2019.10.17.

$$0 = G_i(\underline{\vartheta}) = \frac{4}{\vartheta_i} + \frac{4\alpha+2}{a^2+\vartheta_i} + \frac{4\beta+2}{b^2+\vartheta_i} + \frac{4\gamma+2}{c^2+\vartheta_i} + \sum_{\substack{j=1\\ i\neq j}}^{n} \frac{8}{\vartheta_i-\vartheta_j}$$

$$\frac{\omega^2}{c_0^2} = 2\left(\frac{\alpha}{a^2}+\frac{\beta}{b^2}+\frac{\gamma}{c^2}\right) - \sum_{i=1}^{n}\frac{4}{\vartheta_i} \qquad i=1,\dots,n$$

$$S_n(\underline{r}) = \text{const.} \cdot x_1^{\alpha} x_2^{\beta} x_3^{\gamma} \begin{cases} \prod_{i=1}^{n}\left(1-\frac{x_1^2}{a^2+\vartheta_i}-\frac{x_2^2}{b^2+\vartheta_i}-\frac{x_3^2}{c^2+\vartheta_i}\right) & n\geq 1 \\ 1 & n=0 \end{cases}$$

$$G_i(\underline{\vartheta}) = -\frac{\partial V(\underline{\vartheta})}{\partial \vartheta_i}$$

$\rightsquigarrow$ we can find the ϑ-s with a modified <u>Newton - Rapson method</u>

- it should be stableized so it does not diverge.

$\underline{G}(\underline{\vartheta}^{(\ell)}) \neq 0$ the ℓth approx.

$$\underline{G}(\underline{\vartheta}^{(\ell)}+\delta\underline{\vartheta}) = \underline{G}(\underline{\vartheta}^{(\ell)}) + \left(\frac{\partial \underline{G}}{\partial \underline{\vartheta}}\right)_{\underline{\vartheta}^{(\ell)}} \cdot \delta\underline{\vartheta} = \underline{0}$$

$\rightsquigarrow$ derivative mx.

$$\delta\underline{\vartheta} = -\left(\frac{\partial \underline{G}}{\partial \underline{\vartheta}}\right)^{-1}_{\underline{\vartheta}^{(\ell)}} \underline{G}(\underline{\vartheta}^{(\ell)})$$

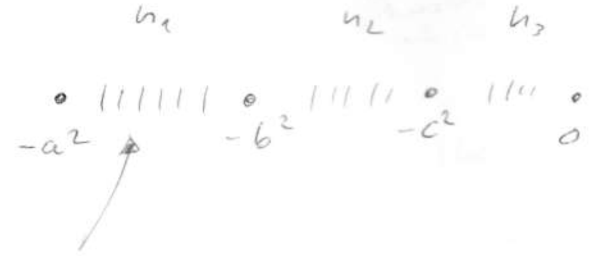
$\rightsquigarrow$ computationally expensive

$$\left(\frac{\partial \underline{G}}{\partial \underline{\vartheta}}\right)^{-1}_{\underline{\vartheta}^{(\ell)}} \delta\underline{\vartheta} = \underline{G}(\underline{\vartheta}^{(\ell)}) \quad \text{is better.}$$

some numerical trickery + Gauss-elimination

$\underline{\vartheta}^{(\ell+1)} = \underline{\vartheta}^{(\ell)} + \delta \underline{\vartheta}$

$\rightsquigarrow$ this can be iterated to get a good. $\underline{\vartheta}$



we put n_1 charges here $\underline{\text{eqidistantly.}}$ $\rightarrow$ "linspace"

- this will be $\underline{\vartheta}^{(0)}$

- components are ordered $\vartheta_1 < \vartheta_2 < \dots$
- after adding $\delta \underline{\vartheta}$ the relation between ϑ-s should stay the same.

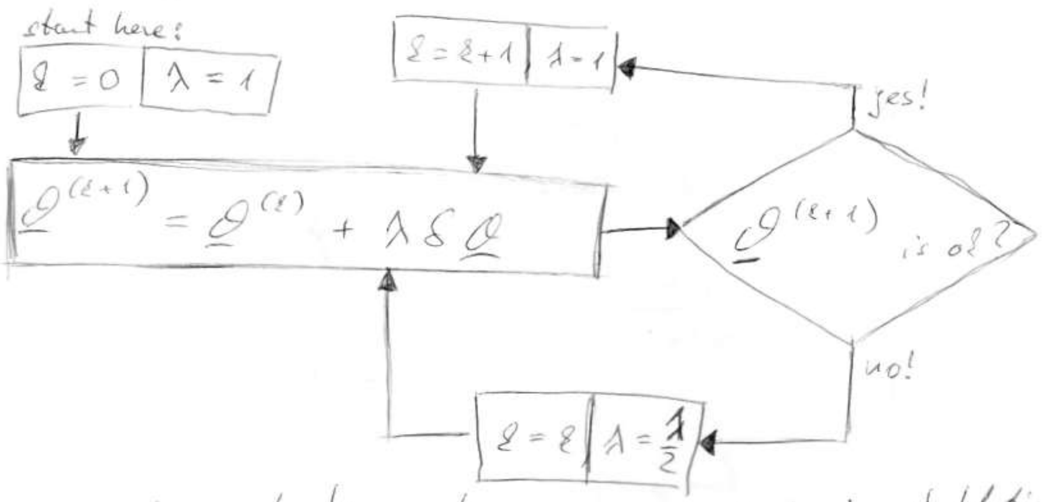
$\downarrow$

$$-a^2 < \vartheta_1 \dots < \vartheta_{n_1} < -b^2 < \dots \quad < \vartheta_{n_1+n_2+n_3} < 0$$

- the "walls" stay at the same place after every iteration!

- if the config is okay Newton-Rapson can be used to refine the solution.

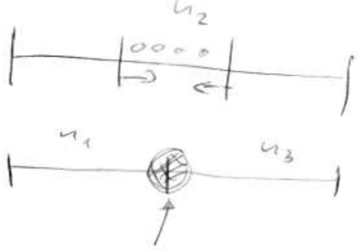
- we can try $\frac{1}{2} \cdot \delta \underline{\vartheta}$ if the previous trial was ~~good~~) bad.



- only a few starting steps are needed to stabilize the process.

a, b, c → a, b

$$\underbrace{\begin{matrix}\omega_\perp = \omega_x, \omega_y \\ \omega_z\end{matrix}}_{\text{cylindrical case}}$$



new, renormalized charge in the middle with new conserved quantity.

It is easier to start from scratch: $x_1^\alpha x_2^\beta x_3^\gamma$

$$\delta n = \rho^{|m|} e^{im\varphi} z^\gamma \begin{cases} \prod_{i=1}^{n} \left(1 - \frac{\rho^2}{a^2+\vartheta_i} - \frac{z^2}{b^2+\vartheta_i}\right) \\ 1 \end{cases}$$

the new pre-factor

⇝ same calc., same equations can be repeated...

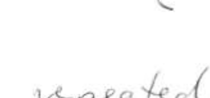
$$\frac{\omega^2}{c_0^2} = 2\left(\frac{|m|}{a^2} + \frac{\gamma}{b^2}\right) - \sum_{i=1}^{n} \frac{4}{\vartheta_i} \qquad \text{trap freq.}$$

$$0 = \frac{4|m|+4}{a^2+\vartheta_i} + \frac{4\gamma+2}{b^2+\vartheta_i} + \frac{4}{\vartheta_i} + \sum_{\substack{j=1 \\ i \neq j}}^{n} \frac{8}{\vartheta_i - \vartheta_j} \qquad \text{force-eq.}$$

a, b → a

$\omega_0 = \omega_x = \omega_y = \omega_z$

spherical case



renormalized value of the fixed charge

$$r^l Y_m^l(\vartheta, \varphi) \begin{Bmatrix} \prod_{i=1}^{n} \left(1 - \frac{r^2}{a^2+\vartheta_i}\right) \\ 1 \end{Bmatrix} = \delta n$$

$$\frac{\omega^2}{c_0^2} = \frac{2l}{a^2} - \sum_{i=1}^{n} \frac{4}{\vartheta_i} \qquad \text{trap freq.}$$

$$0 = \frac{4}{\vartheta_i} + \frac{4\ell + 6}{a^2 + \vartheta_i} + \sum_{\substack{j=1 \\ i \neq j}}^{4} \frac{8}{\vartheta_i - \vartheta_j} \qquad \text{force - eq.}$$

→ this fun gives a new evaluation of a hypergeometric polinomial (which is the known solution for this prob.)

→ it also gives the roots ...

— — — — — — — — —