Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 4

1. Let : $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection across X-axis. Is T a linear transformation? If yes then prove it and find its matrix representation.

2. Find the Standard matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates points counter-clockwise about the origin through $\pi/4$ and then reflects points through the line $x = y$.

3. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Give a geometric description of the transformation $x \to Ax$.

4. For the matrices given below, find a basis for Col(A), Row(A) and a basis for Nul(A).

$$\begin{bmatrix} -5 & 10 & -5 & 4 \\ 8 & 3 & -4 & 7 \\ 4 & -9 & 5 & -3 \\ -3 & -2 & 5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -7 & 3 & 7 & 5 \\ 6 & -8 & 5 & 12 & -8 \\ -7 & 10 & -8 & -9 & 14 \\ 3 & -5 & 4 & 2 & -6 \\ -5 & 6 & -6 & -7 & 3 \end{bmatrix}$$

5. Let $v_1 = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}, v_2 = \begin{bmatrix} 8 \\ 2 \\ -9 \end{bmatrix}$. Determine if v_2 is in the subspace generated by v_1.

6. Let $v_1 = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}, v_2 = \begin{bmatrix} -4 \\ -5 \\ 8 \end{bmatrix}, v_3 = \begin{bmatrix} 8 \\ 2 \\ -9 \end{bmatrix}$. Determine if v_3 is in the subspace generated by v_1, v_2.

7. Assume the three sets include the bounding lines. In each case, decide where the set is a subspace of $\mathbb{R}^2$. Give justification.

a   b   c
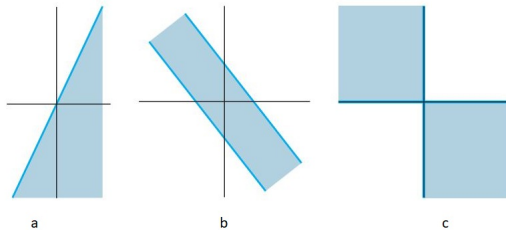
8. Is the following set of vectors a basis of $\mathbb{R}^3$? If no then can you find a basis of $\mathbb{R}^3$ containing some of them?

$$\begin{bmatrix} 1 \\ -6 \\ -7 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 9 \end{bmatrix}$$

9. What is the rank of a 4×5 matrix whose null space is three dimensional?

10. Construct a 4×3 matrix with rank 1.

11. Let $H = \text{Span}\ \{v_1, v_2\}$ and $\mathbb{B} = \{v_1, v_2\}$. Find the co-ordinate vector of x with respect to basis $\mathbb{B}$.

$$\mathbf{v_1} = \begin{bmatrix} 11 \\ -5 \\ 10 \\ 7 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 14 \\ -8 \\ 13 \\ 10 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 19 \\ -13 \\ 18 \\ 15 \end{bmatrix}$$

12. If the subspace of all solutions of $Ax = 0$ has a basis consisting of three vectors and if A is a 5×7 matrix, what is the rank of A?