

# Problem Solving

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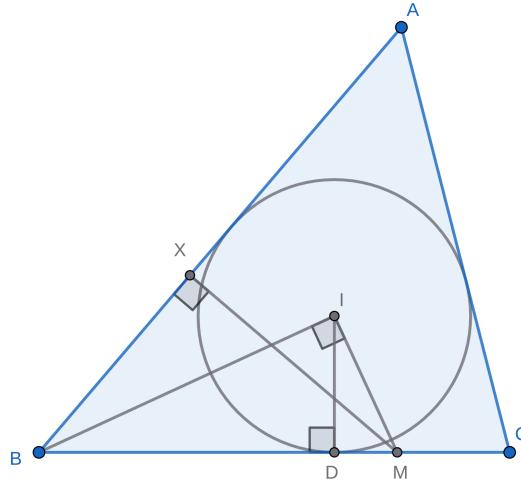
This class by Shreyash Sharma was mainly focused on problem solving.

## §1 Problems

### Problem statement

In the triangle  $ABC$ ,  $I$  is the center of the inscribed circle, point  $M$  lies on the side of  $BC$ , with  $\angle BIM = 90^\circ$ . Prove that the distance from point  $M$  to line  $AB$  is equal to the diameter of the circle inscribed in triangle  $ABC$

### ¶ Solution.



Let  $D$  be the point on the line  $\overline{BC}$  such that  $\overline{ID} \perp \overline{BC}$ . We wish to show that  $2\overline{ID} = \overline{XM}$ . Let  $\angle XBM = \beta$  then

$$\sin \beta = \frac{\overline{XM}}{\overline{BM}} \quad (1)$$

$$\implies \overline{BM} \sin \beta = \overline{XM} \quad (2)$$

Also, in  $\triangle BIM$

$$\cos \frac{\beta}{2} = \frac{\overline{IB}}{\overline{BM}}$$

$$\implies \overline{BM} = \frac{\overline{IB}}{\cos \beta/2}$$

Thus, (2) becomes

$$\overline{XM} = \frac{\overline{IB}}{\cos \beta/2} \sin \beta \quad (3)$$

Now, in  $\triangle IBD$ ,

$$\overline{IB} = \frac{\overline{ID}}{\sin \beta/2}$$

Thus, (3) becomes

$$\begin{aligned} \overline{XM} &= \overline{ID} \frac{\sin \beta}{\cos \beta/2 \sin \beta/2} \\ &\implies \overline{XM} = 2\overline{ID} \end{aligned}$$

as desired.