PH110: Waves and Electromagnetics (3-1-0:4)

Lecture 1

Ajay Nath

Four miracles of Electromagnetic Property

- Lightening

(motors, generators, electric lights, batteries, heaters, telephones, record players, and many other devices emerged)

- Radio wave transmission in 1888, 15 years after Maxwell's predictions

(ability to communicate instantly without wires around the world, not only dots and dashes, but also voice, images, and data.)

- Electronics and photonics:

the ability to electrically manipulate individual electrons and atoms in vacuum and in matter so as to generate, amplify, manipulate, and detect electromagnetic signals.

(vacuum tubes, diodes, transistors, integrated circuits, lasers, and superconductors all vastly extended the capabilities and applications of electromagnetics)

- Cybernetics and Informatics

(the manipulation of electrical signals so complex that entirely new classes of functionality are obtained, such as optimum signal processing, computers, robotics, and artificial intelligence.)

Course Description and Objectives

The objective of this course is to give an idea how the electromagnetic wave behaves. This also provides an understanding of theories of electrostatics, magnetism and electrodynamics with their applications. The course also includes weekly small-group problem solving tutorial session.

On successful completion of this course, students should be able to:

1. Apply vector calculus to analyze simple electrostatic and magnetostatic fields
2. Able to perform calculations involving various differential operators as well as line and surface integrals relating to Gauss and Stoke's theorems.
3. Apply the principles of Coulomb's Law and Gauss's law to electric fields in various coordinate systems.
4. Identify the electrostatic boundary-value problems by application of Poisson's and Laplace's equations.
5. Understand the depth of static and time-varying electromagnetic field as governed by Maxwell's equations.
6. Formulate and analyses problems involving conducting media with planar boundaries using uniform plane waves.

Syllabus

- **Unit 1: Mathematical Foundations**

Vector Calculus- Gradient, Divergence and Curl. Line, Surface and Volume integrals. Gauss's divergence theorem and Stokes' theorem in Cartesian, Spherical polar and cylindrical polar coordinates, Continuity equation.

- **Unit 2: Review of Electrostatics**

Electrostatics in Vacuum-Discrete and Distributed Charges, Electrostatic Force, Scalar & Vector Potentials, Electrostatic Energy, Poisson and Laplace equation and its applications; Electrostatics in Dielectric Medium-Electric Polarization; Electric Displacement Vector, Dielectric Susceptibility, Energy in Dielectric Medium.

- **Unit 3: Review of Magnetostatics**

Magnetic Fields and Forces, Biot-Savart law and Ampere's law, Magnetic Vector Potential, Magnetization-Diamagnetism, Paramagnetism and Ferromagnetism, Ampere's Law in Magnetized Materials-Auxiliary Field H, Magnetic permeability and susceptibility.

- **Unit 4: Review of Electrodynamics**

Electromotive force, Time-varying fields, Faraday's' law of electromagnetic induction, Self and Mutual Inductance, Displacement Current, Maxwell's equations in Free Space & Inside Matter, Energy and Momentum in Electrodynamics.

- **Unit 5: Electromagnetic Waves**

Wave equation, Propagation of Electromagnetic waves in Free Space and in Conducting Medium Reflection and Refraction, Transmission and Dispersion.

Books

Introduction to Electrodynamics	Griffiths. D. J, Prentice Hall, 2007.

- Feynman. R.P, Leighton. R.B, Sands. M, The Feynman Lectures on Physics, Narosa Publishing House, Vol. II, 2008. Hill, 2008.

- Purcell. E.M, Electricity and Magnetism, Berkley Physics Course, V2, Tata McGraw Hill, 2008.

- W. H. Hayt and J. A. Buck, Engineering Electromagnetics, Tata McGraw Hill Education Pvt. Ltd, 2006.

Classes

Time Day	9:15-10:45	11:00-12:30
Tuesday	PH110	
Friday		PH110

Time Day	Lab Batch	1:30 - 4:30 PM
Tuesday, Wednesday, Thursday	A + B + C	PH110 (T) + PH170 (L)

Evaluation

End-semester:	45% (15% online, 15% online, 15% remote)
Mid-semester:	30% (10% online + 10% online + 10% remote)
Continuous Evaluation:	25% (includes quizzes along with surprise tests, assignments etc.)

So, what do we study in electrodynamics

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \text{Gauss's Law} \quad \oint_{surf} \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\nabla \times \mathbf{E} = 0 \quad \text{No Name}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{No Name}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \text{Amperes's Law} \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$$

Maxwell's equations (Electrostatics)

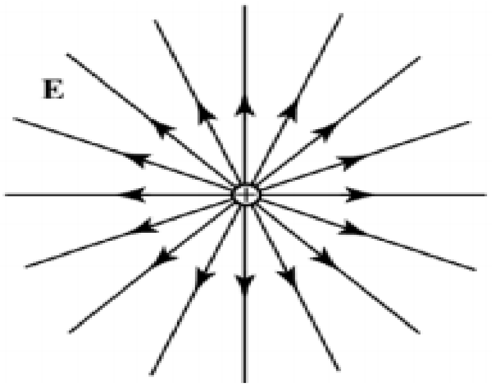
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

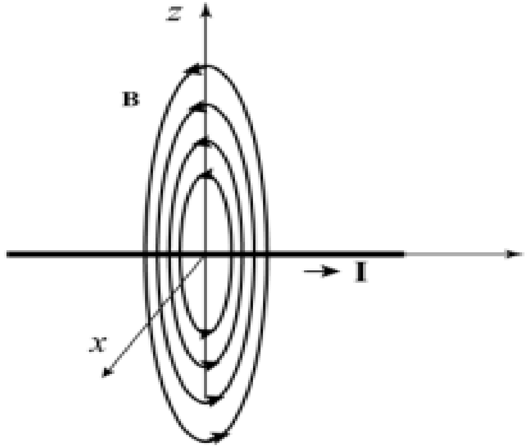
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

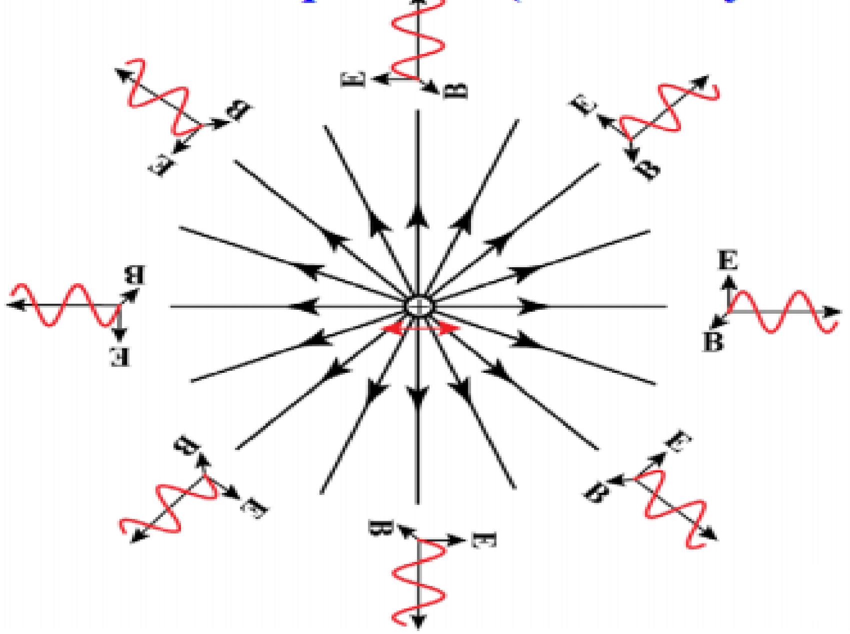
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Maxwell's equations (Electrodynamics)







Why Study Electrodynamics?

- Must for Scientist and Engineers working in ANY field.
- Most everyday equipment involve electrodynamics
  - ✓ Mobile
  - ✓ Computers
  - ✓ Radio
  - ✓ Satellite communications
  - ✓ Lasers
  - ✓ Projectors
  - ✓ Light bulbs
  - ✓ .
  - ✓ .
- Most everyday forces that we feel are of electromagnetic type
  - ✓ Normal Force from the floor or chair
  - ✓ Chemical forces binding a molecule together
  - ✓ Impact force between two colliding objects
  - ✓ .
  - ✓ .

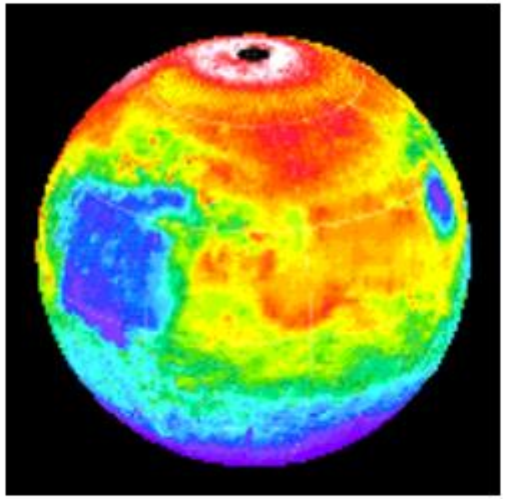
Action at a Distance versus Field Theory

"… In order therefore to appreciate the requirements of the science [of electromagnetism], the student must make himself familiar with a considerable body of most intricate mathematics, the mere retention of which in the memory materially interferes with further progress …"

James Clerk Maxwell [1855]

What is "action at a distance?" It is a worldview in which the interaction of two material objects requires no mechanism other than the objects themselves and the empty space between them. That is, two objects exert a force on each other simply because they are present. Any mutual force between them (for example, gravitational attraction or electric repulsion) is instantaneously transmitted from one object to the other through empty space. There is no need to take into account any method or agent of transmission of that force, or any finite speed for the propagation of that agent of transmission. This is known as "action at a distance" because objects exert forces on one another ("action") with nothing but empty space ("distance") between them. No other agent or mechanism is needed.

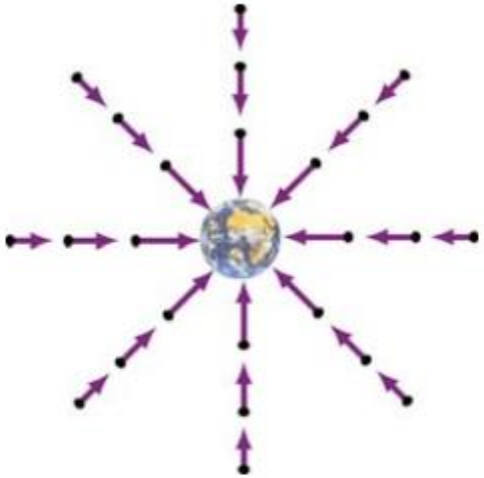
Although the two objects are not in direct contact with one another, they are in direct contact with a medium or mechanism that exists between them. The force between the objects is transmitted (at a finite speed) by stresses induced in the intervening space by the presence of the objects. The "field theory" view thus avoids the concept of "action at a distance" and replaces it by the concept of "action by continuous contact." The "contact" is provided by a stress, or "field," induced in the space between the objects by their presence.



Nighttime temperature map for Mars

$$\phi(x,y,z)=\frac{1}{\sqrt{x^2+(y+d)^2+z^2}}-\frac{1/3}{\sqrt{x^2+(y-d)^2+z^2}}$$

$$\vec{\mathbf{g}}=\lim_{m\to 0}\frac{\vec{\mathbf{F}}_g}{m}=-G\frac{M}{r^2}\hat{\mathbf{r}}$$

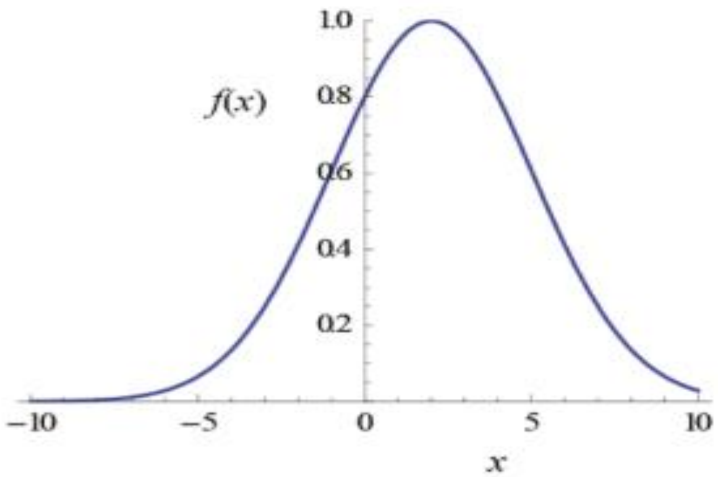


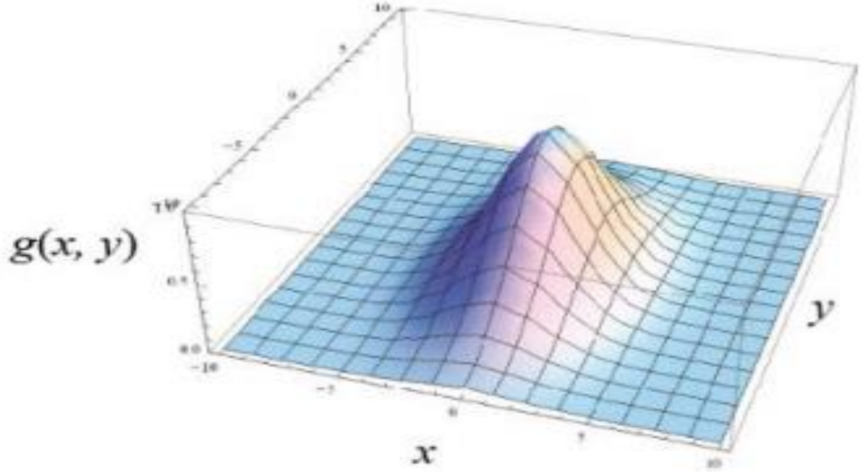
Gravitational field of the Earth.

Scalar and Vector functions/fields

Scalar functions:

- Requires only one function for its description
- Example: $f(x),\ g(x, y),\ t\ (x, y, z)$
- Algebra consists of addition, subtraction, multiplication, etc.
- Calculus consists of differentiation, integration, etc.





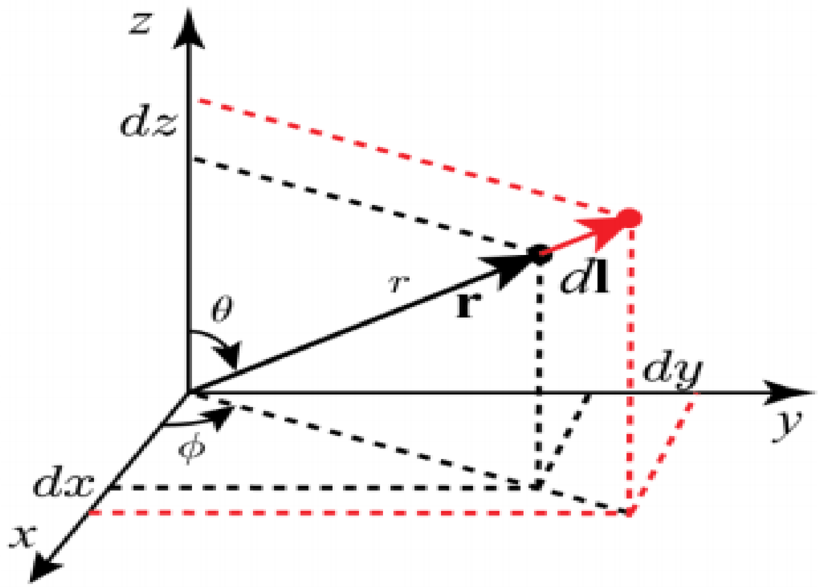
Vector functions:

- Requires more than one scalar functions for its description.
- Example: $\mathbf{A}(x, y, z) = A_x(x, y, z)\hat{\boldsymbol{x}} + A_y(x, y, z)\hat{\boldsymbol{y}} + A_z\ (x, y, z)\hat{\mathbf{z}}$
- The corresponding algebra is called Vector Algebra
- The corresponding calculus is called Vector calculus

Examples of vector functions/fields:

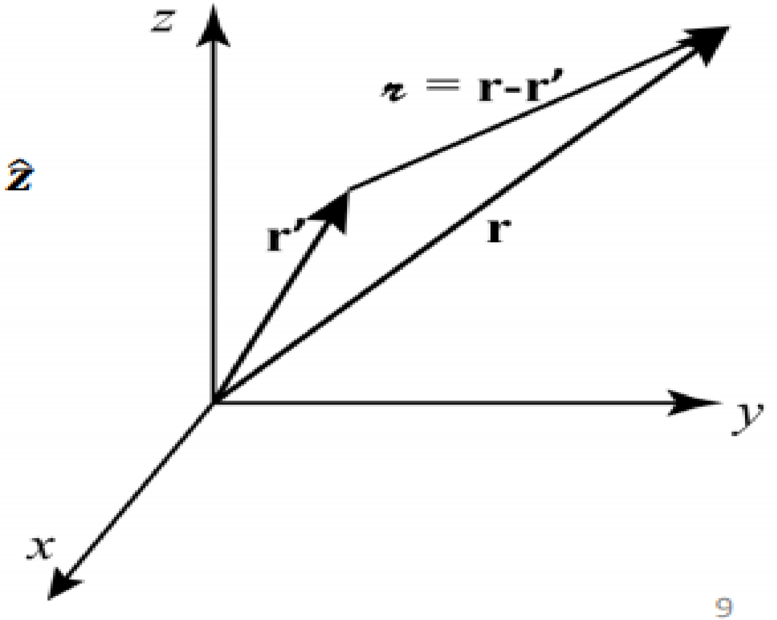
- Position vector: $\mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$

- Displacement vector: $d\mathbf{l} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$



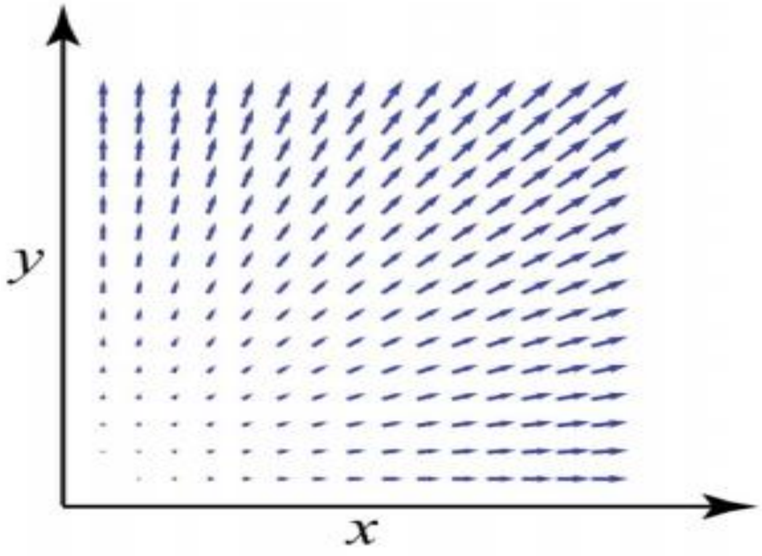
- Separation vector:

$$\hat{\boldsymbol{\mathscr{r}}} = \mathbf{r} - \mathbf{r}' = (x - x')\,\hat{\mathbf{x}} + (y - y')\,\hat{\mathbf{y}} + (z - z')\,\hat{\mathbf{z}}$$

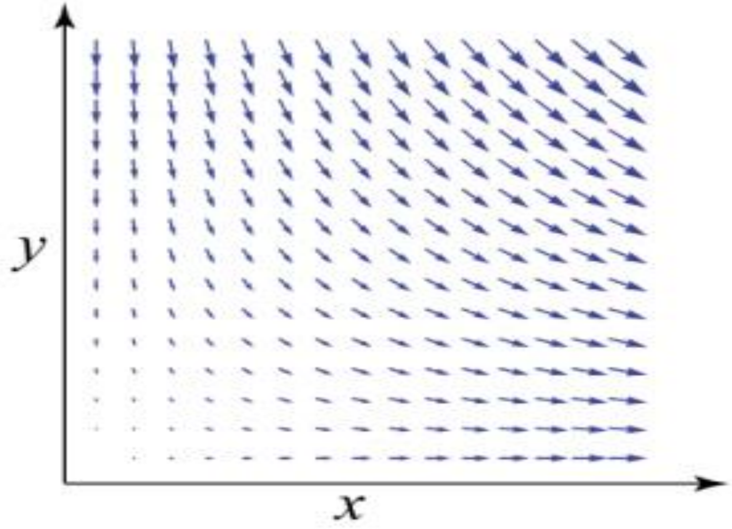


Realization of vector functions/fields:

- Example # 1: $\mathbf{g}(x, y) = \boldsymbol{x}\,\widehat{\boldsymbol{x}} + \boldsymbol{y}\,\widehat{\boldsymbol{y}}$



- Example # 2: $\mathbf{g}(x, y) = \boldsymbol{x}\,\widehat{\boldsymbol{x}} - \boldsymbol{y}\,\widehat{\boldsymbol{y}}$

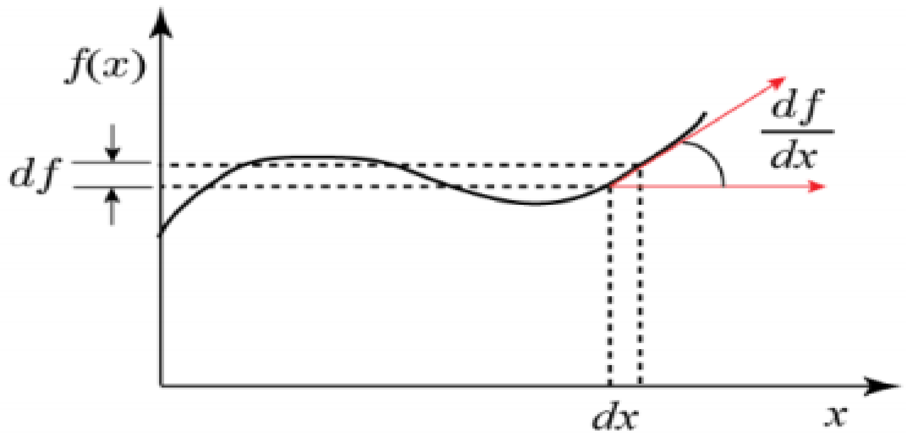


Vector Calculus

When working with functions one also has to study calculus in addition to studying algebra. The calculus dealing with vector function is referred to as the vector calculus. Calculus mainly involves differentiation and integration

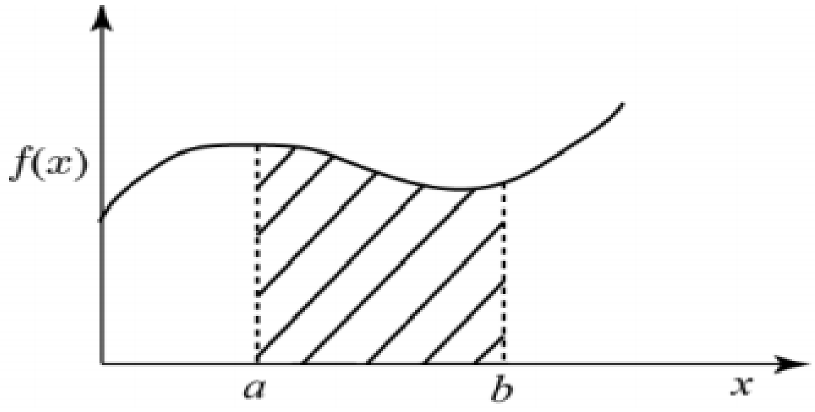
Differential Calculus

$$df = \left(\frac{df}{dx}\right) dx$$



Integral Calculus

$$I = \int_a^b f(x)dx$$



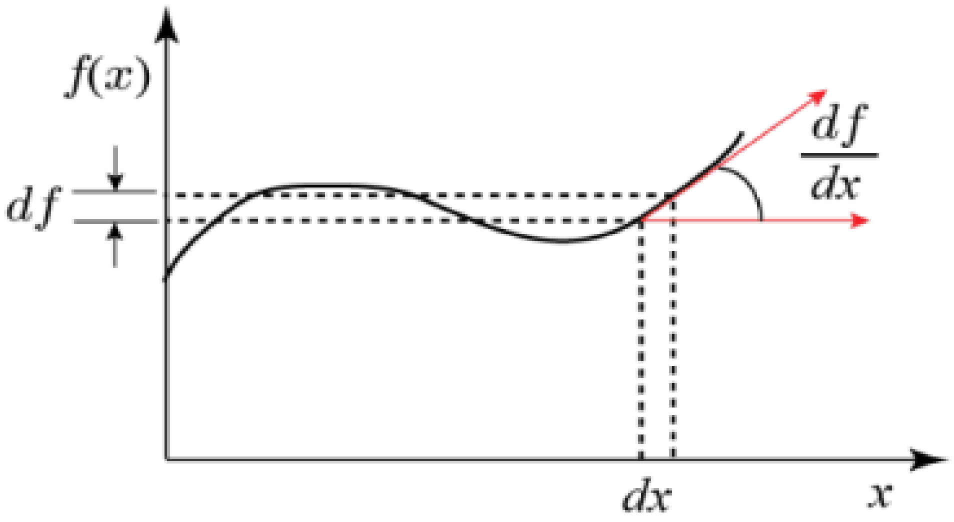
Differential Calculus (of function $f(x)$ of single variable x.)

$$df = \left(\frac{df}{dx}\right) dx$$

Change in $f(x)$

Derivative

Change in x



- The magnitude of $\frac{df}{dx}$ is the rate of change (slope) of function $f(x)$

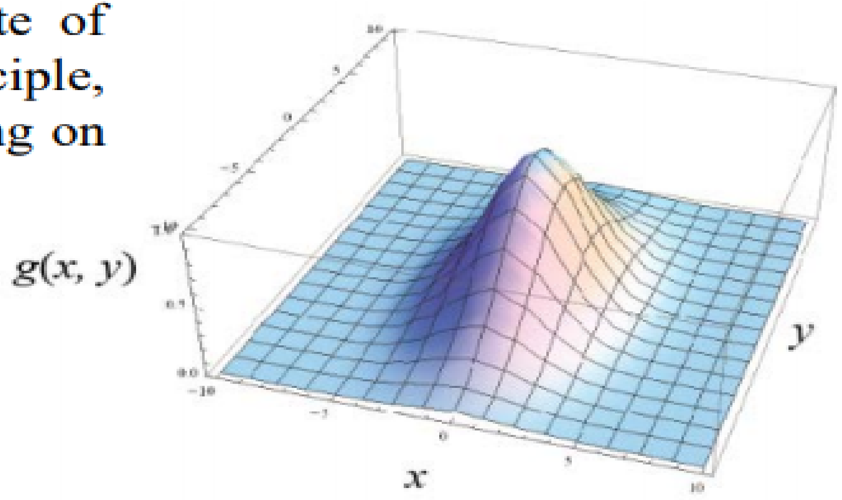
- If $\frac{df}{dx} = 0$ then $df = 0$, this defines the extremum of function $f(x)$.

Vector Calculus

Differential Calculus (of function T(x, y, z) of three variables.)

Q: How do we find out the rate of change of a function of more than one variable?

Issue: For a function of more than one variable the rate of change depends on the direction we move. So, in principle, there can be infinite number of "rate of changes" depending on the direction.



Solution:

It is known from a theorem on partial derivatives that a function of three variables can be written as

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

- This shows how T changes when the three variables are changed by the infinitesimal amounts dx, dy, dz.
- The above representation means that one does not require an infinite number of "slopes" or "rate of changes." It is sufficient to know just three.

Differential Calculus (of function $T(x, y, z)$ of three variables.)

Q: How do we find out the rate of change of a function of more than one variable?

Solution: We can rewrite dT as:

$$dT = \left(\frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}\right) \cdot (dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}})$$

$$dT = (\nabla T) \cdot (d\mathbf{l})$$

Change in T

displacement vector

3-dimensional derivative

$$df = \left(\frac{df}{dx}\right) dx$$

Derivative

- $\nabla T \equiv \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$ is the generalized derivative
- ∇T is called the gradient of T. It is a vector quantity with three components.
- $dT = (\nabla T) \cdot (d\mathbf{l}) = |\nabla T||d\mathbf{l}| \cos\theta$. For a fixed $|d\mathbf{l}|$, dT is maximum when $\theta = 0$
- So, ∇T points in the direction of maximum increase of the function T.
- The magnitude $|\nabla T|$ is the slope or the rate of change along this maximal direction.
- If $\nabla T = 0$, $dT = 0$. So, $\nabla T = 0$ defines the extremum of the function T.

Example # 1

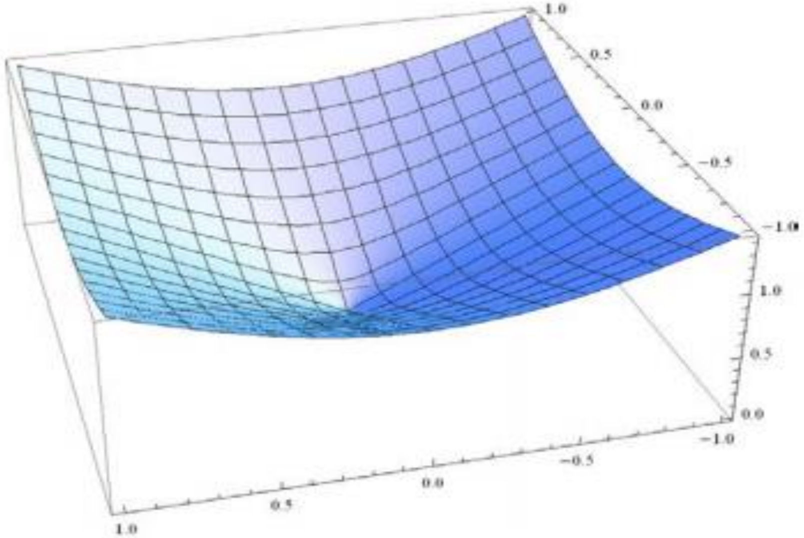
- Find the gradient of $r(x,y) = \sqrt{x^2 + y^2}$

$$\boldsymbol{\nabla} r = \frac{\partial r}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial r}{\partial y}\hat{\boldsymbol{y}}$$

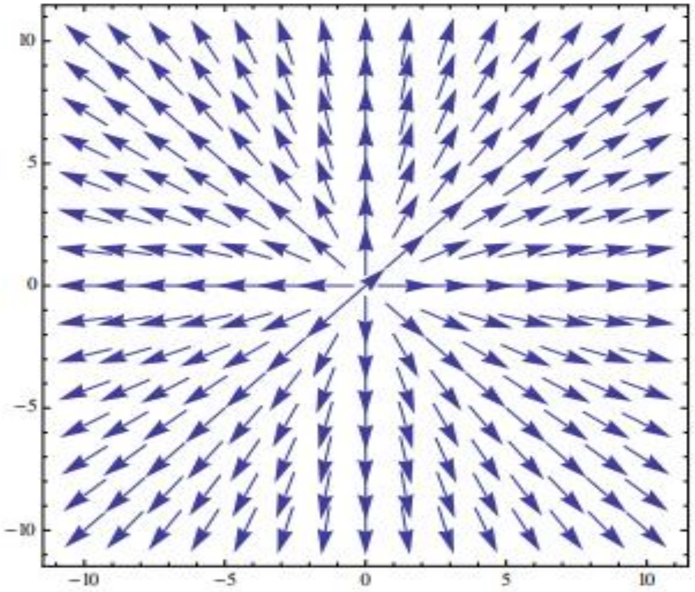
$$= \frac{1}{2}\frac{2x}{\sqrt{x^2 + y^2}}\hat{\boldsymbol{x}} + \frac{1}{2}\frac{y}{\sqrt{x^2 + y^2}}\hat{\boldsymbol{y}}$$

$$= \frac{x}{r}\hat{\boldsymbol{x}} + \frac{y}{r}\hat{\boldsymbol{y}}$$

$$= \hat{\boldsymbol{r}}$$



$\boldsymbol{\nabla} r$

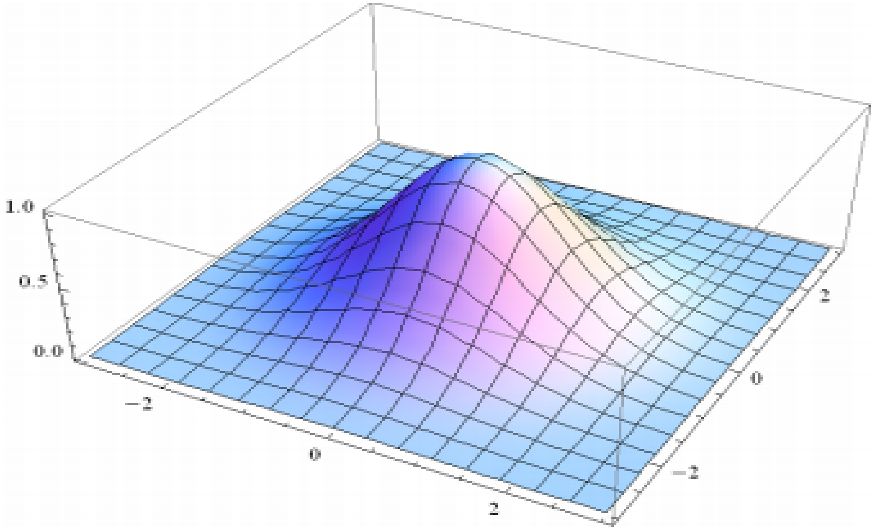


Differential Calculus (of function $T(x, y, z)$ of three variables.)

Example # 2

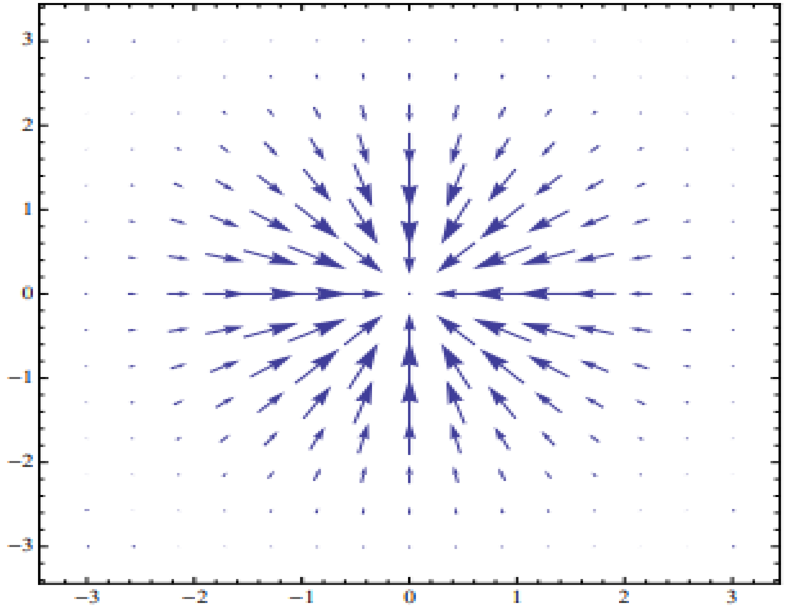
- Find the gradient of $g(x, y) = \exp\left[-\frac{x^2+y^2}{2}\right]$

$$\nabla g = \frac{\partial g}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial g}{\partial y}\hat{\boldsymbol{y}}$$



$$= -x\exp\left[-\frac{x^2+y^2}{2}\right]\hat{\boldsymbol{x}} - y\exp\left[-\frac{x^2+y^2}{2}\right]\hat{\boldsymbol{y}}$$

$\nabla g(x, y)$



- Find the extremum of $g(x, y)$

$\nabla g = 0$ One solution is $(x, y) = (0, 0)$

So, the function has an extremum at $(0, 0)$, which is a maximum.

The gradient operator

$$\nabla T \equiv \frac{\partial T}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial T}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial T}{\partial z}\hat{\boldsymbol{z}} \quad = \left(\frac{\partial}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial}{\partial z}\hat{\boldsymbol{z}}\right)T$$

So, $\nabla \equiv \frac{\partial}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial}{\partial z}\hat{\boldsymbol{z}}$

- ∇ is called the gradient operator.
- ∇ is not a vector in the usual sense, but a vector operator.
- ∇ acts on a scalar function and gives out the generalized derivative

If we are dealing with scalar functions then the differential calculus consists of derivatives only. However, in the case of vector functions/fields, the differential calculus has two more concepts, namely, the two vector derivatives.

Divergence of a vector $\nabla \cdot \mathbf{V}$

Curl of a vector $\nabla \times \mathbf{V}$

The Divergence

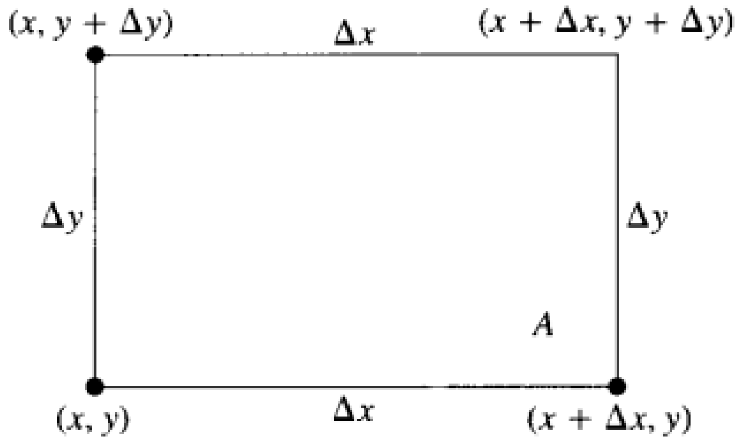
The divergence of a vector **V** is defined as

$$\nabla \cdot \mathbf{V} = \left(\frac{\partial}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}\right) \cdot (v_x\hat{\boldsymbol{x}} + v_y\hat{\boldsymbol{y}} + v_z\boldsymbol{z})$$

$$= \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

- The divergence of a vector is a scalar quantity.
- The divergence measures how much a vector field diverges.

Flux Density at a Point: Divergence



$$\mathbf{F}(x, y) = M(x, y)\,\mathbf{i} + N(x, y)\,\mathbf{j}$$

Top: $\mathbf{F}(x, y + \Delta y) \cdot \mathbf{j}\, \Delta x = N(x, y + \Delta y)\Delta x$

Bottom: $\mathbf{F}(x, y) \cdot (-\mathbf{j})\, \Delta x = -N(x, y)\Delta x$

Right: $\mathbf{F}(x + \Delta x, y) \cdot \mathbf{i}\, \Delta y = M(x + \Delta x, y)\, \Delta y$

Left: $\mathbf{F}(x, y) \cdot (-\mathbf{i})\, \Delta y = -M(x, y)\, \Delta y.$

Top and bottom: $(N(x, y + \Delta y) - N(x, y))\, \Delta x \approx \left(\frac{\partial N}{\partial y}\Delta y\right) \Delta x$

Right and left: $(M(x + \Delta x, y) - M(x, y))\, \Delta y \approx \left(\frac{\partial M}{\partial x}\Delta x\right) \Delta y.$

$$\text{Flux across rectangle boundary} \approx \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) \Delta x\, \Delta y.$$

$$\frac{\text{Flux across rectangle boundary}}{\text{Rectangle area}} \approx \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right).$$

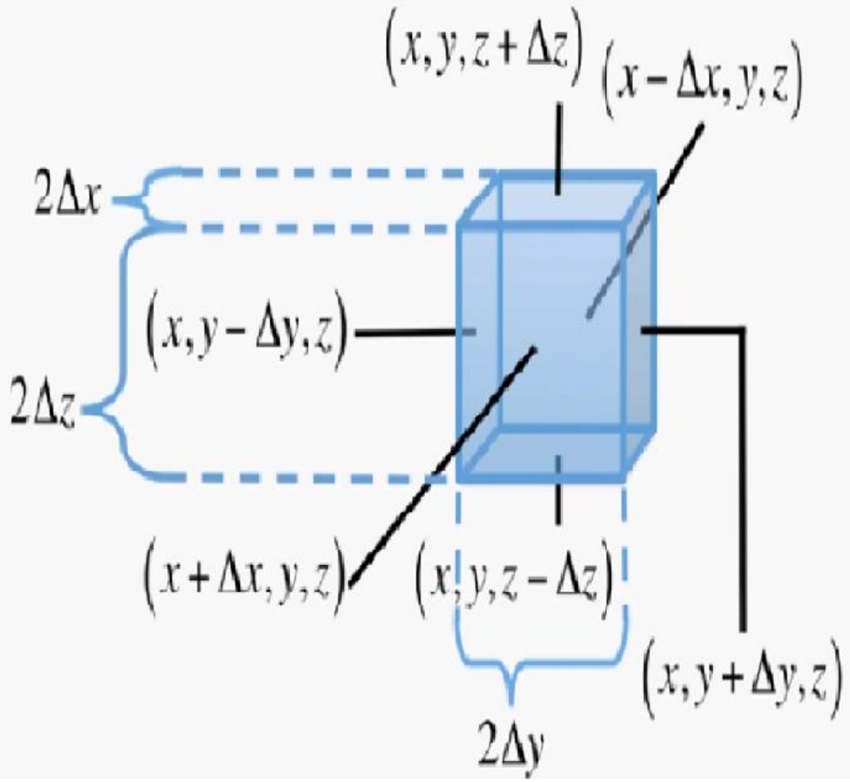
Divergence in 3d

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{W} = f(x,y,z)\hat{i} + g(x,y,z)\hat{j} + h(x,y,z)\hat{k}$$

$$\vec{\nabla}\cdot\vec{W} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

Divergence measures net flux per unit volume through a differential volume element.



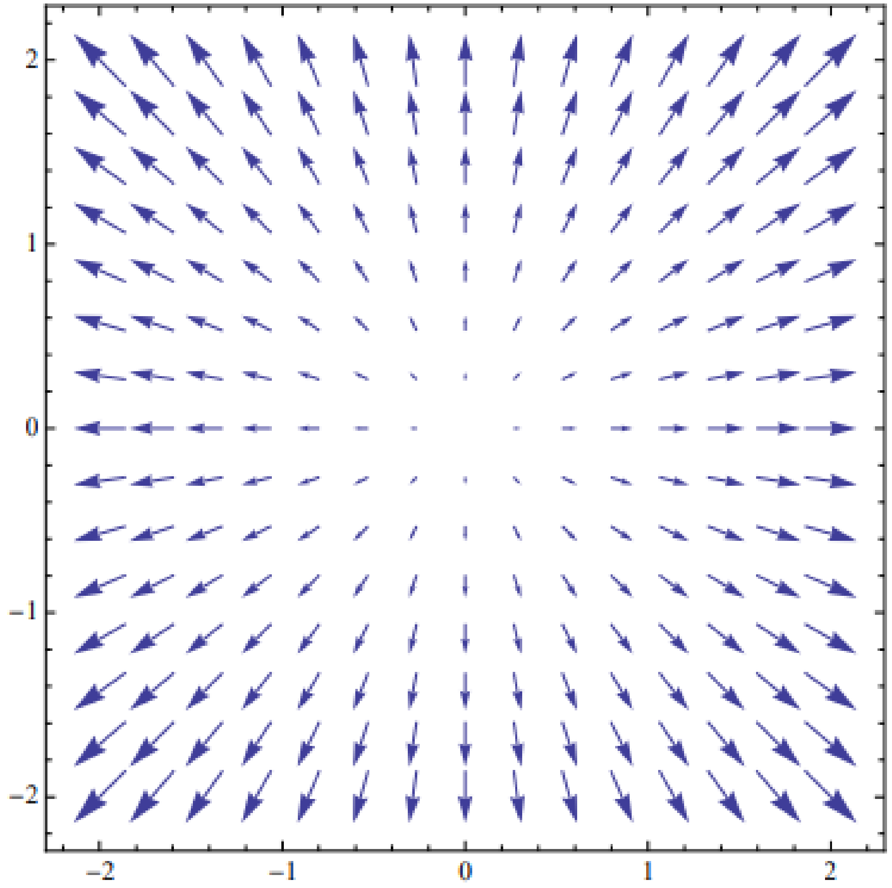
The Divergence

Example # 1

- Find the divergence of $\mathbf{V} = x\,\hat{x} + y\,\hat{y}$

$$\nabla \cdot \mathbf{V} = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$= 1 + 1 = 2$$



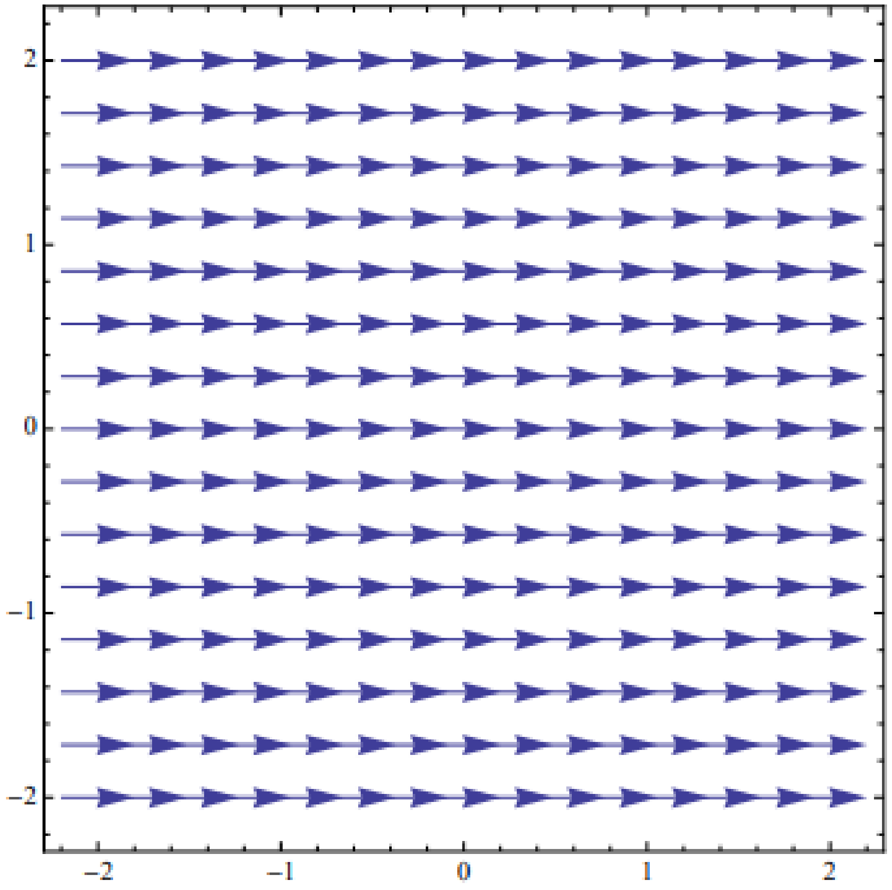
The Divergence

Example # 2

- Find the divergence of $\mathbf{V} = \hat{x}$

$$\nabla \cdot \mathbf{V} = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$= 0$$



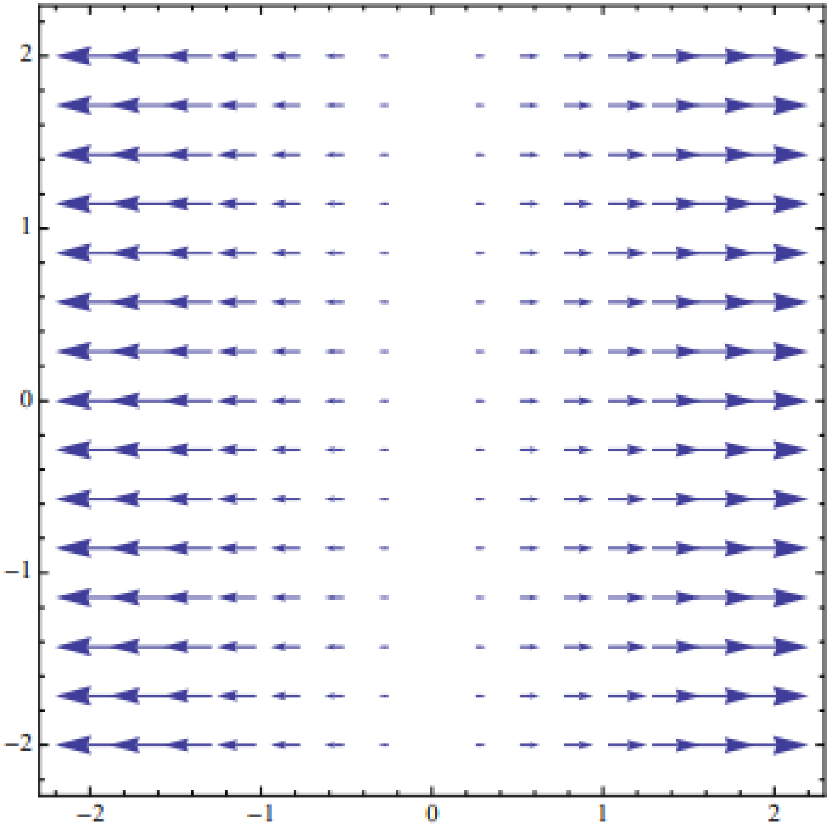
The Divergence

Example # 3

- Find the divergence of $\mathbf{V} = x\,\hat{x}$

$$\nabla \cdot \mathbf{V} = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$= 1$$



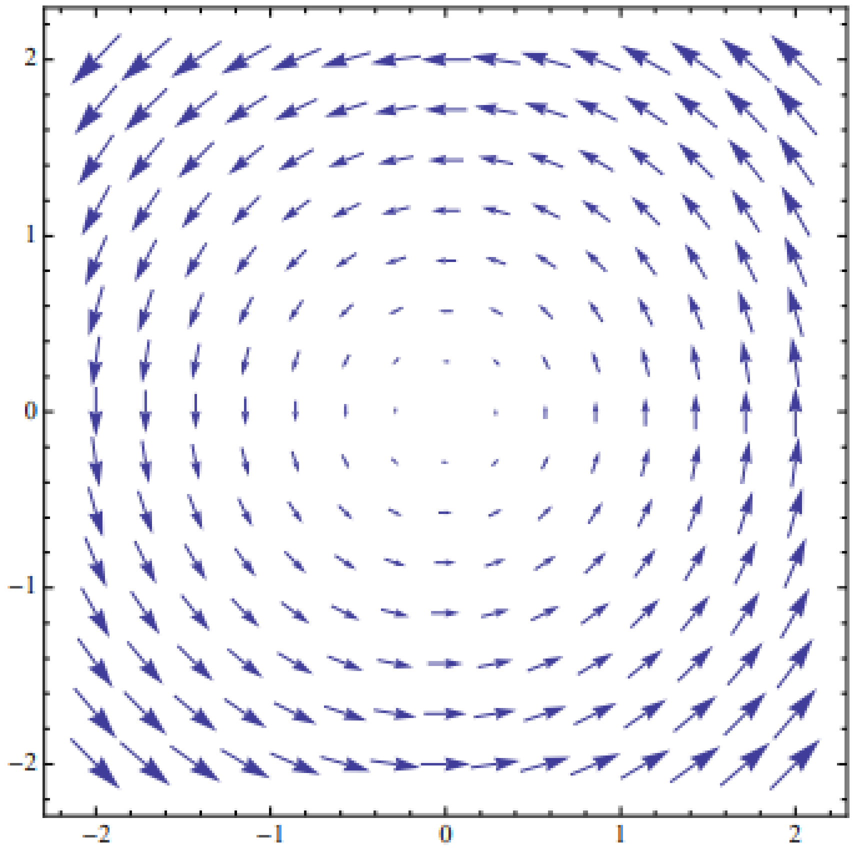
The Divergence

Example #

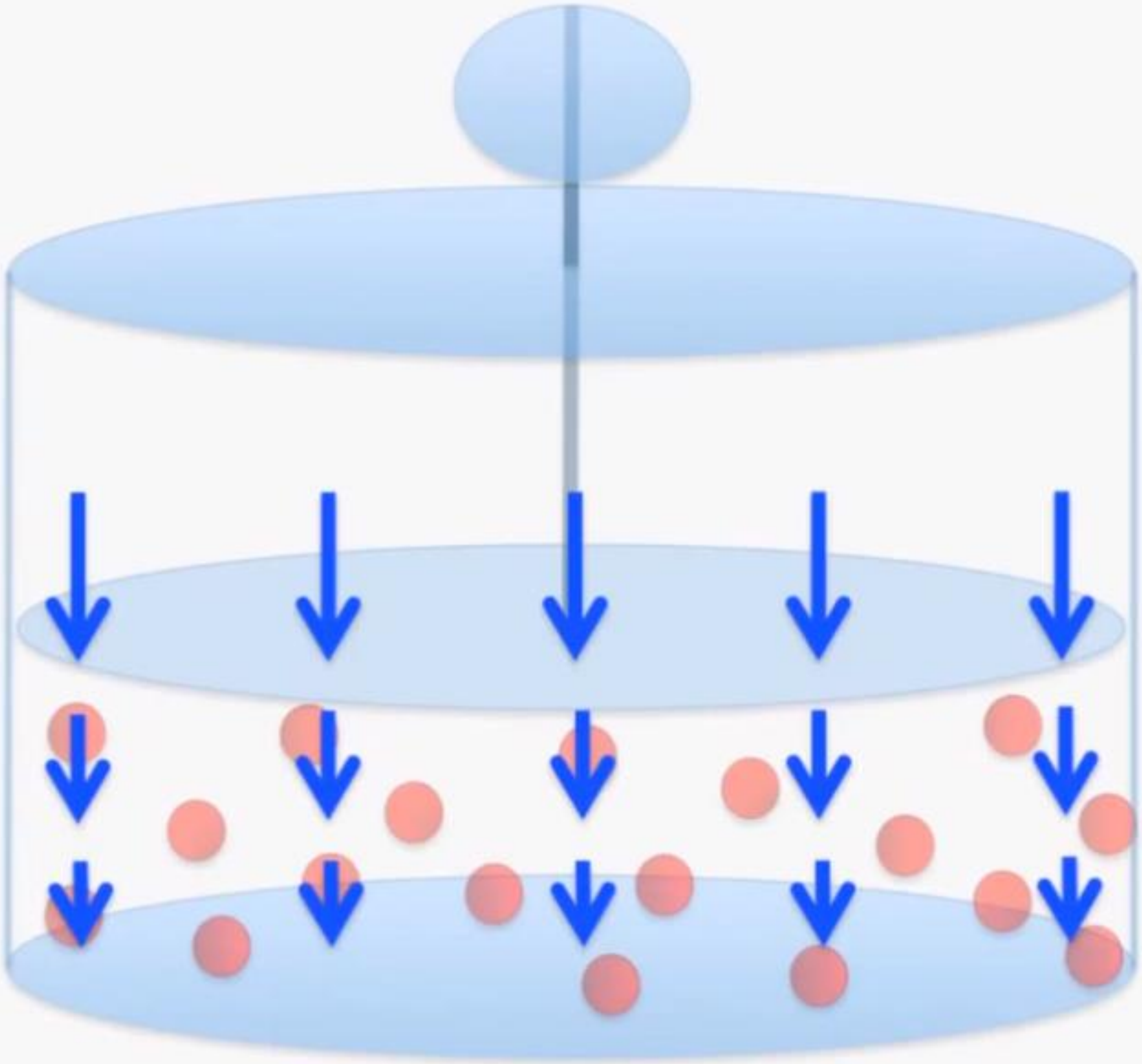
- Find the divergence of $\mathbf{V} = -y\,\hat{x} + x\,\hat{y}$

$$\nabla \cdot \mathbf{V} = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$= 0 + 0 = 0$$



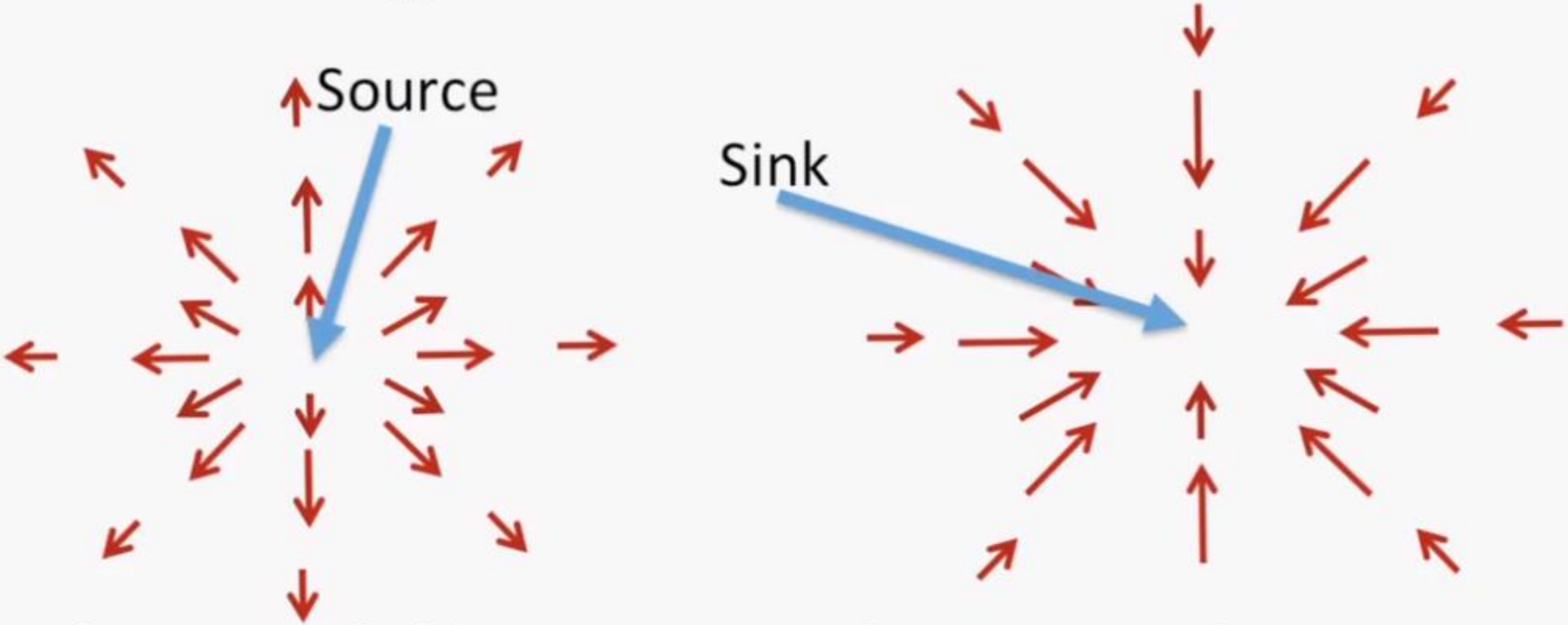
Piston containing a gas



- Increase pressure
- Increase average density

During compression, the divergence of the average velocity field is negative

Divergence at sources and sinks



- The vector field at any source has positive divergence
- The vector field at any sink has negative divergence

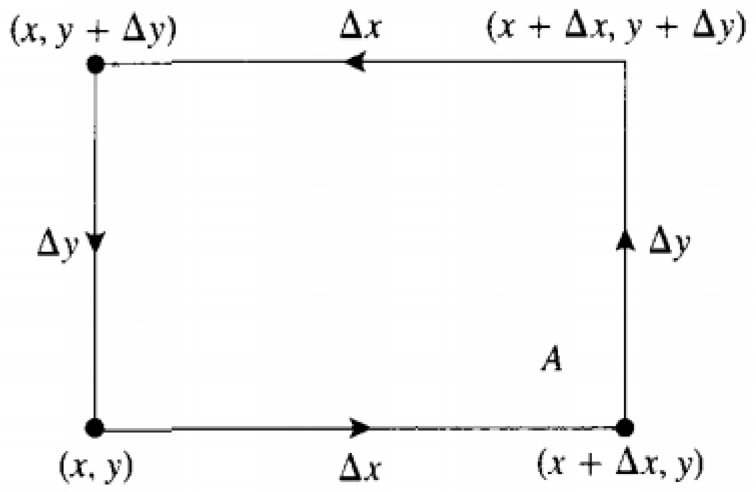
The Curl

The curl of a vector **V** is defined as

$$\nabla \times \mathbf{V} = \left(\frac{\partial}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}\right) \times (v_x\hat{\boldsymbol{x}} + v_y\hat{\boldsymbol{y}} + v_z\mathbf{z})$$

$$= \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

- The curl of a vector is a vector quantity.
- The curl measures how much a vector field curls.



$$\mathbf{F}(x, y) = M(x, y)\,\mathbf{i} + N(x, y)\,\mathbf{j}$$

Top:	$\mathbf{F}(x, y + \Delta y) \cdot (-\mathbf{i})\,\Delta x = -M(x, y + \Delta y)\,\Delta x$
Bottom:	$\mathbf{F}(x, y) \cdot \mathbf{i}\,\Delta x = M(x, y)\,\Delta x$
Right:	$\mathbf{F}(x + \Delta x, y) \cdot \mathbf{j}\,\Delta y = N(x + \Delta x, y)\,\Delta y$
Left:	$\mathbf{F}(x, y) \cdot (-\mathbf{j})\,\Delta y = -N(x, y)\,\Delta y.$

Top and bottom:

$$-(M(x, y + \Delta y) - M(x, y))\,\Delta x \approx -\left(\frac{\partial M}{\partial y}\Delta y\right)\Delta x$$

Right and left:

$$(N(x + \Delta x, y) - N(x, y))\,\Delta y \approx \left(\frac{\partial N}{\partial x}\Delta x\right)\Delta y.$$

$$\frac{\text{Circulation around rectangle}}{\text{Rectangle area}} \approx \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}.$$

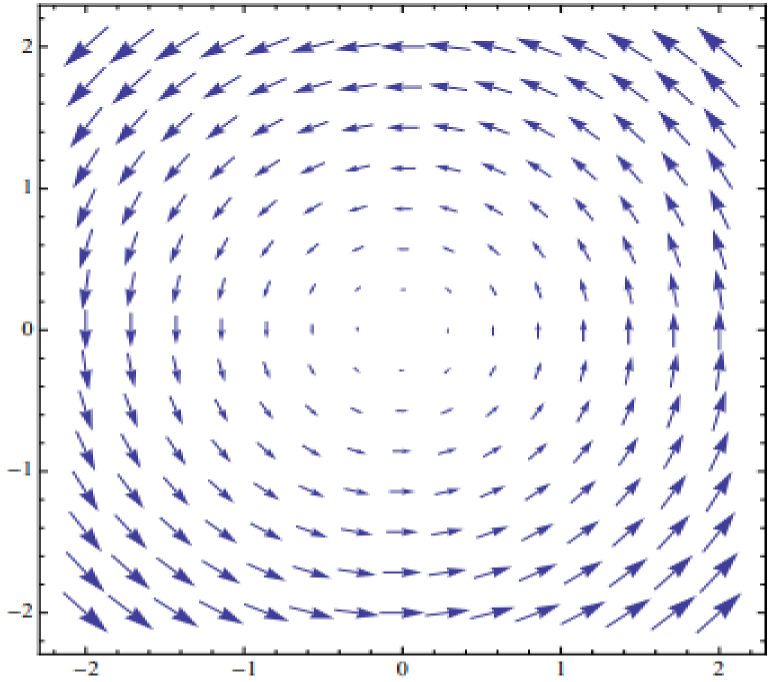
The Curl

Example # 1

- Find the curl of $\mathbf{V} = -y\,\hat{x} + x\,\hat{y}$

$$\nabla \times \mathbf{V} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$= 2\hat{\mathbf{z}}$$



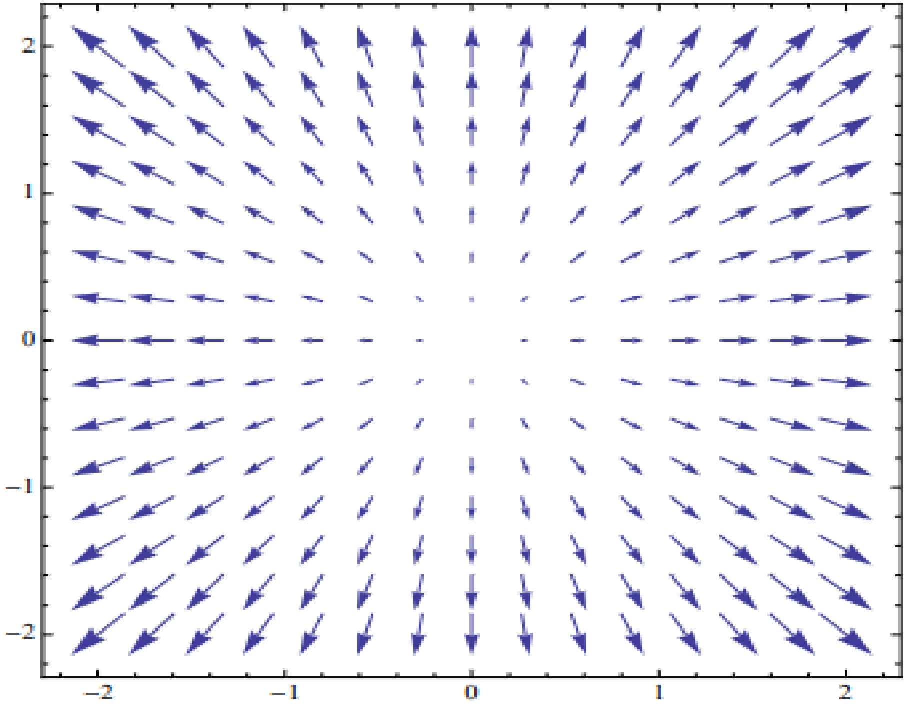
The Curl

Example # 2

- Find the curl of $\mathbf{V} = x\,\hat{x} + y\,\hat{y}$

$$\nabla \times \mathbf{V} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$= 0$$



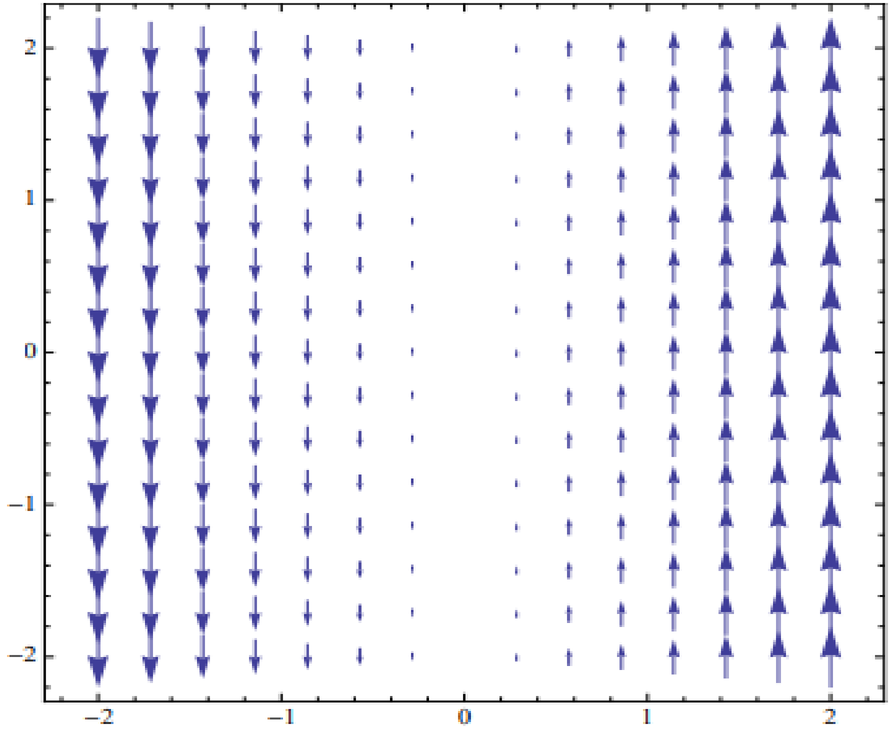
The Curl

Example # 3

- Find the curl of $\mathbf{V} = x\,\hat{y}$

$$\nabla \times \mathbf{V} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\boldsymbol{z}}$$

$$= \hat{\boldsymbol{z}}$$



Thank You