# Assignment 3

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## Question 1

### 1.1

```
data <- tribble(
    ~ color, ~ no_treated, ~ no_contr, ~ avg_treated, ~avg_control,
    "purple", 100, 100, 9, 7,
    "blue", 75, 25, 13, 8,
    "green", 25, 75, 10, 9
)

data %>%
    group_by(color) %>%
    summarize(treatment_effect = avg_treated - avg_control)
```

```
## # A tibble: 3 x 2
##   color  treatment_effect
##   <chr>             <dbl>
## 1 blue                  5
## 2 green                 1
## 3 purple                2
```

### 1.2

The ATE is defined as $\mathbb{E}[\delta] = \mathbb{E}[Y_1^*] - \mathbb{E}[Y_0^*]$ which are the expectations of the potential outcomes. In general, these two variables are not observed. Under the *random assignment* assumption, we assume that $\mathbb{E}[Y_1^*] = \mathbb{E}[Y_1^*|D = 1]$ and $\mathbb{E}[Y_0^*] = \mathbb{E}[Y_0^*|D = 0]$, which can be estimated by their sample by their sample equivalents:

```
data %>%
    summarize(
        e_y1_d_is_1 = (9*100 + 13 * 75 + 10 * 25) / sum(no_treated),
        e_y0_d_is_0 = (7 * 100 + 8 * 25 + 9 * 75) / sum(no_contr)) %>%
    summarize(e_y1_d_is_1 - e_y0_d_is_0)
```

```
## # A tibble: 1 x 1
##   `e_y1_d_is_1 - e_y0_d_is_0`
##                         <dbl>
## 1                        2.75
```

### 1.3

The ATT is defined as $\mathbb{E}[\delta|D = 1] = \mathbb{E}[Y^1|D = 1] - \mathbb{E}[Y^0|D = 1]$. The first term is readily observable. The second term is estimated by us as $\hat{\mathbb{E}}[Y^0|D = 1] = \mathbb{E}[Y^0|D = 0]$. Hence:

```
data_ate <- data %>%
    mutate(n = no_treated + no_contr) %>%
    summarize(e_y1_d_is_1 = (9 * 100 + 13 * 75 + 10 * 25) / sum(no_treated),
              e_y0_d_is_0 = (7 * 100 + 8 * 25 + 9 * 75) / sum(no_contr))

data_ate
```

```
## # A tibble: 1 x 2
##   e_y1_d_is_1 e_y0_d_is_0
##         <dbl>       <dbl>
## 1        10.6        7.88
```

```
data_ate %>%
    summarize(att = e_y1_d_is_1 - e_y0_d_is_0)
```

```
## # A tibble: 1 x 1
##     att
##   <dbl>
## 1  2.75
```

So the $ATE = ATT$ (because of randomization).