Probability Distributions

Basic Probability Laws

- **Union Rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Intersection Rule:** $P(A \cap B) = P(A)P(B|A)$
- **Complement Rule:** $P(A^c) = 1 - P(A)$
- **Law of Total Probability:** $P(A) = \sum P(A|B_i)P(B_i)$
- **Bayes' Theorem:** $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

1. Introduction to Probability Distributions

- **Definition**: A probability distribution describes how probabilities are distributed over different values of a random variable.
- **Types**:
    - Discrete Probability Distributions (e.g., Binomial, Poisson, Bernoulli)
    - Continuous Probability Distributions (e.g., Normal, Chi-Squared, T-Distribution)

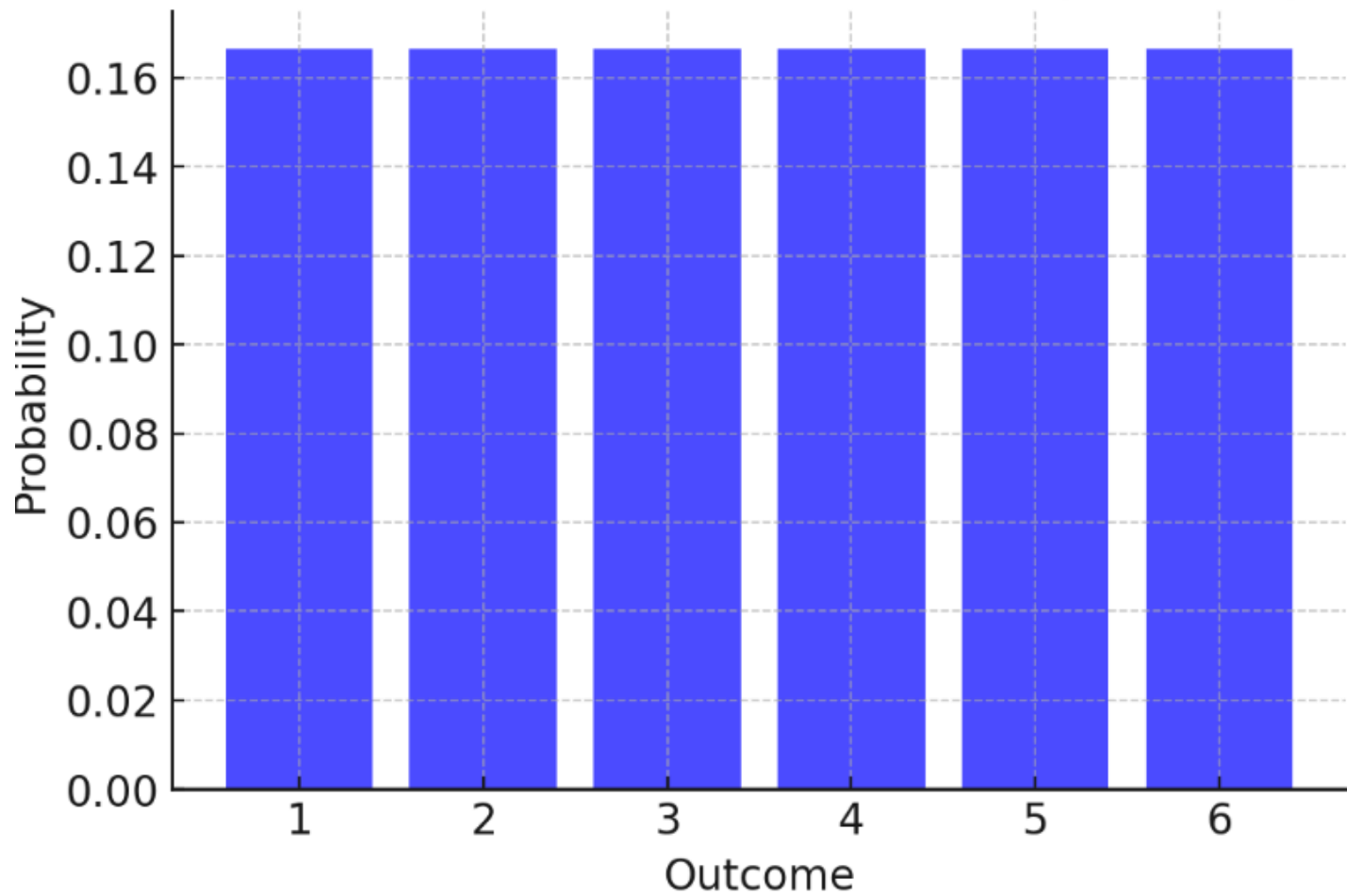# 2. Discrete Uniform Distribution

- **Definition**: A distribution where each possible outcome has an equal probability.

$$P(X = x) = \frac{1}{n}, \quad \text{for } x \in \{x_1, x_2, ..., x_n\}$$

- **Formula**:   where is the number of possible outcomes.

Example: Probability of rolling a fair six-sided die (each outcome has equal probability) .

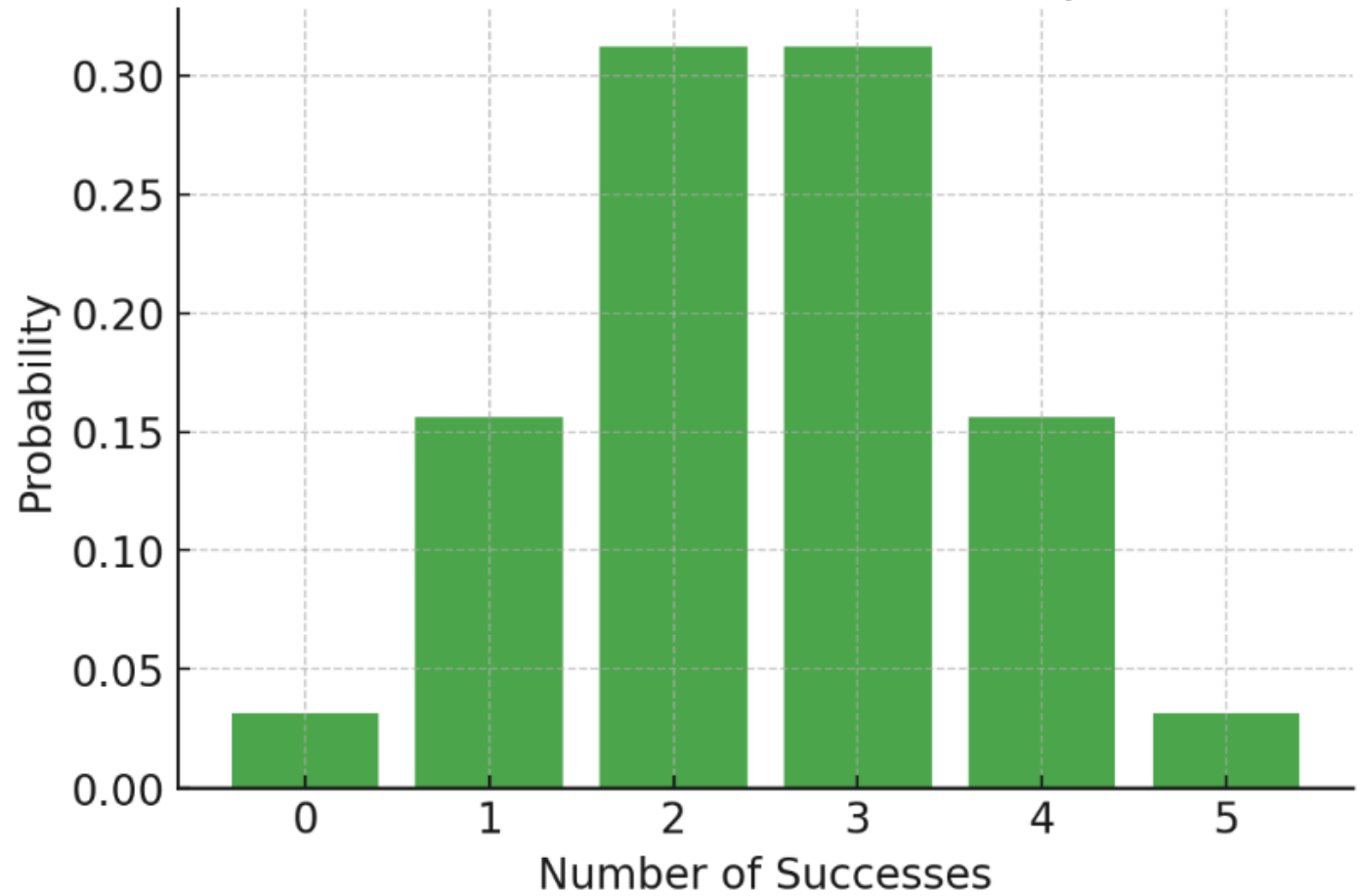Discrete Uniform Distribution (Fair Die)

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3. Binomial Distribution

- **Definition**: Models the number of successes in n independent Bernoulli trials.
- **Formula**:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Example: Probability of getting exactly 3 heads in 5 coin flips (success probability p=0.5)
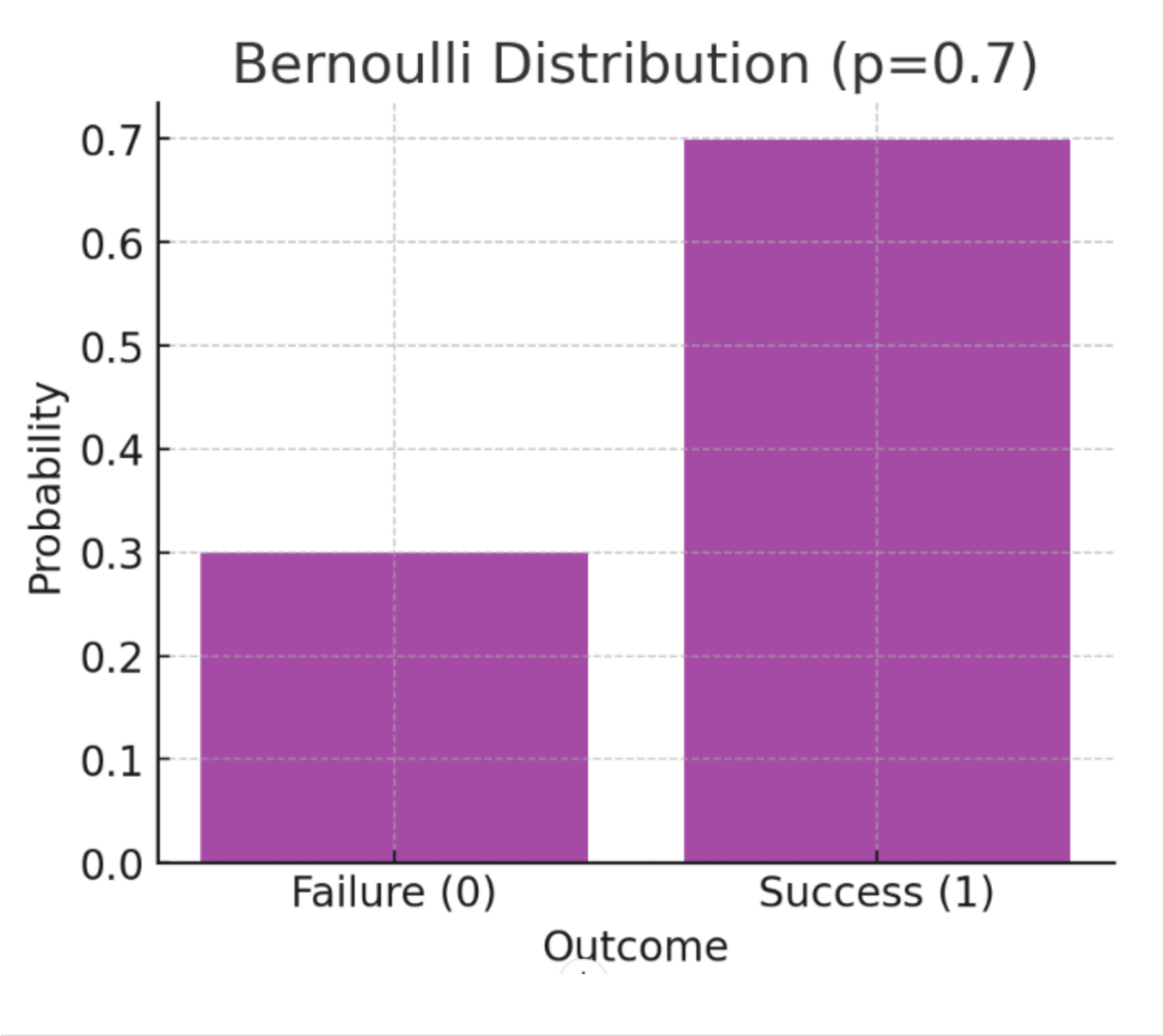

Binomial Distribution (n=5, p=0.5)

4. Bernoulli Distribution

- **Definition**: A special case of the binomial distribution where $n=1$.
- **Formula**:

$$P(X = 1) = p, \quad P(X = 0) = 1 - p$$

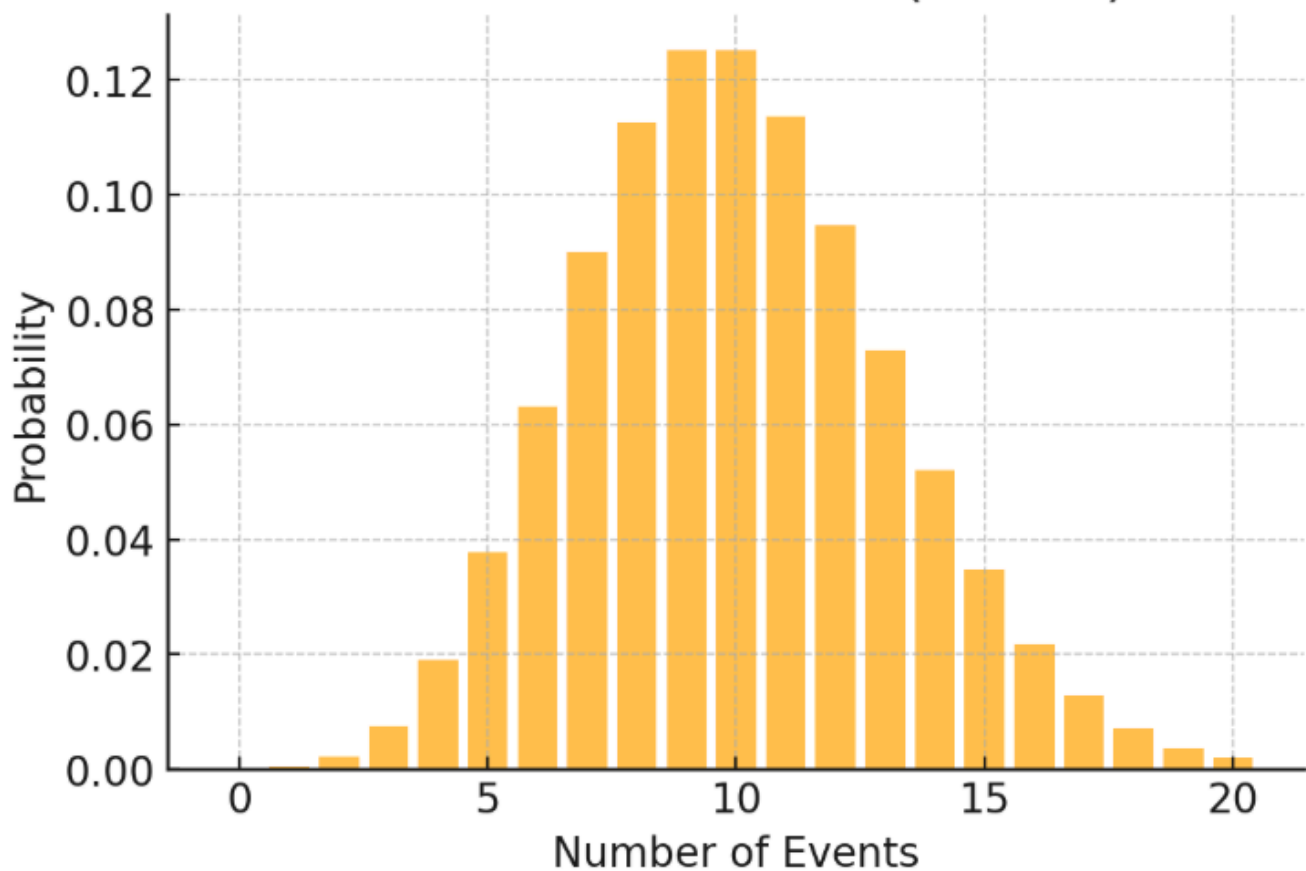**Example**: Success or failure in a penalty kick (e.g., scoring with probability p=0.7).

5. Poisson Distribution

- **Definition**: Models the number of events occurring in a fixed interval of time/space.
- **Formula**:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

Example: Number of customer arrivals per hour in a shop (with an average arrival rate λ=10)
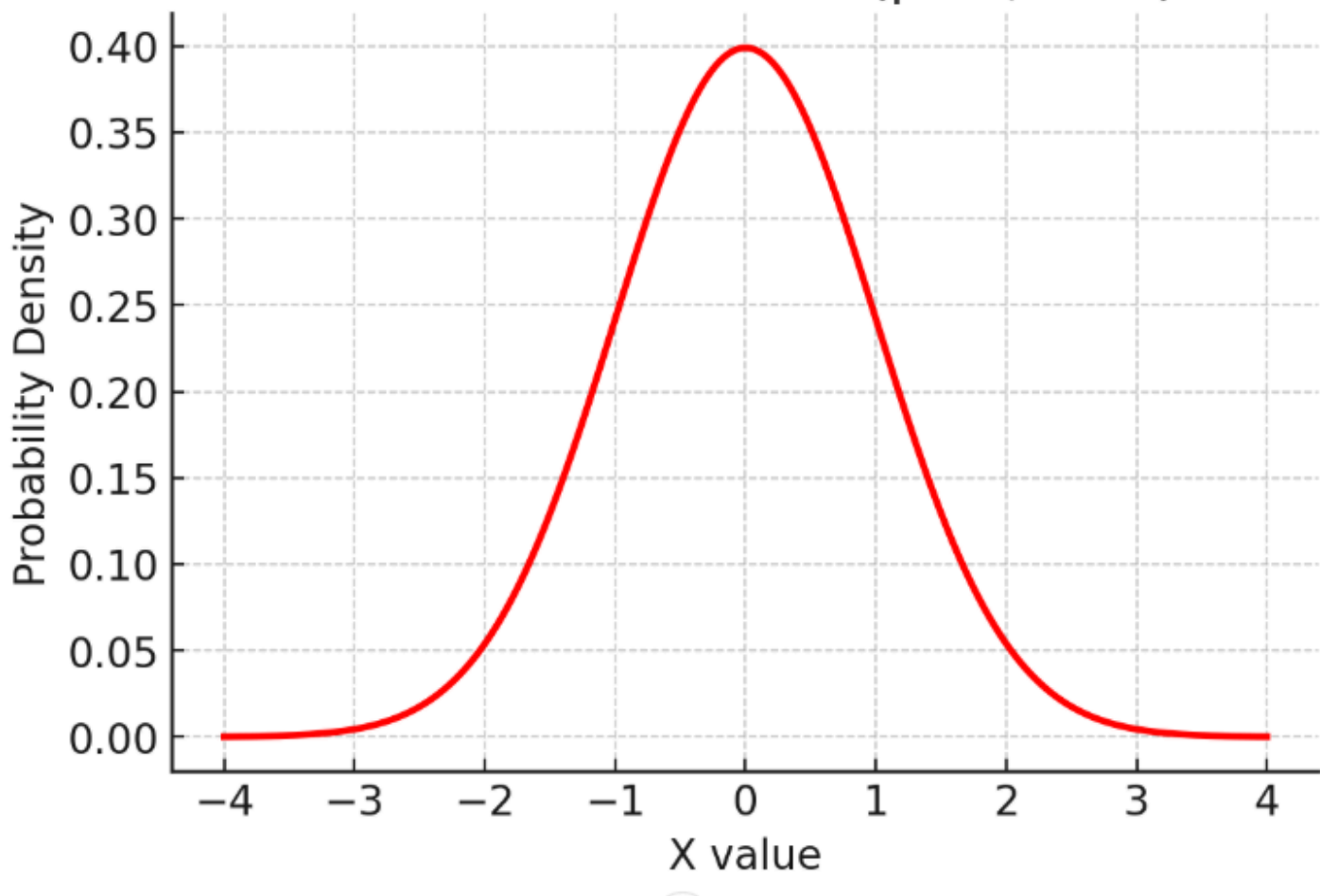
Poisson Distribution ($\lambda = 10$)

6. Normal Distribution

- **Definition**: A continuous distribution describing many natural phenomena.
- **Formula**:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
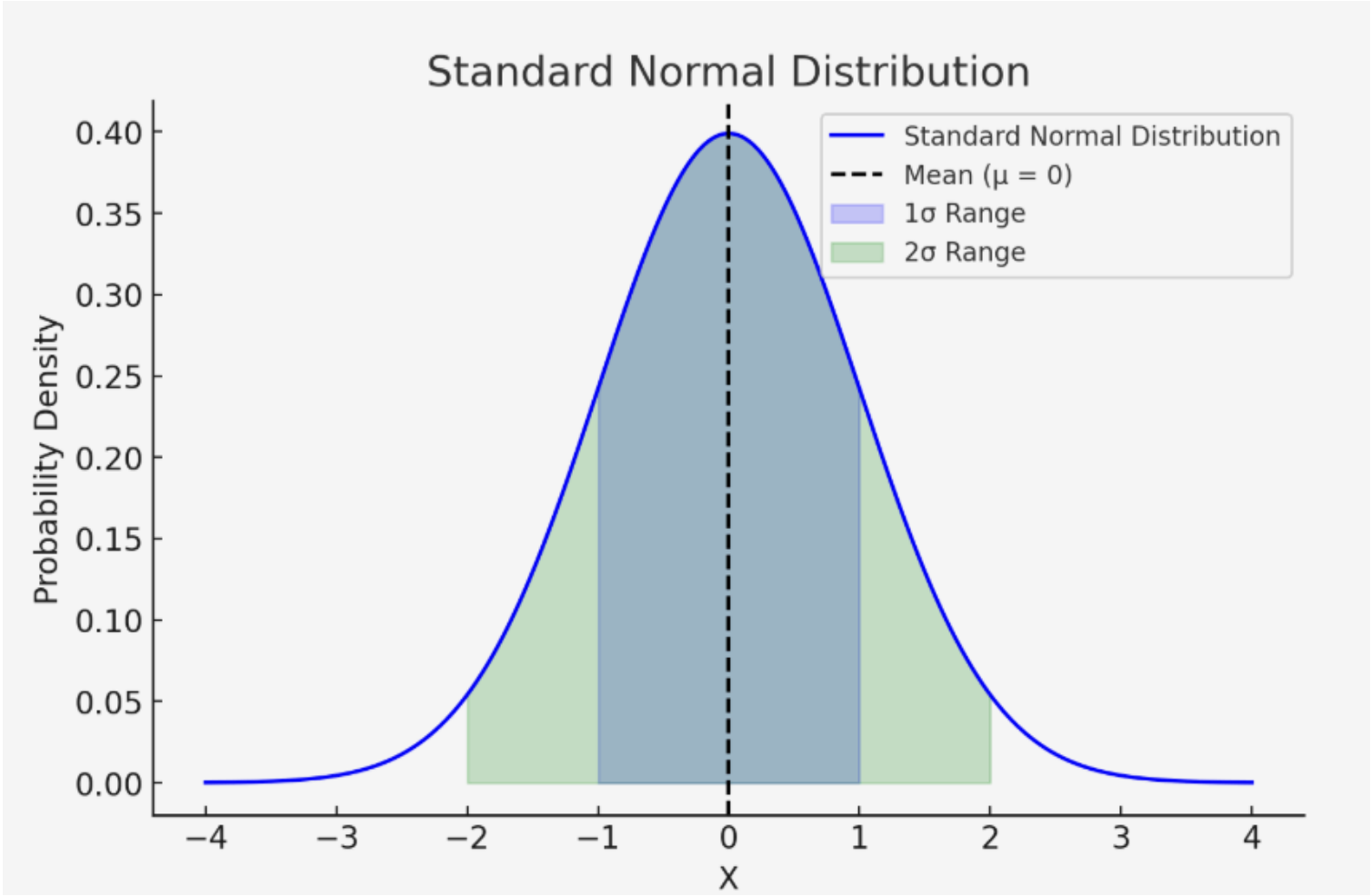
Example: Heights of people in a population (mean $\mu = 170cm\backslash mu = 170cm$, standard deviation $\sigma = 10cm\backslash sigma = 10cm$).

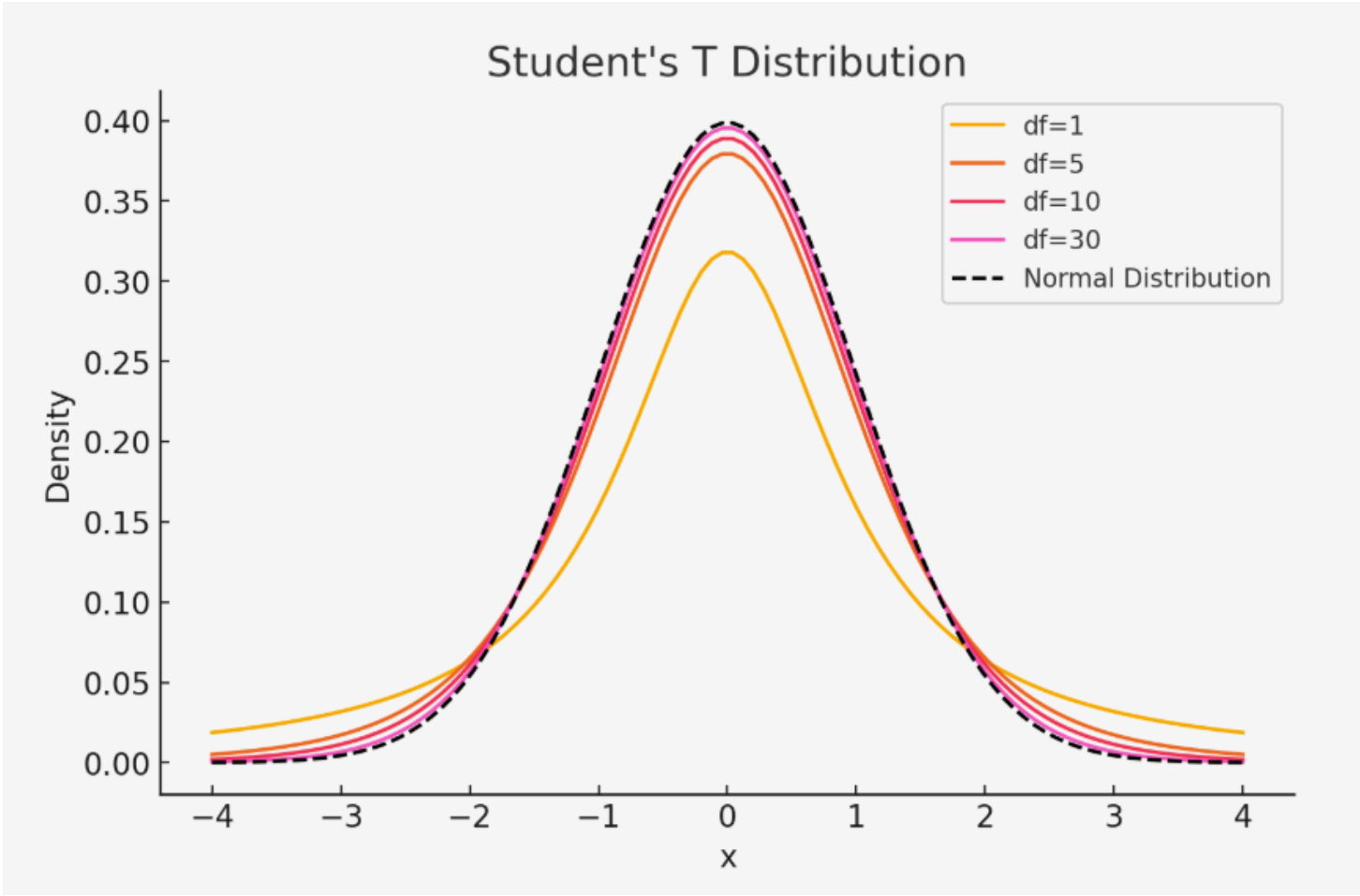Normal Distribution ($\mu=0$, $\sigma=1$)

7. Standard Normal Distribution

- **Definition**: A normal distribution with mean μ=0 and standard deviation σ=1.
- **Usage**: Used for statistical testing.

Standard Normal Distribution

8. Student's T Distribution

- **Definition**: Used for small sample sizes instead of the normal distribution.
- **Formula**:

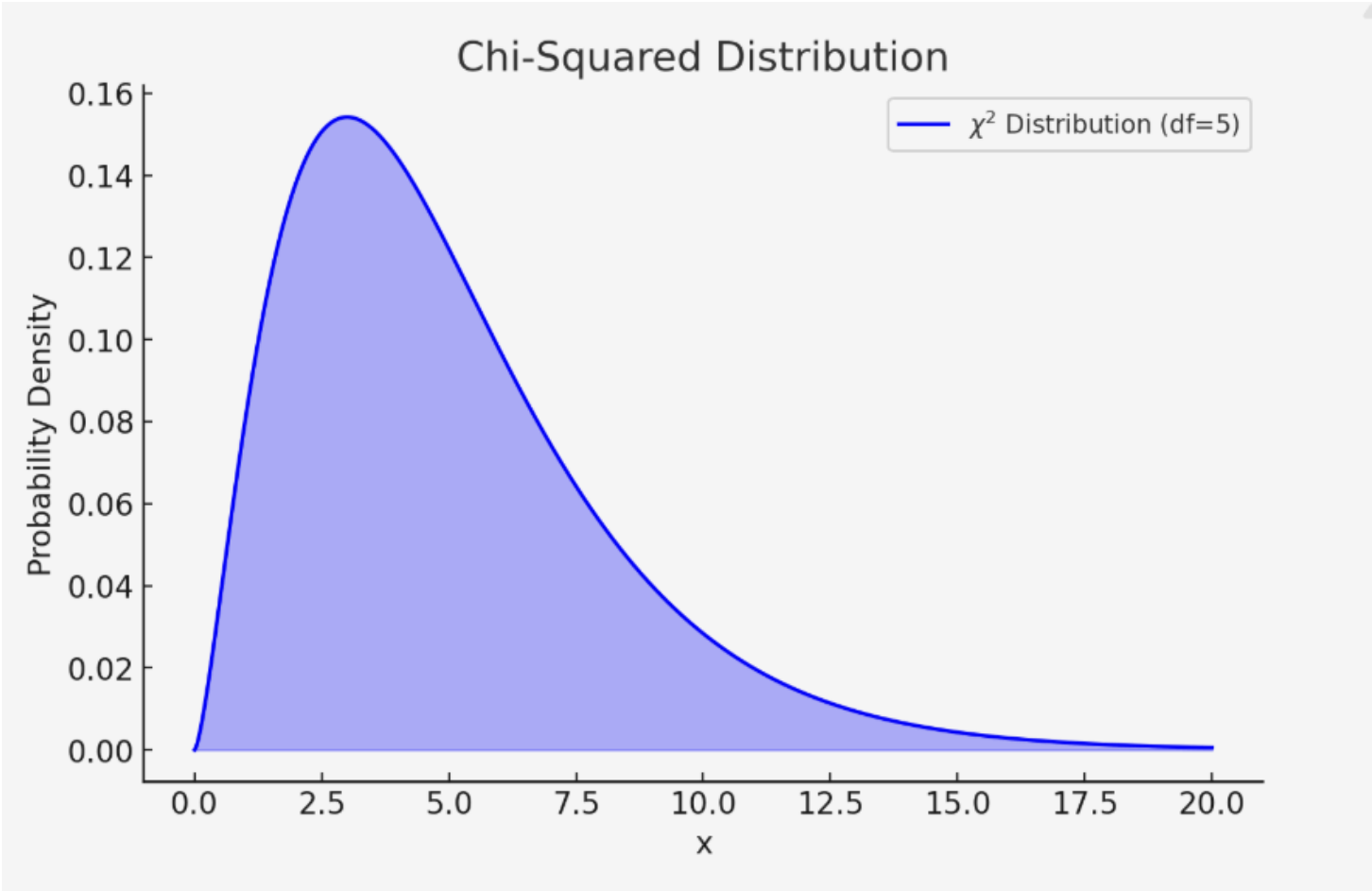$$t = \frac{X - \mu}{s/\sqrt{n}}$$

Student's T Distribution

9. Chi-Squared Distribution

- **Definition**: Used in hypothesis testing and variance estimation.
- **Formula**:

$$: \chi^2 = \sum(X_i^2)$$

Chi-Squared Distribution

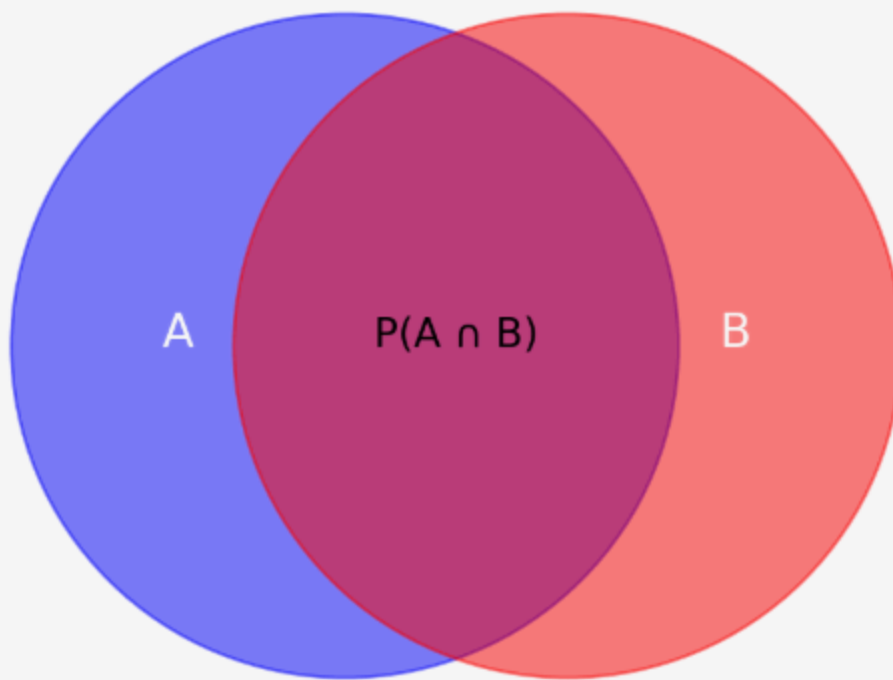# 10. Conditional Probability & Bayes' Theorem

- **Law of Total Probability**:

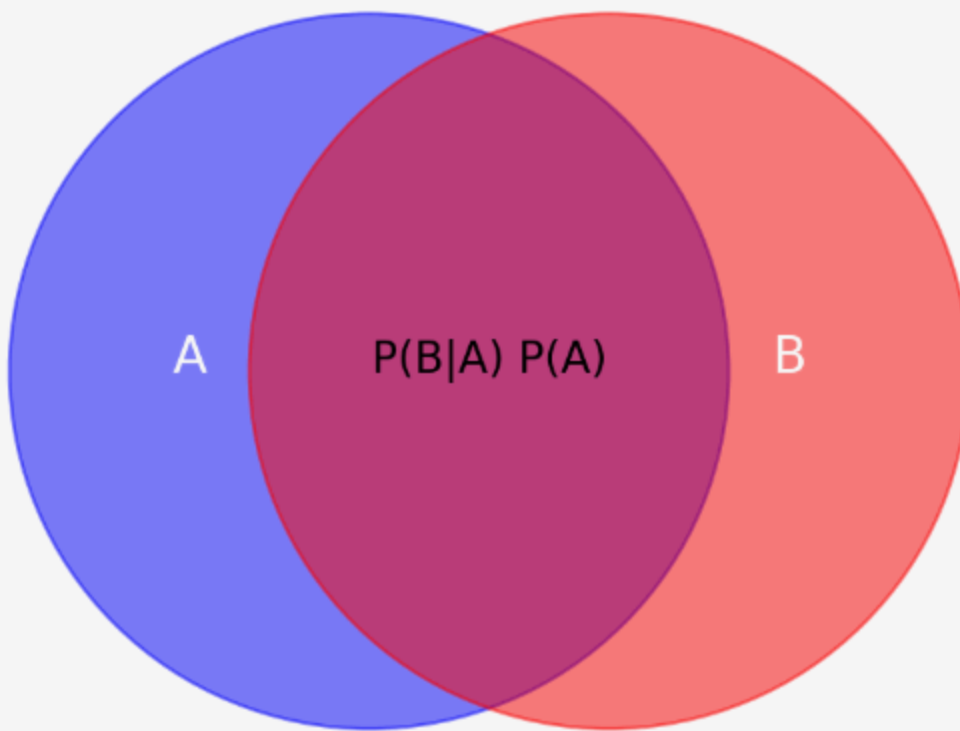$$P(A) = \sum P(A|B_i)P(B_i)$$

- **Bayes' Theorem**:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

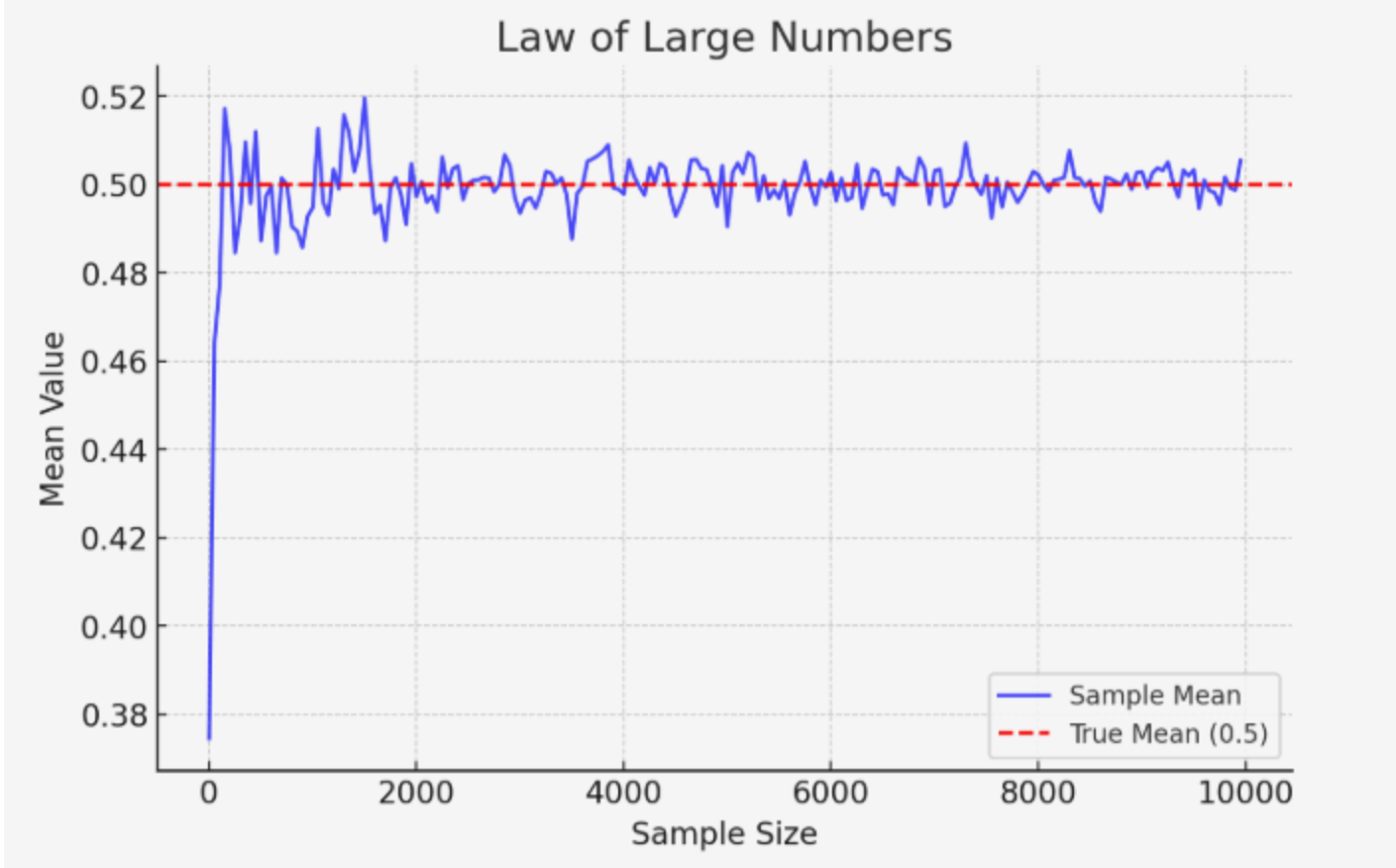Conditional Probability: $P(A|B) = P(A \cap B) / P(B)$
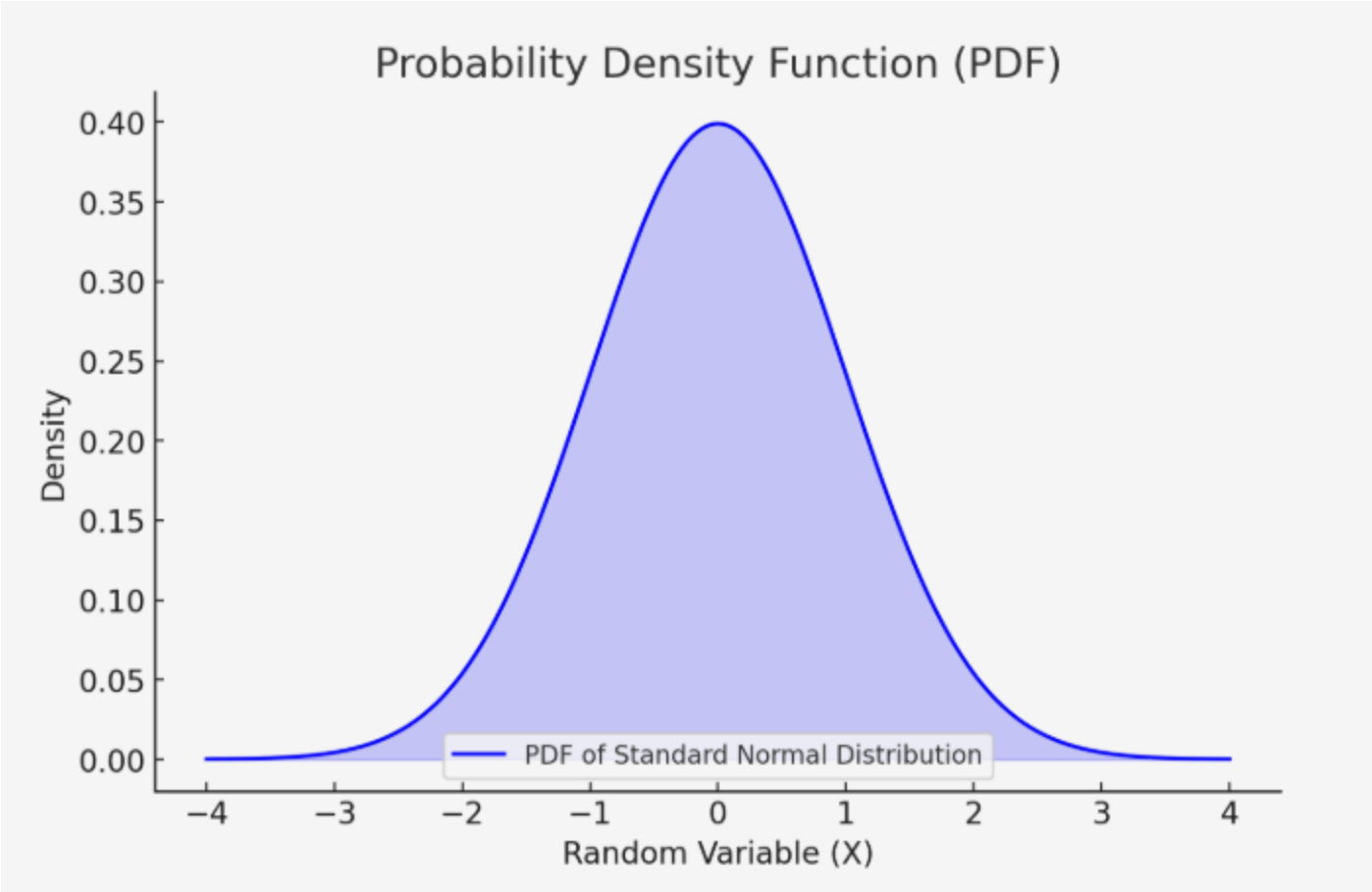
A   P(A ∩ B)   B

Bayes' Theorem: $P(A|B) = P(B|A)\, P(A) / P(B)$

A   P(B|A) P(A)   B

11. Law of Large Numbers

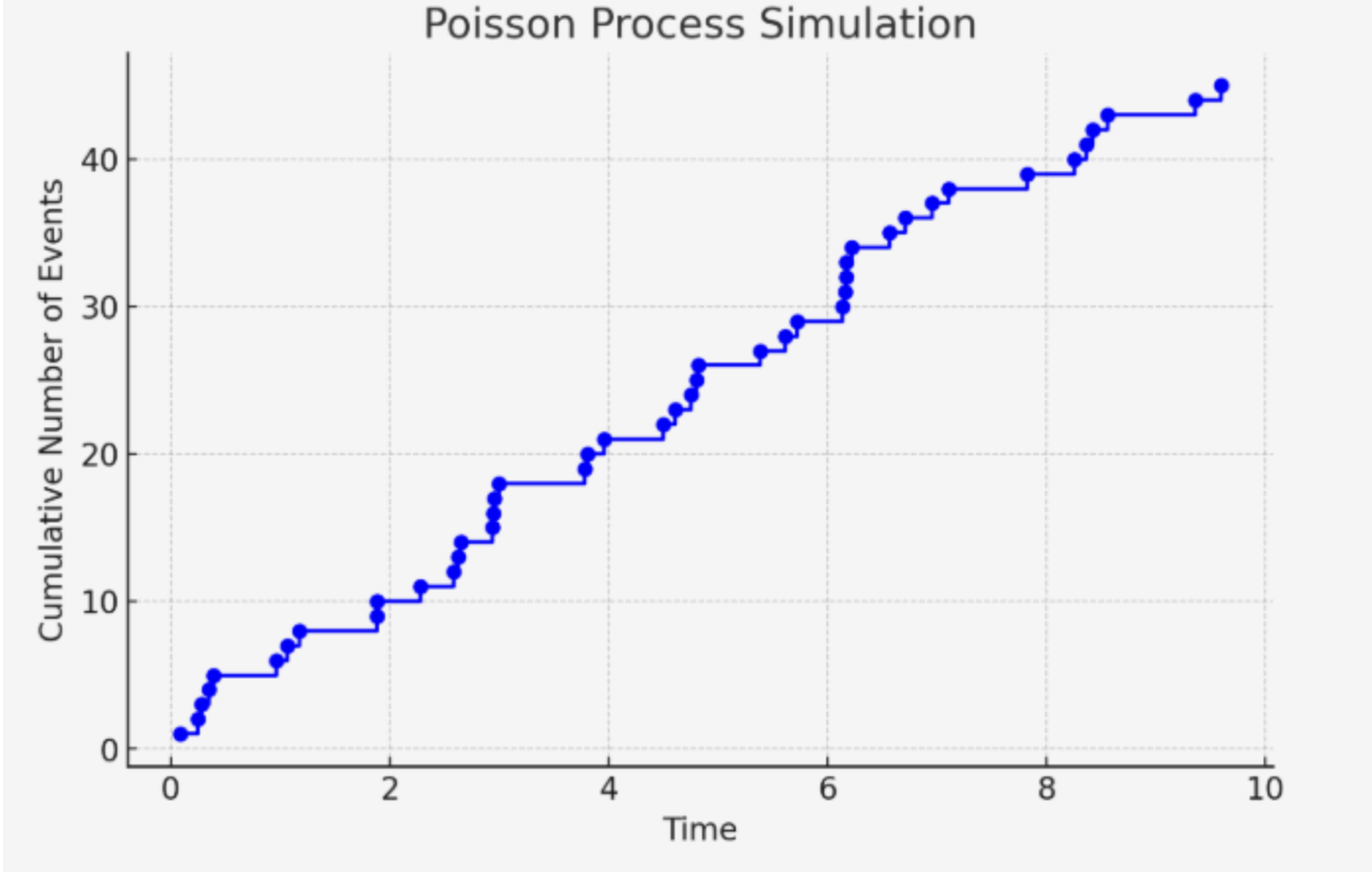- **Definition**: As the number of trials increases, the sample mean approaches the true mean.

Law of Large Numbers

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12. Probability Density Functions

- **Definition**: Describes continuous probability distributions.

Probability Density Function (PDF)

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13. Poisson Process

- **Definition**: Models events occurring randomly over time.

Poisson Process Simulation

14. Discrete & Continuous Random Variables

- **Definition**:
  - Discrete: Countable outcomes (e.g., number of students in a class).
  - Continuous: Infinite possible values (e.g., height of students).
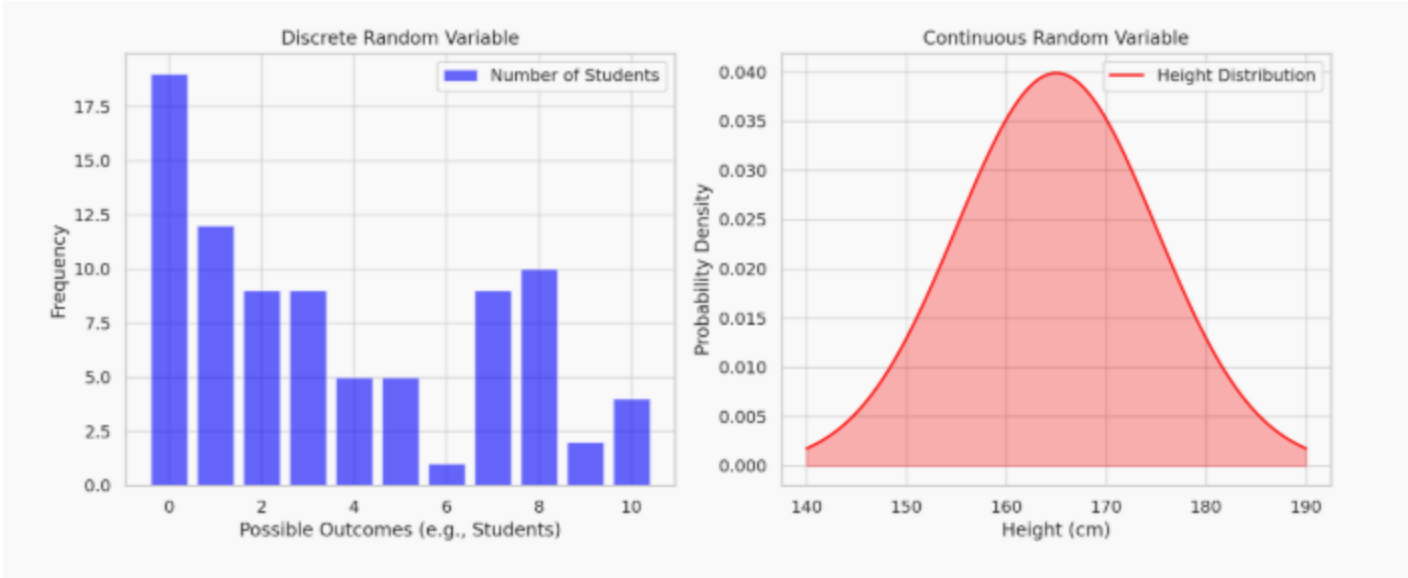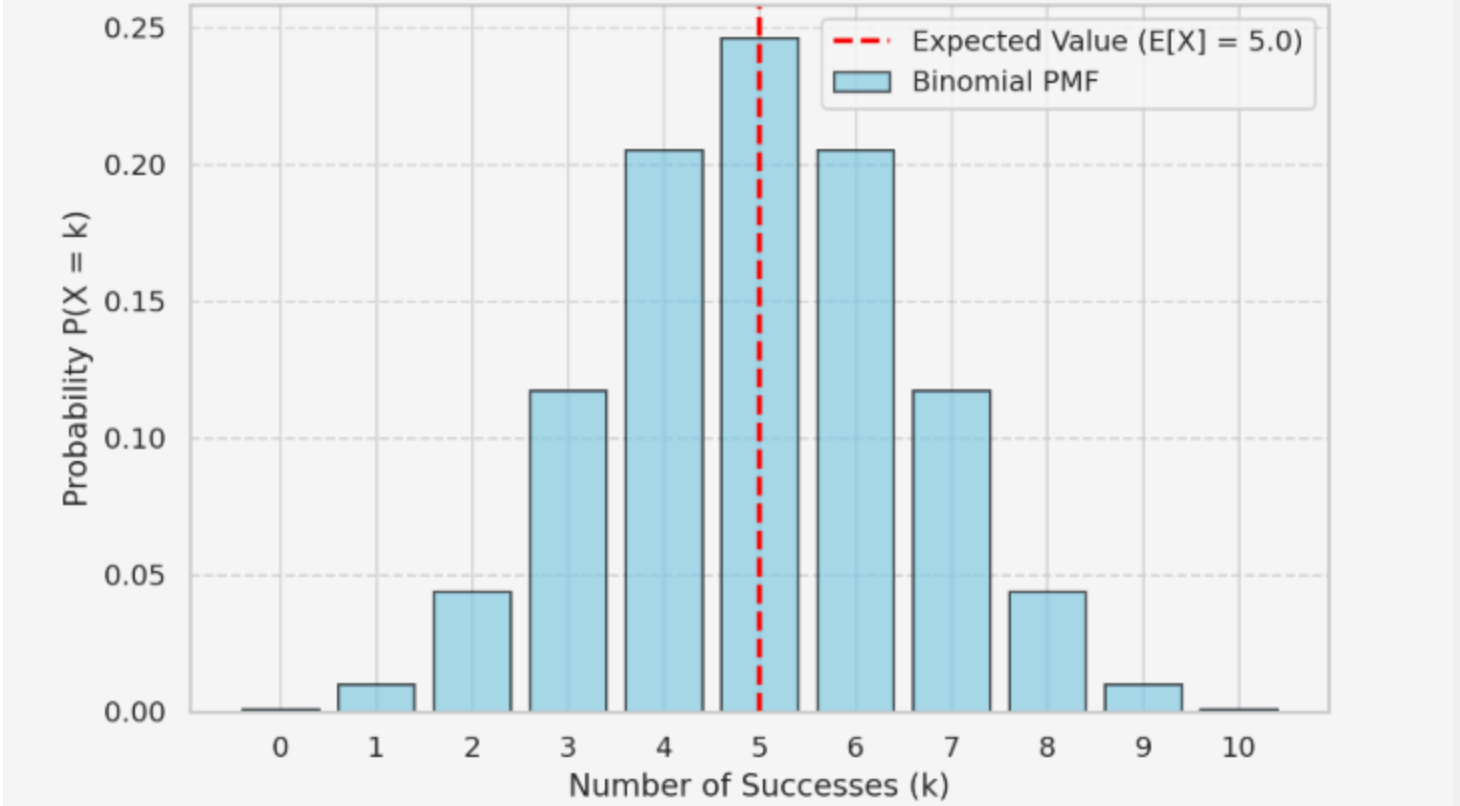

15. Expected Value of Binomial Distribution

- **Formula**: $E(X)=np$

Example: Expected number of heads in 10 coin flips with p=0.5

Expected Value of Binomial Distribution

16. Visualizing a Binomial Distribution