Assignment 2

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Question 1

```
set.seed(2021)
```

```
b0 <- 3
b1 <- 5
b2 <- 8

gamma <- 1

x1 <- rnorm(5000, mean = 1, sd = 1)
x2 <- rnorm(5000, mean = 2, sd = 1)

z <- rgamma(5000, shape = 1.2, scale = 1.1)

sigma_sq <- 1*exp(gamma*z)
epsilon <- rnorm(5000, mean = 0, sd = sqrt(sigma_sq))

y <- b0 + b1*x1 + b2*x2 + epsilon
```

```
model1 <- lm(y ~ x1 + x2)

modelsummary(model1,
             vcov = c("iid", "HC0"),
             gof_map = gm,
             stars = T)
```

```
lmtest::bptest(formula = y ~ x1 + x2)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  y ~ x1 + x2
```

	Model 1	Model 2
(Intercept)	2.609***	2.609***
	(0.375)	(0.315)
x1	5.009***	5.009***
	(0.149)	(0.078)
x2	8.080***	8.080***
	(0.152)	(0.097)
N	5000	5000
Adj. R2	0.44	0.44

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

```
## BP = 0.50429, df = 2, p-value = 0.7771
```

```r
gamma <- 0

sigma_sq <- 1*exp(gamma*z)
epsilon <- rnorm(5000, mean = 0, sd = sqrt(sigma_sq))

y <- b0 + b1*x1 + b2*x2 + epsilon
```

```r
model1 <- lm(y ~ x1 + x2)
model2 <- lm(y ~ x1 + x2, weights = sigma_sq)
```

Question 2

1. Show that the OLS estimator of the parameter β is not consistent.
2. Derive plim (b) where b is the OLS estimator of β. Determine the sign of the magnitude of the inconsistency when $0 < \beta < 1$, that is, the sign of plim(b) - β when $0 < \beta < 1$.

First, we demean the two variables so that the constant-term α equals zero. Then we regress $\tilde{C} = \beta\tilde{D} + \epsilon$. We can do this because of Frisch-Waugh-Lovell. The estimate that we get is:

$$\hat{\beta} = (\tilde{D}^T\tilde{D})^{-1}\tilde{D}^T C_= (\tilde{D}^T\tilde{D})^{-1}(\beta\tilde{D} + \epsilon)$$

and

$$\mathbb{E}[\hat{\beta}] = \beta + (\tilde{D}^T\tilde{D})^{-1}\tilde{D}^T\epsilon$$

Evaluating the probability limit gives:

$$\text{plim}_{n\to\infty}(\hat{\beta}) = \beta + \text{plim}(\frac{1}{n}\tilde{D}^T\tilde{D})^{-1} \cdot \text{plim}(\frac{1}{n}\tilde{D}^T\epsilon)$$

which simplifies to:

$$\beta + \frac{1}{\text{Var}(D)} \cdot \frac{1}{1-\beta}\sigma^2$$

by the fact that variances and covariances are the same after demeaning, and by the reduced form equation for D made explicit below. Under $0 < \beta < 1$, since variances are positives, the right term can only be positive and thus the bias is always positive.

Substituting equation (2) into equation (1) and solving for C gives:

$$C = \frac{\alpha}{1-\beta} + \frac{\beta}{1-\beta}Z_i + \frac{1}{1-\beta}\epsilon_i$$

substituting this back in the definition for D gives:

$$D = \frac{\alpha}{1-\beta} + \left(\frac{\beta}{1-\beta} + 1\right)Z_i + \frac{1}{1-\beta}\epsilon_i$$

From this, we can calculate Cov(D, ϵ_i), which is $\frac{1}{1-\beta}$Var(ϵ).