

Microeconomics II: Assignment 1

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1 Problem 3: The Cournot Game

Without loss of generality, we look at the profits for player 1, and we look at a three player game. The price $p = a - b(q_1 + q_2 + q_3)$ and the costs are $c(q_i) = cq_i$. The profit function of firm 1 is:

$$\begin{aligned}\Pi_1 &= pq_1 - cq_1 \\ &= (a - b(q_1 + q_2 + q_3))q_1 - cq_1 \\ &= -bq_1^2 + (a - c - b(q_2 + q_3))q_1\end{aligned}$$

because from the perspective of firm 1, q_2 and q_3 are constants.

The profit function is a quadratic function of q_1 and has roots $q_1 = 0$ and $q_1 = \frac{a - bq_2 - bq_3 - c}{b}$. Hence, a priori, $S_1 \in [0, \frac{a - bq_2 - bq_3 - c}{b}]$

In the first iteration, we notice that in the best case for player 1, players 2 and 3 produce nothing. Because all other parabolas have their vertices to the left of this best case, we know that all strategies to the right of $\frac{a - c}{2b}$ are strictly dominated.

$$\Pi_1\left(\frac{a - bq_2 - bq_3 - c}{2b}, q_2, q_3\right) > \Pi_1\left(\frac{a - c}{2b}, q_2 = 0, q_3 = 0\right) > \Pi_1\left(\frac{a - c}{2b}, q_2, q_3\right)$$

By symmetry, the same is true for players 2 and 3. Hence, we know after iteration 1, $S_i \in [0, \frac{a - c}{2b}]$ for $i = \{1, 2, 3\}$.

Hence, everybody plays at most $\frac{a - c}{2b}$. Then, the minimum profit, e.g. for player 1, is:

$$\begin{aligned}\Pi_1 &> \Pi_1(q_1, q_2 = \frac{a - c}{2b}, q_3 = \frac{a - c}{2b}) = \\ &= -bq_1^2 + (a - c - b(2\frac{a - c}{2b}))q_1 = -bq_1^2 \leq 0\end{aligned}$$

Hence, in the worst case, $q_1 = 0$, and there are no new strategies that can be ruled out, contrary to the two-player case. We have done only one iteration, and afterwards, it is not possible to rule out new strategies on the basis of strict dominance.

2 Problem 4: The partnership game

First, the explicit profit/pay-off functions are:

$$\begin{aligned}\Pi_1 &= 2(x + y + cxy) - x^2 \\ \Pi_2 &= 2(x + y + cxy) - y^2\end{aligned}$$

Then, in iteration 1: we note that player 1's profit is increasing in the effort of player 2, and player two has to put in at least 0, so:

$$(\Pi_1|Y = 0) = 2x - x^2$$

And the maximum profit for player 1 is the vertex of this parabola: $\frac{-2}{-2} = 1$. That means all strategies that put in lower than 1 effort ($x < 1$) are strictly dominated. By symmetry, $y < 1$ are also strictly dominated.

In iteration 2, we calculate conditioned on iteration 1:

$$(\Pi_1|y \geq 1) \geq 2x + 2 + 2cx - x^2 = -x^2 + (2 + c)x + 1$$

Again, this is a worst-case scenario, in case player 2 does as little as can be rational. In this case, the maximum profit is again equal to the vertex of the parabola:

$$\frac{-(2 + c)}{-2} = 1 + \frac{c}{2}$$

Hence, all strategies smaller than this number $1 + \frac{c}{2}$ are strictly dominated. By symmetry, the same is true for player 2.

In iteration 3, we repeat the procedure by calculating the profit of player 1 at the least rationally possible effort of player 2:

$$(\Pi_1|y \geq 1 + \frac{c}{2}) \geq 2x + 2(1 + \frac{c}{2}) + 2cx(1 + \frac{c}{2}) - x^2 = -x^2 + (2 + 2c + c^2)x + 2 + c$$

That means that the minimal effort that player 1 will undertake is again at the vertex of this parabola:

$$x \geq \frac{-(2 + 2c + c^2)}{-2} = 1 + c + \frac{1}{2}c^2$$

And by symmetry, the same is true for player 2

In iteration 4, we again calculate the profit at the updated minimum effort:

$$\begin{aligned}(\Pi_1|y \geq 1 + c + \frac{1}{2}c^2) &\geq 2x + 2(1 + c + \frac{1}{2}c^2) + 2cx(1 + c + \frac{1}{2}c^2) - x^2 = \\ &\quad -x^2 + (c^3 + 2c^2 + 2c + 2)x + 2 + 2c + c^2\end{aligned}$$

So again in the worst case, the optimal amount is the vertex of the parabola:

$$x \geq 1 + c + c^2 + \frac{1}{2}c^3$$

And the same conclusion holds by symmetry for player 2 and y.

Generalizing this pattern, we find that the solution is:

$$x^* = y^* = \sum_{k=0}^{\infty} c^k = \frac{1}{1-c}$$

3 Problem 6

First, we observe that all strategies $[0, 49]$ are weakly dominated by $S_1 = \{50\}$, because playing 50, you get the same pay-off as you would if the other player played < 50 , and 50 if the other plays ≥ 50 .

Then, for the strategies from $[100, 50]$, start with strategy 100. Conditioned on that all strategies lower than 50 be eliminated, we play the following subgame, with only the outcomes for player 1 in the cells:

p_1/p_2	< 99	99	100
< 99	2,3,4..	98,97..	98,97..
99	2,3,4..	50	99
100	2,3,4..	1	50

Table 1: Payoff matrix

Hence, we can see that playing 99 weakly dominates 100. By symmetry, this is true for both players.

Then, having excluded 100, we can play the following subgame:

p_1/p_2	< 98	98	99
< 98	3,4,..	97,96..	97,96..
98	3,4,..	50	98
99	3,4..	2	50

Table 2: Payoff matrix

This subgame can continue right until we have excluded all strategies higher than 51. Then, we have only two strategies left, $\{50, 51\}$. In the following plot, we can see that 51 weakly dominates 50. Hence, by IEWDS, the equilibrium is $(51, 51)$.

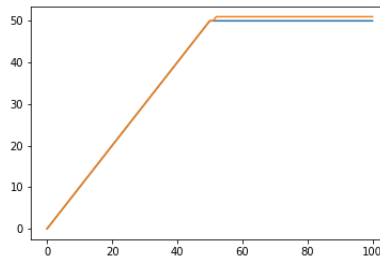


Figure 1: Player 2 plays 50 (Blue), Player 2 plays 51 (Orange)

4 Problem 7

We know that:

$$s_1 = e_1 + 1.5$$

$$s_2 = e_2$$

$$U(s_i) = \begin{cases} 10 - e_i & \text{if } s_i > s_j \\ 8 - e_i & \text{if } s_i < s_j \end{cases}$$

From this, we can calculate the pay-off matrix for all possible strategies $e_1 \times e_2$:

e_1, e_2	0	1	2	3	4	5
0	(10,8)	(10,7)	(8,8)	(8,7)	(8,6)	(8,5)
1	(9,8)	(9,7)	(9,6)	(7,7)	(7,6)	(7,5)
2	(8,8)	(8,7)	(8,6)	(8,5)	(6,6)	(6,5)
3	(7,8)	(7,7)	(7,6)	(7,5)	(7,4)	(5,5)
4	(6,8)	(6,7)	(6,6)	(6,5)	(6,4)	(6,3)
5	(5,8)	(5,7)	(5,6)	(5,5)	(5,4)	(5,3)

Table 3: Payoff matrix

IESDS: Iteration 1: For player 1, we never play $e_1 = \{5, 4, 3\}$, because they are each strictly dominated by one of $\{0, 2, 1\}$. For player 2, we always play $e_2 = \{0, 2\}$, because they strictly dominate all other strategies. We then have a simpler game left, with strategies 0, 1, 2 for player 1, and strategies 0, 2 for player 2.

Iteration 2: We eliminate strategy 2 for player 1, because it is strictly dominated by strategy 1. Now, no more strategies are strictly dominated by other strategies, and the four possible outcomes are:

e_1/e_2	0	2
0	(10,8)	(8,8)
1	(8,8)	(9,6)

Table 4: Payoff Matrix

IEWDS: Iteration 1: We start with player 1, and we never play 5,4,3, and 2, because they are weakly dominated by 0 or 1. Then, player 2 always plays 0, because all others are weakly dominated by 0. Iteration 2: Player 1 can chose between 0 or 1, and player 2 choses 0. In this iteration, player one chooses between 0 with a payoff of 10 or 1 with a payoff of 9. Hence, player one chooses 0, and the equilibrium is (0, 0).