TUTORIAL-4

PROBLEM-1.44

Problem 1.44 Evaluate the following integrals:

(a) $\int_2^6 (3x^2 - 2x - 1)\,\delta(x-3)\,dx$.

(b) $\int_0^5 \cos x\,\delta(x-\pi)\,dx$.

(c) $\int_0^3 x^3\delta(x+1)\,dx$.

(d) $\int_{-\infty}^{\infty} \ln(x+3)\,\delta(x+2)\,dx$.

PROBLEM-1.46

Problem 1.46

(a) Show that

$$x\frac{d}{dx}(\delta(x)) = -\delta(x).$$

[*Hint:* Use integration by parts.]

(b) Let $\theta(x)$ be the **step function**:

$$\theta(x) \equiv \left\{ \begin{array}{ll} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{array} \right\}. \tag{1.95}$$

Show that $d\theta/dx = \delta(x)$.

PROBLEM 2.1

(a) Twelve equal charges, q, are situated at the corners of a regular 12-sided polygon (for instance, one on each numeral of a clock face). What is the net force on a test charge Q at the center?

(b) Suppose one of the 12 q 's is removed (the one at "6 o'clock"). What is the force on Q? Explain your reasoning carefully.

(c) Now 13 equal charges, q, are placed at the comers of a regular 13-sided polygon. What is the force on a test charge Q at the center?

(d) If one of the 13 q's is removed, what is the force on Q? Explain your reasoning.

PROBLEM 2.2

Find the electric field (magnitude and direction) a distance z above the midpoint between equal and opposite charges (±q), a distanced apart (same as Example 2.1, except that the charge at x = +d/2 is -q).

PROBLEM 2.7

Find the electric field a distance z from the center of a spherical surface of radius R (Fig. 2.11) that carries a uniform charge density a. Treat the case z < R (inside) as well as z > R (outside). Express your answers in terms of the total charge q on the sphere. [Hint: Use the law of cosines to write 1- in terms of R and (). Be sure to take the positive sq.root(R2 + z2 - 2Rz)= (R - z) if R > z, but it's (z - R) if R < z.]

PROBLEM 2.8

Use your result in Prob. 2.7 to find the field inside and outside a solid sphere of radius R that carries a uniform volume charge density p. Express your answers in terms of the total charge of the sphere, q. Draw a graph of lEl as a function of the distance from the center.