Lecture 6

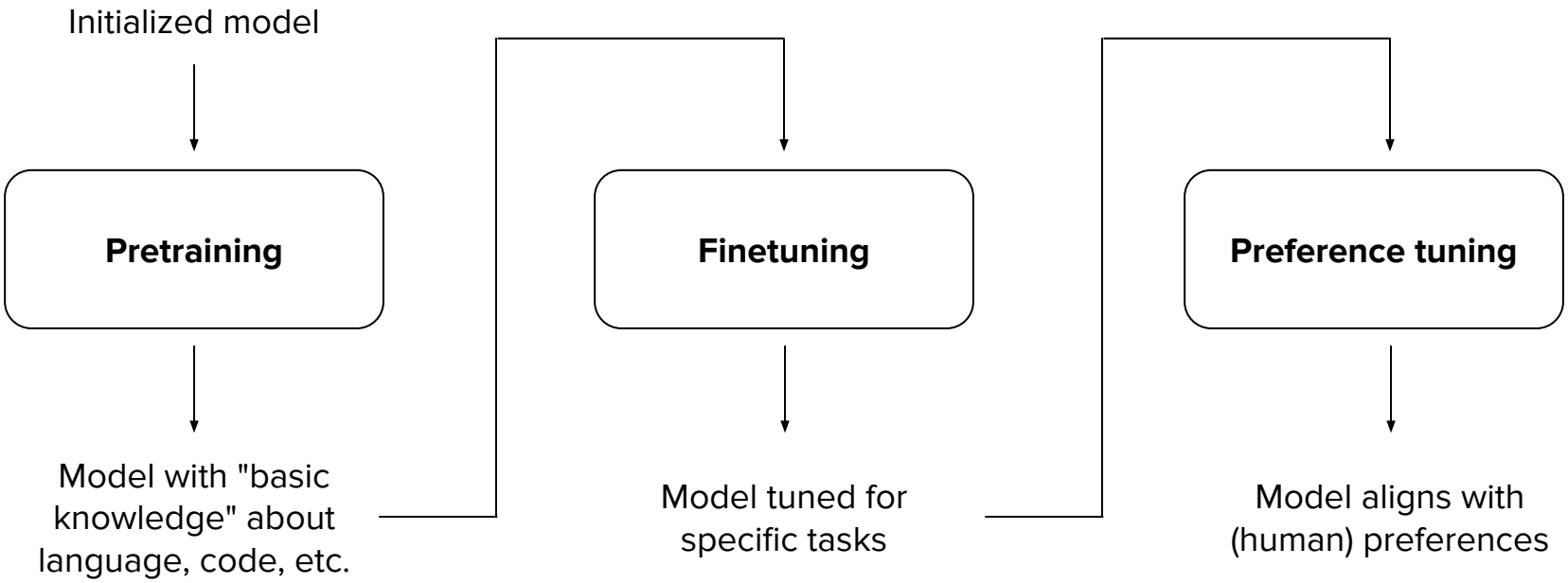
CME 295: Transformers & Large Language Models

Afshine Amidi & Shervine Amidi

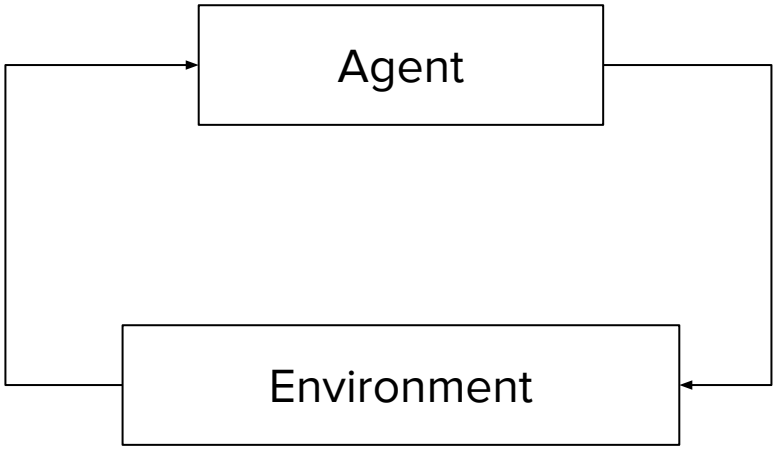
Stanford University

ICME

Recap of last episode...



Recap of last episode...



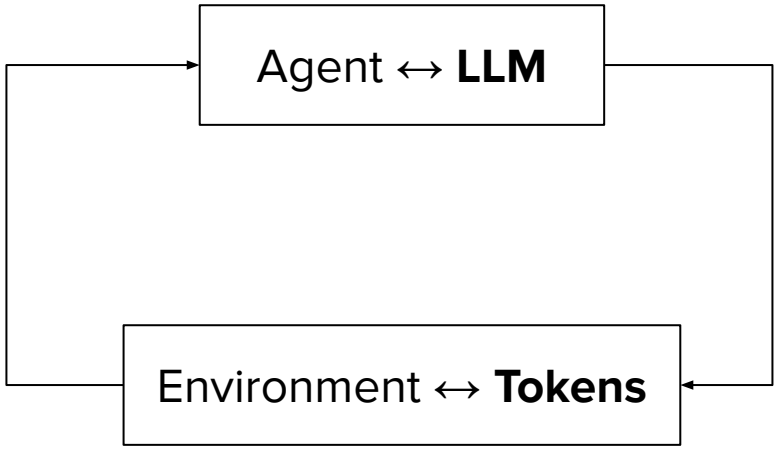
Reward r_t

State s_t

Action a_t

Policy $\pi_\theta(a_t|s_t)$

Recap of last episode...



Reward r_t

↕

Human preference

State s_t

↕

Input so far

Action a_t

↕

Next token

Policy $\pi_\theta(a_t|s_t)$

↕

Probability of next token

Recap of last episode...

$$\mathscr{L}(\theta) = \boxed{\text{Maximize advantages}} + \boxed{\text{Don't deviate too much from old/base model}}$$

Recap of last episode...

$\mathscr{L}(\theta) =$ Maximize advantages $+$ Don't deviate too much from old/base model

PPO-clip

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\left[\min(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t)\right]$$

Recap of last episode...

$$\mathscr{L}(\theta) = \text{Maximize advantages} + \text{Don't deviate too much from old/base model}$$

PPO-clip

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\left[\min(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t)\right]$$

PPO-KL penalty

$$L^{KLPEN}(\theta) = \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}\hat{A}_t - \beta\,\text{KL}[\pi_{\theta_{\text{old}}}(\cdot \mid s_t), \pi_\theta(\cdot \mid s_t)]\right]$$

Practical consideration & caveats of vanilla LLMs

Strengths.

- Great at imitation or idea generation
- Amazing at generating or debugging code

Practical consideration & caveats of vanilla LLMs

Strengths.

- Great at imitation or idea generation
- Amazing at generating or debugging code

Weaknesses.

- Limited reasoning

Practical consideration & caveats of vanilla LLMs

Strengths.

- Great at imitation or idea generation
- Amazing at generating or debugging code

Weaknesses.

- Limited reasoning
- Knowledge is static

Practical consideration & caveats of vanilla LLMs

Strengths.

- Great at imitation or idea generation
- Amazing at generating or debugging code

Weaknesses.

- Limited reasoning
- Knowledge is static
- Cannot perform actions

Practical consideration & caveats of vanilla LLMs

Strengths.

- Great at imitation or idea generation
- Amazing at generating or debugging code

Weaknesses.

- Limited reasoning
- Knowledge is static
- Cannot perform actions
- Hard to evaluate

Practical consideration & caveats of vanilla LLMs

Strengths.

- Great at imitation or idea generation
- Amazing at generating or debugging code

Weaknesses.

- Limited reasoning
- Knowledge is static
- Cannot perform actions ← Focus of lectures 7 & 8
- Hard to evaluate

Practical consideration & caveats of vanilla LLMs

Strengths.

- Great at imitation or idea generation
- Amazing at generating or debugging code

Weaknesses.

- **Limited reasoning** ← **Focus of today**
- Knowledge is static
- Cannot perform actions
- Hard to evaluate

Transformers & Large Language Models

Reasoning models

Scaling with RL

GRPO

Applications

Terminology

Tentative definition

Reasoning = Ability to **solve** a **problem**

Terminology

Tentative definition

Reasoning = Ability to **solve** a **problem**

Not reasoning	Reasoning
"What is the course code of Stanford's Transformers & LLMs class?"	*"The bear was born in 2020. How old is this bear now?"*

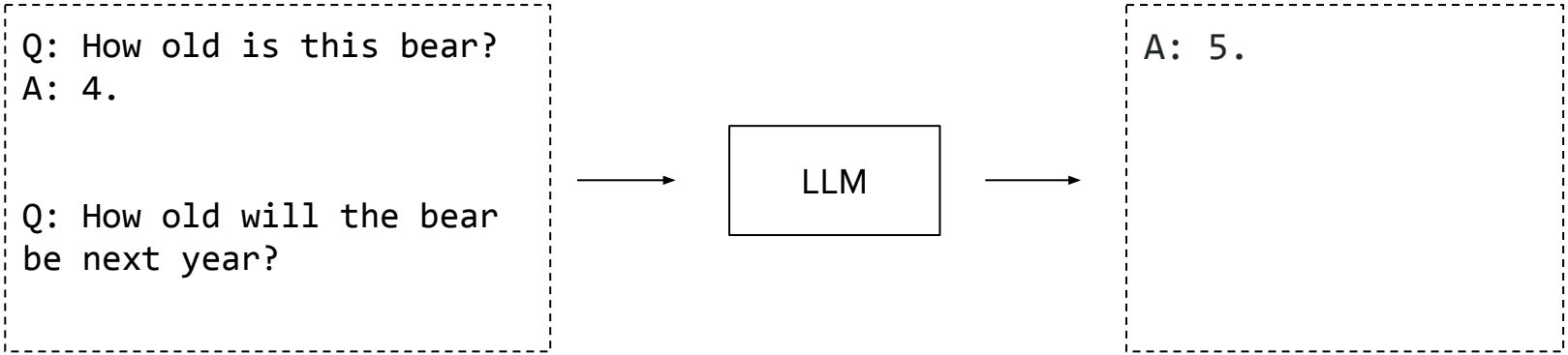
Core idea behind improving reasoning

Strategy. Teach model to explain its reasoning before answering (**C**hain **o**f **T**hought)

"*Chain-of-Thought Prompting Elicits Reasoning in Large Language Models*", *Wei et al., 2022.*

Core idea behind improving reasoning

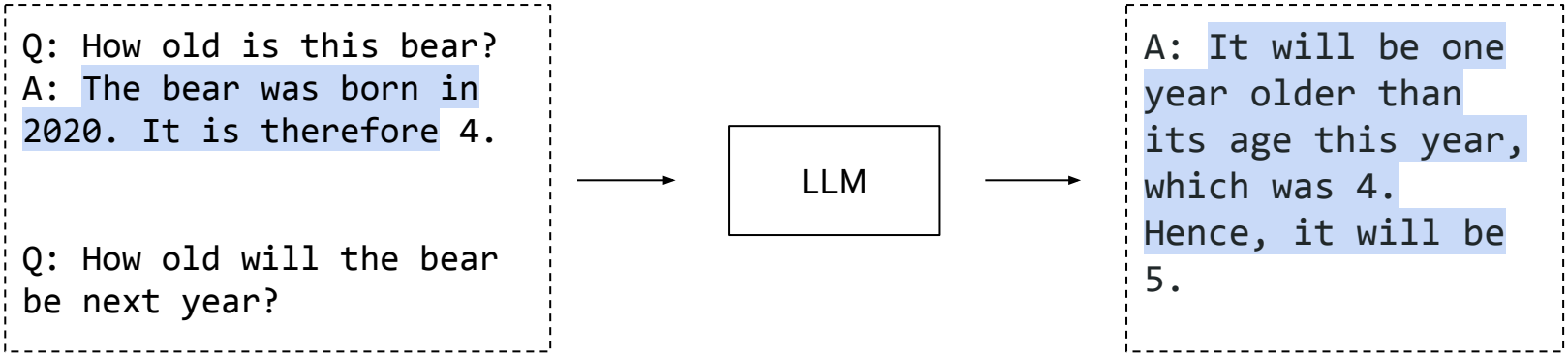
Strategy. Teach model to explain its reasoning before answering (**C**hain **o**f **T**hought)



"*Chain-of-Thought Prompting Elicits Reasoning in Large Language Models*", *Wei et al., 2022.*

Core idea behind improving reasoning

Strategy. Teach model to explain its reasoning before answering (**C**hain **o**f **T**hought)



"*Chain-of-Thought Prompting Elicits Reasoning in Large Language Models*", *Wei et al., 2022.*

Core idea behind improving reasoning

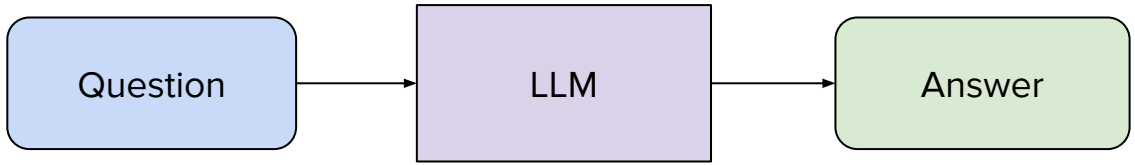
Strategy. Teach model to explain its reasoning before answering (**C**hain **o**f **T**hought)

Idea for reasoning models: Do CoT but at a much larger scale.

"*Chain-of-Thought Prompting Elicits Reasoning in Large Language Models*", *Wei et al., 2022.*

Enhancing model reasoning abilities

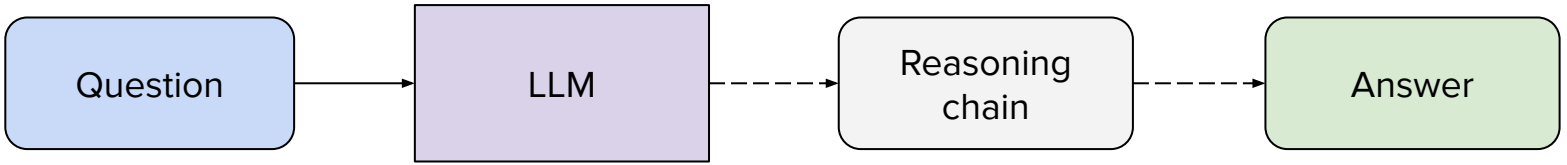
Until now



Enhancing model reasoning abilities

Until now

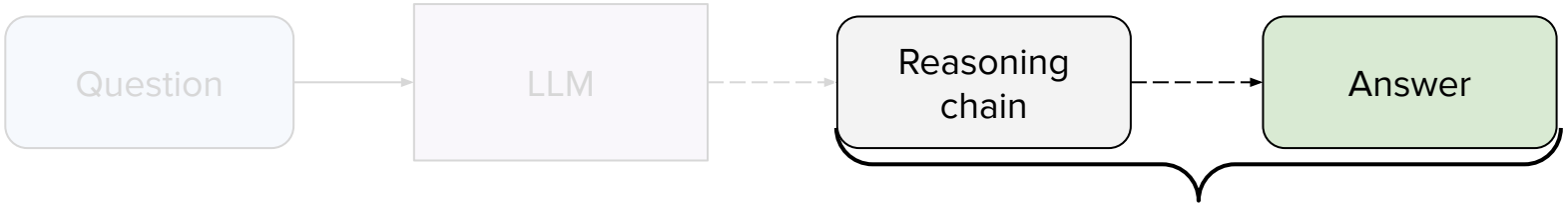
New paradigm



Enhancing model reasoning abilities

Until now

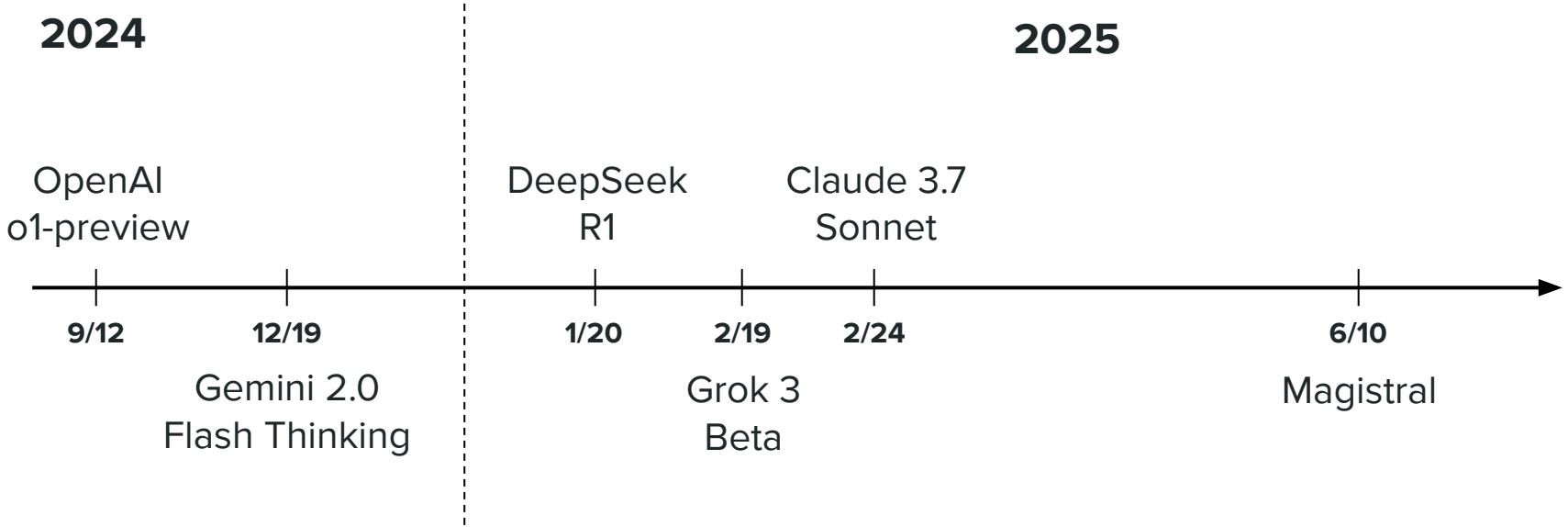
New paradigm



Output = Reasoning + Answer

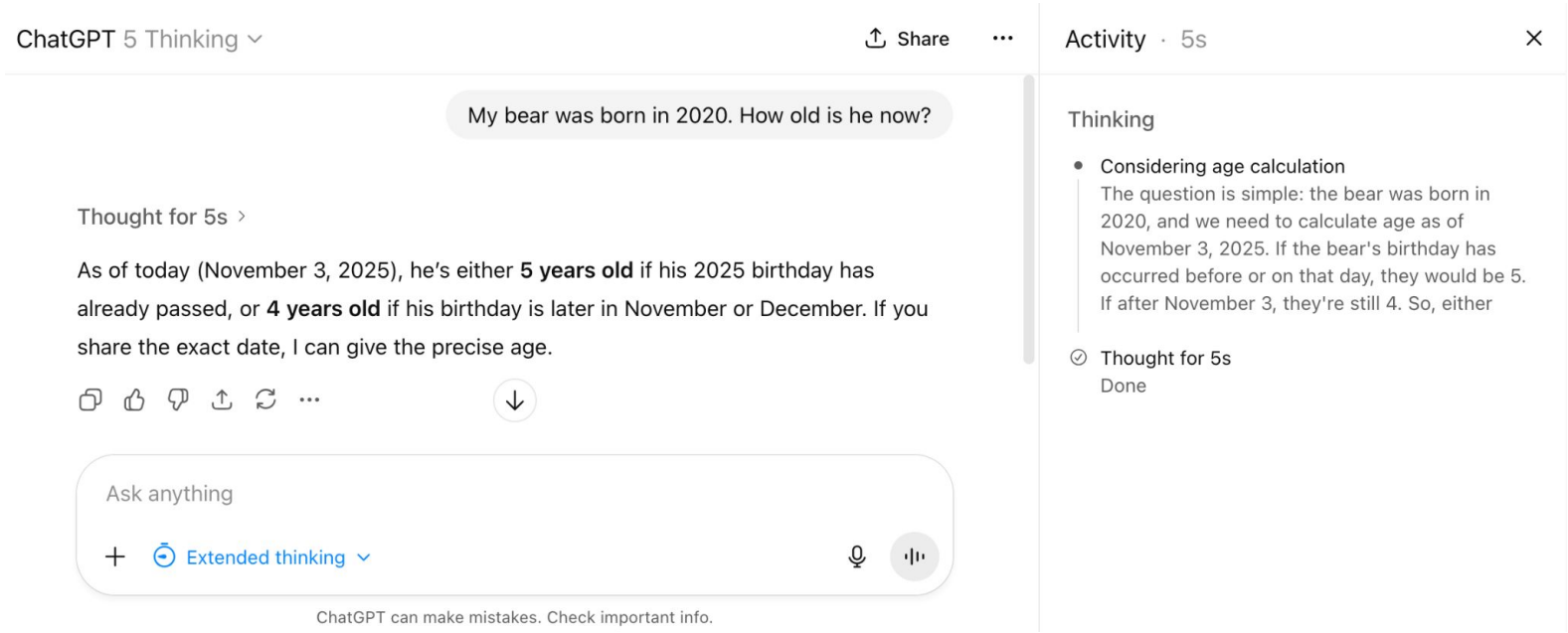
Reasoning model release is trendy

Disclaimer: This timeline is not at scale for better readability. It is also there for illustrative purposes and should not be considered exhaustive. In particular, it shows the first releases of popular AI labs.



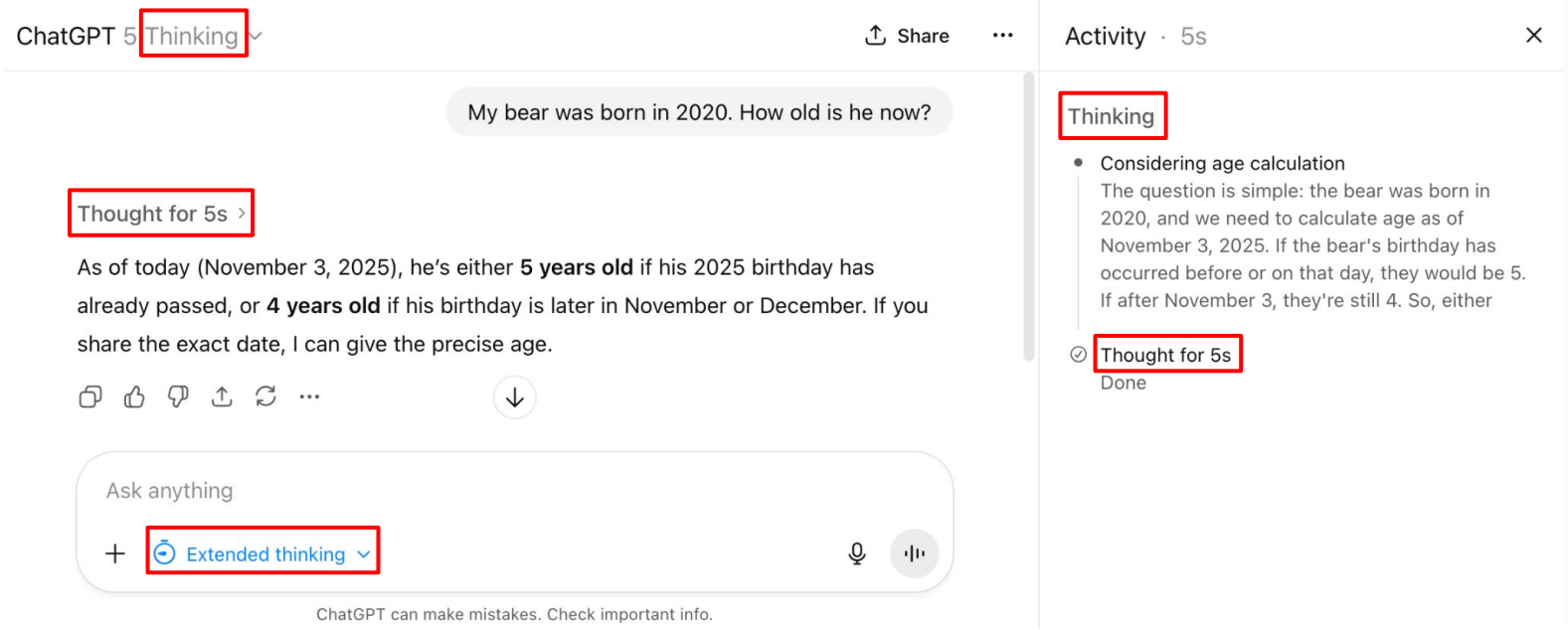
Public model release announcements on 2024-09-12, 2024-12-19, 2025-01-20, 2025-02-19, 2025-02-24, 2025-06-10.

How to spot a reasoning model



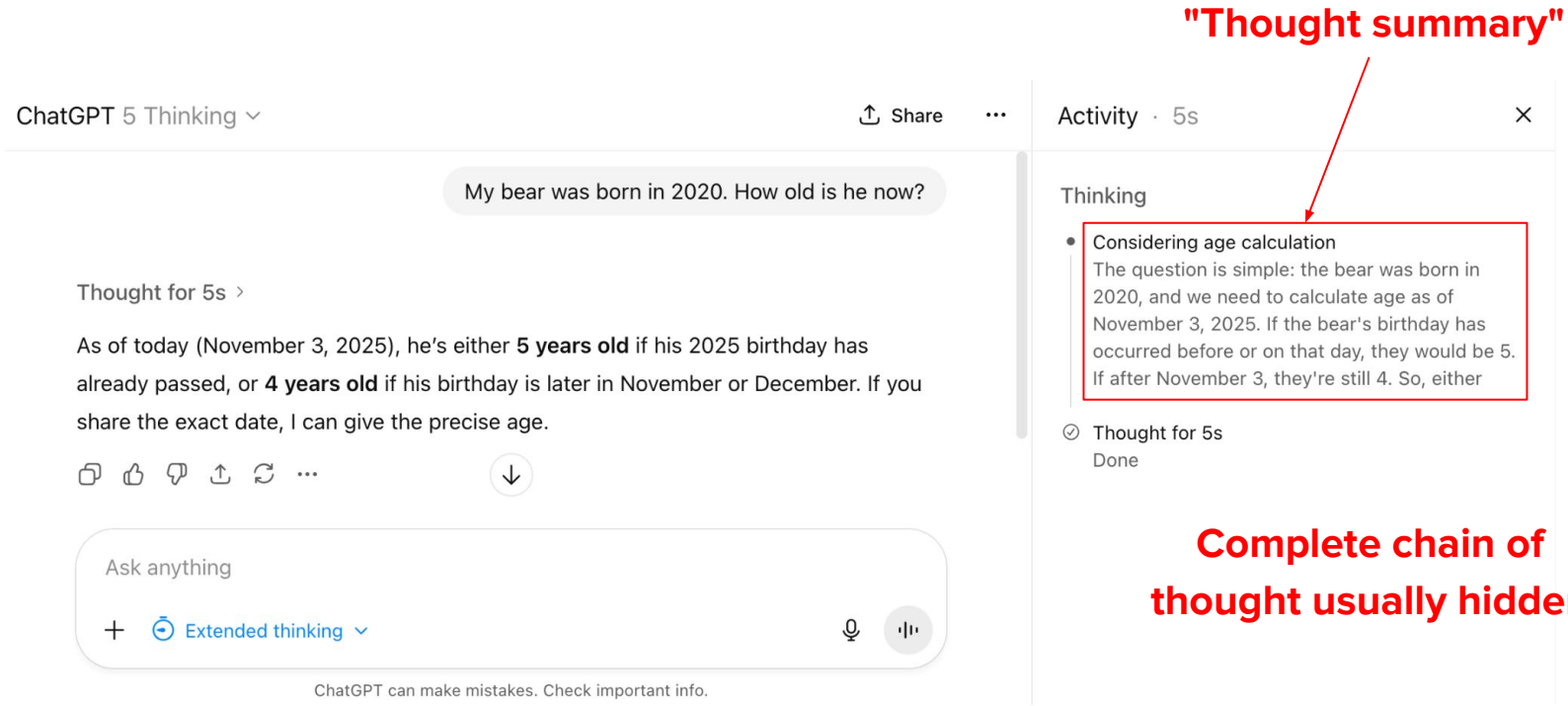
Screenshot of a conversation with ChatGPT 5 Thinking, taken on November 3, 2025.

How to spot a reasoning model



Screenshot of a conversation with ChatGPT 5 Thinking, taken on November 3, 2025.

How to spot a reasoning model



Screenshot of a conversation with ChatGPT 5 Thinking, taken on November 3, 2025.

How to spot a reasoning model

OpenAI

Pricing

Copy page

Text tokens

Prices per 1M tokens.

Batch | Flex | **Standard** | Priority

MODEL	INPUT	CACHED INPUT	OUTPUT
gpt-5	$1.25	$0.125	$10.00
gpt-5-mini	$0.25	$0.025	$2.00
gpt-5-nano	$0.05	$0.005	$0.40
gpt-5-chat-latest	$1.25	$0.125	$10.00
gpt-5-codex	$1.25	$0.125	$10.00

While reasoning tokens are not visible via the API, they still occupy space in the model's context window and are billed as output tokens.

Anthropic

- You're charged for the full thinking tokens generated by the original request, not the summary tokens.
- The billed output token count will **not match** the count of tokens you see in the response.
- The first few lines of thinking output are more verbose, providing detailed reasoning that's particularly helpful for prompt engineering purposes.

Google

Gemini 2.5 Pro

`gemini-2.5-pro`

Try it in Google AI Studio

Our state-of-the-art multipurpose model, which excels at coding and complex reasoning tasks.

Standard | Batch

	Free Tier	Paid Tier, per 1M tokens in USD
Input price	Free of charge	$1.25, prompts <= 200k tokens $2.50, prompts > 200k tokens
Output price (including thinking tokens)	Free of charge	$10.00, prompts <= 200k tokens $15.00, prompts > 200k
Context caching price	Not available	$0.125, prompts <= 200k tokens $0.25, prompts > 200k $4.50 / 1,000,000 tokens per hour (storage price)
Grounding with Google Search	Not available	1,500 RPD (free), then $35 / 1,000 grounded prompts
Grounding with Google Maps	Not available	10,000 RPD (free), then $25 / 1,000 grounded prompts
Used to improve our products	Yes	No

OpenAI pricing page, Anthropic developer guide, Google developer guide. Screenshots taken on November 4th, 2025.

How to spot a reasoning model

OpenAI

Pricing

Copy page

Text tokens

Prices per 1M tokens.

Batch | Flex | **Standard** | Priority

MODEL	INPUT	CACHED INPUT	OUTPUT
gpt-5	$1.25	$0.125	$10.00
gpt-5-mini	$0.25	$0.025	$2.00
gpt-5-nano	$0.05	$0.005	$0.40
gpt-5-chat-latest	$1.25	$0.125	$10.00
gpt-5-codex	$1.25	$0.125	$10.00

While reasoning tokens are not visible via the API, they still occupy space in the model's context window and are billed as output tokens.

Anthropic

- You're charged for the full thinking tokens generated by the original request, not the summary tokens.
- The billed output token count will **not match** the count of tokens you see in the response.
- The first few lines of thinking output are more verbose, providing detailed reasoning that's particularly helpful for prompt engineering purposes.

Google

Gemini 2.5 Pro

`gemini-2.5-pro`

Try it in Google AI Studio

Our state-of-the-art multipurpose model, which excels at coding and complex reasoning tasks.

Standard | Batch

	Free Tier	Paid Tier, per 1M tokens in USD
Input price	Free of charge	$1.25, prompts <= 200k tokens $2.50, prompts > 200k tokens
Output price (including thinking tokens)	Free of charge	$10.00, prompts <= 200k tokens $15.00, prompts > 200k
Context caching price	Not available	$0.125, prompts <= 200k tokens $0.25, prompts > 200k $4.50 / 1,000,000 tokens per hour (storage price)
Grounding with Google Search	Not available	1,500 RPD (free), then $35 / 1,000 grounded prompts
Grounding with Google Maps	Not available	10,000 RPD (free), then $25 / 1,000 grounded prompts
Used to improve our products	Yes	No

Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

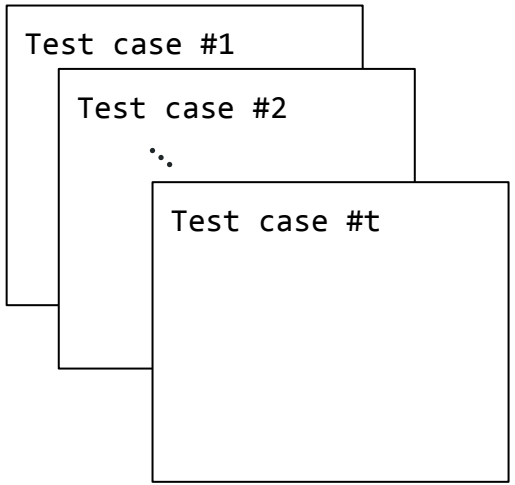
Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

You have n teddy bears in a line. Each bear has a size.

Find the biggest bear that is smaller than the largest bear.

```
def second_biggest_bear(bears):
    largest = max(bears)
    return max(
      b for b in bears
      if b < largest
    )
```



Problem **Solution** **Verification**

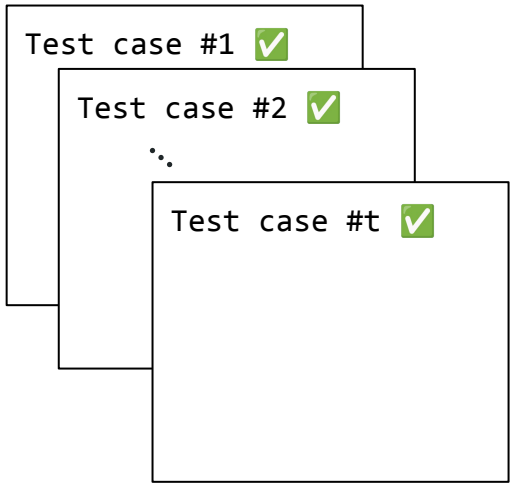
Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

You have n teddy bears in a line. Each bear has a size.

Find the biggest bear that is smaller than the largest bear.

```
def second_biggest_bear(bears):
    largest = max(bears)
    return max(
      b for b in bears
      if b < largest
    )
```



Problem **Solution** **Verification**

Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

task_id string · *lengths* 11↔12 6.1%	prompt string · *lengths* 240↔365 24.4%	canonical_solution string · *lengths* 186↔271 18.9%	test string · *lengths* 455↔624 18.9%

HumanEval/0	from typing import List
	
	def has_close_elements(numbers: List[float],
	threshold: float) -> bool:
	""" Check if in given list of numbers, are any two
	numbers closer to each other than
	given threshold.
	>>> has_close_elements([1.0, 2.0, 3.0], 0.5)
	False
	>>> has_close_elements([1.0, 2.8, 3.0, 4.0, 5.0,
	2.0], 0.3)
	True
	"""
		for idx, elem in enumerate(numbers):
		for idx2, elem2 in enumerate(numbers):
		if idx != idx2:
		distance = abs(elem - elem2)
		if distance < threshold:
		return True
		
		return False
			
			METADATA = {
			'author': 'jt',
			'dataset': 'test'
			}
			
			def check(candidate):
			assert candidate([1.0, 2.0, 3.9, 4.0, 5.0, 2.2], 0.3) == True
			assert candidate([1.0, 2.0, 3.9, 4.0, 5.0, 2.2], 0.05) == Fals
			assert candidate([1.0, 2.0, 5.9, 4.0, 5.0], 0.95) == True
			assert candidate([1.0, 2.0, 5.9, 4.0, 5.0], 0.8) == False
			assert candidate([1.0, 2.0, 3.0, 4.0, 5.0, 2.0], 0.1) == True
			assert candidate([1.1, 2.2, 3.1, 4.1, 5.1], 1.0) == True
			assert candidate([1.1, 2.2, 3.1, 4.1, 5.1], 0.5) == False

Examples: HumanEval, CodeForces, SWE-bench

Screenshot of the HumanEval dataset in HuggingFace Dataset, taken on November 3rd, 2025.

Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

Math. Solve a challenging math problem (e.g. olympiads)

Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

Math. Solve a challenging math problem (e.g. olympiads)

The bear was born in 2020. How old is the bear now?

```
It is 2025 now. Subtract the
birth year from the current
year: 2025-2020=5.

Answer: 5
```

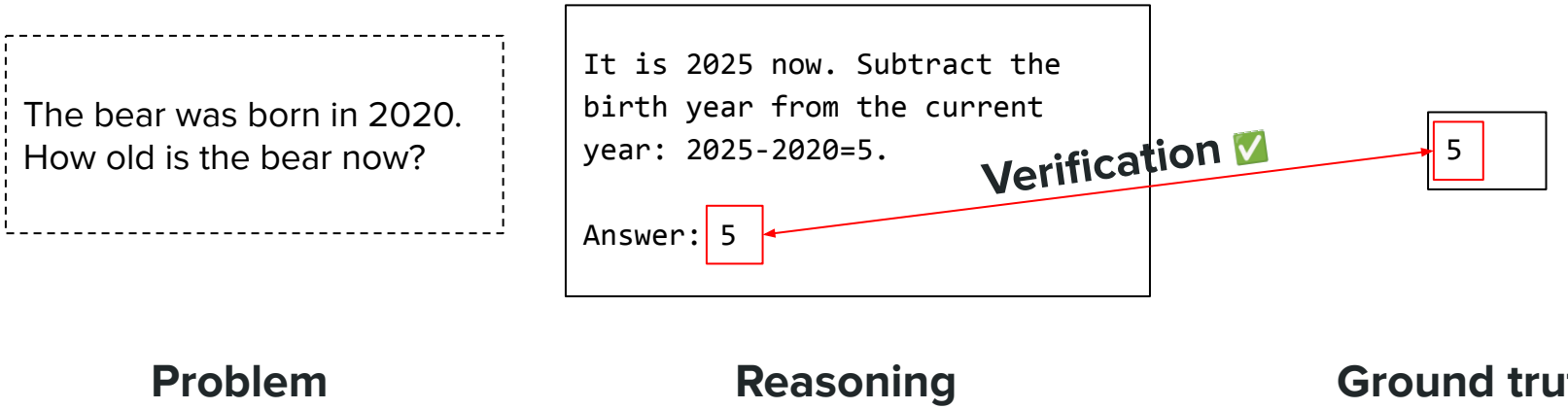
5

Problem **Reasoning** **Ground truth**

Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

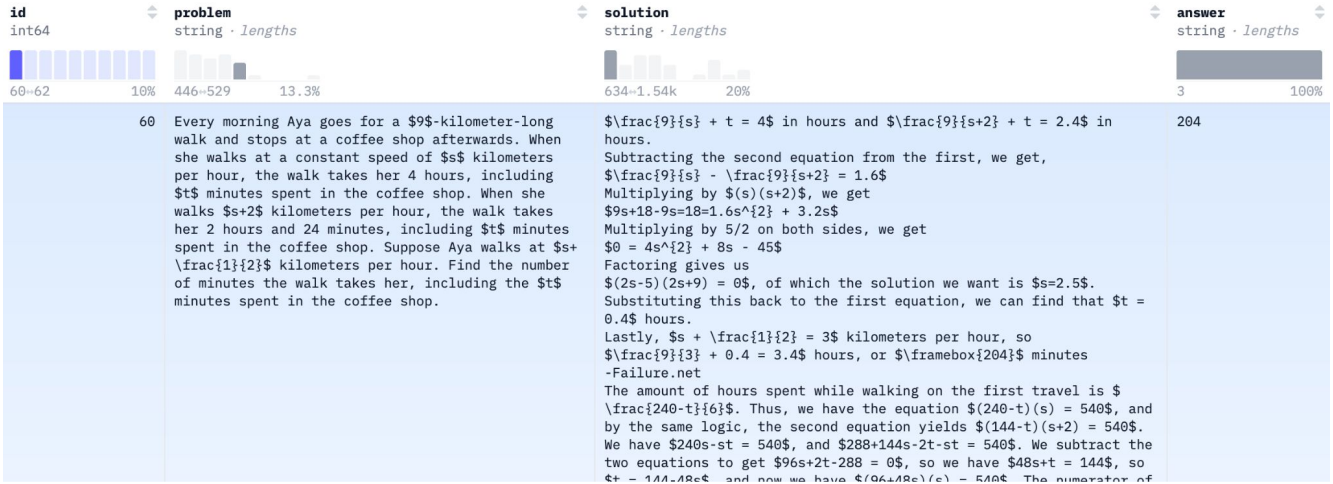
Math. Solve a challenging math problem (e.g. olympiads)



Reasoning-based benchmarks

Coding. Solve a coding problem, fix a bug.

Math. Solve a challenging math problem (e.g. olympiads)



id int64 60–62 10%	problem string · *lengths* 446–529 13.3%	solution string · *lengths* 634–1.54k 20%	answer string · *lengths* 3 100%
60	Every morning Aya goes for a 9-kilometer-long walk and stops at a coffee shop afterwards. When she walks at a constant speed of s kilometers per hour, the walk takes her 4 hours, including t minutes spent in the coffee shop. When she walks $s+2$ kilometers per hour, the walk takes her 2 hours and 24 minutes, including t minutes spent in the coffee shop. Suppose Aya walks at $s+\frac{1}{2}$ kilometers per hour. Find the number of minutes the walk takes her, including the t minutes spent in the coffee shop.	$\frac{9}{s} + t = 4$ in hours and $\frac{9}{s+2} + t = 2.4$ in hours. Subtracting the second equation from the first, we get, $\frac{9}{s} - \frac{9}{s+2} = 1.6$ Multiplying by $(s)(s+2)$, we get $9s+18-9s=18=1.6s^{2} + 3.2s$ Multiplying by 5/2 on both sides, we get $0 = 4s^{2} + 8s - 45$ Factoring gives us $(2s-5)(2s+9) = 0$, of which the solution we want is $s=2.5$. Substituting this back to the first equation, we can find that $t = 0.4$ hours. Lastly, $s + \frac{1}{2} = 3$ kilometers per hour, so $\frac{9}{3} + 0.4 = 3.4$ hours, or $\framebox{204}$ minutes -Failure.net The amount of hours spent while walking on the first travel is $\frac{240-t}{6}$. Thus, we have the equation $(240-t)(s) = 540$, and by the same logic, the second equation yields $(144-t)(s+2) = 540$. We have $240s-st = 540$, and $288+144s-2t-st = 540$. We subtract the two equations to get $96s+2t-288 = 0$, so we have $48s+t = 144$, so $t = 144-48s$, and now we have $(96+48s)(s) = 540$. The numerator of	204

Examples: AIME, GSM8K

Screenshot of the AIME dataset in HuggingFace Dataset, taken on November 3rd, 2025.

Reasoning-based benchmark metrics

Pass@k = "Probability that at least 1 of k attempts succeeds"

"Evaluating Large Language Models Trained on Code", Chen et al., 2021.

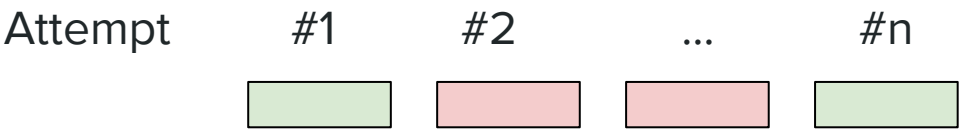
Reasoning-based benchmark metrics

Pass@k = "Probability that at least 1 of k attempts succeeds"

"Evaluating Large Language Models Trained on Code", Chen et al., 2021.

Reasoning-based benchmark metrics

Pass@k = "Probability that at least 1 of k attempts succeeds"



$$n = c + (n - c)$$

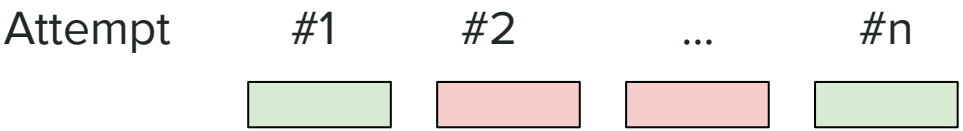
Successful

Unsuccessful

"*Evaluating Large Language Models Trained on Code*", *Chen et al., 2021.*

Reasoning-based benchmark metrics

Pass@k = "Probability that at least 1 of k attempts succeeds"



$$n = c + (n - c)$$

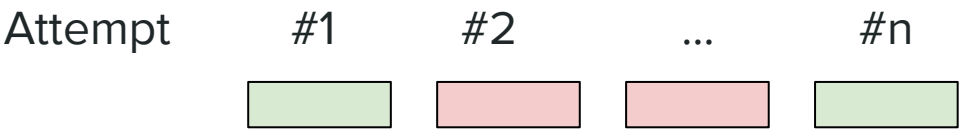
Successful Unsuccessful

$$\text{Pass@}k = \quad ?$$

"*Evaluating Large Language Models Trained on Code*", Chen et al., 2021.

Reasoning-based benchmark metrics

Pass@k = "Probability that at least 1 of k attempts succeeds"



$$n = c + (n - c)$$

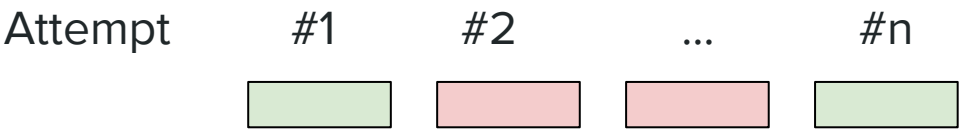
Successful Unsuccessful

$$\text{Pass@}k = 1 - \frac{\binom{n-c}{k}}{\binom{n}{k}}$$

"*Evaluating Large Language Models Trained on Code*", Chen et al., 2021.

Reasoning-based benchmark metrics

Pass@k = "Probability that at least 1 of k attempts succeeds"



$$n = c + (n - c)$$

Successful

Unsuccessful

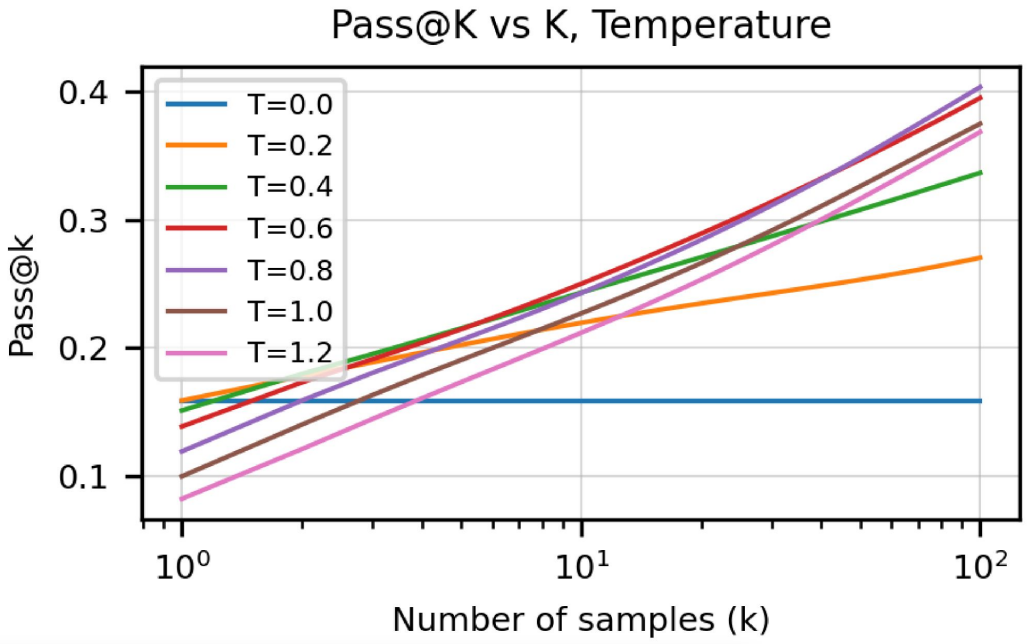
Special case

$$\text{Pass@1} = \frac{c}{n}$$

"*Evaluating Large Language Models Trained on Code*", Chen et al., 2021.

Reasoning-based benchmark metrics

Pass@k = "Probability that at least 1 of k attempts succeeds"



"Evaluating Large Language Models Trained on Code", Chen et al., 2021.

Reasoning-based benchmark metrics

- **Pass@k**. For use cases where checking is easy / can afford higher latency
- **Pass@1**. For use cases where we care about a single generation

"*Evaluating Large Language Models Trained on Code*", *Chen et al., 2021.*

Reasoning-based benchmark metrics

- **Pass@k**. For use cases where checking is easy / can afford higher latency
- **Pass@1**. For use cases where we care about a single generation
- **Cons@k**. "Consensus at k", equivalent to comparing the answer from **majority voting** with the ground truth

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

Transformers & Large Language Models

Reasoning models

Scaling with RL

GRPO

Applications

Develop "test-time scaling"

Idea. Incentivize model to reason before answering.

Develop "test-time scaling"

Idea. Incentivize model to reason before answering.

Considerations.

- Reasoning chain is hard to write from scratch (**SFT data by hand impractical**)

Develop "test-time scaling"

Idea. Incentivize model to reason before answering.

Considerations.

- Reasoning chain is hard to write from scratch (**SFT data by hand impractical**)
- Don't want to limit the model to human-written reasoning

Develop "test-time scaling"

Idea. Incentivize model to reason before answering.

Considerations.

- Reasoning chain is hard to write from scratch (**SFT data by hand impractical**)
- Don't want to limit the model to human-written reasoning
- Natural verifiable reward ("did it solve the problem?" → "yes" or "no")

Develop "test-time scaling"

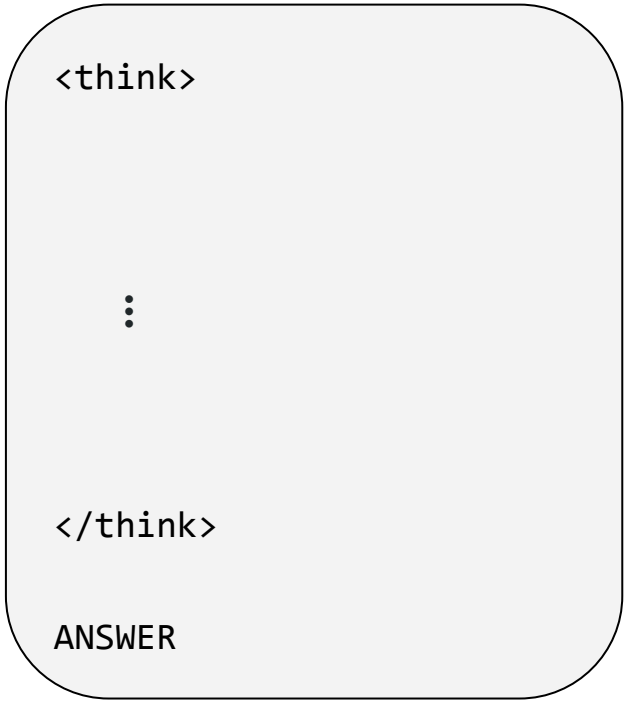
Idea. Incentivize model to reason before answering.

Considerations.

- Reasoning chain is hard to write from scratch (**SFT data by hand impractical**)
- Don't want to limit the model to human-written reasoning
- Natural verifiable reward ("did it solve the problem?" → "yes" or "no")

Let's try RL!

Reward 1: verify that CoT is there



Template

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

Reward 1: verify that CoT is there

```
<think>

   ⋮

</think>

ANSWER
```

Template

Question: If $a > 1$, then the sum of the real solutions of $\sqrt{a - \sqrt{a+x}} = x$ is equal to

Response: <think>

To solve the equation $\sqrt{a - \sqrt{a+x}} = x$, let's start by squaring both $\cdots$

$\left(\sqrt{a - \sqrt{a+x}}\right)^2 = x^2 \implies a - \sqrt{a+x} = x^2.$

Rearrange to isolate the inner square root term:

$(a - x^2)^2 = a + x \implies a^2 - 2ax^2 + (x^2)^2 = a + x \implies x^4 - 2ax^2 - x + (a^2 - a) = 0$

$\ldots$

Wait, wait. Wait. That's an aha moment I can flag here.

Let's reevaluate this step-by-step to identify if the correct sum can be $\cdots$

We started with the equation:

$\sqrt{a - \sqrt{a+x}} = x$

First, let's square both sides:

$a - \sqrt{a+x} = x^2 \implies \sqrt{a+x} = a - x^2$

Next, I could square both sides again, treating the equation: $\cdots$

$\ldots$

Sample response

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

Reward 1: verify that CoT is there

<think>

⋮

</think>

ANSWER

Template

Question: If $a > 1$, then the sum of the real solutions of $\sqrt{a - \sqrt{a+x}} = x$ is equal to

Response: <think>

To solve the equation $\sqrt{a - \sqrt{a+x}} = x$, let's start by squaring both ···

$\left(\sqrt{a - \sqrt{a+x}}\right)^2 = x^2 \implies a - \sqrt{a+x} = x^2.$

Rearrange to isolate the inner square root term:

$(a - x^2)^2 = a + x \implies a^2 - 2ax^2 + (x^2)^2 = a + x \implies x^4 - 2ax^2 - x + (a^2 - a) = 0$

...

Wait, wait. Wait. That's an aha moment I can flag here.

Let's reevaluate this step-by-step to identify if the correct sum can be ···

We started with the equation:

$\sqrt{a - \sqrt{a+x}} = x$

First, let's square both sides:

$a - \sqrt{a+x} = x^2 \implies \sqrt{a+x} = a - x^2$

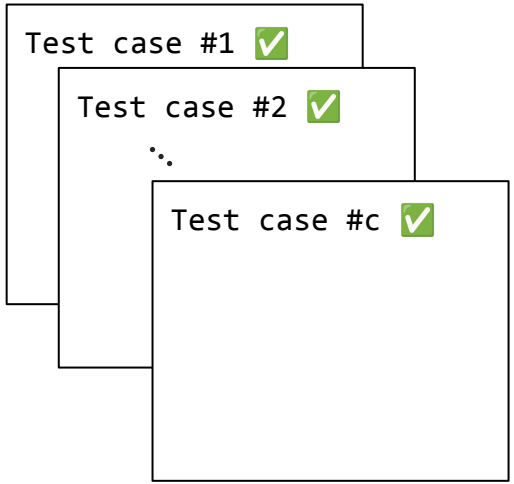
Next, I could square both sides again, treating the equation: ···

...

Sample response

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

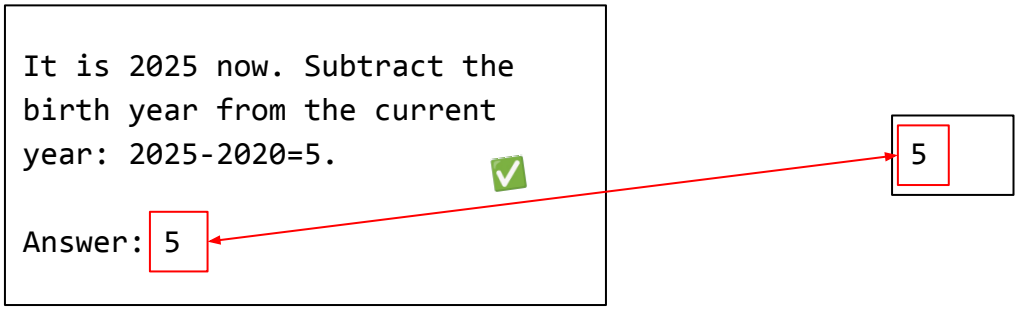
Reward 2: verify that solution is correct



Code verification

Reward 2: verify that solution is correct

Code verification



Math verification

Run RL on "verifiable" rewards

1. **formatting**
(`think` delimiters?)

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

Run RL on "verifiable" rewards

1. **formatting** (`think` delimiters?)

2. **accuracy** (correct solution?)

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

Run RL on "verifiable" rewards

Rewards

||

1 **formatting** (`think` delimiters?)

+

2 **accuracy** (correct solution?)

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

Run RL on "verifiable" rewards

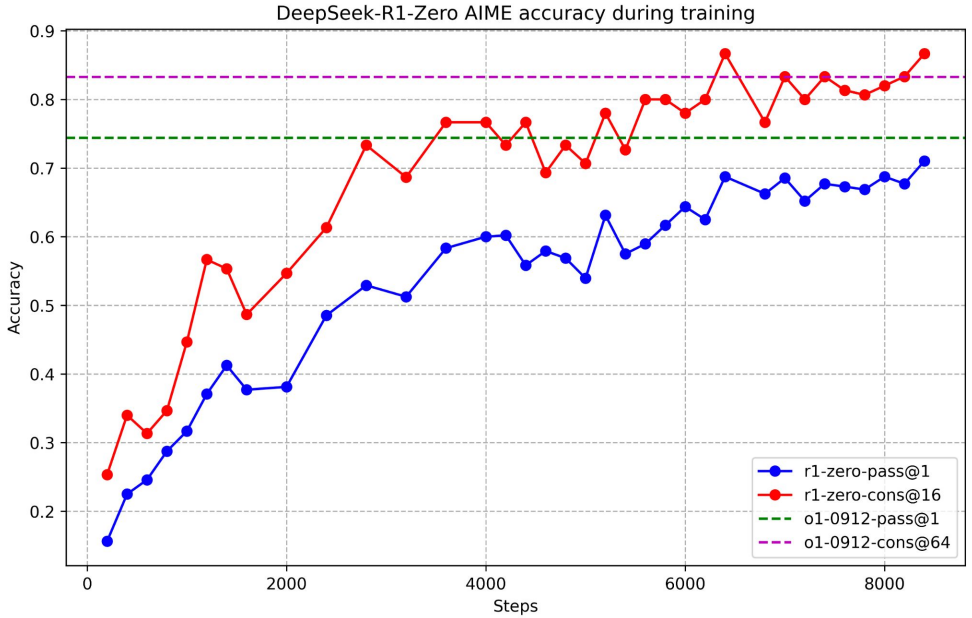
Rewards

||

① **formatting** (`think` delimiters?)

+

② **accuracy** (correct solution?)



"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

Control thinking at inference time

Problem. Not all prompts are equal

Control thinking at inference time

Problem. Not all prompts are equal

Ideas to control "thinking".

- Dynamic budget

Control thinking at inference time

Problem. Not all prompts are equal

Ideas to control "thinking".

- Dynamic budget
- Context awareness

"*Token-Budget-Aware LLM Reasoning*", Han et al., 2024.

Control thinking at inference time

Problem. Not all prompts are equal

Ideas to control "thinking".

- Dynamic budget
- Context awareness
- Budget forcing

"s1: Simple test-time scaling", Muennighoff et al., 2025.

Control thinking at inference time

Problem. Not all prompts are equal

Ideas to control "thinking".

- Dynamic budget
- Context awareness
- Budget forcing
- "Continuous" thoughts

"Training Large Language Models to Reason in a Continuous Latent Space", Hao et al., 2024.

Transformers & Large Language Models

Reasoning models

Scaling with RL

GRPO

Applications

Common RL algorithm for reasoning

GRPO = **G**roup **R**elative **P**olicy **O**ptimization

"*DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models*", *Shao et al., 2024.*

Common RL algorithm for reasoning

GRPO = **G**roup **R**elative **P**olicy **O**ptimization

$$\mathcal{L}(\theta) = \boxed{\text{Maximize } \textbf{advantages}} + \boxed{\text{Don't deviate too much from old / base model}}$$

"*DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models*", *Shao et al., 2024.*

Common RL algorithm for reasoning

GRPO = **G**roup **R**elative **P**olicy **O**ptimization

$$\mathcal{L}(\theta) =$$ Maximize **advantages** $+$ Don't deviate too much from old / base model

Advantage ~ Reward - Avg(reward of group)

"*DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models*", Shao et al., 2024.

Common RL algorithm for reasoning

GRPO = **G**roup **R**elative **P**olicy **O**ptimization

$\mathcal{L}(\theta) =$ Maximize **advantages** $+$ Don't deviate too much from old / base model

Advantage ~ Reward - **Avg(reward of group)**

Big difference compared to PPO!

"*DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models*", *Shao et al., 2024.*

Comparison between GRPO and PPO

GRPO q

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

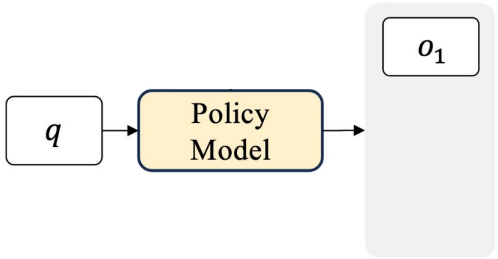


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

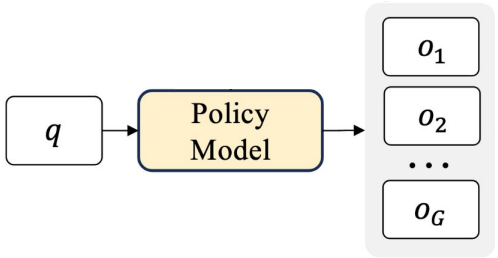


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

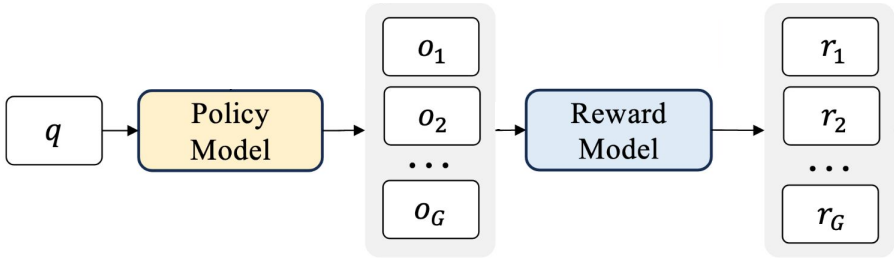
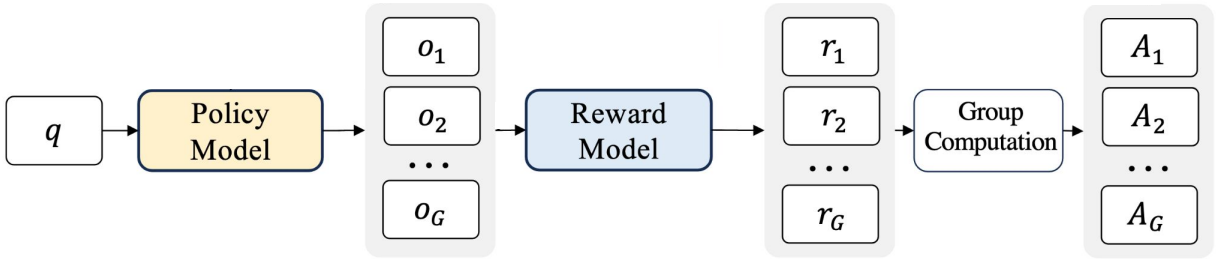


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO



$$A_i = \frac{r_i - \text{mean}(\{r_1, r_2, \cdots, r_G\})}{\text{std}(\{r_1, r_2, \cdots, r_G\})}$$

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

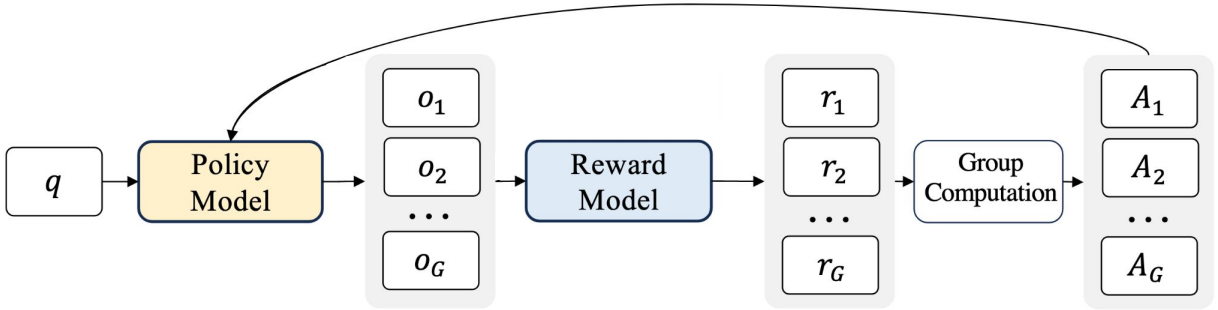


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

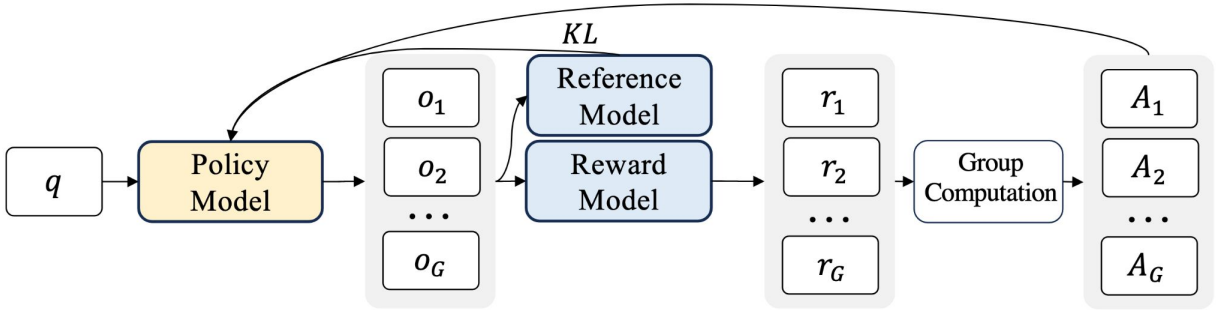


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

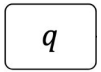


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

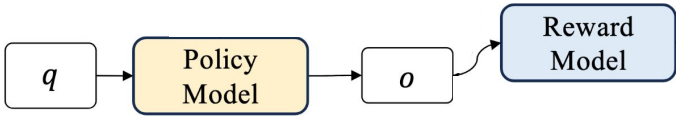


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

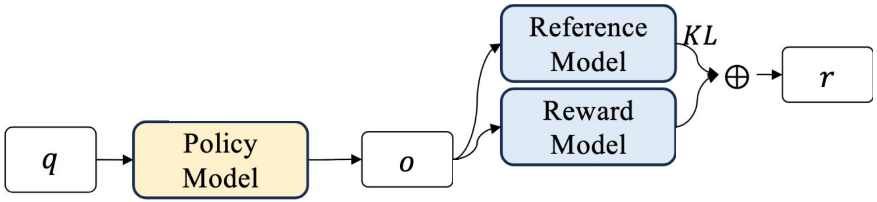


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

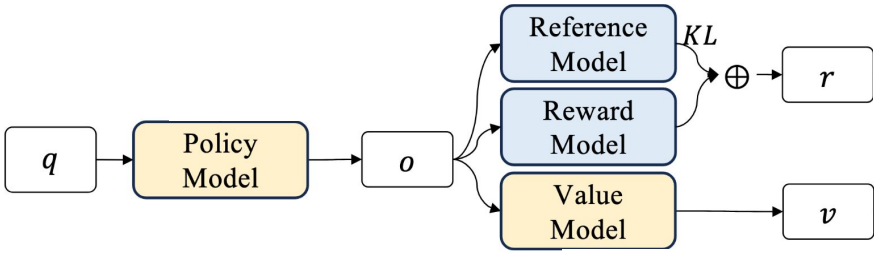


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

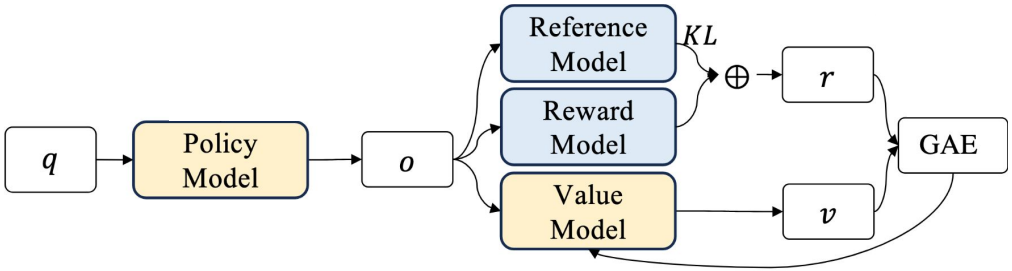


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

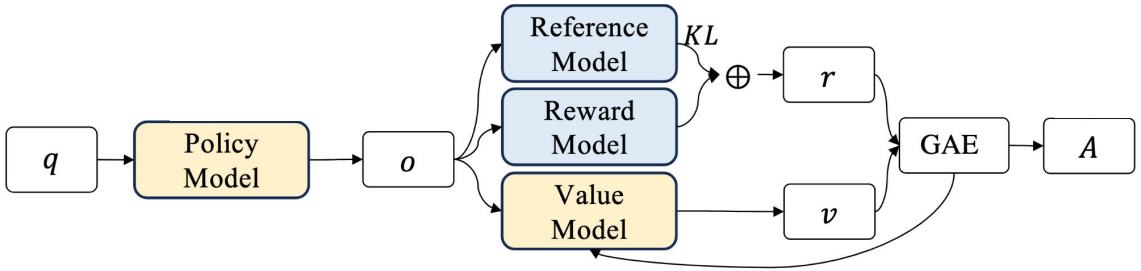


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

PPO

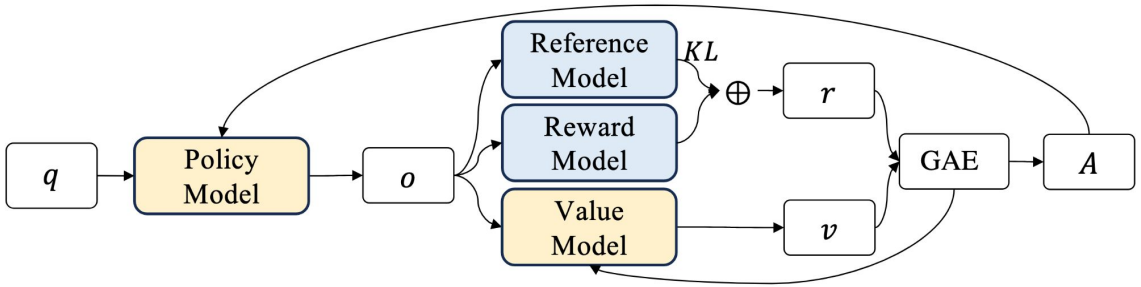


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

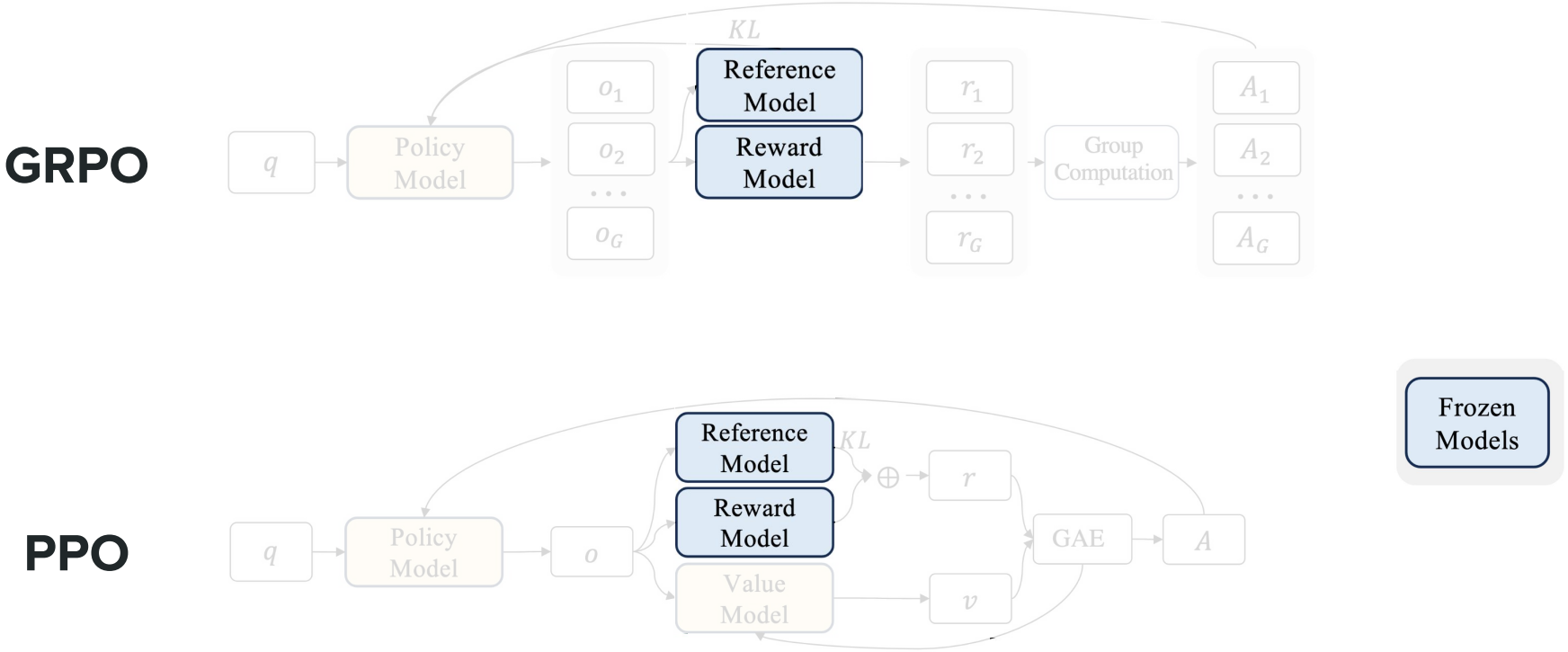


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

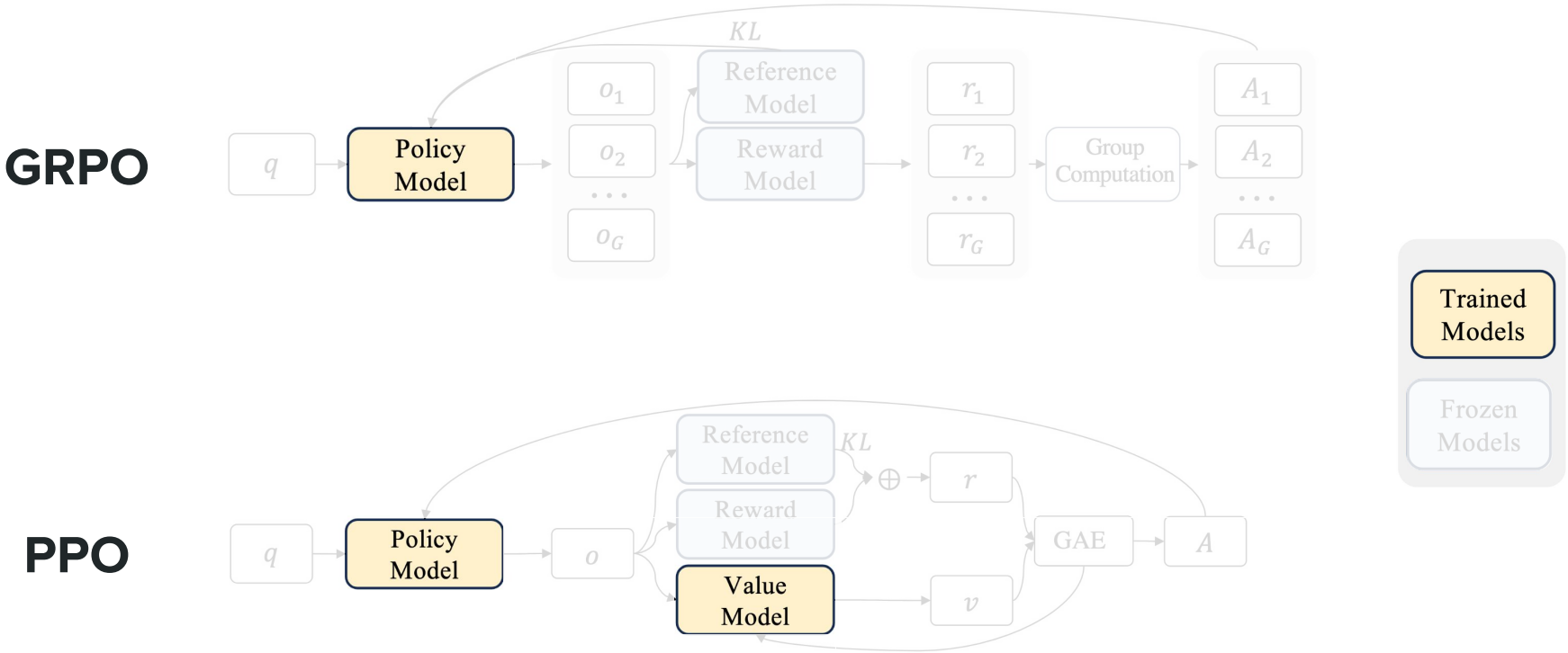


Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

GRPO

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

PPO

$$\mathcal{J}_{PPO}(\theta) = \mathbb{E}[q \sim P(Q), o \sim \pi_{\theta_{old}}(O|q)]\frac{1}{|o|}\sum_{t=1}^{|o|}\min\left[\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}A_t, \text{clip}\left(\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}, 1-\varepsilon, 1+\varepsilon\right)A_t\right]$$

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

Similarities. Ratio

GRPO

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

PPO

$$\mathcal{J}_{PPO}(\theta) = \mathbb{E}[q \sim P(Q), o \sim \pi_{\theta_{old}}(O|q)]\frac{1}{|o|}\sum_{t=1}^{|o|}\min\left[\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}A_t, \text{clip}\left(\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}, 1-\varepsilon, 1+\varepsilon\right)A_t\right]$$

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

Similarities. Ratio, clipping

GRPO

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

PPO

$$\mathcal{J}_{PPO}(\theta) = \mathbb{E}[q \sim P(Q), o \sim \pi_{\theta_{old}}(O|q)]\frac{1}{|o|}\sum_{t=1}^{|o|}\min\left[\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}A_t, \text{clip}\left(\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}, 1-\varepsilon, 1+\varepsilon\right)A_t\right]$$

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

Differences. KL penalty

GRPO

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

PPO

$$\mathcal{J}_{PPO}(\theta) = \mathbb{E}[q \sim P(Q), o \sim \pi_{\theta_{old}}(O|q)]\frac{1}{|o|}\sum_{t=1}^{|o|}\min\left[\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}A_t, \text{clip}\left(\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}, 1-\varepsilon, 1+\varepsilon\right)A_t\right]$$

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

Comparison between GRPO and PPO

Differences. KL penalty, advantage estimation

GRPO

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

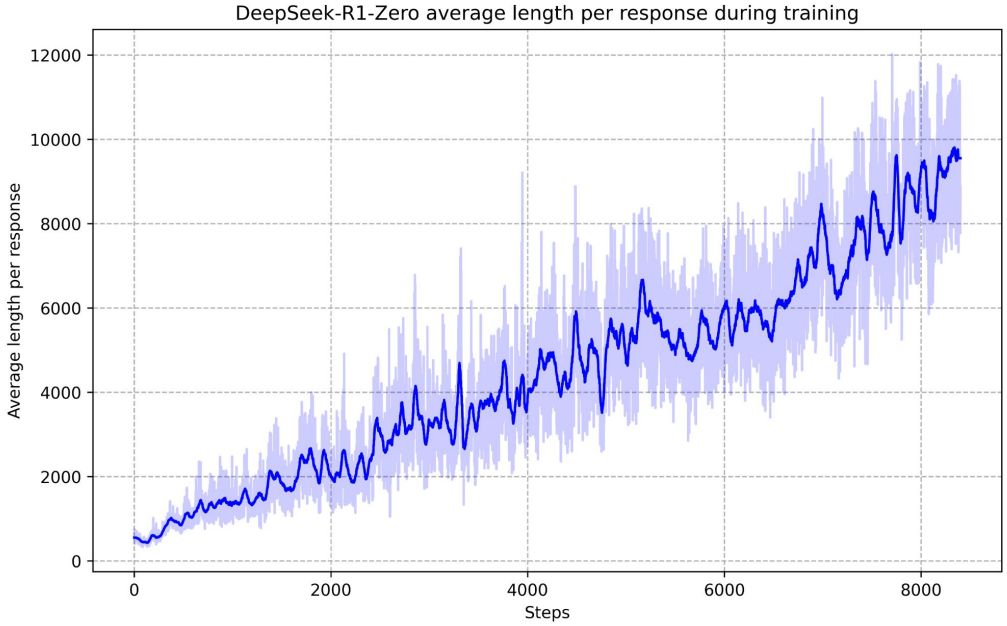
PPO

$$\mathcal{J}_{PPO}(\theta) = \mathbb{E}[q \sim P(Q), o \sim \pi_{\theta_{old}}(O|q)]\frac{1}{|o|}\sum_{t=1}^{|o|}\min\left[\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}A_t, \text{clip}\left(\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}, 1-\varepsilon, 1+\varepsilon\right)A_t\right]$$

Figure from "DeepSeekMath: Pushing the Limits of Mathematical Reasoning in Open Language Models", Shao et al., 2024.

"Increasing output length" phenomenon

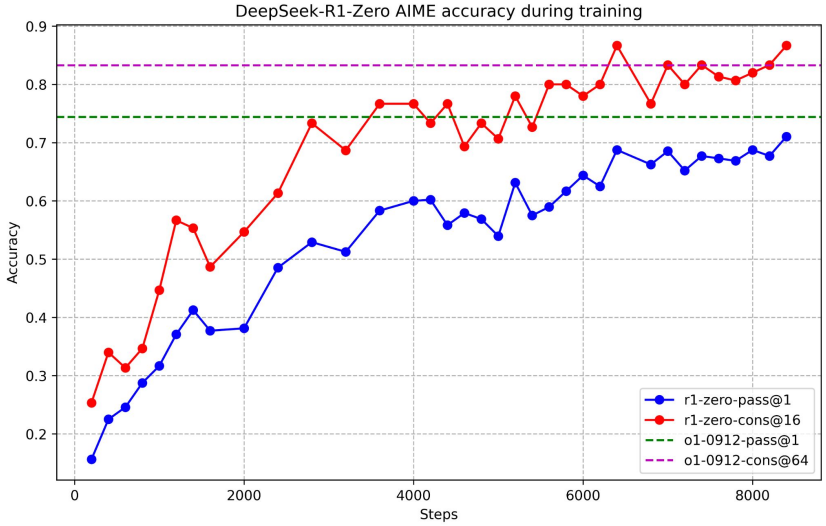
Observation. Response length keeps on increasing with RL training

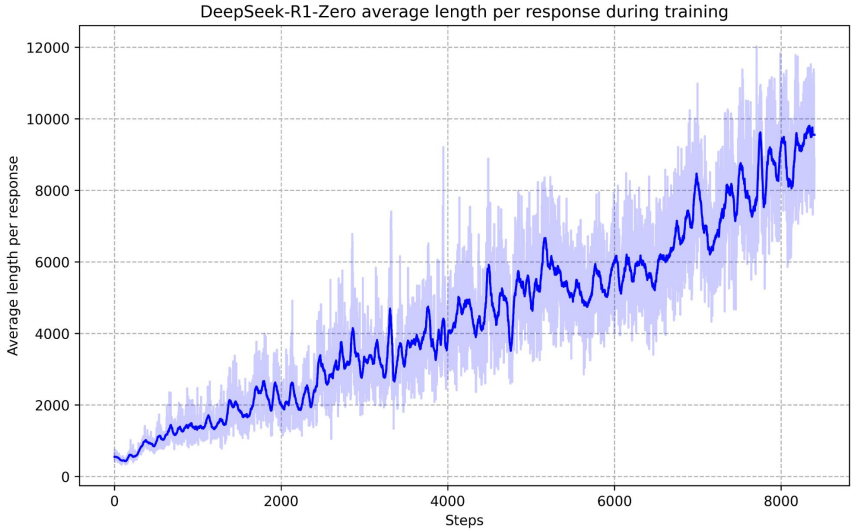


"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

"Increasing output length" phenomenon

Observation. Response length keeps on increasing with RL training

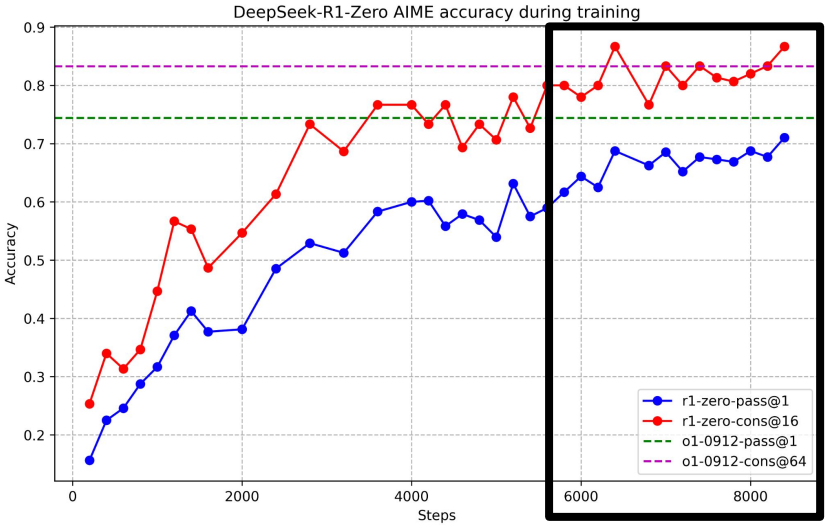


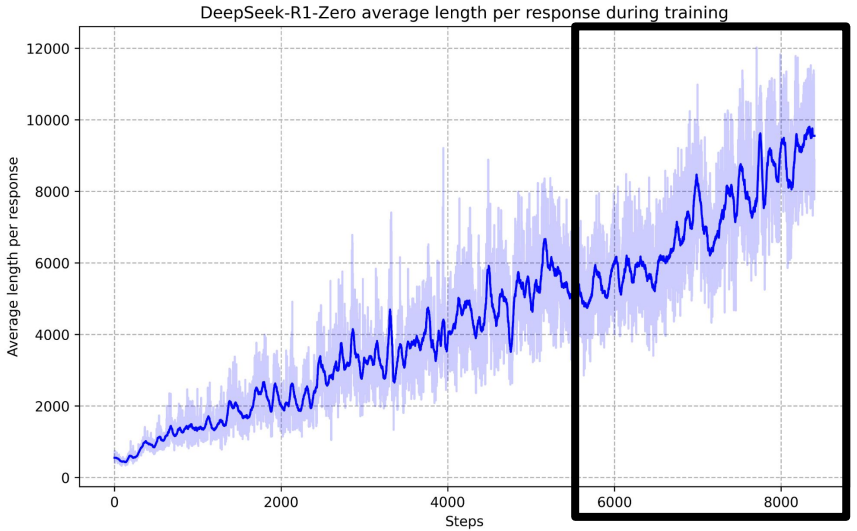


"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

"Increasing output length" phenomenon

Observation. Response length keeps on increasing with RL training





"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}\frac{1}{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

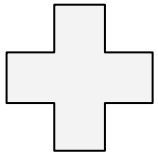
$$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}\frac{1}{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}\frac{1}{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

Small $|o|$



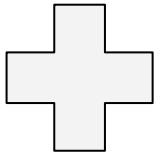
Big $|o|$

"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}\frac{1}{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

Short output

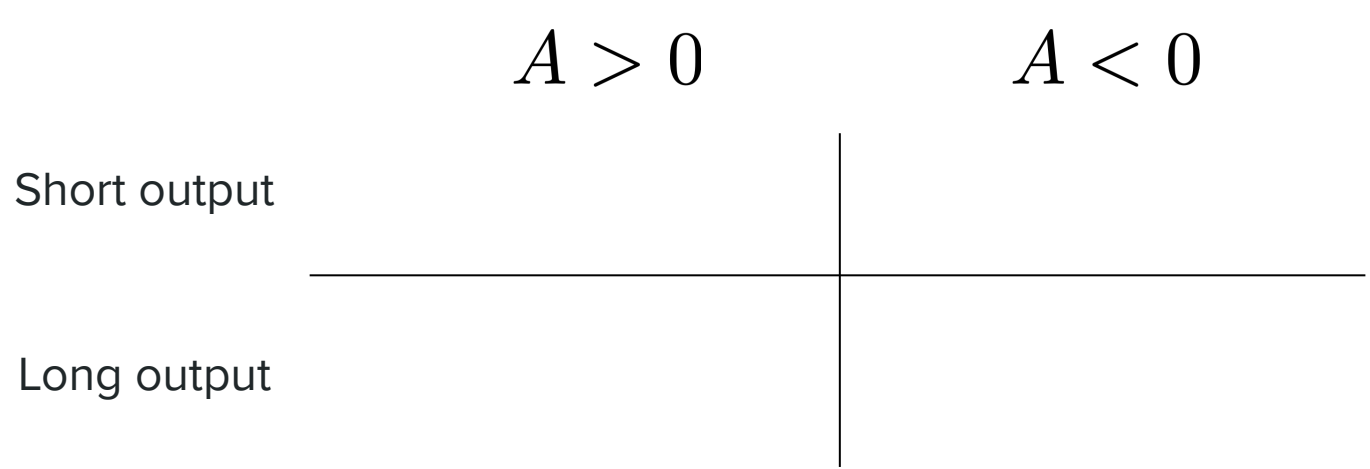


Long output

"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

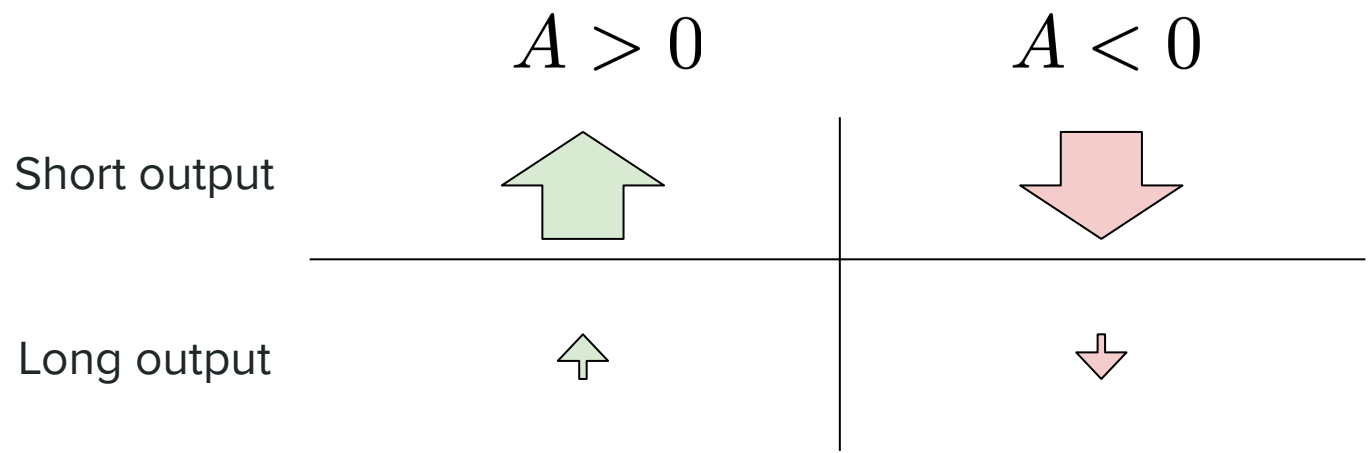
$$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}\frac{1}{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$



"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

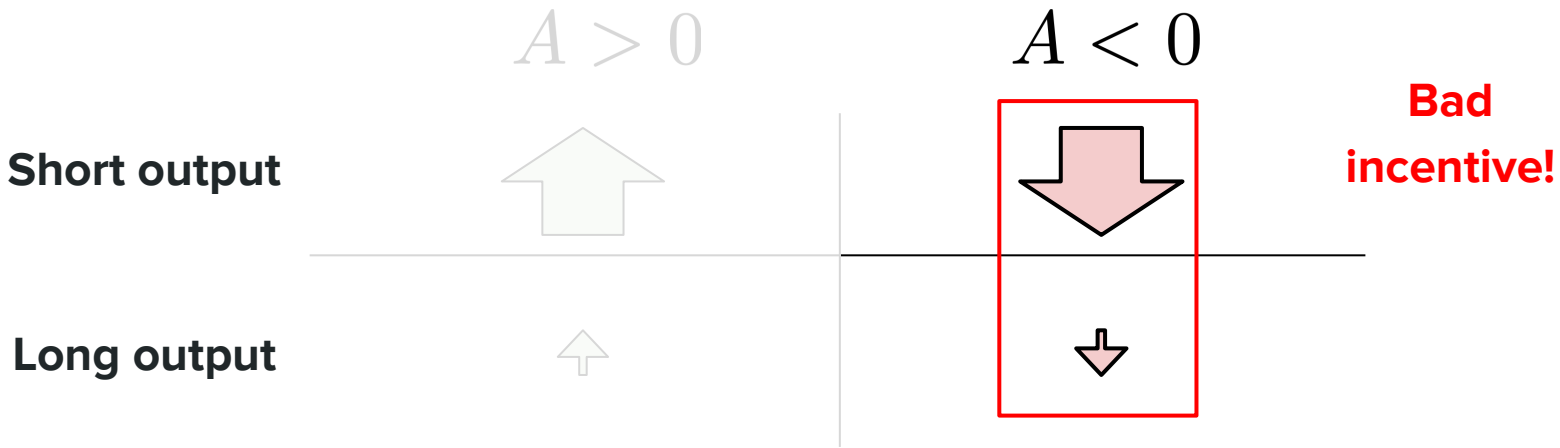
$$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}\boxed{\frac{1}{|o_i|}}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$



"Increasing output length" phenomenon

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

$$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}\boxed{\frac{1}{|o_i|}}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$



Mitigating "increasing length" phenomenon

Problem. $$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|\mathbf{o}_i|}\sum_{t=1}^{|\mathbf{o}_i|}$$

Mitigating "increasing length" phenomenon

Problem. $$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|\mathbf{o}_i|}\sum_{t=1}^{|\mathbf{o}_i|}$$

Remedy. Equalize token-level contributions

Mitigating "increasing length" phenomenon

Problem. $$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|\mathbf{o}_i|}\sum_{t=1}^{|\mathbf{o}_i|}$$

Remedy. Equalize token-level contributions

- DAPO $$\frac{1}{\sum_{i=1}^{G}|o_i|}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}$$

"*DAPO: An Open-Source LLM Reinforcement Learning System at Scale*", Yu et al., 2025.

Mitigating "increasing length" phenomenon

Problem. $$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|\mathbf{o}_i|}\sum_{t=1}^{|\mathbf{o}_i|}$$

Remedy. Equalize token-level contributions

- DAPO $$\frac{1}{\sum_{i=1}^{G}|o_i|}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}$$
- Dr. GRPO $$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|\mathbf{o}_i|}$$

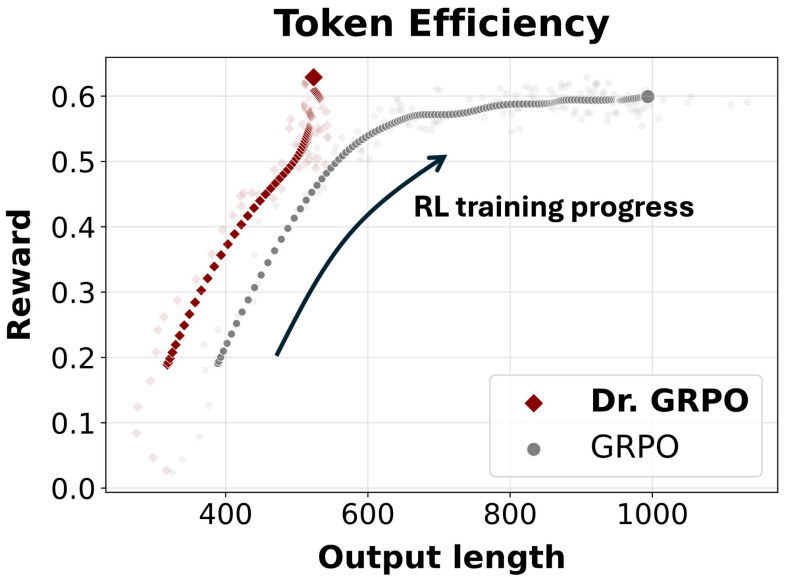
"*Understanding R1-Zero-Like Training: A Critical Perspective*", *Liu et al., 2025.*

Mitigating "increasing length" phenomenon

Problem. $$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|\mathbf{o}_i|}\sum_{t=1}^{|\mathbf{o}_i|}$$

Remedy. Equalize token-level contributions

- DAPO $$\frac{1}{\sum_{i=1}^{G}|o_i|}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}$$
- Dr. GRPO $$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|\mathbf{o}_i|}$$



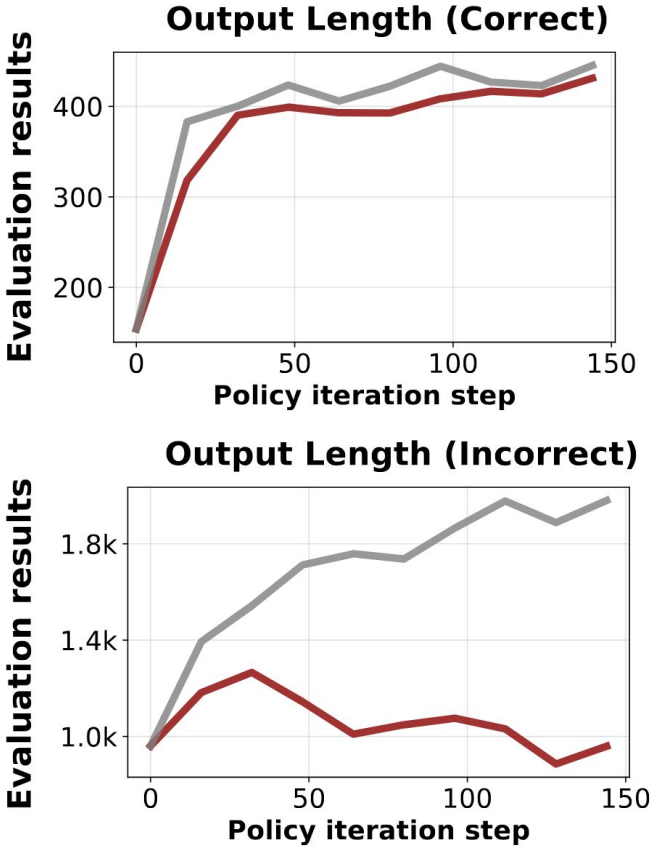
"*Understanding R1-Zero-Like Training: A Critical Perspective*", *Liu et al., 2025.*

Mitigating "increasing length" phenomenon

Problem. $$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|\mathbf{o}_i|}\sum_{t=1}^{|\mathbf{o}_i|}$$

Remedy. Equalize token-level contributions

- DAPO $$\frac{1}{\sum_{i=1}^{G}|o_i|}\sum_{i=1}^{G}\sum_{t=1}^{|o_i|}$$
- Dr. GRPO $$\frac{1}{G}\sum_{i=1}^{G}\sum_{t=1}^{|\mathbf{o}_i|}$$



"*Understanding R1-Zero-Like Training: A Critical Perspective*", Liu et al., 2025.

Exploration of other adjustments

- Bias linked to **level of difficulty**

$$\hat{A}_{i,t} = \frac{R(\mathbf{q}, \mathbf{o}_i) - \text{mean}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}{\text{std}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}$$

"Understanding R1-Zero-Like Training: A Critical Perspective", Liu et al., 2025.

Exploration of other adjustments

- Bias linked to **level of difficulty**

$$\hat{A}_{i,t} = \frac{R(\mathbf{q}, \mathbf{o}_i) - \text{mean}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}{\text{std}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}$$

- Encourage **diversity**

$$\text{clip}\Big(r_{i,t}(\theta), 1-\varepsilon, 1+\varepsilon\Big) \longrightarrow \text{clip}\Big(r_{i,t}(\theta), 1-\varepsilon_{\text{low}}, 1+\varepsilon_{\text{high}}\Big)$$

"*DAPO: An Open-Source LLM Reinforcement Learning System at Scale*", Yu et al., 2025.

Exploration of other adjustments

- Bias linked to **level of difficulty**

$$\hat{A}_{i,t} = \frac{R(\mathbf{q}, \mathbf{o}_i) - \text{mean}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}{\text{std}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}$$

- Encourage **diversity**

$$\text{clip}\Big(r_{i,t}(\theta), 1-\varepsilon, 1+\varepsilon\Big) \longrightarrow \text{clip}\Big(r_{i,t}(\theta), 1-\varepsilon_{\text{low}}, 1+\varepsilon_{\text{high}}\Big)$$

"*DAPO: An Open-Source LLM Reinforcement Learning System at Scale*", Yu et al., 2025.

Exploration of other adjustments

- Bias linked to **level of difficulty**

$$\hat{A}_{i,t} = \frac{R(\mathbf{q}, \mathbf{o}_i) - \text{mean}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}{\text{std}(\{R(\mathbf{q}, \mathbf{o}_1), \ldots, R(\mathbf{q}, \mathbf{o}_G)\})}$$

- Encourage **diversity**

$$\text{clip}\Big(r_{i,t}(\theta), 1-\varepsilon, 1+\varepsilon\Big) \longrightarrow \text{clip}\Big(r_{i,t}(\theta), 1-\varepsilon_{\text{low}}, 1+\varepsilon_{\text{high}}\Big)$$

...among others!

Transformers & Large Language Models

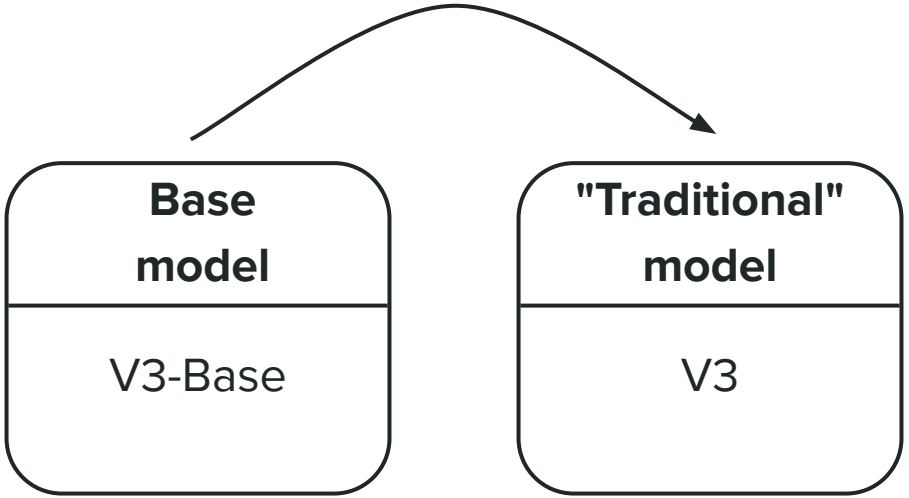
Reasoning models

Scaling with RL

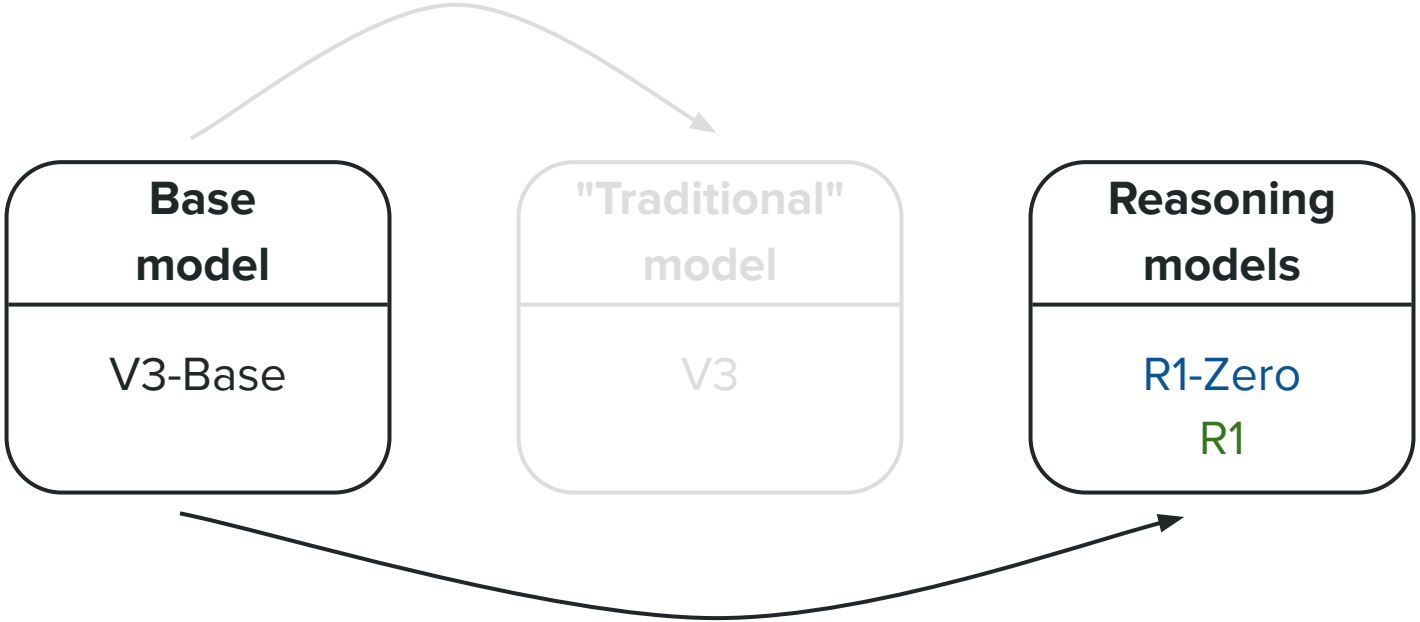
GRPO

Applications

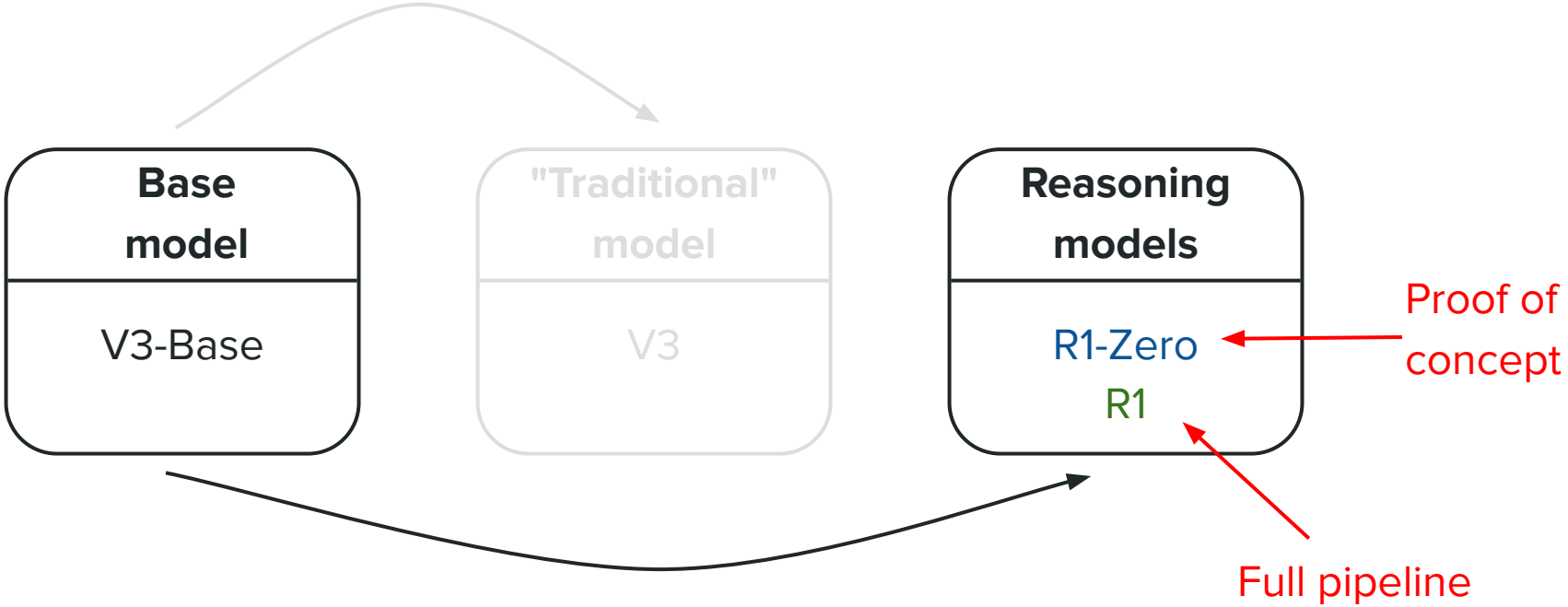
Stitching it all together



Stitching it all together



Stitching it all together



DeepSeek **R1-Zero**'s training recipe

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

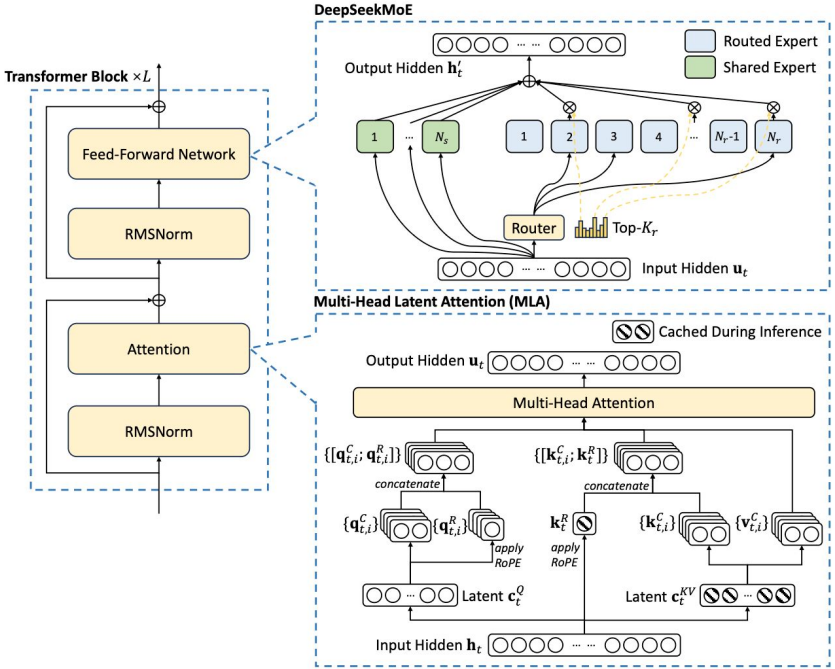
DeepSeek **R1-Zero**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

DeepSeek **R1-Zero**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**



MoE

~671B total, ~37B active

Figure from "DeepSeek-V3 Technical Report", DeepSeek-AI, 2024.
"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1-Zero**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. GRPO with reasoning data: **R1-Zero**

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

DeepSeek R1-Zero's training recipe

1 Pretrain model with "traditional" techniques: **V3-Base**

2 GRPO with reasoning data: **R1-Zero**

```
A conversation between User and Assistant. The user asks a question, and the
Assistant solves it. The assistant first thinks about the reasoning process
in the mind and then provides the user with the answer. The reasoning
process and answer are enclosed within <think> </think> and <answer>
</answer> tags, respectively, i.e., <think> reasoning process here </think>
<answer> answer here </answer>.
User: <this placeholder is replaced by a reasoning query>
Assistant:
```

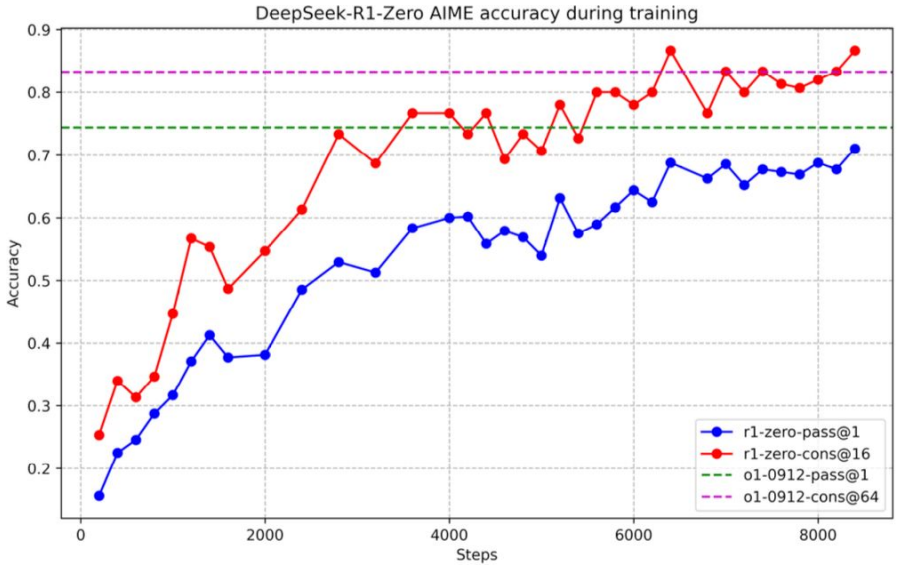
Template

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1-Zero**'s training recipe

1 Pretrain model with "traditional" techniques: **V3-Base**

2 GRPO with reasoning data: **R1-Zero**



"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

DeepSeek R1-Zero's training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. GRPO with reasoning data: **R1-Zero**

Benefits	Challenges
Reasoning abilities without any SFT	Chains of reasoning have formatting and readability issues

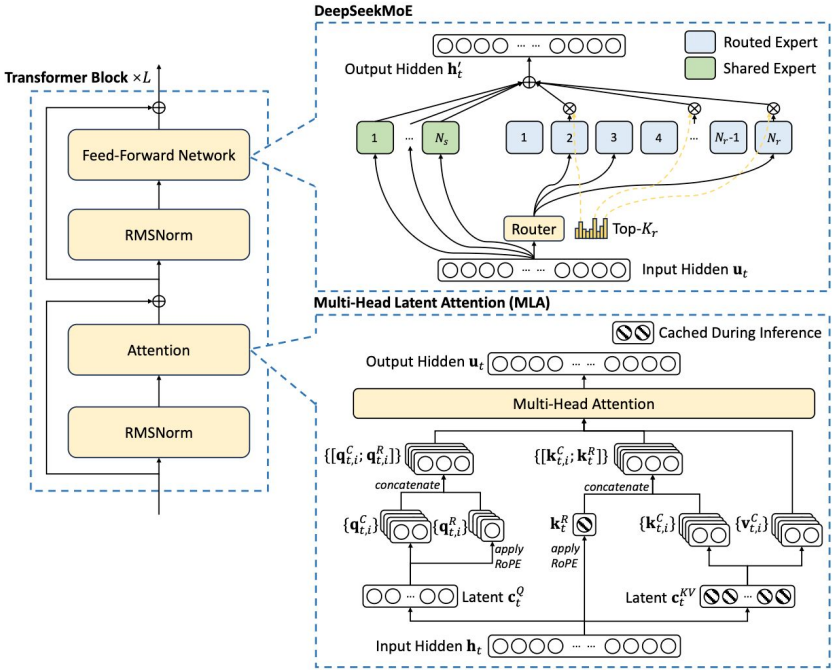
"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek R1's training recipe

Figure from "DeepSeek-V3 Technical Report", DeepSeek-AI, 2024.
"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1**'s training recipe

1 Pretrain model with "traditional" techniques: **V3-Base**



MoE

~671B total, ~37B active

Figure from "DeepSeek-V3 Technical Report", DeepSeek-AI, 2024.
"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

DeepSeek **R1**'s training recipe

1 Pretrain model with "traditional" techniques: **V3-Base**

2 "Small-scale" SFT with reasoning data

Data source: long CoTs generated with R1-Zero and rewritten by humans

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data

~same RL process as with R1-Zero

Reward = Formatting + accuracy + language consistency

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data
4. "Large-scale" SFT with reasoning and non-reasoning data

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data
4. "Large-scale" SFT with reasoning and non-reasoning data

"General" data

Mostly **reuses V3 SFT data**

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data
4. "Large-scale" SFT with reasoning and non-reasoning data

~600k pairs

Maths, coding, logic

Rejection sampling of "R1 so far" responses via rules + V3 judge

~200k pairs

"General" data

Mostly **reuses V3 SFT data**

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data
4. "Large-scale" SFT with reasoning and non-reasoning data
5. GRPO with reasoning and non-reasoning data: **R1**

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data
4. "Large-scale" SFT with reasoning and non-reasoning data
5. GRPO with reasoning and non-reasoning data: **R1**

Maths, coding, logic

Reward = Formatting + accuracy

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

DeepSeek **R1**'s training recipe

1. Pretrain model with "traditional" techniques: **V3-Base**
2. "Small-scale" SFT with reasoning data
3. GRPO with reasoning data
4. "Large-scale" SFT with reasoning and non-reasoning data
5. GRPO with reasoning and non-reasoning data: **R1**

Maths, coding, logic

Reward = Formatting + accuracy

"General" data

Mostly **reuses V3 RL data**

Reward = helpfulness + harmlessness

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek R1's results

	Benchmark (Metric)	Claude-3.5-Sonnet-1022	GPT-4o 0513	DeepSeek V3	OpenAI o1-mini	OpenAI o1-1217	DeepSeek R1
	Architecture	-	-	MoE	-	-	MoE
	# Activated Params	-	-	37B	-	-	37B
	# Total Params	-	-	671B	-	-	671B
English	MMLU (Pass@1)	88.3	87.2	88.5	85.2	**91.8**	90.8
	MMLU-Redux (EM)	88.9	88.0	89.1	86.7	-	**92.9**
	MMLU-Pro (EM)	78.0	72.6	75.9	80.3	-	**84.0**
	DROP (3-shot F1)	88.3	83.7	91.6	83.9	90.2	**92.2**
	IF-Eval (Prompt Strict)	**86.5**	84.3	86.1	84.8	-	83.3
	GPQA Diamond (Pass@1)	65.0	49.9	59.1	60.0	**75.7**	71.5
	SimpleQA (Correct)	28.4	38.2	24.9	7.0	**47.0**	30.1
	FRAMES (Acc.)	72.5	80.5	73.3	76.9	-	**82.5**
	AlpacaEval2.0 (LC-winrate)	52.0	51.1	70.0	57.8	-	**87.6**
	ArenaHard (GPT-4-1106)	85.2	80.4	85.5	92.0	-	**92.3**
Code	LiveCodeBench (Pass@1-COT)	38.9	32.9	36.2	53.8	63.4	**65.9**
	Codeforces (Percentile)	20.3	23.6	58.7	93.4	**96.6**	96.3
	Codeforces (Rating)	717	759	1134	1820	**2061**	2029
	SWE Verified (Resolved)	**50.8**	38.8	42.0	41.6	48.9	49.2
	Aider-Polyglot (Acc.)	45.3	16.0	49.6	32.9	**61.7**	53.3
Math	AIME 2024 (Pass@1)	16.0	9.3	39.2	63.6	79.2	**79.8**
	MATH-500 (Pass@1)	78.3	74.6	90.2	90.0	96.4	**97.3**
	CNMO 2024 (Pass@1)	13.1	10.8	43.2	67.6	-	**78.8**
Chinese	CLUEWSC (EM)	85.4	87.9	90.9	89.9	-	**92.8**
	C-Eval (EM)	76.7	76.0	86.5	68.9	-	**91.8**
	C-SimpleQA (Correct)	55.4	58.7	**68.0**	40.3	-	63.7

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek R1's results

	Benchmark (Metric)	Claude-3.5-Sonnet-1022	GPT-4o 0513	DeepSeek V3	OpenAI o1-mini	OpenAI o1-1217	DeepSeek R1
	Architecture	-	-	MoE	-	-	MoE
	# Activated Params	-	-	37B	-	-	37B
	# Total Params	-	-	671B	-	-	671B
English	MMLU (Pass@1)	88.3	87.2	88.5	85.2	**91.8**	90.8
	MMLU-Redux (EM)	88.9	88.0	89.1	86.7	-	**92.9**
	MMLU-Pro (EM)	78.0	72.6	75.9	80.3	-	**84.0**
	DROP (3-shot F1)	88.3	83.7	91.6	83.9	90.2	**92.2**
	IF-Eval (Prompt Strict)	**86.5**	84.3	86.1	84.8	-	83.3
	GPQA Diamond (Pass@1)	65.0	49.9	59.1	60.0	**75.7**	71.5
	SimpleQA (Correct)	28.4	38.2	24.9	7.0	**47.0**	30.1
	FRAMES (Acc.)	72.5	80.5	73.3	76.9	-	**82.5**
	AlpacaEval2.0 (LC-winrate)	52.0	51.1	70.0	57.8	-	**87.6**
	ArenaHard (GPT-4-1106)	85.2	80.4	85.5	92.0	-	**92.3**
Code	LiveCodeBench (Pass@1-COT)	38.9	32.9	36.2	53.8	63.4	**65.9**
	Codeforces (Percentile)	20.3	23.6	58.7	93.4	**96.6**	96.3
	Codeforces (Rating)	717	759	1134	1820	**2061**	2029
	SWE Verified (Resolved)	**50.8**	38.8	42.0	41.6	48.9	49.2
	Aider-Polyglot (Acc.)	45.3	16.0	49.6	32.9	**61.7**	53.3
Math	AIME 2024 (Pass@1)	16.0	9.3	39.2	63.6	79.2	**79.8**
	MATH-500 (Pass@1)	78.3	74.6	90.2	90.0	96.4	**97.3**
	CNMO 2024 (Pass@1)	13.1	10.8	43.2	67.6	-	**78.8**
Chinese	CLUEWSC (EM)	85.4	87.9	90.9	89.9	-	**92.8**
	C-Eval (EM)	76.7	76.0	86.5	68.9	-	**91.8**
	C-SimpleQA (Correct)	55.4	58.7	**68.0**	40.3	-	63.7

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

DeepSeek R1's results

	Benchmark (Metric)	Claude-3.5-Sonnet-1022	GPT-4o 0513	DeepSeek V3	OpenAI o1-mini	OpenAI o1-1217	DeepSeek R1
	Architecture	-	-	MoE	-	-	MoE
	# Activated Params	-	-	37B	-	-	37B
	# Total Params	-	-	671B	-	-	671B
English	MMLU (Pass@1)	88.3	87.2	88.5	85.2	**91.8**	90.8
	MMLU-Redux (EM)	88.9	88.0	89.1	86.7	-	**92.9**
	MMLU-Pro (EM)	78.0	72.6	75.9	80.3	-	**84.0**
	DROP (3-shot F1)	88.3	83.7	91.6	83.9	90.2	**92.2**
	IF-Eval (Prompt Strict)	**86.5**	84.3	86.1	84.8	-	83.3
	GPQA Diamond (Pass@1)	65.0	49.9	59.1	60.0	**75.7**	71.5
	SimpleQA (Correct)	28.4	38.2	24.9	7.0	**47.0**	30.1
	FRAMES (Acc.)	72.5	80.5	73.3	76.9	-	**82.5**
	AlpacaEval2.0 (LC-winrate)	52.0	51.1	70.0	57.8	-	**87.6**
	ArenaHard (GPT-4-1106)	85.2	80.4	85.5	92.0	-	**92.3**
Code	LiveCodeBench (Pass@1-COT)	38.9	32.9	36.2	53.8	63.4	**65.9**
	Codeforces (Percentile)	20.3	23.6	58.7	93.4	**96.6**	96.3
	Codeforces (Rating)	717	759	1134	1820	**2061**	2029
	SWE Verified (Resolved)	**50.8**	38.8	42.0	41.6	48.9	49.2
	Aider-Polyglot (Acc.)	45.3	16.0	49.6	32.9	**61.7**	53.3
Math	AIME 2024 (Pass@1)	16.0	9.3	39.2	63.6	79.2	**79.8**
	MATH-500 (Pass@1)	78.3	74.6	90.2	90.0	96.4	**97.3**
	CNMO 2024 (Pass@1)	13.1	10.8	43.2	67.6	-	**78.8**
Chinese	CLUEWSC (EM)	85.4	87.9	90.9	89.9	-	**92.8**
	C-Eval (EM)	76.7	76.0	86.5	68.9	-	**91.8**
	C-SimpleQA (Correct)	55.4	58.7	**68.0**	40.3	-	63.7

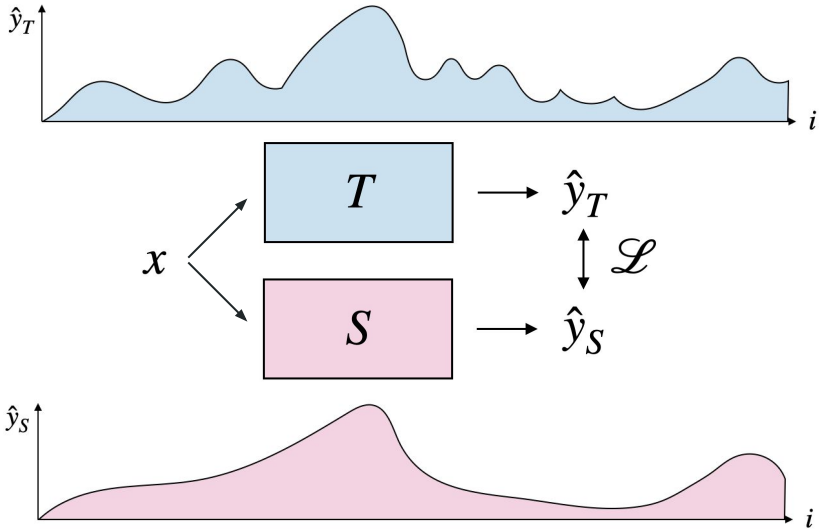
"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

And what about distillation?

Figures from "Super Study Guide: Transformers & Large Language Models", Amidi et al., 2024.

And what about distillation?

Distillation seen in lecture 2

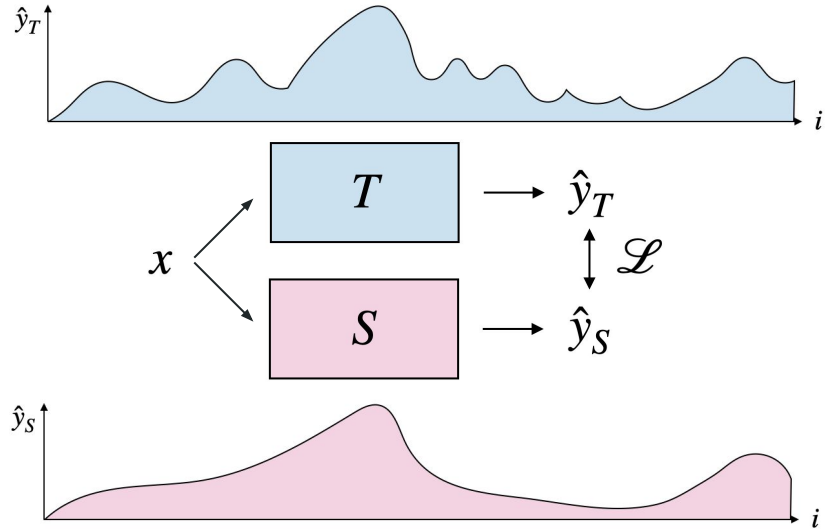


Goal: match next token distribution

Figures from "Super Study Guide: Transformers & Large Language Models", Amidi et al., 2024.

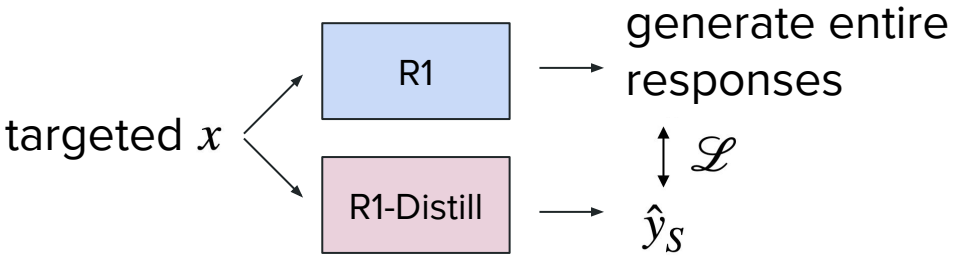
And what about distillation?

Distillation seen in lecture 2



Goal: match next token distribution

Distillation used here



Goal: SFT-learn reasoning traces

Figures from "Super Study Guide: Transformers & Large Language Models", Amidi et al., 2024.

And what about distillation?

Results.

And what about distillation?

Results.

Competitive

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
GPT-4o-0513	9.3	13.4	74.6	49.9	32.9	759
Claude-3.5-Sonnet-1022	16.0	26.7	78.3	65.0	38.9	717
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	**1820**
QwQ-32B-Preview	50.0	60.0	90.6	54.5	41.9	1316
DeepSeek-R1-Distill-Qwen-1.5B	28.9	52.7	83.9	33.8	16.9	954
DeepSeek-R1-Distill-Qwen-7B	55.5	83.3	92.8	49.1	37.6	1189
DeepSeek-R1-Distill-Qwen-14B	69.7	80.0	93.9	59.1	53.1	1481
DeepSeek-R1-Distill-Qwen-32B	**72.6**	83.3	94.3	62.1	57.2	1691
DeepSeek-R1-Distill-Llama-8B	50.4	80.0	89.1	49.0	39.6	1205
DeepSeek-R1-Distill-Llama-70B	70.0	**86.7**	**94.5**	**65.2**	**57.5**	1633

"DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning", DeepSeek-AI, 2025.

And what about distillation?

Results.

Competitive

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
GPT-4o-0513	9.3	13.4	74.6	49.9	32.9	759
Claude-3.5-Sonnet-1022	16.0	26.7	78.3	65.0	38.9	717
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	**1820**
QwQ-32B-Preview	50.0	60.0	90.6	54.5	41.9	1316
DeepSeek-R1-Distill-Qwen-1.5B	28.9	52.7	83.9	33.8	16.9	954
DeepSeek-R1-Distill-Qwen-7B	55.5	83.3	92.8	49.1	37.6	1189
DeepSeek-R1-Distill-Qwen-14B	69.7	80.0	93.9	59.1	53.1	1481
DeepSeek-R1-Distill-Qwen-32B	72.6	83.3	94.3	62.1	57.2	1691
DeepSeek-R1-Distill-Llama-8B	50.4	80.0	89.1	49.0	39.6	1205
DeepSeek-R1-Distill-Llama-70B	70.0	86.7	94.5	65.2	57.5	1633

"Good" use of compute

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCodeBench
	pass@1	cons@64	pass@1	pass@1	pass@1
QwQ-32B-Preview	50.0	60.0	90.6	54.5	41.9
DeepSeek-R1-Zero-Qwen-32B	47.0	60.0	91.6	55.0	40.2
DeepSeek-R1-Distill-Qwen-32B	**72.6**	**83.3**	**94.3**	**62.1**	**57.2**

"*DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning*", *DeepSeek-AI, 2025.*

Thank you for your attention!