CENG 424

HOMEWORK 5

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Question 1)

1.

1. $\exists x \exists y\, (S(x,y) \vee S(y,x))$ Premise
2. $\exists y\, (S(a,y) \vee S(y,a))$ EI : 1
3. $S(a,b) \vee S(b,a)$ EI : 2

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4. $S(a,b)$ Disjunction elimination (Assumption 1)
5. $\exists y\, S(a,y)$ EG : 4
6. $\exists x \exists y\, S(x,y)$ EG : 5

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7. $S(b,a)$ Disjunction elimination (Assumption 2)
8. $\exists y\, S(b,y)$ EG : 7
9. $\exists x \exists y\, S(x,y)$ EG : 8

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10. $\exists x \exists y\, S(x,y)$ Disjunction elimination : 3, 4 – 6, 7 – 9

2.

1. $\forall x\, (P(x) \vee Q(x))$ Premise
2. $\exists x\, \neg Q(x)$ Premise
3. $\forall x\, (R(x) \Rightarrow \neg P(x))$ Premise
4. $P(a) \vee Q(a)$ UI : 1
5. $\neg Q(a) \Rightarrow P(a)$ Mendelson Logic $[(\psi \vee \varphi) \rightarrow (\neg \varphi \Rightarrow \psi)]$ rule : 4
6. $\neg Q(a)$ EI : 2
7. $P(a)$ MP : 5, 6
8. $R(a) \Rightarrow \neg P(a)$ UI : 3
9. $\neg R(a)$ MT : 7, 8
10. $\exists x\, \neg R(x)$ EG : 9

3. We can use Relational Deduction Theorem on this problem because $\exists y\, S(y)$ has no free variable and it can be added to set of premises Δ. And now, we need to deduct $\forall x F(x)$.

i.e. by using Relational Deduction Theorem:

$$\underbrace{\forall x \forall y\,(S(y) \Rightarrow F(x))}_{\Delta} \vdash \underbrace{\exists y\, S(y)}_{\varphi} \Rightarrow \underbrace{\forall x F(x)}_{\psi}$$

can be changed to:

$$\underbrace{\forall x \forall y\,(S(y) \Rightarrow F(x))}_{\Delta} \cup \underbrace{\exists y\, S(y)}_{\{\varphi\}} \vdash \underbrace{\forall x F(x)}_{\psi}$$

Therefore,

1. $\forall x \forall y\,(S(y) \Rightarrow F(x))$ Premise
2. $\exists y\, S(y)$ Premise (from Relational Deduction Theorem)
3. $\forall y\,(S(y) \Rightarrow F(a))$ UI:1
4. $S(b) \Rightarrow F(a)$ UI:3
5. $S(b)$ EI: 2
6. $F(a)$ MP: 4,5
7. $\underline{\underline{\forall x F(x)}}$ UG: 6

Question 2)

Step 1)

- Everyone whom Jane loves is a traveler.

$$\forall x\,(\text{LOVES}(\text{Jane}, x) \Rightarrow \text{TRAVELER}(x))$$

- Any person who does not earn money, does not travel.

  $\forall y((PERSON(y) \land \neg EARN(y)) \Rightarrow \neg TRAVEL(y))$

- Jim is a doctor.

  $DOCTOR(Jim)$

- Every doctor is a person.

  $\forall z(DOCTOR(z) \Rightarrow PERSON(z))$

- Any doctor who does not work, does not earn money.

  $\forall u((DOCTOR(u) \land \neg WORK(u)) \Rightarrow \neg EARN(u))$

- Anyone who does not travel, is not a traveler.

  $\forall v(\neg TRAVEL(v) \Rightarrow \neg TRAVELER(v))$

For the conclusion:

- If Jim does not work, then Jane does not love Jim.

Goal: $\neg WORK(Jim) \Rightarrow \neg LOVES(Jane, Jim)$

Negated Goal: $\neg(\neg WORK(Jim) \Rightarrow \neg LOVES(Jane, Jim))$

$\neg(WORK(Jim) \lor \neg LOVES(Jane, Jim))$

$\neg WORK(Jim) \land LOVES(Jane, Jim)$

Step 2)

* For first sentence:

$\forall x(LOVES(Jane, x) \Rightarrow TRAVELER(x))$

$\forall x(\neg LOVES(Jane, x) \lor TRAVELER(x))$

$\neg LOVES(Jane, x) \lor TRAVELER(x)$

As clause: $\{\neg LOVES(Jane, x), TRAVELER(x)\}$

* For second sentence:

$\forall y\ ((PERSON(y) \wedge \neg EARN(y)) \Rightarrow \neg TRAVEL(y))$

$\forall y\ (\neg (PERSON(y) \wedge \neg EARN(y)) \vee \neg TRAVEL(y))$

$\forall y\ ((\neg PERSON(y) \vee EARN(y)) \vee \neg TRAVEL(y))$

$((\neg PERSON(y) \vee EARN(y)) \vee \neg TRAVEL(y)$

As clause: $\{ \neg PERSON(y), EARN(y), \neg TRAVEL(y) \}$

* For third sentence:

$DOCTOR(Jim)$

As clause: $\{ DOCTOR(Jim) \}$

* For fourth sentence:

$\forall z\ (DOCTOR(z) \Rightarrow PERSON(z))$

$\forall z\ (\neg DOCTOR(z) \vee PERSON(z))$

$\neg DOCTOR(z) \vee PERSON(z)$

As clause: $\{ \neg DOCTOR(z), PERSON(z) \}$

* For fifth sentence:

$\forall u\ ((DOCTOR(u) \wedge \neg WORK(u)) \Rightarrow \neg EARN(u))$

$\forall u\ (\neg (DOCTOR(u) \wedge \neg WORK(u)) \vee \neg EARN(u))$

$\forall u\ ((\neg DOCTOR(u) \vee WORK(u)) \vee \neg EARN(u))$

$(\neg DOCTOR(u) \vee WORK(u)) \vee \neg EARN(u)$

As clause: $\{ \neg DOCTOR(u), WORK(u), \neg EARN(u) \}$

* For sixth sentence:

$\forall v\ (\neg TRAVEL(v) \Rightarrow \neg TRAVELER(v))$

$\forall v\ (TRAVEL(v) \vee \neg TRAVELER(v))$

$TRAVEL(v) \vee \neg TRAVELER(v)$

As clause: $\{TRAVEL(v), \neg TRAVELER(v)\}$

* For negated goal we already found:

$\neg WORK(Jim) \wedge LOVES(Jane, Jim)$

As clause: $\{\neg WORK(Jim)\}$

$\{LOVES(Jane, Jim)\}$

Step 3)

1. $\{\neg LOVES(Jane, x), TRAVELER(x)\}$ Premise
2. $\{\neg PERSON(y), EARN(y), \neg TRAVEL(y)\}$ Premise
3. $\{DOCTOR(Jim)\}$ Premise
4. $\{\neg DOCTOR(z), PERSON(z)\}$ Premise
5. $\{\neg DOCTOR(u), WORK(u), \neg EARN(u)\}$ Premise
6. $\{TRAVEL(v), \neg TRAVELER(v)\}$ Premise
7. $\{\neg WORK(Jim)\}$ Negated Goal
8. $\{LOVES(Jane, Jim)\}$ Negated Goal
9. $\{PERSON(Jim)\}$ 3,4 by substituting $\{z \leftarrow Jim\}$
10. $\{EARN(Jim), \neg TRAVEL(Jim)\}$ 2,9 by substituting $\{y \leftarrow Jim\}$
11. $\{TRAVELER(Jim)\}$ 1,8 by substituting $\{x \leftarrow Jim\}$
12. $\{TRAVEL(Jim)\}$ 6,11 by substituting $\{v \leftarrow Jim\}$
13. $\{EARN(Jim)\}$ 10,12
14. $\{WORK(Jim), \neg EARN(Jim)\}$ 3,5 by substituting $\{u \leftarrow Jim\}$
15. $\{WORK(Jim)\}$ 13,14
16. $\{\ \}$ 7,15

As a result, since we reached empty set $\{\}$ at the end, we proved the goal.

Question 3)

* According to Hint 1, $P(x,y,p)$ represents a transition from x to y with probability p.

* According to Hint 2,

$$P(x,y,p_1) \wedge P(y,z,p_2) \Rightarrow P(x,z,p_1 \times p_2)$$

$$\equiv \neg P(x,y,p_1) \vee \neg P(y,z,p_2) \vee P(x,z,p_1 \times p_2)$$

As a clause: $\{\neg P(x,y,p_1), \neg P(y,z,p_2), P(x,z,p_1 \times p_2)\}$

By using resolution and answer extraction method:

1. $\{P(p,q,0.8)\}$ — $P(p,q,0.8)$ from the tree
2. $\{P(q,t,0.3)\}$ — $P(q,t,0.3)$ from the tree
3. $\{\neg P(x,y,p_1), \neg P(y,z,p_2), P(x,z,p_1 \times p_2)\}$ — from Hint 2
4. $\{\neg P(p,t,w), goal(w)\}$ — $P(p,t,w) \Rightarrow goal(w)$ (to reach the probability w by starting from node p and ending at node t)

5. $\{\neg P(p,y,p_1), \neg P(y,t,p_2), goal(p_1 \times p_2)\}$ — 3, 4 (by substituting $\{x \leftarrow p, z \leftarrow t, w \leftarrow p_1 \times p_2\}$)

6. $\{\neg P(q,t,p_2), goal(0.8 \times p_2)\}$ — 1, 5 (by substituting $\{y \leftarrow q, p_1 \leftarrow 0.8\}$)

7. $\{\underline{\underline{goal(0.8 \times 0.3)}}\}$ — 2, 6 (by substituting $\{p_2 \leftarrow 0.3\}$)

The result is: $0.8 \times 0.3 = 0.24$

Question 4)

We know that universal generalization is $\dfrac{\varphi}{\forall v.\varphi}$.

* Here, if bounded variable (v) of universal quantifier occurs in φ, this rule may not work.

For example, if we think φ as $\varphi(x) = x > 2y$ and if we say that $\dfrac{\varphi}{\forall y\, \varphi}$, it is clearly false

because $\forall y\, \varphi = \forall y\, (y > 2y)$, and this is not sound.

* For another restriction, we can not use universal generalization by starting with constants.

For example, let's say $\varphi = 4 > 0$

Four is greater than zero, but starting with this, we can not say that all numbers are greater than zero.

Again $\dfrac{\varphi}{\forall n\, \varphi}$ is clearly false for this case.

* So, except these cases, universal generalization can work. If we prove generic case c, we can generalize it without doing any other assumption.

For example, let's $\varphi = (x > 1 \Rightarrow x - 1 > 0)$

For this sentence, if we can prove it for arbitrary c, we can easily generalize it by $\dfrac{\varphi}{\forall x\, \varphi}$.