

# **Real Analysis**

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# 1 Real Numbers

## 1.1 Cuts

### Theorem 1.1

No number  $r$  in  $\mathbf{Q}$  has a square equal to 2 i.e  $\sqrt{2} \notin \mathbf{Q}$ .

*Proof.* We can use the standard proof by letting  $(p/q)^2 = 2$  with  $p$  and  $q$  not sharing a factor and deduce  $p, q$  are even.  $\square$

### Definition 1.1

A **cut** in  $\mathbf{Q}$  is a pair of subsets of  $\mathbf{Q}$  such that

1.  $A \cup B = \mathbf{Q}$ ,  $A \neq \emptyset$ ,  $B \neq \emptyset$ ,  $A \cap B = \emptyset$
2. If  $a \in A$  and  $b \in B$  then  $a < b$
3.  $A$  contains no largest element

We denote the cut as  $x = A|B$ .

### Definition 1.2

A **real number** is a cut in  $\mathbf{Q}$ .

### Example 1.1