Tut-05

Problem 2.11 Use Gauss's law to find the electric field inside and outside a spherical shell of radius R that carries a uniform surface charge density σ. Compare your answer to Prob. 2.7.

Problem 2.14 Find the electric field inside a sphere that carries a charge density proportional to the distance from the origin, $\rho = kr$, for some constant k. [*Hint:* This charge density is *not* uniform, and you must *integrate* to get the enclosed charge.]

Problem 2.18 Two spheres, each of radius R and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap (Fig. 2.28). Call the vector from the positive center to the negative center $\mathbf{d}$. Show that the field in the region of overlap is constant, and find its value. [*Hint:* Use the answer to Prob. 2.12.]

Problem 2.20 One of these is an impossible electrostatic field. Which one?

(a) $\mathbf{E} = k[xy\,\hat{\mathbf{x}} + 2yz\,\hat{\mathbf{y}} + 3xz\,\hat{\mathbf{z}}]$;

(b) $\mathbf{E} = k[y^2\,\hat{\mathbf{x}} + (2xy + z^2)\,\hat{\mathbf{y}} + 2yz\,\hat{\mathbf{z}}]$.

Problem 2.21 Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point. Compute the gradient of V in each region, and check that it yields the correct field. Sketch $V(r)$.

Problem 2.28 Use Eq. 2.29 to calculate the potential inside a uniformly charged solid sphere of radius R and total charge q. Compare your answer to Prob. 2.21.

Problem 2.54 Imagine that new and extraordinarily precise measurements have revealed an error in Coulomb's law. The *actual* force of interaction between two point charges is found to be

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\mathscr{r}^2}\left(1 + \frac{\mathscr{r}}{\lambda}\right)e^{-(\mathscr{r}/\lambda)}\,\hat{\boldsymbol{\mathscr{r}}},$$

where λ is a new constant of nature (it has dimensions of length, obviously, and is a huge number—say half the radius of the known universe—so that the correction is small, which is why no one ever noticed the discrepancy before). You are charged with the task of reformulating electrostatics to accommodate the new discovery. Assume the principle of superposition still holds.

(a) What is the electric field of a charge distribution ρ (replacing Eq. 2.8)?

(b) Does this electric field admit a scalar potential? Explain briefly how you reached your conclusion. (No formal proof necessary—just a persuasive argument.)

(c) Find the potential of a point charge q—the analog to Eq. 2.26. (If your answer to (b) was "no," better go back and change it!) Use ∞ as your reference point.

(d) For a point charge q at the origin, show that

$$\oint_{\mathcal{S}} \mathbf{E}\cdot d\mathbf{a} + \frac{1}{\lambda^2}\int_{\mathcal{V}} V\,d\tau = \frac{1}{\epsilon_0}q,$$

where $\mathcal{S}$ is the surface, $\mathcal{V}$ the volume, of any sphere centered at q.

(e) Show that this result generalizes:

$$\oint_{\mathcal{S}} \mathbf{E}\cdot d\mathbf{a} + \frac{1}{\lambda^2}\int_{\mathcal{V}} V\,d\tau = \frac{1}{\epsilon_0}Q_{\text{enc}},$$

for *any* charge distribution. (This is the next best thing to Gauss's Law, in the new "electrostatics.")

(f) Draw the triangle diagram (like Fig. 2.35) for this world, putting in all the appropriate formulas. (Think of Poisson's equation as the formula for ρ in terms of V, and Gauss's law (differential form) as an equation for ρ in terms of $\mathbf{E}$.)

(g) Show that *some* of the charge on a conductor distributes itself (uniformly!) over the volume, with the remainder on the surface. [*Hint:* $\mathbf{E}$ is still zero, inside a conductor.]