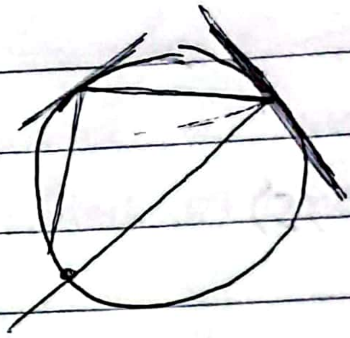


Line equation = $y = mx + C$
Circle equation = $x^2 + y^2 = 100$

inputs
entry point $(1, 9.95)$
first reflection $(8, -6)$

1) given the two points → find the ~~line~~ slope equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{9.95 + 6}{1 - 8} = -2.279$$

2) finding the slope of the tangent at intersection

$$m_T = -x/y$$

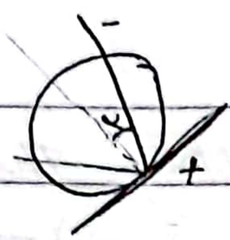
$$m_T = -8/-6 = \frac{4}{3}$$

→ line 1 →

$$y + 6 = -2.279(x - 8)$$

$$y = -2.279x + 12.232$$

3) getting the normal on the tangent



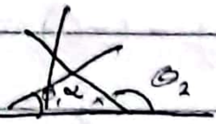
$m_T \cdot m_n = -1$

$\frac{4}{3} \cdot m_n = -1$

$m_n = -\frac{3}{4}$

$\theta_1 + \alpha + x = 180$

$\theta_2 + x = 180$



$\cancel{x} + \theta_1 + \alpha = \theta_2 + \cancel{x}$

$\theta_1 + \alpha =$

4) get the angle between the twos slopes m, m_n
using the tan rule : $\tan\theta_1 = m_1 \rightarrow$
$\tan\theta_2 = m_2 \rightarrow$ this is the angle between the slope and the x-axis

$\tan\alpha = \tan\theta_2 - \tan\theta_1$ $\quad \alpha = \theta_2 - \theta_1$
$= \tan(\theta_2 - \theta_1)$

$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2} \rightarrow \cancel{\tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)}$

$-0.56436 \rightarrow \alpha = -29.4$

5) using the same value for $\tan(\alpha)$
we can find the slope of the refeleded beem

$\cancel{\tan\alpha}$ $\quad -0.56436 = \frac{-\frac{3}{4} - m_r}{1 + -\frac{3}{4} \cdot m_r}$

$-0.56436(1 + -\frac{3}{4} m_r) = -\frac{3}{4} - m_r$

$-0.56436 - 0.42327 m_r = -\frac{3}{4} - m_r$

$0.57673 \; m_r = -0.18564$

$m_r = -0.321883 \rightarrow$ $\quad y + 6 = -0.321883x$

$+$

6) getting the line of m_r using the previous point

$(8,-6) \rightarrow \quad m_r = -0.321883$

$$y-(-6) = -0.321883(x-8)$$

$$y = -0.321883x - 3.425$$

7) finding the next intersection with the circle

$$x^2+y^2 = 100$$

$$x^2+(-0.321883x-3.425)^2 = 100$$

$$x^2+(-0.321883x)^2 + 2(-0.321883x)(-3.425) + 3.425^2 = 100$$

$$\cancel{0.10361x^2 - 0.643766x - 6.85 + 11.730 = 100}$$

$$\cancel{1.10361x^2 - 0.643766x - 95.119 = 0}$$

$$\cancel{x_1 = 9.58} \quad \cancel{x_2 = -8.996}$$

$$1.10361x^2 + 2.2049x - 88.269 = 0$$

$$x_1 = 7.999 \simeq 8$$

$$x_2 = -9.9978$$

$$y_1 = -0.2068$$

$$y_2 = -6$$

8) using x_1 and y_1 as starting points with the m_r and restart calculating to get the next reflection

~~exit~~ exit condition : if $x =$ a certain value exit
$y =$

$$\tan\theta = \frac{m - m_r}{1 + m \cdot m_r}$$

$$\tan\theta(1 + m \cdot m_r) = m - m_r$$

$$\tan\theta + \tan\theta \cdot m \cdot m_r = m - m_r$$

$$\tan\theta \cdot m \cdot m_r = m - m_r - \tan\theta$$

$$\tan\theta \cdot m \cdot m_r + m_r = m - \tan\theta$$

$$m_r(\tan\theta\, m + 1) = m - \tan\theta$$

$$m_r = \frac{m - \tan\theta}{\tan m - 1}$$

$(m_r(x - x_1) - y)^2$

$(m_r(x - x_1))^2 + \cancel{2m_r}$

$-2m_r(x - x_1)y$

$(-y)^2$

$$y = m_r(x - x_1) - y_1$$

$$x^2 + (\cancel{\ldots}\ m_r(x - \underline{x_1}) - \underline{y_1})^2 = 100$$

$\cancel{x^2 + ((m_r\, x))^2}$

$$x^2 + ((m_r(x - x_1))^2 - y_1^2) = 100$$

$$x^2 + ((m_r^2(x^2 + 2x x_1 + x_1^2)) - y_1^2 = 100$$

$$x^2 + ((\underline{m_r^2}\, x^2 + \underline{2}\, x\, \underline{x_1\, m_r^2} + \underline{m_r^2 x_1^2}) - \underline{y^2} = 100$$

$$x^2 + \underbrace{(m_r^2)}_{(1+a)} x^2 + \underbrace{(2x\, m_r^2)}_{b} x + \underbrace{(m_r^2 x_1^2 - y^2 - 100)}_{c}$$

$a = 1 + m_r^2$

$b = 2x_1 m_r^2$

$c = m_r^2 x_1^2 - y_1^2 - 100$

$$y = m_r(x - x_1) - y_1$$

$$y^2 = (m_r(x - x_1))^2 - 2(m_r(x - x_1))y_1 + y^2$$

$$= m_r^2(x^2 - 2xx_1 + x_1^2) - 2(m_r x - m_r x_1)y_1 + y^2 \cancel{=100}$$

$$= (m_r^2 x^2 - 2m_r^2 x x_1 + \overset{m_r^2}{x_1^2})$$

$$- 2y m_r x + 2y m_r x_1 + y^2 \cancel{=100}$$

$$= (m_r^2 \cancel{+1})x^2 - (2m_r^2 x_1)x - (2y m_r)x + 2y m_r x_1 + y^2$$

$$= (m_r^2 \cancel{+1})x^2 - (2m_r^2 x_1 - 2y m_r)x + 2y m_r x_1 + y^2$$

$$x^2 + (m_r^2 x^2) - \underbrace{(2m_r^2 x_1 - 2y m_r)}_{b}x + \underbrace{2y m_r x_1 + y^2 + x_1^2 m_r^2}_{c} = 100$$

$$\underbrace{(m_r^2 + 1)x^2}_{a}$$

a b c

$$y = m_r(x - x_1) + y_1$$