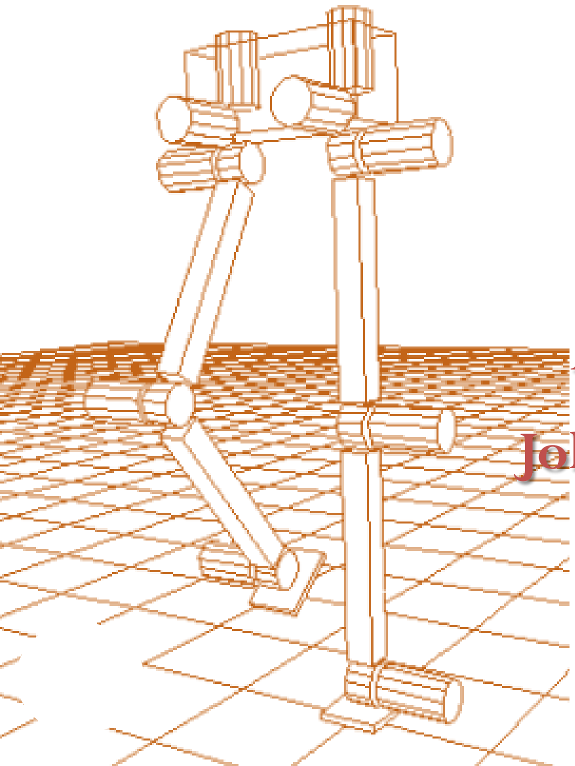
Introduction to Robotics Final Project

Analysis of Dynamics Walking for a Human-riding Biped Robot



Tze-Yuan Cheng and Yan Yan

Mechanical Engineering Department

Johns Hopkins University, Baltimore MD

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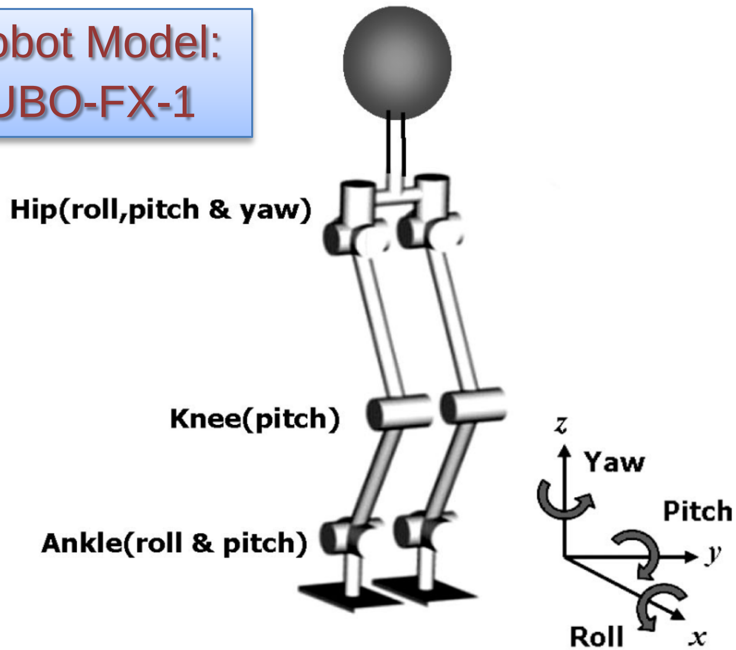
Human-Riding Biped Robot

Robot Model: HUBO-FX-1

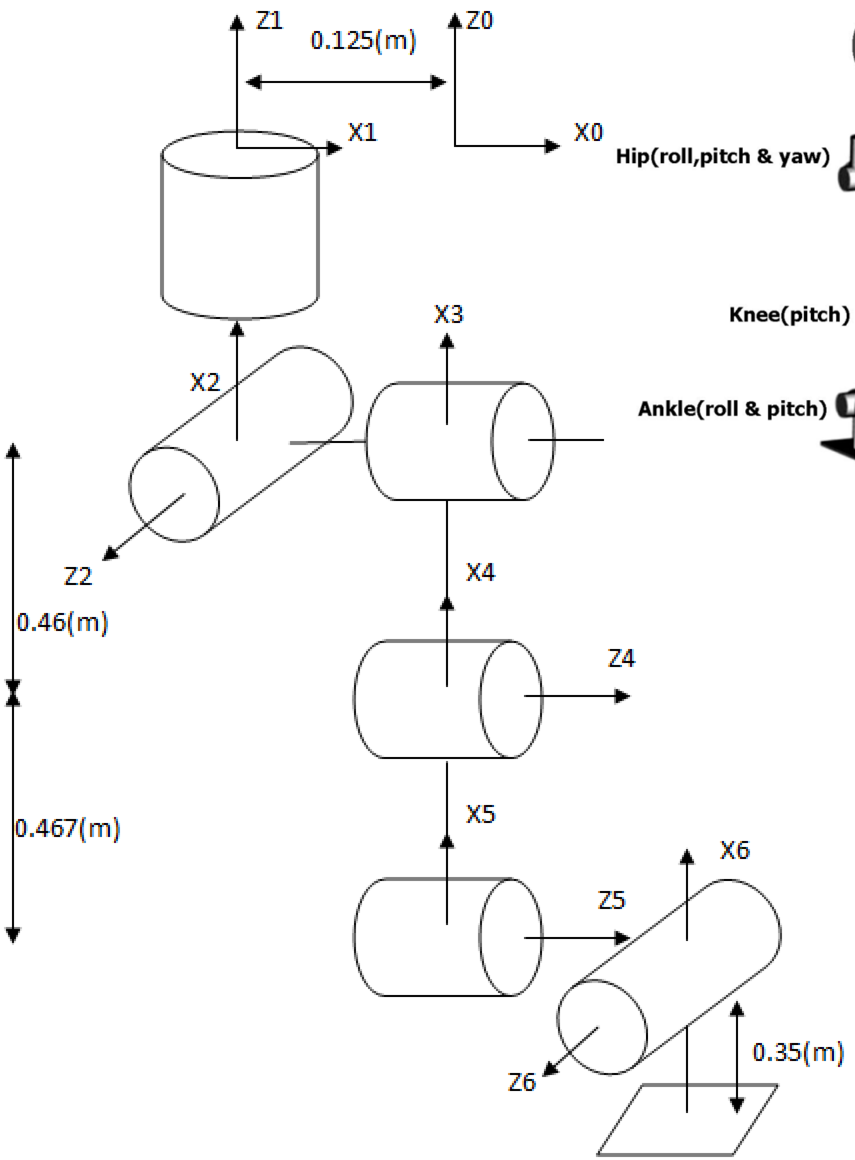


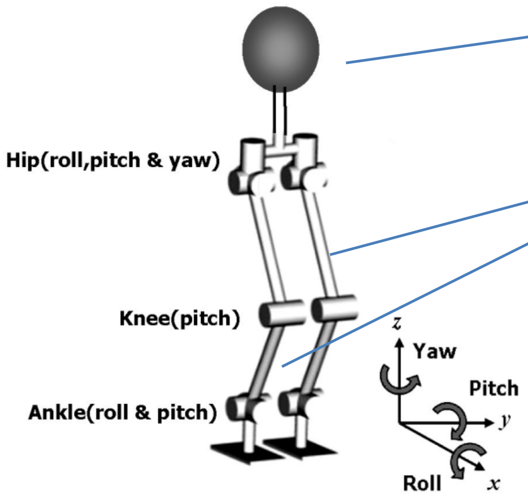
Our project focuses on:

- Kinematics (forward & inverse)
- Dynamics (forward)
- Trajectory generation
- Control (vibration control of the swinging leg)

Leg	hip (roll/pitch/yaw)	3 d.o.f. $\times 2 = 6$ d.o.f.
	knee (pitch)	1 d.o.f. $\times 2 = 2$ d.o.f.
	ankle (roll/pitch)	2 d.o.f. $\times 2 = 4$ d.o.f.
Total		12 d.o.f.
Dimensions	length of thigh	460 mm
	length of shank	467 mm
	length between hip joints	250 mm
	width of sole	230 mm
	length of sole	350 mm

Forward Kinematics





Base frame (assume not moving)

Two legs, two 6R manipulators

Note: During the walking, one is supporting leg, the other is swinging leg. Studying them separately to simplify the problem.

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	±0.125	0	θ_1
2	$\pi/2$	0	0	$\pi/2+\theta_2$
3	$\pi/2$	0	0	θ_3
4	0	-0.46	0	θ_4
5	0	-0.467	0	θ_5
6	$-\pi/2$	0	0	θ_6
7	0	-0.35	0	θ_7

Forward Kinematics (Cont.)

- Transformation Matrix from frame 0 to frame 7 (foot)

$$ {}^{0}_{7}T = \prod_{1}^{7} {}^{i-1}_{i}T(\theta) = \begin{bmatrix} {}^{0}_{7}R & \vdots & {}^{0}_{7}P \\ 0 & & 1 \end{bmatrix} $$

The position of foot w.r.t frame 0

- Jacobian Matrix :

$$ J(\theta) = \begin{bmatrix} \frac{\partial\, {}^{0}_{7}P_x}{\partial \theta_1} & & \frac{\partial\, {}^{0}_{7}P_x}{\partial \theta_6} \\ \frac{\partial\, {}^{0}_{7}P_y}{\partial \theta_1} & \cdots\cdots & \frac{\partial\, {}^{0}_{7}P_y}{\partial \theta_6} \\ \frac{\partial\, {}^{0}_{7}P_z}{\partial \theta_1} & & \frac{\partial\, {}^{0}_{7}P_z}{\partial \theta_6} \end{bmatrix} \longrightarrow \dot{P}(t) = J(\theta) \cdot \dot{\theta}(t) $$

Inverse Kinematics

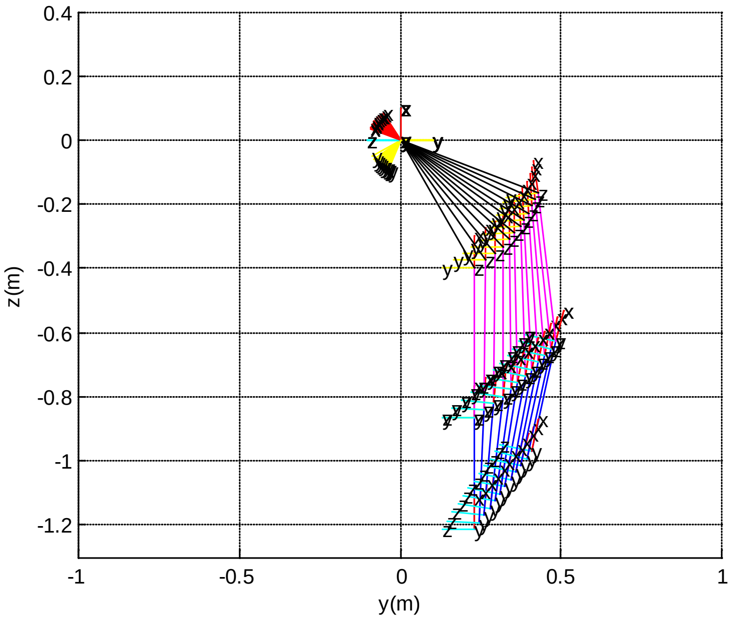
- Goal: Given desired position $\vec{P}_{desired}$ & $\vec{P}_0$ in Cartesian coordinates, find joint angles (generalized coordinates)
- Method: Linear Interpolation (t= 0 to T)

$$\vec{P}(t) = (1-\frac{t}{T})\vec{P}_0 + \frac{t}{T}\vec{P}_{desired} \xrightarrow{\text{Velocity}} \dot{\vec{P}}(t) = \frac{1}{T}(\vec{P}_{desired} - \vec{P}_0)$$

Joint Angle Trajectory:

$$\vec{\theta}(t+\Delta t) = \vec{\theta}(t) + \Delta t \cdot \boxed{J^{-1}}(\vec{\theta}(t)) \cdot \dot{\vec{P}}(t)$$

Pseudo inverse
(underdetermined system) !



Trajectory Generation

- Define walking pose by several nodes
- Generate trajectory by interpolating successive nodes
- Interpolate by using third-order Hermite Polynomials

$$H_n(x) = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2}$$

$$H_0(x) = 1, \quad H_1(x) = x, \quad H_2(x) = x^2 - 1, \quad H_3(x) = x^3 - 3x$$

$$\theta(t) = \sum_{k=0}^{3} a_k H_k(t)$$

From four Boundary Conditions: $\theta(0), \dot{\theta}(0), \theta(t_f), \dot{\theta}(t_f)$

→ Get the coefficients a_0, a_1, a_2 and a_3 for each joint angle

Forward Dynamics

Problem of controlling the manipulator

Given a trajectory point, $\Theta, \dot{\Theta}$, and $\ddot{\Theta}$, find the required vectors of joint torques, τ.

Newton-Euler approach

Step 1: $i : 0 \rightarrow 6$

Outward Iteration:

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\, {}^{i}\omega_{i} + \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1},$$

$$^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\, {}^{i}\dot{\omega}_{i} + {}^{i+1}_{i}R\, {}^{i}\omega_{i} \times \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1},$$

$$^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\dot{\omega}_{i} \times {}^{i}P_{i+1} + {}^{i}\omega_{i} \times ({}^{i}\omega_{i} \times {}^{i}P_{i+1}) + {}^{i}\dot{v}_{i} \right),$$

$$^{i+1}\dot{v}_{C_{i+1}} = {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times \left({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}} \right) + {}^{i+1}\dot{v}_{i+1},$$

$$^{i+1}F_{i+1} = m_{i+1}\, {}^{i+1}\dot{v}_{C_{i+1}},$$

$$^{i+1}N_{i+1} = {}^{C_{i+1}}I_{i+1}\, {}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\, {}^{i+1}\omega_{i+1}.$$

Forward Dynamics (Cont.)

Newton-Euler approach (cont.)

Step 2: $i: 7 \rightarrow 1$

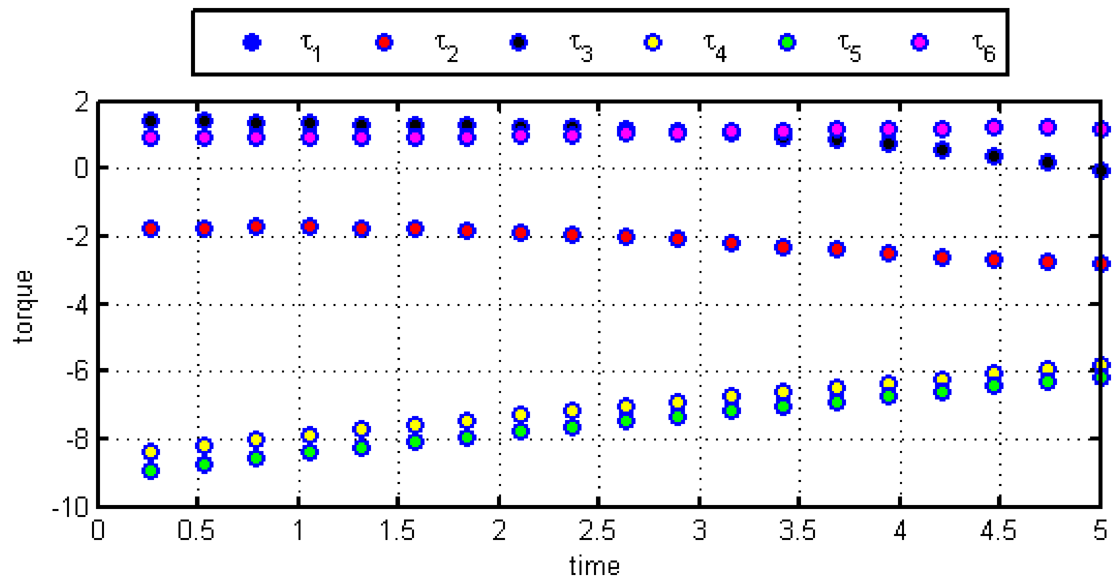
Inward Iteration:

$$^{i}f_{i} = {}^{i}_{i+1}R\,{}^{i+1}f_{i+1} + {}^{i}F_{i},$$

$$^{i}n_{i} = {}^{i}N_{i} + {}^{i}_{i+1}R\,{}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_{i} + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\,{}^{i+1}f_{i+1},$$

$$\tau_{i} = {}^{i}n_{i}^{T}\,{}^{i}\hat{Z}_{i}.$$

One example:
Torque changes during the movement between two poses



Vibration Control of Swinging Leg

In single-leg swing phase, the low damping ratio of hip joint causes vibration, which affect the stability and passenger comfort.

Simplified Mass- Spring Model

$$ml^2\ddot{\theta} = -k(\theta - u)$$

u : control input angle
Θ : actual angle
m : the point mass of a leg
l : distance from the hip joint
k : torsional stiffness of spring

$$G_{\text{roll}}(s) = \frac{469.4}{s^2 + 469.4}, \quad G_{\text{pitch}}(s) = \frac{322.3}{s^2 + 322.3}$$

Applying PD Controller :