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State	Finished
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Time taken	1 hour 14 mins
Grade	18.00 out of 20.00 (90%)

Question 1

Correct

Mark 2.00 out of 2.00

Let X and Y be discrete random variables with joint density

$$f(x,y) = \begin{cases} \frac{x+2y}{18}, & for\ x = 1,2;\ y = 1,2 \\ 0, & otherwise \end{cases}$$

Which of the following statements is not correct?

a. $E(Y) = 29/18$

b. $E(XY) = 15/18$ ✓

c. $E(Y) = 29/18$

d. $Cov(X,Y) = -2/324$

The correct answer is: $E(XY) = 15/18$

Question 2

Correct

Mark 2.00 out of 2.00

For what value of the constant k the function given by

$$f(x,y) = \begin{cases} kxy, & if\ x = 1,2,3;\ y = 1,2,3 \\ 0, & otherwise \end{cases}$$

is a joint probability density function of some random variables X and Y?

- a. 1/36 ✓
- b. 1/12
- c. 1/16
- d. None of these

The correct answer is: 1/36

Question 3

Correct

Mark 2.00 out of 2.00

Let the joint density of the continuous random variables X and Y be

$$f(x,y) = \begin{cases} \frac{6}{5}(x^2 + 2xy), & if\ 0 \le x \le 1; 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

What is the probability of the event $(X \le Y)$?

- a. 1
- b. 2/5 ✓
- c. 5/2
- d. 6/5

The correct answer is: 2/5

Question 4

Correct

Mark 2.00 out of 2.00

Let X and Y be two independent random variables with identical probability density function given by

$$f(x) = \begin{cases} e^{-x}, & if\ x > 0 \\ 0, & otherwise \end{cases}$$

What is the probability density function of $W = \min\{X, Y\}$?

a. $f(w) = \begin{cases} e^{-2w}, & if\ w > 0 \\ 0, & otherwise \end{cases}$

b. $f(w) = \begin{cases} 2e^{-2w}, & if\ w > 0 \\ 0, & otherwise \end{cases}$ ✓

c. $f(w) = \begin{cases} \frac{1}{2}e^{-w}, & if\ w > 0 \\ 0, & otherwise \end{cases}$

d. $f(w) = \begin{cases} 2e^{-w}, & if\ w > 0 \\ 0, & otherwise \end{cases}$

The correct answer is: $f(w) = \begin{cases} 2e^{-2w}, & if\ w > 0 \\ 0, & otherwise \end{cases}$

Question 5

Correct

Mark 2.00 out of 2.00

Let the joint density function of X and Y be given by

$$f(x, y) = \begin{cases} kxy^2, & if\ 0 < x < y < 1 \\ 0, & otherwise \end{cases}$$

What is the value of the constant k?

a. 10 ✓

b. 2/5

c. 5

d. 5/2

The correct answer is: 10

Question 6

Correct

Mark 2.00 out of 2.00

If the joint cumulative distribution function of X and Y is given by

$$F(x,y) = \begin{cases} \frac{1}{5}(2x^3y + 3x^2y^2), & if\ 0 < x, y < 1 \\ 0, & otherwise \end{cases}$$

then what is the joint density of X and Y ?

a. $f(x,y) = \begin{cases} \frac{6}{5}(x^2 + 2xy), & if\ 0 < x, y < 1 \\ 0, & otherwise \end{cases}$ ✓

b. $f(x,y) = \begin{cases} \frac{6}{5}(x^2y + xy^2), & if\ 0 < x, y < 1 \\ 0, & otherwise \end{cases}$

c. None of these

d. $f(x,y) = \begin{cases} \frac{6}{5}(2x^2 + 3xy^2), & if\ 0 < x, y < 1 \\ 0, & otherwise \end{cases}$

The correct answer is: $f(x,y) = \begin{cases} \frac{6}{5}(x^2 + 2xy), & if\ 0 < x, y < 1 \\ 0, & otherwise \end{cases}$

Question 7

Correct

Mark 2.00 out of 2.00

Let X and Y have the joint density function

$$f(x,y) = \begin{cases} e^{-(x+y)}, & if\ 0 < x, y < \infty \\ 0, & otherwise \end{cases}$$

Which of the following statements is not correct?

a. X and Y are independent.

b. None of these ✓

c. $f_X(x) = e^{-x},\ 0 < x < \infty$

d. $f_Y(y) = e^{-y},\ 0 < y < \infty$

The correct answer is: None of these

Question 8

Correct

Mark 2.00 out of 2.00

Let X and Y have the joint density function

$$f(x,y) = \begin{cases} 2x, & if\ 0 < x < 1; 0 < y < 1 \\ 0, & otherwise \end{cases}$$

What is $P(X+Y \leq 1 \mid X \leq \frac{1}{2})$?

- a. 1/6
- b. 2/3 ✓
- c. 1/3
- d. 4/3

The correct answer is: 2/3

Question 9

Correct

Mark 2.00 out of 2.00

Let X and Y have the joint density function

$$f(x,y) = \begin{cases} e^{-(x+y)}, & if\ 0 < x, y < \infty \\ 0, & otherwise \end{cases}$$

Which of the following statements is not correct?

- a. $E(X) = 3/8$
- b. $f_{Y/X}(y/x) = \frac{1}{2x},\ 0 < y < 2x < 1$
- c. $f_X(x) = 24x^2,\ 0 < x < 1/2$
- d. All of the above ✓

The correct answer is: All of the above

Question 10

Incorrect

Mark 0.00 out of 2.00

Let X and Y have the joint density function

$$f(x,y) = \begin{cases} x+y, & if\ 0 \le x \le 1; 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

What is $P(2X \le 1 \mid X+Y \le 1)$?

a. 1/3

b. 11/16

c. 11/48 ✗

d. 13/48

The correct answer is: 11/16