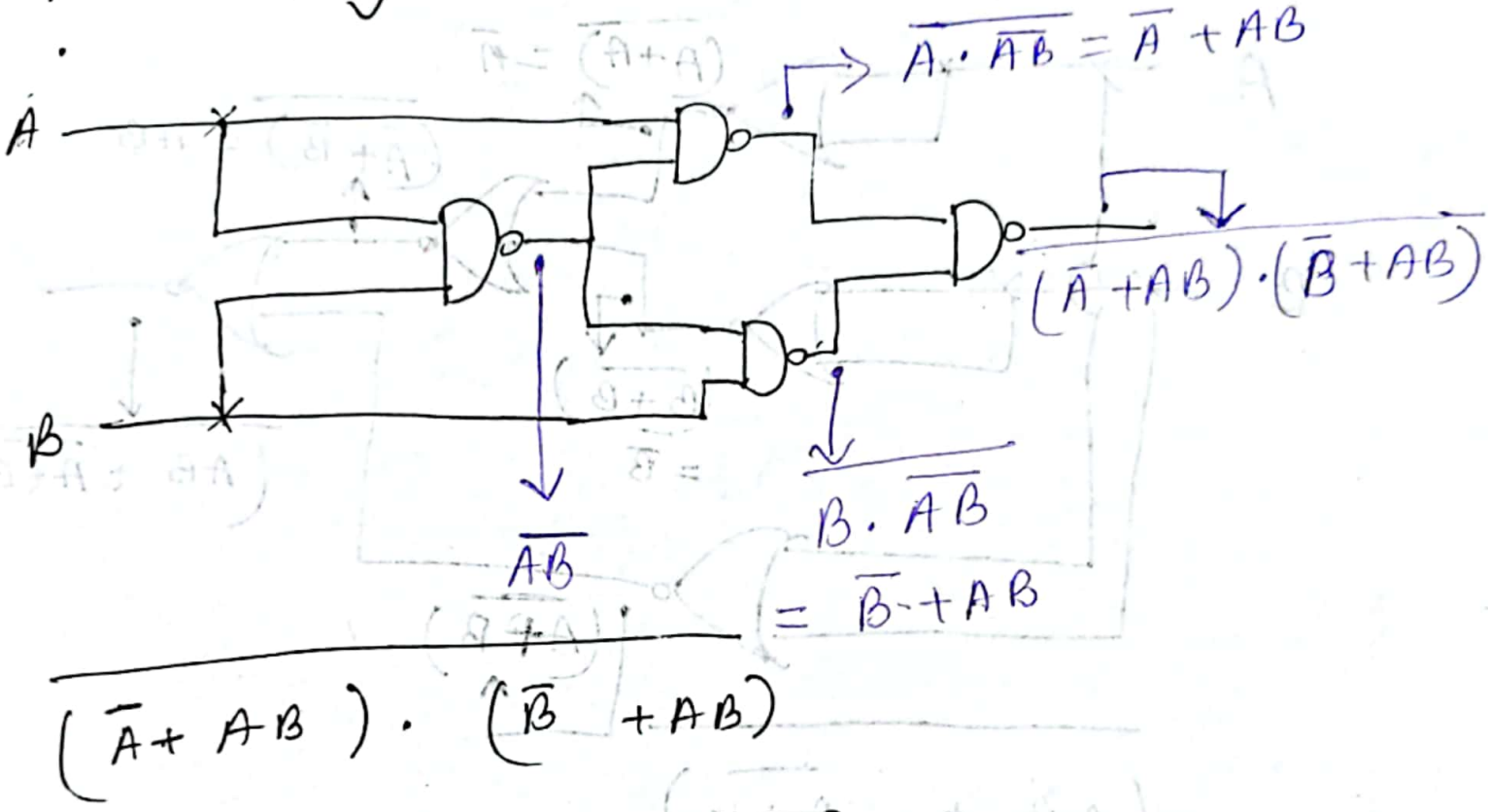
XOR Using NAND:



$$\overline{(\overline{A} + AB) \cdot (\overline{B} + AB)}$$

$$= \overline{(\overline{A} + AB)} + \overline{(\overline{B} + AB)}$$

$$= (\overline{\overline{A}} \cdot \overline{AB}) + (\overline{\overline{B}} \cdot \overline{AB})$$

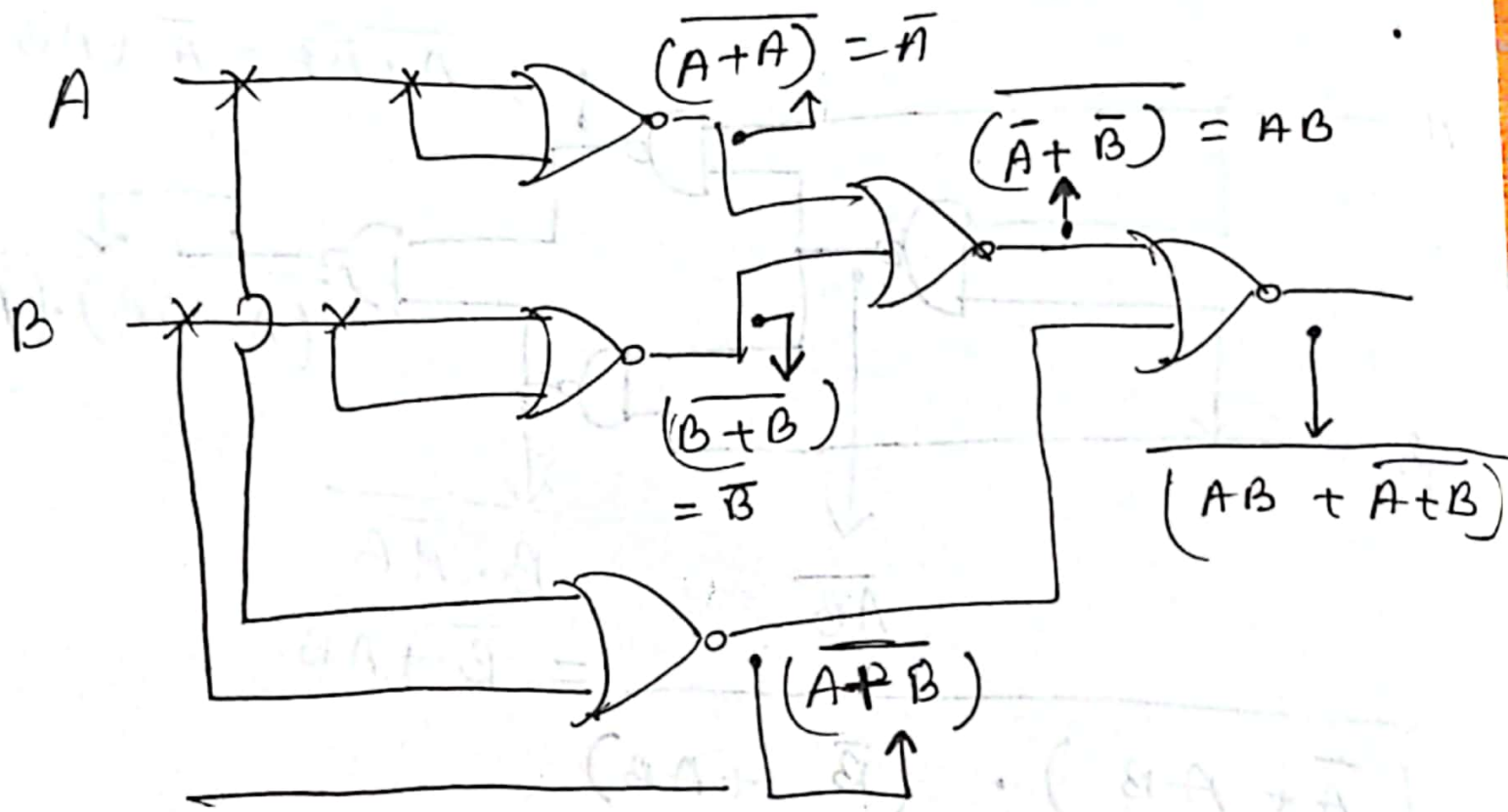
$$= A(\overline{A} + \overline{B}) + B \cdot (\overline{A} + \overline{B})$$

$$= A\overline{A} + A\overline{B} + B\overline{A} + B\overline{B}$$

$$= 0 + A\overline{B} + \overline{A}B + 0$$

$$\boxed{= \overline{A}B + A\overline{B}}$$

XOR Using NOR:



$$\overline{(AB + \overline{A+B})}$$

$$= \overline{AB} \cdot \overline{(\overline{A+B})}$$

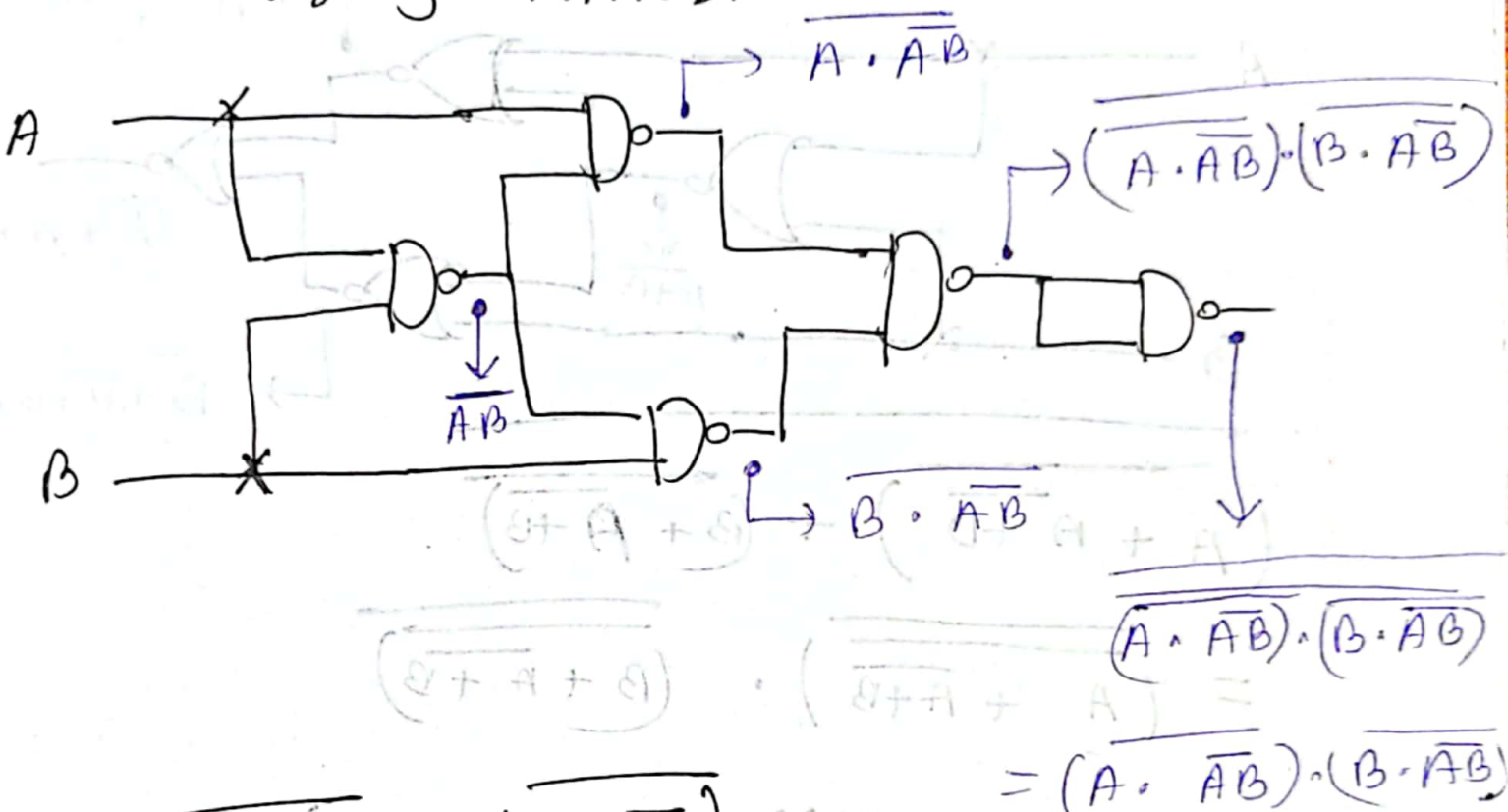
$$= (\bar{A} + \bar{B}) \cdot (A+B)$$

$$= A\bar{A} + \bar{A}B + \bar{B} \cdot A + \bar{B}B$$

$$= 0 + \bar{A}B + A\bar{B} + 0$$

$$\boxed{= \bar{A}B + A\bar{B}}$$

XNOR Using NAND:



$$= \overline{(A \cdot \overline{AB})} \cdot \overline{(B \cdot \overline{AB})}$$

$$= (\overline{A} + \overline{\overline{AB}}) \cdot (\overline{B} + \overline{\overline{AB}})$$

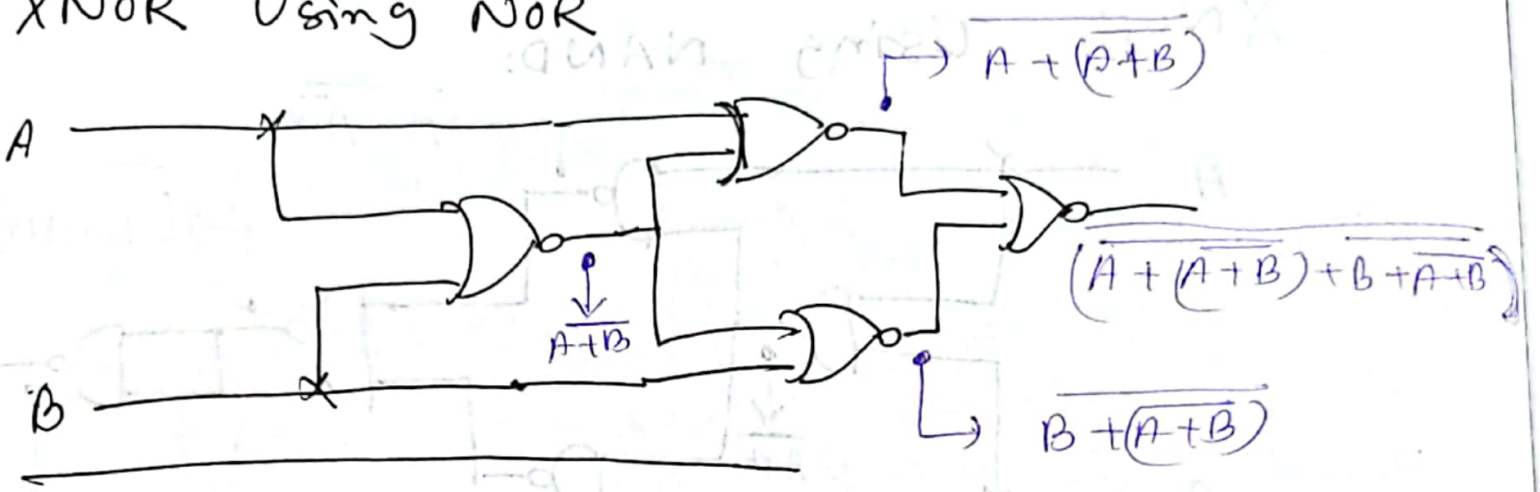
$$= (\overline{A} + AB) \cdot (\overline{B} + AB)$$

$$= \overline{A}\,\overline{B} + \overline{A} \cdot AB + AB \cdot \overline{B} + AB$$

$$= \overline{A}\,\overline{B} + 0 + 0 + AB$$

$$\boxed{= AB + \overline{A}\,\overline{B}}$$

XNOR Using NOR



$$\overline{(A+\overline{A+B})}+\overline{(B+\overline{A+B})}$$

$$=\overline{(A+\overline{A+B})}\cdot\overline{(B+\overline{A+B})}$$

$$=(A+(\overline{A+B}))\cdot(B+(\overline{A+B}))$$

$$=(A+(\bar{A}\,\bar{B}))\cdot(B+(\bar{A}\,\bar{B}))$$

~~$A\bar{A}+A\bar{B}$~~

$$=(A+\bar{A}\,\bar{B})\cdot(B+\bar{A}\,\bar{B})$$

$$=AB+A\bar{A}B+\bar{A}\bar{B}B+\bar{A}\bar{B}$$

$$=AB+\bar{A}\bar{B}$$