Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 10

1. Find maximum value of $Q(x, y, z) = 7x^2 + y^2 + 7z^2 - 8xy - 4xz - 8yz$ subject to $[x\ y\ z]^T$ is a unit vector of $\mathbb{R}^3$.

2. Give examples of matrices each of positive definite, negative definite, positive semi-definite, negative semi-definite.

3. Find the singular values of the matrices $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

4. Find the Singular Value Decomposition of $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

5. Find the SVD of the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

6. Given the SVD of A, find rank, orthonormal basis for null space, column space of A.

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/\sqrt{5} & -2/\sqrt{45} \\ -2/3 & 1/\sqrt{5} & 4/\sqrt{45} \\ 2/3 & 0 & 5/\sqrt{45} \end{bmatrix} \begin{bmatrix} 3\sqrt{2} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

7. Given any real $m \times n$ matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, find $n \times m$ matrix B such that A and B satisfy the following conditions: $ABA = A$, $BAB = B$ and both AB and BA are symmetric. Repeat this exercise for A given in Q. 4. (Hint: try B=$A^T(AA^T)^{-1}$) Note: Such B is called Pseudoinverse/moore-Penrose inverse of A.
   Find b which does not belong to Col(A). Find least square solution to $AX = b$.

8. Given $A = U\Sigma V^T$, find svd of A^{-1}, A^T. Are Σ of A and A^{-1} related?
9. What is the relation between singular values and eigenvalues of a symmetric matrix?
10. Given a $n \times n$ matrix A, show that AA^T and A^TA are similar.
11. If A is square and real matrix, then show that A=0 if and only if every eigenvalue of A is 0.