# Analysis of Kinematics and Dynamics for a Human-Riding Biped Robot, HUBO FX-1

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## A. Introduction

The biped robot has become a representative research topic in robotics in 21th century, since as compared with 4 legs or 6 legs bio-mimic robot, biped robot can be made shorter and lighter along the walking direction and can turn around, and it requires less actuator. Besides, the biped-robot can provide some tasks, in which the social interaction is emphasized, like lifesaving robot at the site of an accident or provide mobility to the physically impaired.

However, as the biped humanoid robot has drawn increasing interest in the robotics field recently, the practical use of a biped walking robot is still limited due to the difficulty of dynamic balance and locomotion design. In addition, during the single-leg supporting phase, a swinging leg was shaking in the air, thus the landing point became inaccurate and the walking stability is lowered [1].

Therefore, out of the curiosity to understand the manipulation of the biped robot, we selected a human-riding biped robot, HUBO FX-1 (see Fig. 1) as our project model. Based on a simplified model, we want to apply all the analytic tools learned in the course to tackle the problems of forward & inverse kinematics, generation of trajectory, forward dynamics, and the vibration control. In the forward kinematics, we can position the location of the end-effecter (foot) by first knowing the joint angle configuration; in the inverse kinematics, we can solve the joint angle configuration at a desired position of end-effecter; For trajectory generation, we can generate a smooth trajectory of joint angle between given initial and final joint angle configurations; In vibration control, we apply PD control-law to critically damp the vibration magnitude.

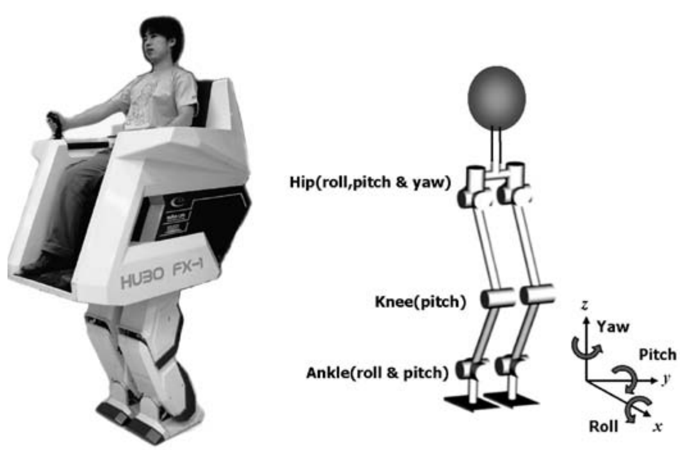



Figure 1. Photograph and joint structure of HUBO FX-1 [1]

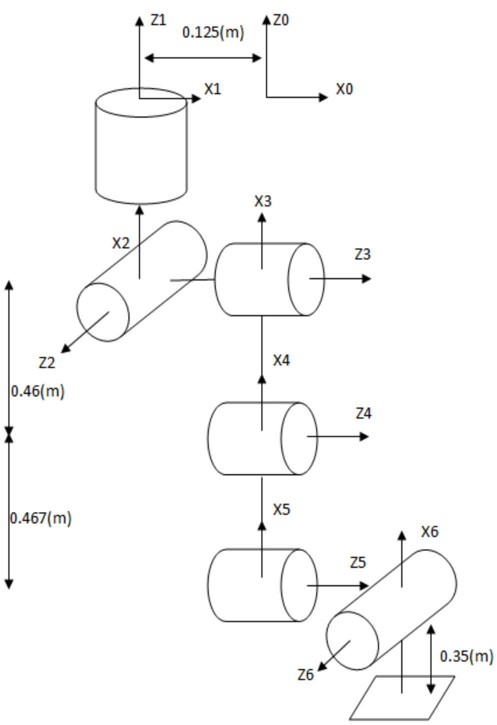
### B. Forward Kinematics

The biped robot in our study has totally 12 degrees of freedom, and each single leg has 6 D.O.F. The 6 D.O.F are achieved by three joints along the single leg: hip, knee, and ankle. As shown in Fig. 1, the 3 rotation axes for the joints are defined as Yaw, Pitch, and Roll.

From the dimension information provided in the Tab.1, we can derive the D-H parameters for a single leg in Fig. 2:

| | | |
|---|---|---|
| Leg | hip (roll/pitch/yaw) | 3 d.o.f. $\times 2 = 6$ d.o.f. |
| | knee (pitch) | 1 d.o.f. $\times 2 = 2$ d.o.f. |
| | ankle (roll/pitch) | 2 d.o.f. $\times 2 = 4$ d.o.f. |
| Total | | 12 d.o.f. |
| Dimensions | length of thigh | 460 mm |
| | length of shank | 467 mm |
| | length between hip joints | 250 mm |
| | width of sole | 230 mm |
| | length of sole | 350 mm |

Table 1. The specification information of the HUBO-FX1 [1].




| | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | ±0.125 | 0 | $\theta_1$ |
| 2 | $\pi/2$ | 0 | 0 | $\pi/2+\theta_2$ |
| 3 | $\pi/2$ | 0 | 0 | $\theta_3$ |
| 4 | 0 | -0.46 | 0 | $\theta_4$ |
| 5 | 0 | -0.467 | 0 | $\theta_5$ |
| 6 | $-\pi/2$ | 0 | 0 | $\theta_6$ |
| 7 | 0 | -0.35 | 0 | $\theta_7$ |

(Length unit: m)

Figure 2. Frame assignment and D-H parameters.

We set our stationary base frame (frame 0) at the central point between two hip joints, and the work frame is at the end-effecter (foot tip). With respect to the base frame, we can derive the transformation matrices for frame 1 to frame 7 using the D-H parameters derived in the Fig. 2. In the Transformation Matrix from frame 7 to frame 0, the upper right entry represents the position of end-effecter as the function of angle configuration, with respect to the base frame.

$$ {}^{0}_{7}T = \prod_{1}^{7} {}^{i-1}_{i}T(\theta) = \begin{bmatrix} {}^{0}_{7}R & \vdots & {}^{0}_{7}P \\ 0 & & 1 \end{bmatrix} $$

The position of foot (w.r.t frame 0)

Therefore the forward kinematics is obtained from ${}^{0}_{7}P$, which describes the transformation relation from joint angle configuration to the end-effecter's position in the work space. Furthermore, by taking the partial derivative of ${}^{0}_{7}P$ with respect to each joint, we can further derive the Jacobian Matrix, which describes the relation between angular velocity of joint angle configuration and linear velocity of the end-effecter, and we will need it for the computation of inverse kinematics later.

$$ J(\theta) = \begin{bmatrix} \dfrac{{}^{0}_{7}P_x}{\partial\theta_1} & & \dfrac{{}^{0}_{7}P_x}{\partial\theta_6} \\ \dfrac{{}^{0}_{7}P_y}{\partial\theta_1} & \cdots\cdots & \dfrac{{}^{0}_{7}P_y}{\partial\theta_6} \\ \dfrac{{}^{0}_{7}P_z}{\partial\theta_1} & & \dfrac{{}^{0}_{7}P_z}{\partial\theta_6} \end{bmatrix} \Longrightarrow \dot{P}(t) = J(\theta)\cdot\dot{\theta}(t) $$

## C. Inverse Kinematics

**Work space:** Before computing the inverse kinematics, we want to have a better idea of the workspace of a single leg, and therefore we can specify a reasonable desired position to compute the corresponding angle configuration. The angle limit for each joint can be summarized as Tab. 2:

| Joint | | Operating range (Degrees) |
|---|---|---|
| Hip | Yaw | -77 to +60 |
| | Pitch | -90 to 90 |
| | Roll | -90 to +38 |
| Ankle | Pitch | -90 to 90 |
| | Roll | -40 to +20 |
| Knee | | 0 to 150 |

Table 2. The range of movement for each joint of the HUBO-FX1 [2].

Based on the information of joint limit, within the range of joint limit, we incremented the angle for each joint in MATAB, and then applied the forward kinematics to generate the position of end-effecter, which will cover the whole workspace of the end-effecter, the results are shown as follows:

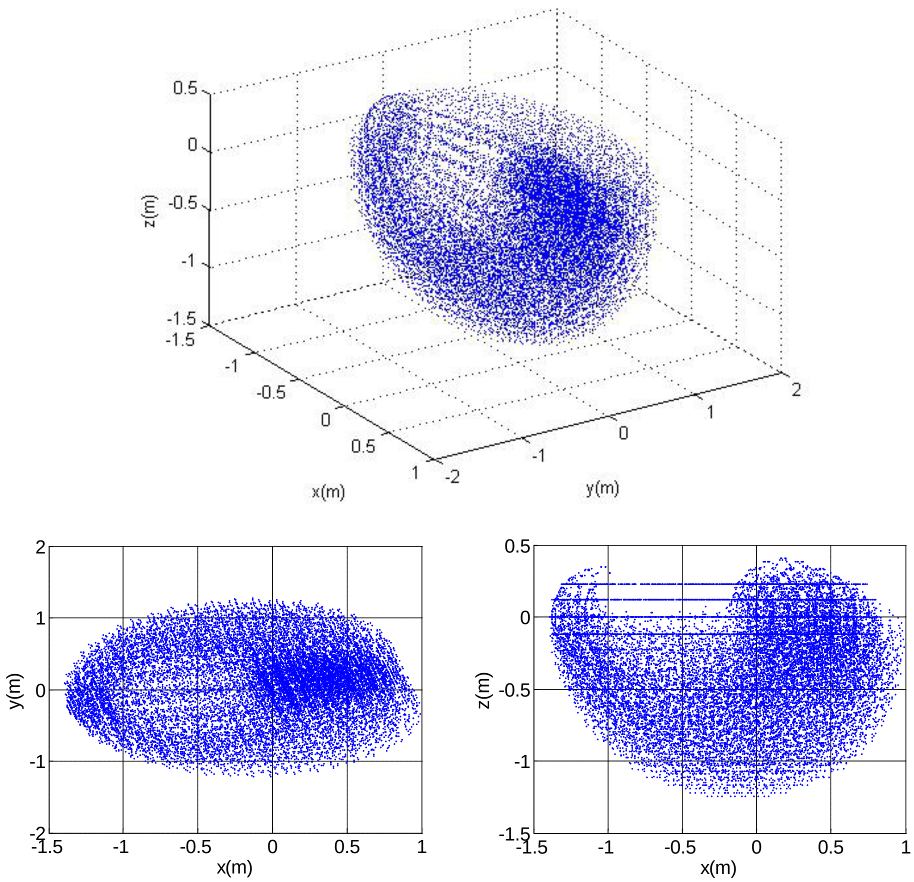



Figure 3. The workspace of a single leg.

In this project, we implemented the inverse kinematics based on inverse Jacobian method, to get the 6 joint angle configurations at certain desired position. To have an analytical form of the end-effecter's position, we apply the linear interpolation to describe the motion of the end-effecter as a straight line function:

$$\vec{P}(t) = (1 - \frac{t}{T})\vec{P}_0 + \frac{t}{T}\vec{P}_{desired}$$

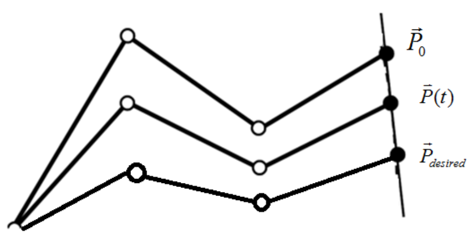



Figure 4. The linear interpolation for inverse kinematics.

Where $\vec{P}_0$ is the initial position of the end-effecter, which is a known position, $\vec{P}_{desired}$ is the desired final position, and T is the total time to achieve that position. If we take the time derivative of the $\vec{P}(t)$, we can get the velocity, which is a constant vector toward the desired position:

$$\dot{\vec{P}}(t) = \frac{1}{T}(\vec{P}_{desired} - \vec{P}_0)$$

After we get the analytic form for the velocity, we can use the inverse Jacobian Matrix to compute angular velocity of each joint.

$$\dot{\vec{\theta}}(t) = J^{-1}(\vec{\theta}(t)) \cdot \dot{\vec{P}}(t)$$

Therefore, starting with the given initial angle configuration at $\vec{P}_0$, we can use the iteration method to compute the $\vec{\theta}(t+\Delta t)$ at each time step :

$$\vec{\theta}(t+\Delta t) = \vec{\theta}(t) + \Delta t \cdot J^{-1}(\vec{\theta}(t)) \cdot \dot{\vec{P}}(t)$$

As the end-effecter reaches the desired position, we can eventually get the angle configuration $\vec{\theta}_d$ at the desired position $\vec{P}_{desired}$ . In our project, since the Jacobian Matrix is not invertible, we applied the Pseudo inverse function in MATLAB to compute the inverse of Jacobian Matrix. The following shows the implementation result when we set the initial position: (-0.125, 0.23, -1.21) to reach the final position (-0.5, -0.6,-0.5).

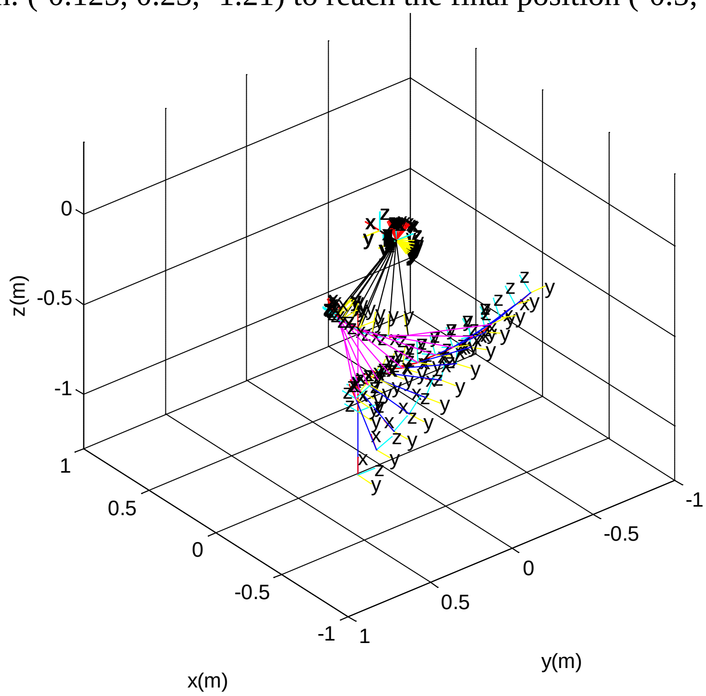



Figure 5. The demonstration of computed angle.

## D. Trajectory Generation and Gait Synthesis

In order to simulate a more realistic pose trajectory, we applied the third-order Hermite polynomial for interpolation. Each term of order n in Hermite polynomial is defined as follows:

$$H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2}$$

For the third order polynomial, we need to find out the coefficients for the terms of order from 0 to 3, and get the angle trajectory by the summation of each term.

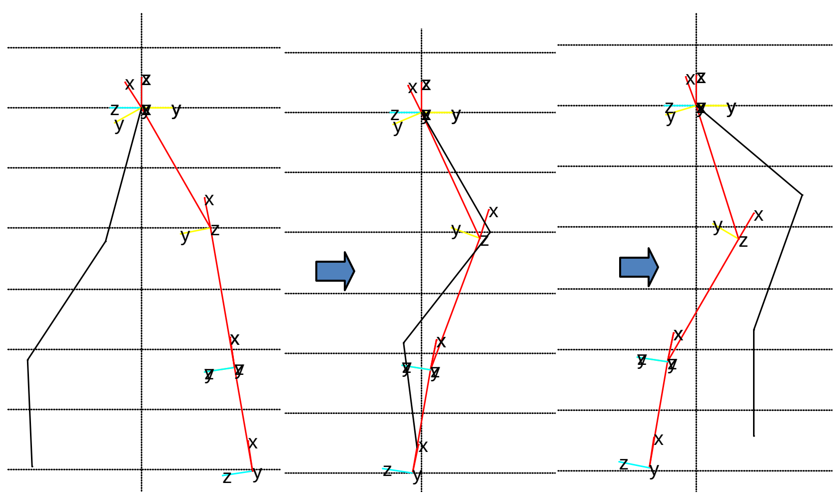
$$\theta(t) = \sum_{k=0}^{3} a_k H_k(t)$$

$$H_0(x) = 1, \quad H_1(x) = x, \quad H_2(x) = x^2 - 1, \quad H_3(x) = x^3 - 3x$$

The four coefficients $a_0, a_1, a_3, a_4$ can be solved by the four boundary conditions: the given initial angle configuration, the final angle configuration, and the zero angular velocity at both initial and desired joint angle configuration:

$$\theta(0) = \theta_0, \theta(t_f) = \theta_f \ ,$$
$$\dot{\theta}(0) = \dot{\theta}(t_f) = 0$$

To simulate the trajectory of end-effecter during walking, we decomposed the walking trajectory into 6 gaits, at each gait the joint angle configuration is shown as follows:



(1) (2) (3)

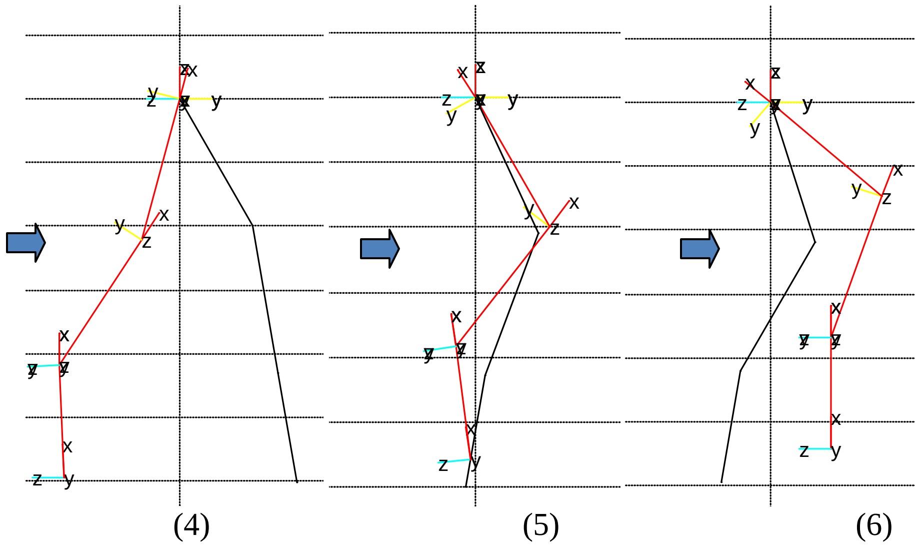


Figure 6. The six decomposed poses during walking motion.

## E. Forward Dynamics

The dynamics system of this biped robot can be either modeled by either Lagrange-Euler equation or calculated by Newton-Euler algorithm. For this 12 degrees of freedom system (or two 6R manipulators), the derivation of analytical dynamics equation by using the Lagrange-Euler equation can be complicated. So in this paper, given a known trajectory, instead of trying to obtain a closed form equations for all the joint torques, we calculated the torques with respect to time numerically by using Newton-Euler algorithm.

Some assumptions were made to simplify the problem.

1. The base frame is not moving. To let the biped robot truly “walking” instead of “swinging” relative to the base frame, we can further attach the base frame to a moving reference frame. But in that case, frictions between the robot feet and the ground should be put into consideration.

2. The links of the robot legs were modeled as solid cylinder. The moment of inertia of the cylinder is

$$I_z = \frac{mr^2}{2}, I_x = I_y = \frac{1}{12}m(3r^2 + h^2).$$

3. For each pose, the desired joint angle configurations are given, and the desired joint angle velocities are set to zero. Complex motion can be decomposed into the movements of several successive poses.

Newton-Euler algorithm can be described as two steps, outward iterations and inward iterations. In outward iterations, we first computed the angular velocity, angular acceleration, linear velocity, and linear acceleration of each link in terms of its preceding link. The initial angular velocity and acceleration of the base frame were zero and assume there was no applied force on the end-effecter. Since gravity force was included, linear acceleration was set to $g$ .These values can be computed in recursive manner, starting from the first moving link and ending at the end-effecter link. The equations can be expressed as follows,

$$i:0\rightarrow 6$$

$$^{i+1}\omega_{i+1}={}^{i+1}_{i}R\,{}^{i}\omega_i+\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1},$$

$$^{i+1}\dot{\omega}_{i+1}={}^{i+1}_{i}R\,{}^{i}\dot{\omega}_i+{}^{i+1}_{i}R\,{}^{i}\omega_i\times\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}+\ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1},$$

$$^{i+1}\dot{v}_{i+1}={}^{i+1}_{i}R\left({}^{i}\dot{\omega}_i\times{}^{i}P_{i+1}+{}^{i}\omega_i\times({}^{i}\omega_i\times{}^{i}P_{i+1})+{}^{i}\dot{v}_i\right),$$

$$^{i+1}\dot{v}_{C_{i+1}}={}^{i+1}\dot{\omega}_{i+1}\times{}^{i+1}P_{C_{i+1}}+{}^{i+1}\omega_{i+1}\times\left({}^{i+1}\omega_{i+1}\times{}^{i+1}P_{C_{i+1}}\right)+{}^{i+1}\dot{v}_{i+1},$$

$$^{i+1}F_{i+1}=m_{i+1}\,{}^{i+1}\dot{v}_{C_{i+1}},$$

$$^{i+1}N_{i+1}={}^{C_{i+1}}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1}+{}^{i+1}\omega_{i+1}\times{}^{C_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1}.$$

In inward iterations, once the velocities and accelerations of the links are found, the joint forces can be computed one link at a time starting from the end-effecter link and ending at the base link.

$$i:7\rightarrow 1$$

$$^{i}f_i={}^{i}_{i+1}R\,{}^{i+1}f_{i+1}+{}^{i}F_i,$$

$$^{i}n_i={}^{i}N_i+{}^{i}_{i+1}R\,{}^{i+1}n_{i+1}+{}^{i}P_{C_i}\times{}^{i}F_i+{}^{i}P_{i+1}\times{}^{i}_{i+1}R\,{}^{i+1}f_{i+1},$$

$$\tau_i={}^{i}n_i^{T}\,{}^{i}\hat{Z}_i.$$

In this paper, the dynamics of the biped robot in four different motions ---“stand”, “Swing”, “Kick” and “Walk” were calculated by using the above algorithm.  A Matlab GUI was built to simulate the forward/inverse kinematics and forward dynamics of these motions.  Both joint variables and end-effecter positions were shown in the textboxes, and the two figures were used to illustrate how the joint torques change during the movement of the robot.

1. “Stand”---initial position or Zero Moment Point (ZMP) in generalized frame. All joint angles are zero. In the simulation result (see Fig. 7), we can see all the join torques are very close to zero.

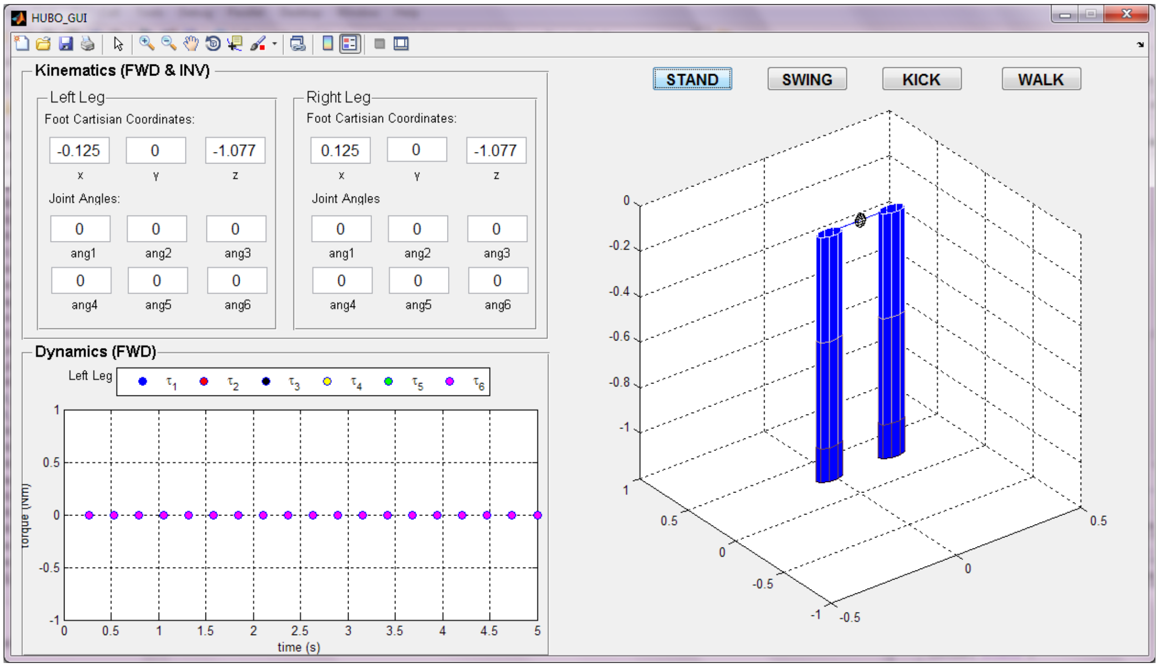



Figure 7. “Stand” motion.

2. “Swing”---forward kinematics. In this motion, the trajectory was generated by given the desired joint configurations (highlighted by the red rectangle). The intuition is that when we swing our leg, we usually have some information about the joint angles of the leg in advance, and after the leg reach the desired joint configurations, we know the foot position in Cartesian coordinates. Although the human’s brain is much intelligent and complicated, we can bring such intuition into the definition of robot motion.

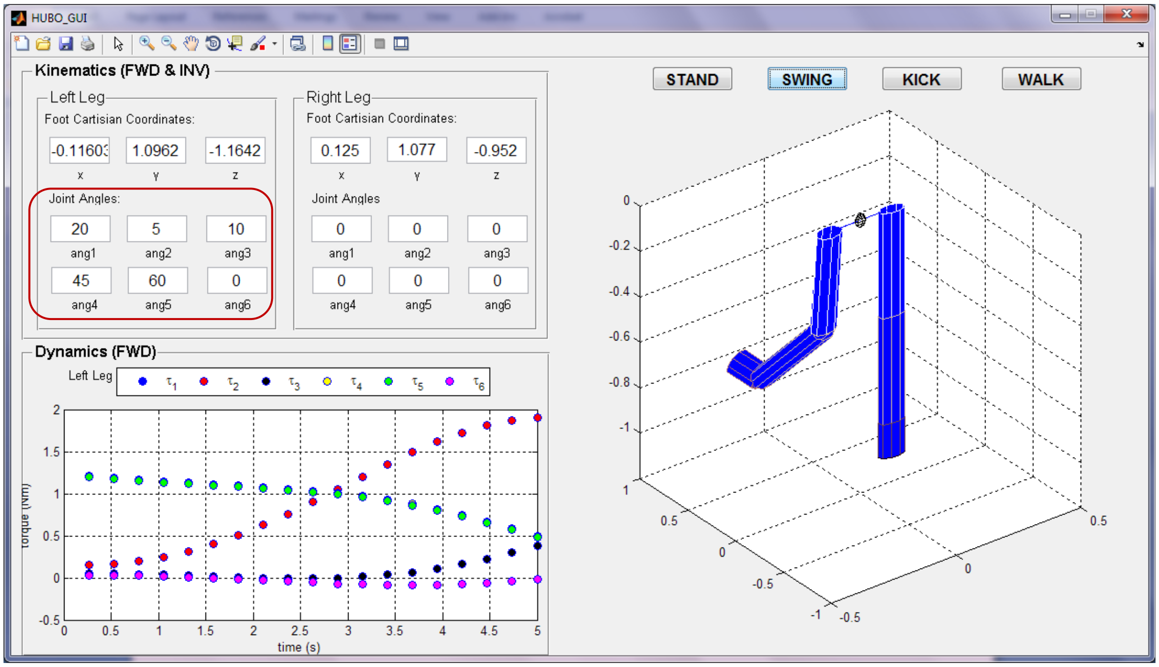



Figure 8. “Swing” motion.

3. “Kick”---inverse kinematics. In this motion, the trajectory was generated by given the desired foot position in Cartesian coordinates (highlighted by the red rectangle). It is like when we kick some target, we know the position of that target first, and then we decide how to “rotate” our leg to reach that target position.

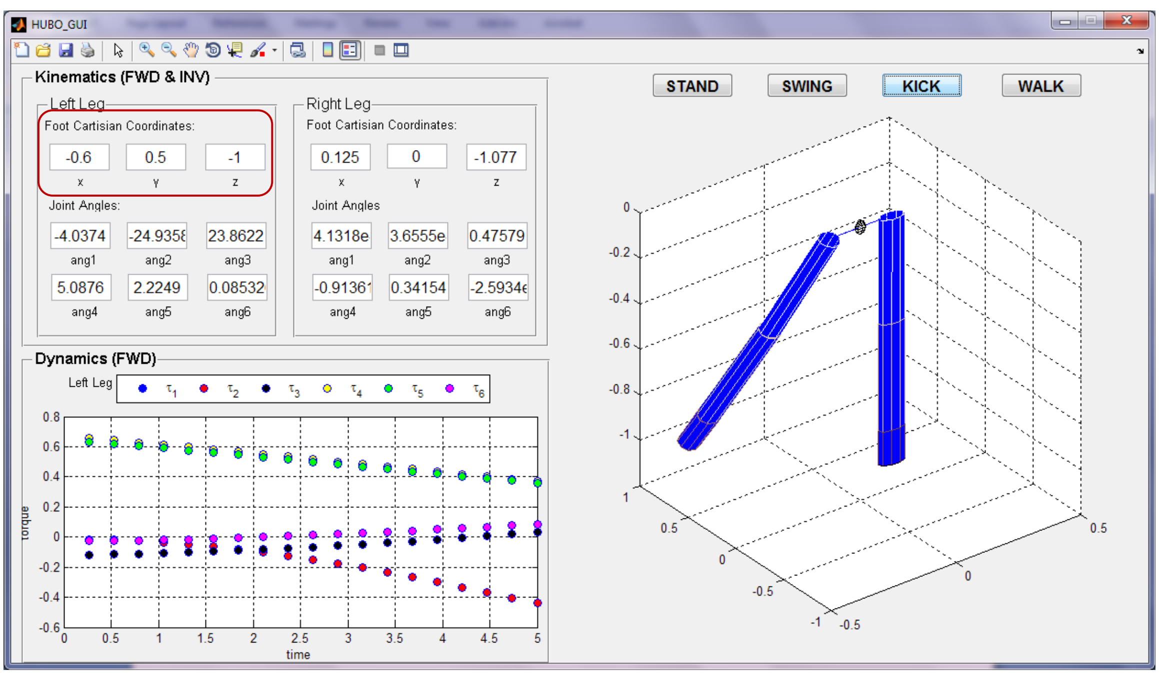



Figure 9. “Kick” motion.

4. “Walk”---a series of pre-defined motion. In each motion, there is one supporting leg and one swinging leg. The six individual poses are shown as follows. We note that only the torques of the left leg were plotted.

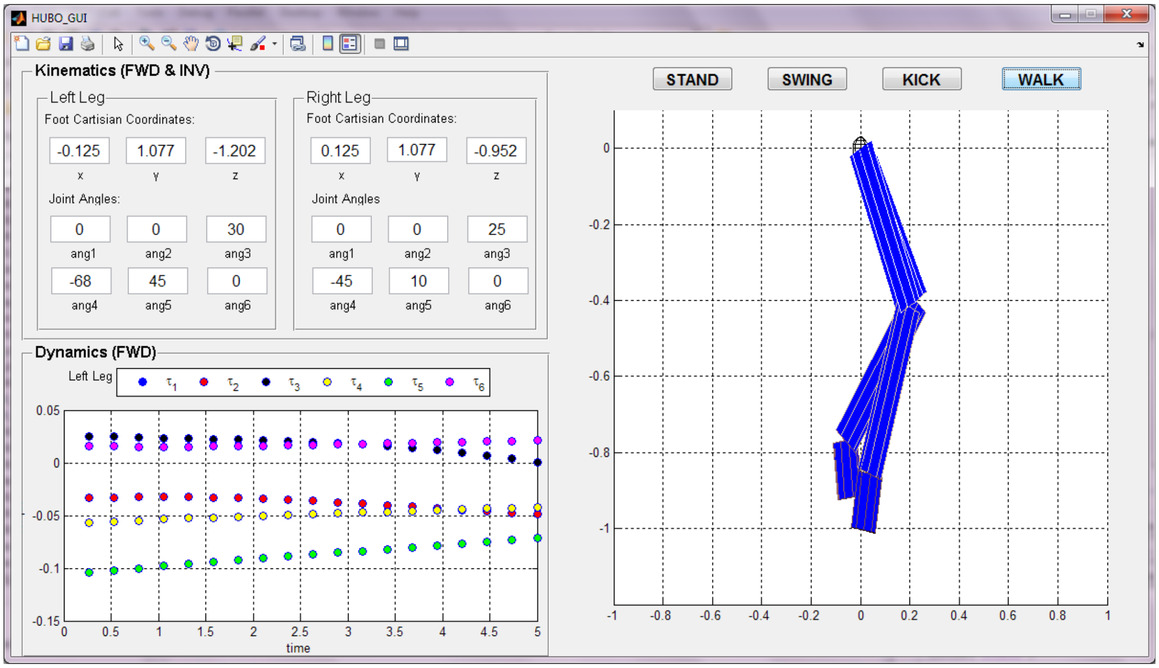



Figure 10. First pose in “Walk” motion.

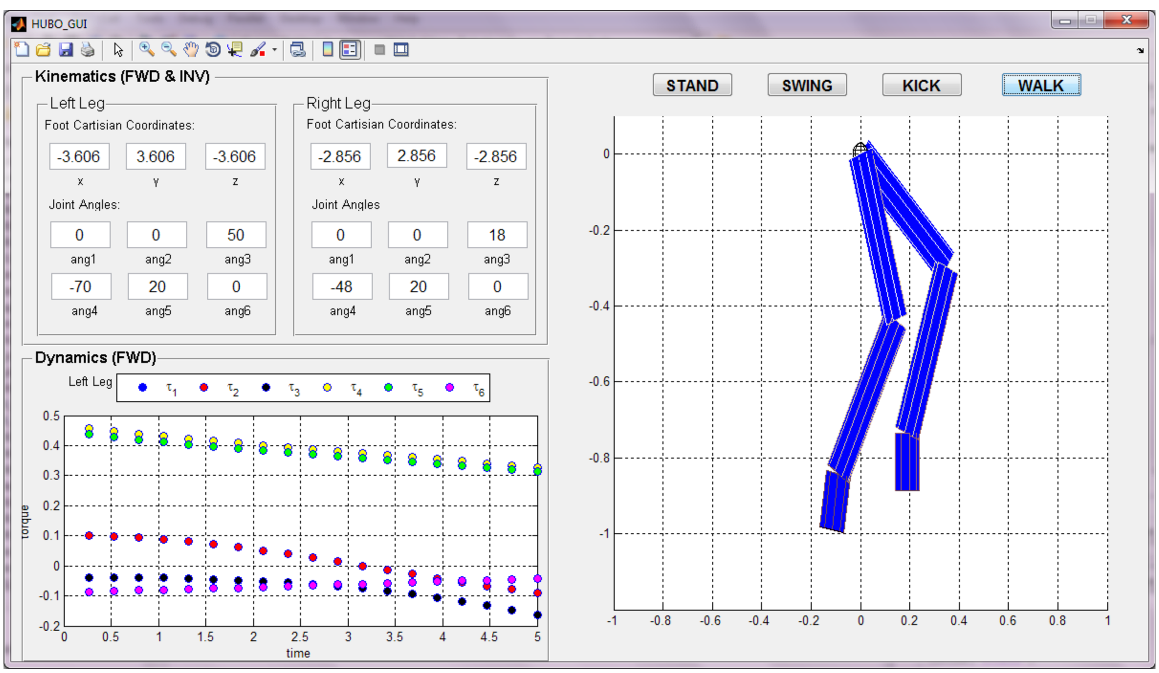



Figure 11. Second pose in “Walk” motion.

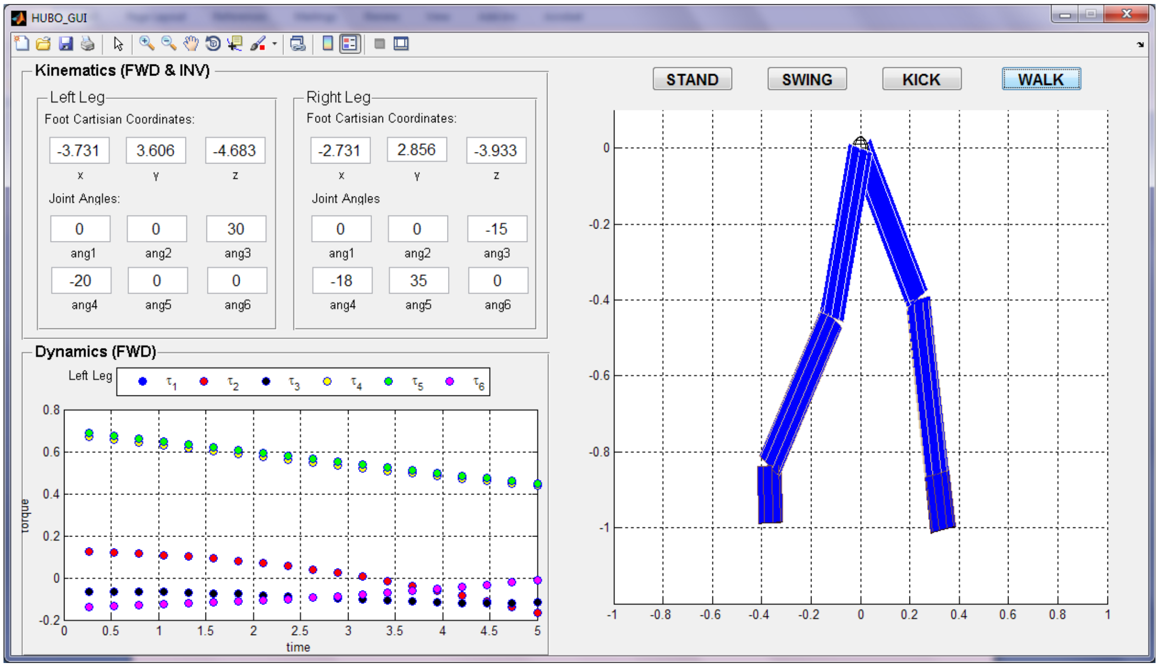



Figure 12. Third pose in “Walk” motion.

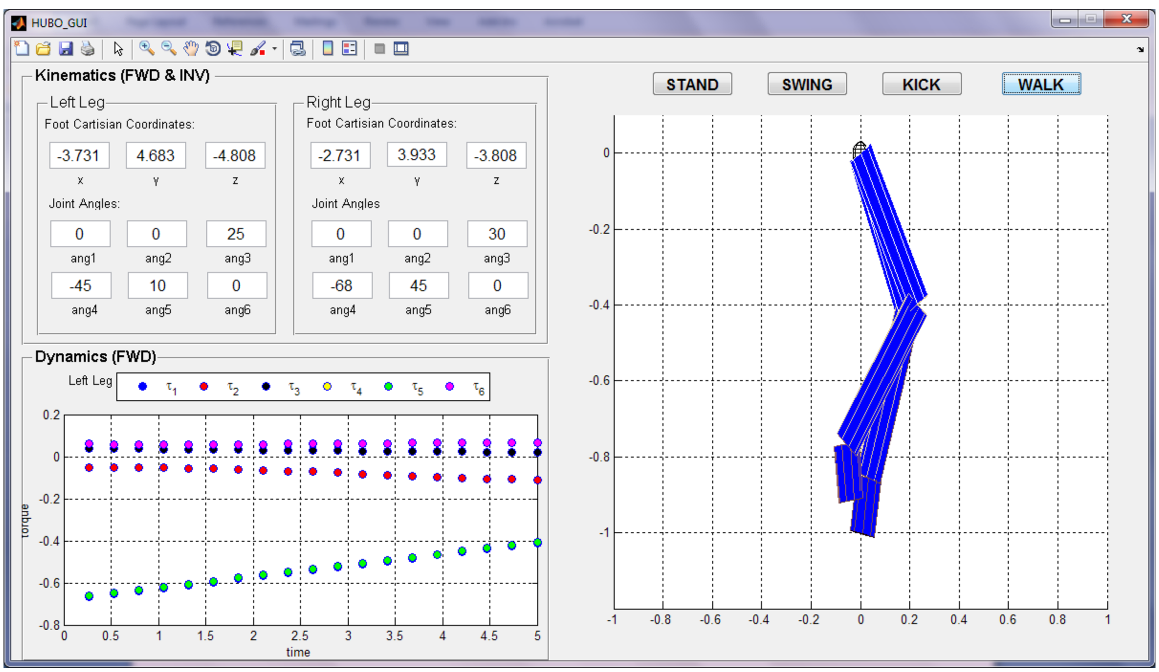



Figure 13. Fourth pose in “Walk” motion.

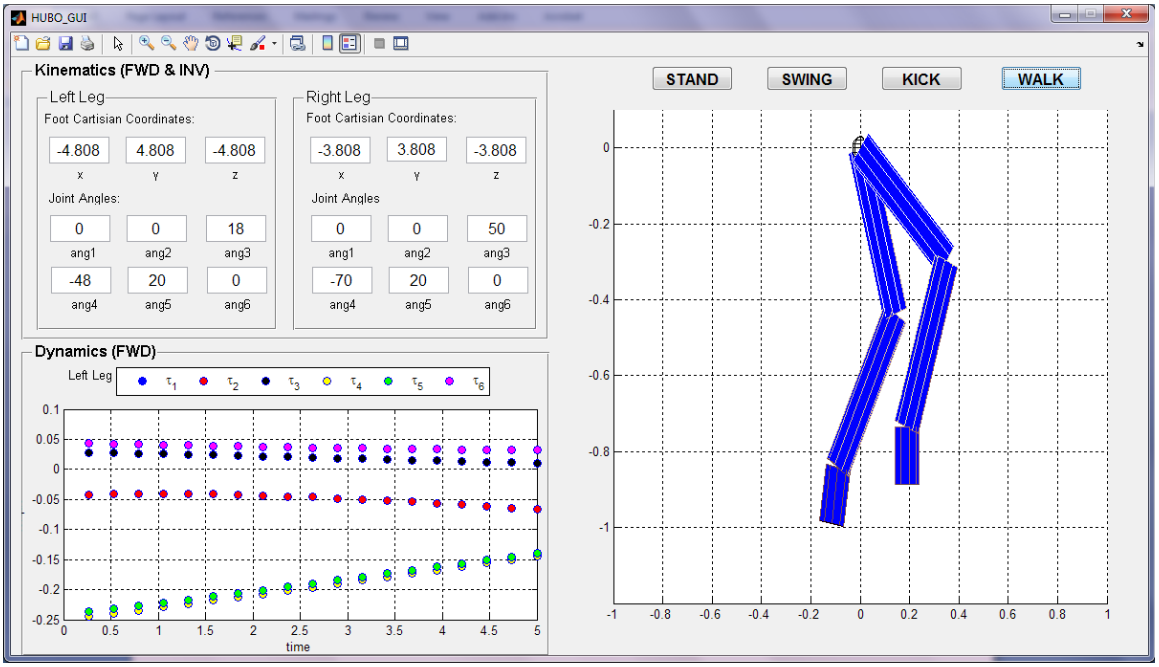



Figure 14. Fifth pose in “Walk” motion.

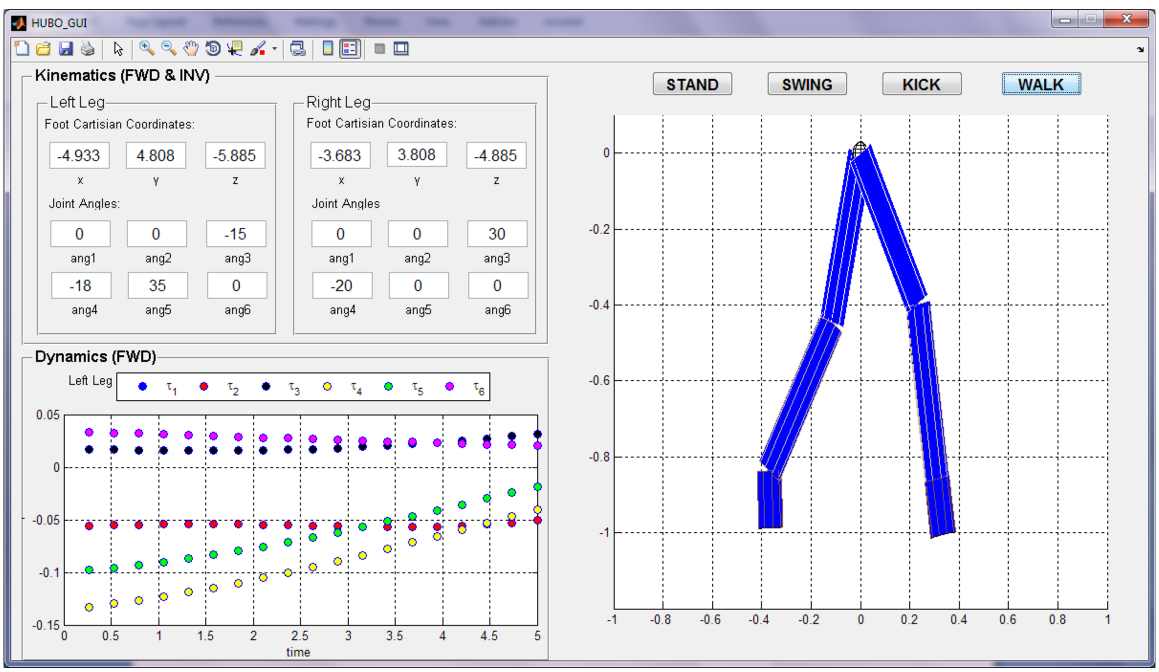



Figure 15. Sixth pose in “Walk” motion.

### F. Vibration Control of the Swinging Leg

Since the body frames of the biped robot are not sufficiently stiff, large structural vibrations are generated during walking. One is the vibration of the swinging leg and the other is the vibration of the torso with respect to the hip joint of the supporting leg. These vibration modes can significantly affect the walking stability and passenger comfort [1]. Therefore, it is very important to reduce the vibrations while walking. In this project, we only study the vibration reduction of the swinging leg.

The vibration of the swinging leg is usually generated due to the hard position control of the joints and flexible frames that connect the leg to the pelvis. The swinging leg is oscillating in rolling and pitching directions during the single support phase [1]. The vibration not only influences the walking stability, but also may cause the swinging foot to collide with the supporting leg. Fig. 16 shows schematics of this vibration.

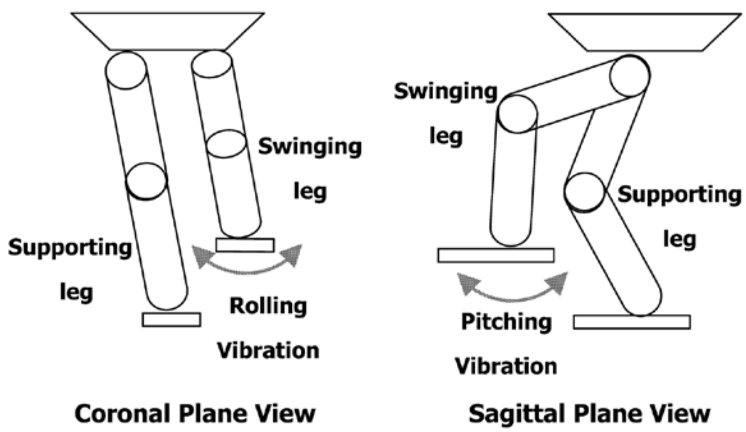



Figure 16. Schematics of the first mode of vibration [1].

The vibration mode of the swinging leg can be described by a spring-mass model. The dynamics equation can be described as

$$ml^2\ddot{\theta} = -(\theta - u)\,.$$

The transfer function of rolling and pitching joints of the hip are written as follows,

$$G_{roll}(s) = \frac{469.4}{s^2 + 496.4}, \text{ and } G_{pitch}(s) = \frac{322.3}{s^2 + 322.3}..$$

A PD controller was designed to achieve the critical damping of the system. The results of vibration reduction were shown in Fig. 17.

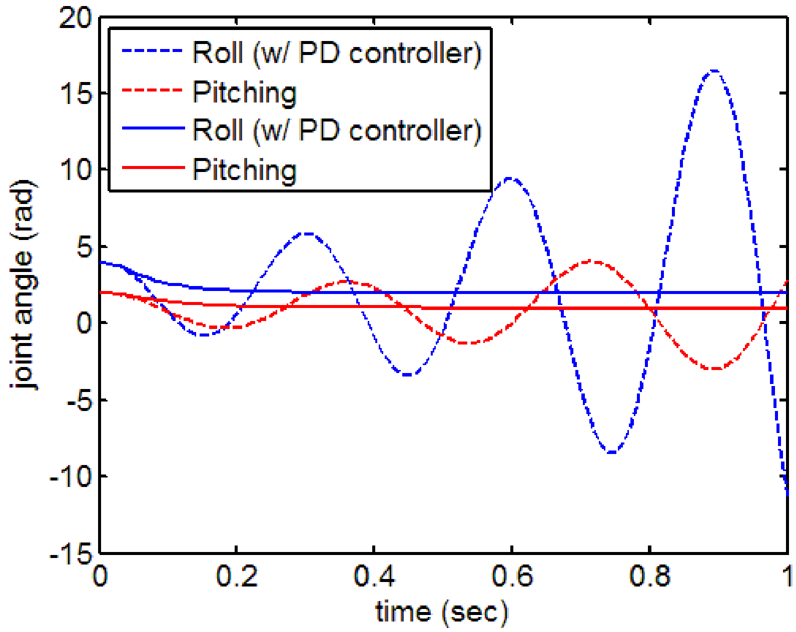



Figure 17. Critically damping roll and pitching angles with the PD controller.

## G. Conclusion

In this project, we studied the forward/inverse kinematics, forward dynamics and vibration control of human-riding biped robot, HUBO FX-1. By given a target joint configuration, the corresponding foot position can be found and vice versa. Trajectory was generated from the initial to the desired configuration by using Hermite polynomials model. The joint torques were calculated numerically by using the Newton-Euler algorithm. To reduce the vibration of the swinging leg, a PD controller was designed, and the critical damping was achieved. The real time balance control and reduction control of other vibration mode were not taken into consideration in this project, but may be studied in the future work.

## Reference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