# Problem Solving

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This class by Shreyash Sharma was focused on problem solving, especially combinatorics problems.

## §1 Problems

> **Problem statement**
>
> The numbers $1, 2, 3, \ldots, 2017$ are on the blackboard. Amelie and Boris take turns removing one of those until only two numbers remain on the board. Amelie starts. If the sum of the last two numbers is divisible by 8, then Amelie wins. Else Boris wins. Who can force a victory?

**¶ Solution.**  Amelie can force a victory. Let us write $1, 2, \ldots, 2017$ as $\pmod 8$, thus we get

$$1, 2, 3, 4, 5, 6, 7, 0, 1, 2, 3, \ldots, 1, 2, 3, 4, 5, 6, 7, 0, 1$$

Amelia starts by picking 2017 and whenever boris chooses a number $z \pmod 8$, amelia removes a number $8 - z \pmod 8$. Since, $\frac{2016}{8} = 252$ is even we'll always have pairs that sums to a multiple of 8 and thus the last two renaming number will always sum to a multiple of 8.

> **Problem statement**
>
> One day Arun and Disha played several games of table tennis. At five points during the day, Arun calculated the percentage of the games played so far that he had won. The results of these calculations were exactly 30%, 40%, 50%, 60% and 70% in some order. What is the smallest possible number of games they played?

**¶ Solution.**  The answer is 30. First, notice $\frac{W_1}{G_1} = \frac{3}{10}$ and $\frac{W_2}{G_2} = \frac{7}{10}$ forces the number of games to be atleast 20 as $10 \mid G_1$ and $10 \mid G_2$. This means that 30% and 70% can only be achieved after playing 10-th game or 20-th game. Suppose the percentages can be achieved with $< 30$ games and suppose 30% is achieved after 10-th game and 70% is achieved after 20-th game. Then that means a total of $14 - 3 = 11$ games were won by arun as the only way to achive 30% after 10-th game is by wining 3 games and only way to achive 70% after 20-th game is by wining 14. But we just won 11 games by only

playing 10 thats impossible. Similar arugment can be repeated to for achieving 30% after 20-th game and 70% after 10-th game. Thus, the total number of games is atleast 30. To prove that 30 works, we need to provide a configuration. A working configuration is provided below,

1. First, arun wins one of the first two matches. (1/2)

2. Then arun wins of the one of the next three matches. (2/5)

3. Then arun wins of the one of the next five matches. (3/10)

4. Then arun wins of the nine of the next ten matches. (12/20)

5. Then arun wins of the nine of the next ten matches again. (21/30)