

# Model

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9/22/2021

## A simple model of voting behavior

I suppose that politicians make decisions according to the utility function:

$$U(p_i, C_i) = -(p_i - p_i^*)^2 + \beta \cdot f(p_i, C_i) + \epsilon_i^p \quad (1)$$

where  $p_i$  is their vote,  $p_i^*$  is the ideal position (referred to as ideological bliss point in e.g. Duggan & Martinelli, 2017) of politician  $i$ , and  $f$  represents a mapping from the personal costs  $C_i$  arising from the acceptance of the law to a politician's utility. Finally,  $\epsilon_i^p$  is a random shock to utility, as in Mian, Sufi, Trebbi (2010).

$p_i^*$  can be interpreted as follows: suppose  $\beta = 0$ , so that the only aspect politicians care about when deciding on a law is their ideological bliss point. Then, the utility function simplifies to:

$$U(p_i) = -(p_i - p_i^*)^2$$

Politicians can only choose to vote in favor of or against a law. Hence, the choice space for politicians is always  $p_i = \{0, 1\}$ . Again in the case of  $\beta = 0$  (and no error terms), politicians would then chose  $p_i = 1$  if  $p_i^* > 0.5$  and  $p_i = 0$  if  $p_i^* < 0.5$ , so as to maximize equation 1. The only factor influencing their decision-making is then their ideology. More generally, politicians would solve the following problem:

$$\max_{p_i \in \{0, 1\}} \{-(p_i - p_i^*)^2 + \beta \cdot f(p_i, C_i) + \epsilon_i^p\}$$

which, after supposing that  $f$  takes on the form:

$$f = \begin{cases} C(W_i) & \text{if } p_i = 1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

makes that politicians vote *in favor of the law* if and only if:

$$p_i = \begin{cases} 1 & \text{if } U(1, C_i) > U(0, C_i) \\ 0 & \text{otherwise} \end{cases}$$

The condition  $U(1, C_i) > U(0, C_i)$  is not deterministically determined, but is determined deterministically only up to the error terms. Specifically, using equation 1 again:

$$U(1, C_i) > U(0, C_i) \Rightarrow -(1 - p_i^*)^2 + \beta C_i + \epsilon_i^1 > -(P_i^*)^2 + \epsilon_i^0$$

Rearranging this gives:

$$-1 + 2p_i^* + \beta C_i > \epsilon_i^1 - \epsilon_i^0$$

If we then assume a distribution for the difference of these  $\epsilon$ 's (a wide range of distributions can apply), we can determine the probability that  $p_i = 1 \iff U(1, C_i) > U(0, C_i)$ . To be concrete, suppose that  $\epsilon_i^1 - \epsilon_i^0 \sim N(0, 1)$ .

Then, the probability  $P(p_i = 1) = P(-1 + 2p_i^* + \beta C_i > \epsilon_i^1 - \epsilon_i^0) = \Phi(-1 + 2p_i^* + \beta C_i)$ , where  $\Phi$  is the cdf of the standard normal distribution. Then, if  $\beta < 0$ , and  $\frac{\partial C_i}{\partial W_i} > 0$ , the term inside  $\Phi$  decreases as  $W_i$  increases, so that the probability that a politician votes  $p_i = 1$  decreases as  $W_i$  increases. (In contrast, the probability of voting in favor increases as a politician's ideology  $p_i^* \rightarrow 1$ ).