

Tutorials

Week 2

Regression Analysis: with time-series data

| Pdf file on Blackboard | Dataset on Blackboard | | Description |
|--|-----------------------|--|---|
| C 10.2: Note: variable “pet” in eq 10.22 is called “gas” in data | barium.dta | Krupp, C. M., & Pollard, P. S. (1996). Market Responses to Antidumping Laws: Some Evidence from the U.S. Chemical Industry. <i>The Canadian Journal of Economics / Revue Canadienne d'Economique</i> , 29(1), 199–227. https://doi.org/10.2307/136159 | <p>This exercise is related to example 10.5 from the book.</p> <p>Understanding the model of an economic situation, time series, adding time trends to the equation to analyze the change in the regression, use of dummies, meaning and test for heteroskedasticity, using of F-test to recognize the joint dummy variables, seasonal dummies, seasonal effects, test for seasonality, check for multicollinearity</p> <p>Extra exercises: serial correlation and FGLS vs. OLS</p> |
| C.12.1 | fertil3.dta | Whittington, L. A., Alm, J., & Peters, H. E. (1990). Fertility and the Personal Exemption: Implicit Pronatalist Policy in the United States. <i>The American Economic Review</i> , 80(3), 545–556. http://www.jstor.org/stable/2006683 | Weak assumption, strict exogeneity, serial correlation |
| C. 12 X | wageprc.dta | | Distributed lag model (DL), testing serial Correlation, correcting standard errors for AR(1) and AR(2), and differences between effects in the long run and short run. |

Relevant concept for Time Series?

- y_t : outcome of y (e.g. inflation) in period t : contemporaneous variable
- y_{t-1} : lag of 1 period : outcome of y in period $t-1$: lagged variable
- y_{t+1} : lead of 1 period : outcome of y in period $t+1$: lead variable
- Stationary vs. Dynamic models
- Trend
- Seasonality
- The assumption of the error term independently and identically distributed (iid)
- Spurious regression:
- Exogeneity:
- Autoregressive Models (AR)
- Autocorrelation (serial correlation)

Spurious regression

- If we don't correct for time, what will happen? It can be interpreted as an omitted variable if it is not added.
- That may lead to **spurious regressions** = the parameters become inconsistent; the parameters are biased if we don't control for time.
- In time series analyses, you should include t
- There might be an effect of x on y if the trend, t , is not included
- *If y and x are time-dependent*: and it is not controlled for time, then the predictions will be biased.
- More about spurious regression in week 3

Contemporaneous exogeneity vs. strict exogeneity

- Assumption of strict exogeneity:

$$E(u_t | X) = 0$$

u_t independent of all x : error term is independent in the past, present, future

- Contemporaneous exogeneity:

$$E(u_t | x_{t1}, \dots, x_{tk}) = 0$$

all x are unrelated to the error term in period t (a specific time)

Weak Dependency

- A stationary time series $\{x_t: t = 1, 2, \dots\}$ is weakly dependent if x_t and x_{t+h} are "almost independent" as $h \rightarrow \infty$. Thus,

$$\text{Corr}(x_t, x_{t+h}) \rightarrow 0 \text{ as } h \rightarrow \infty$$

C. 10.2 Use the data in BARIUM.RAW for this exercise.

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i) Add a linear time trend to equation 10.22 (below). Are any variables, other than the trend, statistically significant?

EXAMPLE 10.5

ANTIDUMPING FILINGS AND CHEMICAL IMPORTS

Krupp and Pollard (1996) analyzed the effects of antidumping filings by U.S. chemical industries on imports of various chemicals. We focus here on one industrial chemical, barium chloride, a cleaning agent used in various chemical processes and in gasoline production. The data are contained in the file BARIUM.RAW. In the early 1980s, U.S. barium chloride producers believed that China was offering its U.S. imports at an unfairly low price (an action known as *dumping*), and the barium chloride industry filed a complaint with the U.S. International Trade Commission (ITC) in October 1983. The ITC ruled in favor of the U.S. barium chloride industry in October 1984. There are several questions of interest in this case, but we will touch on only a few of them. First, were imports unusually high in the period immediately preceding the initial filing? Second, did imports change noticeably after an antidumping filing? Finally, what was the reduction in imports after a decision in favor of the U.S. industry?

To answer these questions, we follow Krupp and Pollard by defining three dummy variables: *befile6* is equal to 1 during the six months before filing, *affile6* indicates the six months after filing, and *afdec6* denotes the six months after the positive decision. The dependent variable is the volume of imports of barium chloride from China, *chnimp*, which we use in logarithmic form. We include as explanatory variables, all in logarithmic form, an index of chemical production, *chempi* (to control for overall demand for barium chloride), the volume of gasoline production, *gas* (another demand variable), and an exchange rate index, *rtwex*, which measures the strength of the dollar against several other currencies. The chemical production index was defined to be 100 in June 1977. The analysis here differs somewhat from Krupp and Pollard in that we use natural logarithms of all variables (except the dummy variables, of course), and we include all three dummy variables in the same regression.

Using monthly data from February 1978 through December 1988 gives the following:

$$\begin{aligned} \widehat{\log(chnimp)} = & -17.80 + 3.12 \log(chempi) + .196 \log(gas) \\ & (21.05) \quad (.48) \quad (.907) \\ & + .983 \log(rtwex) + .060 befile6 - .032 affile6 - .565 afdec6 \quad [10.22] \\ & (.400) \quad (.261) \quad (.264) \quad (.286) \\ n = & 131, R^2 = .305, \bar{R}^2 = .271. \end{aligned}$$

The equation shows that *befile6* is statistically insignificant, so there is no evidence that Chinese imports were unusually high during the six months before the suit was filed. Further, although the estimate on *affile6* is negative, the coefficient is small (indicating about a 3.2% fall in Chinese imports), and it is statistically very insignificant. The coefficient on *afdec6* shows a substantial fall in Chinese imports of barium chloride after the decision in favor of the U.S. industry, which is not surprising. Since the effect is so large, we compute the exact percentage change: $100[\exp(-.565) - 1] \approx -43.2\%$. The coefficient is statistically significant at the 5% level against a two-sided alternative.

The coefficient signs on the control variables are what we expect: an increase in overall chemical production increases the demand for the cleaning agent. Gasoline production does not affect Chinese imports significantly. The coefficient on $\log(rtwex)$ shows that an increase in the value of the dollar relative to other currencies increases the demand for Chinese imports, as is predicted by economic theory. (In fact, the elasticity is not statistically different from 1. Why?)



First we estimate the model:

```
. reg lchnimp lchempi lgas lrtwex befile6 affile6 afdec6
```

| Source | SS | df | MS | Number of obs = 131 | | |
|----------|------------|-----|------------|---------------------|---|--------|
| Model | 19.4051607 | 6 | 3.23419346 | F(6, 124) | = | 9.06 |
| Residual | 44.2470875 | 124 | .356831351 | Prob > F | = | 0.0000 |
| Total | 63.6522483 | 130 | .489632679 | R-squared | = | 0.3049 |
| | | | | Adj R-squared | = | 0.2712 |
| | | | | Root MSE | = | .59735 |

| lchnimp | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|---------|-----------|-----------|-------|-------|----------------------|----------|
| lchempi | 3.117193 | .4792021 | 6.50 | 0.000 | 2.168718 | 4.065668 |
| lgas | .1963504 | .9066172 | 0.22 | 0.829 | -1.598099 | 1.9908 |
| lrtwex | .9830183 | .4001537 | 2.46 | 0.015 | .1910022 | 1.775034 |
| befile6 | .0595739 | .2609699 | 0.23 | 0.820 | -.4569585 | .5761064 |
| affile6 | -.0324064 | .2642973 | -0.12 | 0.903 | -.5555249 | .490712 |
| afdec6 | -.565245 | .2858352 | -1.98 | 0.050 | -1.130993 | .0005028 |
| _cons | -17.803 | 21.04537 | -0.85 | 0.399 | -59.45769 | 23.85169 |

- lchnimp: log(chnimp)
- lgas: log(gas)
- lrtwex: log(rtwex)
- befile6: =1 for all 6 mos before filing
- affile6: =1 for all 6 mos after filing
- afdec6: =1 for all 6 mos after decision

Now the assignment asks us to include a linear trend in the specification:

```
. reg lchnimp lchempi lgas lrtwex befile6 affile6 afdec6 t
```

| Source | SS | df | MS | | | |
|----------|------------|-----|------------|-----------------|--------|--|
| Model | 23.0142898 | 7 | 3.28775569 | Number of obs = | 131 | |
| Residual | 40.6379584 | 123 | .330389906 | F(7, 123) = | 9.95 | |
| Total | 63.6522483 | 130 | .489632679 | Prob > F = | 0.0000 | |
| | | | | R-squared = | 0.3616 | |
| | | | | Adj R-squared = | 0.3252 | |
| | | | | Root MSE = | .5748 | |

| | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|----------|-----------|-----------|-------|-------|----------------------|----------|
| lchnimp | -.6862364 | 1.239711 | -0.55 | 0.581 | -3.140169 | 1.767696 |
| lchempi | .4656786 | .8761779 | 0.53 | 0.596 | -1.268662 | 2.200019 |
| lgas | .0782237 | .47244 | 0.17 | 0.869 | -.8569423 | 1.01339 |
| lrtwex | .09047 | .2512887 | 0.36 | 0.719 | -.4069406 | .5878805 |
| beffile6 | .0970062 | .2573131 | 0.38 | 0.707 | -.4123294 | .6063417 |
| afdec6 | -.3515018 | .2825417 | -1.24 | 0.216 | -.9107758 | .2077722 |
| t | .0127058 | .0038443 | 3.31 | 0.001 | .0050963 | .0203153 |
| _cons | -2.367526 | 20.78216 | -0.11 | 0.909 | -43.50455 | 38.7695 |

- Except for the time trend, we find that none of the explanatory variables yields statistically significant coefficients at 10%.
- A possible explanation is that the results in the original specification were driven by a common trend in our variables.
- Once we explicitly include the trend in the model, there is no statistical evidence for a structural relationship anymore between our explanatory variables and the dependent variable.

ii) In the equation estimated in part i), test for joint significance of all variables except the time trend. What do you conclude?

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Solution:

- The model is:

$$\log(\text{chinimp}_t) = \beta_0 + \beta_1 \log(\text{chempi}_t) + \beta_2 \log(\text{gas}_t) + \beta_3 \log(\text{rtwex}_t) + \beta_4 \text{befile6}_t + \beta_5 \text{affile6}_t + \beta_6 \text{afdec6}_t + \beta_7 t + u_t$$

- Hypotheses:

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$ (not jointly significant)

$H_1: H_0$ is not true. This occurs if at least one of the coefficients listed in H_0 differs from zero.

- If the $F_{\text{stat}} > F_{\text{cv}}$, then reject H_0 .
- $0.54 < 2.17$, we can not reject H_0 at the 5% significance level, and the variables are not jointly significant.

```
. test lchempi lgas lrtwex befile6 affile6 afdec6
```

```
( 1)  lchempi = 0
( 2)   lgas = 0
( 3)  lrtwex = 0
( 4) befile6 = 0
( 5) affile6 = 0
( 6) afdec6 = 0
```

```
F( 6, 123) = 0.54
Prob > F = 0.7767
```

ii) In the equation estimated in part i), test for joint significance of all variables except the time trend. What do you conclude?

Solution:

- Conclusion: the coefficients of the explanatory variables (except the linear trend) do not improve the model in a statistically significant way. They do not have explanatory power.
- The degrees of freedom of the numerator is 6, because we have 6 parameter restrictions in H_0 and the degrees of freedom of the denominator is $131 - 8 = 123$, because in the unrestricted model we estimated 8 parameters with sample size 131.

iii) Add monthly dummy variables to the equation and test for seasonality.

Does including the monthly dummies change any other estimated or their standard error in important ways?

We add 11-month dummies. Remember, one of the dummies must be omitted to avoid the dummy variable trap (perfect multicollinearity). In this case, the month January is excluded.

```
. reg lchnimp lchempi lgas lrtwex befile6 affile6 afdec6 t feb mar apr may jun jul aug sep oct nov dec
```

| Source | SS | df | MS | Number of obs = | 131 |
|----------|------------|-----|------------|-----------------|--------|
| Model | 26.1336807 | 18 | 1.45187115 | F(18, 112) = | 4.33 |
| Residual | 37.5185675 | 112 | .33498721 | Prob > F = | 0.0000 |
| Total | 63.6522483 | 130 | .489632679 | R-squared = | 0.4106 |
| | | | | Adj R-squared = | 0.3158 |
| | | | | Root MSE = | .57878 |

| lchnimp | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|---------|-----------|-----------|-------|-------|----------------------|----------|
| lchempi | -.4516555 | 1.271528 | -0.36 | 0.723 | -2.971026 | 2.067715 |
| lgas | -.820624 | 1.345056 | -0.61 | 0.543 | -3.485679 | 1.844431 |
| lrtwex | -.1971415 | .5295314 | -0.37 | 0.710 | -1.24634 | .852057 |
| befile6 | .1648509 | .2569789 | 0.64 | 0.523 | -.3443198 | .6740216 |
| affile6 | .1534004 | .2719856 | 0.56 | 0.574 | -.3855043 | .692305 |
| afdec6 | -.2950163 | .2994276 | -0.99 | 0.327 | -.8882937 | .298261 |
| t | .0123389 | .0039163 | 3.15 | 0.002 | .0045793 | .0200985 |
| feb | -.3554148 | .293754 | -1.21 | 0.229 | -.9374507 | .2266211 |
| mar | .062566 | .254858 | 0.25 | 0.807 | -.4424025 | .5675345 |
| apr | -.4406149 | .258398 | -1.71 | 0.091 | -.9525974 | .0713676 |
| may | .031299 | .2591998 | 0.12 | 0.904 | -.4822721 | .5448702 |
| jun | -.20095 | .2592134 | -0.78 | 0.440 | -.7145481 | .312648 |
| jul | .0111115 | .2683777 | 0.04 | 0.967 | -.5206446 | .5428675 |
| aug | -.1271137 | .2677917 | -0.47 | 0.636 | -.6577086 | .4034812 |
| sep | -.0751929 | .2583502 | -0.29 | 0.772 | -.5870807 | .4366949 |
| oct | .0797627 | .2570514 | 0.31 | 0.757 | -.4295517 | .5890771 |
| nov | -.2603032 | .2530623 | -1.03 | 0.306 | -.7617136 | .2411073 |
| dec | .0965326 | .2615525 | 0.37 | 0.713 | -.4217002 | .6147654 |
| _cons | 27.30007 | 31.39707 | 0.87 | 0.386 | -34.90919 | 89.50934 |

$$F(11, 112) = 0,85$$

$$\text{Prob} > F = 0,5943$$

$H_0: \delta_2 = 0, \delta_3 = 0, \dots, \delta_{12} = 0$ (no seasonality)

$H_1: H_0$ not true

If $F_{stat} > F_{cv}$, reject H_0

$0,85 < 1.91$ at 5% significance level.

We fail to reject the H_0 , there is no seasonal effect.

These exercises are not part of the book: Now check for homoskedasticity and serial correlation , considering strict exogeneity

```
. predict uhat, residuals
. gen uhat2=uhat^2
. reg uhat2 lchempi lgas lrtwex befile6 affile6 afdec6
```

| Source | SS | df | MS | Number of obs | = | 131 |
|----------|------------|-----|------------|---------------|---|--------|
| Model | 2.55224567 | 6 | .425374279 | F(6, 124) | = | 1.28 |
| Residual | 41.2386262 | 124 | .332569566 | Prob > F | = | 0.2717 |
| | | | | R-squared | = | 0.0583 |
| | | | | Adj R-squared | = | 0.0127 |
| Total | 43.7908719 | 130 | .33685286 | Root MSE | = | .57669 |

| uhat2 | Coefficient | Std. err. | t | P> t | [95% conf. interval] | |
|---------|-------------|-----------|-------|-------|----------------------|-----------|
| lchempi | -1.141681 | .4626243 | -2.47 | 0.015 | -2.057344 | -.2260176 |
| lgas | -.2023653 | .8752532 | -0.23 | 0.818 | -1.934737 | 1.530006 |
| lrtwex | -.3284217 | .3863105 | -0.85 | 0.397 | -1.093038 | .4361951 |
| befile6 | .0064 | .2519417 | 0.03 | 0.980 | -.4922632 | .5050633 |
| affile6 | .0158846 | .2551541 | 0.06 | 0.950 | -.4891368 | .520906 |
| afdec6 | .1584464 | .2759468 | 0.57 | 0.567 | -.3877297 | .7046224 |
| _cons | 11.95703 | 20.31732 | 0.59 | 0.557 | -28.25663 | 52.17069 |

```
. hettest, rhs fstat
```

Breusch-Pagan/Cook-Weisberg test for heteroskedasticity
Assumption: i.i.d. error terms
Variables: All independent variables

H0: Constant variance

F(6, 124) = 0.80
Prob > F = 0.5717

H0: $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = 0$

H1: H0 is not true (heteroskedasticity)

If Ftest > Fcv at 5%, then reject H0

0.80 < 2.10 we fail to reject H0.

Conclusion: homoskedasticity holds.

OLS no longer provides an unbiased estimator in the presence of **serial correlation**, leading to invalid t-statistics.

```
. tsset t
```

Time variable: t, 1 to 131

Delta: 1 unit

```
. reg uhat l.uhat lchempi lgas lrtwex befile6 affile6 afdec6
```

| Source | SS | df | MS | Number of obs | = | 130 |
|----------|------------|-----|------------|---------------|---|--------|
| Model | 3.31972295 | 7 | .474246136 | F(7, 122) | = | 1.41 |
| Residual | 40.9269683 | 122 | .335466954 | Prob > F | = | 0.2058 |
| | | | | R-squared | = | 0.0750 |
| | | | | Adj R-squared | = | 0.0220 |
| Total | 44.2466913 | 129 | .342997607 | Root MSE | = | .5792 |

| uhat | Coefficient | Std. err. | t | P> t | [95% conf. interval] | |
|-------------|-------------|-----------|-------|-------|----------------------|----------|
| uhat L1. | .2772056 | .0881206 | 3.15 | 0.002 | .1027622 | .4516491 |
| lchempi | -.0147775 | .4680184 | -0.03 | 0.975 | -.9412666 | .9117116 |
| lgas | .3318712 | .8927836 | 0.37 | 0.711 | -1.435483 | 2.099225 |
| lrtwex | .1046564 | .3919403 | 0.27 | 0.790 | -.6712285 | .8805414 |
| beffile6 | -.0268293 | .253184 | -0.11 | 0.916 | -.5280324 | .4743737 |
| affile6 | -.0654893 | .2571084 | -0.25 | 0.799 | -.574461 | .4434824 |
| afdec6 | -.0398831 | .2774596 | -0.14 | 0.886 | -.589142 | .5093758 |
| _cons | -7.993576 | 20.79624 | -0.38 | 0.701 | -49.16181 | 33.17466 |

$H_0: \rho = 0$ (no 1st order serial correlation)

$H_1: \rho \neq 0$ (serial correlation)

If $|t| > t_{cv}$, reject H_0 .

$3.15 > 1.980$ at 5%. Therefore, we reject H_0 .

There is evidence of serial correlation.

These exercises are not part of the book:

Apply the Prais-Winsten Test and compare OLS vs. FGLS coefficients:

Can we argue - based on the OLS estimators - that the effect of the Chinese imports (afdec6) after the International Trade Commission's decision is less statistically significant? (t-statistics : -1.98)

- Serial correlation is also called autocorrelation and means that, conditional on X , the error term is correlated over subsequent time periods.

$$\text{Corr}(\varepsilon_t, \varepsilon_{t-1} | X) \neq 0$$

- Positive serial correlation: $\text{Corr}(\varepsilon_t, \varepsilon_{t-1}) > 0$, the error in period t is likely to be positive if the error in period $t-1$ is positive.
- Negative serial correlation: $\text{Corr}(\varepsilon_t, \varepsilon_{t-1}) < 0$, the error in period t is likely to be negative if the error in period $t-1$ is positive.

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \varepsilon_t$$

- Consider the model:

$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$$

- If the error term of this model is serially correlated, we can write:
- ρ shows **the correlation between ε_t and ε_{t-1}**
- Thus $|\rho| \leq 1$
- The closer $|\rho|$ is to 1, the stronger the serial correlation.
- If $\rho = 0$, there is no serial correlation.
- ε_t depends on ε_{t-1} ; u_t is a serially correlated error term with a mean of zero and a constant variance.
- This is first order serial correlation because the error in t depends on the error in $t-1$

Hypothesis for non serial correlation:

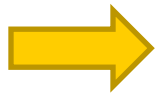
$$H_0 = \rho = 0$$

$$H_1 = \rho \neq 0$$

What are the consequences of serial correlation

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- If serial correlation exists, then $\widehat{\sigma}^2$ and therefore s.e. $\widehat{\beta}_k$ are incorrect.
- Hence, t-test and F-tests are invalid: cannot perform hypothesis tests.
- Important to know:
- Serial correlation in static or finite distributed lag models cause biased standard errors
- However, serial correlation does not cause bias in the estimated coefficients – with one important exception:



Serial correlation in autoregressive models (i.e. models with a lagged dependent variable) cause:



Biased standard errors



Biased coefficient estimates

How to test for serial correlation?

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Two commonly used tests for serial correlation:

- Durbin-Watson test
- Breusch-Godfrey test

What to do if there is serial correlation?

- Add more lags
- In terms of economics, the t statistics are wrong with autocorrelation. Therefore, reconsidering the dynamic structure can help.

Breusch-Godfrey test (BG-test)

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We want to test for first-order serial correlation in the following time series model:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \cdots + \beta_k X_{kt} + \varepsilon_t$$

$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t \quad \text{with } |\rho| < 1$$

Stata commands:

1. Estimate the equation above.
`. reg consumption price income temp`
2. Compute the residuals e_t from the estimated equation
`. predict uhat, resid`
3. Regress e_t on the 1-period lagged residual e_{t-1} and all independent variables from the equation above.
`. reg uhat 1.uhat price income temp`
4. Use the t-value on e_{t-1} to test:

H_0 : $\rho = 0$ no 1st-order serial correlation

H_A : $\rho \neq 0$ 1st-order serial correlation

C. 12.1 In example 11.6, we estimated a finite DL model in first differences:

$$\Delta gfr_t = \gamma_0 + \delta_0 \Delta pe_t + \delta_1 \Delta pe_{t-1} + \delta_2 \Delta pe_{t-2} + u_t$$

Use the data FERTILE3.RAW to test whether there is AR(1) serial correlation in the errors

First run the regression and retrieve the residuals “uhat”

```
. tset t
      time variable: t, 1 to 72
      delta: 1 unit
```

```
. reg D. gfr D.pe L1.D.pe L2.D.pe
```

| Source | SS | df | MS | Number of obs = | 69 |
|----------|------------|----|------------|-----------------|--------|
| Model | 293.259859 | 3 | 97.7532864 | F(3, 65) = | 6.56 |
| Residual | 968.199959 | 65 | 14.895384 | Prob > F = | 0.0006 |
| Total | 1261.45982 | 68 | 18.5508797 | R-squared = | 0.2325 |
| | | | | Adj R-squared = | 0.1971 |
| | | | | Root MSE = | 3.8595 |

| D.gfr | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|-------|-----------|-----------|-------|-------|----------------------|
| pe | | | | | |
| D1. | -.0362021 | .0267737 | -1.35 | 0.181 | -.089673 .0172687 |
| LD. | -.0139706 | .0275539 | -0.51 | 0.614 | -.0689997 .0410584 |
| L2D. | .1099896 | .0268797 | 4.09 | 0.000 | .0563071 .1636721 |
| _cons | -.9636787 | .4677599 | -2.06 | 0.043 | -1.89786 -.0294976 |

```
. predict uhat, residual
(3 missing values generated)
```

11.6 FERTILITY EQUATION

In Example 10.4, we explained the general fertility rate, gfr , in terms of the value of the personal exemption, pe . The first order autocorrelations for these series are very large: $\hat{\rho}_1 = .977$ for gfr and $\hat{\rho}_1 = .964$ for pe . These autocorrelations are highly suggestive of unit root behavior, and they raise serious questions about our use of the usual OLS t statistics for this example back in Chapter 10. Remember, the t statistics only have exact t distributions under the full set of classical linear model assumptions. To relax those assumptions in any way and apply asymptotics, we generally need the underlying series to be $I(0)$ processes.

We now estimate the equation using first differences (and drop the dummy variable, for simplicity):

$$\begin{aligned} \Delta \widehat{gfr} &= -.785 - .043 \Delta pe \\ &\quad (.502) \quad (.028) \\ n = 71, R^2 &= .032, \bar{R}^2 = .018. \end{aligned} \quad [11.26]$$

Now, an increase in pe is estimated to lower gfr contemporaneously, although the estimate is not statistically different from zero at the 5% level. This gives very different results than when we estimated the model in levels, and it casts doubt on our earlier analysis.

If we add two lags of Δpe , things improve:

$$\begin{aligned} \Delta \widehat{gfr} &= -.964 - .036 \Delta pe - .014 \Delta pe_{-1} + .110 \Delta pe_{-2} \\ &\quad (.468) \quad (.027) \quad (.028) \quad (.027) \\ n = 69, R^2 &= .233, \bar{R}^2 = .197. \end{aligned} \quad [11.27]$$

Even though Δpe and Δpe_{-1} have negative coefficients, their coefficients are small and jointly insignificant (p -value = .28). The second lag is very significant and indicates

Perform the **Breusch-Godfrey** test, for first-order autocorrelation:

First, under the assumption of strict exogeneity:

```
. reg uhat L.uhat
```

| Source | SS | df | MS | | | |
|----------|------------|----|------------|-----------------|--------|--|
| Model | 82.0925482 | 1 | 82.0925482 | Number of obs = | 68 | |
| Residual | 885.696544 | 66 | 13.4196446 | F(1, 66) = | 6.12 | |
| Total | 967.789092 | 67 | 14.4446133 | Prob > F = | 0.0160 | |
| | | | | R-squared = | 0.0848 | |
| | | | | Adj R-squared = | 0.0710 | |
| | | | | Root MSE = | 3.6633 | |

| uhat | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|-------------|----------|-----------|------|-------|----------------------|----------|
| uhat L1. | .2918091 | .1179825 | 2.47 | 0.016 | .0562495 | .5273687 |
| _cons | .0180209 | .4442522 | 0.04 | 0.968 | -.8689571 | .904999 |

$$E(u_t | X) = 0.$$

u_t independent of all x: error term is independent in the past, present, future

Second, under the assumption of weak exogeneity:

$$E(u_t | x_{t1}, \dots, x_{tk}) = 0.$$

all x are unrelated to the error term in period t (a specific time)

```
. reg uhat L.uhat D.pe L1.D.pe L2.D.pe
```

| Source | SS | df | MS |
|----------|------------|----|------------|
| Model | 83.1528991 | 4 | 20.7882248 |
| Residual | 884.636193 | 63 | 14.0418443 |
| Total | 967.789092 | 67 | 14.4446133 |

Number of obs = 68
 F(4, 63) = 1.48
 Prob > F = 0.2188
 R-squared = 0.0859
 Adj R-squared = 0.0279
 Root MSE = 3.7472

| uhat | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|-------|-----------|-----------|-------|-------|----------------------|----------|
| uhat | | | | | | |
| → L1. | .29556 | .1214561 | 2.43 | 0.018 | .0528494 | .5382706 |
| pe | | | | | | |
| D1. | -.0071763 | .0261613 | -0.27 | 0.785 | -.0594554 | .0451028 |
| LD. | .0015739 | .0267612 | 0.06 | 0.953 | -.0519041 | .055052 |
| L2D. | -.0009125 | .0261017 | -0.03 | 0.972 | -.0530725 | .0512476 |
| _cons | .0262151 | .4575848 | 0.06 | 0.954 | -.8881952 | .9406254 |

$H_0: \rho = 0$ (no 1st order serial correlation)

$H_1: \rho \neq 0$ (serial correlation)

If $|t| > t_{cv}$, then reject H_0

$2.43 > 2.00$, reject H_0 .

There is evidence for serial correlation.

Also with the p-value:

Pvalue: $0.018 < 0.05$, reject H_0

We reject the H_0 of no first-order autocorrelation at 5% level of significance because lagged uhat is significant at 5% level (p-value = 0,018)

Conclusion: there is empirical evidence in favor of first-order autocorrelation. We need to correct standard errors using e.g. the Newey estimator.

After that, apply the Newey Correction / estimator (OLS + adjusted standard errors):

```
. newey D. gfr D.pe L1.D.pe L2.D.pe, lag(1)
```

Regression with Newey-West standard errors
maximum lag: 1

Number of obs = 69
F(3, 65) = 8.19
Prob > F = 0.0001

| D.gfr | Coef. | Newey-West Std. Err. | t | P> t | [95% Conf. Interval] | |
|-------|-----------|-------------------------|-------|-------|----------------------|----------|
| pe | | | | | | |
| D1. | -.0362021 | .0371648 | -0.97 | 0.334 | -.1104253 | .0380211 |
| LD. | -.0139706 | .0345874 | -0.40 | 0.688 | -.0830465 | .0551052 |
| L2D. | .1099896 | .0276013 | 3.98 | 0.000 | .0548661 | .1651131 |
| _cons | -.9636787 | .5219555 | -1.85 | 0.069 | -2.006096 | .0787384 |

i) Using the data in WAGEPRC.DTA, estimate the distributed lag-model from problem 11.5 . Using regressions (12.14) to test for AR(1) serial correlation.

11.5 For the U.S. economy, let $gprice$ denote the monthly growth in the overall price level and let $gwage$ be the monthly growth in hourly wages. [These are both obtained as differences of logarithms: $gprice = \Delta \log(price)$ and $gwage = \Delta \log(wage)$.] Using the monthly data in WAGEPRC.RAW, we estimate the following distributed lag model:

$$\begin{aligned} \widehat{gprice} = & -.00093 + .119 \, gwage + .097 \, gwage_{-1} + .040 \, gwage_{-2} \\ & (.00057) \quad (.052) \quad \quad (.039) \quad \quad (.039) \\ & + .038 \, gwage_{-3} + .081 \, gwage_{-4} + .107 \, gwage_{-5} + .095 \, gwage_{-6} \\ & (.039) \quad \quad (.039) \quad \quad (.039) \quad \quad (.039) \\ & + .104 \, gwage_{-7} + .103 \, gwage_{-8} + .159 \, gwage_{-9} + .110 \, gwage_{-10} \\ & (.039) \quad \quad (.039) \quad \quad (.039) \quad \quad (.039) \\ & \quad \quad \quad + .103 \, gwage_{-11} + .016 \, gwage_{-12} \\ & \quad \quad \quad (.039) \quad \quad (.052) \\ n = 273, R^2 = .317, \bar{R}^2 = .283. \end{aligned}$$

We first estimate the distributed lagged (DL) model and next, we use the Breusch-Godfrey test for autocorrelation (to test for AR(1) serial correlation)

Just to remember: lags help us to mitigate the statistical problems of autocorrelation.
What we will see are dynamic regression models.

```
tsset t
      time variable: t, 1 to 286
      delta: 1 unit

. reg gprice gwage gwage_1-gwage_12
```

| Source | SS | df | MS | Number of obs | = | 273 |
|----------|------------|-----|------------|---------------|---|--------|
| Model | .000981458 | 13 | .000075497 | F(13, 259) | = | 9.25 |
| Residual | .002113658 | 259 | 8.1608e-06 | Prob > F | = | 0.0000 |
| Total | .003095116 | 272 | .000011379 | R-squared | = | 0.3171 |
| | | | | Adj R-squared | = | 0.2828 |
| | | | | Root MSE | = | .00286 |

| gprice | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] |
|----------|-----------|-----------|-------|-------|----------------------|
| gwage | .1190416 | .0517725 | 2.30 | 0.022 | .0170929 .2209903 |
| gwage_1 | .0972174 | .0390409 | 2.49 | 0.013 | .0203393 .1740954 |
| gwage_2 | .0399518 | .0390717 | 1.02 | 0.307 | -.0369869 .1168905 |
| gwage_3 | .0382652 | .0391513 | 0.98 | 0.329 | -.0388301 .1153605 |
| gwage_4 | .0813362 | .0393483 | 2.07 | 0.040 | .0038528 .1588195 |
| gwage_5 | .106852 | .0391937 | 2.73 | 0.007 | .0296731 .1840308 |
| gwage_6 | .0949731 | .0392186 | 2.42 | 0.016 | .0177451 .1722011 |
| gwage_7 | .1037922 | .0393788 | 2.64 | 0.009 | .0262488 .1813355 |
| gwage_8 | .1025629 | .0394884 | 2.60 | 0.010 | .0248037 .180322 |
| gwage_9 | .1585079 | .0393341 | 4.03 | 0.000 | .0810526 .2359632 |
| gwage_10 | .1104412 | .0392229 | 2.82 | 0.005 | .0332049 .1876776 |
| gwage_11 | .1033206 | .0394388 | 2.62 | 0.009 | .0256591 .180982 |
| gwage_12 | .0156575 | .0518343 | 0.30 | 0.763 | -.0864128 .1177278 |
| _cons | -.0009296 | .0005662 | -1.64 | 0.102 | -.0020445 .0001853 |

Breush-Godfrey test for first-order autocorrelation, assuming weak/contemporaneous exogeneity (and not strict exogeneity):

```
. predict uhat, residual
(13 missing values generated)
```

```
. reg uhat L.uhat gwage L. gwage L2. gwage L3. gwage L4. gwage L5. gwage L6. gwage L7. gwage L8. gwage
```

| Source | SS | df | MS | Number of obs = | 272 |
|----------|------------|-----|------------|-----------------|--------|
| Model | .00054188 | 14 | .000038706 | F(14, 257) = | 6.38 |
| Residual | .00155816 | 257 | 6.0629e-06 | Prob > F = | 0.0000 |
| Total | .002100039 | 271 | 7.7492e-06 | R-squared = | 0.2580 |
| | | | | Adj R-squared = | 0.2176 |
| | | | | Root MSE = | .00246 |

| uhat | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|-------|-----------|-----------|-------|-------|----------------------|----------|
| uhat | | | | | | |
| L1. | .5106527 | .0540557 | 9.45 | 0.000 | .4042042 | .6171013 |
| gwage | | | | | | |
| --. | -.049533 | .0453685 | -1.09 | 0.276 | -.1388743 | .0398084 |
| L1. | -.0102465 | .0337572 | -0.30 | 0.762 | -.0767223 | .0562294 |
| L2. | -.0045952 | .0338253 | -0.14 | 0.892 | -.0712052 | .0620148 |
| L3. | .002263 | .0337474 | 0.07 | 0.947 | -.0641936 | .0687196 |
| L4. | .0003814 | .0340404 | 0.01 | 0.991 | -.0666522 | .0674151 |
| L5. | .0070267 | .0339435 | 0.21 | 0.836 | -.0598162 | .0738696 |
| L6. | -.000841 | .0338047 | -0.02 | 0.980 | -.0674105 | .0657285 |
| L7. | -.0051064 | .0339603 | -0.15 | 0.881 | -.0719824 | .0617696 |
| L8. | .0041929 | .0340396 | 0.12 | 0.902 | -.0628392 | .0712249 |
| L9. | .0008098 | .0339042 | 0.02 | 0.981 | -.0659557 | .0675752 |
| L10. | .0027298 | .0338628 | 0.08 | 0.936 | -.0639541 | .0694136 |
| L11. | .0010463 | .0340397 | 0.03 | 0.976 | -.0659859 | .0680784 |
| L12. | .0310227 | .0451507 | 0.69 | 0.493 | -.0578898 | .1199351 |
| _cons | .0001144 | .0004896 | 0.23 | 0.815 | -.0008497 | .0010786 |

$H_0 = \rho = 0$, no serial correlation

$H_1 = \rho \neq 0$, serial correlation

$|t\text{-value}| > t\text{-critical value}$, reject H_0 .

9.45 > 1.960 (5% significance level)

We reject H_0 , there is significant evidence for serial correlation among the residuals.

Conclude: the empirical evidence is in favor of first order autocorrelation (test based on the results corresponding the uhat L.1)

Also applying the Stata command for the Breush-Godfrey test for first-order autocorrelation

```
. estat bgodfrey
```

Breusch-Godfrey LM test for autocorrelation

| lags(p) | chi2 | df | Prob > chi2 |
|---------|--------|----|-------------|
| 1 | 12.021 | 1 | 0.0005 |

H0: no serial correlation

H_0 = *no serial correlation*

H_1 = *serial correlation*

If $\chi^2 > \chi^2_{cv}$, at 5% significance level, reject H_0

$12.021 > 3.84$, reject H_0 .

There is enough evidence in favor of serial correlation

ii) Reestimate the model using standard errors that are corrected for the presence of first and/or second-order autocorrelation. You may use the “newey” stata-command for this. Does it matter if you correct for AR(1) or AR(2) error terms?

If there is serial correlation, we need them to correct standard error. How? To correct autocorrelation, we need use the Newey estimator of the Prais-Winston. The Newey correction/estimator (OLS + adjust standard errors)

```
newey gprice gwage gwage_1-gwage_12, lag(1)
est store model1
```

Regression with Newey-West standard errors
maximum lag: 1

Number of obs = 273
F(13, 259) = 7.45
Prob > F = 0.0000

| gprice | Coef. | Newey-West Std. Err. | t | P> t | [95% Conf. Interval] | |
|----------|-----------|-------------------------|-------|-------|----------------------|----------|
| gwage | .1190416 | .0613043 | 1.94 | 0.053 | -.0016767 | .2397598 |
| gwage_1 | .0972174 | .0346645 | 2.80 | 0.005 | .0289572 | .1654775 |
| gwage_2 | .0399518 | .0385938 | 1.04 | 0.302 | -.0360458 | .1159493 |
| gwage_3 | .0382652 | .0372509 | 1.03 | 0.305 | -.0350879 | .1116184 |
| gwage_4 | .0813362 | .040698 | 2.00 | 0.047 | .001195 | .1614773 |
| gwage_5 | .106852 | .041456 | 2.58 | 0.011 | .0252183 | .1884856 |
| gwage_6 | .0949731 | .0452216 | 2.10 | 0.037 | .0059243 | .1840219 |
| gwage_7 | .1037922 | .0390687 | 2.66 | 0.008 | .0268595 | .1807248 |
| gwage_8 | .1025629 | .0403154 | 2.54 | 0.012 | .0231751 | .1819506 |
| gwage_9 | .1585079 | .042514 | 3.73 | 0.000 | .0747908 | .2422249 |
| gwage_10 | .1104412 | .0430844 | 2.56 | 0.011 | .0256009 | .1952816 |
| gwage_11 | .1033206 | .0471255 | 2.19 | 0.029 | .0105226 | .1961186 |
| gwage_12 | .0156575 | .0538974 | 0.29 | 0.772 | -.0904754 | .1217904 |
| _cons | -.0009296 | .0006177 | -1.50 | 0.134 | -.002146 | .0002868 |

```
newey gprice gwage gwage_1-gwage_12, lag(2)
est store model2
```

Regression with Newey-West standard errors
maximum lag: 2

Number of obs = 273
F(13, 259) = 5.96
Prob > F = 0.0000

| gprice | Coef. | Newey-West Std. Err. | t | P> t | [95% Conf. Interval] | |
|----------|-----------|-------------------------|-------|-------|----------------------|----------|
| gwage | .1190416 | .0596428 | 2.00 | 0.047 | .001595 | .2364881 |
| gwage_1 | .0972174 | .0360511 | 2.70 | 0.007 | .0262268 | .1682079 |
| gwage_2 | .0399518 | .0390969 | 1.02 | 0.308 | -.0370364 | .11694 |
| gwage_3 | .0382652 | .0381432 | 1.00 | 0.317 | -.0368451 | .1133755 |
| gwage_4 | .0813362 | .0409207 | 1.99 | 0.048 | .0007566 | .1619157 |
| gwage_5 | .106852 | .0412433 | 2.59 | 0.010 | .0256371 | .1880668 |
| gwage_6 | .0949731 | .0455674 | 2.08 | 0.038 | .0052435 | .1847028 |
| gwage_7 | .1037922 | .0399055 | 2.60 | 0.010 | .0252117 | .1823727 |
| gwage_8 | .1025629 | .0424289 | 2.42 | 0.016 | .0190133 | .1861124 |
| gwage_9 | .1585079 | .0442851 | 3.58 | 0.000 | .0713033 | .2457125 |
| gwage_10 | .1104412 | .0438436 | 2.52 | 0.012 | .0241059 | .1967765 |
| gwage_11 | .1033206 | .0461922 | 2.24 | 0.026 | .0123604 | .1942807 |
| gwage_12 | .0156575 | .050763 | 0.31 | 0.758 | -.0843033 | .1156183 |
| _cons | -.0009296 | .0006932 | -1.34 | 0.181 | -.0022946 | .0004355 |

```
est table model1 model2, se
```

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```
. reg uhat l1.uhat l2.uhat gwage gwage_1-gwage_12
```

| Source | SS | df | MS | Number of obs | = | 271 |
|----------|------------|-----|------------|---------------|---|--------|
| Model | .000603529 | 15 | .000040235 | F(15, 255) | = | 6.88 |
| Residual | .00149082 | 255 | 5.8464e-06 | Prob > F | = | 0.0000 |
| | | | | R-squared | = | 0.2882 |
| | | | | Adj R-squared | = | 0.2463 |
| Total | .002094349 | 270 | 7.7568e-06 | Root MSE | = | .00242 |

| uhat | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|----------|-----------|-----------|-------|-------|----------------------|----------|
| uhat | | | | | | |
| l1. | .4038757 | .0617549 | 6.54 | 0.000 | .2822611 | .5254902 |
| l2. | .2082125 | .0614613 | 3.39 | 0.001 | .087176 | .329249 |
| gwage | -.0449009 | .044572 | -1.01 | 0.315 | -.1326771 | .0428753 |
| gwage_1 | -.0218605 | .0334011 | -0.65 | 0.513 | -.0876376 | .0439167 |
| gwage_2 | -.0065564 | .0333377 | -0.20 | 0.844 | -.0722087 | .0590959 |
| gwage_3 | .0011153 | .0332849 | 0.03 | 0.973 | -.064433 | .0666635 |
| gwage_4 | .0007893 | .033429 | 0.02 | 0.981 | -.0650429 | .0666214 |
| gwage_5 | .0080186 | .0334868 | 0.24 | 0.811 | -.0579273 | .0739645 |
| gwage_6 | .0000823 | .0333798 | 0.00 | 0.998 | -.0656529 | .0658175 |
| gwage_7 | -.0060688 | .0333518 | -0.18 | 0.856 | -.0717489 | .0596112 |
| gwage_8 | .0025501 | .0334564 | 0.08 | 0.939 | -.063336 | .0684362 |
| gwage_9 | .0032809 | .0333054 | 0.10 | 0.922 | -.0623077 | .0688695 |
| gwage_10 | .0035026 | .0332565 | 0.11 | 0.916 | -.0619897 | .068995 |
| gwage_11 | .0005685 | .0334864 | 0.02 | 0.986 | -.0653766 | .0665135 |
| gwage_12 | .0271583 | .0443647 | 0.61 | 0.541 | -.0602096 | .1145262 |
| _cons | .000176 | .0004821 | 0.36 | 0.715 | -.0007735 | .0011254 |

- By comparing this table AR(2) with the AR(1) we recognize that the S.E. did change a few, but is not really significant.
- We can conclude that AR(2) shows more empirical evidence in favor of AR(2)
- Does it matter if you correct the error term for AR(1) or AR(2)? No, almost not.



iii) Given the results of ii), what are the long run effects?

These can be obtained using “nlcom”command, but it could be done manually too.

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Short-run effect: The contemporaneous effect (or short-run effect) is the parameter that registers the effect of the independent variable on the dependent one in the same period. It refers to the same period t . Short run effect here is $\delta_1=0.12$

Long-run effect includes the change in the dependent variable in all periods due to changes in the independent variables. effect of the x on y . use the coefficient of the AR(2)

In stata: `nlcom (_b[xt])+ _b([1.x])+ _b([12.x]).....`
but first `reg of AR(2) error terms`

```
. nlcom _b[gwage] + _b[gwage_1] + _b[gwage_2] + _b[gwage_3] + _b[gwage_4] + _b[gwage_5] +  
_b[gwage_6] + _b[gwage_7] + _b[  
> gwage_8] + _b[gwage_9] + _b[gwage_10] + _b[gwage_11] + _b[gwage_12]
```

```
      _nl_1:  _b[gwage] + _b[gwage_1] + _b[gwage_2] + _b[gwage_3] + _b[gwage_4] + _b[  
gwage_5] + _b[gwage_6] + _b[gwage_7  
> ] + _b[gwage_8] + _b[gwage_9] + _b[gwage_10] + _b[gwage_11] + _b[gwage_12]
```

| gprice | Coef. | Std. Err. | z | P> z | [95% Conf. Interval] | |
|-------------|----------|-----------|------|-------|----------------------|----------|
| -----+----- | | | | | | |
| _nl_1 | 1.171919 | .1479148 | 7.92 | 0.000 | .8820116 | 1.461827 |



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