

# Microeconomics II: Assignment 1

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## 1 Problem 3: The Cournot Game

Without loss of generality, we look at the profits for player 1, and we look at a three player game. The price  $p = a - b(q_1 + q_2 + q_3)$  and the costs are  $c(q_i) = cq_i$ . The profit function of firm 1 is:

$$\begin{aligned}\Pi_1 &= pq_1 - cq_1 \\ &= (a - b(q_1 + q_2 + q_3))q_1 - cq_1 \\ &= -bq_1^2 + (a - c - b(q_2 + q_3))q_1\end{aligned}$$

because from the perspective of firm 1,  $q_2$  and  $q_3$  are constants.

The profit function is a quadratic function of  $q_1$  and has roots  $q_1 = 0$  and  $q_1 = \frac{a - bq_2 - bq_3 - c}{b}$ . Hence, a priori,  $S_1 \in [0, \frac{a - bq_2 - bq_3 - c}{b}]$

In the first iteration, we notice that in the best case for player 1, players 2 and 3 produce nothing. Because all other parabolas have their vertices to the left of this best case, we know that all strategies to the right of  $\frac{a - c}{2b}$  are strictly dominated.

$$\Pi_1\left(\frac{a - bq_2 - bq_3 - c}{2b}, q_2, q_3\right) > \Pi_1\left(\frac{a - c}{2b}, q_2 = 0, q_3 = 0\right) > \Pi_1\left(\frac{a - c}{2b}, q_2, q_3\right)$$

By symmetry, the same is true for players 2 and 3. Hence, we know after iteration 1,  $S_i \in [0, \frac{a - c}{2b}]$  for  $i = \{1, 2, 3\}$ .

Hence, everybody plays at most  $\frac{a - c}{2b}$ . Then, the minimum profit, e.g. for player 1, is:

$$\begin{aligned}\Pi_1 &> \Pi_1(q_1, q_2 = \frac{a - c}{2b}, q_3 = \frac{a - c}{2b}) = \\ &= -bq_1^2 + (a - c - b(2\frac{a - c}{2b}))q_1 = -bq_1^2 \leq 0\end{aligned}$$

Hence, in the worst case,  $q_1 = 0$ , and there are no new strategies that can be ruled out, contrary to the two-player case. We have done only one iteration, and afterwards, it is not possible to rule out new strategies on the basis of strict dominance.

## 2 Problem 4: The partnership game

First, the explicit profit/pay-off functions are:

$$\begin{aligned}\Pi_1 &= 2(x + y + cxy) - x^2 \\ \Pi_2 &= 2(x + y + cxy) - y^2\end{aligned}$$

Then, in iteration 1: we note that player 1's profit is increasing in the effort of player 2, and player two has to put in at least 0, so:

$$(\Pi_1|Y = 0) = 2x - x^2$$

And the maximum profit for player 1 is the vertex of this parabola:  $\frac{-2}{-2} = 1$ . That means all strategies that put in lower than 1 effort ( $x < 1$ ) are strictly dominated. By symmetry,  $y < 1$  are also strictly dominated.

In iteration 2, we calculate conditioned on iteration 1:

$$(\Pi_1|y \geq 1) \geq 2x + 2 + 2cx - x^2 = -x^2 + (2 + c)x + 1$$

Again, this is a worst-case scenario, in case player 2 does as little as can be rational. In this case, the maximum profit is again equal to the vertex of the parabola:

$$\frac{-(2 + c)}{-2} = 1 + \frac{c}{2}$$

Hence, all strategies smaller than this number  $1 + \frac{c}{2}$  are strictly dominated. By symmetry, the same is true for player 2.

In iteration 3, we repeat the procedure by calculating the profit of player 1 at the least rationally possible effort of player 2:

$$(\Pi_1|y \geq 1 + \frac{c}{2}) \geq 2x + 2(1 + \frac{c}{2}) + 2cx(1 + \frac{c}{2}) - x^2 = -x^2 + (2 + 2c + c^2)x + 2 + c$$

That means that the minimal effort that player 1 will undertake is again at the vertex of this parabola:

$$x \geq \frac{-(2 + 2c + c^2)}{-2} = 1 + c + \frac{1}{2}c^2$$

And by symmetry, the same is true for player 2

In iteration 4, we again calculate the profit at the updated minimum effort:

$$\begin{aligned}(\Pi_1|y \geq 1 + c + \frac{1}{2}c^2) &\geq 2x + 2(1 + c + \frac{1}{2}c^2) + 2cx(1 + c + \frac{1}{2}c^2) - x^2 = \\ &\quad -x^2 + (c^3 + 2c^2 + 2c + 2)x + 2 + 2c + c^2\end{aligned}$$

So again in the worst case, the optimal amount is the vertex of the parabola:

$$x \geq 1 + c + c^2 + \frac{1}{2}c^3$$

And the same conclusion holds by symmetry for player 2 and y.

Generalizing this pattern, we find that the solution is:

$$x^* = y^* = \sum_{k=0}^{\infty} c^k = \frac{1}{1-c}$$

### 3 Problem 6

First, we observe that all strategies  $[0, 49]$  are weakly dominated by  $S_1 = \{50\}$ , because playing 50, you get the same pay-off as you would if the other player played  $< 50$ , and 50 if the other plays  $\geq 50$ .

Then, for the strategies from  $[100, 50]$ , start with strategy 100. Conditioned on that all strategies lower than 50 be eliminated, we play the following subgame, with only the outcomes for player 1 in the cells:

$p_1/p_2$	$< 99$	99	100
$< 99$	2,3,4..	98,97..	98,97..
99	2,3,4..	50	99
100	2,3,4..	1	50

Table 1: Payoff matrix

Hence, we can see that playing 99 weakly dominates 100. By symmetry, this is true for both players.

Then, having excluded 100, we can play the following subgame:

$p_1/p_2$	$< 98$	98	99
$< 98$	3,4,..	97,96..	97,96..
98	3,4,..	50	98
99	3,4..	2	50

Table 2: Payoff matrix

This subgame can continue right until we have excluded all strategies higher than 51. Then, we have only two strategies left,  $\{50, 51\}$ . In the following subgame, we show the payoffs for both players:

$p_1/p_2$	50	51
50	(50,50)	(50, 49)
51	(49, 50)	(50,50)

Table 3: Last subgame

Therefore, the strategy of playing 50 is weakly dominated to playing 51 for both players. Hence, the equilibrium is  $(50, 50)$ .

## 4 Problem 7

We know that:

$$s_1 = e_1 + 1.5$$

$$s_2 = e_2$$

$$U(s_i) = \begin{cases} 10 - e_i & \text{if } s_i > s_j \\ 8 - e_i & \text{if } s_i < s_j \end{cases}$$

From this, we can calculate the pay-off matrix for all possible strategies  $e_1 \times e_2$ :

$e_1, e_2$	0	1	2	3	4	5
0	(10,8)	(10,7)	(8,8)	(8,7)	(8,6)	(8,5)
1	(9,8)	(9,7)	(9,6)	(7,7)	(7,6)	(7,5)
2	(8,8)	(8,7)	(8,6)	(8,5)	(6,6)	(6,5)
3	(7,8)	(7,7)	(7,6)	(7,5)	(7,4)	(5,5)
4	(6,8)	(6,7)	(6,6)	(6,5)	(6,4)	(6,3)
5	(5,8)	(5,7)	(5,6)	(5,5)	(5,4)	(5,3)

Table 4: Payoff matrix

**IESDS:** Iteration 1: For player 1, we never play  $e_1 = \{5, 4, 3\}$ , because they are each strictly dominated by one of  $\{0, 2, 1\}$ . For player 2, we always play  $e_2 = \{0, 2\}$ , because they strictly dominate all other strategies. We then have a simpler game left, with strategies 0, 1, 2 for player 1, and strategies 0, 2 for player 2.

Iteration 2: We eliminate strategy 2 for player 1, because it is strictly dominated by strategy 1. Now, no more strategies are strictly dominated by other strategies, and the four possible outcomes are:

$e_1/e_2$	0	2
0	(10,8)	(8,8)
1	(8,8)	(9,6)

Table 5: Payoff Matrix

**IEWDS:** Iteration 1: We start with player 1, and we never play 5,4,3, and 2, because they are weakly dominated by 0 or 1. Then, player 2 always plays 0, because all others are weakly dominated by 0. Iteration 2: Player 1 can chose between 0 or 1, and player 2 choses 0. In this iteration, player one chooses between 0 with a payoff of 10 or 1 with a payoff of 9. Hence, player one chooses 0, and the equilibrium is (0, 0).