

# Combinatorics

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This class by Shreyash Sharma was focused on solving combinatorics related problems.

## §1 Problems

### Problem statement

Let  $n$  be a positive integer and consider an arrangement of  $2n$  blocks in a straight line, where  $n$  of them are red and the rest blue. A swap refers to choosing two consecutive blocks and then swapping their positions. Let  $A$  be the minimum number of swaps needed to make the first  $n$  blocks all red and  $B$  be the minimum number of swaps needed to make the first  $n$  blocks all blue. Show that  $A + B$  is independent of the starting arrangement and determine its value.

**¶ Solution.** Let  $R$  be red and  $B$  be blue. Notice that if we have a arrangement and we're trying to swap  $R$  with minimum moves possible then the best way to do it would be to slide the nearest  $R$  as left as directly as possible. This is way to achieve minimum as if we try to do it for any other  $R$ , we would need to swap  $R$  a extra time with another  $R$ .

Now, let  $r_1, \dots, r_n$  the positions of reds such that  $r_1 < \dots < r_n$  then the minimum moves to swap to makes the first  $n$  spot all reds would be

$$A = \sum_{i=1}^n (r_i - i)$$

as to move  $r_i$  to  $i$  we need  $r_i - i$  moves. Similarly let  $b_1, \dots, b_n$  the positions of blue such that  $b_1 < \dots < b_n$  then the minimum moves to swap to makes the first  $n$  spot all blues would be

$$B = \sum_{i=1}^n (b_i - i)$$

Thus,  $A + B = 1 + 2 + \dots + 2n - 2(1 + 2 + \dots + n) = n^2$  as  $r_i$  and  $b_i$  are just permutations of  $1, \dots, 2n$ .