

Indian Institute of Information Technology Vadodara
MA 102: Linear Algebra and Matrices
Tutorial 9

1. Find the equation $y = \alpha + \beta x$ of the least-squares line that best fits the data points $(2, 1)$, $(5, 2)$, $(7, 3)$. Draw the line.
2. Find QR factorization of the following matrix. Use it to find a solution of $AX = b$.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$$

3. Find the minimal polynomial of following matrix and answer diagonalizability and triangulability of the matrix by analysing minimal polynomial.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

4. Find the characteristic and minimal polynomials of the following matrix. Then use the Cayley-Hamilton theorem to find its inverse. Then find an invertible matrix P such that $P^{-1}AP$ is a triangular matrix, if it exists.

$$A = \begin{bmatrix} 4 & -5 & 3 \\ 2 & -3 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

5. Find a 3×3 matrix, A , whose minimal polynomial is λ^2 .
6. Show that every matrix $A \in M_n(\mathbb{R})$ with $A^2 = A$ is diagonalisable over \mathbb{R} .
7. Give an example of a matrix which is not diagonalisable over \mathbb{C} . Show that every square matrix with complex entries is triangulable over \mathbb{C} .
8. A matrix A is nilpotent if there exists some positive natural number k such that $A^k = 0$. Find a nilpotent matrix. Is it triangulable over \mathbb{R} ? Can you generalise the observation?