VRAAG 1 — Specificatie

simple variant:
{* i=N ∧ (found = (∃k : 0≤k≤N : a[k]=x)) ∧ sorted a N *}

VRAAG 2 — Proof simple variant

Bewijzen:
INIT: ⊨ P ⟹ I
PIC: {* I ∧ g *} S {* I *}
PEC: ⊨ I ∧ ¬g ⟹ Q

Invariant (I):
{* 0≤i≤N ∧ (found = (∃k: 0≤k<i : a[k]=x)) ∧ sorted a N *}

PROOF simple variant PIC
A1: i < N
D1: S = found := found ∨ a[i]=x; i:=i+1
D2: I = 0≤i≤N ∧ (found = (∃k: 0≤k<i : a[k]=x))
G : wp S I // note: check if this is the weakest-precondition
BEGIN --

1. { calculate wp }
   wp S I = 0≤i+1≤N ∧ ((found ∨ a[i]=x) = (∃k: 0≤k<i+1 : a[k]=x))

   PROOF calculate wp
   G : wp S I = 0≤i+1≤N ∧ ((found ∨ a[i]=x) = (∃k: 0≤k<i+1 : a[k]=x))
   BEGIN --
       wp (found := found ∨ a[i]=x; i:=i+1) (0≤i≤N ∧ (found = (∃k: 0≤k<i : a[k]=x)))
   1. { wp of statements sequence }
       wp (found := found ∨ a[i]=x) (wp (i:=i+1) (0≤i≤N ∧ (found = (∃k: 0≤k<i : a[k]=x)))
   2. { wp of assignment }
       wp (found := found ∨ a[i]=x) (0≤i+1≤N ∧ (found = (∃k: 0≤k<i+1 : a[k]=x)))
   3. { wp of assignment }
       0≤i+1≤N ∧ ((found ∨ a[i]=x) = (∃k: 0≤k<i+1 : a[k]=x))
   END --

2. { see equality subproof }
   (found ∨ (a[i]=x)) = (∃k: 0≤k<i+1 : a[k]=x)

   PROOF equality
   [SOME i]
   A1: found ∨ (a[i]=x)
   A2: found = (∃k: 0≤k<i : a[k]=x)
   G : (found ∨ (a[i]=x)) = (∃k: 0≤k<i+1 : a[k]=x)
   BEGIN --
       found ∨ (a[i]=x)
   1. { substitute with A2 }
       (∃k: 0≤k<i : a[k]=x) ∨ (a[i]=x)
   2. { introduce ∃-kwantor }
       (∃k: 0≤k<i : a[k]=x) ∨ (∃k: k=i : a[k]=x)
   3. { combine domains }
       (∃k: 0≤k<i+1 : a[k]=x)
   END --

3. { rewrite A1 }
   i+1 ≤ N

4. { combine 2 and 3 }
   (found ∨ (a[i]=x)) = (∃k: 0≤k<i+1 : a[k]=x) ∧ i+1≤N

5. { combine 1 and 4 makes wp True }
   wp S I

END --