Middle East Technical University Department of Computer Engineering

CENG 424

Logic For Computer Science

Fall 2020-2021

Assignment 4

Regulations

1. The homework is due by **25 December 2020**,**23:55**. Late submission is not allowed.
2. Submissions will be via OdtuClass, do not send your homework via e-mail, or do not bring any hardcopy.
3. You can use any typesetting tool (LaTex, Word, etc.) or handwriting while writing the homework. However, you must upload the homework as a pdf file. Other formats will not be considered for grading. A template tex file will be provided to you if you prefer to use LaTex to write your solutions.
4. Name pdf files you will submit as **studentid_hw4** . In case you violate the naming convention, you will receive 5 points penalty (over 100).
5. Send an e-mail to **atakan.garipler@metu.edu.tr** if you need to get in contact.
6. This is an individual homework, which means you have to answer the questions on your own. Any contrary case will be considered as cheating and university regulations about cheating will be applied.

1 Question 1

The following sentences are taken from the Tractacus Logico Philosophicus of Wittgenstein:

1. The world is the totality of facts, not of things.
2. A picture represents a possible situation in logical space.

Your are asked to;

a) Specify each sentence in relational logic. Give definitions of your relation symbols and functions.

b) Specify which variables are free, which ones are bound. Why? Answer seperately for each sentence.

2 Question 2

1. Translate the following argument into a relational logic formula using a suitable set of relation symbols:
   (And give definitions of your relation symbols.)

   If there are any tax payers, then all musicians are tax payers. If there are any philanthropists, then all tax payers are philanthropists. So, if there are any tax-paying philanthropists, then all musicians are philanthropists.

2. Is this formula expressible in ground logic? Why? (If yes, also give the translation.)

3 Question 3

Write sentences (in natural language) that can be represented by the given relational logic sentences. Give definitions of your relation symbols and functions (if used). Write one sentence for each formula.

1. $\forall x D(x, y) \wedge C(y) \Rightarrow \neg A(x)$
2. $\forall x \exists y (A(x) \wedge H(x)) \Rightarrow ((B(y, x) \wedge S(y)) \vee (C(y, x) \wedge Q(y)))$

Example:
Q) $\forall x P(x) \wedge Q(x) \Rightarrow R(x)$
A) Any student who studies logic, gets good grades.
where
P(x): x is a student.
Q(x): x studies logic.
R(x): x gets good grades.

4 Question 4

Let G be a string generator. It works on the alphabet $A = \{a, b, c\}$. Strings that can be generated by G are "$aaab$","$aaac$","$aabb$","$aacc$" and any (finite or infinite) concatenation of these strings. Formally, set of string that G produces can be defined as :

$$L(G) = \{\sigma : \sigma = \text{"}aaab\text{"} \mid \text{"}aaac\text{"} \mid \text{"}aabb\text{"} \mid \text{"}aacc\text{"} \mid \sigma_1 + \sigma_2 \quad where\ \sigma_1 \in L(G),\ \sigma_2 \in L(G)\}$$

where + denotes concatenation.

To illustrate;
"$aaabaaab$","$aabbaabb$","$aaabaaac$","$aaabaaacaabb$","$aaccaaab$","$aaccaabbaaccaaabaaabaabb...$",... are example strings that can be generated by G.

Moreover, you are given two functions; $start(\text{"}aa\text{"})$, a constant function which denotes the beginning of the string, and $sprecede(\sigma_{11}, \sigma_{12})$ which denotes σ_{11} comes right before σ_{12} (i.e. the string is $\sigma_{11}\sigma_{12}$).

Given the restriction that the set of constants you are allowed to use is $\{\epsilon, \text{"}aa\text{"}, \text{"}ab\text{"}, \text{"}ac\text{"}, \text{"}bb\text{"}, \text{"}cc\text{"}\}$ (ϵ: empty string); and only using given functions, write an expression in functional logic that defines $L(G)$:

$start(\text{"}aa\text{"}) \wedge ...$

Remark: You can use the equality symbol (=).

5 Question 5

Compare (logical entailment for) propositional logic and (logical entailment for) relational logic in terms of decidability. Is there a difference? If yes, briefly and intuitively explain the reason behind that difference.