Academic Year 2023-24

Tutorial #09

PH110: Waves and Electromagnetics

1.

(a) Let $\mathbf{F}_1 = x^2\,\hat{\mathbf{z}}$ and $\mathbf{F}_2 = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$. Calculate the divergence and curl of $\mathbf{F}_1$ and $\mathbf{F}_2$. Which one can be written as the gradient of a scalar? Find a scalar potential that does the job. Which one can be written as the curl of a vector? Find a suitable vector potential.

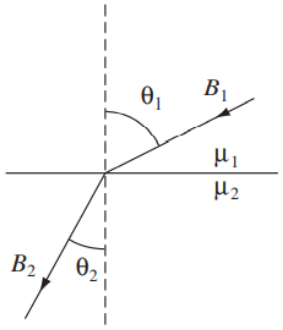
(b) Show that $\mathbf{F}_3 = yz\,\hat{\mathbf{x}} + zx\,\hat{\mathbf{y}} + xy\,\hat{\mathbf{z}}$ can be written both as the gradient of a scalar and as the curl of a vector. Find scalar and vector potentials for this function.

(c) Evaluate $\int_0^5 \cos x\,\delta(x-\pi)\,dx$.

2. (a)

Two positive point charges, q_A and q_B (masses m_A and m_B) are at rest, held together by a massless string of length a. Now the string is cut, and the particles fly off in opposite directions. How fast is each one going, when they are far apart?

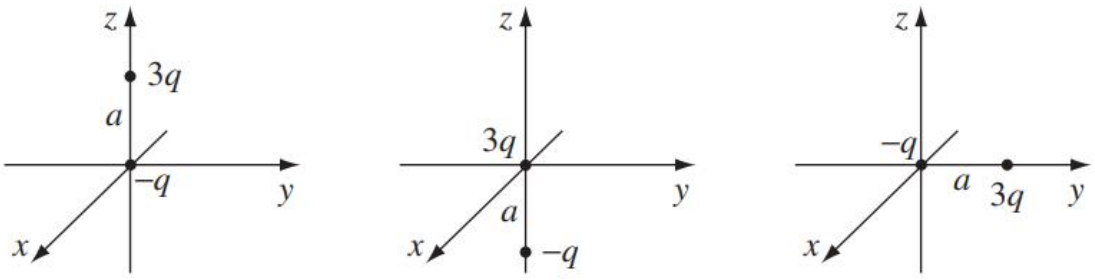
(b) At the interface between one linear magnetic material and another, the magnetic field lines bend (as shown in figure below). Show that $\tan\theta_2/\tan\theta_1 = \mu_2/\mu_1$, assuming there is no free current at the boundary.



3. How bound charges are generated? Physically discuss about volume and surface charges. Utilizing it, deduce Gauss's law in presence of dielectrics. Write down the boundary conditions for the electric displacement field passing through a surface and compare it with electrostatic field is passing through same surface.

4.

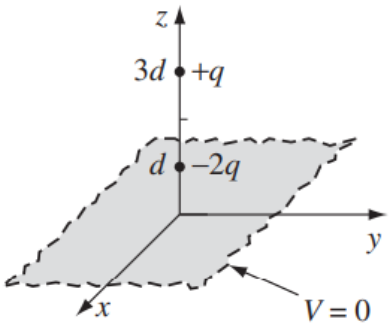
Two point charges, $3q$ and $-q$, are separated by a distance a. For each of the arrangements in Fig. , find (i) the monopole moment, (ii) the dipole moment, and (iii) the approximate potential (in spherical coordinates) at large r (include both the monopole and dipole contributions).



(a) (b) (c)

OR

Find the force on the charge +q in given below figure:



5.

(a) Show that $\vec{F} = yz\,\hat{x} + zx\,\hat{y} + xy\,\hat{z}$ can be expressed as the curl of a vector and as gradient of a scaler. Find the scaler and vector potentials for this function.

(b) Lets consider a function $\Theta(z) \equiv \begin{cases} 1, & \text{if } z > 0 \\ 0, & \text{if } z \leq 0 \end{cases}$. Show the first order derivative of $\Theta(z)$ is equal to $\delta(z)$.

6.

Let $\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \left[1 + \frac{r}{\lambda}\right] e^{-r/\lambda}\,\hat{r}$ REPRESENTS THE FORCE OF ATTRACTION BETWEEN TWO POINT CHARGES AND 'λ' IS A CONSTANT.

(a) USING THIS, CALCULATE ELECTRIC FIELD OF A CHARGE DISTRIBUTION. DOES THIS FIELD ADMIT SCALAR POTENTIAL? EXPLAIN.

(b) WITH THIS MODIFIED FORCE FORM, DOES GAUSS LAW CHANGES? EXPLAIN YOUR ANSWER.

7.

A PURE DIPOLE 'p' IS SITUATED ATTHE ORIGIN, POINTING IN THE Z-DIRECTION. (a) WHAT IS THE FORCE ON A POINT CHARGE "q" AT $(4,0,0)$ CARTESIAN COORDINATES? (b) WHAT IS THE FORCE ON "q" at $(0,0,4)$? HOW MUCH WORK DOES IT TAKE TO MOVE "q" FROM $(4,0,0)$ TO $(0,0,4)$?

8.

(i) DISCUSS THE IMPACT OF ELECTRIC FIELD ON DIELECTRICS. DOES THE GAUSS'S LAW AND BOUNDARY CONDITIONS REMAIN SAME FOR CONDUCTOR AND DIELECTRIC?

(ii) EXPRESS ATOMIC POLARIZIBILITY IN TERMS OF ELECTRICAL SUSCEPTIBILITY. JUSTIFY YOUR ANSWER.