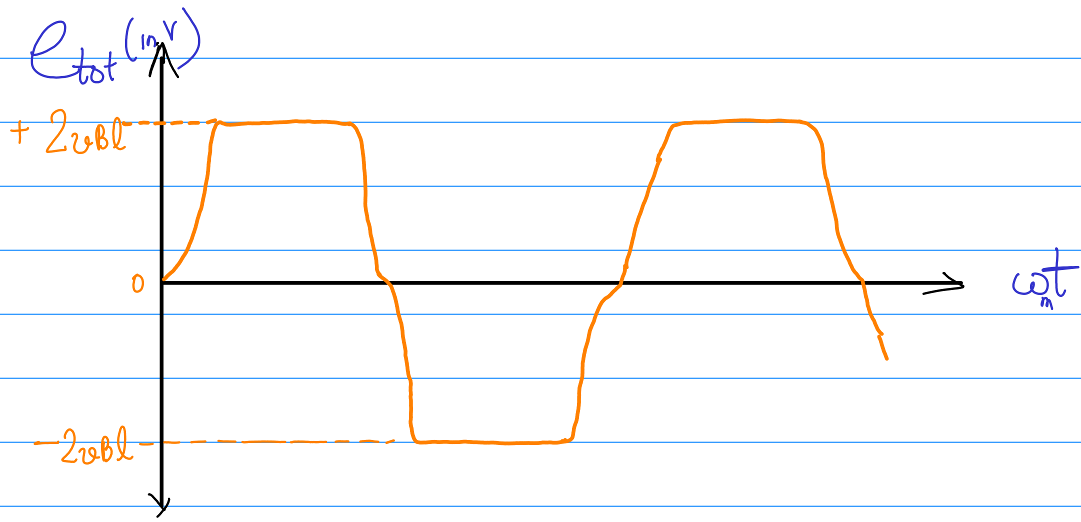
$$e_{ind,\,tot.} = \begin{cases} 2\,vBl & \text{under the pole faces.} \\ 0 & \text{beyond the pole faces} \end{cases}$$

Ques: What will happen when the loop rotates through 180°?

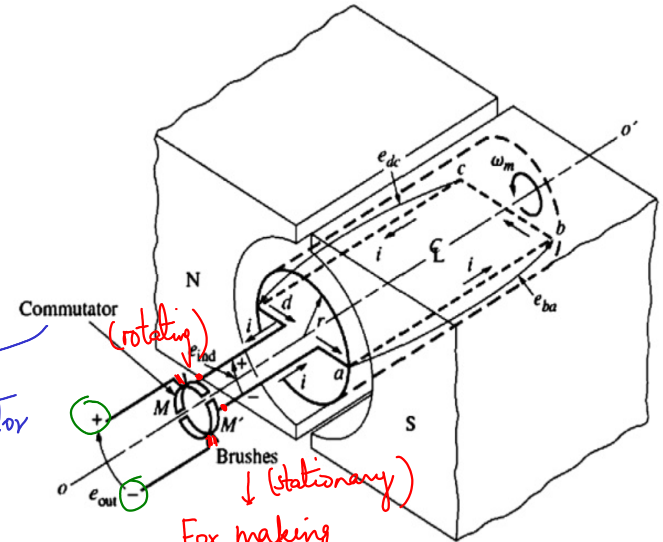
The segment ab (or cd) is under the north (South) pole face instead of the south (north) pole face.

Under this condition (ie, after 180° rotation);

the direction of the induced voltage on the segments ab and cd reverses, however, the magnitude remains constant.



How do we get DC induced voltage out of the rotating loop of wire



Semi-circular conductor connected to the terminals of the loop.

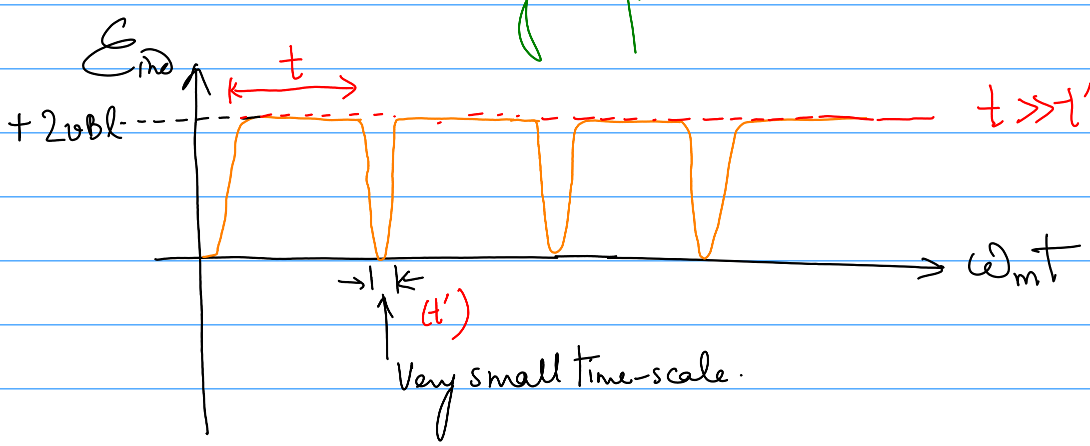
(rotating)

(stationary)

For making electrical contact with the commutator.

After 180° rotation, the commutator M, M′ swaps its position.

Commutator-Brush arrangement is helping in connection switching process.



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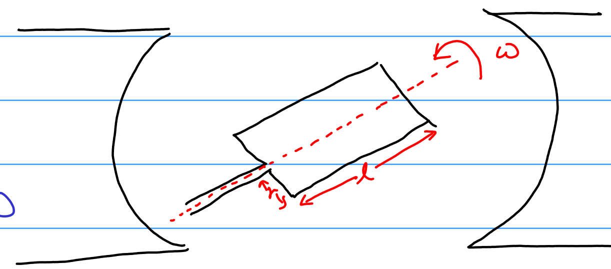


Linear velocity $v = r\omega$

Also, geometrical, the area that the loop swaps is the surface area of the cylender of radius 'r' and length 'ℓ'.

$$A = (2\pi r)\,\ell .$$

Let's ignore the area across the poles edges, so the area of the rotor (loop) under each pole

$$A_p = \pi r \ell$$

therefore, the expression for the induced emf is modifed

$$e_{ind,total} = \begin{cases} 2\,vB\ell & \text{under the pole face,} \\ 0 & \text{beyond the pole face} \end{cases}$$

$$v \to r\omega, \qquad \underbrace{r\omega\;\ell}_{A_p/\pi}$$

$$e_{ind,total} = \begin{cases} \frac{2}{\pi}\,\underbrace{A_p B}_{\phi}\cdot\omega \\ 0 \end{cases}$$

$$e_{ind,tot} = \begin{cases} \left(\frac{2}{\pi}\right)\cdot\phi\,\omega & \text{under each pole faces} \\ 0 & \text{beyond the pole faces} \end{cases}$$

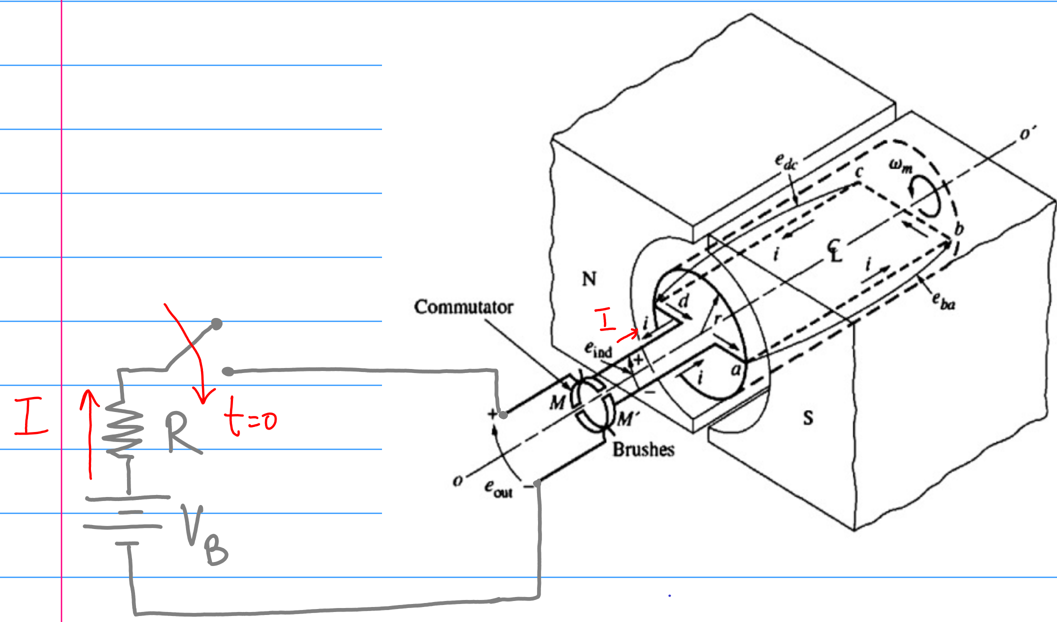
In general, the induced voltage depend on:

(i) The flux in the machine (ϕ).

(ii) The speed of rotation (ω).

(iii) A constant representing the construction of the machine.

(B) Induced Torque in the Current carrying Loop.



- Because of current I, there will be induced force

$$\vec{F}_{ind} = I(\vec{l} \times \vec{B})$$

$$\vec{\tau}_{ind} = \vec{r} \times \vec{F}_{ind}.$$

$$\tau_{ind} = \begin{cases} \frac{2}{\pi}\phi I & \text{under the pole faces} \\ 0 & \text{beyond the pole faces} \end{cases}$$

⇒ We can summarize that the induced Torque depend on

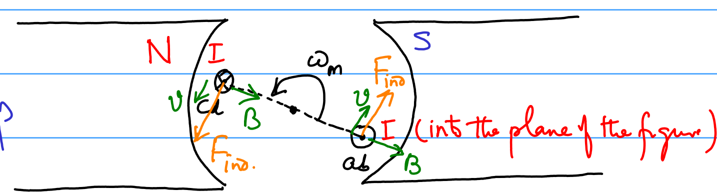
(i) The flux in the machine (ϕ)

(ii) The current in the machine (I)

(iii) The constant ($2/\pi$) representing the construction of the machine.

Problem :

given a rotating loop of wire b/w curved poles connected to an external battery (V_B) and a resistor through a switch.



$r = 0.5\,m$ $\quad\quad$ $l = 1.0\,m$

$R = 0.3\,\Omega$ $\quad\quad$ $B = 0.25\,T$

$V_B = 120V$ (DC)

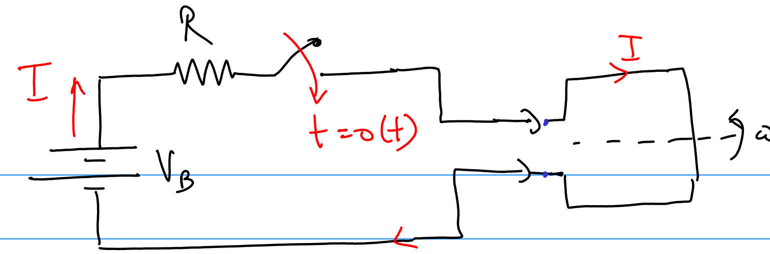
(a) What will happen when the switch is closed? (time at which the switch is Turned-ON).

- As the switch is Turned-ON, there will be current 'I' in the loop.

- Since, at $t=0$, there is no motion (no rotation) at all in the loop;

$$\mathcal{E}_{ind} = 0 \quad (t=0)$$

$$I = \frac{V_B - \mathcal{E}_{ind}}{R}$$



Since, at $t = 0(+)$; $\mathcal{E}_{ind} = 0$

$$I = \frac{V_B}{R} = \frac{120V}{0.3\,\Omega} = 400A \quad ✓$$

$$\tau_{ind} = \frac{2}{\pi}\phi I \qquad \text{CCW}$$

This induced torque produces angular acceleration, and because of this, the loop gains the angular speed.

$$\mathcal{E}_{ind}\uparrow = \left(\frac{2}{\pi}\right)\phi\ \omega\uparrow$$

$$I\downarrow = \frac{V_B - \mathcal{E}_{ind}\uparrow}{R}$$

$$\tau_{ind}\downarrow = \left(\frac{2}{\pi}\right)\phi \cdot I\downarrow$$

Hence, the machine winds up in the steady-state with $\tau_{ind} = 0 \Rightarrow \underbrace{V_B = \mathcal{E}_{ind}}$.

$$V_B = \left(\frac{2}{\pi}\right)\phi\ \omega_{ss}$$

$$\omega_{ss} = \frac{V_B}{(2/\pi)\phi} = \frac{V_B}{(2/\pi)\cdot B\cdot A_p} =$$

$$\omega_{ss} = \frac{120V}{(2/\pi)(0.25T)(\underset{0.5m}{\pi r}\cdot \underset{1.0m}{l})}$$

Here, we didn't talk about any external mechanical load.

$$\boxed{\omega_{ss} = 480 \text{ rad/s}}$$

(No-Load steady-state velocity)

$$\boxed{v_{ss} = r\cdot \omega_{ss}}$$

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(b) What will happen when a mechanical load is attached resulting a load torque = 10 N-m ?

- It will slow down the machine momentarily.
- As a consequence :

$$\omega_{ss}\downarrow \Rightarrow E_{ind}\downarrow \Rightarrow I\uparrow \Rightarrow \tau_{ind}\uparrow \text{ until}$$

$$\tau_{ind} = \tau_{load} = 10 \text{ N-m}$$

$$I' = \frac{\tau_{ind}}{(2/\pi)\phi} = \frac{10\text{N-m}}{(\frac{2}{\pi})\cdot B\cdot A_p} = 40A$$

$$E_{ind}' = V_B - I'R = 108\,V$$

$$\omega' = \frac{E_{ind}'}{\left(\frac{2}{\pi}\right)\phi} = 432 \text{ rad/s.}$$

$$\boxed{\omega_{ss}' = 432 \text{ rad/s}}$$

$\left\{ \omega_{ss} = 480 \text{ rad/s} \right.$

Mechanical power supplied to the rotor's shaft where the load ($\tau_{Load} = 10\,N\text{-}m$) is attached.

$$P_{mech.} = \tau_{ind} \cdot \omega_{ss}' = (10\,N\text{-}m)(432 \text{ rad/s})$$

$$\boxed{P_{mech.} = 4320\,W}$$ Output power

Input power: $P_{Elect.} = V_B \cdot I = (120V)(40A) = 4800\,W$

$$\boxed{P_{Elect.} = 4800\,W}$$

Assumption:
No mechanical loss of power

$$P_{Loss} = I^2 R = (40)^2 \cdot (0.3\,\Omega) = 480\,W$$

$$\boxed{P_{Loss} = 480\,W}$$

$$P_{Elect.} = P_{Mech.} + P_{Loss}$$

DC Motors

(Ref. Chapter 9)

- Beyond the years 1890s, ac power systems took over the dc power systems.
- However, the dc power systems (machines) such as dc motor or generators are still in use.
- In particular, automobile industries, aircraft industries, and many other house hold applications.
- Wide variations in the speed in concerned – DC Motors.
- DC motors are often compared by their speed regulations.

$$SR = \frac{\omega_{NL} - \omega_{FL}}{\omega_{FL}} \times 100 \%$$

$$SR = \frac{n_{NL} - n_{FL}}{n_{FL}} \times 100 \%$$

$SR = +ve \Rightarrow$ motor's speed drops as the mech. load increases

$= -ve \Rightarrow$ motor's speed rises as the mech. load increase.

Types of DC motor :

i) Separately-excited dc motor ✓

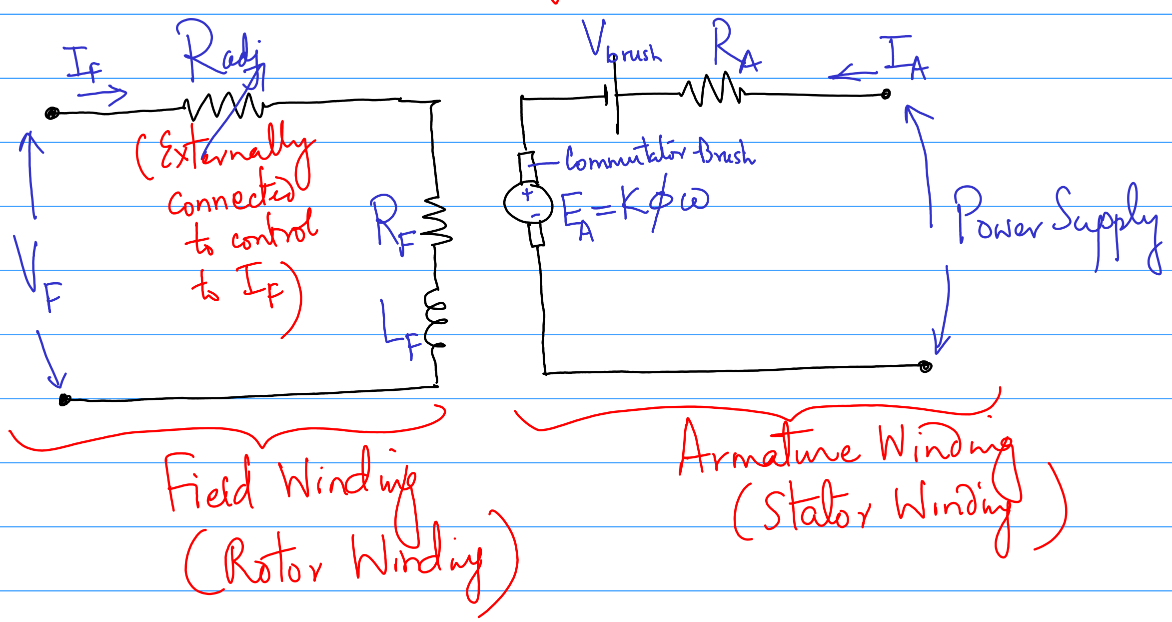
(ii) Shunt dc motor ✓

(iii) Parmanent-magnet dc motor.

(iv) Series dc motor

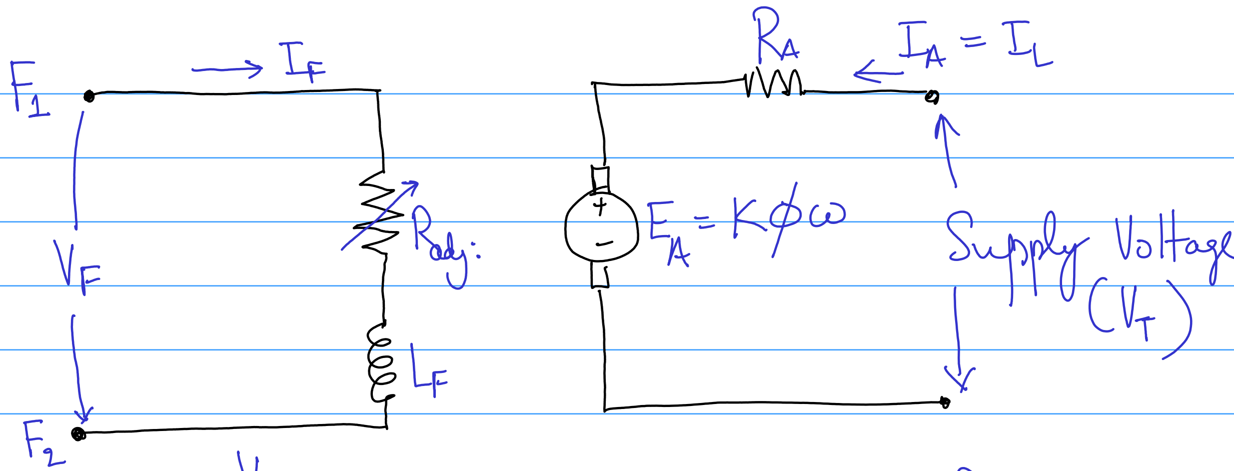
(v) Compounded dc-motor.

Equivalent ckt. model of a dc motor.



⇒ Approximate Equivalent ckt :

Separately excited DC-Motor



$$I_F = \frac{V_F}{R_{adj.}}$$

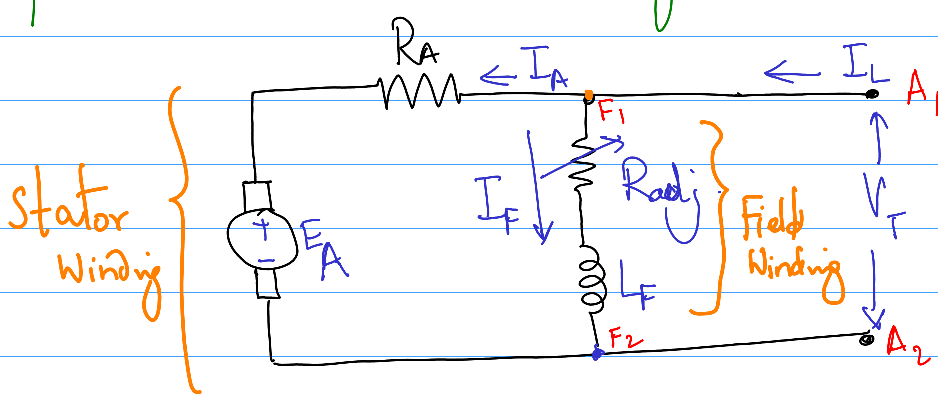
$$V_T = E_A + I_A R_A$$

$$\boxed{\begin{aligned} E_A &= K\phi\omega \\ \tau_{ind} &= K\phi I_A \end{aligned}}$$

$E_A \propto \phi$
$\propto \omega$

$\tau_{ind} \propto \phi$
$\propto \omega$

Equivalent ckt. model of shunt dc-motor.



$$I_F = \frac{V_T}{R_{adj.}} \; ; \quad V_T = E_A + I_A R_A$$

$$I_L = I_A + I_F$$

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Terminal Characteristics of a shunt DC Motor

- Plot of output torque ($\tau_{ind.}$) vs. speed
- Effect of varying load (mech. load)

Let us consider the case where the motor is in steady-state and the more load is attached $\tau_{Load}\uparrow$

$$\Rightarrow \quad \tau_{Load} > \tau_{ind.} \quad \text{thereby} ; \ \omega\downarrow$$

when $\omega\downarrow \Rightarrow E_A\downarrow = K\phi\omega\downarrow$

$$\Rightarrow I_A\uparrow = \frac{V_T - E_A\downarrow}{R_A}$$

$$\Rightarrow \tau_{ind}\uparrow = K\phi I_A\uparrow$$

As an effect, the motor achieves the new steady-state condition; where,

$$\tau_{ind} = \tau_{Load}.$$

But the at the lower speed

$$\boxed{\omega_{ss}' < \omega_{ss} \qquad (\text{Speed in new steady-state cond}^n) < (\text{Speed in the steady-state cond}^n \text{ when the load is increased})}$$

Now, from equivalent ckt. model

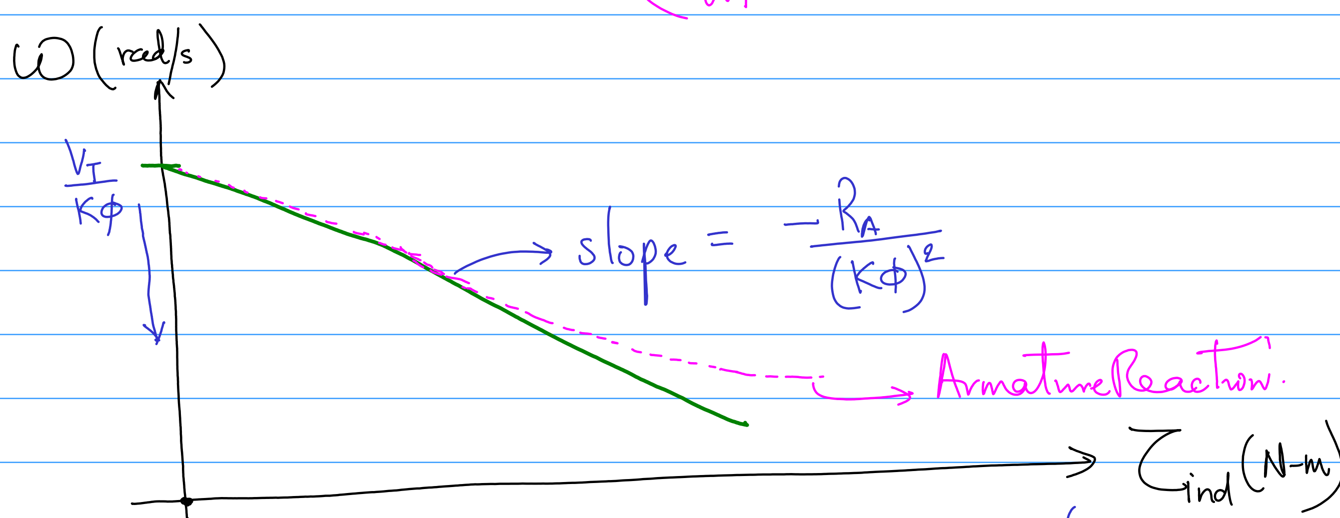
$$V_T = \underbrace{E_A}_{K\phi\omega} + \underbrace{I_A}_{\frac{\tau_{ind}}{K\phi}} R_A$$

$$\Rightarrow \quad V_T = K\phi\omega + \frac{\tau_{ind}}{K\phi} \cdot R_A$$

$$\Rightarrow \quad \omega = \frac{V_T}{K\phi} - \frac{\tau_{ind}}{(K\phi)^2} \cdot R_A$$

$$\boxed{\omega = -\left[\frac{R_A}{(K\phi)^2}\right] \tau_{ind} + \frac{V_T}{K\phi}}$$

$$Y = \left(\text{slope 'm'}\right) X + \left(\text{Y-Intercept } C\right)$$



(Increase in mech. Load)

That is, the speed of the motor vary linearly with the torque provided:

1) the terminal voltage V_T remains constant..

If not, then the shape of the ω Vs τ changes.

2) All other parameters should remains constant.

3) For the st. linear (linear) characteristics, the flux ϕ should also have to remain constant, however, due to armature reaction, there is some reduction in the flux as the load changes (↑↓).

Example Problem:

A 50 hp, 250-V, 1200 rpm dc shunt motor and

$$R_A = 0.06\Omega \quad ; \quad R_{adj} + R_F = 50\Omega$$

$$n_{No-Load} = 1200\text{rpm} \quad ;$$

Input Current	Output Speed n_m
1. 100 A	____
2. 200 A	____
3. 300 A	____

$$E_A = K\phi\omega = K'\phi n_m$$

Let's consider the initial state:
No-load is attached

$$E_{A,i} = K'\phi n_{m,i} \quad — (1)$$

Let's consider a final state: Some load is attached

$$E_{A,f} = K'\phi\, n_{m,f} \qquad \text{— (2)}$$

$$\boxed{\frac{E_{A,f}}{E_{A,i}} = \frac{n_{m,f}}{n_{m,i}}}$$

From the given condⁿ of the problem;

$$\left.\begin{aligned} n_{m,i} &= 1200 \text{ rpm} \\ E_{A,i} &= V_T = 250\text{ V} \end{aligned}\right\} \text{provided in the problem.}$$

We also know from the equivalent ckt. model of dc shunt motor,

$$I_A = I_L - I_F$$

$$\Rightarrow \quad I_A = I_L - \left(\frac{V_T}{R_{adj} + R_F}\right)$$

$$I_L = 100\text{ A} \Rightarrow I_{A,1} = 100\text{A} - \frac{250\text{V}}{50\Omega} = 95\text{A}$$

$$I_L = 200\text{ A} \Rightarrow I_{A,2} = 200\text{A} - \frac{250\text{V}}{50\Omega} = 195\text{A}$$

$$I_L = 300\text{ A} \Rightarrow I_{A,3} = 300\text{A} - 5\text{A} = 295\text{A}$$

Now, $E_{A1} = V_T - I_{A,1} \cdot R_A = 250\text{V} - (95\text{A})(0.06\Omega)$

$$E_{A1} = 250\text{V} - 5.70\text{ V} = 244.3\text{ V}$$

$$E_{A1} = 244.3\ V$$
$$E_{A2} = ?\ \checkmark$$
$$E_{A3} = ?\ \checkmark$$

$$\Rightarrow \frac{n_{m,1}}{n_{m,i}} = \frac{E_{A,1}}{E_{A,i}}$$

$$n_{m1} = \frac{244.3V}{250V} \times 1200\ rpm$$

n_{m2}, n_{m3},

$$P_{conv.} = E_A \cdot I_A = \tau_{ind} \cdot \omega$$

We know that

$$\tau_{ind} = \frac{E_A \cdot I_A}{\omega}$$

$$\tau_{ind,1} = \frac{E_{A1} \cdot I_{A.1}}{\omega_{m,1}} = (\quad)$$

$$\tau_{ind,2} = \frac{E_{A2} \cdot I_{A2}}{\omega_{m,2}} = (\quad)$$

$$\tau_{ind,3} = \frac{E_{A3} \cdot I_{A3}}{\omega_{m,3}} = (\quad)$$