Bilateral Bargaining with Durable Commitments

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Motivation

Motivation

- The paper is about **infite-horizon** bilateral bargaining
  - Infinite horizon captures long-term perspective
- Bilateral bargaining: many important situations in real-life
  - From paper: trade unions vs. firms
  - Politics: Israel-Palestine, US-Russia, India-Pakistan
- Many models imply immediate agreement
  - Real-life agreement only reached after periods
- Literature: delay because of biased beliefs or private information
  - Contribution of this paper: delay because of **commitment decay**

Model Setup

Model Setup

- There are two negotiators, $i = A, B$, who bargain over a fixed surplus of size 1. Players have linear utility in the share of the surplus, but are impatient, with a common discount factor of δ.
- The timing is as follows. In period 1: three stages.

1. *The proposal stage*: each player i chooses either to make a proposal to divide the surplus, or to wait
   - In case of a proposal, $s_1^i \in [0, 1]$ is the offer made by player i to player j(!) in period 1. The waiting action is w, and the set of proposal stage actions is $S = [0, 1] \cup \{w\}$.
   - Players can make a randomized offer ϕ_1^i by offering a probability distribution over s with associated density $p^i(s)$.
   - It is costly to make a proposal (cost c), waiting is free.
   - We say that player i made a *commitment* if i made a proposal $s^i \in [0, 1]$.

Model Setup

- Still in Period 1:

2. *Delay*: After making an offer, time passes before player j can respond. During this time, a commitment can decay. With probability $q \leq 1$, the offer survives until the response stage, with probability $1 - q$, player i instead enters the response stage with $s^i = w$.

Model Setup

3. *The response stage*: each player now observes the proposals made in the proposal stage. Player j observes the *realized* value of ϕ^i, the randomized offer.

- If only player i is committed, player j chooses to Accept or Reject the offer. Acceptance gives payoffs $(1 - s_1^i, s_1^i)$. Rejection transfers the players to the next period.
- If both players are committed:
  - If the outcome is incompatible ($s_1^A + s_1^B < 1$), there is no agreement and players are transferred to the next period.
  - If the outcomes are compatible, one of the players is randomly picked to decide Accept or Reject.
- If no player is committed ($s^A = s^B = w$): player i randomly ($p = \frac{1}{2}$) selected to make a short take-it-or-leave-it offer: randomized offer $\underline{\phi}_1^i$. Other player decides Accept or Reject.

Model Setup

- In period t:
- If negotiations did not end before period t.
  - And if in the beginning of period $t-1$ player i was committed to the offer $s_{t-1}^{i} \in [0,1]$.
  - Then, unless the commitment subsequently decayed, player i remains committed to s_{t-1}^{i}, so $s_{t}^{i} = s_{t-1}^{i}$.
  - If instead player i is flexible at the beginning of period t, a new offer $s^{i} \in S$ is made, and the game proceeds as in round 1 (i.e. *proposal stage*, *delay*, *response stage*).

Solution

Solution to a 1-period Game

- Let $\mathcal{P}$ be the set of all probability distributions on $S = [0,1] \cup w$. Commitment strategies are then functions $\phi^i : \mathcal{H} \to \mathcal{P}$ (long-run strategy), $\underline{\phi}^i : \mathcal{H} \to \underline{\mathcal{P}}$ (short-run strategy).
- A response strategy is $\rho^i : \mathcal{H} \to \{0,1\}$. Hence, an entire strategy for player i is $\sigma_i = (\phi^i, \underline{\phi}^i, \rho^i)$. The set of all subgames is $\mathcal{G}^\infty$.
- Equilibrium concept: subgame perfect Nash equilibrium
- For a small enough δ:

Proposition

There is a unique subgame perfect Nash equilibrium of G^1. At the proposal stage, each player i commits to the offer $s^i = 0$. In case both commitments decay and player i is chosen to be the proposer, player i makes the offer $s^i = 0$. If only player i's commitment decays, or if both commitments decay and player i is chosen to be responder, player i accepts any offer $s^j \in [0,1]$. Since the commitments are incompatible, there is disagreement with probability q^2.

Intuition (Proof Sketch) Behind the SPNE

- Suppose a given commitment s^i:
  - If both players' commitments decay, the game becomes an ultimatum game - in this subgame, it is optimal to offer 0 to your opponent, and to accept any offer.
  - If only i's commitment decays, it is optimal to accept anything - hence, for player j, it is optimal to propose 0.
  - If only j's commitment decays, you receive $1 - s^i$.
  - If no player's commitment decays, there is no agreement.
- The expected payoff of such a strategy is, given $s^j = 0$:

$$\delta\Big[\underbrace{(1-s^i)(1-q)q}_{\text{Payoff if } j \text{ decays}} + \overbrace{\frac{1}{2}(1-q)^2 \cdot 1}^{\text{Payoff if both decay}}\Big] - c$$

- So that the strategies of i and j are best responses to each other.
  - Playing w leads to a lower payoff for a small enough c.
  - Powerful and realistic outcome: in real life, not all conflicts are resolved

Solution to an ∞-period game

- I sketch the (unique) Markov Perfect Equilibrium (Strategies only conditioned on State)
- Only flexible players can make a commitment, a stationary long-run offer strategy only depends on known features of the opponent's current commitment status $s^j_{(t-1)}$. Acceptance decisions depend on the pair of current commitments s_t. c is assumed to be 0, and in period 1, the state is $s_0 = (w, w)$.
- Let $V^i(s^i, s^j|\sigma)$ denote the value to player i at the end of a period's proposal state
- As a function of the (realized) commitment decay, R^i is a player i's acceptance threshold. Player j will take this into account when offering and offers:

$$s^j = V^i(\hat{\phi}^i(w), \hat{\phi}^j(w)|\hat{\sigma}) := \hat{s}^j_n \text{ non-durable} \tag{1}$$

$$s^j = \min_{s^k} V^i(\hat{\phi}^i(s^k), s^k|\hat{\sigma}) := \hat{s}^j \text{ durable}$$

Solution to an ∞-period game

- But, if both players become flexible, stationarity implies that they will commit again to the equilibrium strategy, so that $\hat{s}^j = \hat{s}^j_n$.
- Using the previous result, it is shown that in equilibrium, durable offers **must be incompatible**: $\hat{s}^j < 1 - \hat{s}^i$.
- Then, the authors construct an equilibrium $\sigma^i_S = (s^i_S, \rho^i)$ with:

$$\rho^i = \begin{cases} 0 & \text{if } s^j \leq s^j_S \\ 1 & \text{otherwise} \end{cases}$$

and, whenever flexible at the proposal stage:

$$\phi^i = \begin{cases} w & \text{if } s^j_{t-1} > \tilde{s}^j \\ s^i_S & \text{otherwise} \end{cases}$$

Solution to an ∞-period game

- The threshold $\tilde{s}^j$ solves the equation $\delta(q\tilde{s}^j + \frac{1-q}{2}) = s_S^j$.
- In words, at the proposal stage, player i makes a new incompatible offer s_S^i if the opponent was flexible or committed to an unacceptable offer in the previous period
- The threshold that makes player i indifferent between (i) waiting and accepting $\tilde{s}^j$ at the response stage and (ii) making a new commitment.
- Then, since acceptance thresholds are stationary, the following recursions hold:

$$\begin{pmatrix} V^a \\ V^b \end{pmatrix} = \delta \begin{pmatrix} q^2 V^a + q(1-q)(1-V^b) + (1-q)qV^a + (1-q)^2 \frac{1-V^b+V^a}{2} \\ q^2 V^b + q(1-q)(1-V^a) + (1-q)qV^b + (1-q)^2 \frac{1-V^a+V^b}{2} \end{pmatrix}$$

and the solution yields: $V^a = V^b = \frac{\delta(1-q^2)}{2(1-\delta q^2)}$

- By equation 1, we get the actual proposal values.

Generalizations

Generalizations

- Non-stationary subgame perfect equilibria
  - One player chooses to be flexible, other chooses commitment can be sustained by grim-trigger-like strategies
- Decay rates
  - Payoffs as a function of decay rates in q and δ - efficient outcomes can be sustained
- Asymmetric discounting
  - Payoffs increase in opponents decay and discount rates, and decrease in own decay and discount rates.
- Risk aversion
  - In the stationary equilibrium, a player whose risk-aversion increases will make a more generous offer and receive a less generous offer.

Conclusion

Conclusion

- The paper captures two aspects of bargaining
    - Inflexibility (anchored commitments)
    - Flexibility (possibility to continue negotiating after no agreement)
- Both of these aspects frequently turn-up in real life
- The authors focus on the role of commitment decay
- Commitment might capture important aspects of real-life negotiations:
    - Wavering and contingent military threats
    - Conditional diplomatic support of other countries (Cuban missile crisis)
- Shortcomings: cost independent of proposal