Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 5

1. Find the determinant of following matrices:

$$A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 & a & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & 0 & e & f \\ p & q & r & s & t \\ v & w & x & y & z \end{bmatrix}.$$

2. Find det(A), where $A = [a_{ij}]_{5\times 5}$, where $a_{ij} = i + j$

3. Find the determinant of $n \times n$ identity matrix when its i^{th} row is replaced by $[x_1, x_2, \cdots, x_i, \cdots, x_n]^T$.

4. Find the area of the parallelogram whose corner points are $(2, 2), (5, 2), (3, 6), (6, 6)$.

5. Find a formula for the area of the triangle (in terms of determinant) whose vertices are $0, v_1, v_2$ in $\mathbb{R}^2$.

6. Find a formula for $det(rA)$ when A is an $n \times n$ matrix and r is a real no.

7. Use the concept of area of a parallelogram to write a statement about a 2×2 matrix A that is true if and only if A is invertible.

8. Find matrices A, B, C, D such that $det(\begin{bmatrix} A & B \\ C & D \end{bmatrix}) \neq det(A)det(D) - det(B)det(C)$. If possible, write down $\det(\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix})$ in terms of determinants of A, B, C, D, E, F, G, H?

9. Give a 3×3 invertible matrix with entries from $\mathbb{Z}_2$. Find the total number of 3×3 invertible matrices with entries from $\mathbb{Z}_2$.

10. For any square matrix A, show that $A.adj(A) = det(A).I$.