

Combinatorics

BASU DEV KARKI

26 November 2025

This class by Shreyash Sharma was focused on solving combinatorics related problems.

§1 Problems

Problem statement

Let n be a positive integer and consider an arrangement of $2n$ blocks in a straight line, where n of them are red and the rest blue. A swap refers to choosing two consecutive blocks and then swapping their positions. Let A be the minimum number of swaps needed to make the first n blocks all red and B be the minimum number of swaps needed to make the first n blocks all blue. Show that $A + B$ is independent of the starting arrangement and determine its value.

¶ Solution. Let R be red and B be blue. Notice that if we have a arrangement and we're trying to swap R with minimum moves possible then the best way to do it would be to slide the nearest R as left as directly as possible. This is way to achieve minimum as if we try to do it for any other R , we would need to swap R a extra time with another R .

Now, let r_1, \dots, r_n the positions of reds such that $r_1 < \dots < r_n$ then the minimum moves to swap to makes the first n spot all reds would be

$$A = \sum_{i=1}^n (r_i - i)$$

as to move r_i to i we need $r_i - i$ moves. Similarly let b_1, \dots, b_n the positions of blue such that $b_1 < \dots < b_n$ then the minimum moves to swap to makes the first n spot all blues would be

$$B = \sum_{i=1}^n (b_i - i)$$

Thus, $A + B = 1 + 2 + \dots + 2n - 2(1 + 2 + \dots + n) = n^2$ as r_i and b_i are just permutations of $1, \dots, 2n$.