Real Analysis

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Content

1 Real Numbers

1.1 Cuts

> **Theorem 1.1**
>
> No number r in $\mathbf{Q}$ has a square equal to 2 i.e $\sqrt{2} \notin \mathbf{Q}$.

Proof. We can use the standard proof by letting $(p/q)^2 = 2$ with p and q not sharing a factor and deduce p, q are even. $\qquad\square$

> **Definition 1.1**
>
> A **cut** in $\mathbf{Q}$ is a pair of subsets of A and B of $\mathbf{Q}$ such that
>
> 1. $A \cup B = \mathbf{Q}$, $A \neq \emptyset$, $B \neq \emptyset$, $A \cap B = \emptyset$
>
> 2. If $a \in A$ and $b \in B$ then $a < b$
>
> 3. A contains no largest element
>
> We denote the cut as $x = A|B$.

> **Example 1.1**
>
> Here are some examples of a cut.
>
> (i) $A|B = \{r \in \mathbf{Q} : r < 1\}|\{r \in \mathbf{Q} : r \geq 1\}$.
>
> (ii) $A|B = \{r \in \mathbf{Q} : r^2 < 2\}|\{r \in QQ : r^2 \geq 2\}$.

> **Definition 1.2**
>
> A **real number** is a cut in $\mathbf{Q}$.