Model

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A simple model of voting behavior

I suppose that politicians make decisions according to the utility function:

$$U(p_i, C_i) = -(p_i - p_i^*)^2 + \beta \cdot f(p_i, C_i) + \epsilon_i^p \tag{1}$$

where p_i is their vote, p_i^* is the party line (an ideological bliss point for the political party as in e.g. Duggan & Martinelli, 2017) of politician i, and f represents a mapping from the personal costs C_i arising from the acceptance of the law to a politician's utility. Finally, ϵ_i^p is a random shock to utility, as in Mian, Sufi, Trebbi (2010).

p_i^* can be interpreted as follows: suppose $\beta = 0$, so that the only aspect politicians care about when deciding on a law is their ideological bliss point. Then, the utility function simplifies to:

$$U(p_i) = -(p_i - p_i^*)^2$$

Politicians can only choose to vote in favor of or against a law. Hence, the choice space for politicians is always $p_i = \{0, 1\}$. Again in the case of $\beta = 0$ (and no error terms), politicians would then chose $p_i = 1$ if $p_i^* > 0.5$ and $p_i = 0$ if $p_i^* < 0.5$, so as to maximize equation 1. The only factor influencing their decision-making is then their party alignment. More generally, politicians would solve the following problem:

$$\max_{p_i \in \{0,1\}} \{-(p_i - p_i^*)^2 + \beta \cdot f(p_i, C_i) + \epsilon_i^p\}$$

which, after supposing that f takes on the form:

$$f = \begin{cases} C(W_i) & \text{if } p_i = 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

makes that politicians vote *in favor of the law* if and only if:

$$p_i = \begin{cases} 1 & \text{if } U(1, C_i) > U(0, C_i) \\ 0 & \text{otherwise} \end{cases}$$

The condition $U(1, C_i) > U(0, C_i)$ is not deterministically determined, but is determined deterministically only up to the error terms. Specifically, using equation 1 again:

$$U(1, C_i) > U(0, C_i) \Rightarrow -(1 - p_i^*)^2 + \beta C_i + \epsilon_i^1 > -(P_i^*)^2 + \epsilon_i^0$$

Rearranging this gives:

$$-1 + 2p_i^* + \beta C_i > \epsilon_i^1 - \epsilon_i^0$$

If we then assume a distribution for the difference of these ϵ's (a wide range of distributions can apply), we can determine the probability that $p_i = 1 \iff U(1, C_i) > U(0, C_i)$. To be concrete, suppose that $\epsilon_i^1 - \epsilon_i^0 \sim N(0, 1)$.

Then, the probability $P(p_i = 1) = P(-1 + 2p_i^* + \beta C_i > \epsilon_i^1 - \epsilon_i^0) = \Phi(-1 + 2p_i^* + \beta C_i)$, where Φ is the cdf of the standard normal distribution. Suppose also that $C_i = C(W_i) = \log W_i$. Then, if $\beta < 0$, the term inside Φ decreases as W_i increases, so that the probability that a politician votes $p_i = 1$ decreases as $\log W_i$ increases. (In contrast, the probability of voting in favor increases as a function of a party's ideology $p_i^* \to 1$).

Empirically, a regression of $P(p_i = 1) = f(p_i^*, W_i)$ on W_i, controlled for p_i^* would then identify the coefficient β.