- parabolic confinement, non-int. atoms

$$E_{n_x n_y n_z} = \sum_{i=1}^{3} \hbar\omega_i \left(n_i + \frac{1}{2}\right)$$

if $\omega_x = \omega_y = \omega_z$

$$E_{n,l,m} = \hbar\omega_0 \left(\frac{3}{2} + \underbrace{2n + l}_{\text{shell-quantum-number}}\right)$$

⤳ lots of degeneracies.

⤳ we fill up every lvl. with N

⤳ Ψ is a single Slater-det.

⤳ exact density $\langle \Psi | n | \Psi \rangle$

⤳ $n(\vec{r}) = 2 \sum_i |\varphi_i(\vec{r})|^2$

⤳ the diffrence between the exact density and the local-density-approx is less than 1%

↓

it's worth to use for big N-s.

$$n(r) = \frac{1}{3\pi^2} \left(\frac{2m(\mu - V(r))}{\hbar^2}\right)^{3/2} \Theta(\mu - V(r)) \qquad \text{(fermions)}$$

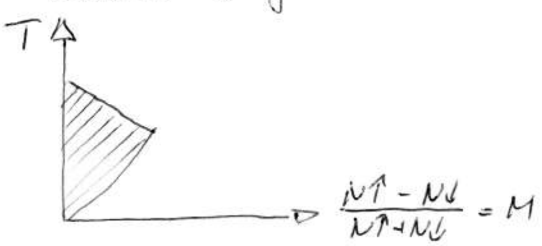
$$n(r) = \frac{(\mu - V(r))}{g} \Theta(\mu - V(r)) \qquad \text{(TF profile for bosons)}$$

- Feshbach-ressonance can ~~help~~ change between the two
- (They have very diffrent limits...)

2019.11.28.

BCS - BEC transition

- ArxiV. 0706.3360
- erős Sz. eset → minta mágnesezettsége nem okoz problémát, ellentben a fémekkel (nem mivs...)



- Hamilton op.:

$$H = \sum_{\sigma=\uparrow\downarrow} \int d^3\vec{r}\ \Psi^+_\sigma(\vec{r}) \left(-\frac{\hbar^2}{2m}\Delta + V_{\sigma,ext}(\vec{r}) - \mu_\sigma \right) \Psi_\sigma(\vec{r}) \ + \ \circledast$$

$\rightsquigarrow T = 0$

$\rightsquigarrow \mu_\sigma$ eltérhet 2 spinbeállásra

$$\circledast \ + \int d^3\vec{r}\, d^3\vec{r}' \ V_{\uparrow\downarrow}(\vec{r}-\vec{r}')\, \Psi^+_\uparrow(\vec{r})\, \Psi^+_\downarrow(\vec{r}')\, \Psi_\downarrow(\vec{r}')\, \Psi_\uparrow(\vec{r})$$

$\rightsquigarrow$ csak szinglet szórás, tripletet elhanyagoljuk

$\rightsquigarrow$ nyugalmas

$\rightsquigarrow$ $\left.\begin{matrix} \uparrow\uparrow\downarrow\uparrow \\ \uparrow\uparrow\uparrow\uparrow \end{matrix}\right\}$ elhanyagoljuk

$\hookrightarrow$ Hartree - Fock - like terms.

$\rightsquigarrow$ nincs $\frac{1}{2}$: $\downarrow\uparrow\uparrow\downarrow$ ugyanazt hozza, mint a fenti.

- fermionic - operators:

$$\left\{ \Psi_\sigma(\vec{r}),\ \Psi^+_{\sigma'}(\vec{r}') \right\} = \delta_{\sigma\sigma'}\, \delta(\vec{r}-\vec{r}')$$

- to fix μ_σ $\rightsquigarrow$ $N_\sigma = \int d^3\vec{r}\ \Psi^+_\sigma(\vec{r})\, \Psi_\sigma(\vec{r})$

  $\rightsquigarrow$ if $N_\uparrow \neq N_\downarrow \rightarrow \mu_\uparrow \neq \mu_\downarrow$

- we will consider the simplest case
- homogenious system

  $V_{\sigma, ext} = 0$

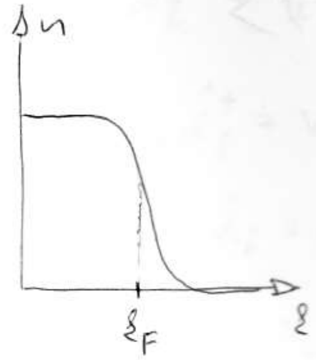
  $N_\uparrow = N_\downarrow \longrightarrow \mu_\uparrow = \mu_\downarrow$

- in non-int systems:

  $n = 2n_\uparrow = 2n_\downarrow$

  $k_F = (3\pi^2 n)^{1/3}$

- if we have Cooper-pairs $\rightsquigarrow$ no sharp Fermi-surface



$$E_F = \frac{\hbar^2 \varepsilon_F^2}{2m} = \frac{\hbar^2 (3\pi^2 n)^{2/3}}{2m} = \varepsilon T_F$$ $\rightsquigarrow$ defines a charachteristic temperature.

$\underline{T_c > T}$

$$F(R,s) = \langle \Psi_\downarrow (\vec{R} + \tfrac{\vec{s}}{2}) \Psi_\uparrow (\vec{R} - \tfrac{\vec{s}}{2}) \rangle = \frac{m}{4\pi \hbar^2} \Delta(R) \left(\frac{1}{s} - \frac{1}{a} \right)$$

this is the order parameter.

$\rightsquigarrow$ correlation func.

$\rightsquigarrow$ in homogenious sys it is independent of R

$\rightsquigarrow$ for small s $\sim \frac{1}{s}$

$\rightsquigarrow$ a is the s-wave scattering length.

- interaction term:

$$\int d^3r \int d^3r' \; v(r-r') \Psi_\uparrow^+(r) \Psi_\downarrow^+(r') \Psi_\downarrow(r') \Psi_\uparrow(r)$$

$\rightsquigarrow$ no spin reversing

$\rightsquigarrow$ no parallel spins.

- let's introduce center of mass and relative coordinates.

$$\vec{R} = \frac{\vec{r} + \vec{r}'}{2} \qquad d^3r \, d^3r' = d^3R \, d^3s$$

$$\vec{s} = \vec{r} - \vec{r}' \qquad \rightsquigarrow \text{Jakobian is } 1$$

$$\int d^3R \int d^3s \; v(s) \Psi_\uparrow^+(R + \tfrac{s}{2}) \Psi_\downarrow^+(R - \tfrac{s}{2}) \Psi_\downarrow(R - \tfrac{s}{2}) \Psi_\uparrow(R + \tfrac{s}{2}) \approx$$

$$\approx - \int d^3R \; \Delta(R) \left(\Psi_\uparrow^+(R) \Psi_\downarrow^+(R) + \text{H.c.} \right)$$

real

and $\Delta(R) = -\int d^3s\, v(s) \left\langle \Psi_\downarrow \left(R - \frac{s}{2}\right) \Psi_\uparrow \left(R + \frac{s}{2}\right) \right\rangle$

- delta must be fixed self-consistently.
- if the system is translationally invariant:

$$\Delta(R) = \Delta$$

- in homogenious system:

$$-\int d^3R\, \Delta\, \hat{\Psi}^+_\uparrow(R)\, \hat{\Psi}^+_\downarrow(R) =$$

$$\left.\hat{\Psi}_\sigma(R) = \sum_{k} \frac{e^{i\vec{k}\vec{R}}}{\sqrt{V}}\, \hat{a}_{k,\sigma} \right\}\text{ plane-waves}$$

$$= -\Delta \frac{1}{V} \sum_{\vec{k},\vec{k}'} \hat{a}^+_{k\uparrow} \hat{a}^+_{k'\downarrow} \underbrace{\int d^3R\, e^{-ikR} e^{-ik'R}}_{\delta_{k+k',0} \cdot V} = -\Delta \sum_{\vec{k}} \hat{a}^+_{\vec{k}\uparrow} \hat{a}^+_{-\vec{k}\downarrow}$$

$$H^{MF}_{BCS} = \sum_{\sigma=\uparrow\downarrow} \int d^3\vec{R}\, \hat{\Psi}^+_\sigma(\vec{R}) \left(-\frac{\hbar^2}{2m}\Delta_R - \mu\right) \hat{\Psi}_\sigma(\vec{R}) - \int d^3\vec{R}\, \Delta \left(\hat{\Psi}^+_\uparrow(\vec{R}) \hat{\Psi}^+_\downarrow(\vec{R}) + \text{h.c.}\right)$$

$$\boxed{K^{MF}_{BCS\text{-}BEC} = \sum_{\vec{k}} \left(\frac{\hbar^2 k^2}{2m} - \mu\right)\left(\hat{a}^+_{\vec{k}\uparrow} \hat{a}_{\vec{k}\uparrow} + \hat{a}^+_{\vec{k}\downarrow} \hat{a}_{\vec{k}\downarrow}\right) - \Delta \sum_{\vec{k}} \left(\hat{a}^+_{\vec{k}\uparrow} \hat{a}^+_{-\vec{k}\downarrow} + \hat{a}_{-\vec{k}\downarrow} \hat{a}_{+\vec{k}\uparrow}\right)}$$

- new notation:

$$\eta_{\vec{k}} = \frac{\hbar^2 \vec{k}^2}{2m} - \mu$$

- every quadratic Hamiltonian can be diagonalized by a generalized Bogoliubov-transformation.

$$\left.\begin{aligned}
\hat{a}_{\vec{k}\uparrow} &= u_k \hat{\alpha}_{\vec{k}} + v_k \hat{\beta}^+_{-\vec{k}} \\
\hat{a}^+_{\vec{k}\uparrow} &= u_k \hat{\alpha}^+_{\vec{k}} + v_k \hat{\beta}_{-\vec{k}} \\
\hat{a}^+_{-k\downarrow} &= -v_k \hat{\alpha}_{\vec{k}} + u_k \hat{\beta}^+_{-\vec{k}} \\
\hat{a}_{-k\downarrow} &= -v_k \hat{\alpha}^+_{\vec{k}} + u_k \hat{\beta}_{-\vec{k}}
\end{aligned}\right\} \text{ new creation - annihilation operators.}$$

$U_k^2 + V_k^2 = 1 \rightsquigarrow$ criteria for U_k, V_k

- canonical commutation relations:

$\{,\}$	$a_{k\uparrow}$	$a_{+k\downarrow}$	$a^+_{k\uparrow}$	$a^+_{k\downarrow}$
$a_{k'\uparrow}$	0	0	$\delta_{kk'}$	0
$a_{k'\downarrow}$	0	0	0	$\delta_{kk'}$
$a^+_{k'\uparrow}$	$\delta_{kk'}$	0	0	0
$a^+_{k'\downarrow}$	0	$\delta_{kk'}$	0	0

$\Downarrow$ provided $u_k^2 + v_k^2 = 1$ is fulfilled.

$\{,\}$	α_k	β_k	α_k^+	β_k^+
$\alpha_{k'}$	0	0	$\delta_{kk'}$	0
$\beta_{k'}$	0	0	0	$\delta_{kk'}$
$\alpha_{k'}^+$	$\delta_{kk'}$	0	0	0
$\beta_{k'}^+$	0	$\delta_{kk'}$	0	0

$$K = \sum_\ell \Big\{ \eta_\ell \left(u_\ell \alpha_\ell^+ + v_\ell \beta_{-\ell}\right)\left(u_\ell \alpha_\ell + v_\ell \beta_{-\ell}^+\right) +$$
$$+ \eta_\ell \left(-v_\ell \alpha_\ell + u_\ell \beta_{-\ell}^+\right)\left(-v_\ell \alpha_\ell^+ + u_\ell \beta_{-\ell}\right) -$$
$$- \Delta \left(u_\ell \alpha_\ell^+ + v_\ell \beta_{-\ell}\right)\left(-v_\ell \alpha_\ell + u_\ell \beta_{-\ell}^+\right) -$$
$$- \Delta \left(-v_\ell \alpha_\ell^+ + u_\ell \beta_{-\ell}\right)\left(u_\ell \alpha_\ell^+ + v_\ell \beta_{-\ell}^+\right)\Big\} =$$

$$= \sum_\ell \Big\{ \alpha_\ell^+ \alpha_\ell \left(\eta_\ell u_\ell^2 - \eta_\ell v_\ell^2 + \Delta u_\ell v_\ell + \Delta u_\ell v_\ell\right) +$$
$$+ \beta_{-\ell}^+ \beta_{-\ell} \left(-\eta_\ell v_\ell^2 + \eta_\ell u_\ell^2 + \Delta u_\ell v_\ell + \Delta u_\ell v_\ell\right) +$$
$$+ \alpha_\ell \beta_{-\ell} \left(-\eta_\ell u_\ell v_\ell - \eta_\ell u_\ell v_\ell - \Delta v_\ell^2 + \Delta u_\ell^2\right) +$$
$$+ \alpha_\ell^+ \beta_{-\ell}^+ \left(\eta_\ell u_\ell v_\ell + \eta_\ell u_\ell v_\ell - \Delta u_\ell^2 + \Delta v_\ell^2\right) +$$
$$+ \left(2\eta_\ell v_\ell^2 - 2\Delta u_\ell v_\ell\right)\Big\}$$

this last constant term does not matter in normal averages.
(just shifts the grund. state energy)

- we can choose u_ℓ, v_ℓ so the cross-terms die:

$$\Delta\left(u_\ell^2 - v_\ell^2\right) = 2\eta_\ell u_\ell v_\ell$$

Normalization: $u_\ell^2 + v_\ell^2 = 1$

Statement:

$$\begin{pmatrix} \eta_\ell & \Delta \\ \Delta & -\eta_\ell \end{pmatrix}\begin{pmatrix} u_\ell \\ v_\ell \end{pmatrix} = E_\ell \begin{pmatrix} u_\ell \\ v_\ell \end{pmatrix}$$