

Started on	Monday, 29 April 2024, 11:07 PM
State	Finished
Completed on	Tuesday, 30 April 2024, 12:07 AM
Time taken	1 hour
Grade	16.00 out of 20.00 (80%)

Question 1

Correct

Mark 2.00 out of 2.00

If the random variable X has a Gamma(1,1), then what is the probability that X is between its mean and median?

- ☐ a. $\frac{e-2}{e}$
- ☐ b. $\frac{2e-1}{e}$
- ☐ c. $\frac{2e}{e-2}$
- ☒ d. $\frac{e-2}{2e}$ ✓

The correct answer is: $\frac{e-2}{2e}$

Question 2

Correct

Mark 2.00 out of 2.00

Suppose that X is a random variable with density function $f(x) = \begin{cases} \frac{3}{8}x^2 & \text{for } 0 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$

Let $Y = mX^2$, where m is a fixed positive number. What is the density function of Y where nonzero?

- ☐ a. $f(y) = \begin{cases} \frac{3y\sqrt{y}}{16m\sqrt{m}}, & 0 < y < 4m^2 \\ 0, & \text{otherwise} \end{cases}$
- ☐ b. $f(y) = \begin{cases} \frac{3y\sqrt{y}}{16m\sqrt{m}}, & 0 < y < 4m \\ 0, & \text{otherwise} \end{cases}$
- ☒ c. $f(y) = \begin{cases} \frac{3\sqrt{y}}{16m\sqrt{m}}, & 0 < y < 4m \\ 0, & \text{otherwise} \end{cases}$ ✓
- ☐ d. $f(y) = \begin{cases} \frac{3\sqrt{y}}{16m\sqrt{m}}, & 0 < y \\ 0, & \text{otherwise} \end{cases}$

The correct answer is:

$$f(y) = \begin{cases} \frac{3\sqrt{y}}{16m\sqrt{m}}, & 0 < y < 4m \\ 0, & \text{otherwise} \end{cases}$$

Question 3

Correct

Mark 2.00 out of 2.00

If $Z \sim N(0, 1)$, what is the value of the constant c such that $P(|Z| \leq c) = 0.95$?

- ☒ a. 1.96 ✓
- ☐ b. 0.68
- ☐ c. 2.17
- ☐ d. 1.41

The correct answer is: 1.96

Question 4

Correct

Mark 2.00 out of 2.00

If the random variable X has a Gamma(1,2), then what is the probability density function of the random variable $Y = e^X$?

- ☒ a. $f(y) = \frac{1}{2y\sqrt{y}}, y \geq 1$ ✓
- ☐ b. $f(y) = \frac{1}{2\sqrt{y}}, y \geq 1$
- ☐ c. $f(y) = \frac{1}{y\sqrt{y}}, y > 1$
- ☐ d. $f(y) = \frac{1}{y\sqrt{y}}, y \geq 1$

The correct answer is: $f(y) = \frac{1}{2y\sqrt{y}}, y \geq 1$

Question 5

Incorrect

Mark 0.00 out of 2.00

Let X be a random variable with density function $f(x) = \begin{cases} \frac{1}{3} & \text{for } -1 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$
Find the density function of $Y = |X|$.

- ☐ a. $f(y) = \begin{cases} \frac{1}{3} & 0 \leq y < 1 \\ \frac{1}{3} & 1 \leq y \leq 2 \end{cases}$
- ☐ b. None of these
- ☐ c. $f(y) = \begin{cases} \frac{2}{3} & 0 \leq y < 1 \\ \frac{1}{3} & 1 \leq y \leq 2 \end{cases}$
- ☒ d. $f(y) = \begin{cases} \frac{1}{3} & 0 \leq y < 1 \\ \frac{2}{3} & 1 \leq y \leq 2 \end{cases}$ ✗

The correct answer is: $f(y) = \begin{cases} \frac{2}{3} & 0 \leq y < 1 \\ \frac{1}{3} & 1 \leq y \leq 2 \end{cases}$

Question 6

Incorrect

Mark 0.00 out of 2.00

Consider one meter long string which cut into two unequal pieces at a random point along its length. Find the probability that the longer piece is at least twice the length of the shorter.

- ☒ a. $1/3$ ✖
- ☐ b. $1/2$
- ☐ c. None of these
- ☐ d. $2/3$

The correct answer is: $2/3$

Question 7

Correct

Mark 1.00 out of 1.00

If X has a Uniform distribution on the interval from 0 to 10, then what is $P(X + \frac{10}{X} \geq 7)$?

- ☐ a. $10/7$
- ☐ b. None of these
- ☒ c. $7/10$ ✔
- ☐ d. $3/10$

The correct answer is: $7/10$

Question 8

Correct

Mark 2.00 out of 2.00

The probability density function of the random variable X is shown in the table below.

x	-2	-1	0	1	2	3	4
$f(x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$

Let $Y = X^2$, then what is $E(Y)$?

- ☐ a. 1.9
- ☒ b. 6.1 ✓
- ☐ c. 1.3
- ☐ d. 5.3

The correct answer is: 6.1

Question 9

Correct

Mark 2.00 out of 2.00

Let $Y = -\ln X$. If $X \sim \text{Uniform}(0, 1)$, then which of the following is correct?

- ☐ a. $Y \sim \text{Exp}(1/2)$
- ☒ b. $Y \sim \text{Exp}(1)$ ✓
- ☐ c. $E(Y) = 3/2$
- ☐ d. $V(Y) = 1/4$

The correct answer is: $Y \sim \text{Exp}(1)$

Question 10

Correct

Mark 1.00 out of 1.00

Let X be a random variable with cumulative distribution function $F(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ 1 - e^{-x}, & \text{if } x > 0 \end{cases}$.

What is $P(0 \leq e^X \leq 4)$?

- ☐ a. $1/e$
- ☐ b. $1/4$
- ☐ c. $1/2$
- ☒ d. $3/4$ ✓

The correct answer is: $3/4$

Question 11

Correct

Mark 2.00 out of 2.00

If the random variable X has Uniform distribution on the interval $[0, a]$. Find $P(X > X^2)$.

- ☐ a. a^2
- ☒ b. $1/a$ ✓
- ☐ c. $2a$
- ☐ d. $1/a^2$

The correct answer is: $1/a$