

# Graph Signal Processing Exercise Set 1

## Graphs - Signals - Shifts

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### Learning Goals

By the end of this exercise, you should be able to...

- Identify graph signals
- Understand alternate graph shift normalizations

### Image signals

Explain how (greyscale) images can be interpreted as signals on a rectangular graph. That is,

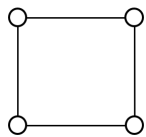
- (a) define a rectangular graph (e.g., do it on a small example and write down the adjacency matrix),
- (b) explain how the greyscale defines a signal value on each node.

### Transportation network

Consider a city's public transportation system with 6 bus stops. The daily passenger count boarding at each stop is recorded for a week. Model this situation as a graph and graph signal. Draw one simple example graph with 6 nodes and assign realistic passenger count values. Then calculate both, normalized adjacency and Laplacian shift, and apply them to your signal. What could you learn from the results? What additional data could you collect to make this application even more useful?

### Graph shift

Consider the graph  $C_4$



with signal  $\mathbf{s} = [0 \ 1 \ 1 \ 0]^T$ . Calculate the adjacency  $A$  and Laplacian  $L$  matrix and apply both to the signal. Discuss the differences between the results.

### *More shift normalizations*

For the adjacency matrix from the lecture

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

compare the normalization by the degree matrix  $D$  from the left  $D^{-1} \cdot A$  (normalization used in consensus network) and normalization from the right  $A \cdot D^{-1}$  (normalization used in physical distribution network), for example by applying it to a signal.

The Laplacian can also be normalized from the right, but more interesting is normalizing it symmetrically from both sides with the square root of the degrees, i.e.,

$$\mathcal{L} = D^{-1/2} L D^{-1/2}.$$

Calculate this symmetric normalized Laplacian for above graph and study it's behavior.