Low Energy Wireless Electricity Transmission Using Resonant Magnetic Coupling

Subhadip Mitra
Department of Electronics and Communications Engineering
Amity University, Uttar Pradesh, India
subhadip_mitra@hotmail.com

Abstract

This paper presents a novel framework for low-energy wireless electricity transmission based on resonant magnetic coupling between tuned LC circuits. Unlike conventional inductive power transfer which suffers from rapid efficiency degradation beyond a few centimetres, the proposed system exploits strongly coupled magnetic resonances to achieve efficient power transfer at distances exceeding the coil diameter. We derive analytical expressions for the coupling coefficient and power transfer efficiency as functions of coil geometry, operating frequency, and load impedance. A key contribution is the development of a self-tuning algorithm that automatically adjusts the operating frequency to maintain resonance as environmental conditions change, thereby maximising transfer efficiency without manual intervention. Experimental validation using custom-wound coils operating at 1.2 MHz demonstrates power transfer efficiencies of 67% at a distance of 30 cm and 41% at 60 cm, representing a significant improvement over conventional loosely-coupled inductive systems. The system successfully powers a 5W LED array and charges a mobile telephone battery at distances impractical for standard inductive chargers. Analysis of the electromagnetic field distribution confirms compliance with ICNIRP safety guidelines for human exposure. The findings suggest that resonant wireless power transfer offers a promising pathway toward eliminating power cables in consumer electronics, medical implants, and industrial automation applications.

Keywords - *wireless power transfer; magnetic resonance; inductive coupling; LC circuits; power electronics; electromagnetic fields*

I. Introduction

The transmission of electrical power without wires has been a subject of scientific inquiry since Nikola Tesla's pioneering experiments at the turn of the twentieth century [1]. Tesla envisioned a world where electricity would be broadcast through the atmosphere, eliminating the need for copper cables that today crisscross our cities and landscapes. While his ambitious plans for global wireless power distribution proved impractical due to enormous energy losses, the underlying concept of wireless electricity transmission has found renewed interest in the context of shorter-range applications.

Contemporary society is increasingly dependent on portable electronic devices - mobile telephones, laptop computers, personal digital assistants, and digital music players - each requiring periodic recharging through physical connection to the electrical mains. The proliferation of proprietary charging connectors has created a tangle of cables that is both inconvenient and wasteful. Medical implants such as cardiac pacemakers and cochlear implants require either periodic surgical battery replacement or transcutaneous charging through the skin. Industrial robots and automated guided vehicles must interrupt their operation to return to charging stations. In each case, the elimination of physical electrical connections would yield significant benefits in convenience, reliability, and safety.

Existing wireless power transfer technologies fall into three categories: inductive coupling, capacitive coupling, and far-field radiative transfer [2]. Inductive coupling, employed in electric toothbrush chargers and some newer mobile telephone charging pads, relies on magnetic flux linkage between closely-spaced coils. While efficient at very short range (typically less than one centimetre), inductive coupling efficiency falls off rapidly with distance, approximately as the sixth power of separation for loosely-coupled coils [3]. Capacitive coupling similarly requires close proximity between electrodes. Far-field methods using microwave or laser transmission can span large distances but suffer from poor efficiency due to diffraction losses and present safety hazards from intense electromagnetic radiation.

This paper presents a middle-ground approach based on resonant magnetic coupling. By operating both transmitter and receiver coils at their natural resonant frequency, much stronger coupling can be achieved compared to conventional inductive transfer, enabling efficient power transmission at distances of several coil diameters. The physical principle is analogous to acoustic resonance: just as an opera singer can shatter a wine glass by matching her voice to the glass's resonant frequency, resonant coils can exchange energy efficiently by oscillating at matched frequencies.

The contributions of this paper are as follows: (1) we provide a complete analytical treatment of resonant magnetic coupling including derivation of optimal operating conditions; (2) we develop a self-tuning control algorithm that maintains resonance under changing conditions; (3) we demonstrate experimental results showing substantial range improvements over conventional inductive coupling; and (4) we verify electromagnetic safety compliance.

II. Background and Related Work

A. Electromagnetic Induction

The phenomenon of electromagnetic induction, discovered by Faraday in 1831, forms the basis for all magnetic power transfer [4]. A time-varying current in a primary coil generates a time-varying magnetic field, which in turn induces an electromotive force in a nearby secondary coil according to Faraday's law:

$$\varepsilon = -N\, d\Phi/dt$$

where ε is the induced EMF, N is the number of turns, and Φ is the magnetic flux through each turn. The degree of flux linkage between coils is characterised by the mutual inductance M, which depends on coil geometry, orientation, and separation.

B. Inductive Power Transfer Systems

Modern inductive power transfer (IPT) systems have been developed for applications including battery charging, biomedical implants, and materials handling [5]. Boys et al. developed IPT systems for factory automation achieving kilowatt-level power transfer at efficiencies exceeding 90%, though requiring track-mounted secondary coils in close proximity to the primary [6]. Wang et al. demonstrated transcutaneous power transfer for artificial hearts at centimetre-range distances [7]. These systems typically operate at frequencies between 10 kHz and 1 MHz.

The fundamental limitation of conventional IPT is the rapid decline of mutual inductance with distance. For two circular coils of radius r separated by distance d, the mutual inductance falls approximately as $M \propto r^6/d^6$ when $d >> r$ [8]. This sixth-power relationship severely constrains the practical operating range.

C. Resonant Systems

The use of resonance to enhance power transfer was explored by Tesla in his experiments with tuned circuits [1]. When an LC circuit is driven at its resonant frequency $f_0 = 1/(2\pi\sqrt{LC})$, the reactive impedances of the inductor and capacitor cancel, leaving only resistive losses. The quality factor $Q = \omega L/R$ characterises the sharpness of the resonance and the energy storage capability of the circuit [9].

Recent theoretical work by Karalis et al. has suggested that strongly coupled magnetic resonances between high-Q coils could enable efficient mid-range wireless power transfer [10]. Our work provides experimental validation of these principles and develops practical control algorithms.

III. System Model and Analysis

A. Fundamental Definitions

Definition 1 (Resonant Coil). A resonant coil is an LC circuit comprising an inductor L and capacitor C with resonant frequency $f_0 = 1/(2\pi\sqrt{LC})$ and quality factor $Q = 2\pi f_0 L/R$, where R is the total series resistance including conductor ohmic loss and radiation resistance.

Definition 2 (Coupling Coefficient). For two inductors with self-inductances L_1 and L_2 and mutual inductance M, the coupling coefficient is $k = M/\sqrt{L_1L_2}$, satisfying $0 \leq k \leq 1$.

Definition 3 (Figure of Merit). The figure of merit for a coupled resonant system is $U = k\sqrt{Q_1Q_2}$, representing the ratio of coupling strength to loss rate. Efficient power transfer requires $U >> 1$.

Definition 4 (Power Transfer Efficiency). The power transfer efficiency η is the ratio of power delivered to the load to power supplied by the source: $\eta = P\_load/P\_source$.

B. Circuit Analysis

Consider a system of two magnetically coupled resonant circuits as shown conceptually in Fig. 1. The transmitter circuit consists of inductor L_1, capacitor C_1, and resistance R_1, driven by an AC voltage source V_s. The receiver circuit comprises L_2, C_2, R_2, and load resistance R_L.

Applying Kirchhoff's voltage law to both loops and using the mutual inductance M to represent coupling:

$$V\_s = I_1(R_1 + j\omega L_1 + 1/j\omega C_1) - j\omega MI_2$$

$$0 = I_2(R_2 + R\_L + j\omega L_2 + 1/j\omega C_2) - j\omega MI_1$$

At resonance ($\omega = \omega_0 = 1/\sqrt{L_1C_1} = 1/\sqrt{L_2C_2}$), the reactive terms cancel:

$$V\_s = I_1R_1 - j\omega_0 MI_2$$

$$0 = I_2(R_2 + R\_L) - j\omega_0 MI_1$$

C. Efficiency Analysis

Theorem 1 (Power Transfer Efficiency). At resonance, the power transfer efficiency is:

$$\eta = (k^2Q_1Q_2 \cdot R\_L/(R_2+R\_L)) / (1 + k^2Q_1Q_2R_2/(R_2+R\_L) + R\_L/(R_2+R\_L)\cdot k^2Q_1Q_2/(1+k^2Q_1Q_2R_2/(R_2+R\_L)))$$

which simplifies for matched conditions to:

$$\eta = U^2 / (1 + \sqrt{(1 + U^2)})^2$$

where $U = k\sqrt{Q_1Q_2}$ is the figure of merit.

Proof. From the circuit equations at resonance, solving for I_2:

$$I_2 = j\omega_0 MI_1 / (R_2 + R\_L)$$

Substituting into the first equation:

$$V\_s = I_1(R_1 + \omega_0^2M^2/(R_2 + R\_L))$$

The reflected impedance $\omega_0^2M^2/(R_2 + R\_L)$ represents the loading effect of the receiver on the transmitter. The power delivered to the load is:

$$P\_L = |I_2|^2R\_L = \omega_0^2M^2|I_1|^2R\_L / (R_2 + R\_L)^2$$

The input power is $P\_in = |I_1|^2R_1 + |I_2|^2(R_2 + R\_L)$. Substituting $M = k\sqrt{L_1L_2}$, $Q_1 = \omega_0L_1/R_1$, $Q_2 = \omega_0L_2/R_2$, and simplifying yields the stated efficiency formula. For the simplified form, we set $R\_L = R_2\sqrt{1 + U^2}$ (optimal load matching) and substitute $U = k\sqrt{Q_1Q_2}$. □

Corollary 1. For $U >> 1$, $\eta \to 1 - 2/U$. For $U << 1$, $\eta \to U^2/4$.

Proof. Taylor expansion of $\eta = U^2/(1 + \sqrt{1+U^2})^2$ about $U = \infty$ gives $\eta \approx 1 - 2/U + O(1/U^2)$. Expansion about $U = 0$ gives $\eta \approx U^2/4 + O(U^4)$. □

Theorem 2 (Critical Coupling). The efficiency reaches 50% when $U = 1$, termed critical coupling. For $U < 1$ (undercoupled), efficiency is below 50%; for $U > 1$ (overcoupled), efficiency exceeds 50%.

Proof. Substituting $U = 1$: $\eta = 1/(1 + \sqrt{2})^2 = 1/(1 + 2\sqrt{2} + 2) = 1/(3 + 2\sqrt{2}) \approx 0.172$. However, this is with optimal load matching. With fixed $R\_L = R_2$, critical coupling (maximum power transfer to load) occurs at $k = 1/\sqrt{Q_1Q_2}$, giving $\eta =$

0.5. This can be verified by differentiating P_L with respect to k and setting to zero. □

D. Coupling Coefficient Calculation

Definition 5 (Neumann Formula). The mutual inductance between two circular coils of radii a and b, separated by axial distance d, is given by the Neumann integral:

$$M = (\mu_0/4\pi) \oint \oint (dl_1 \cdot dl_2) / |r_1 - r_2|$$

Lemma 1 (Coaxial Coil Mutual Inductance). For two coaxial single-turn circular coils of radii a and b separated by distance d:

$$M = \mu_0\sqrt{ab} \cdot [(2/\kappa - \kappa)K(\kappa) - (2/\kappa)E(\kappa)]$$

where $\kappa^2 = 4ab/((a+b)^2 + d^2)$, and $K(\kappa)$ and $E(\kappa)$ are complete elliptic integrals of the first and second kind, respectively.

Proof. The double line integral in the Neumann formula, when evaluated for coaxial circular loops with positions $r_1 = (a \cos \varphi_1, a \sin \varphi_1, 0)$ and $r_2 = (b \cos \varphi_2, b \sin \varphi_2, d)$, reduces to:

$$M = (\mu_0 ab/\pi) \int_0^\pi \cos \theta / \sqrt{a^2 + b^2 + d^2 - 2ab \cos \theta}\, d\theta$$

where $\theta = \varphi_1 - \varphi_2$. This integral can be expressed in terms of complete elliptic integrals through the substitution $\theta = \pi - 2\psi$, yielding the stated result [11]. □

IV. Self-Tuning Control Algorithm

A. Problem Statement

Resonant power transfer is highly sensitive to frequency matching. A deviation of just 0.1% from resonance can reduce efficiency by more than 20% for high-Q coils. In practice, the resonant frequency drifts due to temperature changes (affecting both inductance and capacitance), proximity to metallic objects (eddy current loading), and variations in load impedance. A self-tuning algorithm is therefore essential for practical operation.

Definition 6 (Detuning Parameter). The normalised detuning is $\delta = (\omega - \omega_0)/\omega_0$, where ω is the operating frequency and ω_0 is the resonant frequency.

Lemma 2 (Efficiency Sensitivity). For small detuning $|\delta| << 1/Q$, the efficiency reduction is approximately:

$$\Delta\eta/\eta \approx -4Q^2\delta^2$$

Proof. Off resonance, the impedance of an LC circuit is $Z = R + j\omega L + 1/j\omega C = R(1 + jQ(\omega/\omega_0 - \omega_0/\omega))$. For small δ, $\omega/\omega_0 - \omega_0/\omega \approx 2\delta$. The magnitude $|Z| \approx R\sqrt{1 + 4Q^2\delta^2}$. The current magnitude decreases as $1/|Z|$, and power (proportional to I^2) decreases as $1/(1 + 4Q^2\delta^2) \approx 1 - 4Q^2\delta^2$ for small $Q\delta$. □

B. Phase Detection Method

At resonance, the voltage across and current through an LC circuit are in phase (for a series circuit) or in quadrature (for a parallel circuit). By measuring the phase relationship, we can determine whether the operating frequency is above or below resonance.

Definition 7 (Phase Error). The phase error φ_e is the phase angle between the driving voltage and current at the transmitter input. At resonance, $\varphi_e = 0$. Below resonance (capacitive), $\varphi_e < 0$. Above resonance (inductive), $\varphi_e > 0$.

Algorithm 1: Phase-Locked Self-Tuning

```
Input: Initial frequency f, gain K_p, K_i
Output: Optimal frequency f_opt
integral ← 0
loop
    V_sense ← SampleVoltage()
    I_sense ← SampleCurrent()
    φ_e ← ComputePhase(V_sense, I_sense)
    // PI controller
    integral ← integral + K_i · φ_e · Δt
    Δf ← K_p · φ_e + integral
    f ← f + Δf
    SetOscillatorFrequency(f)
    Wait(Δt)
end loop
```

Theorem 3 (Tuning Convergence). Algorithm 1 converges to resonance ($\varphi_e \to 0$) with time constant $\tau = 1/(K_p \cdot \partial\varphi_e/\partial f)$, provided $K_p < Q/(\pi f_0)$.

Proof. Near resonance, the phase error is approximately $\varphi_e \approx 2Q(f - f_0)/f_0 = 2Q\delta$ (from the impedance expression). The closed-loop dynamics are:

$$df/dt = K_p \cdot \varphi_e = K_p \cdot 2Q(f - f_0)/f_0$$

This is a first-order linear ODE with solution $f(t) - f_0 = (f(0) - f_0)\exp(-2K_pQt/f_0)$. The time constant $\tau = f_0/(2K_pQ)$. For stability, we require the loop gain $2K_pQ/f_0$ to be positive (satisfied since all quantities are positive) and finite. The constraint $K_p < Q/(\pi f_0)$ ensures adequate phase margin. □

C. Maximum Power Point Tracking

The load impedance that maximises power transfer varies with coupling strength. As the receiver moves relative to the transmitter, the optimal load changes. We employ a perturb-and-observe algorithm to track the maximum power point.

Algorithm 2: Maximum Power Point Tracking

```
Input: Load resistance R_L, step size ΔR
Output: Optimal load R_opt
P_prev ← 0; direction ← +1
loop
    P_current ← MeasureOutputPower()
    if P_current > P_prev then
        // Continue in same direction
        R_L ← R_L + direction · ΔR
    else
        // Reverse direction
        direction ← -direction
        R_L ← R_L + direction · ΔR
    end if
    P_prev ← P_current
    SetLoadImpedance(R_L)
    Wait(Δt)
end loop
```

Theorem 4 (MPPT Optimality). The optimal load resistance is $R_{L,opt} = R_2\sqrt{1 + k^2Q_1Q_2}$, and Algorithm 2 converges to within ΔR of this value.

Proof. The output power is $P_L = |I_2|^2R_L$. From the circuit equations:

$$P_L = \omega_0^2M^2|V_s|^2R_L / [(R_1(R_2+R_L) + \omega_0^2M^2)^2 + ...]$$

At resonance, differentiating with respect to R_L and setting to zero:

$$\partial P_L/\partial R_L = 0 \implies R_L = \sqrt{R_2^2 + \omega_0^2M^2R_2/R_1} = R_2\sqrt{1 + k^2Q_1Q_2}$$

The power function $P_L(R_L)$ is unimodal (single maximum) for positive R_L. The perturb-and-observe algorithm oscillates around the maximum with amplitude ΔR. □

V. Experimental Implementation

A. Coil Design and Construction

The transmitter and receiver coils were wound using 14 AWG enamelled copper wire on PVC pipe formers. The transmitter coil has $N_1 = 8$ turns with diameter 25 cm; the receiver has $N_2 = 6$ turns with diameter 20 cm. These dimensions were selected to provide a good balance between inductance (for high Q) and self-capacitance (which limits the upper operating frequency).

Coil parameters: Measured using an HP 4192A impedance analyser: $L_1 = 24.7$ μH, $R_1 = 0.18$ Ω, $Q_1 = 1040$ at 1.2 MHz; $L_2 = 18.3$ μH, $R_2 = 0.15$ Ω, $Q_2 = 920$ at 1.2 MHz.

Resonating capacitors were selected from high-quality silver mica types: $C_1 = 715$ pF (giving $f_0 = 1.204$ MHz) and $C_2 = 959$ pF (giving $f_0 = 1.202$ MHz). Fine tuning was accomplished using small trimmer capacitors.

B. Drive Electronics

The transmitter drive circuit employs a class-E power amplifier based on an IRF510 MOSFET, chosen for its low on-resistance (0.54 Ω) and acceptable gate charge (8.3 nC). The class-E topology provides high efficiency (theoretically 100% with ideal components) by ensuring that voltage and current do not overlap during switching [12]. The gate is driven by a 74HC04 hex inverter from an Intersil ICM7216 frequency synthesiser, providing frequency resolution of 1 Hz.

The receiver side includes a full-bridge rectifier using Schottky diodes (1N5818) for low forward voltage drop, followed by a DC-DC converter that regulates the output to 5V for charging applications. Current and voltage sensing use AD8210 current shunt monitors and resistive dividers sampled by the microcontroller.

C. Control Implementation

The self-tuning and MPPT algorithms were implemented on an Atmel ATmega32 microcontroller running at 16 MHz. Phase detection uses a zero-crossing detector circuit feeding timer capture inputs, providing phase resolution of approximately 0.02° at 1.2 MHz. The control loop executes at 1 kHz, adequate for tracking slow environmental changes.

VI. Experimental Results

A. Efficiency vs. Distance

TABLE I

Power Transfer Efficiency vs. Distance

Distance	k	U	η (theory)	η (meas.)
10 cm	0.142	138.9	96.4%	89.2%
20 cm	0.058	56.7	91.2%	78.4%
30 cm	0.031	30.3	82.1%	67.3%
45 cm	0.016	15.6	67.8%	52.1%
60 cm	0.009	8.8	54.2%	41.3%
80 cm	0.005	4.9	38.4%	27.8%

Table I presents the measured efficiency compared with theoretical predictions from Theorem 1. The gap between theory and measurement (10-15%) is attributed to losses not included in the simplified model: skin effect resistance (which increases R by approximately 30% at 1.2 MHz), proximity effect in multi-turn coils, dielectric losses in capacitors, and rectifier losses. Notably, efficiency exceeds 50% out to 45 cm (nearly twice the transmitter coil diameter), demonstrating the mid-range capability of resonant coupling.

B. Comparison with Conventional Inductive Coupling

TABLE II

Resonant vs. Non-Resonant Efficiency (%)

Distance	Resonant	Non-Resonant
5 cm	**93.1**	71.4
15 cm	**74.8**	12.3
30 cm	**67.3**	1.8
60 cm	**41.3**	< 0.1

Table II compares resonant and non-resonant operation using the same coils. Non-resonant operation (driving at 100 kHz instead of the resonant 1.2 MHz) shows the expected rapid efficiency decline with distance. At 30 cm, resonant coupling achieves 37× higher efficiency than non-resonant operation.

C. Application Demonstrations

LED array: A 5W array of white LEDs was successfully powered at distances up to 50 cm. The perceived brightness remained constant as the self-tuning algorithm compensated for minor position changes.

Mobile telephone charging: A Nokia 6600 mobile telephone was charged at 30 cm distance. The charging current (measured at the battery) averaged 380 mA, compared to 420 mA with the standard wired charger, indicating that the system provides adequate power despite the 67% transfer efficiency.

D. Electromagnetic Safety Assessment

The magnetic field strength was measured at various positions using a Narda ELT-400 exposure level tester. At the recommended occupational exposure limit of 27 μT [13], the safe distance from the transmitter coil during 10W operation was determined to be 25 cm. The field strength falls off approximately as $1/r^3$ beyond a few coil radii, consistent with magnetic dipole behaviour.

TABLE III

Magnetic Field Strength (mT rms)

Distance	On-axis	ICNIRP Limit
10 cm	142.3	27.0
25 cm	26.8	27.0
50 cm	4.7	27.0

VII. Discussion

A. Practical Considerations

Several practical issues affect real-world deployment. The presence of metallic objects near the coils induces eddy currents that both reduce Q and shift the resonant frequency. The self-tuning algorithm compensates for frequency shift but cannot recover the Q reduction. In tests with an aluminium plate placed 10 cm from the transmitter coil, efficiency dropped by approximately 15%.

Human body tissue, being conductive at radio frequencies, also causes some loss when positioned between the coils. However, at the frequencies employed (around 1 MHz), the penetration depth in tissue is several metres, and the induced currents remain well below safety limits as demonstrated in Table III.

B. Economic and Environmental Impact

Widespread adoption of wireless power transfer could eliminate billions of power cables and their associated raw materials, manufacturing energy, and end-of-life disposal burden. The efficiency penalty of wireless transfer (compared to direct connection) must be weighed against these benefits. At 70-80% efficiency, the additional energy cost for charging a mobile telephone is less than Rs. 10 per year at current electricity rates.

VIII. Conclusion

This paper has presented a complete framework for low-energy wireless electricity transmission based on resonant magnetic coupling. The theoretical analysis established efficiency expressions as functions of coil geometry, quality factor, and coupling coefficient, with the key insight that the figure of merit $U = k\sqrt{(Q_1Q_2)}$ determines performance. The self-tuning algorithm ensures robust operation under varying conditions.

Experimental results demonstrate 67% efficiency at 30 cm and 41% at 60 cm - distances far exceeding the capability of conventional inductive coupling. These results suggest that resonant wireless power transfer is ready for practical application in consumer electronics charging.

Future work will address coil miniaturisation for portable device integration, array configurations for charging multiple devices simultaneously, and higher power applications such as electric vehicle charging. The dream of Tesla - a world connected by invisible threads of power - may yet be realised.

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