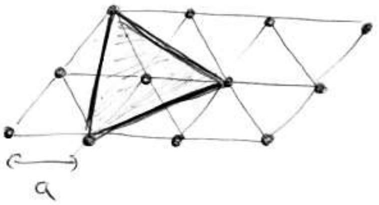
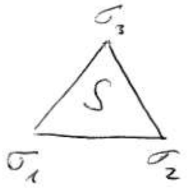
Ising-model $d=2$ △-rácson (2019. 12. 11.)

új rács: △ rács $\sqrt{3}a$ rácsállandóval





$S=1$ ↑↑↑ | ↑↑↓ | ↑↓↑ | ↓↑↑

$S=-1$ ↓↓↓ | ↓↓↑ | ↓↑↓ | ↑↓↓

$$\sigma_i = \pm 1 \qquad \mathcal{H} = -K\sum_{\langle ij\rangle}\sigma_i\sigma_j - h\sum_i \sigma_i$$

$$K = \frac{J}{k_B T}, \quad h = \frac{H}{k_B T}$$

blokkok indexe: I, J

$$e^{-\mathcal{H}'(\tilde{s})} = \sum_{\{\tilde{\sigma}|\tilde{s}\}} e^{-\mathcal{H}(\tilde{\sigma})}$$

$$\mathcal{H} = \underbrace{-K\sum_I \Big(\sum_{\substack{\langle ij\rangle \\ i,j\in I}}\sigma_i\sigma_j\Big)}_{\mathcal{H}_0} \underbrace{- K\sum_{\langle IJ\rangle}\Big(\sum_{\substack{\langle ij\rangle \\ i\in I \\ j\in J}}\sigma_i\sigma_j\Big) - h\sum_I\Big(\sum_{i\in I}\sigma_i\Big)}_{V} = \mathcal{H}_0 + V$$

$$e^{-\mathcal{H}} = e^{-\mathcal{H}_0}e^{-V}$$

$$e^{-\mathcal{H}'} = \sum_{\{\tilde{\sigma},\tilde{s}\}} e^{-\mathcal{H}_0}e^{-V} \qquad Z_0 = \sum_{\{\tilde{\sigma}|\tilde{s}\}} e^{-\mathcal{H}_0}$$

$$\langle \ldots \rangle_0 := \frac{1}{Z_0}\sum_{\{\tilde{\sigma}|\tilde{s}\}} \ldots e^{-\mathcal{H}_0} \longrightarrow e^{-\mathcal{H}'} = Z_0 \cdot \langle e^{-V}\rangle_0$$

$$Z_0 = \sum_{\{\sigma_i s\}} e^{-\mathcal{H}_0} = \prod_I \sum_{\{\sigma_1\sigma_2\sigma_3 | s_I\}} e^{K\langle\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\rangle} = \circledast$$

$\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$

ha $s_I = +1$

$\uparrow\uparrow\uparrow$	3	e^{3K}
$\uparrow\uparrow\downarrow$	-1	e^{-K}
$\uparrow\downarrow\uparrow$	-1	e^{-K}
$\downarrow\uparrow\uparrow$	-1	e^{-K}

$\Big\} \; e^{3K} + 3e^{-K}$

ha $s_I = -1$ ⇝ ugyan ez...

$\{s_I\}$ = a blokkspin értéke végértéke +1 v. −1

$$\circledast = \prod_I \left(e^{3K} + 3e^{-K}\right) = \left(e^{3K} + 3e^{-K}\right)^{N/3}$$ ⇝ termodinamikai határeset.

$$\langle e^{-V}\rangle_0 \approx e^{-\langle V\rangle_0}$$

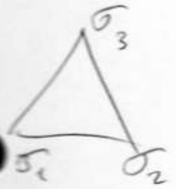
x: valósz. változó

$$\underbrace{\langle e^{x}\rangle = e^{\langle x\rangle + \frac{1}{2}\left[\langle x^2\rangle - \langle x\rangle^2\right] + \dots}}$$

akkor igaz ha a várható érték konst.

$$-\langle V\rangle_0 = K \sum_{\langle IJ\rangle} \Big(\sum_{\substack{(ij)\\ i\in I\\ j\in J}} \langle\sigma_i\sigma_j\rangle_0 \Big) + h \sum_I \Big(\sum_{i\in I} \langle\sigma_i\rangle_0 \Big)$$

I. blokkban:



$$\langle\sigma_1\rangle_0 = \frac{\sum_{\sigma_1\sigma_2\sigma_3} e^{K(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}\,\sigma_1}{\sum_{\substack{\sigma_1\sigma_2\sigma_3\\ (s_I = 1)}} e^{K(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}}$$

$$\sigma_1 e^{-K(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

$$\left.\begin{array}{ll} \uparrow\uparrow\uparrow & e^{3K}\cdot 1 \\ \uparrow\uparrow\downarrow & e^{-K}\cdot 1 \\ \uparrow\downarrow\uparrow & e^{-K}\cdot 1 \\ \downarrow\uparrow\uparrow & e^{-K}\cdot 1 \end{array}\right\} e^{3K} + e^{-K}$$

$$\langle \sigma_1 \rangle_0 = \frac{e^{3K} + e^{-K}}{e^{3K} + 3e^{-K}} \cdot S_I$$

ha $S_I = -1$ minden szorzódik (-1)-el

$$\langle \sigma_1 \rangle_0 = \langle \sigma_2 \rangle_0 = \langle \sigma_3 \rangle_0$$

$$\langle \sigma_i \sigma_j \rangle_0 = \langle \sigma_i \rangle_0 \langle \sigma_j \rangle_0 = g(K) S_I \, g(K) S_J$$

$i \in I$
$j \in J$

a $\langle\rangle_0$ átlagolásra a blokkok függetlenek

$$-\langle V \rangle_0 = K \sum_{IJ}' 2 g^2(K) S_I S_J + h \sum_I 3 g(K) S_I$$

$$e^{-\mathcal{H}'} = \exp\left\{ -\frac{N}{3} \ln\left(e^{3K} + 3e^{-K}\right) + K' \sum_{i,J}' S_i S_J + h' \sum_I S_I \right\}$$

$$\left.\begin{array}{l} K' = 2 g^2(K) K \\ h' = 3 g(K) h \end{array}\right\}$$

magasabb kumulánsok:

$\langle V^2 \rangle_0 - \langle V \rangle_0^2 \rightsquigarrow$ ebben megjelenik a spin kh.

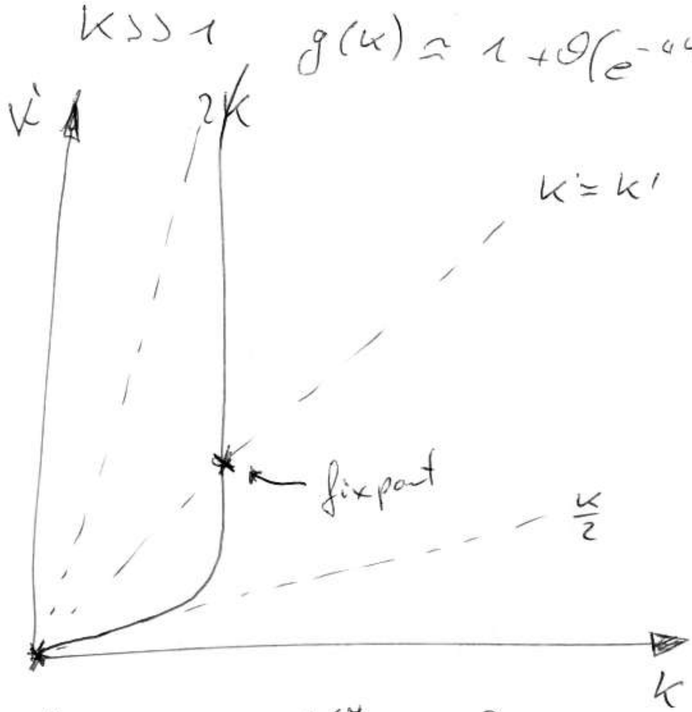
→ rekurziós összefüggés vizsgálata

$$g(K) = \frac{e^{3K} + e^{-K}}{e^{3K} + 3e^{-K}} = \frac{1 + e^{-4K}}{1 + 3e^{-4K}} \qquad K \ll 1 \quad g(K) \simeq \frac{1}{2} + \mathcal{O}(K)$$

$$K \gg 1 \quad g(K) \simeq 1 + \mathcal{O}(e^{-4K})$$

$$2Kg^2(K) = \xrightarrow{K \ll 1} \frac{K}{2}$$

$$\xrightarrow{K \gg 1} 2K$$



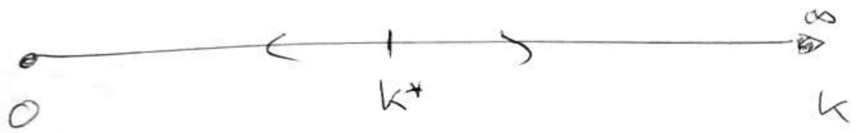
fixpontok: $\left.\begin{matrix} K^* = 0 \\ K = 0 \end{matrix}\right\}$

$$K^* = 2K^* g^2(K^*)$$

$$g(K') = \sqrt{\frac{1}{2}}$$

$$K^* = \frac{1}{4} \ln \frac{3 - \sqrt{2}}{\sqrt{2} - 1} = 0.336$$

→ transzformáció



→ linearizált rekurzió:

$$K' = K + \delta K \, 2Kg^2(K) = 2Kg^2(K^*) + \frac{d}{dK}\left(2Kg^2(K)\right)_* \overbrace{(K - K^*)}^{\delta K}$$

$$2g^2(K) + 4K\frac{dg}{dl} + g(K)\Big|_{K^*} = \underbrace{2g^2(K)}_{1/2} + \frac{8e^{4K}}{(e^{4K}+3)^2}\Bigg|_{K^*} = 1.624$$

90

$\delta K' = 1{,}624\, \delta K$

$\delta K' = b^{y_1} \delta K \qquad b = \sqrt{3} \qquad (\sqrt{3})^{y_1} = 1{,}624$

$$y_1 = \frac{\ln 1.624}{\ln \sqrt{3}} = 0.883$$

$\rightsquigarrow$ a másik rel. összefüggés

$h' = 3g(K)h$

$3g(K) > 3/2 > 1$

$h^* = 0$

linearizálás:

$h' = 3\, g(K^*)\, h = \frac{3}{\sqrt{2}} h = b^{y_H} h$

$b = \sqrt{3} \qquad y_H = \frac{\ln 3/\sqrt{2}}{\ln \sqrt{3}} = 1.369$

$\frac{1}{\nu} = y_1 \qquad \frac{1}{\mu} = y_H \qquad K^* = \frac{J}{k_B T_c}$

	számolás	egzakt (Δ rács)	MF
K^*	0.336	0.274	$\frac{1}{z} = \frac{1}{6} = 0.166$
$\frac{1}{\nu}$	0.883	1	2
$\Delta = \frac{\nu}{\mu}$	1.55	$\frac{15}{8} = 1.875$	$3/2$

a fluktuációk figyelembevétele csökkent: a krit. hőmérsékletet

Variációs RG-transzformáció

↳ melyik a legjobb módszer a blokk választásra.

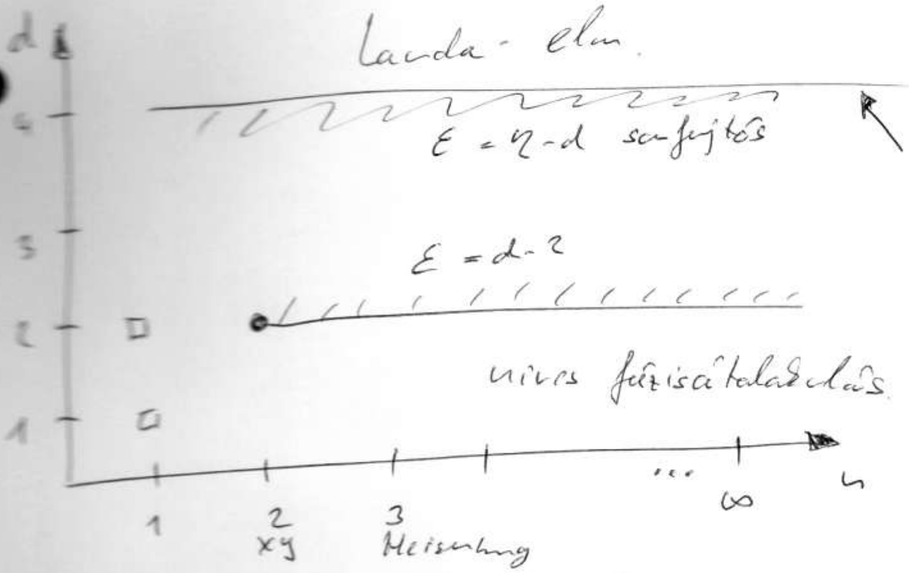
$d=2$ Ising: $d\nu = 1{,}988$

$\delta = 15{,}04$

Migdal transzformáció

M.C. ren. csoport tr. $d=2$ Ising $y_t = 1.00$

$y_h = 1.87$



$d=4$

L + ln korrekciók

↳ $\chi \sim \frac{1}{t} |\ln t|^{\frac{n+2}{n+8}}$

□ : = $d=1,2$ Ising modell

● : Kosterlitz-Thouless-tr.