Macroeconomics I: Problem Set 2

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1 Problem 1: Preference Shocks and the Stock Market

1.1 Write down the sequential problem of the representative household.

The sequential problem of the household is:

$$\max_{\{c[z_t],\theta[z_t]\}} \{U(\{c[z_t]\}_{z^t \in \mathcal{Z}, t \in \mathcal{T}})\} \text{ s.t.}$$

$$a[z(0)|z^t] = 0 \text{ for all } z \in \mathcal{Z}, t \in \mathcal{T}$$

$$\theta[z^t] \ge 0 \text{ for all } t$$

$$c^h(z^t) \le \theta(z^{t-1}) \cdot d(t) - (\theta(z^t) - \theta(z^{t-1}))P(z^t)$$

1.2 Carefully define a sequential competitive equilibrium for this economy.

A sequential equilibrium occurs when:

- 1. $\{c(z^t), \theta(z^t)\}_{z^t \in \mathcal{Z}(z^0)}$ solves the sequential household problem;
- 2. Markets clear:

$$c(z^t) = 1$$
 for all $z^t \in \mathcal{Z}(z^0)$
 $\theta(z^t) = 1$ for all $z^t \in \mathcal{Z}(z^0)$

1.3 An optimal plan for the household must satisfy the first-order condition [equation in assignment]. Provide an economic interpretation of this condition.

The term on the left-hand side provides the expected marginal utility from one infinitesimal unit of consumption should one decide to sell an infinitesimal additional share of the firm. At the optimum, the agent is indifferent between this number, and keeping that infinitesimal share of the firm and not consuming it, and saving it to the next period, receiving the dividends from the firm, and then the same problem resurfaces again. In this first order condition, this dilemma is seen from the perspective of period 0, hence the discounting.

1.4 Impose market clearing in the first-order condition. Guess that the resulting equation has a solution for stock prices where the stock price only depends on the current event. Show that given this guess the stock prices P(g) and P(b) must satisfy the following system of two linear equations in two unknowns [see equations in assignment].

We impose market clearing in the first-order condition by setting $c[z(t)] = c[z^t, z(t+1)] = 1 \ \forall z^t \in \mathcal{Z}$. This results in a new first order condition given by:

$$\beta^t q[z^t|z^0]\psi(z^t)u_c(1)P[z_t] = \sum_{z(t+1)\in\mathcal{Z}} \beta^{t+1}q[z^t,z(t+1)|z^0]\psi[z(t+1)]u_c(1)(P[z^t,z(t+1)]+1)$$

Then, after realizing that marginal utility is the same and can thus be canceled out, we can simplify the FOC to:

$$\beta^t q[z^t|z^0]\psi(z^t)P[z_t] = \sum_{z(t+1)\in\mathcal{Z}} \beta^{t+1} q[z^t, z(t+1)|z^0]\psi[z(t+1)](P[z^t, z(t+1)] + 1)$$

which after summing over the possible histories at $t+1 \in \{b, g\}$ gives:

$$\beta^t q[z^t | z^0] \psi(z^t) P[z_t] = \beta^{t+1} q[z^t, b | z^0] \psi(b) [P(b) + 1] + \beta^{t+1} q[z^t, g | z^0] \psi(g) [P(g) + 1]$$

using the stationarity of prices, i.e. P(g) is the same regardless of preceding states, and likewise for P(b).

Then, guessing that the prices in a particular z^t only depend on the particular event in that period z(t), and that z(t) can take only two values, evaluating them for both $z(t) = \{g\}$ and $z(t) = \{b\}$, and writing the sum over the future period histories explicitly gives us, first for $z(t) = \{g\}$:

$$\beta^{t}q(g)\psi(g)P[g] = \beta^{t+1}q(g)q(g)\psi(g)[P(g)+1] + \beta^{t+1}q(g)q(b)\psi(b)[P(b)+1]$$

After realizing that $\beta^{t+1} = \beta \cdot \beta^t$, dividing both sides by $\beta^t q(g) \psi(g)$, and using the definitions of q and ρ gives:

$$P[g] = \beta \left[\phi(P[g] + 1) + (1 - \phi)\rho(P[b] + 1) \right]$$

And secondly for $z(t) = \{b\}$:

$$\beta^t q(b)\psi(b)P[b] = \beta^{t+1}q(b)q(q)\psi(q)[P(q)+1] + \beta^{t+1}q(b)q(b)\psi(b)[P(b)+1]$$

Now dividing both sides by $\beta^t q(b)\psi(b)$ and using the definitions of ρ and q gives the desired answer:

$$P[b] = \beta \left[\phi \frac{1}{\rho} (P[g] + 1) + (1 - \phi)(P[b] + 1) \right]$$

1.5 Does a switch to state b cause a crash or a boom in the stock price in terms of units of current consumption? Discuss the economic intuition for your result.

At the optimum, the first order condition tells us that the stock price in a particular state must be such that the household is indifferent between consuming extra, and forgoing consumption for at least one period and facing the same dilemma. If we are in a bad state, the consumer experiences a preference shock, such that the value for present-day consumption is higher. At the old stock price, then, the consumer would now prefer to sell the entire firm and consume immediately. To offset this increased tendency to consume, the stock price must also be higher to make the consumer indifferent again. Hence, a switch to state b causes a boom in the stock price.

2 Problem 2: Piketty's Second Fundamental Law of Capitalism

2.1 Show that

$$\dot{K}(t) = s^{net}(t)Y^{net}(t)$$

and give an economic interpretation of this equation.

We are given that

$$\begin{split} \dot{K}(t) &= Y(t) - C(t) - \Delta(t) \\ &= Y^{net} - C(t), \quad \text{where } Y^{net} = Y(t) - \Delta(t) \\ &= \frac{Y^{net}(t) - C(t)}{Y^{net}} Y^{net}(t) \\ &= s^{net}(t) Y^{net}(t) \quad \text{where } s^{net} = \frac{Y^{net}(t) - C(t)}{Y^{net}}. \end{split}$$

This result should be interpreted as the net growth rate multiplied by net output. That is, the flow value of capital equals the amount saved in period t. Intuitively, this makes sense. All savings are transmitted into capital, we can define the increase in capital in period t as the savings ratio multiplied by the net output.

2.2 Differentiate the definition of $k^{net}(t)$ and use the definitions above to show that

$$\dot{k}^{net}(t) = s^{net}(t) - g^{net}_{tot}(t)k^{net}(t).$$

Give an economic interpretation of this equation.

First note that
$$\frac{\partial k^{net}(t)}{\partial t} = \dot{k}^{net}(t) = \frac{\partial}{\partial t} \frac{K(t)}{Y^{net}(t)}$$
. Then:

$$\begin{split} \frac{\partial}{\partial t} \frac{K(t)}{Y^{net}(t)} &= \frac{s^{net}(t)Y^{net}(t)}{Y^{net}} - \frac{K(t)\dot{Y}^{net}(t)}{(Y^{net}(t))^2} \\ &= s^{net}(t) - \frac{K(t)\dot{Y}^{net}}{(Y^{net}(t))^2} \\ &= s^{net}(t) - \frac{K(t)}{Y^{net}(t)}g^{net}_{tot}(t) \\ &= s^{net}(t) - k^{net}(t)g^{net}_{tot}(t). \end{split}$$

Note that the flow value for capital-net income ratio is a function of the net savings rate, capital-net income ratio and the growth rate of net income. This implies that if the net savings rate is higher (lower) than the product of the capital-net income ratio and the growth rate of net income, the capital-net income ratio will decrease (increase). Also note that in equilibrium, $k^{net}(t)$ is constant. That is, $k^{net}(t) = 0$.

This result can be interpreted as follows. When out of equilibrium, a too high (low) net savings rate relative to the product of capital-net income ratio and the growth rate of net income will lead to an increase (decrease) in capital. This can be explained by the fact that if there is a lot of capital in the economy, the profit per additional unit of capital will decrease. However, is there merely is a low amount of capital, it is profitable to save and invest in capital. If so, the savings rate will increase until the equilibrium $\dot{k}^{net*}(t) = 0$ is reached.

2.3 Show that on the balanced growth path

$$k^{net*} = \frac{s^{net*}}{g_{tot}^{net*}}$$

and provide an economic interpretation of this relationship. (Here you may find it helpful to use a numerical example.)

Using the previous equation, on a balanced growth path, we need that $\dot{k}^{net} = 0$. Setting $\dot{k}^{net} = 0$ in the equation given in 2.2, we obtain:

$$0 = s^{net}(t) - g_{tot}^{net}(t) \cdot k^{net}(t)$$
$$\Leftrightarrow k^{net} = \frac{s^{net}}{g_{tot}^{net}}$$

We will now interpret this relationship. In equilibrium, the capital-net income ratio is equal to the equilibrium net savings rate divided by the equilibrium growth rate of net income. That is, ceteris paribus, for a higher net savings rate, the capital-net income ratio will be higher. Also, for a lower growth rate of net income, the capital-net income ratio will increase. This relationship also holds intuitively. Once people start to save more, the amount of capital will increase. On the other hand, if economic growth is higher, absolute savings will also increase, but as national income growths disproportionally faster, the long-run equilibrium capital-to-income ratio will be lower, ceteris paribus.

2.4 Read the extract from the book and discuss the interpretation and prediction within your team. Specifically, consider the statement "Taking as given that his predictions concerning growth are correct, Piketty's simulations of what will happen to capital accumulation are plausible." Do you agree or disagree with this statement (here you can also express differences in views between team members)? Explain your reasoning in a way that demonstrates that you have read the extract and have some understanding of the relevant concepts.

We agree with the statement. In particular, his argument does not hinge on various price differentials between asset classes, because, as Piketty notes, price variations tend to balance over the long-run. This is, however, an empirical argument and is not implied in his theoretical model. If we take this for granted, then the capital accumulation is only determined by the savings rate and the growth rate, and the equilibrium marks the point where the aggregate actors in the economy do not have any incentive to change their behavior (i.e. savings-consumption trade-offs), so that the capital stock in the country necessary converges to that ratio.

One possible counterargument is, however, that structural parameters are not that structural: i.e., if n, the demographic growth rate which Piketty mentions, or technology, preferences or other sources of g change over time, the economy still approaches a steady-state, but a different one. Particularly, historically, changes in n, e.g. due to migration, and in A, due to technological progress, have been quite common. In these cases, Piketty's model might be too simple to realistically model these changes. Furthermore, the changes in parameters might occur at a frequency that the economy is never in a steady state, but keeps bouncing from a process of convergence to one steady-state, to a process of convergence to another.

2.5 Show that $\tilde{k^*}$ in terms of the parameters of the model is given by

$$\tilde{k^*} = \left(\frac{\alpha}{\delta + \rho + \theta g}\right)^{\frac{1}{1 - \alpha}}$$

The model's Euler equation (under CRRA utility) is given in the notes, eq. 4.53. On the balanced growth path, technology-corrected consumption growth is zero, hence the left-hand side of the Euler equation is zero. This then gives us:

$$0 = \frac{1}{\theta} \left[f'(\tilde{k}^*) - \delta - \rho - \theta g \right]$$

Taking the functional form of f, evaluating the derivative and then isolating \tilde{k}^* gives the desired answer:

$$\tilde{k}^* = \left(\frac{\alpha}{\delta + \rho + \theta q}\right)^{\frac{1}{1 - \alpha}}$$

2.6 Show that the net saving rate along the balanced growth path in terms of parameters of the model is given by

$$s^{net*} = \frac{(n+g)\alpha}{(1-\alpha)\delta + \rho + \theta g}.$$

Starting with the definition of net savings:

$$S^{net}(t) = \frac{Y^{net}(t) - C(t)}{Y^{net}(t)} = \frac{Y(t) - \delta K(t) - C(t)}{Y(t) - \delta K(t)}$$

Then, writing this in per-capita, per-unit-of-technology terms gives:

$$s^{net}(t) = \frac{y(t) - \delta k(t) - c(t)}{y(t) - \delta k(t)}$$

Evaluated at the balanced-growth path, this gives:

$$s^{net*}(t) = \frac{y(t) - \delta \tilde{k}^*(t) - c(t)}{y(t) - \delta \tilde{k}^*(t)}$$

Then, substituting the production function for $y(t) = \tilde{k}^{\alpha}$, and realizing that along the balanced growth path, $\tilde{c}(t) = f(\tilde{k}(t)) - (n + \delta + g)\tilde{k}(t)$ (eq. 4.51 and $\dot{\tilde{k}} = 0$), we can write this equation as:

$$s^{net*} = 1 - \left(\frac{\tilde{k}^{\alpha} - \delta\tilde{k} - (n+g)\tilde{k}}{\tilde{k}^{\alpha} - \delta\tilde{k}}\right)$$

After substituting the definition for \tilde{k} found in the previous question, and some algebra, this simplifies to the desired answer:

$$s^{net*} = \frac{\alpha(n+g)}{(1-\alpha)\delta + \rho + \theta g}$$

2.7 Explain why $g_{tot}^{net*} = n + g$.

As explained in Piketty's chapter, the overall growth rate g_{tot} of national income is the sum of the per capita growth rate g and the population growth rate n. Since there are no direct costs associated with growth, g_{tot}^{net*} is thus n+g. Another way to see this is to proceed from the definition of $g_{tot}^{net} = \frac{Y^{net}}{Y^{net}}$. In the balanced growth path, Y^{net} is just large enough to keep it at the same level, corrected for population growth and technological change. Hence, $\dot{Y}^{net} = (n+g)Y^{net}$. Substituting this in the definition for \dot{Y}^{net} gives that $g_{tot}^{net*} = n+g$.

2.8 Show that

$$k^{net*} = \frac{\alpha}{(1-\alpha)\delta + \rho + \theta g}.$$

We know that $k^{*net} = \frac{s^{*net}}{g_{tot}^{net}}$. Using g_{tot}^{net*} , we can rewrite k^{*net} :

$$k^{*net} = \frac{\frac{\alpha(n+g)}{(1-\alpha)\delta + \rho + \theta g}}{(n+g)}$$

which simplifies to the desired answer:

$$k^{*net} = \frac{\alpha}{(1-\alpha)\delta + \rho + \theta g}$$

2.9 How does a drop in the population growth rate from 0.01 to 0.005 affect k^{net*} ? How does this compare to Piketty's interpretation and prediction? What is the mechanism underlying any discrepancies you observe (examine the corresponding changes in s^{net*} and g_{tot}^{net*})?

If n decreases from 0.01 to 0.005 in the equation from 2.8, we see that k^{*net} is unaffected. This is because the numerator of s^{*net} changes with the same rate as g_{tot}^{net} does.

In Piketty's book chapter, the effect of a drop in the population growth rate on the capital/income ratio β depends on the per capita growth rate and the savings rate. For instance, if s is 10 percent, and the total growth rate is 2 percent, the capital/income ratio is 5. If the population growth rate decreases from 1 to 0.5 percent, and thus the total growth rate is now 1.5 percent, then the capital/income ratio becomes \approx 6.6 percent. This reflects why countries with very similar growth rates per capita can have very different capital/income ratios, which is not reflected in our k^{*net} .

2.10 How does a drop in the rate of technological progress from 0.02 to 0.01 affect k^{net*} ? How does this compare to Piketty's interpretation and prediction? What is the mechanism underlying any discrepancies you observe (examine the corresponding changes in s^{net*} and g_{tot}^{net*})?

From the specification of k^{*net} , we can see that if g decreases, k^{*net} increases. This is because the growth rate is also reflected in the denominator of s^{*net} . This is concordant with Piketty's model, where a slow-down in technological progress would also need to an increase in the net capital stock in terms of national income. There, a decrease in the rate of growth thus also increases the long-term capital/income ratio. This also explains why after the very high growth rates in the second half of the twentieth century, the capital/income ratio is becoming very high now.

2.11 Reflect on your answer to part 4. Does the analysis in parts change the views in your team about the statement "Taking as given that his predictions concerning growth are correct, Piketty's simulations of what will happen to capital accumulation are plausible." Explain your reasoning.

In 2.4, we said that we agreed with the statement. However, we did mention the counterargument that structural parameters, such as the demographic growth rate of the technological growth rate, are not that structural. After doing the analysis in parts, we realise that this counterargument may be more important than we thought before.

Since the capital/income ratio is a long-term ratio, it takes long before it is reached. If then, the 'structural' parameters (that may sometimes be hard to predict) change at a higher rate than the long-term capital/income ratio is reached, then the economy will never actually get to the steady state. Since we saw in the questions 2.9 and 2.10 that even small changes in the population growth rate and the rate of technological progress can have large effects on the long-term capital/income ratio, we do not expect that Piketty's simulations of what will happen to capital accumulation will come true.

3 Problem 3: Spot the Neoclassical Growth Model

3.1 Find N ($n_{group}=3$) academic research papers that, based on your search, appear to be the most relevant and high quality papers that analyze this research question with an economic model and with a macroeconomic perspective. (...) Briefly explain how you found the papers and how you determined that they appear to be the most relevant and high quality papers.

We have chosen to research the question "What are the effects of income inequality on a country's growth?", related to the topic "Inequality and Wealth Inequality/Redistribution". We used Google Scholar and searched on the key terms "Income Inequality" and "Growth". We did not limit ourselves to a particular study context. In addition, we used the snowballing method, and thus also looked at important references (evaluated by the introduction of the paper) in the papers we found at Google Scholar.

First, we made a list of papers that included papers that could give an answer to our research question based on the title, the abstract and the introduction. After this, we shortened the list and selected the top 3 papers by selecting relevant papers based on the quality of the journal they were published in (either an impact factor of at least 3...) and the number of citations (...or at least 100). We also made sure that we chose diverse papers in terms of authors, time written and journal, so that we could answer our research question with insights from different angles.

We were left with 3 papers:

- Garcia-Penalosa, C. & Turnovsky, S.J. (2006) Growth and Income Inequality: A Canonical Model, Economic Theory, 28(1), 25-49. {195 citations}
- Aghion, F. (2002) Schumpeterian Growth Theory and the Dynamics of Income Inequality, Econometrica, 70(3), 855-882. {385 citations}
- Barro, R.J. (1999) Inequality, Growth and Investment, NBER Working Paper, 7038. {772 citations}
- 3.2 (...) Can you spot the neoclassical growth model or general equilibrium theory in these papers? For each paper, answer the following questions:
 - 1. What features (if any) of the neoclassical growth model or general equilibrium theory are retained in the model? Do you think that retaining these features is a sensible modeling decision, given the aims of the paper?
 - 2. How does the paper depart from the neoclassical growth model or general equilibrium theory? Do you think that these departures are a sensible modeling decision, given the aims of the paper?

In general, we are successful in spotting the neoclassical growth model in the papers we selected. All the papers are concerned with growth and have retained some features of the neoclassical growth model, such as the use of representative firms or households, assuming an infinite time horizon and using utility maximization. However, the papers all include some kind of extension to the neoclassical growth model in order to answer their research questions.

Paper 1: Growth and Income Inequality: A Canonical Model

The model in this paper builds on Romer's endogenous growth model, in which it is acknowledged that inequality and growth are jointly determined and dependent on various structural changes (such as civil liberty and openness, found by Lundberg and Squire). In essence, the endogenous distribution of income is determined by the positive equilibrium relationship that can be derived from the agents' relative wealth (capital) and their relative allocation of time between work and leisure. The relative value of their capital endowments endogenously determines the income distribution. The results show that structural changes that lead to growth generally increase income inequality.

- 1. There is a representative firm j which has one production function, and thus all firms will choose identical levels of employment L and capital stock K. This has the properties of positive but diminishing marginal physical products and constant returns to scale in capital K_j and labor measured in efficiency unites L_jK . This makes sense, because it is necessary for the model to be consistent with competitive equilibrium theory in order to reach an equilibrium state. Consumers i are identical in all respects except for their initial capital endowments K_{i0} . The consumers maximize expected lifetime utility in accordance with an isoelastic utility function. The optimality conditions are standard. Each agent chooses the same growth rate for their shadow value of capital, irrespective of their capital endowment. These properties are also sensible to ensure that the market clears. Just as with a neoclassical model with identical agents, the economy is always on its balanced growth path. The model ignores human capital and education, but this is also ignored in neoclassical growth theory.
- 2. The basis of the model is an AK endogenous growth model. This contrasts the neoclassical growth model in which long run growth is determined by exogenous factors. It is sensible to choose this as a basis, because the goal of the model is to measure the efficiency of the labor demand by the firm.

Furthermore, Individuals i are different in their initial capital endowments K_{i0} , and are thus heterogeneous in terms of capital. This makes sense in this model, since these heterogeneous capital endowments create the income inequality in the model. Wealthier agents have a lower marginal utility of wealth, and thus choose to consume more leisure and reduce their labor supply, which in turn determines the income distribution. This offsets the growth rate, which is therefore independent of the capital distribution.

Paper 2: Schumpeterian Growth Theory and the Dynamics of Income Inequality

In Schumpeterian Growth Theory, long-run growth is the result of a sequence of innovations. these innovations destroy the rents generated by earlier innovations. In this article, Aghion gives insight on the relationship between wage inequality and growth in developed economies by exploring the relation between endogenous technical change arising from this earlier-mentioned innovation and the dynamics in wage structure. He explores both wage inequality between educational groups, as well as within educational and age groups. He states that this second wage inequality has been largely ignored in economic literature. In this description we will focus on this within-group inequality. The within-group wage inequality decreases with adaptability σ , but increases with the rate of technological progress γ and the knowledge transferability τ .

- 1. The basic framework contains relatively many features of the neoclassical growth model. There is an infinite horizon discrete time model, which is necessary to get the market to clear. The model works with one good (one-good economy), and there are sequential productivity-improving innovations. The model assumes that machines last for only two periods, but do not depreciate in the first period. This corresponds to the overlapping generations model in which it is possible to 'mimic' the real-life origin and the depreciation of technology without having to assume a finite time horizon. Furthermore, households' preferences are denoted by a logarithmic specification. In this model, the firms set their demand for labor by setting the marginal product of labor equal to the wage level (which is given). This corresponds to the neoclassical growth model in some sense, other than that the wage level is in this case given rather than decided on by the market.
- 2. Rather than a constant technology, the model assumes that every new technology is $(1 + \gamma)$ times more productive than the previous technology. The main constraint in the model is the adaptability constraint; only a fraction σ of all workers can use the leading technology at time t. This is sensible, because it creates this within-group wage inequality. Furthermore, workers can only transfer some of their knowledge $((1 + \tau))$, where τ < full rate of learning η in previous periods) to the new period. In contrast to the common elastic wage, workers supply one unit of labor inelastically to the market. They thus only have to decide whether they stay in their job or move to another job. This makes sense, because this is necessary to determine the wage distribution in stationary equilibrium.

Paper 3: Inequality, Growth and Investment

Empirical evidence seems to show that higher inequality tends to decrease growth in poor countries, and encourage growth in richer countries. The Kuznets-curve does only explain that inequality changes in different phases of economic development, but does not explain why the effects vary across countries or over time. In this paper, Barro uses a framework for the determinants of economic growth and extends it to income inequality. The results show that income-equalizing policies can increase growth in poor countries, and in richer countries the choice between growth and income redistribution looks more like a trade-off.

1. The framework in this article is an extension of the neoclassical growth model, and thus contains many similar features. In general, the growth rate of per capital output Dy is a function of the current level of per capital output y and the long-run per capital output y^* . There are diminishing returns to capital accumulation, thus Dy is inversely related to the level of development y, as in the neoclassical model. Given that we need this diminishing marginal returns to show that policy changes only affect y, and not Dy, as we specify below, it is sensible to retain these features.

2. Rather than the growth rate of per capita output just being dependent on the current level of per capita output, this property applies in a conditional sense, for a given value of long-run per capita out y^* . This reflects that if some government policy increases y^* , Dy increases for given y. Then y increases gradually over time, and due to diminishing returns, the growth rate Dy is eventually consistent with the long-run rate of technological progress. This is different from the neoclassical model, in which this is determined outside of the model. This extension is sensible, since it reflects that in the long run, improved policy only affects the level of per capita output, and not the growth rate. However, since these transitions tend to be lengthy, effects of policy on growth persist for a long time and may explain the variation across countries.

3.3 (...) Overall, does a good understanding of the neoclassical growth model and/or general equilibrium theory appear to be an important basis for understanding state-of-the-art research on this question?

We see that all papers build on the neoclassical growth model as a basis for their understanding the relationship between income inequality and economic growth. Not only do they use it as a basis for their model, in the papers not all features are explicitly being described as features of the neoclassical growth model, thus also as a reader of academic articles it is important to have a good understanding of the neoclassical growth model. The three papers that we selected here were in that respect not so different from other papers on our 'long list'. So, even though the study of income inequality always requires some changes to the neoclassical growth model, it is still essential to have a good basic knowledge of the neoclassical growth model in order to read current state-of-the-art research.

4 References

Aghion, F. (2002) Schumpeterian Growth Theory and the Dynamics of Income Inequality, Econometrica, 70(3), 855-882.

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Garcia-Penalosa, C. & Turnovsky, S.J. (2006) Growth and Income Inequality: A Canonical Model, Economic Theory, 28(1), 25-49.