

Invariant I: // note: same as simple variant  
 $\{ * 0 \leq i \leq N \wedge (\text{found} = (\exists k: 0 \leq k < i : a[k]=x)) \wedge \text{sorted } a \ N * \}$

Statement S is a sequence S1;S2:

S1:  $\text{found} := \text{found} \vee (a[i]=x);$

S2:  $i := i + 1;$

Guard g:

$\{ * i < N \wedge \neg \text{found} \wedge a[i] \leq x * \}$

Proof PIC:

$\{ * I \wedge g * \} S \{ * I * \}$

PROOF PIC

A1:  $\text{found} = (\exists k: 0 \leq k < i : a[k]=x)$  // from Invariant I

A2:  $0 \leq i < N$  // from Guard g

G : wp S I // prove that the weakest precondition holds

BEGIN -----

1. { see calculate wp }  
 $\text{wp } S \ I = 0 \leq i+1 \leq N \wedge ((\text{found} \vee a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x))$

PROOF calculate wp

BEGIN -----

- wp  $(\text{found} := \text{found} \vee a[i]=x; i:=i+1) (0 \leq i \leq N \wedge (\text{found} = (\exists k: 0 \leq k < i : a[k]=x)))$
1. { wp of statements sequence }  
 $\text{wp } (\text{found} := \text{found} \vee a[i]=x) (\text{wp } (i:=i+1) (0 \leq i \leq N \wedge (\text{found} = (\exists k: 0 \leq k < i : a[k]=x))))$
2. { wp of assignment }  
 $\text{wp } (\text{found} := \text{found} \vee a[i]=x) (0 \leq i+1 \leq N \wedge (\text{found} = (\exists k: 0 \leq k < i+1 : a[k]=x)))$
3. { wp of assignment }  
 $0 \leq i+1 \leq N \wedge ((\text{found} \vee a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x))$

END -----

2. { see subproof equality }  
 $(\exists k: 0 \leq k < i : a[k]=x) \vee (a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x)$

PROOF equality

[some i]

[some x]

A1:  $(\exists k: 0 \leq k < i : a[k]=x) \vee (a[i]=x)$

G :  $(\exists k: 0 \leq k < i : a[k]=x) \vee (a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x)$

BEGIN -----

- $(\exists k: 0 \leq k < i : a[k]=x) \vee (a[i]=x)$
1. { introduce  $\exists$ -kwantor }  
 $(\exists k: 0 \leq k < i : a[k]=x) \vee (\exists k: k=i : a[i]=x)$
2. { combine domains }  
 $(\exists k: 0 \leq k < i+1 : a[k]=x)$
3. { we have proven equality }  
 $(\exists k: 0 \leq k < i : a[k]=x) \vee (a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x)$

END -----

3. { reversed substitution of A1 in 2 }  
 $((\text{found} \vee a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x))$
4. { rewrite A2 }  
 $0 \leq i+1 \leq N$
5. { combine 3 and 4 }  
 $0 \leq i+1 \leq N \wedge ((\text{found} \vee a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x))$
6. { we have proved wp by equality on 1 and 5 }  
 $\text{wp } S \ I = 0 \leq i+1 \leq N \wedge ((\text{found} \vee a[i]=x) = (\exists k: 0 \leq k < i+1 : a[k]=x))$

END -----