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Tutorials
Week 3

Regression Analysis with time series data II

Pdf file on Blackboard	Dataset on Blackboard	Papers related to the data	Description
C 18.2	hseinv.dta	McFadden, D., 1994. Demographics, the housing market, and the welfare of the elderly. In: D.A. Wise, ed. 1994. Studies in the Economics of Aging. Chicago: University of Chicago Press, pp.225-285.	Use of lagged variables, test for unit root (Dickey Fuller test), use of ADF (augmented DF) test, consequences of unit root, meaning and consequences of co-integration.
C 18.3	volat.dta	Hamilton, J. D., & Lin, G. (1996). Stock Market Volatility and the Business Cycle. Journal of Applied Econometrics, 11(5), 573–593. http://www.jstor.org/stable/2285217	AR(3) model, Granger causality, VAR, Wald Test, logit, forecast, MAE.
C.18.3	fertil3.dta	Whittington, L. A., Alm, J., & Peters, H. E. (1990). Fertility and the Personal Exemption: Implicit Pronatalist Policy in the United States. *The American Economic Review*, *80*(3), 545–556. http://www.jstor.org/stable/2006683	Random walk with drift, AR(2), forecasting, MAE

C.18.2 Use the data HSEINV.RAW for this exercise.

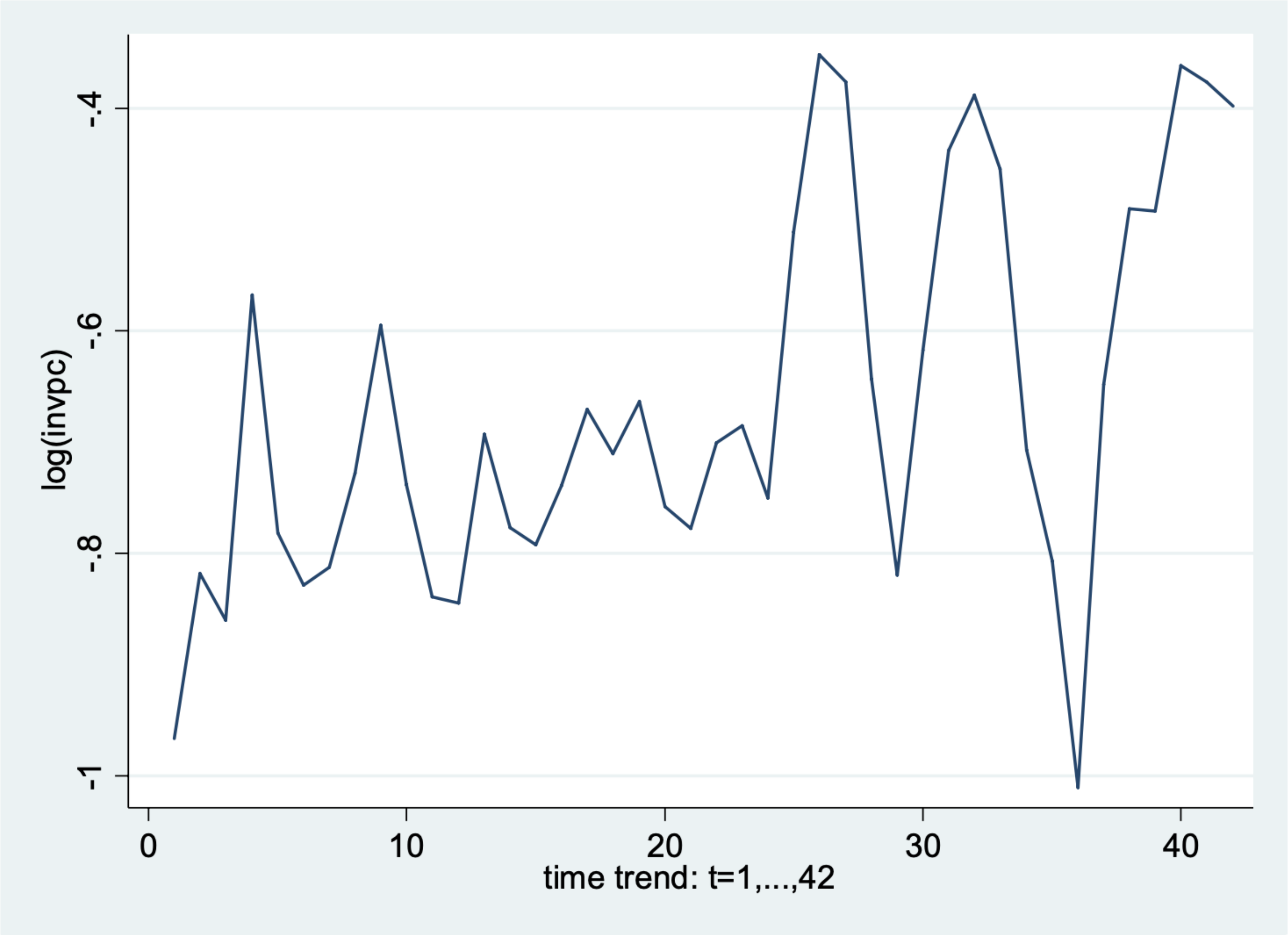
Demographics, the Housing Market, and the Welfare of the Elderly

Variables:

invpc: per capita investment: inv/pop
price: housing price index; 1982=1
year:1947-1988
t: time trend: t=1,…,42

Before we check if log(*invpc)* has a unit root, plot log(invpc) and time. Discuss the graph.

```
. line linvpc t
```



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C.18.2 Use the data HSEINV.RAW for this exercise.

i) Test for unit root in log(invpc), including a linear time trend and two lags of $\Delta\log(invpc)$. Use a 1% significance level.

- Perform the augmented Dickey-Fuller test for unit root separately for all variables of the regression equation.

$$\Delta\log(invpc_{t)} = \alpha_0 + \alpha_1\log(invpc_{t-1}) + \alpha_2\Delta\log(invpc_{t-1)} + \alpha_3\Delta\log(invpc_{t-2)} + \alpha_4 t + \varepsilon_t$$

- Where: $\Delta\log(invpc_{t)} = log(invpc_t - \log(invpc_{t-1})$

- In Stata: declare the dataset as time-series:

```
. tsset t
        time variable:  t, 1 to 42
                delta:  1 unit
```

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$$\Delta\log(invpc_t) = \alpha_0 + \alpha_1\log(invpc_{t-1}) + \alpha_2\Delta\log(invpc_{t-1}) + \alpha_3\Delta\log(invpc_{t-2}) + \alpha_4 t + e_t$$

Then estimate the test regression.

```
. reg d.linvpc l.linvpc dl(1/2).linvpc t

      Source |       SS       df       MS              Number of obs =      39
-------------+------------------------------           F(  4,    34) =    6.59
       Model |  .34943844     4  .08735961           Prob > F      =  0.0005
    Residual | .450566441    34  .013251954           R-squared     =  0.4368
-------------+------------------------------           Adj R-squared =  0.3705
       Total | .800004881    38   .02105276           Root MSE      =  .11512

------------------------------------------------------------------------------
    D.linvpc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      linvpc |
        L1. |  -.9557867   .1977787    -4.83   0.000    -1.357721   -.5538521
        LD. |    .531659   .1615547     3.29   0.002     .2033404    .8599776
       L2D. |   .2900152   .1646455     1.76   0.087    -.0445847    .6246151
            |
          t |     .00676   .0021276     3.18   0.003     .0024361    .0110839
      _cons |  -.7863716   .1699981    -4.63   0.000    -1.131849   -.4408939
------------------------------------------------------------------------------
```

- The test statistics of interest is the t-statistics of the lagged linvpc , $\boldsymbol{\alpha_1}\log(\boldsymbol{invpc_{t-1}})$ which is -4.83.
- We will test:

  $H_0 : \ \theta = 0 \ \leftrightarrow \ \rho = 1$ unit root (non stationary)

  $H_A : \ \theta < 0 \ \leftrightarrow \ \rho < 1$ no unit root (stationary)

- If t-value < t critical value, → reject Ho

Check table 18.3

- -4,83 < -3,96 → we reject Ho at a 1% significance level. That means, the variable log(invpc) does not have a unit root. It is trend stationary.

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Application of the aug. Dickey Fuller test in Stata

```
. dfuller linvpc, lag(2) trend reg

Augmented Dickey-Fuller test for unit root         Number of obs   =        39

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -4.833            -4.251            -3.544            -3.206
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0004

------------------------------------------------------------------------------
D.linvpc     |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      linvpc |
         L1. |  -.9557867   .1977787    -4.83   0.000    -1.357721   -.5538521
         LD. |    .531659   .1615547     3.29   0.002     .2033404    .8599776
        L2D. |   .2900152   .1646455     1.76   0.087    -.0445847    .6246151
      _trend |     .00676   .0021276     3.18   0.003     .0024361    .0110839
       _cons |  -.7796116   .1683262    -4.63   0.000    -1.121692   -.4375316
------------------------------------------------------------------------------
```

- The only difference you can observe is that Stata will not use the same critical values.
- This test is called an Augmented DF test, as lags have been added, requiring different critical values. Use these critical values instead of the ones from Table 18.3.

- Test:

$$H_0 : \theta = 0 \leftrightarrow \rho = 1$$ unit root (non stationary)

$$H_A : \theta < 0 \leftrightarrow \rho < 1$$ no unit root (stationary)

- If t value < t critical value → reject Ho
- -4,83 < -4,251 → reject Ho at 1% sig. level, which means the variable log(invpc) does not have a unit root. And it is stationary.

ii) Use the approach from part i) to test for a unit root in log(price)

Variables:
invpc: per capita investment: inv/pop
price: housing price index; 1982=1
year:1947-1988
t: time trend: t=1,…,42

```
. dfuller lprice , lag(2) trend reg

Augmented Dickey-Fuller test for unit root         Number of obs   =        39

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -2.409            -4.251            -3.544            -3.206
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.3749

------------------------------------------------------------------------------
D.lprice     |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      lprice |
         L1. |  -.2216337    .092006    -2.41   0.022    -.4086124   -.0346549
         LD. |    .327572   .1551807     2.11   0.042      .012207     .642937
        L2D. |   .1300876   .1491206     0.87   0.389     -.172962    .4331372
      _trend |    .000971   .0004867     1.99   0.054    -.0000182    .0019602
       _cons |   -.039384   .0190149    -2.07   0.046    -.0780269   -.0007412
------------------------------------------------------------------------------
```

- Test:

$$H_0: \ \theta = 0 \ \leftrightarrow \ \rho = 1 \ \text{ unit root (non stationary)}$$

$$H_A: \ \theta < 0 \ \leftrightarrow \ \rho < 1 \ \text{ no unit root (stationary)}$$

- If t value < t critical value → reject Ho
- -2,41 > - 4,251 → we can not reject Ho. The variable log(price) has a unit root. It follows a non-stationary process, so not weakly dependent.

iii) Given the outcomes in parts i) and ii), does it make sense to test for cointegration between log(invpc) and log(price)?

- If one variable is non-stationary – unit root- (I(1)), and the other is stationary – no unit root - (I(0)), then it does not make sense to co-integrate them.
- Both variables need to have a unit root (be non-stationary) to be co-integrated.

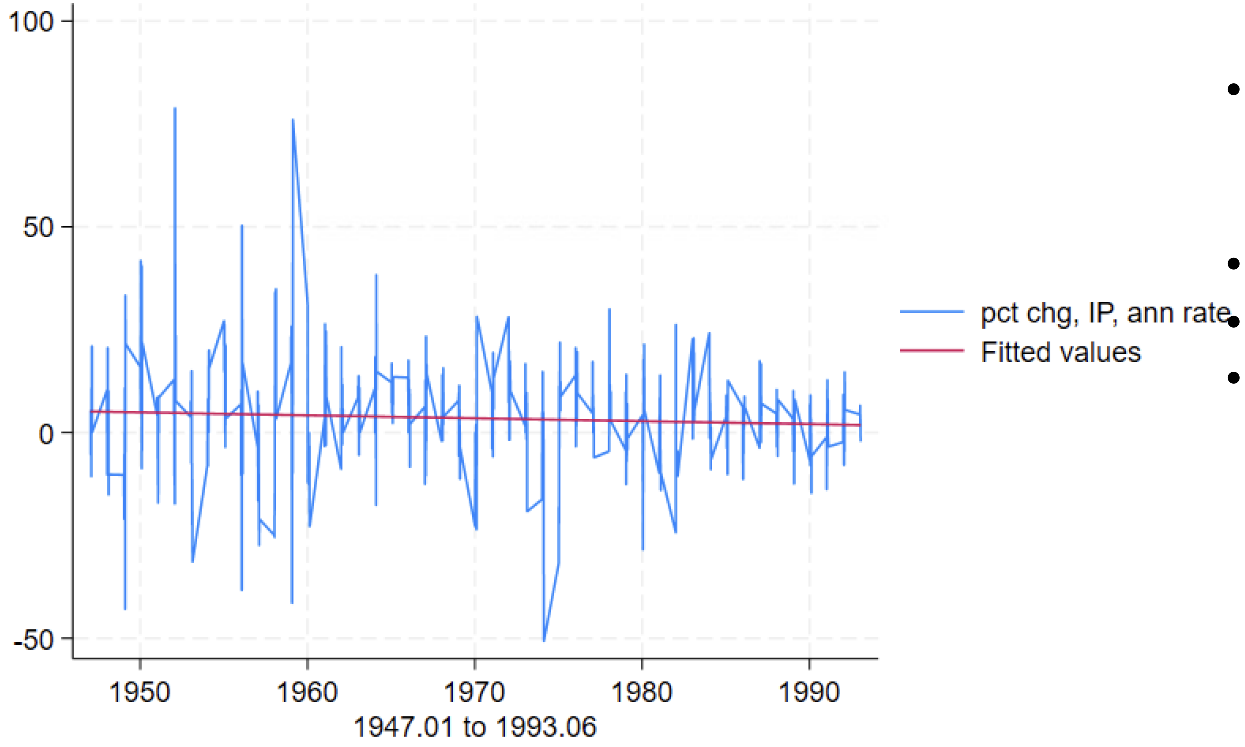
- The answer is no. Cointegration makes sense between two non-stationary processes (unit roots) integrated in the same order.
- If we take any nontrivial linear combination of an I(0) process (which may have a trend) and an I(1) process, the result will be an I(1) process (possibly with drift).

C.18.3 Use the data in VOLAT.RAW for this exercise.
Graph pcip against time. Does it contain a clear upward or downward trend over the entire sample period?

gen time = _n
tsset time

We plot PCIP against time, in this case, date.

line pcip date ||lfit pcip date



Variables:

Pcip: annualized percentage change in the Industrial Production index.
date: 1947.01 to 1993.06

- The graph shows significant fluctuations in the annual percentage change of industrial production over the years.
- There is plenty of volatility
- Clear upward and downward trends over time.
- However, the trend appears to stabilize around the zero line, especially in the later years. That means that the PCIP (annual % change in industrial production) does not show a strong upward or downward trend in the long term.

C.18.3 Use the data in VOLAT.RAW for this exercise.

Check for unit root in pcip : annualized percentage change in the Industrial Production index.

```
. dfuller pcip, lags(3) trend

Augmented Dickey-Fuller test for unit root

Variable: pcip                              Number of obs  = 553
                                            Number of lags =   3

H0: Random walk with or without drift

                                    Dickey-Fuller
                  Test      ---------- critical value ----------
             statistic           1%           5%           10%
--------------------------------------------------------------------
 Z(t)           -9.053       -3.960       -3.410        -3.120
--------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000.
```

- Test:

H_0 : $\theta = 0 \leftrightarrow \rho = 1$ unit root (non stationary)

H_A : $\theta < 0 \leftrightarrow \rho < 1$ no unit root (stationary)

- If |t value| < t critical value → reject Ho
- -9.053 < -3.960 → reject Ho at 1% sig. level (and at all other levels). pcip does not have a unit root. It is stationary.

C.18.3 Use the data in VOLAT.RAW for this exercise.
i) Estimate an AR(3) model for pcip. Now, add a fourth lag and verify that it is very insignificant.

$$pcip_t = \beta_0 + \beta_1(pcip_{t-1}) + \beta_2(pcip_{t-2}) + \beta_3(pcip_{t-3})$$

Frist generate a time trend:

```
. gen time = _n
```

Then declare the data as time-series:

```
. tsset time
        time variable:  time, 1 to 558
                delta:  1 unit
```

We now estimate the AR(3) specification:

```
. reg pcip l(1/3).pcip

      Source |       SS       df       MS              Number of obs =     554
-------------+------------------------------           F(  3,   550) =   36.40
       Model |  16126.3579     3  5375.45264           Prob > F      =  0.0000
    Residual |  81224.8954   550  147.681628           R-squared     =  0.1657
-------------+------------------------------           Adj R-squared =  0.1611
       Total |  97351.2533   553  176.042049           Root MSE      =  12.152

------------------------------------------------------------------------------
        pcip |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pcip |
         L1. |   .3491232   .0425232     8.21   0.000     .2655954    .4326509
         L2. |   .0707984   .0449501     1.58   0.116    -.0174965    .1590932
         L3. |   .0673713   .0425274     1.58   0.114    -.0161647    .1509073
             |
       _cons |   1.804189   .5480442     3.29   0.001     .7276729    2.880704
```

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We test for heteroskedasticity and autocorrelation in the error term

Test for heteroskedasticity (Breush-Pagan Test)

```
. hettest

Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: Normal error terms
Variable: Fitted values of pcip

H0: Constant variance

    chi2(1) =  62.06
Prob > chi2 = 0.0000
```

The error term is heteroscedastic at 1%.
For the exam, you have to write all the statistical steps.

If chi2-stat > CVchi2, then reject Ho.
62.06 > 6.63, reject Ho. There is Heteroskedasticity in the error term at 1%.

Table G.4.

Test for serial correlation (Breusch Godfrey Test)

```
. bgodfrey, lags(1/3)

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          0.007               1                   0.9357
       2     |          0.890               2                   0.6408
       3     |          1.312               3                   0.7263
---------------------------------------------------------------------------
                        H0: no serial correlation
```

But we find no evidence for serial correlation in the error-term.

Ho: $\rho = 0$ (no 1st order serial correlation)
H_1: $\rho \neq 0$ (serial correlation)

If pvalue < 0.10 or < 0.05; then reject Ho.
P values are not less than 0.10 or 0.05.
Therefore, we can not reject the Ho. So, there is no serial correlation.

We use the rob option

```
. reg pcip l(1/3).pcip, rob

Linear regression                                      Number of obs =     554
                                                       F(  3,   550) =   16.24
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.1657
                                                       Root MSE      =  12.152

------------------------------------------------------------------------------
             |               Robust
        pcip |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pcip |
         L1. |   .3491232   .0623237     5.60   0.000     .2267015    .4715448
         L2. |   .0707984   .0488291     1.45   0.148     -.025116    .1667127
         L3. |   .0673713   .0414654     1.62   0.105    -.0140786    .1488212
             |
       _cons |   1.804189   .6400428     2.82   0.005     .5469612    3.061416
------------------------------------------------------------------------------
```

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Now, add a fourth lag and verify that it is very insignificant. And also use the rob option

```
. reg pcip l(1/4).pcip, rob

Linear regression                                      Number of obs =     553
                                                       F(  4,   548) =   12.21
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.1659
                                                       Root MSE      =  12.173

------------------------------------------------------------------------------
             |               Robust
        pcip |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pcip |
         L1. |    .349382   .0632797     5.52   0.000     .2250816    .4736823
         L2. |   .0702363   .0483599     1.45   0.147    -.0247571    .1652298
         L3. |   .0657502   .0443265     1.48   0.139    -.0213205    .1528209
         L4. |   .0043168   .0587231     0.07   0.941    -.1110331    .1196667
             |
       _cons |   1.787332   .6639847     2.69   0.007     .4830655    3.091599
------------------------------------------------------------------------------
```

- When $pcip_{t-4}$ is added, its coefficient is 0.0043 with a t-statistics of about 0.10.
- The t-statistics of the fourth lag is very small (0.07); the coefficient is close to zero.
- We conclude that the fourth lag is insignificant (Pvalue is greater than 0.10).

ii) To the AR(3) model from part i), add three lags of pcsp to test whether pcsp Granger causes pcip. Carefully, state your conclusion.

The Granger test specification:

$$pcip_t = \delta_0 + \alpha_1 pcip_{t-1} + \alpha_2 pcip_{t-2} + \alpha_3 pcip_{t-3} + \gamma_1 pcsp_{t-1} + \gamma_2 pcsp_{t-2} + \gamma_3 pcsp_{t-3} + u_t,$$

Variables: **pcsp:** = %change, sp500, ann rate. **pcip:** annualized %change in the industrial production index.

Test the Granger-causality in the following way:

If *pcsp* Granger causes *pcip* then, its past values should explain the current value of *pcip* in a statistically significant way.

H_o: $\gamma_1 = \gamma_2 = \gamma_3 = 0$ *pcsp* does not have Granger causality on *pcip*.

H_1: At least one of the tested coefficients is not zero. In this case, *pcsp* Granger causes *pcip*.

Let us estimate the equation (since heteroskedasticity exists in the error term, we use robust estimation).

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```
. reg pcip l(1/3).pcip l(1/3).pcsp, rob

Linear regression                               Number of obs =        554
                                                F(  6,   547) =      14.13
                                                Prob > F      =     0.0000
                                                R-squared     =     0.1895
                                                Root MSE      =      12.01

------------------------------------------------------------------------------
             |               Robust
        pcip |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pcip |
         L1. |   .3258447   .0621686     5.24   0.000     .2037263    .4479631
         L2. |   .0691003   .0475705     1.45   0.147    -.0243429    .1625434
         L3. |   .0799492   .0410298     1.95   0.052    -.0006461    .1605444
             |
        pcsp |
         L1. |   .0234479   .0134366     1.75   0.082    -.0029458    .0498416
         L2. |   .0323316    .012825     2.52   0.012     .0071394    .0575238
         L3. |   .0195941   .0137932     1.42   0.156       -.0075    .0466883
             |
       _cons |   1.245541   .6232294     2.00   0.046     .0213254    2.469757
------------------------------------------------------------------------------
```

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We now test the exclusion restrictions for the lags of the *pcsp*:

```
. test l.pcsp l2.pcsp l3.pcsp

 ( 1)  L.pcsp = 0
 ( 2)  L2.pcsp = 0
 ( 3)  L3.pcsp = 0

       F(  3,   547) =    5.71
            Prob > F =    0.0007
```

- H_o: $\beta_4 = \beta_5 = \beta_6 = 0$ *pcsp* does not have Granger causality on *pcip*.
- H_1 At least one of the tested coefficients is not zero. In this case, *pcsp* Granger causes *pcip*.
- $If\ Fstat > Fcv, reject\ Ho$
- 5.71 > 3.78, reject Ho, at 1% significance level. Table G.3.c
- Pcsp (pct chg, sp500, ann rate) does Granger cause pcip (pct chg, IP, ann rate). That means that past values of the change of the Standard and Poor's 500 index (pcsp) can predict changes in the current value of industrial production growth rate (pcip).

iii) To the model in part ii), add three lags of the change in i3, the three-month T-bill rate. Does pcsp Granger cause pcip conditional on past $\Delta i3$?

Model from part ii)

$$pcip_t = \delta_0 + \alpha_1 pcip_{t-1} + \alpha_2 pcip_{t-2} + \alpha_3 pcip_{t-3} + \gamma_1 pcsp_{t-1} + \gamma_2 pcsp_{t-2} + \gamma_3 pcsp_{t-3} + u_t,$$

Solution:

- i3: 3 mo. T-bill annualized rate
- Difference of the interest rate for 3 months = Δi_3
- 3 lags of the change of i3: $\Delta i_{t-1} + \Delta i_{t-2} + \Delta i_{t-3}$
- Granger Causality: if the difference of the interest rate on the past 3 months will still be considered on the Granger causality of pCSP on pCIP

- We need to include three lags of the change of i3 (3 month Treasury Bill annualized interest rate) and retest Granger causality from pcsp to pcip. That means, test joint significance of $pcsp_{t-1}; pcsp_{t-2}; pcsp_{t-3}$.

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```
. reg pcip l(1/3).pcip l(1/3).pcsp dl(1/3).i3, rob

Linear regression                               Number of obs =        554
                                                F(  9,    544) =     11.93
                                                Prob > F       =    0.0000
                                                R-squared      =    0.1959
                                                Root MSE       =    11.995

------------------------------------------------------------------------------
             |               Robust
        pcip |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pcip |
         L1. |   .3145074   .0649439     4.84   0.000     .1869358     .442079
         L2. |   .0621721   .0487805     1.27   0.203    -.0336492    .1579934
         L3. |   .0789091   .0411188     1.92   0.056     -.001862    .1596801
             |
        pcsp |
         L1. |    .028815   .0137396     2.10   0.036     .0018258    .0558042
         L2. |   .0314511   .0128079     2.46   0.014     .0062921    .0566102
         L3. |   .0141627   .0139711     1.01   0.311    -.0132812    .0416067
             |
          i3 |
         LD. |   1.519901   1.237951     1.23   0.220    -.9118499    3.951651
        L2D. |   1.268064   1.151717     1.10   0.271    -.9942935    3.530422
        L3D. |  -.7773987    1.14427    -0.68   0.497    -3.025127    1.470329
             |
       _cons |   1.311842   .6380201     2.06   0.040     .0585574    2.565127
------------------------------------------------------------------------------
. test l.pcsp l2.pcsp l3.pcsp

 ( 1)  L.pcsp = 0
 ( 2)  L2.pcsp = 0
 ( 3)  L3.pcsp = 0

       F(  3,   544) =    5.18
            Prob > F =    0.0016
```

Solution:

This was added: $\Delta i_{t-1} + \Delta i_{t-2} + \Delta i_{t-3}$

Retest Granger causality from pcsp to pcip. That means, test joint significance of $pcsp_{t-1}; pcsp_{t-2}; pcsp_{t-3}$.

H_o: $\gamma_1 = \gamma_2 = \gamma_3 = 0$

H_1: At least one of the tested coefficients is not zero. In this case, *pcsp* Granger causes *pcip*.

$If\ Fstat > Fcv, reject\ Ho$

5.18 > 3.78, reject Ho at level 1%

Conclusion: Past $\Delta \boldsymbol{i3}$ is considered on the Granger causality of pCSP on pCIP.

Additional material

It is customary that Granger tests are carried out in all directions. That is, in this particular example, we should not only test id pcsp Granger causes pcip, but also vice versa. We can do this either in a single equation framework, just as we did before, or we can use a VAR (Vector Autoregression) model instead.

If you use the following command:

```
. var pcip pcsp d.i3, lag(1/3) small

Vector autoregression

Sample:  5 - 558                                    No. of obs      =       554
Log likelihood = -5298.088                          AIC             =  19.23498
FPE            =  45313.31                          HQIC            =   19.3263
Det(Sigma_ml)  =  40661.66                          SBIC            =  19.46876

Equation           Parms      RMSE     R-sq        F       P > F
----------------------------------------------------------------
pcip                 10     11.9954   0.1959   15.00105   0.0000
pcsp                 10     38.5903   0.0966   6.579677   0.0000
D_i3                 10     .455928   0.1582   11.57126   0.0000
----------------------------------------------------------------
```

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
pcip						
pcip						
L1.	.3145074	.0427499	7.36	0.000	.2305323	.3984825
L2.	.0621721	.0444846	1.40	0.163	-.0252105	.1495547
L3.	.0789091	.0423072	1.87	0.063	-.0041963	.1620145
pcsp						
L1.	.028815	.013294	2.17	0.031	.0027011	.0549289
L2.	.0314511	.0136768	2.30	0.022	.0045853	.058317
L3.	.0141627	.01336	1.06	0.290	-.0120808	.0404063
i3						
LD.	1.519901	1.135269	1.34	0.181	-.7101465	3.749948
L2D.	1.268064	1.160591	1.09	0.275	-1.011725	3.547854
L3D.	-.7773987	1.13927	-0.68	0.495	-3.015306	1.460509
_cons	1.311842	.5563367	2.36	0.019	.2190109	2.404673
pcsp						
pcip						
L1.	.021078	.1375305	0.15	0.878	-.2490778	.2912338
L2.	-.0968251	.1431111	-0.68	0.499	-.3779432	.184293
L3.	-.0490966	.1361062	-0.36	0.718	-.3164547	.2182615
pcsp						
L1.	.2345174	.0427681	5.48	0.000	.1505065	.3185284
L2.	-.0691706	.0439997	-1.57	0.117	-.1556007	.0172595
L3.	.0608425	.0429805	1.42	0.157	-.0235856	.1452705
i3						
LD.	-14.60858	3.652268	-4.00	0.000	-21.78285	-7.434302
L2D.	1.383237	3.733734	0.37	0.711	-5.951065	8.717538
L3D.	.0436589	3.665141	0.01	0.991	-7.155904	7.243222
_cons	6.796643	1.789789	3.80	0.000	3.2809	10.31239

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
D_i3						
pcip						
L1.	.0030131	.0016249	1.85	0.064	-.0001787	.0062049
L2.	.0032639	.0016908	1.93	0.054	-.0000574	.0065852
L3.	-.0003259	.001608	-0.20	0.839	-.0034846	.0028328
pcsp						
L1.	.0007001	.0005053	1.39	0.166	-.0002924	.0016927
L2.	.0017094	.0005198	3.29	0.001	.0006882	.0027305
L3.	-.0006568	.0005078	-1.29	0.196	-.0016543	.0003407
i3						
LD.	.3029408	.04315	7.02	0.000	.2181797	.3877019
L2D.	-.1863638	.0441125	-4.22	0.000	-.2730155	-.0997121
L3D.	-.0050184	.0433021	-0.12	0.908	-.0900782	.0800415
_cons	-.0304459	.0211456	-1.44	0.150	-.0719829	.0110911

- We estimate all relevant equations in one step. Observe that the VAR command has no robust option.
- VAR assumes that the error terms are stationary.
- Now, we only need to ask STATA to perform the exclusion tests. This allows us to determine whether certain variables should be excluded from the VAR model because they do not contribute significantly.
- To perform the exclusion test for **VAR analysis: Wald Test (which is an F-test)**

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```
. vargranger

  Granger causality Wald tests
 +------------------------------------------------------------------+
 |          Equation           Excluded |   F      df    df_r  Prob > F |
 |--------------------------------------+---------------------------|
 |              pcip               pcsp | 5.1703     3     544   0.0016 |
 |              pcip               D.i3 | 1.4741     3     544   0.2206 |
 |              pcip                ALL |  3.479     6     544   0.0022 |
 |--------------------------------------+---------------------------|
 |              pcsp               pcip | .28184     3     544   0.8385 |
 |              pcsp               D.i3 | 5.6388     3     544   0.0008 |
 |              pcsp                ALL | 3.2168     6     544   0.0041 |
 |--------------------------------------+---------------------------|
 |              D_i3               pcip | 3.6886     3     544   0.0119 |
 |              D_i3               pcsp | 5.3425     3     544   0.0012 |
 |              D_i3                ALL | 4.9576     6     544   0.0001 |
 +------------------------------------------------------------------+
```

Now, we can interpret the above equations. The following line, for example:

```
 |          Equation           Excluded |   F      df    df_r  Prob > F |
 |--------------------------------------+---------------------------|
```

is the result of the F-test of the joint significance of the three lags of the variable pcsp in the equation with pcip as dependent variable. We can see that the p-value is less than 0.01, hence we can say that at 1% pcsp Granger causes pcip, with the past values of Δi3 fixed. We find that the change of interest rates does not Granger-cause pcip.

We find, however, that the change in S&P 500 index is not Granger caused by the changes in the industrial production, but is Granger caused by the change of interest rate.

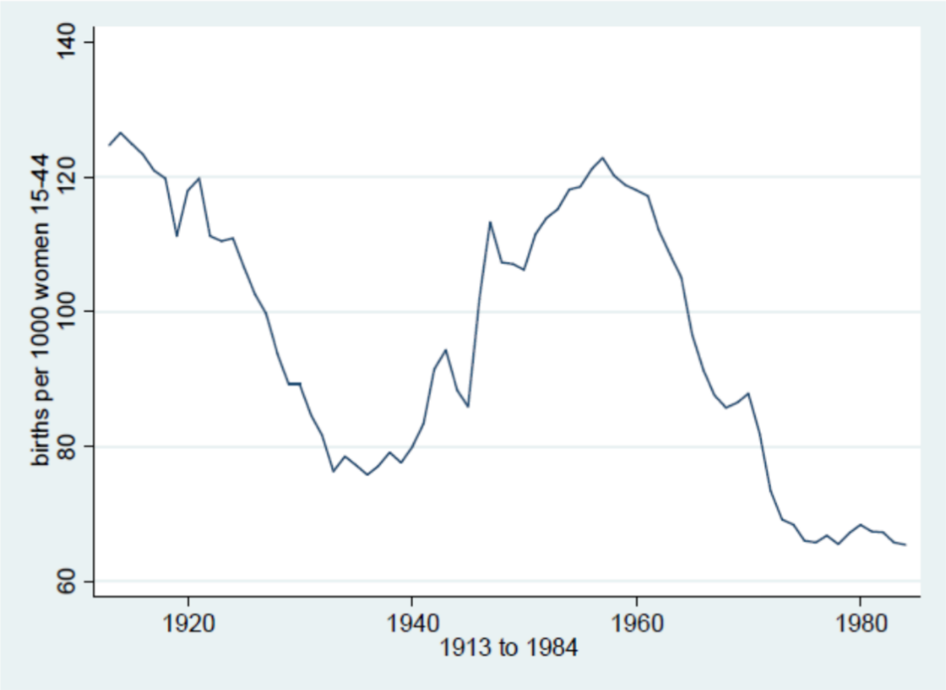
Finally, the change of the 3 month Treasury Bill interest rates is Granger caused by both pcip and pcsp at 5%.

C.18.8 Use the data in FERTIL3.RAW.

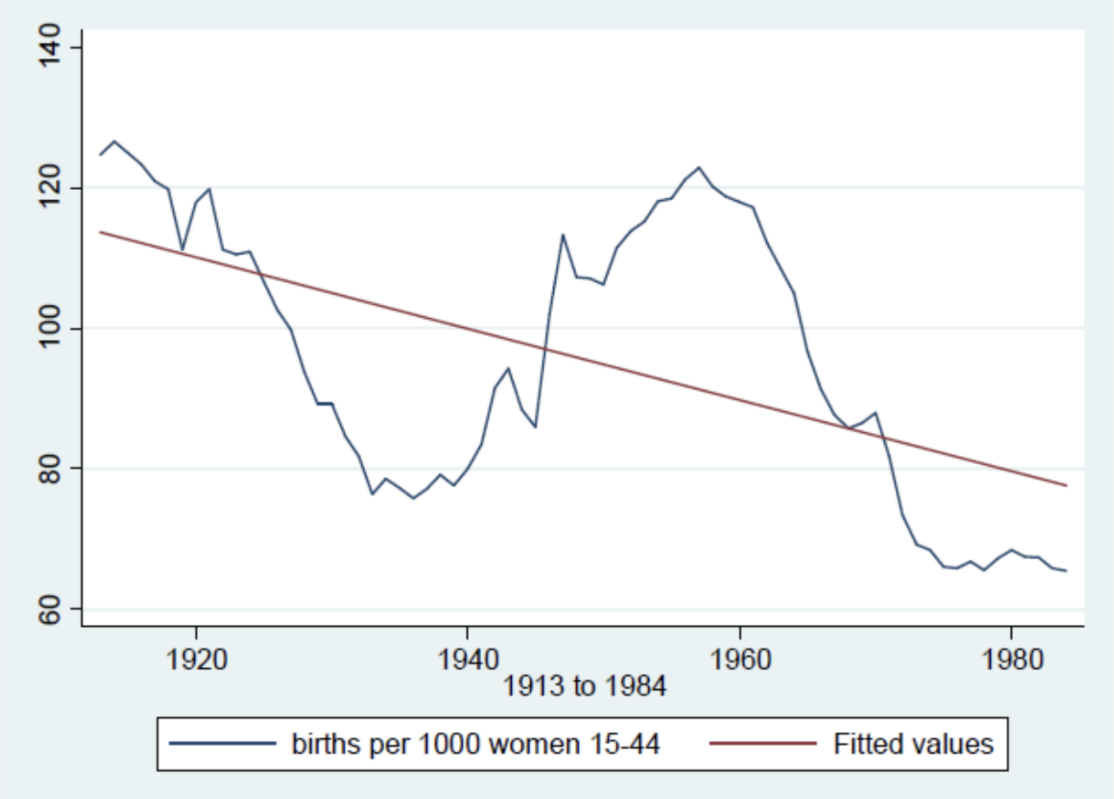
i)Graph gfr against time. Does it contain a clear upward or downward trend over the entire sample period?

(i) We plot gfr (gross fertility rate) against time.

```
. line gfr year
```




```
. line gfr year||lfit gfr year
```



There is a negative long-term in the data. We can even make this more apparent by introducing a linear time trend in the plot

But the negative trend is not clear since there is a period of increasing fertility after circa 1940 until the late 1950s.

(ii) Using the data through 1979, estimate a cubic time trend model for *gfr* (that is, regress *gfr* on t, $t^2{}_2$, and t^3 along with an intercept). Comment on the *R*-squared of the regression.

```
. reg gfr t tsq tcu if year<1980

      Source |       SS       df       MS              Number of obs =      67
-------------+------------------------------           F(  3,    63) =   59.47
       Model |   17288.927     3  5762.97566           Prob > F      =  0.0000
    Residual |  6104.84329    63  96.9022745           R-squared     =  0.7390
-------------+------------------------------           Adj R-squared =  0.7266
       Total |  23393.7703    66  354.451065           Root MSE      =  9.8439

------------------------------------------------------------------------------
         gfr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           t |  -6.904217   .6438123   -10.72   0.000    -8.190773   -5.617661
         tsq |   .2426157   .0219125    11.07   0.000     .1988271    .2864042
         tcu |  -.0024194   .0002119   -11.42   0.000    -.0028429   -.0019959
       _cons |   148.7082   5.092812    29.20   0.000     138.5311    158.8854
------------------------------------------------------------------------------
```

If we use the usual *t* critical values, all terms are very statistically significant, and the *R*-squared indicates that this curve-fitting exercise tracks gfr_t pretty well, at least up through 1979.

```
. hettest

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of gfr

         chi2(1)      =     0.38
         Prob > chi2  =   0.5401
```

Heteroscedasticity is no problem.

```
. bgodfrey, lags(1/3)

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         50.801               1                   0.0000
       2     |         51.222               2                   0.0000
       3     |         51.496               3                   0.0000
---------------------------------------------------------------------------
                        H0: no serial correlation
```

But there is autoregression in the error-term signifying specification problems.

iii) **Using this model $gfr_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$ compute the mean absolute error of the one step ahead forecast error for the years 1980 through 1984**

Solution

Manually calculation

Year	GFR actual y	Predicted GFR ("y-hat")	Absolute forecast error
1979	67.2	47.6	
1980	68.4	40.3	28.1
1981	67.4	32.6	34.8
1982	67.3	24.4	42.9
1983	65.8	15.6	50.2
1984	65.4	6.3	59.1

STATA

```
. predict forecast if year>1979, xb
(67 missing values generated)

. gen abserror=abs(gfr-forecast)
(67 missing values generated)

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
abserror_iii |          5    43.01686     12.2686   28.06189   59.11368
```

The MAE is 43.02 (see STATA outcome), and the model performs badly in forecasting.
A high R2 is no guarantee of a good forecasting capability.

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iv) Using the data through 1979, regress Δgfr_i on a constant only. Is the constat statistically sgnificant different from zero? Does it make sense to assume that any drift term is zero, if we assume that gfr_i follows a random walk?

We should estimate the Δgfr_i against a consant , using data up through 1and and decide if a drift is needed if we assume that gfr follows a random walk process.

```
. reg d.gfr if year<1980

      Source |       SS       df       MS              Number of obs =      66
-------------+------------------------------           F(  0,    65) =    0.00
       Model |           0     0           .           Prob > F      =       .
    Residual |  1264.15569    65   19.448549           R-squared     =  0.0000
-------------+------------------------------           Adj R-squared =  0.0000
       Total |  1264.15569    65   19.448549           Root MSE      =  4.4101

------------------------------------------------------------------------------
       D.gfr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |  -.8712121   .5428397    -1.60   0.113    -1.955338    .2129137
------------------------------------------------------------------------------
```

The R-squared is identically zero since there are no explanatory variables. But $\hat{\sigma}$ which estimates the standard deviation of the error, is comparable to that in part (ii), and we see that it is much smaller here.) The t statistic for the intercept is about –1.60, which is not significant at the 10% level against a two-sided alternative. Therefore, it is legitimate to treat gfrt as having no drift, if it is indeed a random walk. (That is, if gfrt = α_0 + $gfrt_{t-1} + e_t$, where {et} is zero-mean, serially uncorrelated process, then we cannot reject H0: α_0 = 0.)

v) Now, forecast, gfr for 1980 through 1984, using a random walk model: the forecast of gfr_{n+1} is simply gfr_n. Find the MAE. How does it compare with the MAE from part iii). Which method of forecasting do you prefer?

Solution: Now we should do the forecasting for 1980-84, but this time under the assumption that GFR is a random walk process without drift. In other words, our forecast for period t+1 is the value in t.

Year	GFR actual (1)	GFR forecast (2)	Diff (1-2)	Absolute forecast error
1979	67.2			
1980	68.4	67.2	1.2	1.2
1981	67.4	68.4	-1	1.0
1982	67.3	67.4	-0.1	0.1
1983	65.8	67.3	-1.5	1.5
1984	65.4	65.8	-0.4	0.4

The MAE is 0.84, that is, the random-walk model outperforms the deterministic trend model in terms of forecasting accuracy. With stata, we can calculate the absolute forecasting errors as the absolute value of the differenced gfr series, since the gfr(t) is the actuals series and gfr(t-1) is the forecast for period t.

That is the forecast error for period t is: $\hat{e}_t = y_t - y_{t-1} = \Delta y_t$

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Hence the mean absolute error of the forecast is:

$$\frac{1}{5}\sum_{i=1980}^{1984}\hat{e}_i=\frac{1}{5}\sum_{i=1980}^{1984}\Delta y_i$$

We can calculate this in stata:

```
. gen absfe=abs(d.gfr) if year>1979
(67 missing values generated)

. sum absfe

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       absfe |         5    .8400009    .5770624   .0999985        1.5
```

vi) Now, estimate an AR(2) model for gfr, again using the data only through 1979. Is the second lag significant?

Solution: We need to estimate an AR(2) model for gfr until 1979.

```
. reg gfr l(1/2).gfr if year<1980

      Source |       SS       df       MS              Number of obs =      65
-------------+------------------------------           F(  2,    62) =  571.67
       Model |  20669.7236     2  10334.8618           Prob > F      =  0.0000
    Residual |  1120.86601    62   18.078484           R-squared     =  0.9486
-------------+------------------------------           Adj R-squared =  0.9469
       Total |  21790.5896    64  340.477963           Root MSE      =  4.2519

------------------------------------------------------------------------------
         gfr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gfr |
         L1. |   1.272076   .1203391    10.57   0.000     1.031522    1.512631
         L2. |  -.3113864   .1213988    -2.56   0.013    -.5540592   -.0687136
             |
       _cons |   3.215658   2.924166     1.10   0.276    -2.629667    9.060983
```

The second lag is significant. (Recall that its *t* statistic is valid even though *gfrt* apparently contains a unit root: the coefficients on the two lags sum to .961.) The standard error of the regression is slightly below that of the random walk model.

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vii) Obtain the MAE for 1980 through 1984, using the AR(2) model. Does this more general model work better out-of-sample than the random walk model?

Solution: Now we are asked to calculate the MAE for the forecasts from the AR(2) model.

Year	GFR actual (1)	GFR forecast (2)	Diff (1-2)	Absolute forecast error
1978	65.5			
1979	67.2			
1980	68.4	68.3	0.1	0.1
1981	67.4	69.3	-1.9	1.9
1982	67.3	67.7	-0.4	0.4
1983	65.8	67.8	-2.0	2.0
1984	65.4	66.0	-0.6	0.6

The out-of-sample forecasting performance of the AR(2) model is worse than the random walk without drift: the MAE for 1980 through 1984 is about .991 for the AR(2) model.

And the MAE is 1, which is slightly higher than the MAE from our forecast under the random walk assumption. Even though the AR(2) model seems more sophisticated than the random-walk, it cannot outperform the random-walk in terms of forecasting.

With stata:

```
. predict forecastar2 if year>1979, xb
(67 missing values generated)

. gen absfe2=abs(forecastar2-gfr) if year>1979
(67 missing values generated)

. mean absfe2

Mean estimation                     Number of obs    =       5

--------------------------------------------------------------
             |       Mean   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
      absfe2 |   .9905746   .4070746     -.1396457    2.120795
--------------------------------------------------------------
```