Modular Arithemetic(Class 5)

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The class by shreyash sharma was focused on modular arithmetic which was the continuation of the previous session on modular arithmetic.

§1 Theory

Proposition 1.1

Let $s(n)$ be the sum of digits of number n. Then

$$s(n) \equiv n \pmod 9$$

Proof. Write $n = 10^n a_n + \cdots a_0$ where a_i are the digits of n, then take $\pmod 9$. □

Proposition 1.2

The following statement are true:

1. $a^2 \equiv 2 \pmod 2$
2. $(odd)^2 \equiv 1 \pmod 8$
3. $a^2 \equiv \{0, 1\} \pmod 4$
4. $a^2 \equiv \{0, +1, -1\} \pmod 4$
5. $a^3 \equiv \{0, +1, -1\} \pmod 7$
6. $a^3 \equiv \{0, +1, -1\} \pmod 9$

Proof. Omitted. □

Proposition 1.3

Let p be a prime such that $\gcd(a, p) = 1$ then

$$a^{p-1} \equiv 1 \pmod p$$

Proof. Omitted. □

§2 Probelms

Problem statement

Find all n such that $\frac{5^n-1}{3}$ is a prime or a perfect sqaure.

Claim — No such n exists such that the $\frac{5^n-1}{3}$ prime.

Proof. Say there is a n

$$5^n - 1 \equiv 3p \pmod 4$$
$$p \equiv 0 \pmod 4$$

□

Claim — $n = 0$ is the only one integer such that $\frac{5^n-1}{3} = x^2$

Proof. Assume $n \geqslant 1$ then

$$5^n - 1 \equiv 3x^2 \pmod 5$$
$$-1 \equiv 3x^2 \pmod 5$$

Which leads us to the conclusion that there is no sol for $n \geqslant 1$ as $x^2 \in \{0, 1, -1\} \pmod 5$. □

Problem statement

Find the remainder

1. 2^{72} is divided by 73.
2. 2^{331} is divided by 31

Use **Proposition 1.3**

Problem statement

Find $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60} \pmod 7$

Using **Proposition 1.3** we get

$$2^{20} \equiv 4 \pmod 7$$
$$3^{30} \equiv 1 \pmod 7$$
$$4^{40} \equiv 4 \pmod 7$$
$$5^{50} \equiv 4 \pmod 7$$
$$6^{60} \equiv (-1)^{60} \equiv 1 \pmod 7$$

Adding them up we get our desired answer.