$$N = \int\limits_{\mu > V(r)} d^3r \, \frac{1}{3\pi^2} \left(\frac{2m\left(\mu - \frac{1}{2}\omega_1 x^2 - \frac{1}{2}\omega_2 y^2 - \frac{1}{2}\omega_3 z^2\right)}{\hbar^2} \right)^{3/2} =$$

new length:

$$a = \sqrt{\frac{2\mu}{m\omega_1^2}} \qquad b = \sqrt{\frac{2\mu}{m\omega_2^2}} \qquad c = \sqrt{\frac{2\mu}{m\omega_3^2}}$$

$$= \int\limits_{\mu > V(r)} d^3r \, \frac{1}{3\pi^2} \left(\frac{2m\mu_{TF}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}\right)}{\hbar^2} \right)^{3/2} =$$

$$x' = \frac{x}{a} \quad dx = a\,dx' \qquad y' = \frac{y}{b} \quad dy = b\,dy' \qquad z' = \frac{z}{c} \quad dz = c\,dz'$$

$$= \frac{1}{3\pi^2} \left(\frac{2\cancel{m}\mu_{TF}}{\hbar^2} \right)^{3/2} \left(\frac{2\mu}{\cancel{m}\omega_1^2} \right)^{1/2} \left(\frac{2\mu}{\cancel{m}\omega_2^2} \right)^{1/2} \left(\frac{2\mu}{\cancel{m}\omega_3^2} \right)^{1/2} \cdot$$

$$\cdot \int\limits_{1 > x'^2 + y'^2 + z'^2} dx'dy'dz' \left(1 - x'^2 - y'^2 - z'^2\right)^{3/2} =$$

integral is rotationally invariant:

$$\underbrace{4\pi \int_0^1 r'^2 dr' \left(1 - r'^2\right)^{3/2}}_{\frac{\pi}{32}}$$

$$= \frac{1}{3} \frac{\mu_{TF}^3}{\underbrace{\hbar^3 \omega_1 \omega_2 \omega_3}_{\hbar^3 \bar{\omega}^3}} = N$$

where $\bar{\omega} = (\omega_1 \omega_2 \omega_3)^{1/3}$

$$\boxed{\mu_{TF} = (3N)^{1/3} \cdot \hbar\bar{\omega}}$$

2019. 11. 21.

- $^6Li : (1s)^2(2s)$

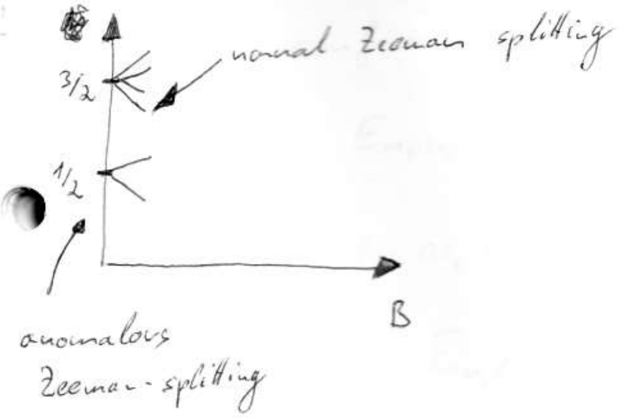
$\rightsquigarrow L = 0$
$\rightsquigarrow S = \frac{1}{2}$ } J

$\rightsquigarrow J = \frac{1}{2}$

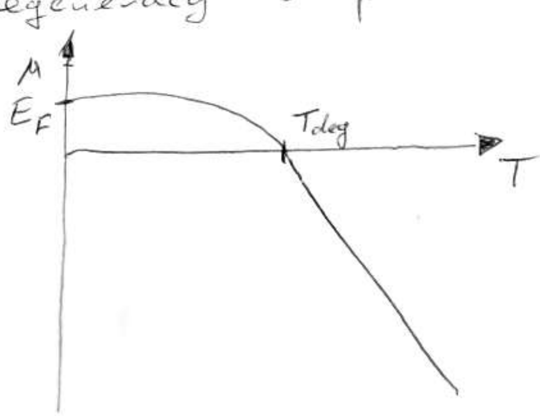
$\longrightarrow \vec{F} = \vec{L} + \vec{s} + \vec{i} \Longrightarrow F = \frac{1}{2}, \frac{3}{2}$ this is a fermionic atoms

nuclear ang. mom.
$i = 1$

$\hookrightarrow$ even the grnd. state is split by the hyperfine interaction.
$a \cdot \vec{L} \cdot \vec{J}$



- cooling Fermions ⇝ BCS - transition
- $N \approx 10^5$, $T \sim 100\,nK$
- degeneracy - temperature ⇝ $\mu = 0$



- deviation from Boltzman - dist.
- can see the effects of Fermi-dist.

- critical temperature ⇝ much lower for p-waves than it is for s-waves.
  (experimental problem for spin polarized sample ↑↑↑↑...)
  ↓
  evaporative cooling is also problematic...

- adiabatic heating (cooling)
  ⇝ strong confining potential

⇝ opening it up, atoms will cool down.

- boson - fermion mix. (sympathetic cooling)
  ⇝ bosons cool with s-wave
  ⇝ they can interact with fermions → they cool them down.
  ⇝ remove all the bosons in the end.

- two spin-components in the system help avoid the problems above.

  s-wave ✓ → higher crit. temp.

- k_F in non-int systems:

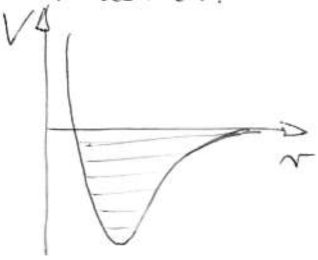
  $$n = \frac{1}{3\pi^2} k_F^3, \quad E_F = \frac{\hbar^2 k_F^2}{2m} \rightarrow \Delta \sim E_F \, e^{-\frac{\pi}{2}\frac{1}{k_F a}} \sim k_B T_C$$

  (density)

  T_c is low!!

- Flashbach-ressonance

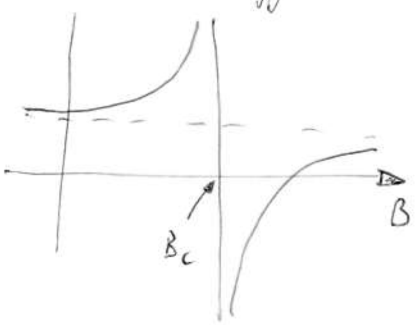
  → interaction:



→ the shape can be changed by external params.

→ there can be a bound-state, that goes to the asymptotic part..

↳ at that engy s-wave scattering length diverges



$$a(B) = a_0 \left(1 - \frac{\Delta}{B - B_c}\right)$$

→ scattering len. can be changed between atoms

→ when $a \rightarrow \pm\infty$ the T_c increases drastically.

- parabolic confinement, non-int. atoms

$$E_{n_x n_y n_z} = \sum_{i=1}^{3} \hbar\omega_i \left(n_i + \frac{1}{2}\right)$$

if $\omega_x = \omega_y = \omega_z$

$$E_{n,\ell,m} = \hbar\omega_0 \left(\frac{3}{2} + \underbrace{2n + \ell}_{}\right)$$

shell-quantum-number
⤳ lots of degeneracies.

⤳ we fill up every lvl. with N

⤳ Ψ is a single Slater-det.

⤳ exact density $\langle \Psi | n | \Psi \rangle$

⤳ $n(\vec{r}) = 2 \sum_i |\varphi_i(\vec{r})|^2$

⤳ the diffrence between the exact density and the local-density-approx is less than 1%

↓

it's worth to use for big N-s.

$$n(r) = \frac{1}{3\pi^2} \left(\frac{2m(\mu - V(r))}{\hbar^2}\right)^{3/2} \Theta(\mu - V(r)) \qquad \text{(fermions)}$$

$$n(r) = \frac{(\mu - V(r))}{g} \Theta(\mu - V(r)) \qquad \text{(TF profile for bosons)}$$

- Feshbach-ressonance can ~~help~~ change between the two
- (They have very diffrent limits...)