

Problem Solving

BASU DEV KARKI

13 November 2025

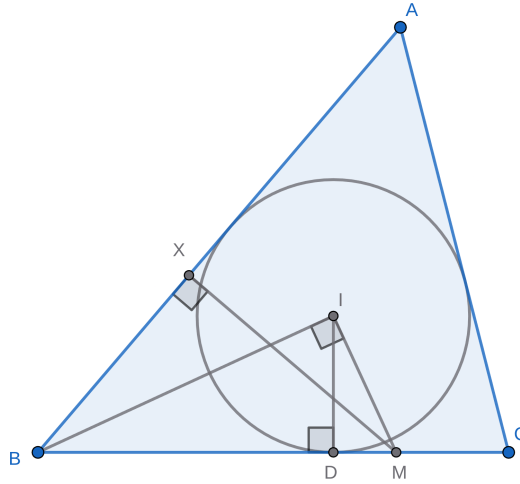
This class by Shreyash Sharma was mainly focused on problem solving.

§1 Problems

Problem statement

In the triangle ABC , I is the center of the inscribed circle, point M lies on the side of BC , with $\angle BIM = 90^\circ$. Prove that the distance from point M to line AB is equal to the diameter of the circle inscribed in triangle ABC

¶ Solution.



Let D be the point on the line \overline{BC} such that $\overline{ID} \perp \overline{BC}$. We wish to show that $2\overline{ID} = \overline{XM}$. Let $\angle XBM = \beta$ then

$$\sin \beta = \frac{\overline{XM}}{\overline{BM}} \quad (1)$$

$$\implies \overline{BM} \sin \beta = \overline{XM} \quad (2)$$

Also, in $\triangle BIM$

$$\cos \frac{\beta}{2} = \frac{\overline{IB}}{\overline{BM}}$$

$$\implies \overline{BM} = \frac{\overline{IB}}{\cos \beta/2}$$

Thus, (2) becomes

$$\overline{XM} = \frac{\overline{IB}}{\cos \beta/2} \sin \beta \tag{3}$$

Now, in $\triangle IBD$,

$$\overline{IB} = \frac{\overline{ID}}{\sin \beta/2}$$

Thus, (3) becomes

$$\begin{aligned} \overline{XM} &= \overline{ID} \frac{\sin \beta}{\cos \beta/2 \sin \beta/2} \\ \implies \overline{XM} &= 2\overline{ID} \end{aligned}$$

as desired.