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Invariant I:
{* 0≤i≤N ∧ (found = (∃k: 0≤k<i : a[k]=x)) ∧ sorted a N *}

Statement S is a sequence S1;S2:
S1: found := found ∨ (a[i]=x);
S2: i := i + 1;

Guard g:
{* i < N ∧ ¬found ∧ a[i]≤x *}

Postcondition Q:
{* 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x) *}

Proof PEC:
⊨ I ∧ ¬g ⟹ Q

Proof PIC:
{* I ∧ g *} S {* I *}



PROOF PEC
A1: (found = (∃k: 0≤k<i : a[k]=x))    // from Invariant I
A2: 0 ≤ i ≤ N                        // from Invariant I
A3: i ≥ N ∨ found ∨ a[i] > x          // from Guard ¬g
A4: (∀i: 0≤i<N : (∀j : i≤j<N : a[i]≤a[j])) // from Invariant I
G : 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))  // prove Q
BEGIN ------------------------------------------

1. { see subproof breakWithCounter }
    i≥N ⟹ 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))

    PROOF breakWithCounter
    A1: found = (∃k : 0≤k<i : a[k]=x)
    A2: i ≥ N
    A3: 0 ≤ i ≤ N
    G : 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))
    BEGIN ----------------------------------------
    1. { combine A2 and A3 }
        i = N
    2. { replace in A1, i with N }
        found = (∃k : 0≤k<N : a[k]=x)
    3. { combine A3 and 2 }
        0≤i≤N ∧ (found = (∃k: 0≤k<N : a[k]=x))
    END ------------------------------------------

2. { see subproof breeakWithFound }
    found ⟹ 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))

    PROOF breakWithFound
    A1: found = (∃k : 0≤k<i : a[k]=x)
    A2: found
    A3: 0 ≤ i ≤ N
    G : 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))
    BEGIN ----------------------------------------
    1. { substitute found in A1 with A2 }
        (∃k : 0≤k<i : a[k]=x)
    2. { domain expanding }
        (∃k : 0≤k<i ∨ i≤k<N : a[k]=x)
    3. { domain combine }
        (∃k : 0≤k<N : a[k]=x)
    4. { equality of 3 and A2 }
        (found = (∃k : 0≤k<N : a[k]=x))
    5. { combine with A3 }
        0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))
    END ------------------------------------------

3. { see subproof breakWithValue }
    a[i]>x ⟹ 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))

    PROOF breakWithValue
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    A1: found = (∃k : 0≤k<i : a[k]=x)
    A2: a[i]>x
    A3: 0 ≤ i ≤ N
    A4: (∀i: 0≤i<N : (∀j: i≤j<N : a[i]≤a[j]))
    G : 0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))
    BEGIN ----------------------------------------
    1. { kwantor elimintation in A4 } [ANY i]
        0≤i<N ⟹ (∀j: i≤j<N : a[i]≤a[j])
    2. { from A3 and 1 }
        (∀j: i≤j<N : a[i]≤a[j])
    3. { from 2 and A2 }
        ¬(∃j: i≤j<N : a[j]=x)
    4. { flip domain of 3 }
        (∃j: j<i ∨ j≥N : a[j]=x)
    5. { split kwantor from 4 }
        (∃j: j<i : a[j]=x) ∨ (∃j: j≥N : a[j]=x)
    6. { remove part second part from 5 }
        (∃j: j<i : a[j]=x)
    7. { by equality of 6 with A1 }
        (found = (∃k : 0≤k<i : a[k]=x))
    8. { combine 7 and A3 }
        0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))
    END ------------------------------------------


4. { A3 with 1,2 and 3 will prove G }
    0≤i≤N ∧ (found = (∃k : 0≤k<N : a[k]=x))
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