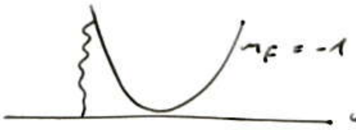
- using RF-ressonance can put atoms to differnt m_F

  ↳ most energetic ones can be removed



↳ "evaporation"

- modulus potential: spontaneous majorana flip: $m_F = -1 \to m_F = 0$
  → problem...

  → using HO potential
  → apply rotating pot. on the modulus

} solution

  ⇒ the effective pot will be H.O.

2019.03.04.

- $T = T_c$ (non-int., homogenious sys)

$$N = V \underbrace{\left(\frac{m k_B T_c}{2\pi \hbar^2}\right)^{3/2}}_{\lambda_{dB}^{-3}(T_c)} \cdot \zeta(3/2)$$

$\lambda_{dB}^{-3}(T_c)$ ~ thermal deBroigle wavelength

$\mathcal{O}(10)!$ ~ characteristic length

$$k_B T = \frac{2\pi}{m} \frac{\hbar^2}{\lambda_{dB}^2(T)}$$ ~ this defines it. (Kinetic energy)

$$\boxed{\zeta(3/2) = \underbrace{n \lambda_{dB}^3(T_c)}_{\text{phase-space density}}}$$ ~ critical combination

this is the good check for BEC

- ζ func. has 0-s at even ⊖ numbers /Mathematica/

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- explaining the deviation on the N_0/N - T_c/T plot:

$\boxed{T > T_c}$

$$N = \left(\frac{k_B T}{\hbar\bar{\omega}}\right)^3 F_-\left(3, \frac{\varepsilon_0 - \mu}{k_B T}\right) + \gamma_1 \left(\frac{k_B T}{\hbar\bar{\omega}}\right)^2 F_-\left(2, \frac{\varepsilon_0 - \mu}{k_B T}\right)$$

$T < T_c$

(new stuff)

- big degeneracy on grund state $\rightsquigarrow N_0$
- $\mu = \varepsilon_0 \rightsquigarrow$ 0-s in F_-!

$$N = N_0 + \left(\frac{k_B T}{\hbar\bar\omega}\right)^3 \zeta(3) + \gamma_1\left(\frac{k_B T}{\hbar\bar\omega}\right)^2 \zeta(2)$$

$$1 - \frac{N_0}{N} = \left(\frac{k_B T}{\hbar\bar\omega}\right)^3 \frac{\zeta(3)}{N} + \gamma_1\left(\frac{k_B T}{\hbar\bar\omega}\right)^2 \frac{\zeta(2)}{N}$$

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_0}\right)^3 \underbrace{\left(\frac{k_B T_0}{\hbar\bar\omega}\right)^3 \frac{\zeta(3)}{N}}_{1} - \gamma_1\left(\frac{T}{T_0}\right)^2 \underbrace{\left(\frac{k_B T_0}{\hbar\bar\omega}\right)^2}_{\left(\frac{N}{\zeta(3)}\right)^{2/3}} \frac{\zeta(2)}{N}$$

explains why it is <u>underneath</u>

$\sim N^{-1/3}$ behaviour
$\to$ finite size corr.
$\to$ gets smaller with $N \to \infty$

- - - - - - - - -

- depending on ω-s: $\underset{\text{disc type}}{\omega_z < \omega_r}$; $\underset{\text{cigar type condensate}}{\omega_z > \omega_r}$
- oscillator length: $d = \sqrt{\frac{\hbar}{m\omega}} \rightsquigarrow$ depends on direction
  $\rightsquigarrow$ usual condensate size
- What is the half-width of c., if the non-int. modell is correct?

• grund. state wf:

$$\varphi_0(\vec{r}_1)\varphi_0(\vec{r}_2)\ldots\varphi_0(\vec{r}_N) = \Psi(\vec{r}_1,\ldots\vec{r}_N)$$

$$\hat{n}(\vec{r}) = \sum_{i=1}^{N}\delta(\vec{r}-\vec{r}_i) \rightsquigarrow \text{density op. in 1st quantization.}$$

$\downarrow$

$$n(\vec{r}) = \langle\Psi|\hat{n}|\Psi\rangle \rightsquigarrow \text{density func.!}$$

$$\left(\int|\varphi_0(r)|^2 d^3r = 1\right)$$

$$n(r) = \sum_i \int d^3r_1 \int d^3r_2 \ldots \int d^3r_N\, \varphi_0^*(r_1)\ldots\varphi_0^*(r_N)\,\delta(r-r_i)\,\varphi_0(r_1)\ldots\varphi_0(r_N)$$

$$= \sum_{i=1}^{N}|\varphi_0(r)|^2 = \underline{\underline{N|\varphi_0(r)|^2 = n(r)}}$$

if we know the grund. state non-int. wf.

• non-int. modell for harmonically trapped atoms:

$$\hat{H} = \left(-\frac{\hbar^2}{2m}\Delta + V(r)\right)$$

$$V(r) = \frac{1}{2}m\left(\omega_1^2x_1^2+\omega_2^2x_2^2+\omega_3^2x_3^2\right)$$

$$\rightsquigarrow \varphi_0(r) = \left(\frac{m\bar{\omega}}{\pi\hbar}\right)^{3/2} e^{-\frac{1}{2}\left(\frac{x_1}{d_1}\right)^2-\frac{1}{2}\left(\frac{x_2}{d_2}\right)^2-\frac{1}{2}\left(\frac{x_3}{d_3}\right)^2}$$

and $d_i = \sqrt{\frac{\hbar}{m\omega_i}}$

• total density is same φ_0 and $n(r)$

$\downarrow$

half width is in the order of d $\rightsquigarrow$ <u>few microns</u>

• however the measured half width is in the order of 100 μm

$\Longrightarrow$ INTERACTION IS NON-NEGLIGABLE

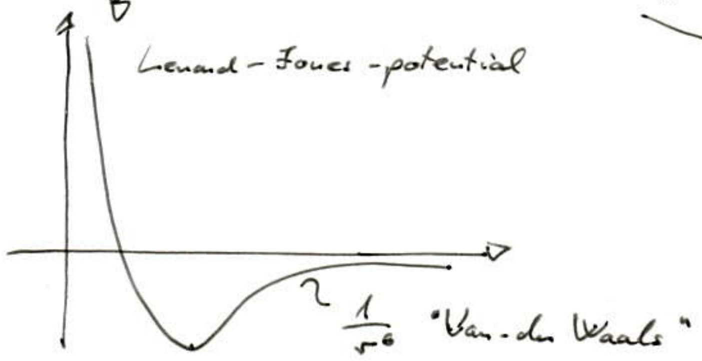
Groß-Pitajevski-eq.: Inclusion of the interaction

^{4}He ~ spin-less boson

- wf for rotating BEC

external, HO potential

$$\hat{H} = \sum_{i=1}^{N}\left(-\frac{\hbar^2}{2m}\Delta_i + V_{ext}(\vec{r}_i)\right) + \frac{1}{2}\sum_{i\neq j=1}^{N} v(\vec{r}_i - \vec{r}_j)$$



$v \to v(|\vec{r}\,|)$

- it acts weakly

- we can still use product ansatz for the whole wf.

$$\Psi(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N) = \varphi(\vec{r}_1)\cdot \ldots \cdot \varphi(\vec{r}_N)$$

$\rightsquigarrow$ constraint: $1 = \int |\varphi|^2 d^3r$ $\rightsquigarrow$ Ψ is normalized, too.

- Minimize:

$$\frac{\delta}{\delta\varphi^*(r)}\left(\langle\Psi|H|\Psi\rangle - \varepsilon\int d^3r\, |\varphi(r)|^2\right) \overset{!}{=} 0$$

Lagrange-multiplier...

(really similar to Hartree-Fock, but we only have φ, not a Slater...)

$$\langle\Psi|\hat{H}|\Psi\rangle = \sum_{i=1}^{N}\int d^3r_1 \ldots d^3r_N\, \varphi^*(r_1)\ldots\varphi^*(r_N)\left(-\frac{\hbar^2}{2m}\Delta_i + V(r_i)\right)\varphi(r_1)\ldots\varphi(r_N)$$

$$+ \frac{1}{2}\sum_{i\neq j}\int d^3r_1 \ldots d^3r_N\, \varphi^*(r_1)\ldots\varphi^*(r_N)\, v(\vec{r}_i - \vec{r}_j)\,\varphi(r_1)\ldots\varphi(r_N)$$

$$= \sum_{i=1}^{N} \int d^3r_i\, \varphi^*(\vec{r}_i)\left(-\frac{\hbar^2}{2m}\Delta_i + V(\vec{r}_i)\right)\varphi(\vec{r}_i) + \frac{1}{2}\sum_{\substack{i,j=1\\ i\neq j}}^{N} \int d^3r_i\, d^3r_j\, \varphi^*(\vec{r}_i)\varphi^*(\vec{r}_j)\cdot$$

$$\cdot V(\vec{r}_i - \vec{r}_j)\,\varphi(\vec{r}_i)\,\varphi(\vec{r}_j) =$$

$r_i = r'$
$r_j = r''$ } change variable → summations are doable...

$$= N\cdot \int d^3r'\, \varphi^*(r')\left(-\frac{\hbar}{2m}\Delta' + V(r')\right)\varphi(r') + \frac{N(N-1)}{2}\int d^3r'\, d^3r''\, \varphi^*(r')\varphi^*(r'')\cdot$$

$$\cdot V(r'-r'')\,\varphi(r')\,\varphi(r'')$$

$$\frac{\delta}{\delta\varphi^*(r)}\left(\langle\Psi|\hat{H}|\Psi\rangle - E\int d^3r'\, \varphi^*(r')\varphi(r')\right) =$$

$$\left[\frac{\delta\varphi(r')}{\delta\varphi^*(r)} = 0\ ;\ \frac{\delta\varphi^*(r')}{\delta\varphi^*(r)} = \delta(r-r')\right]$$

↳ can be used for integration.

$$= N\int d^3r'\, \delta(r-r')\left(-\frac{\hbar^2}{2m}\Delta' + V(r')\right)\varphi(r') +$$

$$+ \frac{N(N-1)}{2}\int d^3r'\, d^3r''\left(\delta(r-r')\varphi^*(r'') + \delta(r-r'')\varphi^*(r')\right)V(r'-r'')\,\varphi(r')\varphi(r'')$$

$$- E\int d^3r'\, \delta(r-r')\,\varphi(r') =$$

symmetric... $(V(|r|))$

$$= N\left(-\frac{\hbar^2}{2m}\Delta + V(r)\right)\varphi(r) + N(N-1)\int d^3r'\, \varphi^*(r')\, V(r'-r)\,\varphi(r')\varphi(r) -$$

$$- E\varphi(r)$$

$$0 = N\left(-\frac{\hbar^2}{2m}\Delta + V(r)\right)\varphi(r) + N(N-1)\left(\int d^3r'\, \varphi^*(r')\, v(r'-r)\, \varphi(r')\right)\varphi(r) - E\,\varphi(r)$$

⇝ non-linear-eq.

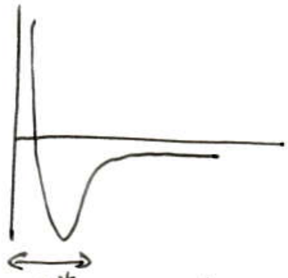
$\Psi_0(r) = N^{1/2}\varphi(r)$ ⇝ condensate-wf.

$\int |\Psi_0(r)|^2 d^3r = N$ ⇝ $n_c(r) = |\Psi_0(r)|^2$ condensate-density

$\mu = \frac{E}{N}$ ⇝ chemical potential

$$\mu\,\Psi_0(r) = \left(-\frac{\hbar^2}{2m}\Delta + V(r)\right)\Psi_0(r) + \underbrace{\frac{N-1}{N}}_{\substack{\to 1 \\ \text{if } N\to\infty}}\varphi(r)\int d^3r'\, v(r-r')\,|\varphi(r')|^2$$

⇝ non-local form of Gross-Pitaevski-equation



R^* ⇝ char. length of pot ⇝ if the $\overset{\text{dist. of atoms}}{R} \gg R^*$

and $T \to \infty$ and big density

$v(r-r')$ can be replaced with: $\frac{4\pi\hbar a}{m}\delta(r-r')$

where a is the s-wave scattering length.

Then:

$$\boxed{\mu\,\Psi_0(r) = \left(-\frac{\hbar^2}{2m}\Delta + V(r)\right) + \frac{4\pi\hbar^2 a}{m}|\Psi_0(r)|^2\Big)\Psi_0(r)}$$