Kidney Exchange

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Introduction

Kidney Exchange over the Years

- Kidney exchange involves the transplantation of kidneys from donors to patients
- Life-saving operation

TABLE I
U. S. KIDNEY TRANSPLANTS

Year	Cadaveric donors	Cadaveric transplants	Live donors	All wait-list patients	New wait-list additions
1992	4,276	7,202	2,535	22,063	15,224
1993	4,609	7,509	2,851	24,765	16,090
1994	4,797	7,638	3,009	27,258	16,538
1995	5,003	7,690	3,377	30,590	17,903
1996	5,038	7,726	3,649	34,000	18,328
1997	5,083	7,769	3,912	37,438	19,067
1998	5,339	8,017	4,361	40,931	20,191
1999	5,386	8,023	4,552	43,867	20,986
2000	5,490	8,089	5,324	47,596	22,269
2001	5,528	8,202	5,924	51,144	22,349
2002	5,630	8,534	6,233	54,844	23,494

The data for years 1992–2001 are constructed from the annual report of UNOS/OPTN, the data for 2002 are constructed from the national database of UNOS/OPTN. National database numbers are slightly higher than the annual report numbers due to continuous updating regarding previous years. Number of registrations may have multiple counts of patients since one patient may have registered in multiple centers for the wait-list.

Figure 1: Kidney Exchange in the US (Roth et al., 2004)

Kidney Exchange over the Years

- Almost 15,000 individuals have been waiting for over 3 years to find a kidney transplant (in the US) as of 2004
  - As of 2017, more than 100,000 individuals await kidney transplants (Biro et al., 2019)
  - 13 people die each day while waiting for a life-saving kidney transplant
- In the Netherlands, the waiting time was 4-5 years according to de Klerk et al. (2005)
- In principle, markets could equilibrate demand and supply
  - Market trading of kidneys not seen as ethically permissible
- A consistent lack of donors to satisfy demand
  - Not specific to US

Kidney Exchange over the years

Cijfers van 10 maart 2022, wachtlijst per 1 maart 2022

	Mrt-2022	Mrt-2021	Verschil
Alvleesklier	22	25	-12%
Hart	130	143	-9%
Hart met long	1	-	-
Heart Liver	1	-	-
Lever	63	106	-41%
Long	163	159	3%
Nier	880	791	11%
Nier met alvleesklier	21	23	-9%
Dunne darm	1	-	-
Totaal	1282	1247	3%

Figure 2: Waiting List Netherlands

Cijfers van 10 maart 2022, rapportage bevat data t/m 28 februari 2022

	2022	2021	Verschil	Totaal 2022	Totaal 2021	Verschil
Januari	18	23	-22 %	18	23	-22 %
Februari	26	19	37 %	44	42	5 %
Maart	-	17	-	-	59	-
April	-	20	-	-	79	-
Mei	-	18	-	-	97	-
Juni	-	25	-	-	122	-
Juli	-	26	-	-	148	-
Augustus	-	28	-	-	176	-
September	-	22	-	-	198	-
Oktober	-	19	-	-	217	-
November	-	25	-	-	242	-
December	-	29	-	-	271	-

Figure 3: How Many Additions to Cadavre Queue, 2021-2022

When is kidney exchange possible?

- Two medical characteristics determine whether kidney exchange is possible
  - Blood type and tissue type
- **Blood type**:
  - Type A $\rightarrow$ Type A, Type B or Type AB
  - Type B $\rightarrow$ Type A, Type B or Type AB
  - Type AB $\rightarrow$ Type AB
  - Type O $\rightarrow$ Type A, Type B, Type AB, Type O
- So if you're type O, you can only get a kidney from a type O donor
  - This is one reason why people are opposed to a centralized kidney exchange mechanism
- **Tissue type**: Kidney cannot be transplanted if the recipient has antibodies against the donor's tissue (positive crossmatch)

Status Quo

- When a cadaveric kidney becomes available for transplantation, the priority of each patient on the waiting list is determined by a point system based on factors including the blood type, HLA antigen-match, time spent on the waiting list, the region the kidney is harvested, etc.
  - The kidney is offered to the patient with the highest priority.
  - If that patient declines, the kidney is offered to the patient with the next highest priority, and so on.
- Living donor kidney grafts have superior survival rates (and their availability can also avoid the long waiting time for a cadaver kidney).
- Typically, a patient identifies a healthy willing donor (a spouse, for example)
  - If the transplant is feasible on medical grounds, it is carried out.
  - However, potential living donors can be eliminated from consideration due to incompatibility of the potential donor kidney with the intended recipient.

Possibilities of Exchange

- Rapaport (1986): creation of a living donor pool for paired exchange
  - Involves transactions between two pairs ('couples') who are individually incompatible, but pairwise compatible
  - Example:
    - Patient 1: Type O, Donor 1: Type A
    - Patient 2: Type A, Donor 2: Type O
  - Solution: Patient 1 gets kidney from donor 2, patient 2 gets kidney from donor 1
- Ross and Woodle (2000): exchange between individually incompatible couple and cadaver queue:
  - Incompatible couple donates to the cadaver queue, patient gets a high position on the cadaver queue
  - Widely seen as disadvantaging type O patients in the cadaver queue
    - They lose priority to type O patients whose incompatible donors donate to the cadaver queue
    - Will not be compensated by extended supply of type O donors on the cadaver queue (since type O donors can donate to anyone)

Theory

How should kidney exchange be organized?

- Roth (2004), QJE, Kidney Exchange
- Based on a generalized version of the "top trading cycle" algorithm by Shapley and Scarf (1974)
- Each agent endowed with one object, and has a preference relationship over all objects
- Algorithm:
  - Step 0: each agent points at their most preferred object
    - There is at least 1 cycle
    - The cycles represent exchanges and are implemented, and the agents who exchange are removed from the algorithm
  - Step k: each agent points at their most preferred object among the remaining objects
    - There is at least 1 cycle, the cycles are implemented and the agents removed from the pool.
- Mechanism is **incentive compatible** and yields **unique Pareto efficient outcome** as a function of the endowments

Some Notation

- A donor-recipient pair is (k_i, t_i), kidney k_i, patient t_i.
  - Some kidneys have no particular patient (kidneys from the queue), and some patients have no paired kidney.
- K is the set of live donor kidneys
- Preferences over K come from maximizing the prob. of a successful transplant: part of K is outside the feasible set because of blood type and tissue type. Preferences are assumed to be strict.
- Let K_i be the set of living donor kidneys that are compatible with patient t_i. Let w be the option of entering the waiting list with priority reflecting the preferences (P_i) over $K_i \cup \{k_i, w\}$.
  - If a patient t_i ranks k_i at the top of their preferences, they do not want to exchange .
  - If a patient t_i ranks k_i higher than w, the patients does not want to exchange with the cadaver queue.

Problem Setting

- A *static kidney exchange problem* is then defined as a set of donor-recipient pairs $\{(k_1, t_1), \ldots, (k_n, t_n)\}$, a set of compatible kidneys $K_i \in K = \{k_1, \ldots, k_n\}$ for each patient t_i, and a strict preference relation P_i over $K_i \cup \{k_i, w\}$ for each patient t_i.
- The *outcome* of a kidney exchange problem is a matching of kidneys and the wait list option to patients such that each patient t_i is assigned a kidney in $K_i \cup \{k_i\}$ or the wait list option w, and no kidney can be assigned to more than one patient, although the wait-list option w can be assigned to several patients.
- A *kidney exchange mechanism* creates a matching for each kidney exchange problem.

Kidney Exchange Mechanism

- Roth, Sonmez and Unver (2004) consider the *Top Trading Cycles and Chains* mechanism.
- Makes use of two concepts: cycles and w-chains
- A *cycle* an ordered list of kidneys and patients $(k'_1, t'_1, \ldots, k'_m, t'_m)$ such that kidney patient t'_1 points to kidney k'_2, patient t'_2 to kidney k'_3, etc., and patient t'_m to kidney k'_1. Each kidney or patient can be part of at most one cycle and no two cycles intersect.
- A *w-chain* is an ordered list where t'_1 points to k'_2, etc., and t'_m points to w. Kidney's can be part of multiple w-chains.
- In the algorithm, when a w-chain is selected, the patients are assigned the kidneys they point at, and patient t'_m receives high priority for the next compatible kidney in the cadaver queue. Kidney k'_1 is offered to the queue or to another patient with a paired donor.

Top Trading Cycle and Chains Mechanism

- 1. All kidneys are available, and all agents are *active*. At each stage of the procedure:
    - Each active patient points to their most preferred available kidney or to the wait-list option w
    - Each passive patient continues to point to their assignment, and each remaining kidney points to its paired recipient t_i
- 2. There is either a cycle, a w-chain, or both
    - Proceed to step 3 if there are no cycles. Otherwise, carry out the cycle and remove all patients in the cycle. Each remaining patient points at there top choice among remaining kidneys. Repeat until no cycle exists.
- 3. If there are no pairs left, the algorithm is done. Otherwise, each pair is the tail of a w-chain. Select only one of the chains.
    - The w-chain is removed and the transactions implemented, or the w-chain is kept and each patient turns passive.
- 4. After a w-chain is selected, new cycles may form. Repeat steps 2 and 3 with the remaining active patients and unassigned kidneys until no patient is left.

Leading Example

Leading Example

- In the leading example, we will consider a particular *chain selection rule*:
  - Choose the longest w-chain
  - In case it is not unique, choose the w-chain with the highest priority patient
  - If the highest priority patient belongs to more than one w-chains, choose the one with the second highest priority patient, etc.
  - Patients are ordered in terms of priority by index (so t_1 is the patient with the highest priority in this example)
  - Fix the longest w-chain, by making the agents in the w-chain passive, but the kidney at the tail of the w-chain can still be allocated among the remaining active participants.

Leading Example

- Consider the following preferences with 12 pairs $(k_1, t_1), \ldots, (k_{12}, t_{12})$:
  - Relevant part of the preferences P_i is only until one's own donor kidney or until the queue.

t_1 :	k_9	k_{10}	k_1				
t_2 :	k_{11}	k_3	k_5	k_6	k_2		
t_3 :	k_2	k_4	k_5	k_6	k_7	k_8	w
t_4 :	k_5	k_9	k_1	k_8	k_{10}	k_3	w
t_5 :	k_3	k_7	k_{11}	k_4	k_5		
t_6 :	k_3	k_5	k_8	k_6			

t_7 :	k_6	k_1	k_3	k_9	k_{10}	k_1	w
t_8 :	k_6	k_4	k_1	k_2	k_3	k_8	
t_9 :	k_3	k_{11}	w				
t_{10} :	k_{11}	k_1	k_4	k_5	k_6	k_7	w
t_{11} :	k_3	k_6	k_5	k_{11}			
t_{12} :	k_{11}	k_3	k_9	k_8	k_{10}	k_{12}	

Round 1

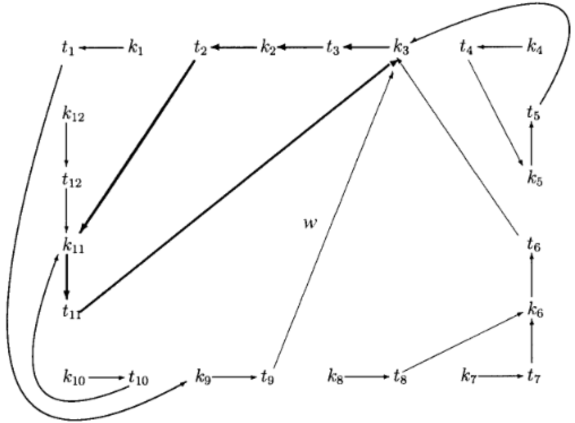

FIGURE I

Example 1, Round 1

There is a single cycle $C_1 = (k_{11}, t_{11}, k_3, t_3, k_2, t_2)$. Remove the cycle by assigning k_{11} to t_2, k_3 to t_{11}, and k_2 to t_3.

Figure 4: 1st round

Round 2

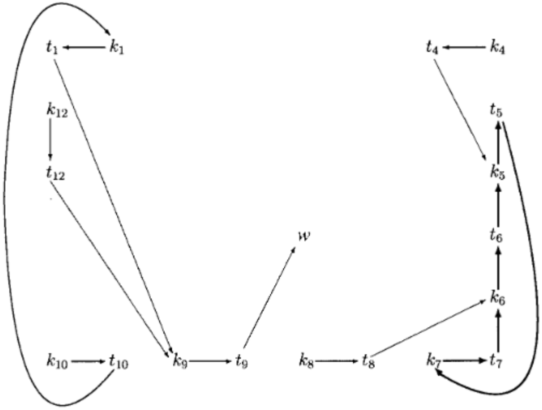

FIGURE II
Example 1, Round 2
Upon removing cycle C_1, a new cycle $C_2 = (k_7, t_7, k_6, t_6, k_5, t_5)$ forms. Remove it by assigning k_7 to t_5, k_6 to t_7, and k_5 to t_6.

Figure 5: 2nd round

Round 3

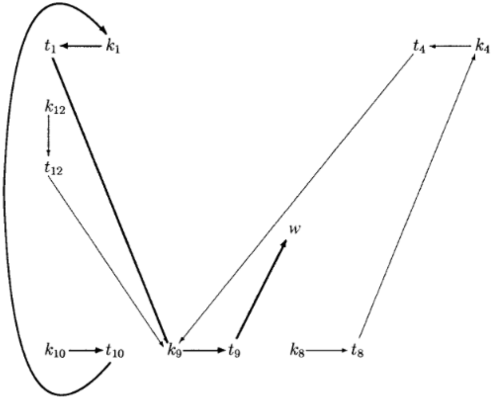

FIGURE III

Example 1, Round 3

No new cycle forms, and hence each kidney-patient pair starts a w-chain. The longest w-chains are $W_1 = (k_8, t_8, k_4, t_4, k_9, t_9)$ and $W_2 = (k_{10}, t_{10}, k_1, t_1, k_9, t_9)$. Since t_1, the highest priority patient, is in W_2 but not in W_1, choose and fix W_2. Assign w to t_9, k_9 to t_1, and k_1 to t_{10} but do not remove them. Kidney k_{10}, the kidney at the tail of W_2, remains available for the next round.

Figure 6: 3rd round

Round 4

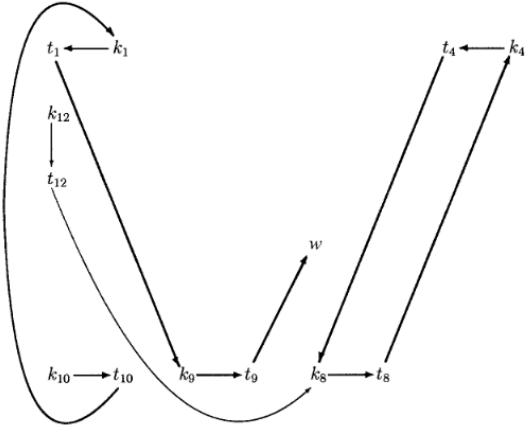

FIGURE IV
Example 1, Round 4
Upon fixing the w-chain W_2, a new cycle $C_3 = (k_4, t_4, k_8, t_8)$ forms. Remove it by assigning k_4 to t_8 and k_8 to t_4.

Figure 7: 4th round

Round 5

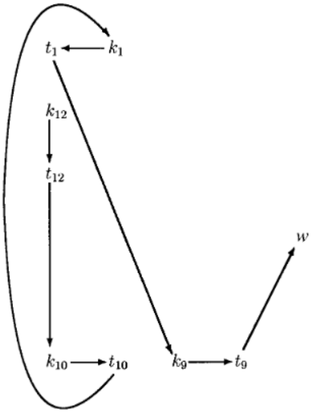

FIGURE V

Example 1, Round 5

No new cycles form, and the pair (k_{12},t_{12}) "joins" W_2 from its tail to form the longest w-chain $W_3 = (k_{12},t_{12},k_{10},t_{10},k_1,t_1,k_9,t_9)$. Fix W_3, and assign k_{10} to t_{12}. Since no patient is left, w-chain W_3 is removed, and kidney k_{12} at its tail is offered to the highest priority patient at the cadaveric waiting list.

Figure 8: 5th round

How well does this work?

- The final matching is:

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
k_9	k_{11}	k_2	k_8	k_7	k_5	k_6	k_4	w	k_1	k_3	k_{10}

Theorem

Consider a TTCC mechanism with any w-chain rule that stipulates that the w-chain remain in the procedure, such that kidney in the tail remains available for the next round. Any such mechanism is Pareto-efficient.

How well does this work?

- **Proof**: Let the mechanism be implemented with a chain selection rule such that any w-chain selected at a nonterminal round remains in the procedure and the kidney at its tail remains available for the next round.
  - Any patient whose assignment is done in Round 1 received its top choice and cannot be made better off.
  - Any patient whose assignment is done in Round 2 received its top choice among the kidneys not already assigned and cannot be made better off without hurting a patient whose assignment was done in Round 1.
    - Since the kidney at the tail remains available
  - In a similar way, any patient whose assignment is done in round k cannot be made better off without hurting a patient whose assignment is finalized in round $k-1$.
  - Therefore, in the terminal round, the allocation reached is Pareto efficient.

How well does this work?

- Corollary: Chain selection rules that remove a selected w-chain before termination may yield Pareto-inefficient outcomes.
- Roth et al. (2004) suggest that there is a trade-off between efficiency and equity for Type-O patients in practice.
  - Efficiency can be sacrificed by removing a w-chain if the w-chain with highest priority pair has a type-O donor
  - Then, the w-chain should be removed
- The results of Roth (1982) also apply for several versions of the chain selection rule: the TTCC mechanism is **strategy proof**.
  - But: not for chain selection rules that choose among the longest w-chain!
  - The result holds for rules that prioritize patient-donor pairs, and remove/keep it, and for minimal w-chain $\rightarrow$ remove rules.

How well does this work?

- Simulation results in Roth et al. (2004) with utility functions from empirical research determining preferences over kidneys
  - Under no exchange, 55% of living-donor kidneys are utilized, compared to 89% under TTCC
- TTCC decreases the HLA (tissue type) mismatch
  - Effect larger for larger pools
- Type O patients without living donors benefit from the TTCC mechanism: it reduces the incidence of type O patients with potential donors who are forced to rely on the cadaver queue.

Generalizations

Generalization

- Kidney Exchange just a special case of non-market allocation
  - But also more general because takes into account individuals without endowments
  - But also endowments without individuals!
- Unver (2010): Dynamic Kidney Exchange:
  - Dynamically evolving agent pool with time- and compatibility-based preferences.
  - Derives the dynamically efficient two-way and multi-way exchange mechanisms that maximize total discounted exchange surplus
- Abdulkadiroglu and Sonmez (1999): University rooms allocation
  - Some students already have a room, others are new and have no room
  - There are also vacant rooms in addition to the occupied rooms
  - Very similar to this setting

Conclusion

Conclusion

- Kidney exchange is a politically charged topic with constraints on the allocation mechanism
  - Markets and auctions, although efficient and incentive compatible, probably wouldn't work
  - Hence the search for alternative mechanisms: efficient, incentive compatible, and ..fair?
- Roth et al. (2004) found one such mechanism
  - They do not claim to have found all possible mechanisms
  - Mostly based on Shapley and Scarf (1974) top trading cycle and the "leftovers"
  - Simulation results very promising
- Difficult to implement because of difficulties eliciting preferences and ensuring participation in practice
- Simpler methods with more modest gains might find more goodwill among patients
  - And changing the "default option" increases the cadaver queue