ALG2 Assignment 2

Answer the questions in the document. Don’t just give the final answer but show how you got to that answer. If you are solving problems on paper, you may include (readable) pictures of this as well.

Quiz

1. Convert the following Boolean formulas into a CNF:
   a. $((a \wedge b) \vee (e \wedge c \wedge d))$
      $(a \vee e) \wedge (b \vee e) \wedge (a \vee c) \wedge (b \vee c) \wedge (a \vee d) \wedge (b \vee d)$

   b. $(((a \vee x) \wedge (b \vee y)) \vee (c \wedge d))$
      $(a \vee x) \wedge (b \vee y \vee c) \wedge (b \vee y \vee d)$

   c. $(a \vee (b \wedge (c \vee (d \wedge e))))$
      $a \vee b \vee d) \wedge (a \vee b \vee e) \wedge (a \vee c \vee d) \wedge (a \vee c \vee e)$

   d. $(\overline{(a \vee \overline{b})} \vee (\overline{c} \Rightarrow d))$
      $(\neg a \vee d) \wedge (\neg b \vee d) \wedge (\neg c \vee d)$

   e. $(a \Leftrightarrow ((b \wedge c) \Rightarrow \overline{d}))$
      $(\neg a \vee \neg b \vee d) \wedge (\neg a \vee \neg c \vee d) \wedge (\neg (b \wedge c) \vee d \vee a)$

2. Convert the following Boolean formula into 3CNF:
   A 3CNF means that ever clause

   $$\overline{a} \wedge (a \vee \overline{e} \vee f) \wedge (b \vee \overline{c})$$

   Already in 3CNF?????

3. Reducing a problem X to a problem Y in polynomial time implies:
   a. X is at least as hard to solve as Y
   b. Y is at least as hard to solve as X
   c. If X can be solved in polynomial time, then so can Y
   d. If Y can be solve in polynomial time, then so can X

4. Which statement(s) are true:
   a. There can be only one NP-complete problem
   b. If there is a polynomial time algorithm for NP-complete problem(s) then there are polynomial time algorithms for all problems in NP
   c. If Vertex cover is an NP-complete problem, then so is Clique

5. Now that we know that SAT is an NP-complete problem how could we show that vertex cover, independent set and clique problems are NP-complete?

   a. Show that any input for SAT can be transformed into an input for one of these problems
   b. Show that one of these three problems can be expressed as an input to SAT

   c. Show that all three problems can be expressed as an input to SAT

6. Is the following Boolean formula satisfiable?

$$\left(x_1 \vee x_3\right) \wedge \left(x_1 \wedge \left(x_2 \vee \overline{x_3}\right)\right) \wedge \left(\overline{x_2} \vee \overline{x_3}\right) \wedge \left(\overline{x_1} \vee \overline{x_2}\right)$$

No

7. When we talk about running time of a SAT problem which of the following parameters should be taken into account
   a. the length of the (input) formula
   b. the number of the variables in the (input) formula
   c. the number of the times each variable occurs in the (input) formula

8. We are considering the following Boolean formula with 6 variables

$$\left(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6\right) \wedge$$

$$\left(\overline{x_1} \vee \overline{\left(x_2 \vee x_3 \vee x_4 \vee x_5 \vee x_6\right)}\right) \wedge$$

$$\left(\overline{x_2} \vee \overline{\left(x_1 \vee x_3 \vee x_4 \vee x_5 \vee x_6\right)}\right) \wedge$$

$$\left(\overline{x_3} \vee \overline{\left(x_1 \vee x_2 \vee x_4 \vee x_5 \vee x_6\right)}\right) \wedge$$

$$\left(\overline{x_4} \vee \overline{\left(x_1 \vee x_2 \vee x_3 \vee x_5 \vee x_6\right)}\right) \wedge$$

$$\left(\overline{x_5} \vee \overline{\left(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_6\right)}\right) \wedge$$

$$\left(\overline{x_6} \vee \overline{\left(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5\right)}\right)$$

   What is true for this formula (more answers may be correct):
   a. It is not satisfied, i.e. for no assignments of the variables the formula evaluates to true
   b. It has several satisfying assignments
   c. it can only be satisfied if exactly one variable is assigned to true
   d. In general case of n variables the formula has size $O(n^2)$ for n variables

9. When an algorithm is executed on a non-deterministic RAM a configuration (a snapshot) of the RAM is determined (there might be more correct answers):
   a. only by the input
   b. only by the current step of the algorithm
   c. only the current content of the read/write (working) memory
   d. by the input and the current content of the read/write memory
   e. by the input, the current step of the algorithm and the content of the read/write memory

10. A polynomial time algorithm is run on a non-deterministic RAM on an input with length n. How much read/write memory is needed :
    a. exponential in n
    b. <u>polynomial in n</u>
    c. logarithmic in n
    d. constant
    e. depends on the input

11. A polynomial time algorithm is run on a non-deterministic RAM on an input with length n. At every single moment of execution, the non-deterministic RAM is in a certain configuration (snapshot). Which of the following is true (more answers may be correct):
    a. each configuration is in size polynomial in n
    b. there can be exponential numbers of configurations
    c. all configurations together have polynomial size

12. The execution of a polynomial time algorithm on a non-deterministic RAM on input of size n is represented as a Boolean formula according to the method use in the proof of the Cook-Levin theorem. Which properties does the Boolean formula have (there might be more correct answers):
    a. Its size is polynomial in n
    b. it has only one satisfying assignment
    c. every satisfying assignment presents a potential configuration of the non-deterministic RAM during the execution

13. If you can show that SAT is solvable in polynomial time on a deterministic RAM then you:
    a. have proved P=NP
    b. have proved P=/= NP
    c. have proved that P = NPC
    d. None of the above

14. If we can show that one NP-complete problem is solvable in polynomial time on a deterministic RAM then we (there might be more correct answers):
    a. Some problems in NP can be solved in polynomial time
    b. All NP-complete problems are solvable in polynomial time
    c. All problems in NP are solvable in polynomial time

15. What happens if you show that one NP-complete problem is only solvable in exponential time (there might be more correct answers):
    a. Some problems in NP are only solvable in exponential time
    b. All NP-complete problems are only solvable in exponential time
    c. All problems in NP are only solvable in exponential time