Linear Algebra Questions

(a) $A \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}_{2\times3}$

$B \begin{bmatrix} -6 & 2 & 10 \\ 3 & -3 & 4 \\ -5 & -11 & 9 \\ 1 & -1 & 1 \end{bmatrix}_{4\times3}$

$C \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \end{bmatrix}_{1\times5}$

$D \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}_{2\times2}$

$F \begin{bmatrix} 3 \\ 5 \\ -11 \\ 7 \end{bmatrix}_{4\times1}$

$G \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}_{3\times3}$

$E \begin{bmatrix} 3 \end{bmatrix}_{1\times1}$

(b) D & G

(c) E

(D) B

(E) A, B

(F) F, E

(g) C, E

(h) $A^T = \begin{bmatrix} -1 & 0 \\ 23 & -2 \\ 10 & -11 \end{bmatrix}$

$B^T \begin{bmatrix} -6 & 3 & -5 & 1 \\ 2 & -3 & -11 & -1 \\ 10 & 4 & 9 & 9 \end{bmatrix}$

$C^T \begin{bmatrix} -3 \\ 2 \\ 9 \\ -5 \\ 7 \end{bmatrix}$

$E^T \begin{bmatrix} 3 \end{bmatrix}$

$G^T \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$

Date / / No

(2) (a) AB

$$A\begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}_{2\times 3} \qquad B\begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}_{3\times 3}$$

$$AB = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & -2 \\ -1 & 4 & -5 \end{bmatrix}_{2\times 3}$$

(b) BC

$$\begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}_{[3\times 3]} \begin{bmatrix} 3 & 2 & 9 & -5 & 7 \end{bmatrix}_{1\times 5}$$

not possible.

(3) $M = \begin{vmatrix} 15 & 10 \\ 3 & 2 \end{vmatrix} = 2\times 15 - 3\times 10 = 0$

$$M \begin{vmatrix} 2 & -3 & 1 \\ -1 & 2 & 3 \\ 3 & 2 & -1 \end{vmatrix}$$

(+ − +)

$= 2(-2-3) - 3(1-9) + 1(-2-6)$

$= -10 + 24 - 8 = 6$

(4) $A \begin{vmatrix} -3 & -2 \\ 3 & 3 \end{vmatrix}$

$|A| = -9 + 6 = -3$

$$A^{-1} = \frac{-1}{3}\begin{vmatrix} 3 & 2 \\ -3 & -3 \end{vmatrix} = \begin{vmatrix} -1 & \frac{-2}{3} \\ +1 & +1 \end{vmatrix}$$

Date / / No

$$A \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad A^{-1}$$

$$\Delta = \begin{vmatrix} + & - & + \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1(0) + 1(-1) = -1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 0 \qquad \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = -1 \qquad \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = -1$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = -1 \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 0 \qquad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = -1 \qquad \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = 1 \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$$

$$-1 * \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} 0 & -1 & +1 \\ -1 & 0 & +1 \\ +1 & +1 & -1 \end{bmatrix}$$

(5) b

(6) $f(x,y) = x^T A y + x^T B x - C y + D$

$x \in R^M \rightarrow M \times 1$

$y \in R^N \rightarrow N \times 1$

$R^M \times R^N \rightarrow R$

$x^T \rightarrow 1 \times M$

$Ay \rightarrow N \times 1 \quad \therefore M \times N \ni A$

$Bx \rightarrow M \times 1 \quad \therefore M \times M \ni B$ Matrix Square

$Cy \rightarrow N \times 1 \quad \therefore 1 \times N \ni C$

$D \rightarrow R$ scalar