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Grade	16.00 out of 20.00 (80%)

Question 1

Correct

Mark 2.00 out of 2.00

Let X and Y be random variables with $\mathrm{Var}(X) = 4$, $\mathrm{Var}(Y) = 9$ and $\mathrm{Var}(X - Y) = 16$. Then, $\mathrm{Cov}(X, Y) = \cdots$.

Answer: -1.5 ✓

The correct answer is: -1.5

Question 2

Correct

Mark 2.00 out of 2.00

Let $X_1, X_2, \ldots, X_{50}$ be a random sample of size 50 from a distribution with density

$$f(x, y) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & for\ 0 \leq x < \infty \\ 0 & otherwise \end{cases}$$

What variance of the sample mean $\bar{X}$?

- a. 2/25 ✓
- b. 4/5
- c. 4
- d. 2

The correct answer is: 2/25

Question 3

Correct

Mark 2.00 out of 2.00

Let X be a random variable with mean 2. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be unbiased estimators of the second and third moments, respectively, of X about the origin. Find an unbiased estimator of the third moment of X about its mean in terms of $\hat{\theta}_1$ and $\hat{\theta}_2$.

a. None of these

b. $\hat{\theta}_2 - \hat{\theta}_1 + 5$

c. $3\hat{\theta}_2 - 2\hat{\theta}_1 + 8$

d. $\hat{\theta}_2 - 6\hat{\theta}_1 + 16$ ✓

The correct answer is: $\hat{\theta}_2 - 6\hat{\theta}_1 + 16$

Question 4

Correct

Mark 2.00 out of 2.00

Let the two random variable X and Y both are independent if and only if $\mathrm{Cov}(X,Y) = 0$. True/False

True

False ✓

The correct answer is 'False'.

Question 5

Correct

Mark 2.00 out of 2.00

Let $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with density function

$$f(x;\theta) = \begin{cases} 3\theta x^2 e^{-\theta x^3} & for\ 0 < x < \infty \\ 0 & otherwise \end{cases}$$

What is the Cramer-Rao lower bound for the variance of unbiased estimator of the parameter θ?

a. $\frac{\theta}{n}$

b. $\frac{\theta}{2n}$

c. $\frac{\theta^2}{2n}$

d. $\frac{\theta^2}{n}$ ✓

The correct answer is: $\frac{\theta^2}{n}$

Question 6

Correct

Mark 2.00 out of 2.00

Let $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with density function

$$f(x;\beta) = \begin{cases} \frac{x^6 e^{-\frac{x}{\beta}}}{\Gamma(7)\beta^7} & for\ 0 < x < \infty \\ 0 & otherwise \end{cases}$$

then what is the maximum likelihood estimator of β?

a. $7\bar{X}$

b. $6\bar{X}$

c. $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

d. $\bar{X}/7$ ✓

The correct answer is: $\bar{X}/7$

Question 7

Correct

Mark 2.00 out of 2.00

Let X and Y be continuous random variables with joint density function $f(x,y) = \begin{cases} e^{-y} & for\ 0 < x < y < \infty \\ 0 & otherwise \end{cases}$. What is the conditional variance of Y given the knowledge that $X = x$?

a. None of these ✓

b. $x^2 + 1$

c. $(1+x)^2 + 1$

d. $x + 1$

The correct answer is: None of these

Question 8

Incorrect

Mark 0.00 out of 2.00

Let X_1, X_2, X_3 be a random sample of size 3 from a population with density

$$f(x;\lambda) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & for\ x = 0, 1, 2, \ldots, \infty \\ 0 & otherwise \end{cases}$$

where λ is a parameter. Which of the following statements is not correct?

a. $\widehat{\lambda}_2 = \frac{1}{9}(4X_1 + 3X_2 + 2X_3)$ is unbaised estimator of λ.

b. None of these ✗

c. $\widehat{\lambda}_1$ is efficient than $\widehat{\lambda}_2$.

d. $\widehat{\lambda}_1 = \frac{1}{4}(X_1 + 2X_2 + X_3)$ is unbaised estimator of λ.

The correct answer is: $\widehat{\lambda}_1$ is efficient than $\widehat{\lambda}_2$.

Question 9

Incorrect

Mark 0.00 out of 2.00

Let $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with density function

$$f(x;\beta) = \begin{cases} (1-\theta)x^{-\theta} & for\ 0 < x < 1 \\ 0 & otherwise \end{cases}$$

then what is the maximum likelihood estimator of β?

a. $1 + \frac{1}{\overline{\ln X}}$

b. None of these

c. $\frac{1}{\overline{\ln X}}$

d. $\overline{\ln X} = \frac{1}{n}\sum_{i=1}^{n} \ln X_i$ ✗

The correct answer is: $1 + \frac{1}{\overline{\ln X}}$

Question 10

Correct

Mark 2.00 out of 2.00

Let X and Y have the joint density function $f(x,y) = \begin{cases} x+y & for\ 0 < x, y < 1 \\ 0 & otherwise \end{cases}$.

What is the conditional mean $E(Y|X = \frac{1}{3})$?

a. 1/2

b. 6/5

c. 1/3

d. 3/5 ✓

The correct answer is: 3/5