CENG 424 - Logic For Computer Science

HOMEWORK 4

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Question 1)

1. a-)

$$\boxed{\forall x\,(fact(x) \land \neg thing(x) \Rightarrow world(x))}$$

- x is a variable.
- fact(x) is unary relation. It means "x is a fact."
- thing (x) is unary relation. It means "x is a thing."
- world (x) is unary relation. It means "x is a member of world."

1. b-)

The only variable "x" is bound by universal quantifier.

2. a-)

$$\boxed{\exists y\,(represent(picture(x),y) \Rightarrow (\exists u \exists w\,(possible(situation(u)) \land logical(space(w)) \land ternary(y,u,w))))}$$

- x is a variable.
- picture (x) is unary function. It means "picture of x."
- y is a variable.
- represent (x,y) is binary relation. It means "x represents y."
- u is a variable.
- situation (x) is unary function. It means "situation of x"
- possible (x) is unary relation. It means "x is possible."
- w is a variable.
- space (x) is unary function. It means "space of x"
- logical (x) is unary relation. It means "x is logical"

- ternary (x,y,z) is ternary relation.

It means "x is a y in z."

2. b-)

- x is a free variable because it is not bounded by any quantifier.
- y is a bound variable because it is bounded by leftmost existential quantifier.
- u is a bound variable because it is bounded by middle existential quantifier.
- w is a bound variable because it is bounded by rightmost existential quantifier according to my relational sentence.

Question 2)

1.

$$\boxed{\begin{array}{l}[\exists x\,(tpayer(x) \Rightarrow \forall y\,(musician(y) \Rightarrow tpayer(y))) \;\wedge \\ \;\exists x\,(phila(x) \Rightarrow \forall y\,(tpayer(y) \Rightarrow phila(y)))] \Rightarrow \\ \;\exists x\,(tpayer(x) \wedge phila(x) \Rightarrow \forall y\,(musician(y) \Rightarrow phila(y)))\end{array}}$$

- x and y are variables.
- tpayer(x) is unary relation. It means "x is tax payer"
- musician(x) is unary relation. It means "x is musician"
- phila(x) is unary relation. It means "x is philanthropist"

2.

It can be expressed in ground logic without using any variable and quantifier. However, this process has important limitation.

$\Downarrow$

[tpayer(Joe) ⇒ (musician(Mary) ⇒ tpayer(Mary)) v -- (check Joe and others)

tpayer(Mary) ⇒ (musician(Arnold) ⇒ tpayer(Arnold)) v ... (check Mary and others) and so on

check for all people in universe of discourse

tpayer(Scott) ⇒ (musician(Lisa) ⇒ tpayer(Lisa))]

$\wedge$

[phila(Joe) ⇒ (tpayer(Mary) ⇒ phila(Mary)) v

phila(Mary) ⇒ (tpayer(Arnold) ⇒ phila(Arnold)) v

check for all people in universe of discourse

phila(Scott) ⇒ (tpayer(Lisa) ⇒ phila(Lisa))]

$\Longrightarrow$

[tpayer(Joe) ∧ phila(Joe) ⇒ (musician(Mary) ⇒ phila(Mary)) v .... (check Joe and others)

tpayer(Mary) ∧ phila(Mary) ⇒ musician(Arnold) ⇒ phila(Arnold)) v.... (check Mary and others)

continue for checking all people in universe of discourse.

]

* As a result, if the universe of discourse is large, it can be impossible to write every case one by one without quantifiers and variables.

And, this is very impractical.

Question 3)

1. $O(x,y)$: x loves y.

$C(y)$: y dies.

$A(x)$: x is happy.

"Every person is unhappy when his loved one dies."

2. $A(x)$: x is girl.

$H(x)$: x is beautiful.

$B(y,x)$: y sees x

$S(y)$: y is excited

$C(y,x)$: y talks with x.

$D(y)$: y is happy.

" For every beautiful girl, there is someone who is excited to see her or is happy to talk with her."

Question 4)

x = start("aa") $\wedge$ sprecede (ϵ, "ab")

y = start ("aa") $\wedge$ sprecede (ϵ, "ac")

z = start ("aa") $\wedge$ sprecede (ϵ, "bb")

u = start ("aa") $\wedge$ sprecede (ϵ, "cc")

p = sprecede ("aa", "ab")

q = sprecede ("aa", "ac")

r = sprecede ("aa", "bb")

s = sprecede ("aa", "cc")

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$$\boxed{w = x \lor y \lor z \lor u}$$

$$\boxed{\begin{aligned} t = \ & sprecede(\epsilon, \epsilon) \lor sprecede(\epsilon, p) \lor sprecede(\epsilon, q) \\ & \lor sprecede(\epsilon, r) \lor sprecede(\epsilon, s) \end{aligned}}$$

Answer $\boxed{L(G): \quad w \land t \land t \land t \land t \land \ldots\ldots}$

Question 5)

We know that logical entailment holds

i.e. $\Delta \models \varphi$ when models of Δ (premise) is the subset of models of φ (result).

Therefore, clearly, YES, there is a difference about logical entailment for propositional logic and relational logic in terms of decidability.

For example, in propositional logic, for checking logical entailment, we can simply use truth table method to see whether interpretations that hold premise also hold the result or not, and we can easily say that Δ entails φ or Δ does not entail φ. Thus, logical entailment for propositional logic is <u>decidable</u>.

However, in relational logic, this process may not halt. For example,

$$\forall n \ \overbrace{divisibleBySix(n)}^{\Delta} \Rightarrow \overbrace{divisibleByTwo(n) \land divisibleByThree(n)}^{\varphi}$$

and domain is natural numbers i.e. $n \in \{0, 1, 2, 3, 4, 5, \ldots\ldots\}$

Clearly, there is no way to show that $\Delta \models \varphi$ or not in finite amount of time. For this reason, logical entailment for relational logic is <u>undecidable</u>.