$$\lim_{s\to 0}\left[F(s) - \frac{m\Delta}{4\pi\hbar^2 s}\right]$$

$$\lim_{s\to 0} \Delta \int \frac{d^3 l}{(2\pi)^3} e^{-ils}\left(\frac{1}{2E_l} - \frac{m}{\hbar^2 l^2}\right) = -\frac{m\Delta}{4\pi\hbar^2 a}$$

$$F(R,s) = \frac{m}{4\pi\hbar^2}\Delta(R)\left(\frac{1}{s} - \frac{1}{a}\right) + \mathcal{O}(s)$$ ← leading behaviour of F.

- this is the regularized gap-eq.

$$\boxed{\Delta \int \frac{d^3 l}{(2\pi)^3}\left(\frac{1}{2E_l} - \frac{m}{\hbar^2 l^2}\right) = -\frac{m\Delta}{4\pi\hbar^2 a}}$$

2019. 12. 12.

- több-féle közelítés is erre vezet.
- BCS - BEC - átalakulás:

$$n_\uparrow = \frac{1}{2}\int \frac{d^3 l}{(2\pi)^3}\left(1 - \frac{\eta_l}{E_l}\right) \qquad \eta_l = \frac{\hbar^2 l^2}{2m} - \mu$$

$$+\frac{m}{4\pi\hbar^2 a} = \int \frac{d^3 l}{(2\pi)^3}\left(\frac{m}{\hbar^2 l^2} - \frac{1}{2E_l}\right) \qquad E_l = \sqrt{\left(\frac{\hbar^2 l^2}{2m} - \mu\right)^2 + \Delta^2}$$

- BCS → Feschbach pontig

$a < 0$

$\mu > 0$

$$n_\uparrow = \frac{1}{2}\int \frac{d^3 l}{(2\pi)^3}\left(1 - \frac{\frac{\hbar^2 l^2}{2m} - \mu}{\sqrt{\left(\frac{\hbar^2 l^2}{2m} - \mu\right)^2 + \Delta^2}}\right)$$

$$\frac{\Delta}{\mu} := t$$

$$\frac{4\pi}{16\pi^3}\int_0^\infty \ell^2 d\ell \left(1 - \frac{\frac{\hbar^2\ell^2}{2m\mu} - 1}{\sqrt{\left(\frac{\hbar^2\ell^2}{2m\mu} - 1\right)^2 + t^2}}\right) =$$

$$x^2 = \frac{\hbar^2\ell^2}{2m\mu} \rightsquigarrow \ell = \sqrt{\frac{2m\mu}{\hbar^2}}\, x$$

$$d\ell = \sqrt{\frac{2m\mu}{\hbar^2}}\, dx$$

$$= \frac{1}{4\pi^2}\left(\frac{2m\mu}{\hbar^2}\right)^{3/2} \underbrace{\int_0^\infty x^2 dx \left(1 - \frac{x^2 - 1}{\sqrt{(x^2-1)^2 + t^2}}\right)}_{I_1(t)}$$

$I_1(t) \rightsquigarrow$ csak t-től függ az elvégzett integrál

$$n_\uparrow = \frac{1}{4\pi^2}\left(\frac{2m\mu}{\hbar^2}\right)^{3/2} \cdot I_1(t)$$

$$a < 0$$

$$\downarrow$$

$$\frac{m}{4\pi\hbar^2|a|} = \frac{1}{2}\int \frac{d^3\ell}{(2\pi)^3}\left(-\frac{1}{\frac{\hbar^2\ell^2}{2m}} + \frac{1}{\sqrt{\left(\frac{\hbar^2\ell^2}{2m} - \mu\right)^2 - \Delta^2}}\right) =$$

$$= \frac{1}{2\mu}\,\frac{4\pi}{8\pi^3}\left(\frac{2m\mu}{\hbar^2}\right)^{3/2} \underbrace{\int_0^\infty x^2 dx \left(\frac{1}{\sqrt{(x^2-1)^2 - t^2}} - \frac{1}{x^2}\right)}_{I_2(t)}$$

$$\frac{\cancel{4\pi}}{4\cancel{\pi}\, t_1^2\, |a|} = \frac{1}{\cancel{2\mu}} \left(\frac{\cancel{2}\, \cancel{\mu}}{\cancel{t}^2}\right)\left(\frac{2m\mu}{t_1^2}\right)^{1/2} \cdot \frac{1}{2\pi\chi} \cdot I_1(t)$$

$$1 = \frac{2}{\pi} |a| \sqrt{\frac{2m\mu}{t_1^2}}\, I_2(t)$$

- $I_1(t), I_2(t)$ visszavezethetők elliptikus integrálokra

némi #magic után.

↳ ált. $\frac{1}{\sqrt{\mathcal{O}(x^4)}}$ -el ezt meg lehet csinálni...

- Def: $K(\ell) = \int_0^{\pi/2} \frac{1}{\sqrt{1-\ell^2 \sin^2\alpha}}\, d\alpha$ "elsőfajú teljes elliptikus integrál"

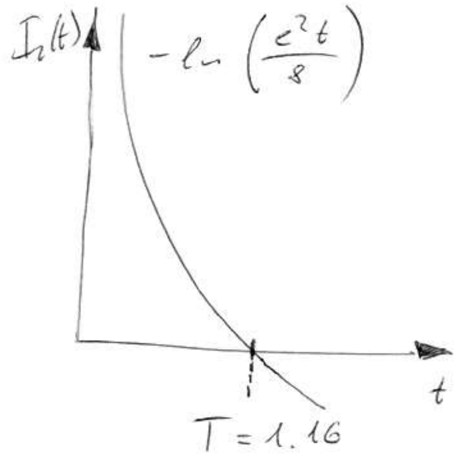
  ⇝ akkor teljes, ha a felső határ $\frac{\pi}{2}$

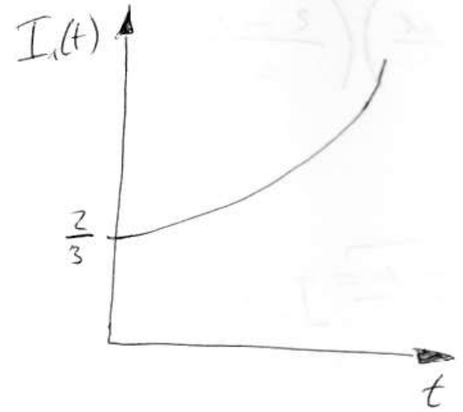
  ⇝ $\ell = 1$ -ben $K(\ell) \to \infty$!

  $E(\ell) = \int_0^{\pi/2} \sqrt{1-\ell^2\sin^2\alpha}\, d\alpha$ "másodfajú teljes ..."

$$I_2(t) = \sqrt[4]{1+t^2}\left[K\left(\sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{1+t^2}}\right)}\right) - 2E\left(\sqrt{\ldots}\right)\right]$$

$$I_1(t) = -\frac{1}{3} I_2(t) + \frac{1}{3}(1+t^2) \underbrace{\int_0^\infty \frac{dx}{\sqrt{(x^2-1)^2+t^2}}}_{\frac{1}{\sqrt[4]{1+t^2}} K\left(\sqrt{\frac{1}{2}\left(1+\frac{1}{\sqrt{1+t^2}}\right)}\right)}$$





$k_F = (2\pi^2 n)^{1/3}$ ⇝ nem-kölcsönható Fermi-gázban

$$\mu_F = \frac{\hbar^2 k_F^2}{2m}$$

$$\mu = \tilde{\mu} \cdot \mu_F$$

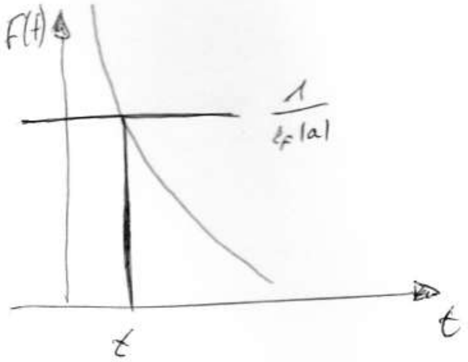
dimenziótlan szám, nem kh. rsz.-re $\hat{\mu} = 1$

$$n_\uparrow = \frac{1}{6}\pi^2 k_F^3 = \frac{1}{4\pi^2}\underbrace{\left(\frac{2m\mu}{\hbar^2}\right)^{3/2}}_{\left(\hat{\mu} k_F^2\right)^{3/2}} I_1(t)$$

$\hat{\mu} = \left(\frac{2}{3}\,\frac{1}{I_1(t)}\right)^{2/3}$ ⇝ $I_1(t)$ meghatározza az aktuális kémiai potenciált.

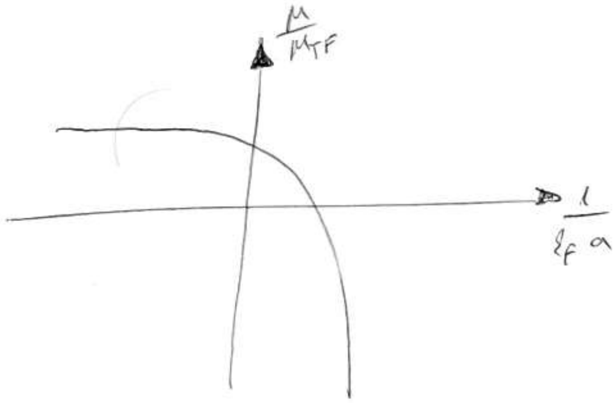
$$1 = \frac{2}{\pi}|a|\left(\hat{\mu} k_F^2\right)^{1/2} I_2(t)$$

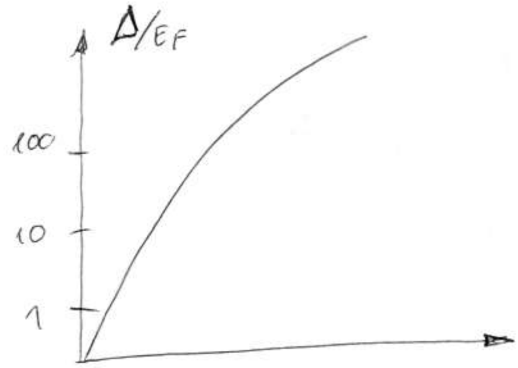
$$\frac{1}{k_F|a|} = \frac{2}{\pi}\sqrt{\hat{\mu}(t)}\, I_2(t) = \frac{2}{\pi}\left(\frac{2}{3}\,\frac{1}{I_1(t)}\right)^{1/3} I_2(t) =: F(t)$$



↪ megadja $\hat{\mu}$-t

↪ $\Delta = t\mu = t\hat{\mu}\mu_F$ megadja a gap-et.





• Gyenge BCS

$$a \to 0^- \iff \frac{1}{k_F|a|} \to -\infty$$

$$\frac{1}{k_F|a|} = \underbrace{\frac{2}{\pi}(-)\ln\left(\frac{e^2 t}{8}\right)}_{\approx I_2(t)} \cdot \underbrace{1}_{\approx\left(\frac{2}{3}\frac{1}{I_1(t)}\right)^{1/3}}$$

$$t \simeq \frac{8}{e^2} e^{-\frac{\pi}{2}\frac{1}{k_F|a|}}$$

$\frac{1}{k_F|a|} \to -\infty$

$\hat{\mu} \to 1 \rightsquigarrow \mu \simeq \mu_F$

$$\Delta = t\mu \approx \underline{\frac{8}{e^2}\mu_F e^{-\frac{\pi}{2}\frac{1}{k_F|a|}}}$$

⇝ exp. kicsi gap mint normál BCS-ben.

- Feshbach - rezonancia

$$\frac{1}{k_F|a|} \longrightarrow 0$$

$$\frac{1}{k_F|a|} = \frac{2}{\pi} I_2(t) \left(\frac{2}{3}\frac{1}{I_1(t)}\right)^{1/3}$$

$$0 = \;—\quad a \quad—$$

$\downarrow$

csak úgy elégíthető ki, ha

$$I_2(t) = 0$$

$\Downarrow$

$$I_2(T) = 0 \qquad T \approx 1.16$$

$$\hat{\mu} = \left(\frac{2}{3}\frac{1}{I_1(T)}\right)^{2/3} \lesssim 1 \quad \text{a Feshbach-ponton}$$

$$\mu = \hat{\mu}(T)\cdot\mu_F$$

$$\Delta = T\cdot\mu(T) = T\cdot\hat{\mu}(T)\cdot\mu_F$$

$$\underline{\underline{\frac{\Delta}{\mu_F} = T\,\hat{\mu}(T)}} \quad \text{univerzális szám}$$

- MF modell kvalitatíve helyesen írja le, de számokat nem jósol jól.

$\rightsquigarrow$ pl. monte-carlo (vagy kísérlet) mást ad...