Prime Factorization(Class 12)

Basu Dev Karki

3 November 2025

Class 12 by Shreyash Sharma was mainly focused on number theory, specially on primes factorization.

§1 Theory

Proposition 1.1

Suppose $a, b \in \mathbb{N}$ such that $a = p_1^{a_1} \cdots p_n^{a_n}$ and $b = p_1^{b_1} \cdots p_n^{b_n}$ where p_i's are primes with a_i and b_i being non-negative integers then

$$\gcd(a, b) = p_1^{\min\{a_1, b_1\}} \cdots p_n^{\min\{a_n, b_n\}}$$

and

$$\operatorname{lcm}(a, b) = p_1^{\max\{a_1, b_1\}} \cdots p_n^{\max\{a_n, b_n\}}$$

Proof. Omitted. □

§2 Problems

Problem statement

Find positive integers n are there such that n is a multiple of 5 and

$$\operatorname{lcm}(5!, n) = 5 \gcd(10!, n)$$

¶ **Solution.** Let us note that $5! = 2^3 \times 3 \times 5$ and $10! = 2^8 \times 3^4 \times 5^2 \times 7$. Notice that if a prime $p > 7$ divides n then p divides the left side but doesn't divide the right side. Thus, our n takes the form of $2^a \times 3^b \times 5^c \times 7^d$. Now, if we apply our theory we get

$$\max\{3, a\} = \min\{8, a\}$$

If $a = 0, 1, 2$ then obliviously it doesn't work. If $a = 3, 4, 5, 6, 7, 8$ then we can see that it works. And if $a > 8$ then it doesn't work. Thus, a can be any of $\{3, 4, 5, 6, 7, 8\}$. Now, if we apply the same theory for 3 we get similar results. For 5, its slightly different as the right side has a extra factor of 5. If we take that into account, we get the equation,

$$\max\{1, c\} = 1 + \min\{2, c\}$$

and we apply similar things to get the number of possible c. Similarly for 7 as well. Once we get all the possible values of a, b, c and d, we can count the how many n can be formed with those values, which comes down to be **96**.

Problem statement

Find all $a, b, c, d, e, f \in \mathbb{N}$ such that

$$abc = 70$$

$$cde = 71$$

$$efg = 72$$

¶ **Solution.** Notice that 71 is a prime which forces $c, e = 1$. Thus, it simplifies to solving $ab = 70$, $fg = 72$, which we can do it by hand.

§3 Exercises

Exercise 3.1. For $n \in \mathbb{N}$, $100n^3$ has 110 divisors, how many divisors does $81n^4$ have?