Group-A (10X2=20)

1. Verify Rolle's theorem for the function $f(x) = \frac{x^3}{3} - 3x$ on the interval [-3,3]

Here the given function is $f(x) = \frac{x^3}{3} - 3x$

The given interval is [-3,3]

According to Rolle's theorem,

$$f(-3) = \frac{(-3)^3}{3} - 3X(-3) = 0$$

$$f(3) = \frac{(3)^3}{3} - 3X(3) = 0$$

Now, $\mathrm{f}'(\mathrm{x}) = \frac{3\mathrm{x}^2}{3} - 3 = x^2 - 3$

We have, $\mathrm{f}'(\mathrm{c}) = 0$

or, $c^2 - 3 = 0$

or, $c = \sqrt{3}$

Here, c falls between [-3,3]. Thus, Rolle's theorem is verified.

2. Obtain the area between two curves $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{4}$

Let $f(x) = \sec^2 x$; $0 \le x \le \frac{\pi}{4}$

$g(x) = \sin x$; $0 \le x \le \frac{\pi}{4}$

we know that, $A = \int_a^b [f(x) - g(x)]dx$

or, $A = \int_0^{\frac{\pi}{4}} (\sec^2 x - \sin x)dx$

or, $A = [\tan x + \cos x]_0^{\frac{\pi}{4}}$

or, $A = [\tan\frac{\pi}{4} + \cos\frac{\pi}{4} - \tan 0 - \cos 0]$

or, $A = 1 + \frac{1}{\sqrt{2}} - 0 - 1$

or, $A = \frac{1}{\sqrt{2}}$

3. Test the convergence of p-series $\sum_{n=1}^{\infty} = 1\,\frac{1}{np}$ for $p > 1$

Solution: If $p > 1$, then $f(x) = \frac{1}{x^p}$ is a positive decreasing function of x. Since

$$\int_1^{\infty} \frac{1}{xp}\,dx = \int_1^{\infty} x^{-p}\,dx = \lim_{b\to\infty}\left[\frac{x^{-p+1}}{-p+1}\right]_1^b$$

$$= \frac{1}{1-p}\lim_{b\to\infty}\left(\frac{1}{b^{p-1}} - 1\right) = \frac{1}{1-p}(0-1) = \frac{1}{p-1}$$

Here, $b^{p-1} \to \infty$ as $b \to \infty$ because $p - 1 > 0$. Thus, the series converges by the Integral test.

4. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$

Here the given hyperbola is

$9x^2 - 16y^2 = 144$

or, $\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$

or, $\frac{x^2}{16} - \frac{y^2}{9} = 1$

which is in the form of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

where, $a^2 = 16 \to a = 4$ and $b^2 = 9 \to b = 3$

now, $c = \sqrt{a^2 + b^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Eccentricity of the hyperbola is given as;

$$e = \frac{c}{a} = \frac{5}{4}$$

5. Find a vector perpendicular to the plane of $\boldsymbol{P(1, -1, 0), Q(2, 1, -1), R(-1, 1, 2)}$

Here, $P(1, -1,0), Q(2,1, -1), R(-1,1,2)$

The vector $\overrightarrow{PQ} X \overrightarrow{PR}$ is perpendicular to the plane because it is perpendicular to both the vector.

$$\overrightarrow{PQ} = (2 - 1)\vec{\imath} + (1 + 1)\vec{\jmath} + (-1 - 0)\vec{k} = \vec{\imath} + 2\vec{\jmath} - \vec{k}$$

$$\overrightarrow{PR} = (-1 - 1)\vec{\imath} + (1 + 1)\vec{\jmath} + (2 - 0)\vec{k} = -2\vec{\imath} + 2\vec{\jmath} + 2\vec{k}$$

Now, vector perpendicular to the plane is given as;

$$\overrightarrow{PQ} X \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix}$$

or, $\overrightarrow{PQ} X \overrightarrow{PR} = \begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} \vec{\imath} - \begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} \vec{\jmath} + \begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix} \vec{k}$

or, $\overrightarrow{PQ} X \overrightarrow{PR} = (4 + 2)\vec{\imath} - (2 - 2)\vec{\jmath} + (2 + 4)\vec{k}$

or, $\overrightarrow{PQ} X \overrightarrow{PR} = 6\vec{\imath} + 6\vec{k}$

6. Find the area enclosed by the curve $\boldsymbol{r^2 = 4\cos 2\theta}$

Here, the equation of the curve is

$$r^2 = 4\cos 2\theta \;\; \rightarrow \;\; r = 2\sqrt{\cos 2\theta}$$

If we draw typical ray from origin, we get

$0 \le r \le 2\sqrt{\cos 2\theta}$ and $0 \le \theta \le \frac{\pi}{4}$

Now,

Area (A) $= 4\int_0^{\frac{\pi}{4}}\int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta$

$= 4\int_0^{\frac{\pi}{4}}[\frac{r^2}{2}]\begin{matrix}2\sqrt{\cos 2\theta}\\0\end{matrix}\,d\theta$

$= 4\int_0^{\frac{\pi}{4}}\left[\frac{(2\sqrt{\cos 2\theta})^2}{2}\right]d\theta$

$= 4\int_0^{\frac{\pi}{4}} 2\cos 2\theta\,d\theta$

$= 8[\frac{\sin 2\theta}{2}]\begin{matrix}\frac{\pi}{4}\\0\end{matrix}$

$= 8\ \frac{\sin\frac{\pi}{2}}{2}$

$= 4$

7. Obtain the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4,-5) if $f(x,y) = x^2 + 3xy + y - 1$

Here, $f(x,y) = x^2 + 3xy + y - 1$

Now, $(\frac{\partial f}{\partial x})$ at point (4,-5) is given as

$$\frac{\partial f}{\partial x} = \frac{\partial(x^2+3xy+y-1)}{\partial x} = 2x + 3y = 2X4 + 3X(-5) = -7$$

Also, $(\frac{\partial f}{\partial y})$ at point (4,-5) is given as

$$\frac{\partial f}{\partial y} = \frac{\partial(x^2+3xy+y-1)}{\partial y} = 3x + 1 = 3X4 + 1 = 13$$

8. Using partial derivatives, find $\frac{dy}{dx}$ if $x^2 + \cos y - y^2 = 0$

Here, $f(x, y) = x^2 + \cos y - y^2$

Now, $fx = 2x$ and $fy = -\sin y - 2y$

we have, $\frac{dy}{dx} = \frac{fx}{fy} = \frac{-2x}{-\sin\, y - 2y} = \frac{2x}{\sin\, y + 2y}$

9. Find the partial differential equation of the function $(x - a)^2 + (y - b)^2 + z^2 = c^2$

Here, the given function is $(x - a)^2 + (y - b)^2 + z^2 = c^2$ ---------- (i)

Differentiating equation (i) w.r.t. x keeping y constant.

$$\frac{\delta f}{\delta x} = 2(x - a) + 2z\frac{\partial z}{\partial x} = 0$$

or, $2(x - a) = -2z\frac{\partial z}{\partial x}$

or, $x - a = -zp$; $[p = \frac{\partial z}{\partial x}]$

or, $a = x + zp$

Differentiating equation (i) w.r.t. y keeping x constant.

$$\frac{\delta f}{\delta y} = 2(y - b) + 2z\frac{\partial z}{\partial y} = 0$$

or, $2(y - b) = -2z\frac{\partial z}{\partial y}$

or, $y - b = -zq$; $[q = \frac{\partial z}{\partial y}]$

or, $b = y + zq$

Putting the values of a and b in equation (i), we get

$$(x - x - zp)^2 + (y - y - zq)^2 + z^2 = c^2$$

or, $z^2p^2 + z^2q^2 + z^2 = c^2$

or, $z^2(p^2 + q^2 + 1) = c^2$

10. Solve the partial differential equation $x^2p + q = z^2$

Here, the given partial differential equation is

$$x^2p + q = z^2$$

Comparing it with: $Pp + Qq = R$

we get; $P = x^2$; $Q = 1$; $R = z^2$

Now, $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \rightarrow \frac{dx}{x^2} = \frac{dy}{1} = \frac{dz}{z^2}$

Taking first two equations, we have

$$\frac{dx}{x^2} = \frac{dy}{1}$$

Integrating both sides

$$\int \frac{1}{x^2} dx = \int 1\, dy$$

or, $-\frac{1}{x} = y + c1$

or, $c1 = y + \frac{1}{x}$

Taking last two equations, we have

$$\frac{dy}{1} = \frac{dz}{z^2}$$

Integrating both sides

$$\int 1\, dy = \int \frac{dz}{z^2}$$

or, $\quad y = -\frac{1}{z} + c2$

or, $\quad c2 = y + \frac{1}{z}$

Thus, $(c1, c2) = (y + \frac{1}{x}\ ,\ y + \frac{1}{z})$

Group-B (5X4=20)

11. State and prove the Mean Value Theorem for a differentiable function

Statement:

If f(x) is continuous on [a, b] and differentiable on (a, b), then there exists a point c in (a, b) such that

$$f(b) - f(a)$$

$$f(b) - f(a) = fo(c)(b - a) \rightarrow fo(c) = b - a$$

Proof:

Consider the function h(x) = f (x)(b − a) − x(f (b) − f (a)), noting that
h(x) is continuous on [a, b] because f (x) is. We compute

$$ho(x) = fo(x)(b - a) - \big(f(b) - f(a)\big)$$

which is differentiable on (a, b) since f (x) is. Note that

$$h(a) = b.f(a) - a.f(a) - a.f(b) + a.f(a)$$

$$= b.f(a) - a.f(b)$$

$$= b.f(b) - a.f(b) - b.f(b) + b.f(a)$$

$$= f(b)(b - a) - b\big(f(b) - f(a)\big)$$

$$= h(b)$$

12. Find the length of the asteroid $x = cos^3 t, y = sin^3 t$ for $0 \leq t \leq 2\pi$

Here, $x = cos^3 t, y = sin^3 t \; ; \; 0 \leq t \leq 2\pi$

$$(\frac{dx}{dt})^2 = [3\, cos^2 t\,(-\sin t)]^2 = 9\, cos^4 t\,.\, sin^2 t$$

$$\left(\frac{dy}{dt}\right)^2 = [3\, sin^2\, t\,.\cos t]^2 = 9\, sin^4\, t\,.\, cos^2\, t$$

Length of astroid $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \sqrt{9\, cos^4\, t\,.\, sin^2\, t + 9\, sin^4\, t\,.\, cos^2\, t}$

$= 3\cos t\,.\sin t$

13. Define a curvature of a curve. Prove that the curvature of a circle of radius a is $\frac{1}{a}$

Statement:

If T is the unit tangent vector of smooth curve, the curvature function of the curve is $k = |\frac{dt}{ds}|$

Proof:

$\vec{r}(\theta) = (a\cos\theta)\,\vec{\imath} + (a\sin\theta)\,\vec{\jmath}$ --------------- (i)

Substituting $\theta = \frac{s}{a}$ in equation (i) to parameterize in term of arc length s.

$$\vec{r} = (a\cos\frac{s}{a})\,\vec{\imath} + (a\sin\frac{s}{a})\,\vec{\jmath}$$

Now,

$$T = \frac{d\,\vec{r}}{ds} = \left(-\sin\frac{s}{a}\right)\vec{\imath} + \left(\cos\frac{s}{a}\right)\vec{\jmath}$$

$$\frac{dT}{ds} = \left(-\frac{1}{a}\cos\frac{s}{a}\right)\vec{\imath} - \left(\frac{1}{a}\sin\frac{s}{a}\right)\vec{\jmath}$$

Now,

$$k = \left|\frac{dT}{ds}\right| = \sqrt{\left(-\frac{1}{a}\cos\frac{s}{a}\right)^2 + \left(\frac{1}{a}\sin\frac{s}{a}\right)^2} = \sqrt{\frac{1}{a^2}} = \frac{1}{|a|} = \frac{1}{a}$$

14. What is meant by direction derivative in the plane? Obtain the derivative of the function $f(x,y) = x^2 + xy$ at $P(1,2)$ in the direction of the unit vector $v = \left(\frac{1}{\sqrt{2}}\right)\vec{i} + \left(\frac{1}{\sqrt{2}}\right)\vec{j}$

Statement:

The derivative of f at $P_0(x_0, y_0)$ in the direction of unit vector $u = u_1\vec{i} + u_2\vec{j}$ is

$$\left(\frac{df}{ds}\right)_{u,P_0} = \lim_{s\to 0}\frac{f(x_0 + su_1, y_0 + su_2)}{s}$$

provided the limit exists. It is also denoted by$(D_u f)_{P_0}$.

Solution:

$$\left(\frac{df}{ds}\right)_{u,f} = \lim_{s\to 0}\frac{f(x_0 + su_1, y_0 + su_2) - f(x_0, y_0)}{s}$$

$$= \lim_{s\to 0}\frac{f\left(1 + \frac{s}{\sqrt{2}}, 2 + \frac{s}{\sqrt{2}}\right) - f(1,2)}{s}$$

$$= \lim_{s\to 0}\frac{\left(1 + \frac{s}{\sqrt{2}}\right)^2 + \left(1 + \frac{s}{\sqrt{2}}\right)X\left(2 + \frac{s}{\sqrt{2}}\right) - (1^2 + 1X2)}{s}$$

$$= \lim_{s\to 0}\frac{5\frac{s}{\sqrt{2}} + 2\frac{s^2}{2}}{s} = \lim_{s\to 0}\frac{s\left(\frac{5}{\sqrt{2}} + s\right)}{s} = \lim_{s\to 0}\left(\frac{5}{\sqrt{2}} + s\right) = \frac{5}{\sqrt{2}}$$

15. Find the centre of mass of a solid of constant density δ, bounded below by the disk: $x^2 + y^2 = 4$ in the plane $z = 0$ and above by the paraboid $z = 4 - x^2 - y^2$

By symmetry, $\bar{x} = \bar{y} = 0$. To find $\bar{z}$, we first calculate

$$M_{xy} = \int \int_R \int_{z=0}^{z=4-x^2-y^2} z \ \delta \ dz.dy.dx$$

$$= \int \int_R \left[\frac{z^2}{2}\right]_{z=0}^{z=4-x^2-y^2} \delta \ dy.dx$$

$$= \frac{\delta}{2} \int \int_R (4 - x^2 - y^2)^2 \ dy.dx$$

$$= \frac{\delta}{2} \int_0^{2\pi} \int_0^2 (4 - r^2)^2 \, r \ dr.d\theta$$

$$= \frac{\delta}{2} \int_0^{2\pi} \left[-\frac{1}{6}(4 - r^2)^3\right]_{r=0}^{r=2} d\theta$$

$$= \frac{16\delta}{3} \int_0^{2\pi} d\theta = \frac{32\pi\delta}{3}$$

A similar calculation gives

$$M = \int \int_R \int_0^{4-x^2-y^2} \delta \ dz.dy.dx = 8\pi\delta$$

Therefore, $\bar{z} = \left(\frac{M_{xy}}{M}\right) = \frac{4}{3}$, and the center of mass is $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{4}{3}\right)$

Group C (8X5=40)

16. Graph the function $f(x) = -x^3 + 12x + 5$ for $-3 \leq x \leq 3$

17. Define Taylor's Polynomial of order n. Obtain Taylor's Polynomial and Taylor's Series generated by the function $f(x) = e^x$ at $x = 0$

Statement:

Solution:

$$f(x) = e^x \;;\; f(0) = 1$$

$$f'(x) = e^x \;;\; f'(0) = 1$$

$$f''(x) = e^x \;;\; f''(0) = 1$$

$$f^n(x) = e^x \;;\; f^n(0) = 1$$

Therefore, Taylor's series is given as;

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Therefore, Taylor's polynomial is given as;

$$\sum_{k=1}^{\infty} \frac{n^k}{k!}$$

18. Obtain the centroid of the region in the first quadrant that is bounded above by the line $y = x$ and below by the parabola $y = x^2$

Here,

$$M = \int_0^1 \int_{x^2}^{x} 1\, dy.dx = \int_0^1 [y]_{y=x^2}^{y=x} dx = \int_0^1 (x - x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$$

$$M_x = \int_0^1 \int_{x^2}^{x} y\, dy.dx = \int_0^1 \left[\frac{y^2}{2}\right]_{y=x^2}^{y=x} dx = \int_0^1 \left(\frac{x^2}{2} - \frac{x^4}{2}\right)dx = \left[\frac{x^3}{6} - \frac{x^5}{10}\right]_0^1 = \frac{1}{15}$$

$$M_y = \int_0^1 \int_{x^2}^{x} x\, dy.dx = \int_0^1 [xy]_{y=x^2}^{y=x} dx = \int_0^1 (x^2 - x^3)dx = \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{12}$$

From these values of M, M_x & M_y, we find

$$\bar{x} = \frac{M_y}{M} = \frac{\left(\frac{1}{12}\right)}{\left(\frac{1}{6}\right)} = \frac{1}{2} \quad \& \quad \bar{y} = \frac{M_x}{M} = \frac{\left(\frac{1}{15}\right)}{\left(\frac{1}{6}\right)} = \frac{2}{5}$$

The centroid is the point $\left(\frac{1}{2}, \frac{2}{5}\right)$

19. Find the maximum and minimum values of $f(x,y) = 2xy - 2y^2 - 5x^2 + 4x + 4y - 4$. Also find the saddle point if it exist.

Here,

$$f(x,y) = 2xy - 2y^2 - 5x^2 + 4x + 4y - 4$$

$$f_x = 2y - 10x + 4 \;;\; f_{xx} = -10$$

$$f_y = 2x - 4y + 4 \;;\; f_{yy} = -4$$

$$f_{xy} = 2 \;;\; f_{yx} = 2$$

Now,

$$f_{xx}.f_{yy} - f_{xy}^2 = (-10)X(-4) - 4 = 36$$

we have, $f_{xx} < 0$; $f_{xx}.f_{yy} - f_{xy}^2 > 0$. Thus there exists local maximum value.

$$2y - 10x + 4 = 0 \ldots\ldots\ldots.(i)$$

$$2x - 4y + 4 = 0 \ \ldots\ldots\ldots.(ii)$$

Solving equation (i) and (ii), we get;

$$x = \frac{2}{3} \ \& \ y = \frac{4}{3}$$

Now,

$$f\left(\frac{2}{3},\frac{4}{3}\right) = 2 \times \frac{2}{3} \times \frac{4}{3} - 2 \times \frac{16}{9} - 5 \times \frac{4}{9} + 4 \times \frac{2}{3} + 4 \times \frac{4}{3} - 4 = 0$$

OR 19. Evaluate the integral $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz.dx.dy$

Here,

$$\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz.dx.dy$$

$$= \int_0^{\sqrt{2}} \int_0^{3y} 8 - x^2 - y^2 - x^2 - 3y^2 \;\; dx.dy$$

$$= \int_0^{\sqrt{2}} \int_0^{3y} 8 - 2x^2 - 4y^2 \;\; dx.dy$$

$$= \int_0^{\sqrt{2}} \left[8x - \frac{2x^3}{3} - 4xy^2\right]_0^{3y} dy$$

$$= \int_0^{\sqrt{2}} \left(8 \times 3y - \frac{2}{3} \times 27y^3 - 4y^2 \times 3y\right) dy$$

$$= \int_0^{\sqrt{2}} 24y - 30y^3 \; dy$$

$$= \left[\frac{24y^2}{2} - \frac{30y^4}{4}\right]_0^{\sqrt{2}}$$

$$= 12 \times \sqrt{2}^2 - 30 \times \frac{\sqrt{2}^4}{4}$$

$$= -6$$

20. What do you mean by D'Alembert's solution of the one-dimensional wave equation? Derive it.

Solution:

The one dimensional wave equation is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} \;\; \ldots\ldots\ldots.(i)$$

whose solution is $y(x,t) = f(x+ct) + g(x-ct)$ ……….(ii)

The general solution can be obtained by introducing new variables $\xi = x - c\,t$ and $\eta = x + c\,t$, and applying the chain rule to obtain

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta}$$
$$= \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta}$$
$$\frac{\partial}{\partial t} = \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial t}\frac{\partial}{\partial \eta}$$
$$= -c\frac{\partial}{\partial \xi} + c\frac{\partial}{\partial \eta}.$$

Now,

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta}\right)\left(\frac{\partial y}{\partial \xi} + \frac{\partial y}{\partial \eta}\right)$$
$$= \frac{\partial^2 y}{\partial \xi^2} + 2\frac{\partial^2 y}{\partial \xi\,\partial \eta} + \frac{\partial^2 y}{\partial \eta^2}$$
$$\frac{\partial^2 y}{\partial t^2} = \left(-c\frac{\partial}{\partial \xi} + c\frac{\partial}{\partial \eta}\right)\left(-c\frac{\partial y}{\partial \xi} + c\frac{\partial y}{\partial \eta}\right)$$
$$= c^2\frac{\partial^2 y}{\partial \xi^2} - 2c^2\frac{\partial^2 y}{\partial \xi\,\partial \eta} + c^2\frac{\partial^2 y}{\partial \eta^2},$$
$$\frac{\partial^2 y}{\partial \xi\,\partial \eta} = 0.$$

This partial differential equation has general solution

$$y(x,t) = f(\xi) + g(\eta)$$
$$= f(x - c\,t) + g(x + c\,t), \qquad \text{(i)}$$

where f and g are arbitrary functions, with f representing a right-traveling wave and g a left-traveling wave.

The initial value problem for a string located at position $y(x, t=0) = y_0(x)$ as a function of distance along the string x and vertical speed $\partial y/\partial t|_{t=0} = v_0(x)$ can be found as follows. From the initial condition and equation (i),

$$y_0(x) = f(x) + g(x). \qquad \text{(ii)}$$

Taking the derivative with respect to t then gives

$$\begin{aligned} v_0(x) &= f'(x)\frac{\partial(x-ct)}{\partial t} + g'(x)\frac{\partial(x+ct)}{\partial t} \\ &= -c f'(x) + c g'(x), \end{aligned}$$

and integrating gives

$$\int_a^x v_0(s)\,ds = -c f(x) + c g(x). \qquad \text{(iii)}$$

Solving equation (ii) and (iii) simultaneously for f and g immediately gives

$$\begin{aligned} f(x) &= \tfrac{1}{2} y_0(x) - \frac{1}{2c}\int_a^x v_0(s)\,ds \\ g(x) &= \tfrac{1}{2} y_0(x) + \frac{1}{2c}\int_a^x v_0(s)\,ds, \end{aligned}$$

so plugging these into (ii) then gives the solution to the wave equation with specified initial conditions as

$$y(x,t) = \tfrac{1}{2} y_0(x-ct) + \tfrac{1}{2} y_0(x+ct) + \frac{1}{2c}\int_{x-ct}^{x+ct} v_0(s)\,ds.$$

OR 20. Find the particular integral of the equation $(D^2 - D^1)z = 2y - x^2$ where $D = \frac{\partial}{\partial z}, D' = \frac{\partial}{\partial y}$

Solution:

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 2y - x^2$$

ie; $(D^2-D'^2)z=2y-x^2$

$$PI = \frac{1}{D^2 - D'^2} 2y - x^2$$

$$= \frac{\left[1 - \left(\frac{D'}{D}\right)^2\right]^{-1}}{D^2} (2y - x^2)$$

$$= \frac{1}{D^2} [2y - x^2 + \frac{D'^2}{D^2} (2y - x^2]$$

$$= \frac{1}{D^2} \left[2y - x^2 + \frac{2yD'}{D^2} - \frac{D'}{D^2} x^2\right]$$

$$= \frac{2y}{D^2} - \frac{x^2}{D^2} + \frac{2yD'}{D^4} - \frac{D'}{D^4} x^2$$

$$= \frac{2x^2 y}{2} - \frac{x^4}{12} + \frac{2x^4}{24} - 0$$

$$= x^2 y - \frac{x^4}{12} + \frac{x^4}{12}$$

$$= \boldsymbol{x^2 y}$$