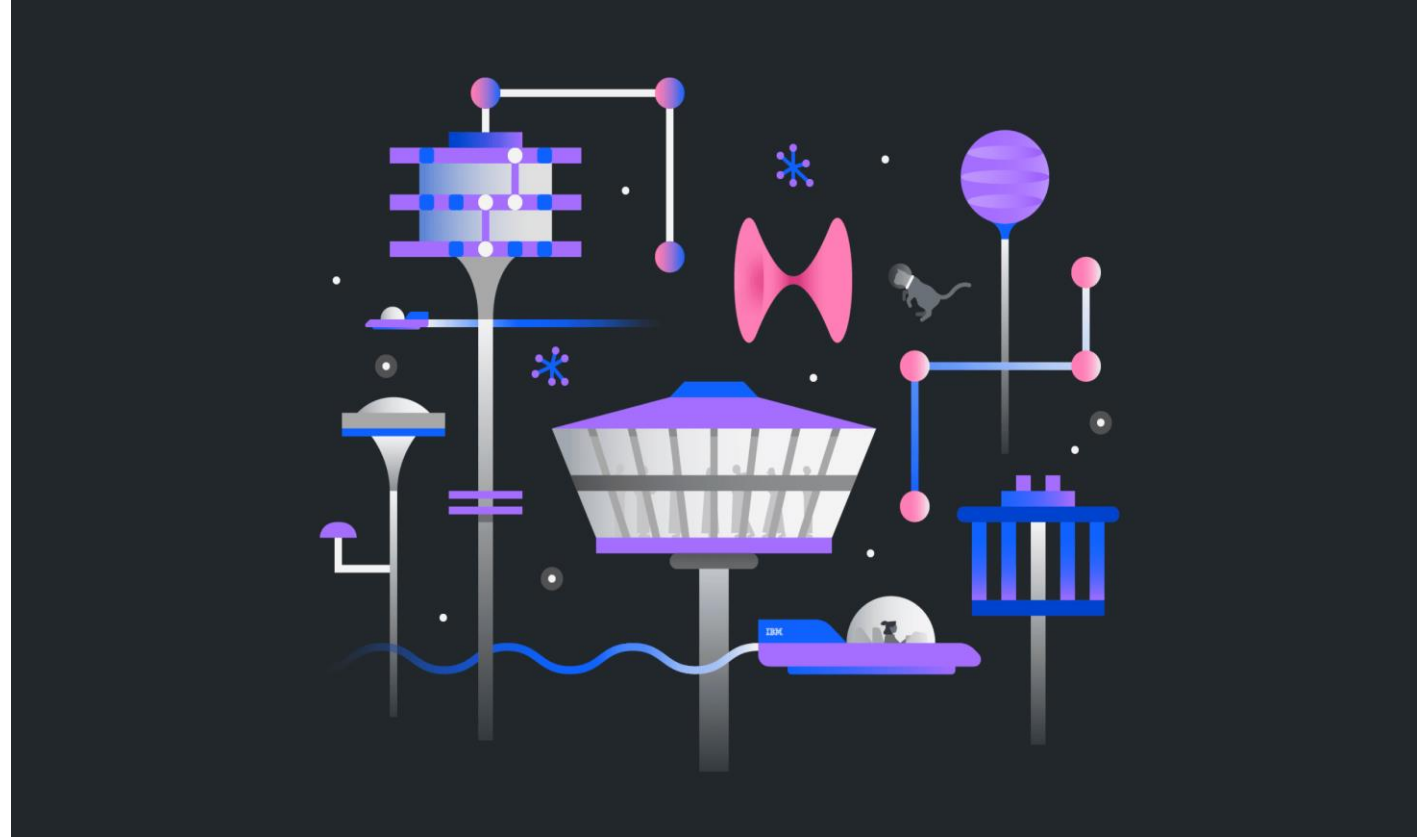


Implementation of the Jaynes-Cummings Hamiltonian on an IBM quantum computer

Jonathan Sepúlveda
PhD Student

Supervisors: Rubén Esteban, and Javier Aizpurua

IBM Collaborators: Niall Robertson, Sergei Zhuk,
and Martin Mevissen

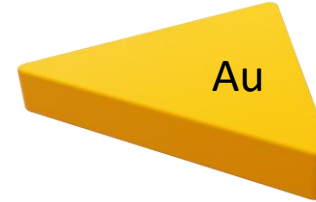
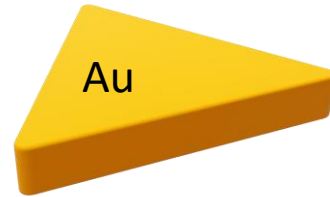


Introduction and motivation

Metallic optical nanoresonators

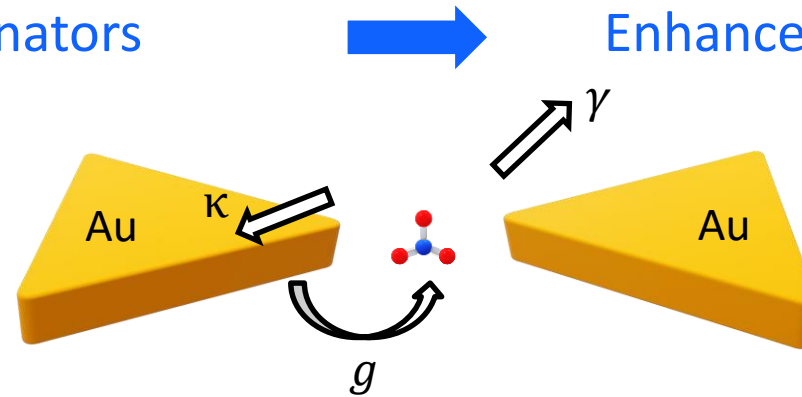


Enhance and concentrate the optical near fields to small regions



Introduction and motivation

Metallic optical nanoresonators



Enhance and concentrate the optical near fields to small regions

g : Coupling strength

γ : Radiation losses

κ : Nanoantenna losses

This system can be described by

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar \frac{\omega_0}{2} \hat{\sigma}_z + \hbar g (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger)$$

\hat{a}^\dagger : Creation operator

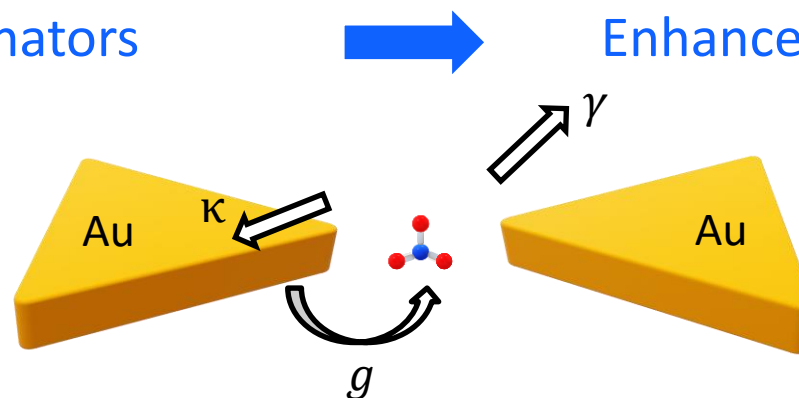
\hat{a} : Annihilation operator

$\hat{\sigma}_+$, $\hat{\sigma}_-$, $\hat{\sigma}_z$: Pauli operators

Vincenzo Giannini et al. *Chem. Rev.* 373, 111, 6, 3888 – 3912 (2011).
Francisco J. Garcia-Vidal et al. *Science* eabd0336 (2021).

Introduction and motivation

Metallic optical nanoresonators



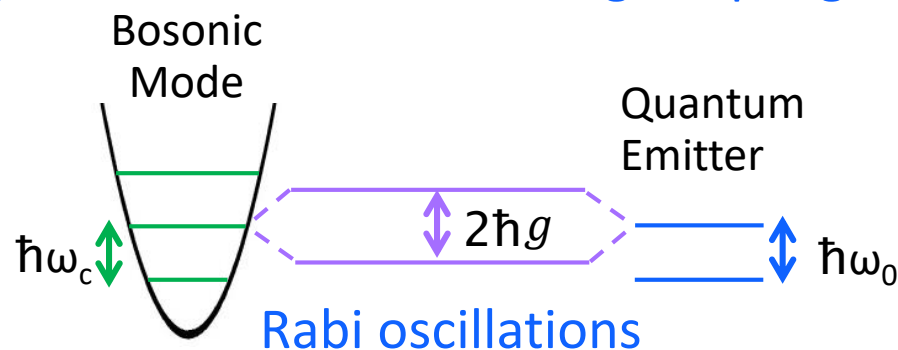
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The system can reach the strong coupling regime



Strong coupling criteria

$$\frac{\kappa + \gamma}{4} < g$$

\hat{a}^\dagger : Creation operator

\hat{a} : Annihilation operator

$\hat{\sigma}_+$, $\hat{\sigma}_-$, $\hat{\sigma}_z$: Pauli operators

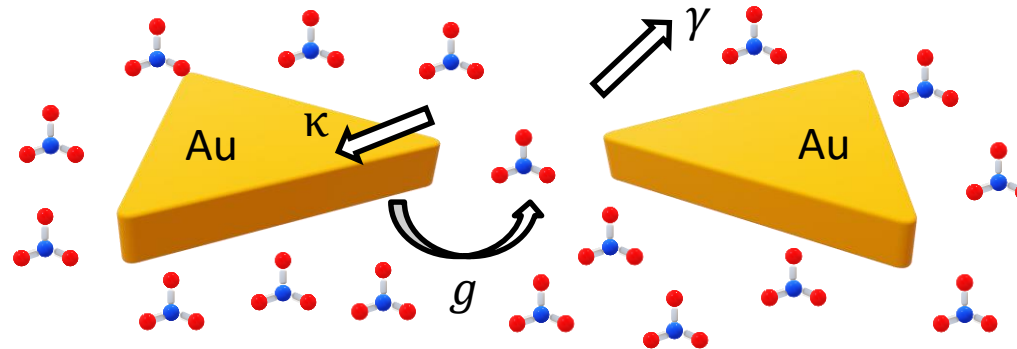
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BasQ
 Basque Quantum



Introduction and motivation

Extension to include more molecules



$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar \sum_j \frac{\omega_{0,i}}{2} \hat{\sigma}_{z,j} + \hbar \sum_j g (\hat{\sigma}_{+,j} \hat{a} + \hat{\sigma}_{-,j} \hat{a}^\dagger)$$

The study of these systems can be useful in molecular sensing and spectroscopic techniques, and also possibly in polaritonic chemistry

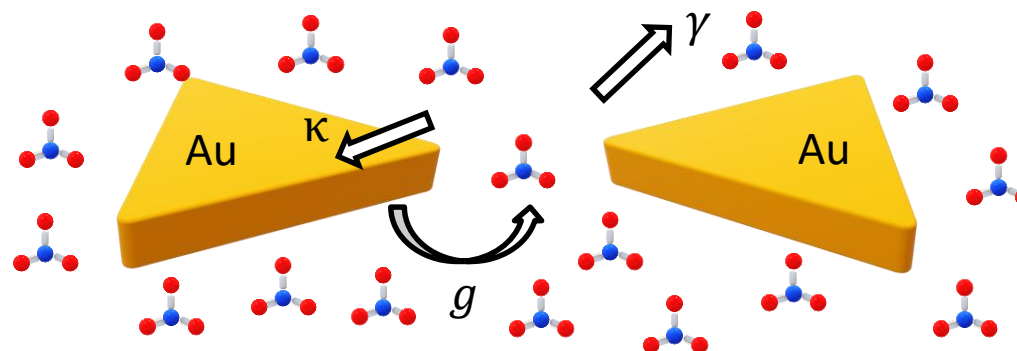
Tomáš Neuman, et. al. J. Phys. Chem. C. 119, 26652, (2015).
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Introduction and motivation

Extension to include more molecules



Simulations become very demanding
in classical computers



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BasQ
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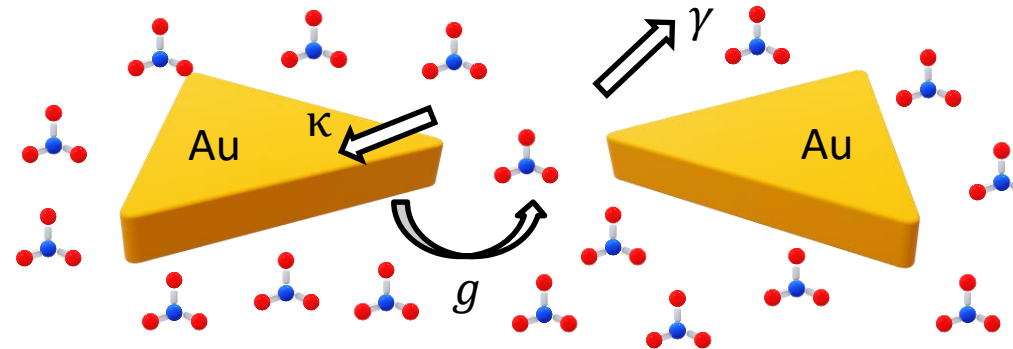


Introduction and motivation

Extension to include more molecules



Simulations become very demanding
in classical computers



Explore the implementation
of these systems in a
quantum computer

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar \sum_j \frac{\omega_{0,j}}{2} \hat{\sigma}_{z,j} + \hbar \sum_j g (\hat{\sigma}_{+,j} \hat{a} + \hat{\sigma}_{-,j} \hat{a}^\dagger)$$

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BasQ
Basque Quantum



Qubitization of the optical (bosonic) mode

Example of mapping 4 Fock states

$$|0\rangle_p \rightarrow |1_0 0_1 0_2 0_3\rangle$$

$$|1\rangle_p \rightarrow |0_0 1_1 0_2 0_3\rangle$$

$$|2\rangle_p \rightarrow |0_0 0_1 1_2 0_3\rangle$$

$$|3\rangle_p \rightarrow |0_0 0_1 0_2 1_3\rangle$$

Qubitization of the optical (plasmonic) mode

Example of mapping 4 Fock states

$$\begin{aligned} |0\rangle_p &\rightarrow |1_0 0_1 0_2 0_3\rangle \\ |1\rangle_p &\rightarrow |0_0 1_1 0_2 0_3\rangle \\ |2\rangle_p &\rightarrow |0_0 0_1 1_2 0_3\rangle \\ |3\rangle_p &\rightarrow |0_0 0_1 0_2 1_3\rangle \end{aligned}$$



$$\hat{a}^\dagger \rightarrow \tilde{a}^\dagger = \sum_{n=0}^{N_{Fock}-1} \sqrt{n+1} \hat{\sigma}_-^n \hat{\sigma}_+^{n+1}$$



$$\begin{aligned} \tilde{a}^\dagger |1_0 0_1 0_2 0_3\rangle &= |0_0 1_1 0_2 0_3\rangle \\ \tilde{a}^\dagger |0_0 0_1 0_2 1_3\rangle &= 0 \\ \tilde{a} |1_0 0_1 0_2 0_3\rangle &= 0 \\ \tilde{a} |0_0 1_1 0_2 0_3\rangle &= |1_0 0_1 0_2 0_3\rangle \end{aligned}$$

Qubitization of the optical (plasmonic) mode

Example of mapping 4 Fock states

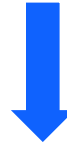
$$\begin{aligned} |0\rangle_p &\rightarrow |1_0 0_1 0_2 0_3\rangle \\ |1\rangle_p &\rightarrow |0_0 1_1 0_2 0_3\rangle \\ |2\rangle_p &\rightarrow |0_0 0_1 1_2 0_3\rangle \\ |3\rangle_p &\rightarrow |0_0 0_1 0_2 1_3\rangle. \end{aligned}$$



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$$\begin{aligned} \hat{H} = & \hbar\omega_c \left[\left(\frac{1 + \hat{\sigma}_{z'}^0}{2} \right) \left(\frac{1 - \hat{\sigma}_{z'}^1}{2} \right) + 2 \left(\frac{1 + \hat{\sigma}_{z'}^1}{2} \right) \left(\frac{1 - \hat{\sigma}_{z'}^2}{2} \right) + 3 \left(\frac{1 + \hat{\sigma}_{z'}^2}{2} \right) \left(\frac{1 - \hat{\sigma}_{z'}^3}{2} \right) \right] + \hbar \frac{\omega_0}{2} \hat{\sigma}_{z'}^{QE} + \\ & + \frac{\hbar g}{4} \left[\hat{\sigma}_x^{QE} (\hat{\sigma}_x^1 \hat{\sigma}_x^0 + \hat{\sigma}_y^1 \hat{\sigma}_y^0) + \hat{\sigma}_y^{QE} (\hat{\sigma}_y^1 \hat{\sigma}_x^0 - \hat{\sigma}_x^1 \hat{\sigma}_y^0) + \sqrt{2} \hat{\sigma}_x^{QE} (\hat{\sigma}_x^2 \hat{\sigma}_x^1 + \hat{\sigma}_y^2 \hat{\sigma}_y^1) + \sqrt{2} \hat{\sigma}_y^{QE} (\hat{\sigma}_y^2 \hat{\sigma}_x^1 - \hat{\sigma}_x^2 \hat{\sigma}_y^1) + \right. \\ & \left. + \sqrt{3} \hat{\sigma}_x^{QE} (\hat{\sigma}_x^3 \hat{\sigma}_x^2 + \hat{\sigma}_y^3 \hat{\sigma}_y^2) + \sqrt{3} \hat{\sigma}_y^{QE} (\hat{\sigma}_y^3 \hat{\sigma}_x^2 - \hat{\sigma}_x^3 \hat{\sigma}_y^2) \right] \end{aligned}$$

R. Somma. Quantum Computation, Complexity and many-body physics. Phd Thesis, 2005.

R. Somma, et. al., Proc. SPIE 5105, Quantum Information and Computation 96, 2003.

P. Cordero Encinar, A. Agustí, and C. Sabín. Phys. Rev. A 104, 052609, 2021.

Trotterization for one quantum emitter

After trotterization for $\omega_c = \omega_0$ (zero detuning) the time dynamics is given by

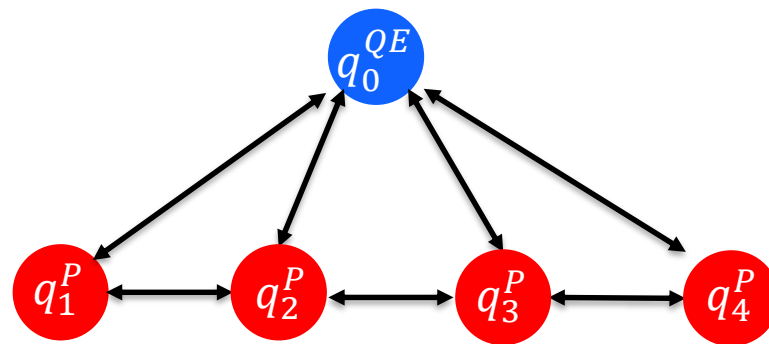
$$e^{-\frac{it}{\hbar}\hat{H}^{RF}} = e^{-\frac{itg}{4}\hat{\sigma}_x^{QE}\hat{\sigma}_x^1\hat{\sigma}_x^0} e^{-\frac{itg}{4}\hat{\sigma}_x^{QE}\hat{\sigma}_y^1\hat{\sigma}_y^0} e^{-\frac{itg}{4}\hat{\sigma}_y^{QE}\hat{\sigma}_y^1\hat{\sigma}_x^0} e^{\frac{itg}{4}\hat{\sigma}_y^{QE}\hat{\sigma}_x^1\hat{\sigma}_y^0} e^{-\sqrt{2}\frac{itg}{4}\hat{\sigma}_x^{QE}\hat{\sigma}_x^2\hat{\sigma}_x^1} e^{-\sqrt{2}\frac{itg}{4}\hat{\sigma}_x^{QE}\hat{\sigma}_y^2\hat{\sigma}_y^1} e^{-\sqrt{2}\frac{itg}{4}\hat{\sigma}_y^{QE}\hat{\sigma}_y^2\hat{\sigma}_x^1} e^{\sqrt{2}\frac{itg}{4}\hat{\sigma}_y^{QE}\hat{\sigma}_x^2\hat{\sigma}_y^1} \\ e^{-\sqrt{3}\frac{itg}{4}\hat{\sigma}_x^{QE}\hat{\sigma}_x^3\hat{\sigma}_x^2} e^{-\sqrt{3}\frac{itg}{4}\hat{\sigma}_x^{QE}\hat{\sigma}_y^3\hat{\sigma}_y^2} e^{-\sqrt{3}\frac{itg}{4}\hat{\sigma}_y^{QE}\hat{\sigma}_y^3\hat{\sigma}_x^2} e^{\sqrt{3}\frac{itg}{4}\hat{\sigma}_y^{QE}\hat{\sigma}_x^3\hat{\sigma}_y^2}$$

Trotterization for one quantum emitter

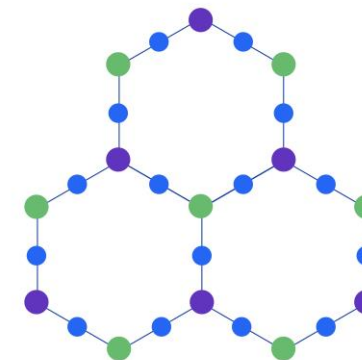
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Non-local Hamiltonian



Topology of all active IBM quantum devices

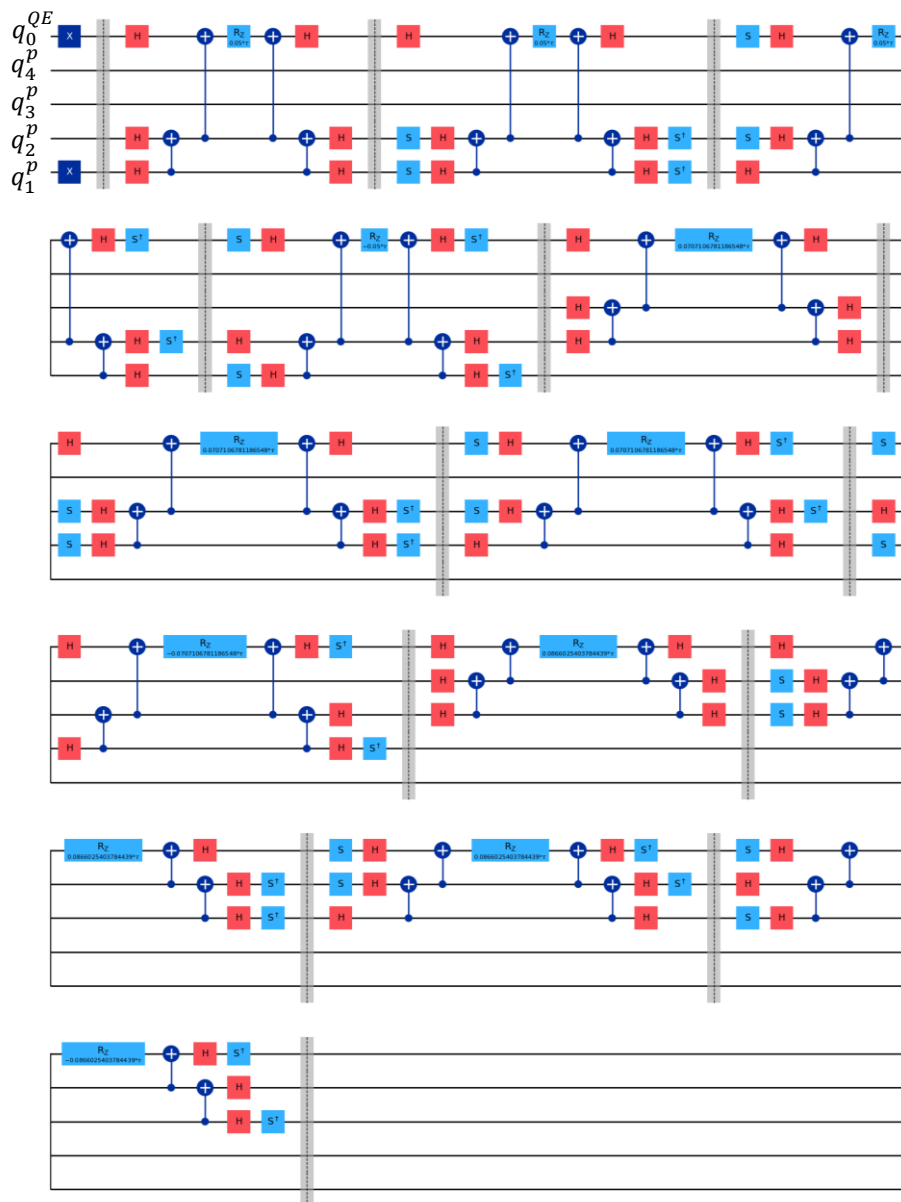


Heavy hex
lattice

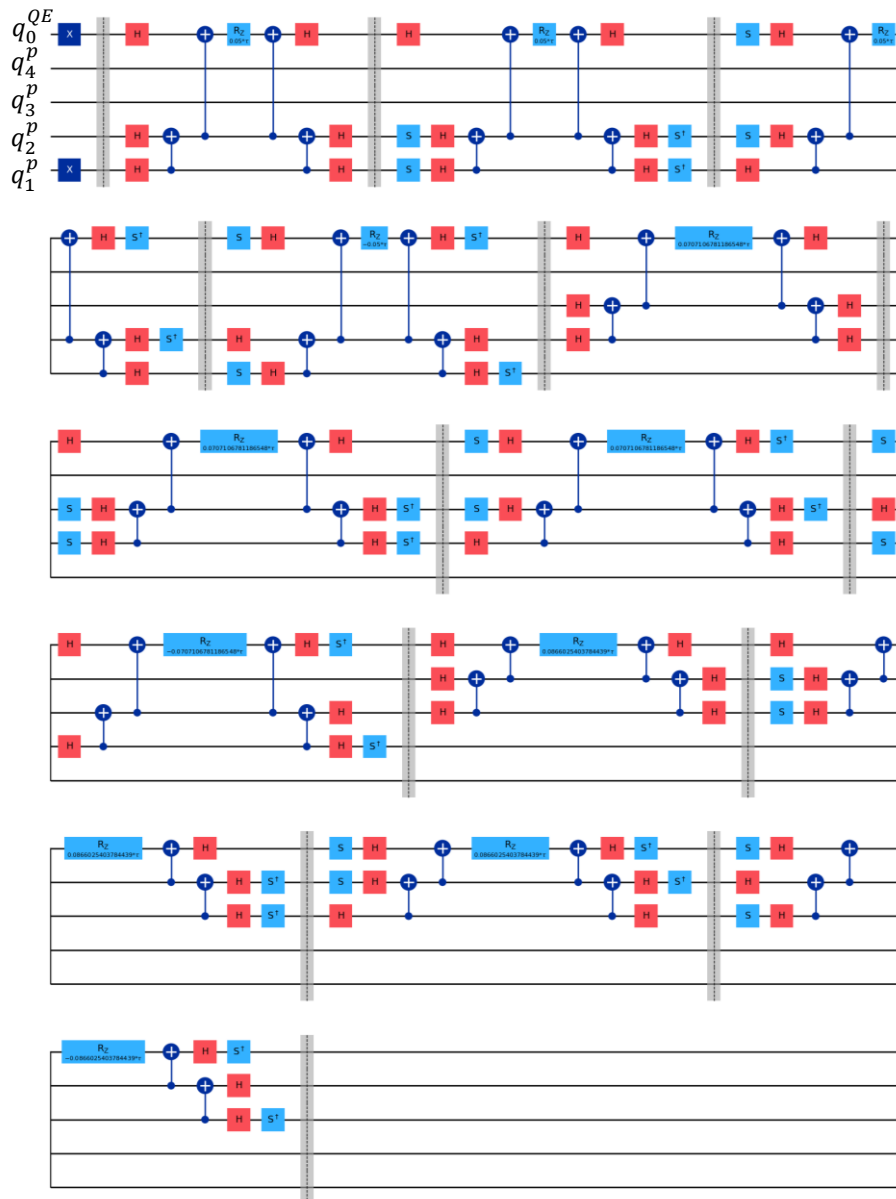
●: Qubits that represent the quantum emitter

●: Qubits that represent the boson

Quantum circuit for one quantum emitter



Quantum circuit for one quantum emitter



Example of mapping 4 Fock states



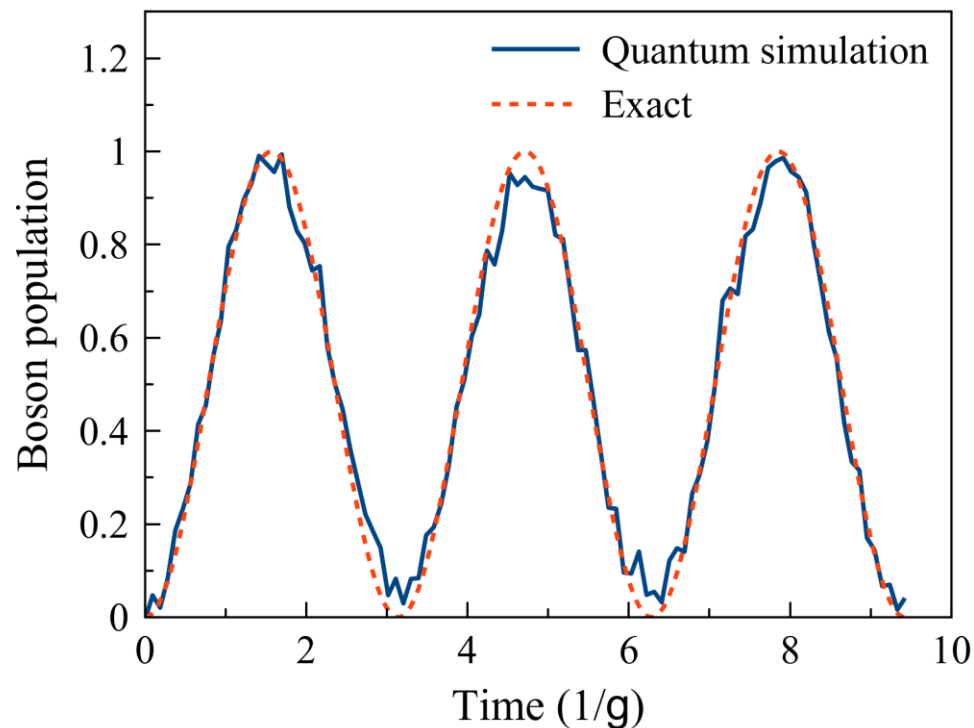
For 5 qubits we obtain a 2-qubit depth of 48 (for 4 Fock states, before transpilation)

Quantum simulation with IBM Torino

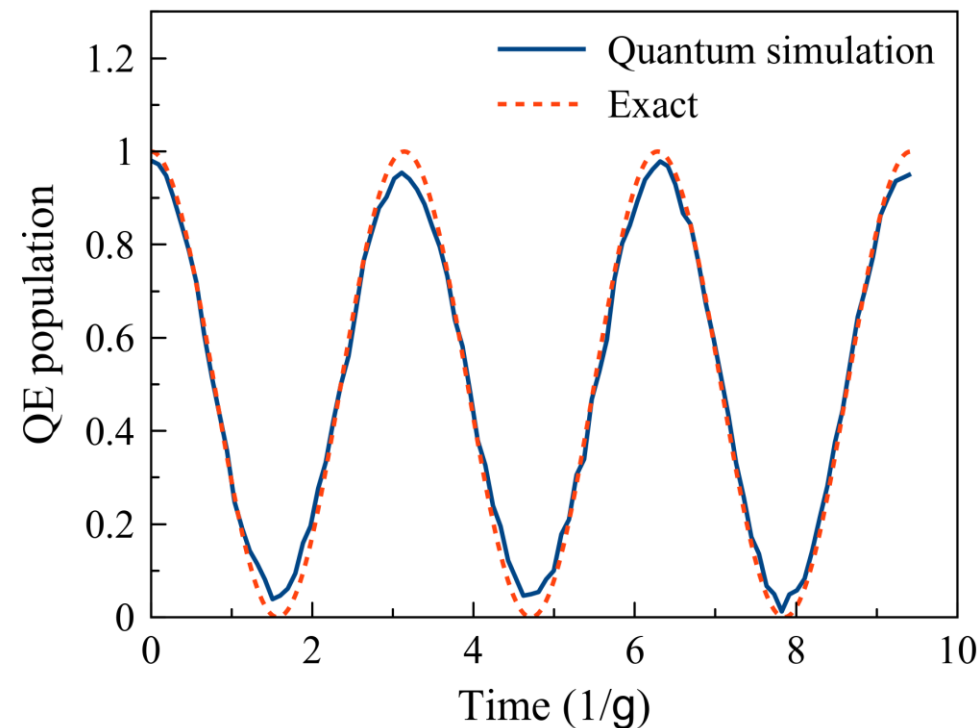
Estimator primitive: We have used a resilience level 2 and optimization level 1

Initial state: $(|0\rangle_p \otimes |1\rangle_{QE} = |1_0 0_1 0_2 0_3 1_{QE}\rangle)$

Population of the boson



Population of the emitter

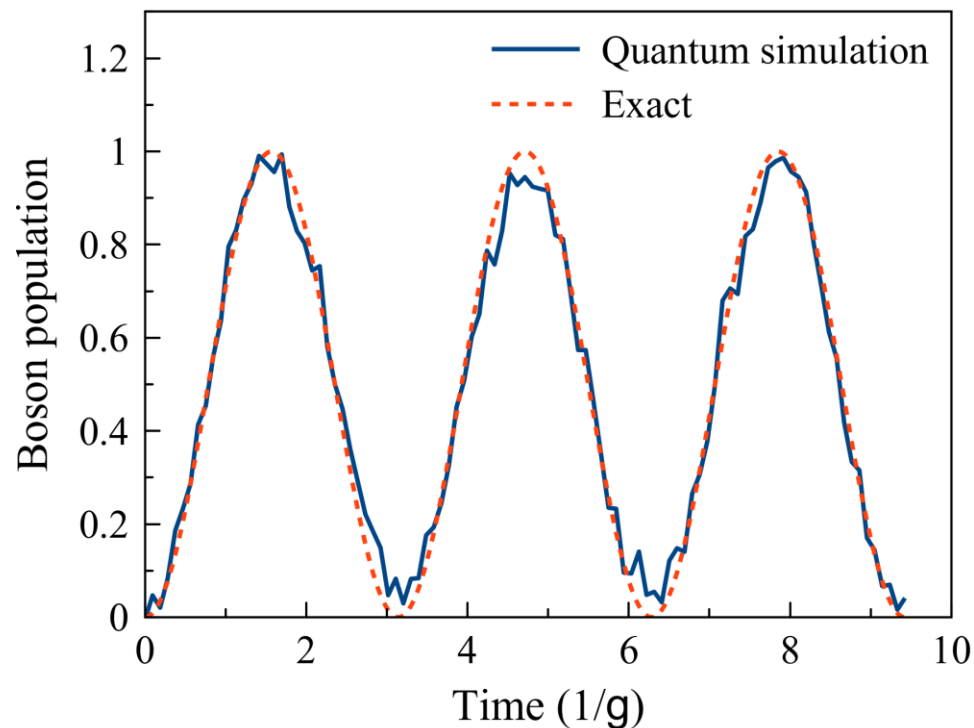


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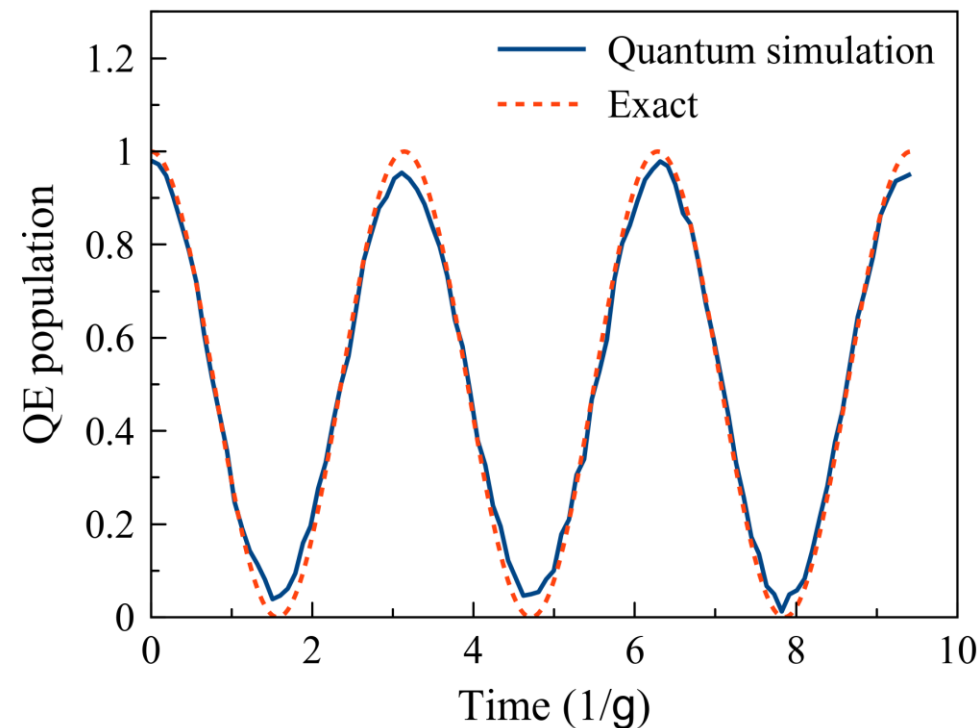
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Population of the boson



Population of the emitter



We obtain an excellent agreement between the classical and quantum simulations

We observe 3 clear Rabi oscillations

Extension to many quantum emitters

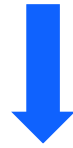
The really interesting part is including j quantum emitters ($\omega_c = \omega_0$)

$$\hat{H}_T^{RF} = \hbar g \sum_j (\hat{\sigma}_{+,j} \hat{a} + \hat{\sigma}_{-,j} \hat{a}^\dagger)$$

Extension to many quantum emitters

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$$\hat{H}_T^{RF} = \hbar g \sum_j (\hat{\sigma}_{+,j} \hat{a} + \hat{\sigma}_{-,j} \hat{a}^\dagger)$$



$$e^{-\frac{it}{\hbar} \hat{H}_T^{RF}} = \sum_j e^{-\frac{it}{\hbar} \hat{H}_j^{RF}} =$$

$$\prod_j \left[e^{-\frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_x^1 \hat{\sigma}_x^0} e^{-\frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_y^1 \hat{\sigma}_y^0} e^{-\frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_y^1 \hat{\sigma}_x^0} e^{\frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_x^1 \hat{\sigma}_y^0} e^{-\sqrt{2} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_x^2 \hat{\sigma}_x^1} e^{-\sqrt{2} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_y^2 \hat{\sigma}_y^1} \right. \\ \left. e^{-\sqrt{2} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_y^2 \hat{\sigma}_x^1} e^{\sqrt{2} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_x^2 \hat{\sigma}_y^1} e^{-\sqrt{3} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_x^3 \hat{\sigma}_x^2} e^{-\sqrt{3} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_y^3 \hat{\sigma}_y^2} e^{-\sqrt{3} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_y^3 \hat{\sigma}_x^2} e^{\sqrt{3} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_x^3 \hat{\sigma}_y^2} \right]$$

We need to add more terms

Extension to many quantum emitters

The really interesting part is including j quantum emitters ($\omega_c = \omega_0$)

$$\hat{H}_T^{RF} = \hbar g \sum_j (\hat{\sigma}_{+,j} \hat{a} + \hat{\sigma}_{-,j} \hat{a}^\dagger)$$



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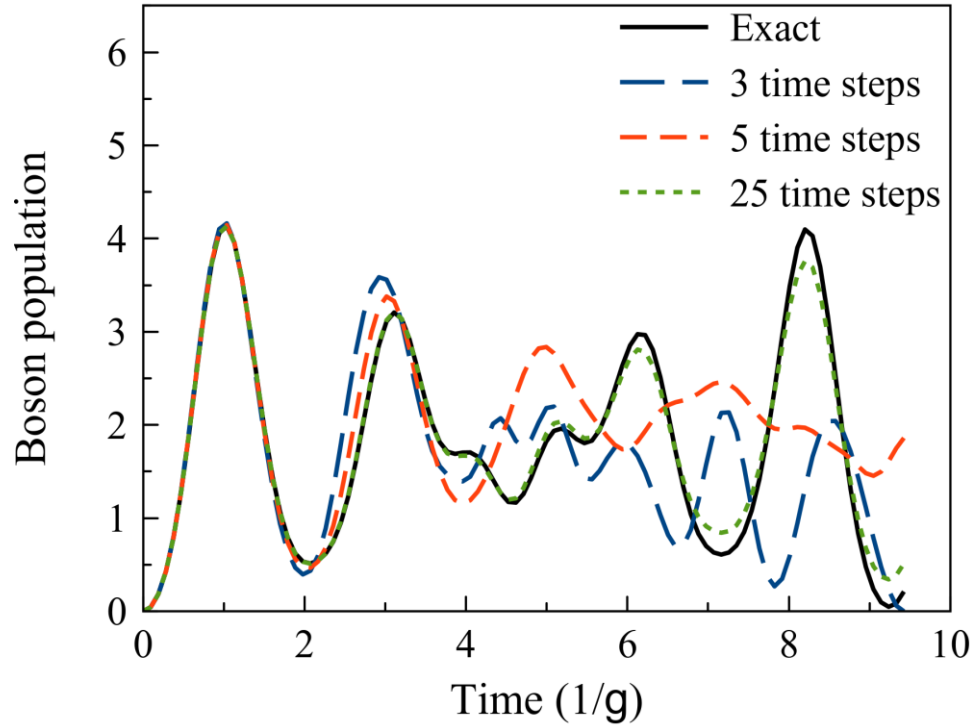
$$\prod_j \left[e^{-\frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_x^1 \hat{\sigma}_x^0} e^{-\frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_y^1 \hat{\sigma}_y^0} e^{-\frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_y^1 \hat{\sigma}_x^0} e^{\frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_x^1 \hat{\sigma}_y^0} e^{-\sqrt{2} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_x^2 \hat{\sigma}_x^1} e^{-\sqrt{2} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_y^2 \hat{\sigma}_y^1} \right. \\ \left. e^{-\sqrt{2} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_y^2 \hat{\sigma}_x^1} e^{\sqrt{2} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_x^2 \hat{\sigma}_y^1} e^{-\sqrt{3} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_x^3 \hat{\sigma}_x^2} e^{-\sqrt{3} \frac{itg}{4} \hat{\sigma}_x^{QEj} \hat{\sigma}_y^3 \hat{\sigma}_y^2} e^{-\sqrt{3} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_y^3 \hat{\sigma}_x^2} e^{\sqrt{3} \frac{itg}{4} \hat{\sigma}_y^{QEj} \hat{\sigma}_x^3 \hat{\sigma}_y^2} \right]$$

We need to add more terms

Strongly non-local Hamiltonian

Results for 5 quantum emitters (classical simulation)

Automatic circuit



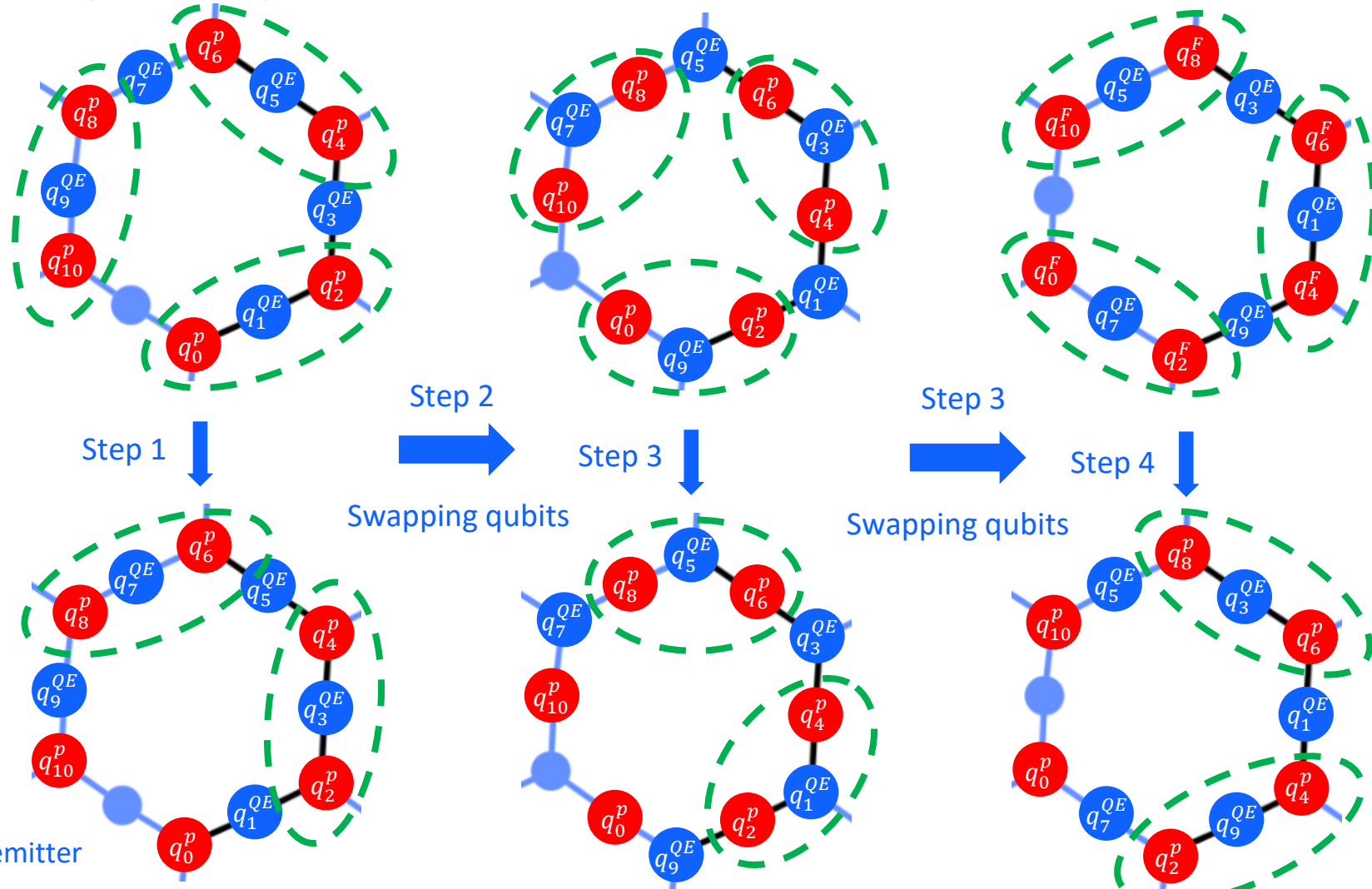
For 1 time step:

2-qubit depth = 208

We need to include more time steps

Parallelize the quantum circuit

Example of 5 quantum emitters and 6 bosonic states

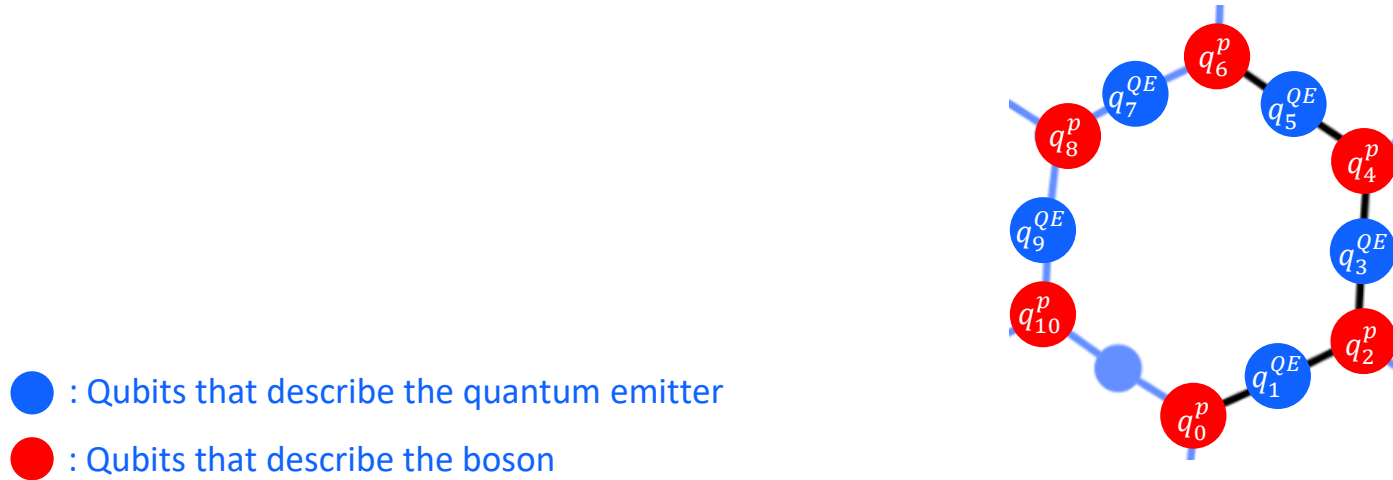


● : Qubits that describe the quantum emitter

● : Qubits that describe the boson

Parallelize the quantum circuit

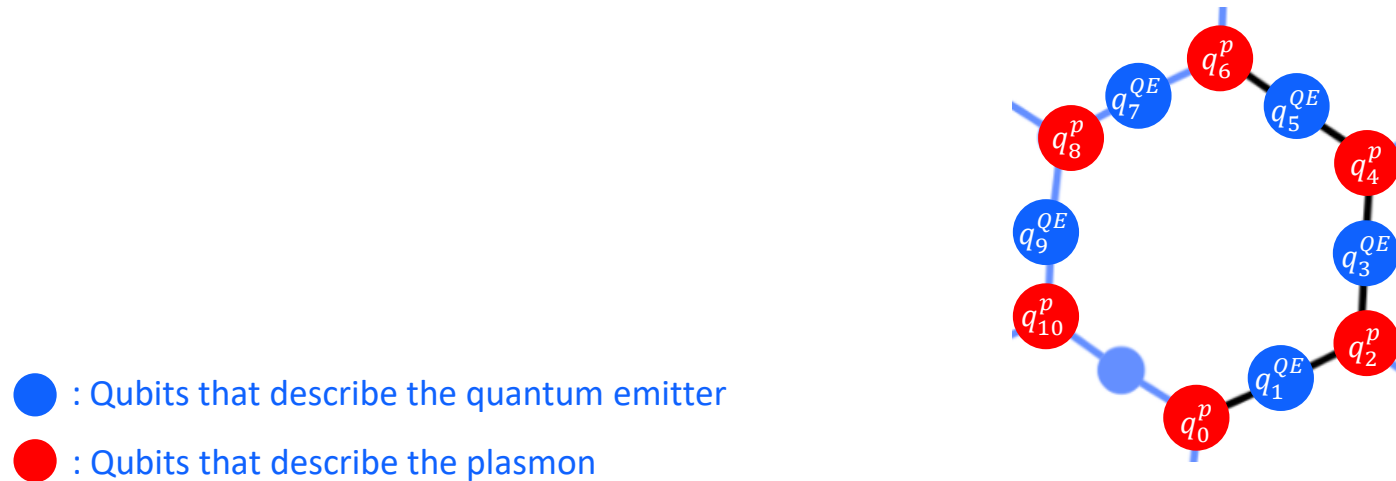
Example of 5 quantum emitters and 6 bosonic states



$$\text{2-qubit Depth} = \left(N_{\text{qubits}} - N_{QE} - (N_P - 2) \right) (N_{QE} - 1) N_{\text{steps}} + 32 N_{QE} N_{\text{steps}}$$

Parallelize the quantum circuit

Example of 5 quantum emitters and 6 bosonic states



$\approx 1 \text{ or } 3$

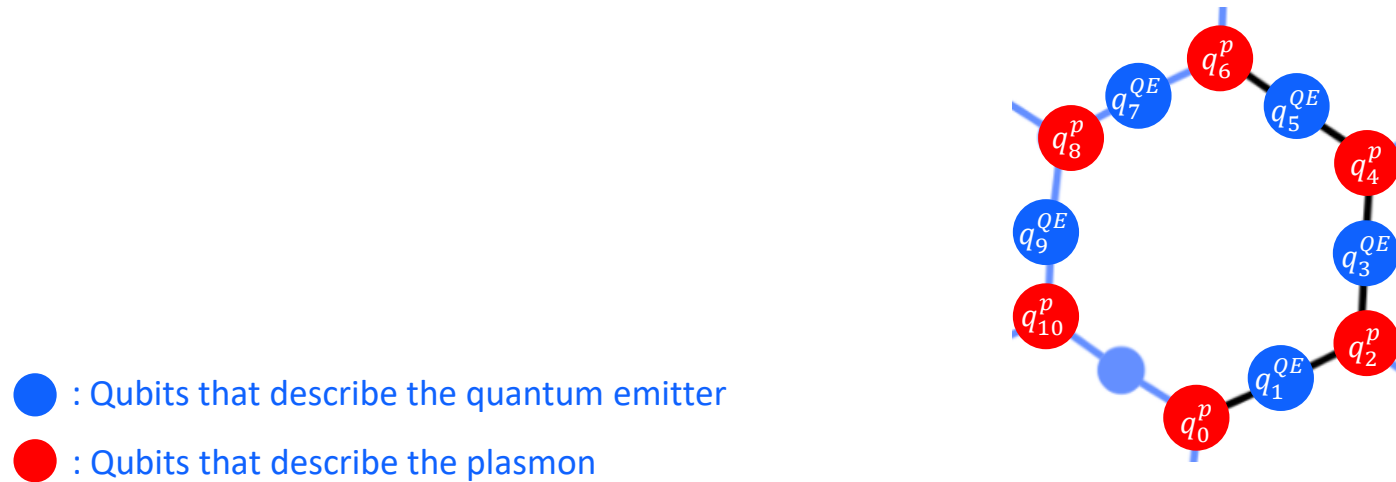
Scaling of the quantum circuit

$$\text{2-qubit Depth} = \underbrace{\left(N_{\text{qubits}} - N_{QE} - (N_P - 2) \right) (N_{QE} - 1) N_{\text{steps}}}_{\text{Number of swaps gates}} + \underbrace{32 N_{QE} N_{\text{steps}}}_{\text{Scaling of the quantum circuit}}$$

Number of swaps gates

Parallelize the quantum circuit

Example of 5 quantum emitters and 6 plasmonic states

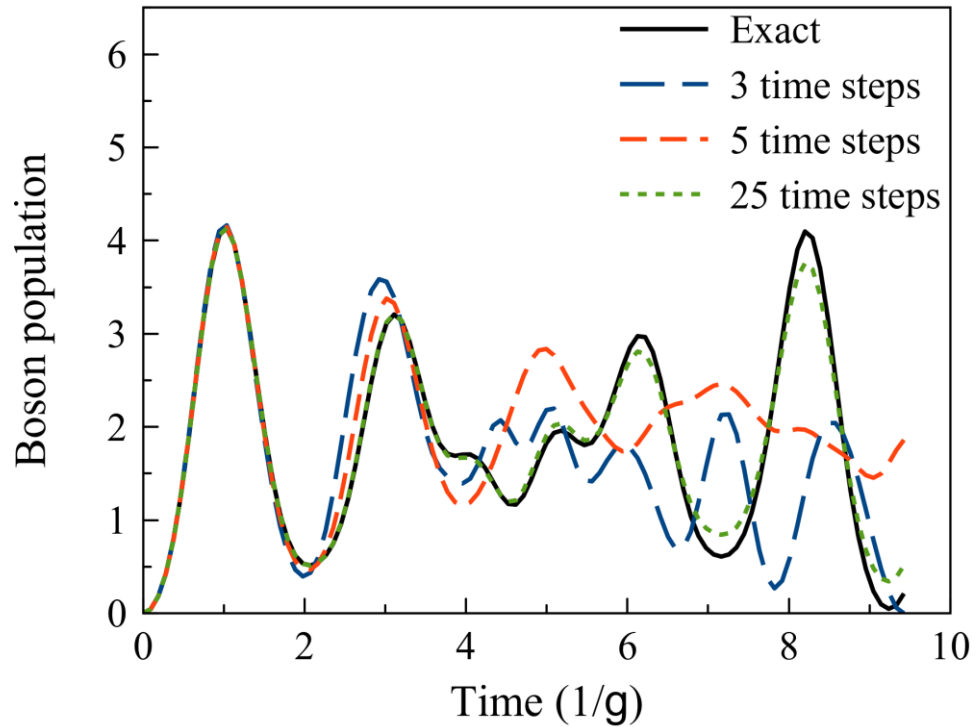


$$\text{2-qubit Depth} = \left(N_{qubits} - N_{QE} - (N_P - 2) \right) (N_{QE} - 1) N_{steps} + 32 N_{QE} N_{steps}$$

This equation scales linearly with the number of the quantum emitters and the number of time steps

Results for 5 quantum emitters (classical simulation)

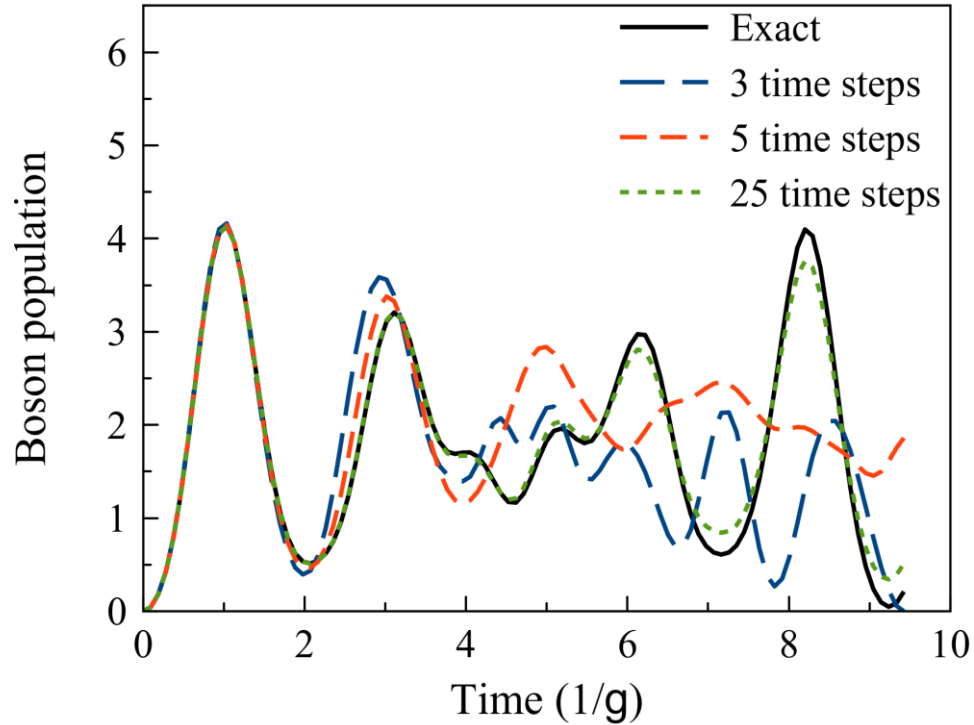
Automatic circuit



For 1 time step:
2-qubit depth = 208

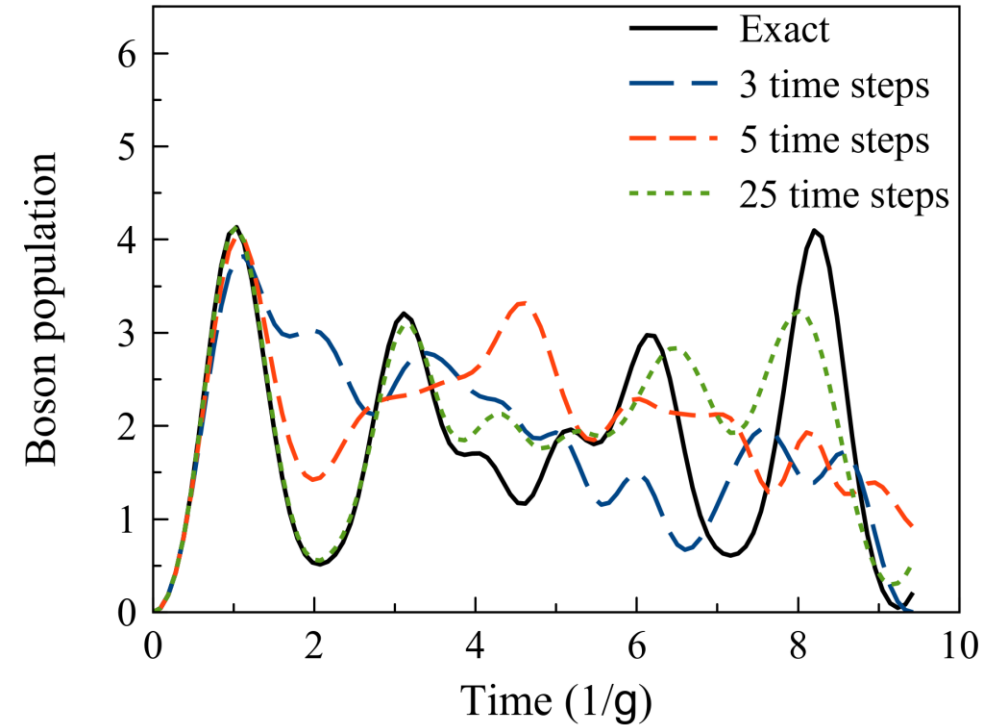
Results for 5 quantum emitters (classical simulation)

Automatic circuit



For 1 time step:
2-qubit depth = 208

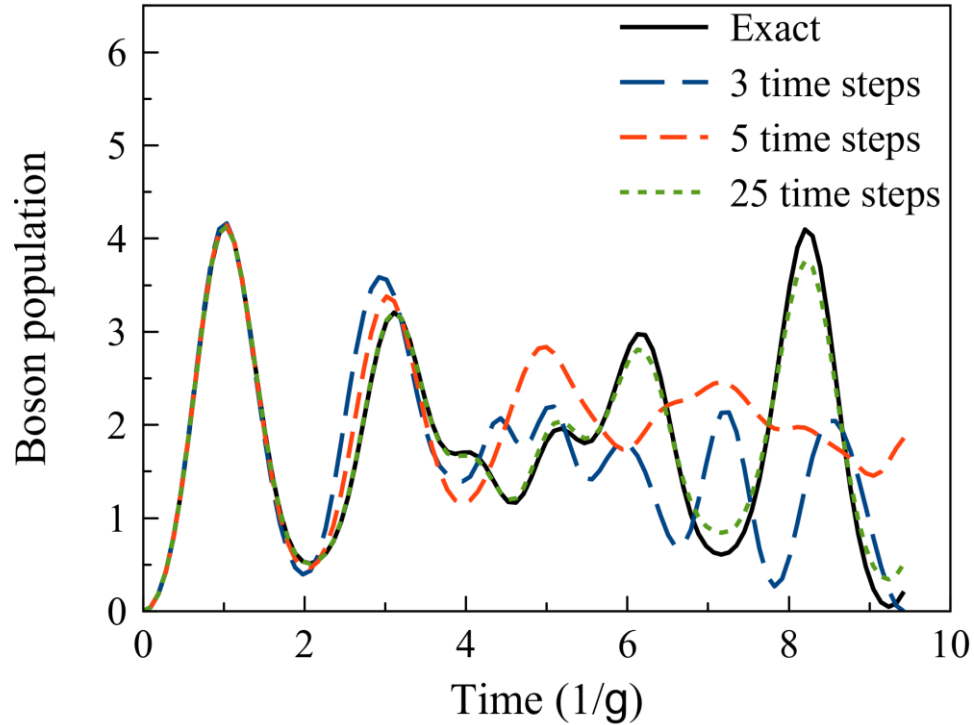
Parallelize circuit



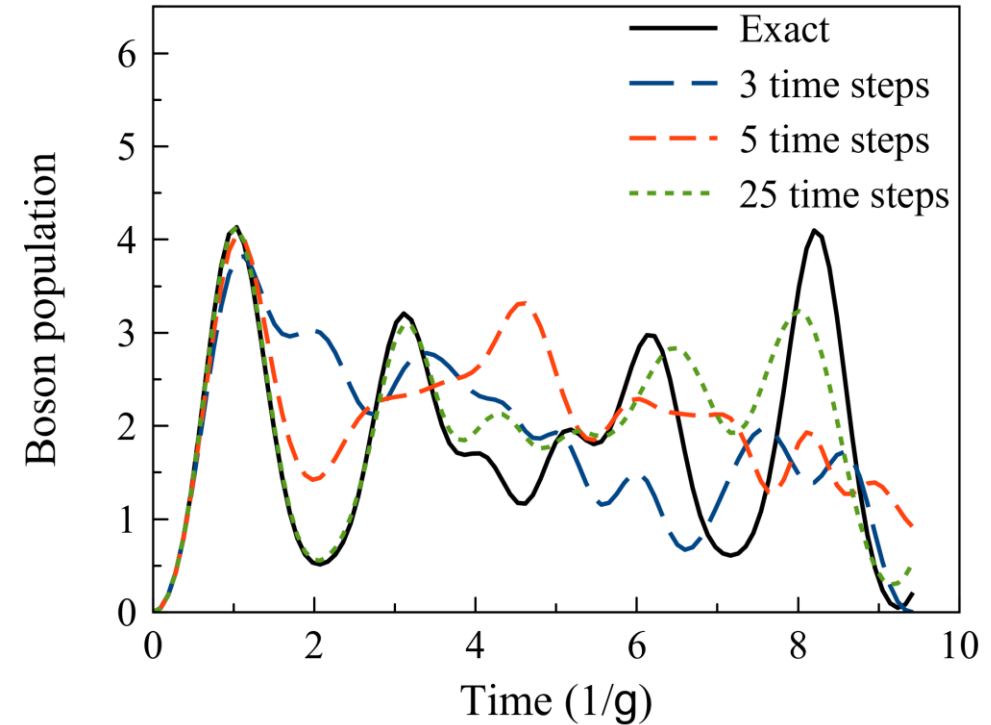
For 1 time step:
2-qubit depth = 160

Results for 5 quantum emitters (classical simulation)

Automatic circuit



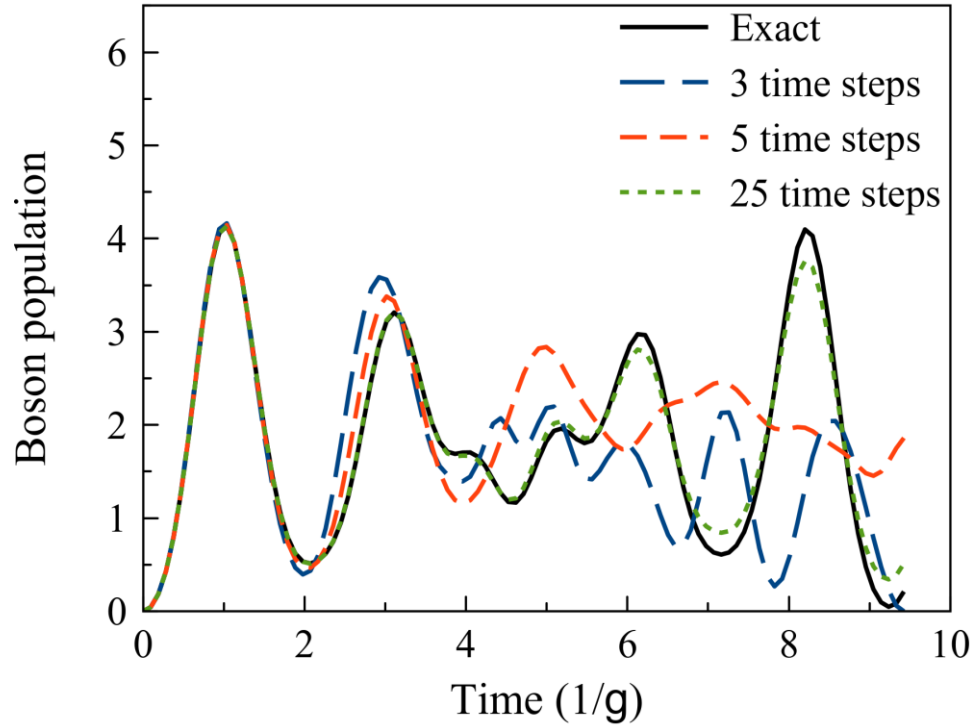
Parallelize circuit



Quantum emitters	Automatic circuit 2-qubit depth	Parallelize circuit 2-qubit depth
5	208	160
7	304	224
10	448	320
15	688	480

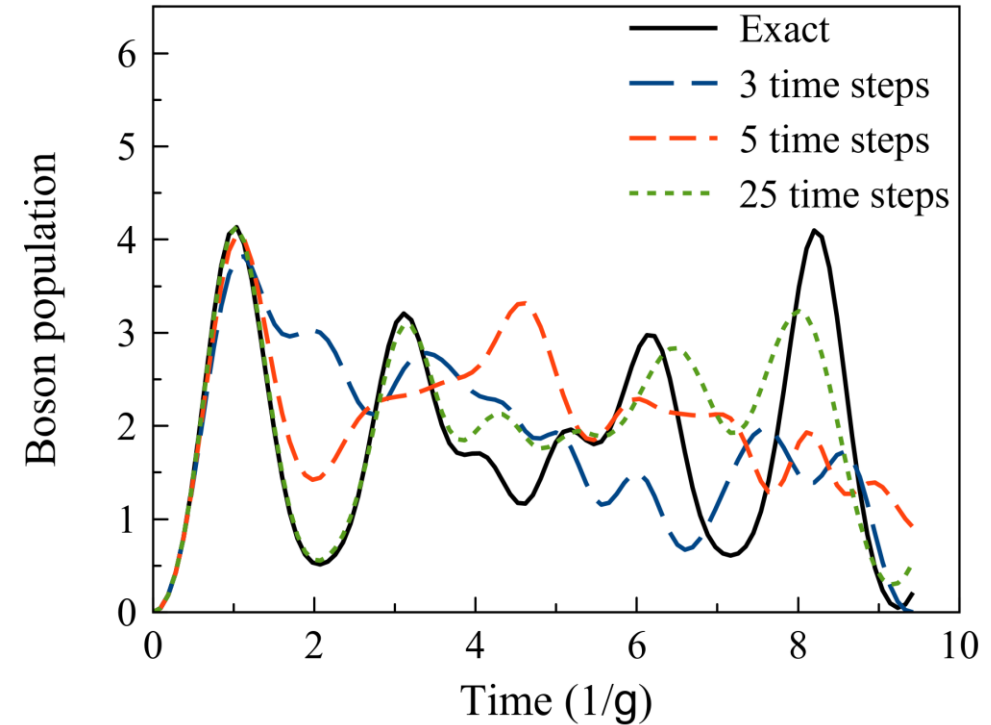
Results for 5 quantum emitters (classical simulation)

Automatic circuit



For 1 time step:
2-qubit depth = 208

Parallelize circuit



For 1 time step:
2-qubit depth = 160

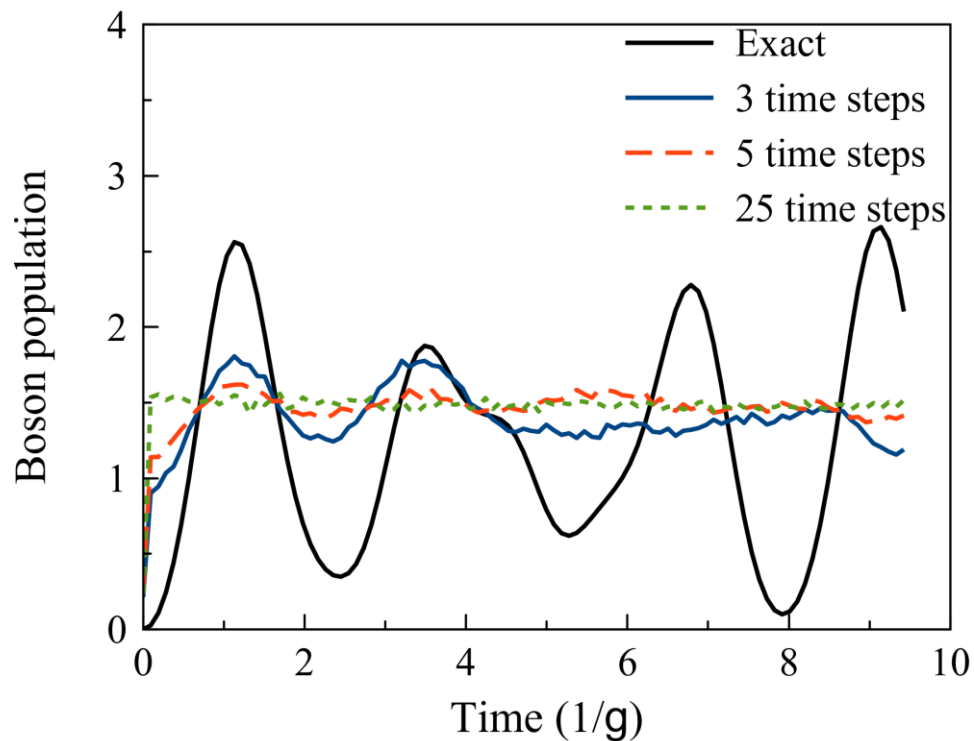
We reduce the 2-qubit depth but at the cost of
increasing the Trotter error

Quantum simulation with IBM Torino

Quantum simulation with 3 molecules

Estimator primitive: We have used a resilience level 2 and optimization level 1

Automatic circuit

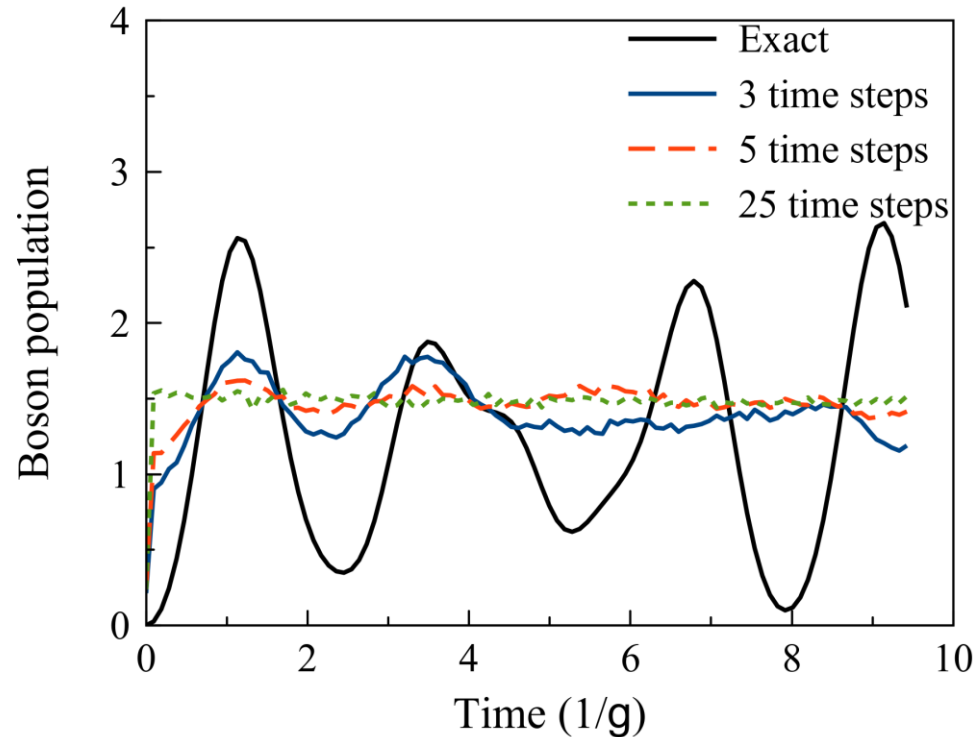


Quantum simulation with IBM Torino

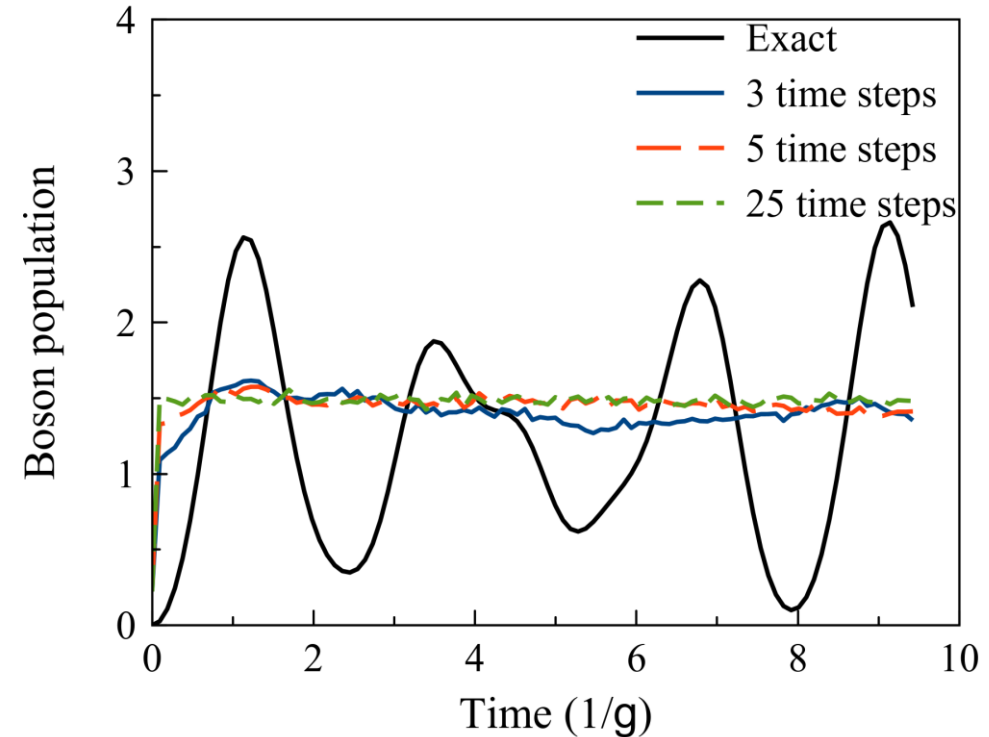
Quantum simulation with 3 molecules

Estimator primitive: We have used a resilience level 2 and optimization level 1

Automatic circuit



Parallelize circuit



Quantum simulation becomes very demanding

We want to implement the Qiskit Addons Multi-Product Formula (MPF) to obtain lower trotter errors

BasQ
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Conclusions

We have implemented the Jaynes-Cumming Hamiltonian in a quantum circuit.

- Excellent agreement with the exact results.
- We observe three clear Rabi oscillations.
- We have extended the model to include more quantum emitters
 - We obtain very deep circuits.
 - Linear scaling with the number of emitters and time steps.

Acknowledgements

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- Material Physics Center predoctoral research grant.



Thank you for your attention

An interesting finding

The terms of the Hamiltonian do **not commute**. However, for zero detuning $\Delta = \omega_0 - \omega_c = 0$

$$\hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar \frac{\omega_0}{2} \hat{\sigma}_z \xrightarrow{\text{Rotating frame}} 0$$

Rotating frame

Further, if we write the remaining terms in the order

$$\hbar g \left(\hat{\sigma}_-^{(\text{QE})} \hat{a}^\dagger + \hat{\sigma}_+^{(\text{QE})} \hat{a} \right) = \frac{\hbar g}{4} \left[\hat{\sigma}_x^{(\text{QE})} \hat{\sigma}_x^1 \hat{\sigma}_x^0 + \hat{\sigma}_x^{(\text{QE})} \hat{\sigma}_y^1 \hat{\sigma}_y^0 + \hat{\sigma}_y^{(\text{QE})} \hat{\sigma}_y^1 \hat{\sigma}_x^0 - \hat{\sigma}_y^{(\text{QE})} \hat{\sigma}_x^1 \hat{\sigma}_y^0 + \right. \\ \left. \sqrt{2} \hat{\sigma}_x^{(\text{QE})} \hat{\sigma}_x^2 \hat{\sigma}_x^1 + \sqrt{2} \hat{\sigma}_x^{(\text{QE})} \hat{\sigma}_y^2 \hat{\sigma}_y^1 + \sqrt{2} \hat{\sigma}_y^{(\text{QE})} \hat{\sigma}_y^2 \hat{\sigma}_x^1 - \sqrt{2} \hat{\sigma}_y^{(\text{QE})} \hat{\sigma}_x^2 \hat{\sigma}_y^1 + \sqrt{3} \hat{\sigma}_x^{(\text{QE})} \hat{\sigma}_x^3 \hat{\sigma}_x^2 + \sqrt{3} \hat{\sigma}_x^{(\text{QE})} \hat{\sigma}_y^3 \hat{\sigma}_y^2 + \sqrt{3} \hat{\sigma}_y^{(\text{QE})} \hat{\sigma}_y^3 \hat{\sigma}_x^2 - \sqrt{3} \hat{\sigma}_y^{(\text{QE})} \hat{\sigma}_x^3 \hat{\sigma}_y^2 \right]$$

We find **zero trotterization error**

PRELIMINARY EXPLANATION

Terms $\{H_1, H_2, H_3, H_4\}$ commute, as well as $\{H_5, H_6, H_7, H_8\}$ terms and $\{H_9, H_{10}, H_{11}, H_{12}\}$ terms.

$H_5 + H_6 + H_7 + H_8$ commute with $H_1 + H_2 + H_3 + H_4$ and $H_9 + H_{10} + H_{11} + H_{12}$.

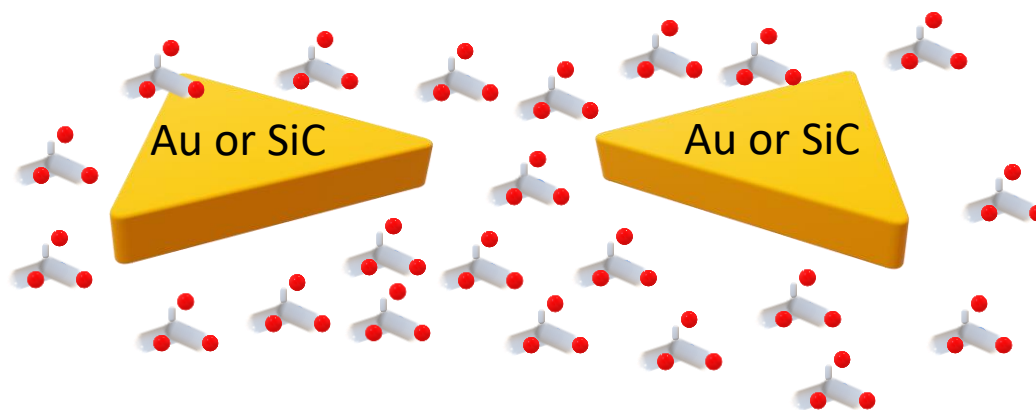
$H_1 + H_2 + H_3 + H_4$ 'almost commutes' with $H_9 + H_{10} + H_{11} + H_{12}$ (the result applied to any plasmonic state gives zero)

$$e^{-iHt} = e^{-i(H_1+H_2+H_3+H_4+H_5+H_6+H_7+H_8+H_9+H_{10}+H_{11}+H_{12})t} = \\ e^{-i(H_1+H_2+H_3+H_4)t} e^{-i(H_5+H_6+H_7+H_8)t} e^{-i(H_9+H_{10}+H_{11}+H_{12})t} = \\ e^{-iH_1t} e^{-iH_2t} e^{-iH_3t} e^{-iH_4t} e^{-iH_5t} e^{-iH_6t} e^{-iH_7t} e^{-iH_8t} e^{-iH_9t} e^{-iH_{10}t} e^{-iH_{11}t} e^{-iH_{12}t}$$



Basque Quantum





Scaling

$$\left(N_{qubits} - N_{QE} - (N_{Fock} - 2)\right) (N_{QE} - 1) N_{steps} + 32 N_{QE} N_{steps}$$

