Real Analysis

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Abstract

I really don't feel like doing analysis the way I did group theory and linear algebra, where I type out my notes on a latex file. Instead, I'll type out problems and their solutions here. I'm reading **Principle of Mathematical Analysis** by *Walter Rudin*.

Content

1 Basic Topology

> **Problem 1.0**
>
> A complex number z is said to be algebraic if there exists integers $a_0, \ldots, a_n$ not all zero such that
> $$a_0 z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n = 0$$
> Prove that all the algebraic numbers are countable.

Solution

For a given positive integer N, we know that there are finitely many tuples $(n, a_0, \ldots, a_n)$ that satisfy,
$$n + |a_0| + |a_1| + \cdots + |a_n| = N$$

Let Q_N be the set of polynomials with degree n and coefficient $a_0, \ldots, a_n$ such that they satisfy the above condition, i.end

$$Q_N = \{P \in \mathbb{Z}[x] \mid \deg(P) = n \text{ and } n + |a_0| + |a_1| + \cdots + |a_n| = N\}$$

Then define

$$A_N = \{\#\text{all the algebraic solutions to the polynomials in } Q_N\}$$

Since Q_N is finite and each polynomial has at most n roots, A_N is also finite. Since A_N is finite we know its countable as well. Define

$$A = \bigcup_{i=1}^{\infty} A_i$$

Thus A is the set of all algebraic numbers. Since A is the union of countable sets, A is also countable.