CENG 424

HOMEWORK 3

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Question 1-)

* Case 1-)

p: Your christmas gift is pink.

q: Your christmas gift is black.

r: Your christmas gift is white.

s: Your christmas gift is a panda.

t: Your christmas gift is a cat.

$$(p \wedge s) \vee (q \wedge r \wedge t)$$

* Case 2-)

p: The pandemic will end.

q: The population develops herd immunity.

r: A vaccine is found.

$$(\neg q \vee \neg r) \Rightarrow \neg p$$

* Case 3-)

p: Process A enters the critical region.

q: Process B enters the critical region.

r: The room will lack air.

s: Process C will die.

t: Process C has a variable named air.

$$(p \wedge q) \Rightarrow (r \wedge (\neg t \Rightarrow s))$$

* Case 4-)

p: They will accept your offer.

q: Interest rates go down.

r: Their market share increases.

$$(q \vee r) \Rightarrow p$$

* Case 5-)

p: A formula is valid.

q1: A formula computes T for its first valuation.

q2: A formula computes T. for its second valuation.

q3: A formula computes T for its third valuation.

q4: " " " " " " fourth "

⋮

goes like this

r: A formula is satisfiable.

$$(p \Leftrightarrow \underbrace{(q1 \wedge q2 \wedge q3 \wedge \dots)}_{s}) \wedge (r \Leftrightarrow \underbrace{(q1 \vee q2 \vee q3 \vee \dots)}_{t})$$

s: A formula computes T for all its valuations.

t: A formula computes T for at least one of its valuations.

$$(p \Leftrightarrow s) \wedge (r \Leftrightarrow t)$$

$$\boxed{((p \Rightarrow s) \wedge (s \Rightarrow p)) \wedge ((r \Rightarrow t) \wedge (t \Rightarrow r))}$$

Question 2-)

1.

1.	s	Premise
2.	$p \Rightarrow (s \Rightarrow q)$	Premise
3.	p	Premise (according to Deduction Theorem)
4.	$s \Rightarrow q$	MP 2,3
5.	q	MP 1,4
6.	$\underline{q \wedge s}$	(And introduction) 1,5

2.

1.	$q \Rightarrow p$	Premise
2.	$\neg s \Rightarrow \neg p$	Premise
3.	$p \wedge s \Rightarrow t$	Premise
4.	q	Premise (according to Deduction Theorem)
5.	p	MP 1,4
6.	s	MT 2,5
7.	$p \wedge s$	(And introduction) 5,6
8.	$\underline{t}$	MP 3,7

3.

1.	$p \Rightarrow \neg q$	Premise
2.	$s \Rightarrow t$	Premise
3.	$p \wedge s$	Premise (according to Deduction Theorem)
4.	p	(And elimination) 3
5.	s	(And elimination) 3
6.	$\neg q$	MP 1,4
7.	t	MP 2,5
8.	$\underline{\neg q \wedge t}$	(And introduction) 6,7

Question 3-)

1. Premise : $\neg(p \wedge q)$

Negated Goal : $\neg(\neg p \vee \neg q) \equiv (p \wedge q)$

for premise, we have $\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\{\neg p, \neg q\}$

For negated goal, we have $(p \wedge q)$

$\{p\}$

$\{q\}$

1.	$\{\neg p, \neg q\}$	Premise
2.	$\{p\}$	Negated goal
3.	$\{q\}$	Negated goal
4.	$\{\neg q\}$	1,2
5.	$\{\}$	3,4

Since we reach empty set $\{\}$, we prove the unsatisfiability of negated goal, i.e. we proved the satisfiability of goal.

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2. $(s \Rightarrow t) \wedge ((p \wedge q) \Rightarrow \neg t)$ is premise.

$\equiv (\neg s \vee t) \wedge (\neg(p \wedge q) \vee \neg t)$

$\equiv (\neg s \vee t) \wedge (\neg p \vee \neg q \vee \neg t)$

As clause, we have $\{\neg s, t\}$

$\{\neg p, \neg q, \neg t\}$

$\neg((p \wedge q) \Rightarrow \neg s)$ is negated goal.

$\equiv \neg(\neg(p \wedge q) \vee \neg s)$

$\equiv ((p \wedge q) \wedge s)$

$\equiv (p \wedge q \wedge s)$ As clause, we have $\{p\}$

$\{q\}$

$\{s\}$

1. $\{\neg s, t\}$ Premise

2. $\{\neg p, \neg q, \neg t\}$ Premise

3. $\{p\}$ Negated goal

4. $\{q\}$ Negated goal

5. $\{s\}$ Negated goal

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6. $\{\neg q, \neg t\}$ 2,3

7. $\{\neg t\}$ 4,6

8. $\{t\}$ 1,5

9. $\{\}$ 7,8

Since we reached $\{\}$ empty set, our goal is satisfiable.

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3. $(p \Rightarrow q) \wedge (s \Rightarrow t)$ is premise.

$\equiv (\neg p \vee q) \wedge (\neg s \vee t)$

As clause, we have $\{\neg p, q\}$

$\{\neg s, t\}$

$\neg((p \vee s) \Rightarrow q \wedge t)$ is negated goal.

$\equiv \neg(\neg(p \vee s) \vee q \wedge t)$

$\equiv ((p \vee s) \wedge \neg q \vee \neg t)$ (Distribute $\vee$ over $\wedge$)

$\equiv (\neg t \vee p \vee s) \wedge (\neg t \vee \neg q)$

As clause, we have $\{\neg t, p, s\}$

$\{\neg t, \neg q\}$

1. $\{\neg p, q\}$ Premise

2. $\{\neg s, t\}$ Premise

3. $\{\neg t, p, s\}$ Negated Goal

4. $\{\neg t, \neg q\}$ Negated Goal

By using DPLL:

Choose literal t and simplify

t is T

1. $\{\neg p, q\}$
3. $\{p, s\}$
4. $\{\neg q\}$

5. $\{q, s\}$ 1,3
6. $\{\neg p\}$ 1,4
7. $\{s\}$ 3,6

choose literal p and simplify

p is T

1. $\{q\}$
4. $\{\neg q\}$
5. $\{q, s\}$
7. $\{s\}$

8. $\{\}$ 1,4

Since we reached empty set $\{\}$, our goal is satisfiable.

t is ⊥

1. $\{\neg p, q\}$
2. $\{\neg s\}$

p is ⊥

Question 4-)

* $\neg s \vee \neg r \Rightarrow \neg t$

$(s \wedge r) \vee \neg t$

$(\neg t \vee s) \wedge (\neg t \vee r)$

we have $\{\neg t, s\}$

$\{\neg t, r\}$

* $s \Rightarrow p$

$\neg s \vee p$

we have $\{\neg s, p\}$

* $p \wedge s \Rightarrow q$

$\neg(p \wedge s) \vee q$

$\neg p \vee \neg s \vee q$

we have $\{\neg p, \neg s, q\}$

* $\neg(t \Rightarrow q)$

$\neg(\neg t \vee q)$

$t \wedge \neg q$

we have $\{t\}$

$\{\neg q\}$

Hence,

1. $\{\neg t, s\}$
2. $\{\neg t, r\}$
3. $\{\neg s, p\}$
4. $\{\neg p, \neg s, q\}$
5. $\{t\}$
6. $\{\neg q\}$

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7. $\{s\}$ 1,5
8. $\{r\}$ 2,5
9. $\{\neg p, \neg s\}$ 4,6
10. $\{\neg p\}$ 7,9
11. $\{p\}$ 3,7
12. $\{\}$ 10,11

Question 5-)

We say that $\dfrac{\{p, q\} \quad \{\neg p, t\}}{\{q, t\}}$ in propositional resolution.

If $p = 1$, t must be 1. If $p = 0$, q must be 1. At the end, no matter what p is, resulting clause $\{q, t\}$ is always true. Thus, propositional resolution works.