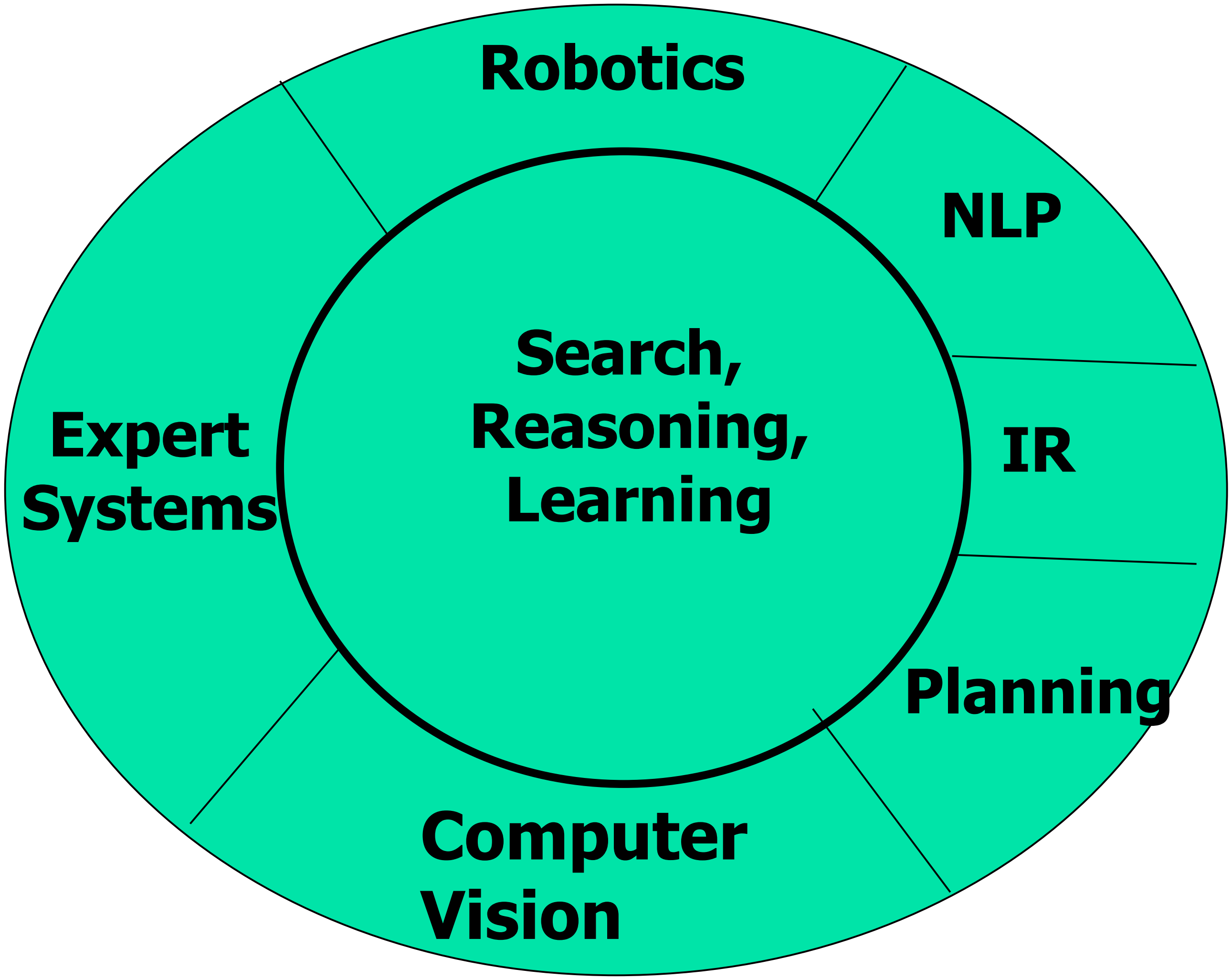
AI Perspective (post-web)



Two paradigms of AI

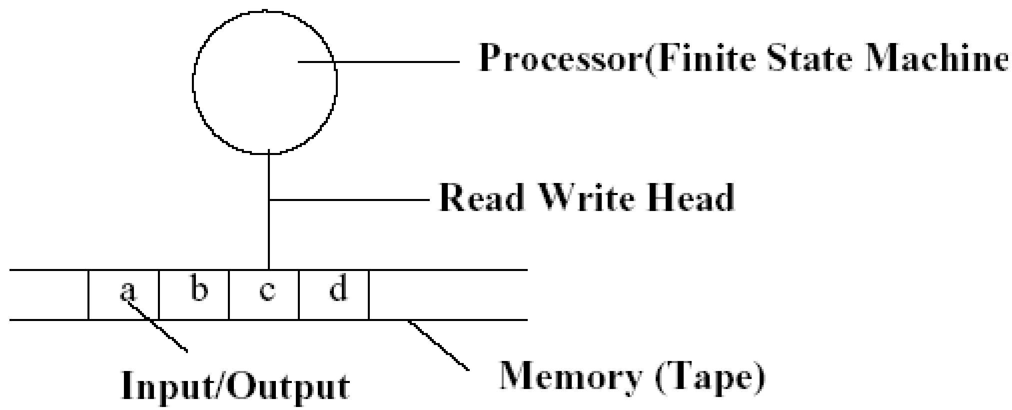
- Symbolic and Connectionist
- **Symbolic AI**
  - Physical Symbol System Hypothesis
  - Every intelligent system can be constructed by storing and processing symbols and nothing more is necessary
- **Connectionist AI**
  - Distributed method to represent knowledge
  - Inspired by human brains

Symbolic AI

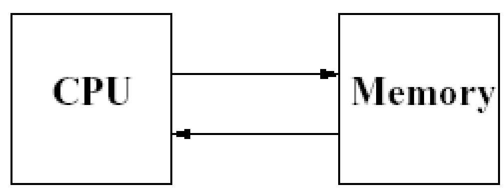
Symbolic AI has a bearing on models of computation such as

Turing Machine
Von Neumann Machine
Lambda calculus

Turing Machine & Von Neumann Machine



Turing machine



VonNeumann Machine

Challenges to Symbolic AI

Motivation for challenging Symbolic AI

- A large number of computations and information process tasks that living beings are comfortable with, are not performed well by computers!

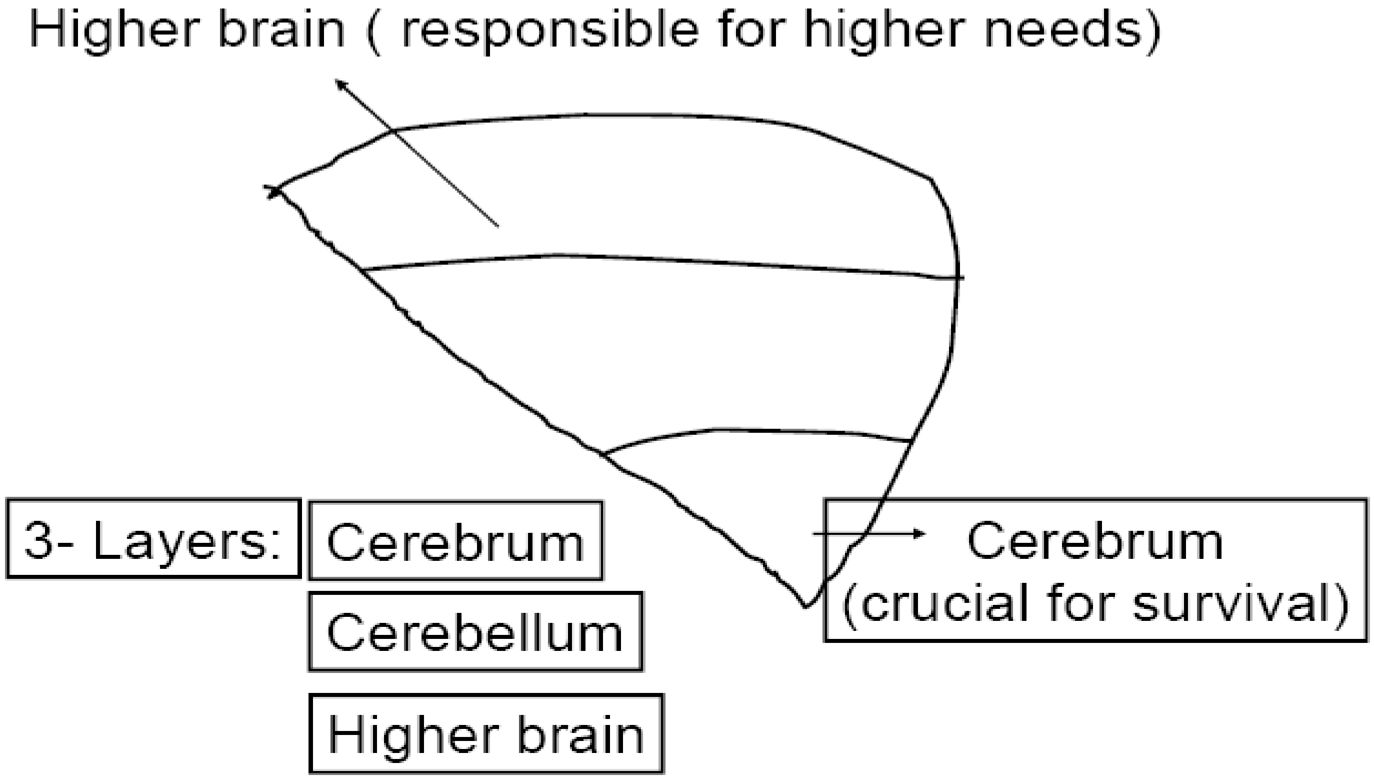
The Differences

<u>Brain computation in living beings</u>	<u>TM computation in computers</u>
Pattern Recognition	Numerical Processing
Learning oriented	Programming oriented
Distributed & parallel processing	Centralized & Serial processing
Content addressable	Location addressable

The human brain

Seat of consciousness and cognition

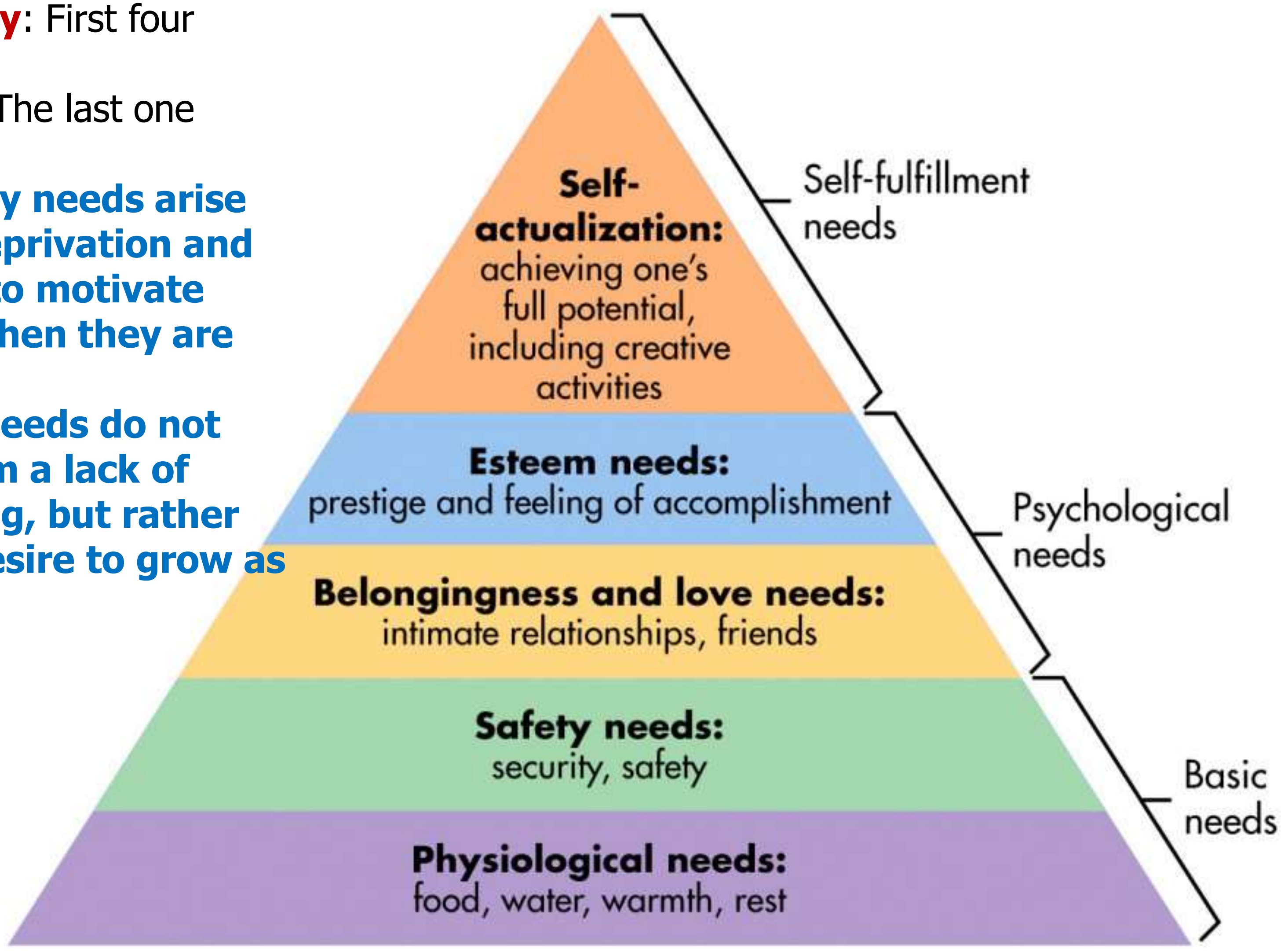
Perhaps the most complex information processing machine in nature


Higher brain (responsible for higher needs)
3- Layers:
Cerebrum
Cerebellum
Higher brain
Cerebrum
(crucial for survival)

Maslow Hierarchy of Need- 5 tier (1943 work "A Theory of Human Motivation" and later book Motivation and Personality, Maslow originally presented the notion of a hierarchy of needs)

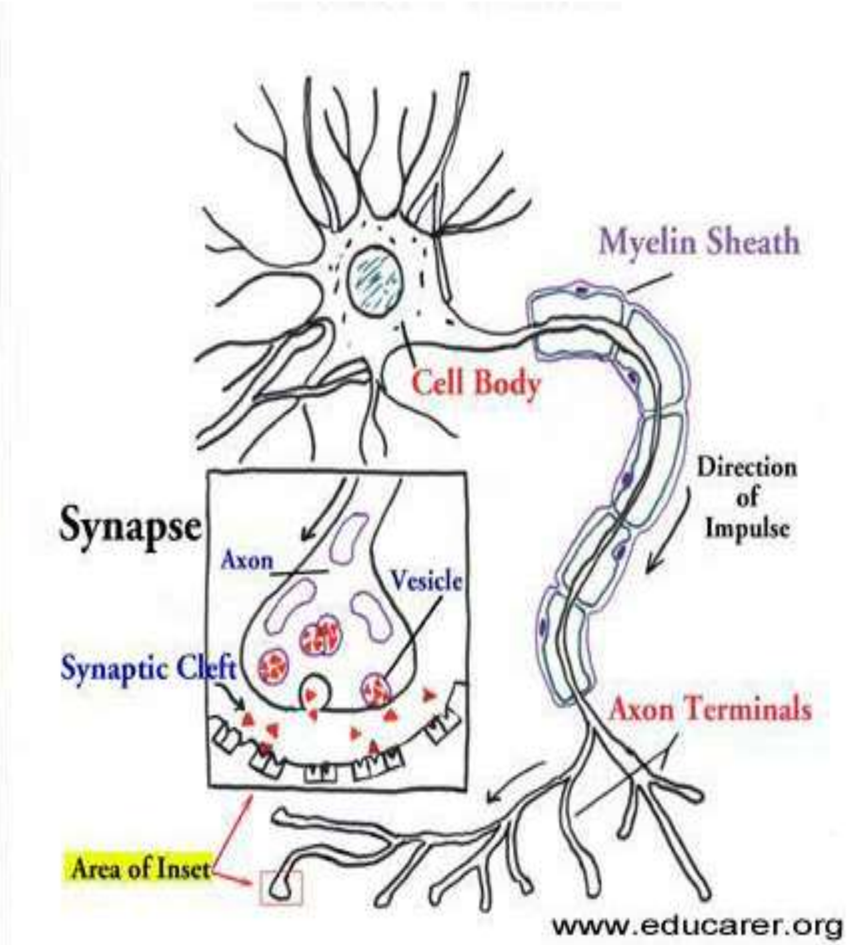
- **Deficiency**: First four levels
- **Growth**: The last one

- **Deficiency needs arise due to deprivation and are said to motivate people when they are unmet**
- **Growth needs do not stem from a lack of something, but rather from a desire to grow as a person**



Neuron - "classical": Fundamental Units of Human Brain

- **Dendrites**
  - Receiving stations of neurons
  - Don't generate action potentials
- **Cell body**
  - Site at which information received is integrated
- **Axon**
  - Generate and relay action potential
  - Terminal
    - Relays information to next neuron in the pathway

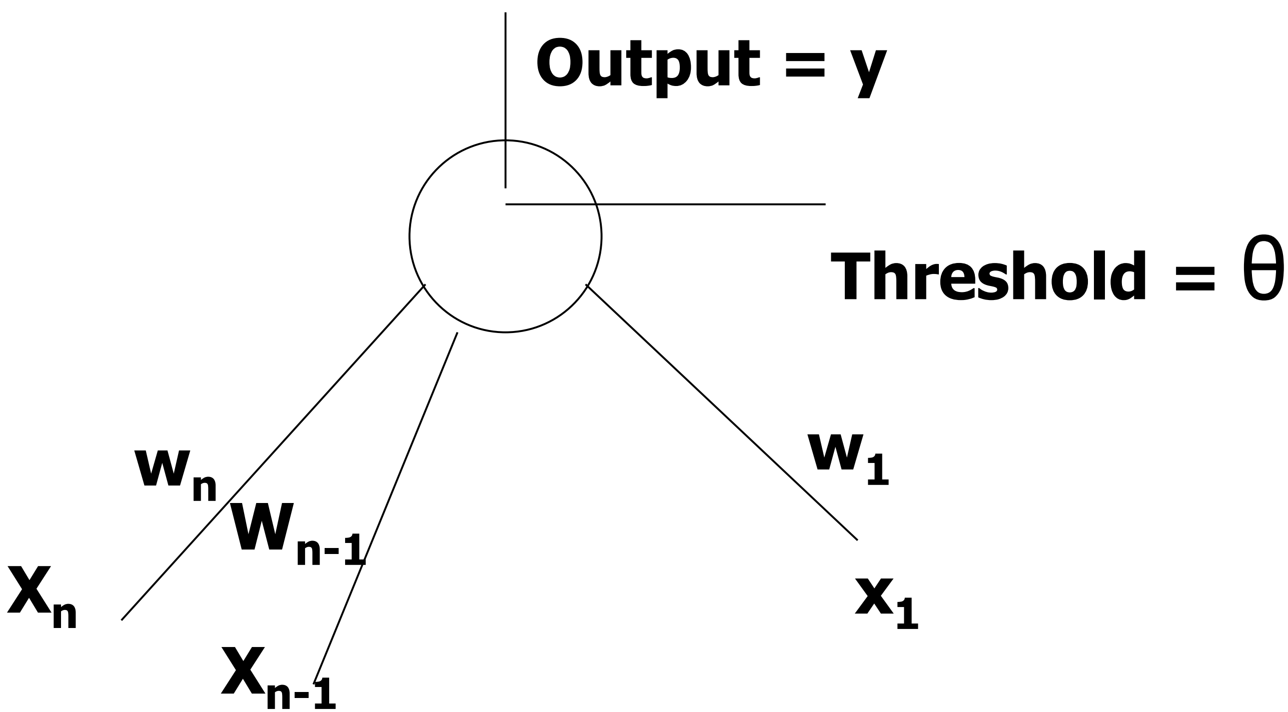


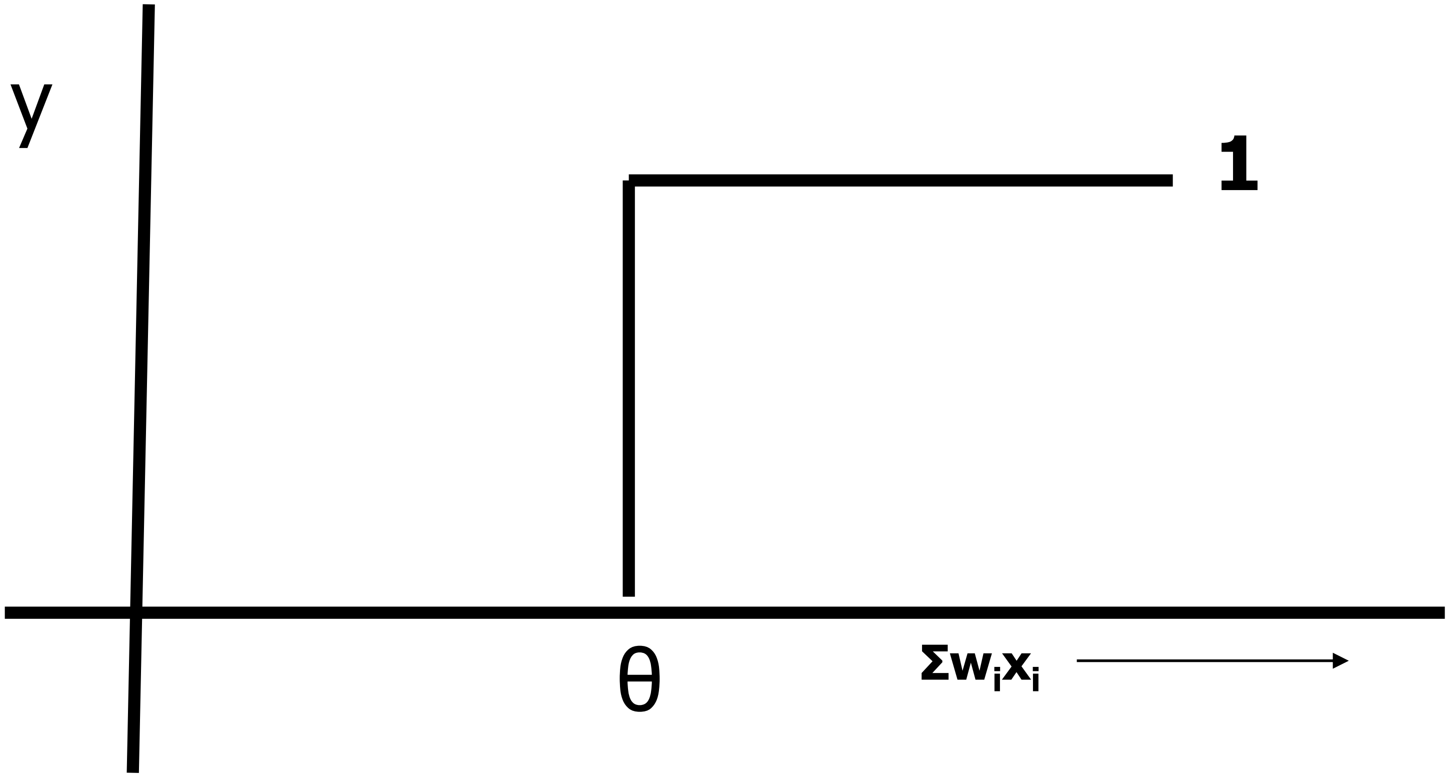
http://www.educarer.com/images/brain-nerve-axon.jpg

Perceptron

The Perceptron Model

A perceptron is a computing element with input lines having associated weights and the cell having a threshold value. The perceptron model is motivated by the biological neuron





Step function / Threshold function

y = 1 for $\Sigma w_i x_i >= \theta$

=0 otherwise

Features of Perceptron

- Input-output behavior is discontinuous and the derivative does not exist at $\Sigma w_i x_i = \theta$

- $\Sigma w_i x_i - \theta$ is the net input denoted as net

- Referred to as a linear threshold element - linearity because of **x** appearing with power **1**

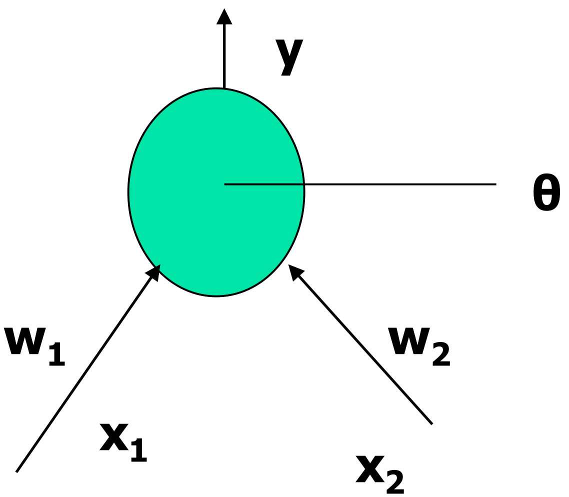
y= f(net): Relation between y and net is non-linear

Computation of Boolean functions

AND of 2 inputs

X1	x2	y
0	0	0
0	1	0
1	0	0
1	1	1

The parameter values (weights & thresholds) need to be found



Computing parameter values

$w1 * 0 + w2 * 0 \; <= \theta \Rightarrow \theta >= 0$; since y=0

$w1 * 0 + w2 * 1 \; <= \theta \Rightarrow w2 <= \theta$; since y=0

$w1 * 1 + w2 * 0 \; <= \theta \Rightarrow w1 <= \theta$; since y=0

$w1 * 1 + w2 *1 > \theta \Rightarrow w1 + w2 > \theta$; since y=1

$w1 = w2 = \theta = 0.5$

satisfy these inequalities and find parameters to be used for computing AND function

Other Boolean functions

- OR can be computed using values of $w1 = w2 = 1$ and $\theta = 0.5$

- XOR function gives rise to the following inequalities:

$$w1 * 0 + w2 * 0 <= \theta \Rightarrow \theta >= 0$$

$$w1 * 0 + w2 * 1 > \theta \Rightarrow w2 > \theta$$

$$w1 * 1 + w2 * 0 > \theta \Rightarrow w1 > \theta$$

$$w1 * 1 + w2 * 1 <= \theta \Rightarrow w1 + w2 <= \theta$$

No set of parameter values satisfy these inequalities

Threshold functions

n	# Boolean functions (2^2^n)	#Threshold Functions (2^{n^2})
1	4	4
2	16	14
3	256	128
4	64K	1008

- Functions computable by perceptrons - threshold functions
- #TF becomes negligibly small for larger values of #BF
- For n=2, all functions except XOR and XNOR are computable

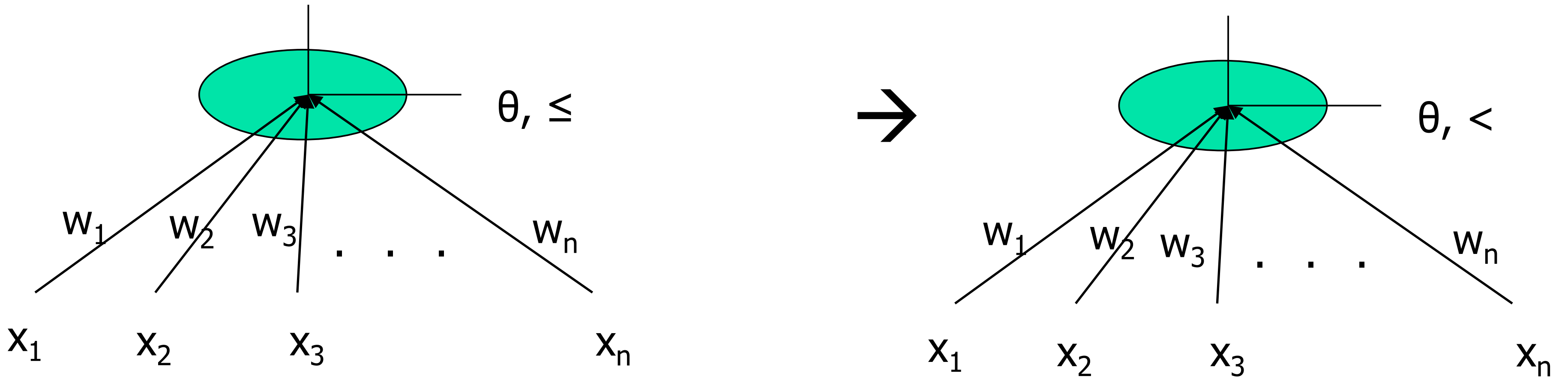
Perceptron training

Perceptron Training Algorithm (PTA)

Preprocessing:

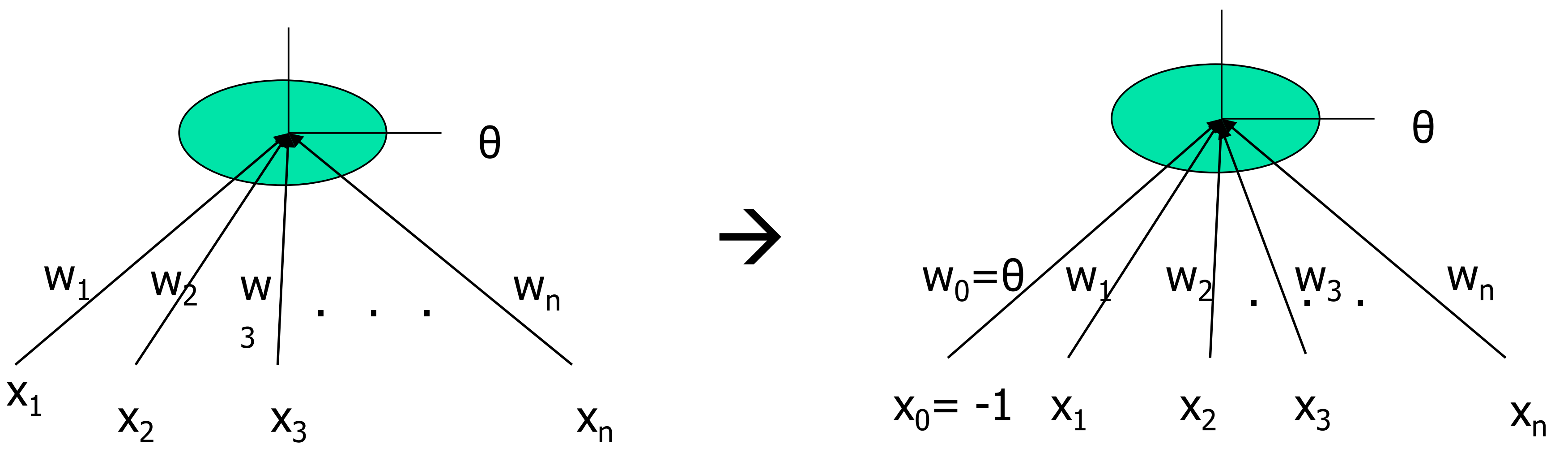
1. The computation law is modified to

$$y = 1 \text{ if } \sum w_i x_i > \theta$$
$$y = 0 \text{ if } \sum w_i x_i < \theta$$



PTA – preprocessing cont…

2. Absorb θ as a weight



3. Negate all the zero-class examples

Example to demonstrate preprocessing

- **OR perceptron**

1-class <1,1> , <1,0> , <0,1>

0-class <0,0>

Augmented x vectors:-

1-class <-1,1,1> , <-1,1,0> , <-1,0,1>

0-class <-1,0,0>

Negate 0-class:- <1,0,0>

Example to demonstrate preprocessing cont..

Now the vectors are

	x_0	x_1	x_2
X_1	-1	0	1
X_2	-1	1	0
X_3	-1	1	1
X_4	1	0	0

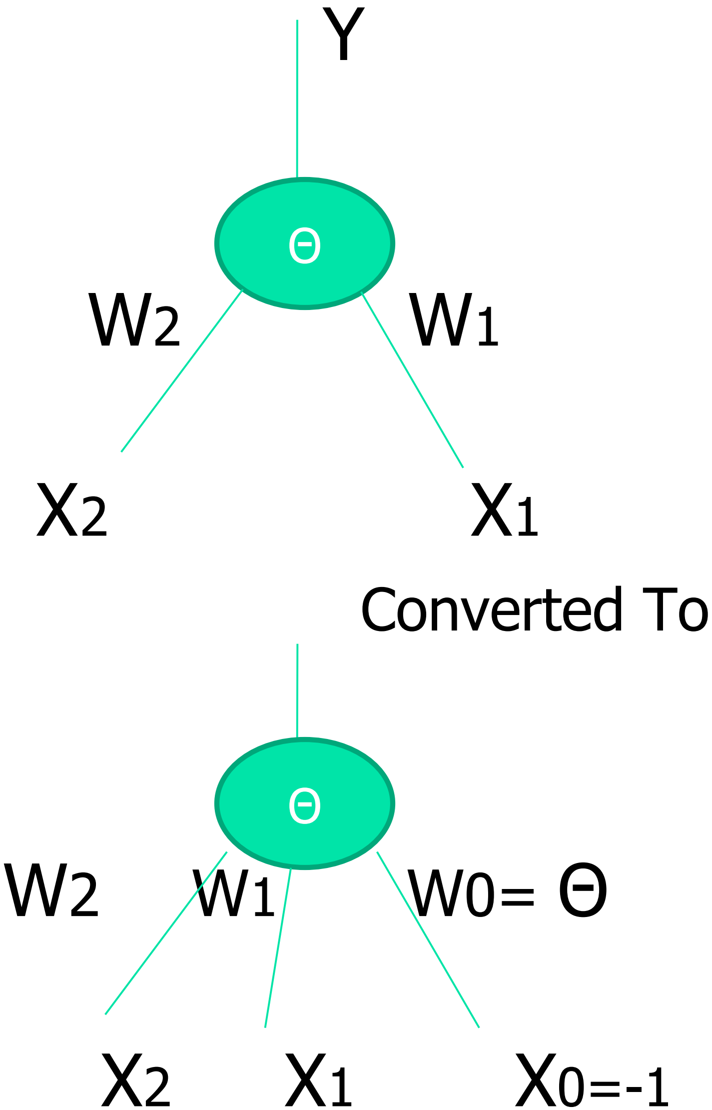
Perceptron Training Algorithm

1. Start with a random value of w
   ex: <0,0,0…>
2. Test for $wx_i > 0$
   If the test succeeds for i=1,2,…n
   then return w
3. Modify w, $w_{next} = w_{prev} + x_{fail}$

PTA on NAND

NAND:

X_2	X_1	Y
0	0	1
0	1	1
1	0	1
1	1	0



Preprocessing

NAND Augmented:

X_2	X_1	X_0	Y
0	0	-1	1
0	1	-1	1
1	0	-1	1
1	1	-1	0

NAND-0 class Negated

	X_2	X_1	X_0
V_0:	0	0	-1
V_1:	0	1	-1
V_2:	1	0	-1
V_3:	-1	-1	1

Vectors for which
W=<W2 W1 W0> has to
be found such that
W. Vi > 0

PTA Algo steps

Algorithm:

1. Initialize and Keep adding the failed vectors until W. Vi > 0 is true

Step 0: $W = <0, 0, 0>$

$W_1 = <0, 0, 0> + <0, 0, -1>$ {V_0 Fails}

$= <0, 0, -1>$

$W_2 = <0, 0, -1> + <-1, -1, 1>$ {V_3 Fails}

$= <-1, -1, 0>$

$W_3 = <-1, -1, 0> + <0, 0, -1>$ {V_0 Fails}

$= <-1, -1, -1>$

$W_4 = <-1, -1, -1> + <0, 1, -1>$ {V_1 Fails}

$= <-1, 0, -2>$

Trying convergence

$W_5 = \langle -1, 0, -2 \rangle + \langle -1, -1, 1 \rangle$ {V_3 Fails}

$= \langle -2, -1, -1 \rangle$

$W_6 = \langle -2, -1, -1 \rangle + \langle 0, 1, -1 \rangle$ {V_1 Fails}

$= \langle -2, 0, -2 \rangle$

$W_7 = \langle -2, 0, -2 \rangle + \langle 1, 0, -1 \rangle$ {V_0 Fails}

$= \langle -1, 0, -3 \rangle$

$W_8 = \langle -1, 0, -3 \rangle + \langle -1, -1, 1 \rangle$ {V_3 Fails}

$= \langle -2, -1, -2 \rangle$

$W_9 = \langle -2, -1, -2 \rangle + \langle 1, 0, -1 \rangle$ {V_2 Fails}

$= \langle -1, -1, -3 \rangle$

Trying convergence

$W_{10} = \langle -1, -1, -3 \rangle + \langle -1, -1, 1 \rangle$ $\{V_3 \text{ Fails}\}$

$= \langle -2, -2, -2 \rangle$

$W_{11} = \langle -2, -2, -2 \rangle + \langle 0, 1, -1 \rangle$ $\{V_1 \text{ Fails}\}$

$= \langle -2, -1, -3 \rangle$

$W_{12} = \langle -2, -1, -3 \rangle + \langle -1, -1, 1 \rangle$ $\{V_3 \text{ Fails}\}$

$= \langle -3, -2, -2 \rangle$

$W_{13} = \langle -3, -2, -2 \rangle + \langle 0, 1, -1 \rangle$ $\{V_1 \text{ Fails}\}$

$= \langle -3, -1, -3 \rangle$

$W_{14} = \langle -3, -1, -3 \rangle + \langle 0, 1, -1 \rangle$ $\{V_2 \text{ Fails}\}$

$= \langle -2, -1, -4 \rangle$

$W15 = \langle -2, -1, -4 \rangle + \langle -1, -1, 1 \rangle$ {V3 Fails}

$= \langle -3, -2, -3 \rangle$

$W16 = \langle -3, -2, -3 \rangle + \langle 1, 0, -1 \rangle$ {V2 Fails}

$= \langle -2, -2, -4 \rangle$

$W17 = \langle -2, -2, -4 \rangle + \langle -1, -1, 1 \rangle$ {V3 Fails}

$= \langle -3, -3, -3 \rangle$

$W18 = \langle -3, -3, -3 \rangle + \langle 0, 1, -1 \rangle$ {V1 Fails}

$= \langle -3, -2, -4 \rangle$

$W2 = -3, \quad W1 = -2, \quad W0 = \Theta = -4$

Succeeds for all vectors

Statement of Convergence of PTA

- Statement:

  Whatever be the initial choice of weights and whatever be the vector chosen for testing, PTA converges if the vectors are from a linearly separable function.

Proof of Convergence of PTA

- Suppose w_n is the weight vector at the n^{th} step of the algorithm.
- At the beginning, the weight vector is w_0
- Go from w_i to w_{i+1} when a vector X_j fails the test $w_i X_j > 0$ and update w_i as

$$w_{i+1} = w_i + X_j$$

- Since Xjs form a linearly separable function,

$$\exists\ w^* \text{ s.t. } w^* X_j > 0\ \forall j$$

Proof of Convergence of PTA (cntd.)

- Consider the expression

$$G(w_n) = \frac{w_n \cdot w^*}{|w_n|}$$

where w_n = weight at nth iteration

- $$G(w_n) = \frac{|w_n| \cdot |w^*| \cdot \cos\theta}{|w_n|}$$

where θ = angle between w_n and w^*

- $G(w_n) = |w^*| \cdot \cos\theta$
- $G(w_n) \le |w^*|$ (as $-1 \le \cos\theta \le 1$)

Behavior of Numerator of G

$$w_n \,.\, w^* = (w_{n-1} + X^{n-1}_{fail}) \,.\, w^*$$

- $= w_{n-1} \,.\, w^* + X^{n-1}_{fail} \,.\, w^*$
- $= (w_{n-2} + X^{n-2}_{fail}) \,.\, w^* + X^{n-1}_{fail} \,.\, w^* \,.....$
- $= w_0 \,.\, w^* + (X^0_{fail} + X^1_{fail} + ... + X^{n-1}_{fail}).\, w^*$

  $w^*.X^i_{fail}$ is always positive: *note carefully*

- Suppose $|X_j| \geq \delta$, where δ is the minimum magnitude.
- Num of G $\geq |w_0 \,.\, w^*| + n\, \delta \,.\, |w^*|$
- So, numerator of G grows with n.

Behavior of Denominator of G

- $|w_n| = \sqrt{w_n . w_n}$
- $= \sqrt{(w_{n-1} + X^{n-1}_{fail})^2}$
- $= \sqrt{(w_{n-1})^2 + 2. w_{n-1.} X^{n-1}_{fail} + (X^{n-1}_{fail})^2}$
- $\leq \sqrt{(w_{n-1})^2 + (X^{n-1}_{fail})^2}$ (as $w_{n-1.} X^{n-1}_{fail} \leq 0$)
- $\leq \sqrt{(w_0)^2 + (X^0_{fail})^2 + (X^1_{fail})^2 + ... + (X^{n-1}_{fail})^2}$

- $|X_j| \leq \rho$ (max magnitude)
- So, Denom $\leq \sqrt{(w_0)^2 + n\rho^2}$

Some Observations

- Numerator of G grows as n
- Denominator of G grows as $\sqrt{n}$

  => Numerator grows faster than denominator
- If PTA does not terminate, $G(w_n)$ values will become unbounded.

Some Observations contd.

- But, as $|G(w_n)| \leq |w^*|$ which is finite, this is impossible!
- Hence, PTA has to converge.
- Proof is due to Marvin Minsky.

A Problem that can be solved using the proof of PTA

Problem: *If a weight repeats while training the perceptron, then the function is not linearly separable.*

Proof

Let us prove first $w_n . w^*$ is an increasing function.

From the proof of convergence of PTA, we can write

$$w_n . w^* = (w_{n-1} + X^{n-1}{}_{fail}) . w^*$$

$$= w_{n-1} . w^* + w^* . X^{n-1}{}_{fail}$$

Since w^* is optimal weight vector therefore:

$$w^* . X^{n-1}{}_{fail} > 0$$

Proof cntd.

Because in every iteration we are adding +ve number $w^*. X^{n-1}{}_{fail}$

Therefore:

$$w_n . w^* > w_{n-1} . w^*$$

(1)

Hence $w_n . w^*$ is an increasing function.

According to the claim made by theorem, if weight repeat then the weight w_i at a given iteration i, will be equal to the weight w_{i+k} at a given iteration $(i+k)$ where k is a +ve number

$$w_{i=} \ w_{i+k}$$

Proof cntd.

Therefore:

$$w_i \cdot w^* = w_{i+k} \cdot w^* \qquad (2)$$

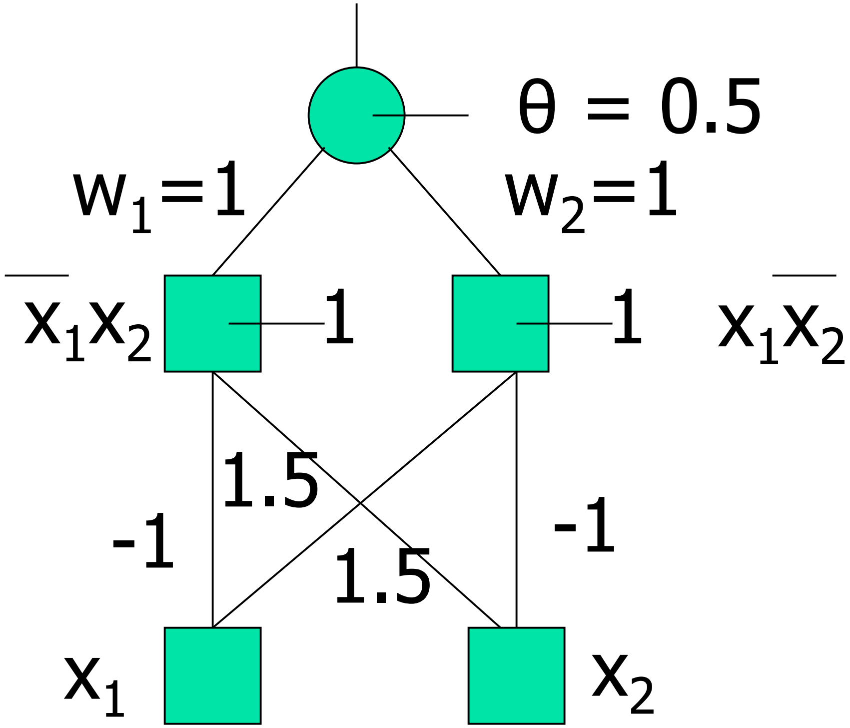
(2) contradicts the (1)

Hence no w* exists

So function is not linearly separable.

Feedforward Network and Backpropagation

Example - XOR



Gradient Descent Technique

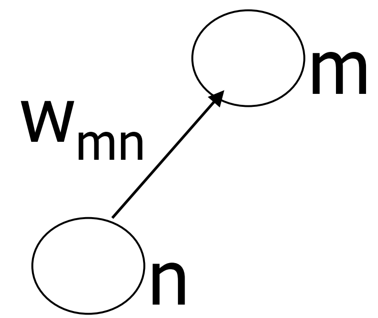
- Let E be the error at the output layer

$$E = \frac{1}{2}\sum_{j=1}^{p}\sum_{i=1}^{n}(t_i - o_i)_j^2$$

- t_i = target output; o_i = observed output
- i is the index going over n neurons in the outermost layer
- j is the index going over the p patterns (1 to p)
- Ex: XOR:– p=4 and n=1

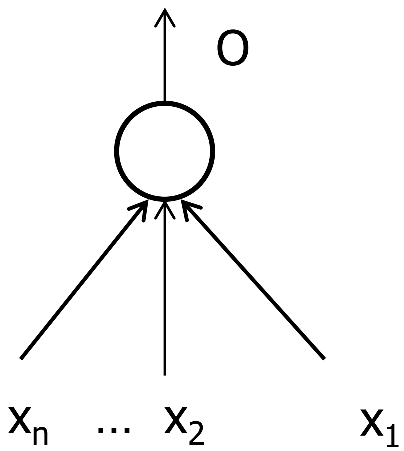
Weights in a FF NN

- w_{mn} is the weight of the connection from the n^{th} neuron to the m^{th} neuron
- E *vs* $\overline{W}$ surface is a complex surface in the space defined by the weights w_{ij}
- $-\frac{\delta E}{\delta w_{mn}}$ gives the direction in which a movement of the operating point in the w_{mn} co-ordinate space will result in maximum decrease in error



$$\Delta w_{mn} \propto -\frac{\delta E}{\delta w_{mn}}$$

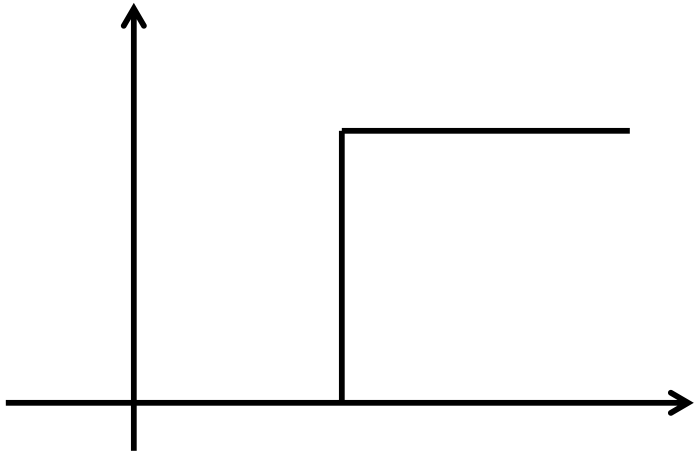
Step function v/s Sigmoid function



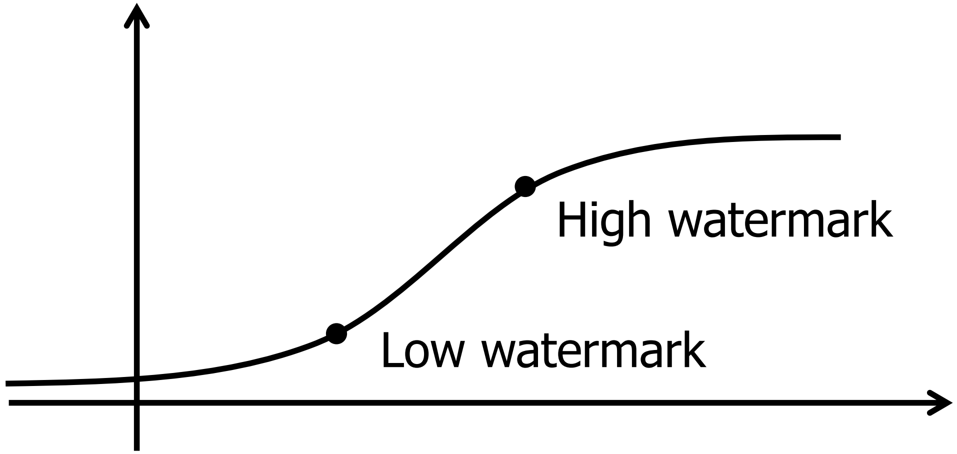
$$O = f(\sum w_i x_i)$$
$$= f(net)$$

So partial derivative of O w.r.t. net is

$$\frac{\delta O}{\delta net}$$

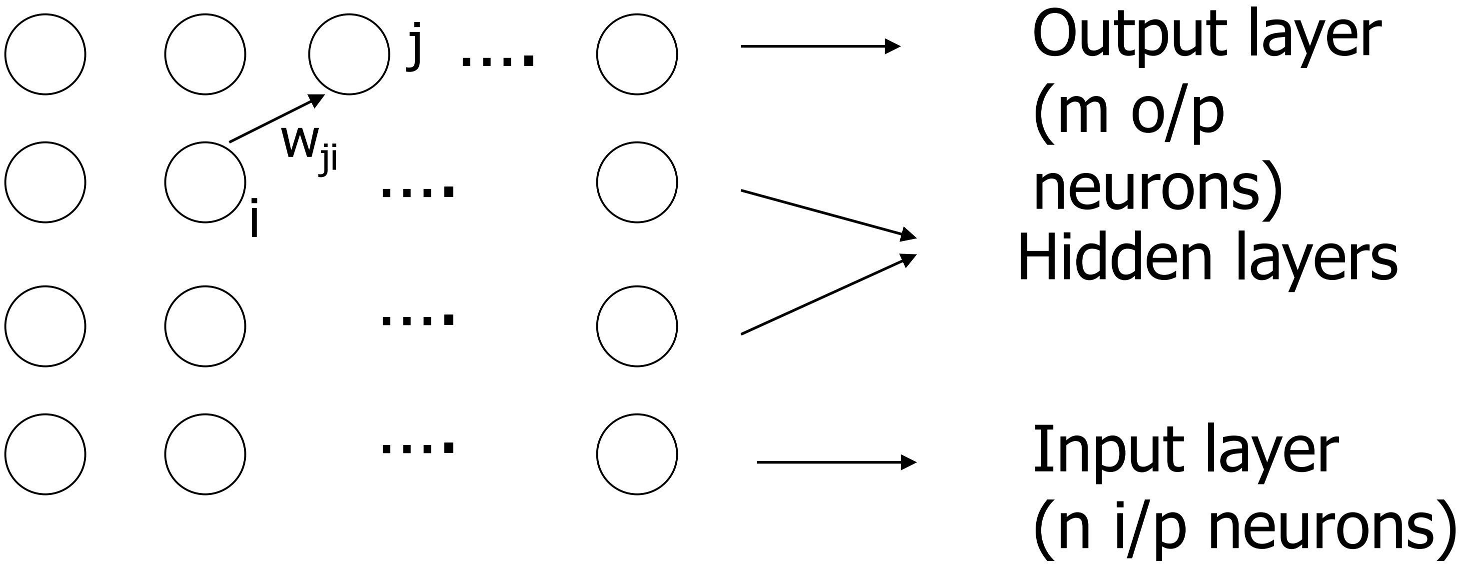


Non-differentiable



Differentiable

Backpropagation algorithm



- Fully connected feed forward network
- Pure FF network (no jumping of connections over layers)

Gradient Descent Equations

$$\Delta w_{ji} = -\eta \frac{\delta E}{\delta w_{ji}} (\eta = \text{learning rate}, 0 \le \eta \le 1)$$

$$\frac{\delta E}{\delta w_{ji}} = \frac{\delta E}{\delta net_j} \times \frac{\delta net_j}{\delta w_{ji}} (net_j = \text{input at the j}^{th} \text{ layer})$$

$$\frac{\delta E}{\delta net_j} = -\delta j$$

$$\Delta w_{ji} = \eta \delta j \frac{\delta net_j}{\delta w_{ji}} = \eta \delta j o_i$$

Backpropagation – for outermost layer

$$\delta j = -\frac{\delta E}{\delta net_j} = -\frac{\delta E}{\delta o_j} \times \frac{\delta o_j}{\delta net_j} (net_j = \text{input at the j}^{th} \text{ layer})$$

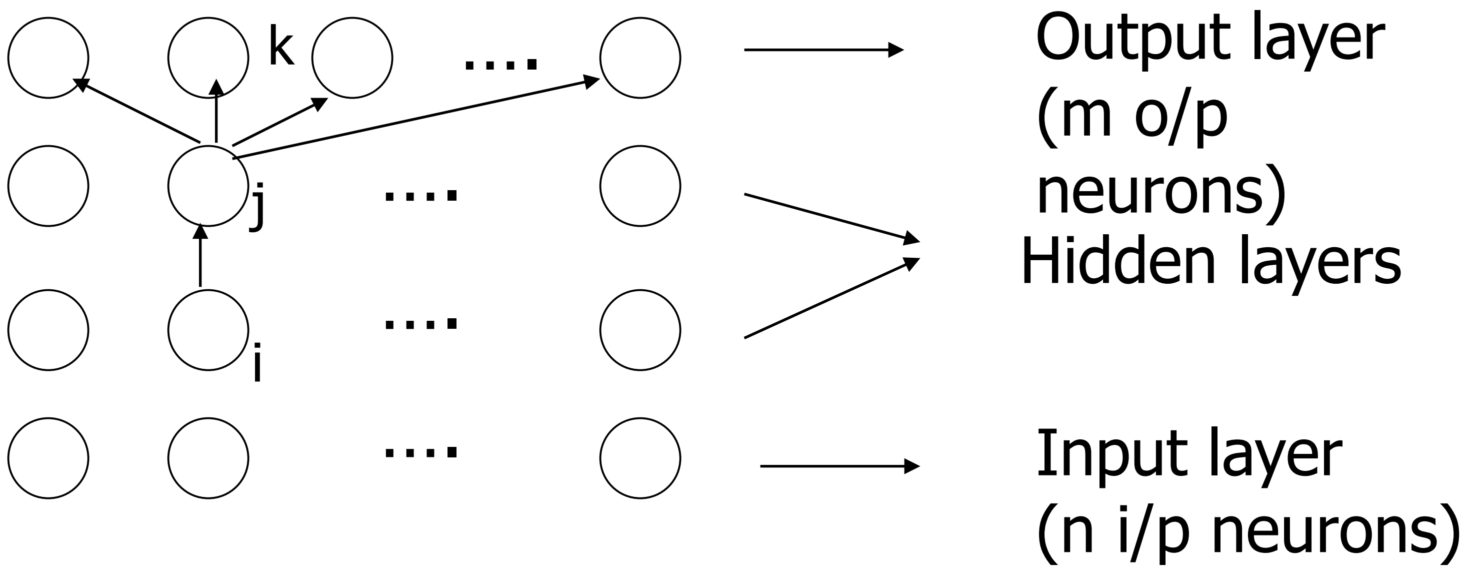
$$E = \frac{1}{2}\sum_{p=1}^{m}(t_p - o_p)^2$$

$$\text{Hence, } \delta j = -(-(t_j - o_j)o_j(1 - o_j))$$

$$\Delta w_{ji} = \eta(t_j - o_j)o_j(1 - o_j)o_i$$

Oj: Sigmoid function

Backpropagation for hidden layers



δ_k is propagated backwards to find value of δ_j

Backpropagation – for hidden layers

$$\Delta w_{ji} = \eta \delta j o_i$$

$$\delta j = -\frac{\delta E}{\delta net_j} = -\frac{\delta E}{\delta o_j} \times \frac{\delta o_j}{\delta net_j}$$

$$= -\frac{\delta E}{\delta o_j} \times o_j (1 - o_j)$$

This recursion can give rise to vanishing and exploding Gradient problem

$$= -\sum_{k \in \text{next layer}} \left(\frac{\delta E}{\delta net_k} \times \frac{\delta net_k}{\delta o_j}\right) \times o_j (1 - o_j)$$

$$\text{Hence, } \delta_j = -\sum_{k \in \text{next layer}} (-\delta_k \times w_{kj}) \times o_j (1 - o_j)$$

$$= \sum_{k \in \text{next layer}} (w_{kj} \delta_k) o_j (1 - o_j)$$

General Backpropagation Rule

- General weight updating rule:

$$\Delta w_{ji} = \eta \delta j o_i$$

- Where

$$\delta_j = (t_j - o_j) o_j (1 - o_j) \quad \text{for outermost layer}$$

$$= \sum_{k \in \text{next layer}} (w_{kj} \delta_k) o_j (1 - o_j) o_i \quad \text{for hidden layers}$$

References

- Pattern Recognition and Machine Learning. Christopher M. Bishop, Springer

Thank you