Assignment 1

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Assignment 1

At the beginning of the code,set the random seed to 810 using set.seed(). Failure to do so will be penalised.

```
set.seed(810)
```

a. Simulate 100,000 observations from the DGP

```
x <- rnorm(100000, mean = 6, sd = sqrt(3))
alpha <- 5
e <- rnorm(100000, 0, 1)

y <- alpha + 0.3*x + 0.1*x^2 + e

dataset <- data.frame(x = x, x_sq = x^2, y = y)
```

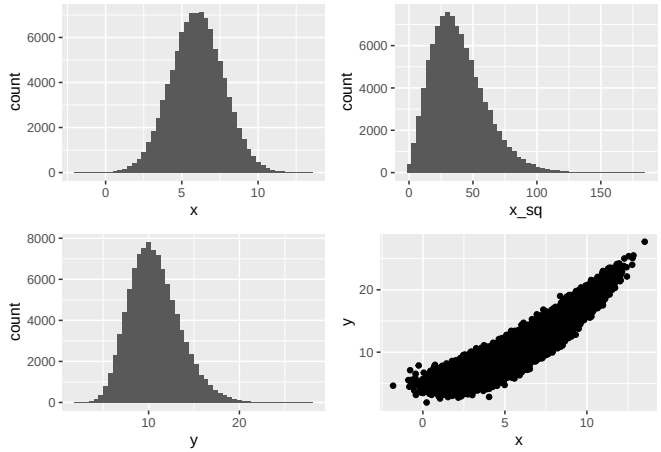
```
p1 <- dataset %>%
    ggplot(aes(x = x)) + geom_histogram(bins = 50)

p2 <- dataset %>%
    ggplot(aes(x = x_sq)) + geom_histogram(bins = 50)

p3 <- dataset %>%
    ggplot(aes(x = y)) + geom_histogram(bins = 50)

p4 <- dataset %>%
    ggplot(aes(x = x, y = y)) + geom_point()

cowplot::plot_grid(p1, p2, p3, p4, nrow = 2, ncol = 2)
```



b. Break the data into 1000 datasets of 100 observations sequentially (i.e. dataset 1 comprises observations 1-100 from your simulation, dataset 2 comprises observations 101-200 and so on

```
hundred <- dataset %>%
   group_by((row_number()-1) %/% (n()/1000)) %>%
   nest %>% pull(data)
```

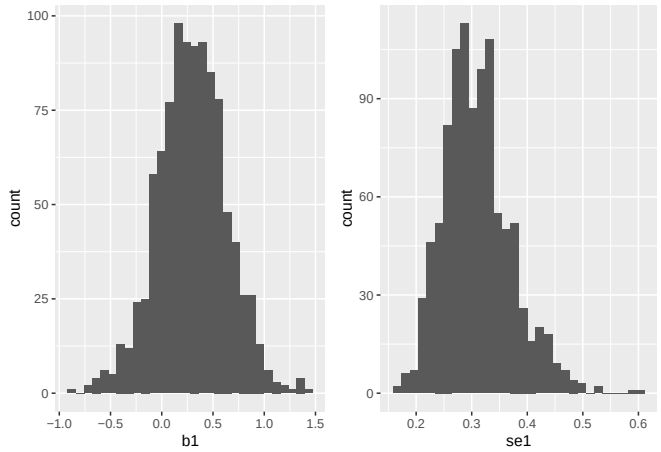
```
beta1 <- map_dbl(hundred, ~ lm(data = .x, formula = y ~ x + x_sq) %>%
    .$coefficients %>%
    .[2])

se1 <- map_dbl(hundred, ~ lm(data = .x, formula = y ~ x + x_sq) %>%
                  summary() %>%
                  .$coefficients %>%
                  .[2,2])

p1_een <- data.frame(b1 = beta1, se1 = se1) %>%
    ggplot(aes(x = b1)) + geom_histogram()

p2_een <- data.frame(b1 = beta1, se1 = se1) %>%
    ggplot(aes(x = se1)) + geom_histogram()

cowplot::plot_grid(p1_een, p2_een)
```



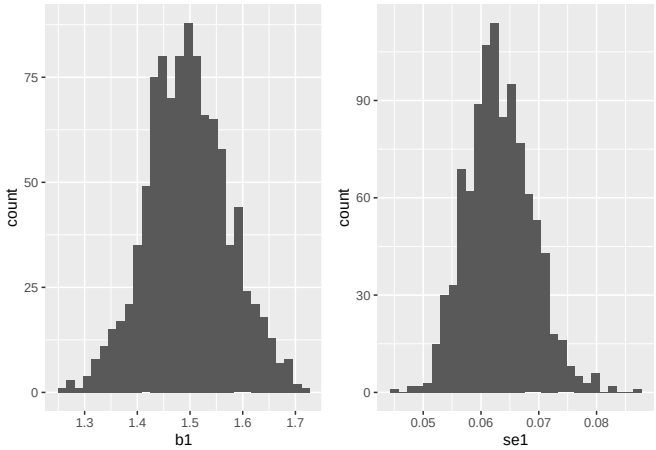
```
beta_omv <- map_dbl(hundred, ~ lm(data = .x, formula = y ~ x) %>%
    .$coefficients %>%
    .[2])

se_omv <- map_dbl(hundred, ~ lm(data = .x, formula = y ~ x) %>%
                  summary() %>%
                  .$coefficients %>%
                  .[2,2])

p1_omv <- data.frame(b1 = beta_omv, se1 = se_omv) %>%
    ggplot(aes(x = b1)) + geom_histogram()

p2_omv <- data.frame(b1 = beta_omv, se1 = se_omv) %>%
    ggplot(aes(x = se1)) + geom_histogram()

cowplot::plot_grid(p1_omv, p2_omv)
```



We see that the coefficients from the fully specified model (without omitted variable biased) are normally distributed around the true coefficient value from the DGP, whereas the coefficient in the wrongly specified model is noisy distributed around a wrong value.

c. Imagine we had instead generated $X_i = c$ for all x_i and tried to perform our simulation above. This would fail because we would violate one of the necessary conditions for computing the least squares estimator. Which one, and how?

$$\sum_{i=1}^{N}(x_i - \bar{x})^2 \geq 0$$

is not met. If we have no finite sum of squares, we have $\text{Var}X = 0$, and hence, we cannot estimate the OLS estimator.

d. Simulate another dataset as in a), but with 1,000,000 observations. Repeat part b), but now using 1000 observations per model. Only fit the equation $Y_i = \alpha + \beta_1 X_i + \beta_2 X_i^2 + \epsilon$ this time. Plot the histograms of the estimate of β_1 and $se(\beta_1)$ and compare them to the estimates from the same model in (b). What happens to the distribution of the coefficient estimates and standard errors when we increase the sample size per model?

```
x2 <- rnorm(1000000, mean = 6, sd = sqrt(3))
alpha2 <- 5
e2 <- rnorm(1000000, 0, 1)

y2 <- alpha2 + 0.3*x2 + 0.1*x2^2 + e2

dataset2 <- data.frame(x = x2, x_sq = x2^2, y = y2)
```

```
thousand <- dataset %>%
   group_by((row_number()-1) %/% (n()/1000)) %>%
   nest %>% pull(data)
```

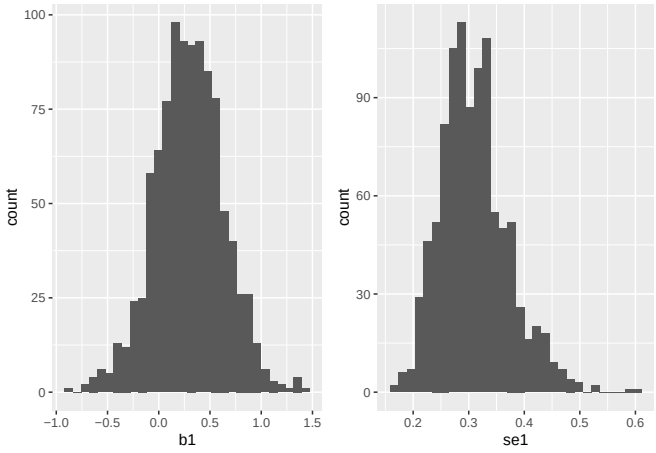
```
beta_ef <- map_dbl(thousand, ~ lm(data = .x, formula = y ~ x + x_sq) %>%
    .$coefficients %>%
    .[2])

se_ef <- map_dbl(thousand, ~ lm(data = .x, formula = y ~ x + x_sq) %>%
                  summary() %>%
                  .$coefficients %>%
                  .[2,2])

p1_ef <- data.frame(b1 = beta_ef, se1 = se_ef) %>%
    ggplot(aes(x = b1)) + geom_histogram()

p2_ef <- data.frame(b1 = beta_ef, se1 = se_ef) %>%
    ggplot(aes(x = se1)) + geom_histogram()

cowplot::plot_grid(p1_ef, p2_ef)
```



e. Now, generate a new variable C_i that is correlated with X_i. Do this by creating a vector of observations drawn from N(1,2) and adding them to X_i. Add C_i the dataset from the previous questions (i.e with 100,000 observations total).

```
c <- rnorm(100000, mean = 1, sd = sqrt(2)) + x
dataset <- data.frame(x = x, x_sq = x^2, y = y, c = c)

hundred <- dataset %>%
   group_by((row_number()-1) %/% (n()/1000)) %>%
   nest %>% pull(data)
```

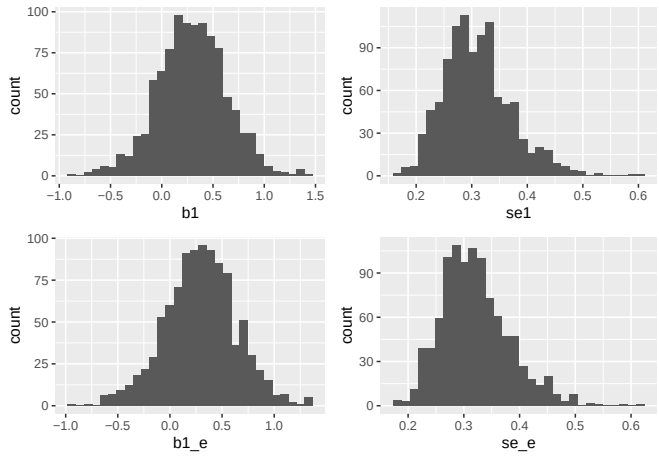
```
beta_twee <- map_dbl(hundred, ~ lm(data = .x, formula = y ~ x + x_sq + c) %>%
    .$coefficients %>%
    .[2])

se_twee <- map_dbl(hundred, ~ lm(data = .x, formula = y ~ x + x_sq + c) %>%
                   summary() %>%
                   .$coefficients %>%
                   .[2,2])

p1_twee <- data.frame(b1_e = beta_twee,
           se_e = se_twee) %>%
    ggplot(aes(x = b1_e)) + geom_histogram()

p2_twee <- data.frame(b1_e = beta_twee,
           se_e = se_twee) %>%
    ggplot(aes(x = se_e)) + geom_histogram()

cowplot::plot_grid(p1_een, p2_een,
        p1_twee, p2_twee,
        nrow = 2, ncol = 2)
```



Re-run the regressions as $Y_i = \alpha + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 C_i + \epsilon_i$ and plot the distribution of these coefficient estimates and standard errors for β_1 next tothe coefficient estimates from the second part. What happens to the coefficient estimates, and why?

The coefficients are not impacted, because there is no omitted variable bias. There is no relationship between Y and C, hence, the OLS coefficient for β_1 is still unbiased. We do pay a small penalty in terms of efficiency for adding an unnecessary variable, but with $N = 1000$, this is negligable.

Question 2

You are a labor economist trying to estimate the gender wage gap within occupations for women with children - that is, the effect of gender on wages given occupational choice. You have access to a dataset containing a set of wages,W_i, a gender dummy D_i, a set of occupational dummiesO_{ij} and hours spent on childcare C_i for a sample of men and women with children. Assume that there is a positive covariance between each of your regressors, some covariance between each of the regressors and wages,and that gender at least partially determines occupational choice and hours spent on childcare.

$y = \alpha W_i + \beta D_i + \gamma O_i + \delta C_i + \epsilon.$

a. Derive the expected value of the least-squares estimator for the coefficient on the gender dummy without controlling for either occupation or hoursspent on childcare

We have that $b = (D^T D)^{-1} D^T y$ and

$$\mathbb{E}[b] = \mathbb{E}[(D^T D)^{-1} D^T y] = \mathbb{E}[(D^T D)^{-1} D^T (\alpha W_i + \beta D_i + \gamma O_i + \delta C_i + \epsilon)]$$