

CENG 424

Logic For Computer Science

Fall 2020-2021

Assignment 3

Regulations

1. The homework is due by 23:55 on **12 December 2020**. Late submission is not allowed.
2. Submissions will be via OdtuClass, do not send your homework via e-mail, or do not bring any hardcopy.
3. You can use any typesetting tool (LaTeX, Word, etc.) or handwriting while writing the homework. However, you must upload the homework as a pdf file. Other formats will not be considered for grading.
4. Send an e-mail to *atakan.garipler@metu.edu.tr* if you need to get in contact.
5. This is an individual homework, which means you have to answer the questions on your own. Any contrary case will be considered as cheating and university regulations about cheating will be applied.

1 Question 1

Use \neg , \Rightarrow , \wedge and \vee to express the following declarative sentences in propositional logic; in each case state what your respective propositional atoms (p , q , etc.) mean:

1. Your Christmas gift is a pink panda or a black and white cat.
2. The pandemic will not end unless the population develops herd immunity or a vaccine is found.
3. If process A and process B enter the critical region, the room will lack air and process C will die if it does not have a variable named air.
4. They will accept your offer if interest rates go down or their market share increases.
5. A formula is valid iff it computes T for all its valuations; it is satisfiable iff it computes T for at least one of its valuations.

2 Question 2

Using rules of inferences, standard axiom schemata and metatheorems (deduction theorem, substitution theorem and chaining theorem);

1. Given the set of premises $\{s, p \Rightarrow (s \Rightarrow q)\}$ show that $p \Rightarrow (q \wedge s)$ is provable.
2. Given the set of premises $\{q \Rightarrow p, \neg s \Rightarrow \neg p, p \wedge s \Rightarrow t\}$ show that $q \Rightarrow t$ is provable.
3. Given the set of premises $\{p \Rightarrow \neg q, s \Rightarrow t\}$ show that $p \wedge s \Rightarrow \neg q \wedge t$ is provable.

3 Question 3

Use propositional resolution to prove or disprove each of the following sentences:

1. $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q$
2. $(s \Rightarrow t) \wedge ((p \wedge q) \Rightarrow \neg t) \Rightarrow ((p \wedge q) \Rightarrow \neg s)$
3. $(p \Rightarrow q) \wedge (s \Rightarrow t) \Rightarrow ((p \vee s) \Rightarrow q \wedge t)$

4 Question 4

Given $\Delta = \{\neg s \vee \neg r \Rightarrow \neg t, s \Rightarrow p, p \wedge s \Rightarrow q, \neg(t \Rightarrow q)\}$ show that $\Delta \vdash \{\}$ using propositional resolution.

5 Question 5

Why does propositional resolution work? Give a brief intuitive explanation.