

Real Analysis

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1 Real Numbers

1.1 Cuts

Theorem 1.1

No number r in \mathbb{Q} has a square equal to 2 i.e $\sqrt{2} \notin \mathbb{Q}$.

Proof. We can use the standard proof by letting $(p/q)^2 = 2$ with p and q not sharing a factor and deduce p, q are even. \square

Definition 1.1

A **cut** in \mathbb{Q} is a pair of subsets of A and B of \mathbb{Q} such that

1. $A \cup B = \mathbb{Q}$, $A \neq \emptyset$, $B \neq \emptyset$, $A \cap B = \emptyset$
2. If $a \in A$ and $b \in B$ then $a < b$
3. A contains no largest element

We denote the cut as $x = A|B$.

Definition 1.2

A **real number** is a cut in \mathbb{Q} .

Example 1.1