Microeconomics II: Assignment 1

Ilse van der Voort
Matthijs van Tiggelen
Bas Machielsen

October 30, 2020

1 Problem 3: The Cournot Game

Without loss of generality, we look at the profits for player 1, and we look at a three player game. The price $p = a - b(q_1 + q_2 + q_3)$ and the costs are $c(q_i) = cq_i$. The profit function of firm 1 is:

$$\begin{aligned}\Pi_1 &= pq_1 - cq_1 \\ &= (a - b(q_1 + q_2 + q_3)q_1 - cq_1 \\ &= -bq_1^2 + (a - c - b(q_2 + q_3))q_1\end{aligned}$$

because from the perspective of firm 1, q_2 and q_3 are constants.

The profit function is a quadratic function of q_1 and has roots $q_1 = 0$ and $q_1 = \frac{a-bq_2-bq_3-c}{b}$. Hence, a priori, $S_1 \in [0, \frac{a-bq_2-bq_3-c}{b}]$, and the same goes for S_2 and S_3.

We notice that the profit of player 1 is decreasing in the production of players 2 and 3, so, in the first iteration, we notice that in the best case for player 1, players 2 and 3 produce nothing. Because all other parabolas (representing higher production levels for players 2 and 3) have their vertices to the left of this best case, we know that all strategies to the right of $\frac{a-c}{2b}$ are strictly dominated. Formally, and in parallel to the lecture slides:

$$\Pi_1\left(\frac{a - bq_2 - bq_3 - c}{2b}, q_2, q_3\right) > \Pi_1\left(\frac{a-c}{2b}, q_2, q_3\right) > \Pi_1\left(q_1, q_2, q_3\right)$$

for all $q_1 > \frac{a-c}{2b}$. By symmetry, the same is true for players 2 and 3. Hence, we know after iteration 1, $S_i \in [0, \frac{a-c}{2b}]$ for $i = \{1, 2, 3\}$.

Hence, everybody plays at most $\frac{a-c}{2b}$. In iteration 2, we look at what "low outputs" we can rule out. In the case that players 2 and three play $\frac{a-c}{2b}$, the minimum profit for player 1, is:

$$\begin{aligned}\Pi_1(q_1, q_2, q_3) > \Pi_1(q_1, q_2 = \frac{a-c}{2b}, q_3 = \frac{a-c}{2b}) = \\ -bq_1^2 + (a - c - b(2\frac{a-c}{2b}))q_1 = -bq_1^2 \leq 0\end{aligned}$$

Hence, in the worst case, the optimum profit is $q_1 = 0$, and there are no new strategies that can be ruled out, contrary to the two-player case. We have done only one iteration, and afterwards, it is not possible to rule out new strategies on the basis of strict dominance.

In the case where $n > 3$, the results are similar: the minimum profit in the second iteration is always 0, so there are no strategies to be ruled out in that iteration.

2 Problem 4: The partnership game

First, the explicit profit/pay-off functions are:

$$\Pi_1 = 2(x + y + cxy) - x^2$$
$$\Pi_2 = 2(x + y + cxy) - y^2$$

Then, in iteration 1: we note that player 1's profit is increasing in the effort of player 2, and player two has to put in at least 0, so:

$$(\Pi_1|Y = 0) = 2x - x^2$$

And the maximum profit for player 1 is the vertex of this parabola: $\frac{-2}{-2} = 1$. That means all strategies that put in lower than 1 effort ($x < 1$) are strictly dominated. By symmetry, $y < 1$ are also strictly dominated.

In iteration 2, we calculate conditioned on iteration 1:

$$(\Pi_1|y \geq 1) \geq 2x + 2 + 2cx - x^2 = -x^2 + (2 + c)x + 1$$

Again, this is a worst-case scenario, in case player 2 does as little as can be rational. In this case, the maximum profit is again equal to the vertex of the parabola:

$$\frac{-(2 + c)}{-2} = 1 + \frac{c}{2}$$

Hence, all strategies smaller than this number $1 + \frac{c}{2}$ are strictly dominated. By symmetry, the same is true for player 2.

In iteration 3, we repeat the procedure by calculating the profit of player 1at the least rationally possible effort of player 2:

$$(\Pi_1|y \geq 1 + \frac{c}{2}) \geq 2x + 2(1 + \frac{c}{2}) + 2cx(1 + \frac{c}{2}) - x^2 = -x^2 + (2 + 2c + c^2)x + 2 + c$$

That means that the minimal effort that player 1 will undertake is again at the vertex of this parabola:

$$x \geq \frac{-(2 + 2c + c^2)}{-2} = 1 + c + \frac{1}{2}c^2$$

And by symmetry, the same is true for player 2

In iteration 4, we again calculate the profit at the updated minimum effort:

$$(\Pi_1 | y \geq 1 + c + \frac{1}{2}c^2) \geq 2x + 2(1 + c + \frac{1}{2}c^2) + 2cx(1 + c + \frac{1}{2}c^2) - x^2 = \\ -x^2 + (c^3 + 2c^2 + 2c + 2)x + 2 + 2c + c^2$$

So again in the worst case, the optimal amount is the vertex of the parabola:

$$x \geq 1 + c + c^2 + \frac{1}{2}c^3$$

And the same conclusion holds by symmetry for player 2 and y.

Generalizing this pattern, we find that the solution is:

$$x^* = y^* = \sum_{k=0}^{\infty} c^k = \frac{1}{1-c}$$

3 Problem 6

First, we observe that all strategies $[0, 49]$ are weakly dominated by $S_1 = \{50\}$, because playing 50, you get the same pay-off as you would if the other player played < 50, and 50 if the other plays ≥ 50.

Then, for the strategies from $[100, 50]$, start with strategy 100. Conditioned on that all strategies lower than 50 be eliminated, we play the following subgame, with only the outcomes for player 1 in the cells:

p_1/p_2	< 99	99	100
< 99	2,3,4..	98,97..	98,97..
99	2,3,4..	50	99
100	2,3,4..	1	50

Table 1: Payoff matrix

Hence, we can see that playing 99 weakly dominates 100. By symmetry, this is true for both players.

Then, having excluded 100, we can play the following subgame:

p_1/p_2	< 98	98	99
< 98	3,4,..	97,96..	97,96..
98	3,4,..	50	98
99	3,4..	2	50

Table 2: Payoff matrix

This subgame can continue right until we have excluded all strategies higher than 51. Then, we have only two strategies left, $\{50, 51\}$. In the following subgame, we show the payoffs for both players:

p_1/p_2	50	51
50	(50,50)	(50, 49)
51	(49, 50)	(50,50)

Table 3: Last subgame

Therefore, the strategy of playing 50 is weakly dominated to playing 51 for both players. Hence, the equilibrium is $(50, 50)$.

4 Problem 7

We know that:

$$
\begin{aligned}
s_1 &= e_1 + 1.5 \\
s_2 &= e_2 \\
U(s_i) &= \begin{cases} 10 - e_i \text{ if } s_i > s_j \\ 8 - e_i \text{ if } s_i < s_j \end{cases}
\end{aligned}
$$

From this, we can calculate the pay-off matrix for all possible strategies $e_1 \times e_2$:

e_1, e_2	0	1	2	3	4	5
0	(10,8)	(10,7)	(8,8)	(8,7)	(8,6)	(8,5)
1	(9,8)	(9,7)	(9,6)	(7,7)	(7,6)	(7,5)
2	(8,8)	(8,7)	(8,6)	(8,5)	(6,6)	(6,5)
3	(7,8)	(7,7)	(7,6)	(7,5)	(7,4)	(5,5)
4	(6,8)	(6,7)	(6,6)	(6,5)	(6,4)	(6,3)
5	(5,8)	(5,7)	(5,6)	(5,5)	(5,4)	(5,3)

Table 4: Payoff matrix

IESDS: Iteration 1: For player 1, we never play $e_1 = \{5, 4, 3\}$, because they are each strictly dominated by one of $\{0, 2, 1\}$. For player 2, we always play $e_2 = \{0, 2\}$, because they strictly dominate all other strategies. We then have a simpler game left, with strategies $0, 1, 2$ for player 1, and strategies $0, 2$ for player 2.

Iteration 2: We eliminate strategy 2 for player 1, because it is strictly dominated by strategy 1. Now, no more strategies are strictly dominated by other strategies, and the four possible outcomes are:

e_1/e_2	0	2
0	(10,8)	(8,8)
1	(8,8)	(9,6)

Table 5: Payoff Matrix

IEWDS: Iteration 1: We start with player 1, and we never play 5,4,3, and 2, because they are weakly dominated by 0 or 1. Then, player 2 always plays 0, because all others are weakly dominated by 0. Iteration 2: Player 1 can chose between 0 or 1, and player 2 choses 0. In this iteration, player one chooses between 0 with a payoff of 10 or 1 with a payoff of 9. Hence, player one chooses 0, and the equilibrium is $(0, 0)$.