Graph Signal Processing Exercise Set 3

Graph Fourier transform - Frequency response - PCA

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Learning Goals

By the end of this exercise, you should be able to...

- Interpret a graph spectrum and graph Fourier transformation of a graph signal
- Understand the frequency response of filters
- Know how graph Fourier transform generalizes PCA

Graph Fourier spectrum

Consider the graph with adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

. Calculate it's graph Fourier transformation and spectrum and determine, which frequencies are high and low using total variation. Apply the graph Fourier transform to the signals $\mathbf{s}_1 = [\, 1\ 1\ 1\ 1 \,]$ and $\mathbf{s}_2 = [\, 0\ 1\ -2\ 3 \,]$ and compare their graph Fourier transforms.

Frequency response of low-pass filter

Consider again the graph with adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

Calculate the frequency response of the low-pass filter $H = \frac{1}{2}A^0 + \frac{1}{2}A$ and argue, that it is really a low-pass filter.

PCA (Karhunen-Loeve transform) as graph Fourier transform

Principal components of a set of data points (represented as a vector X) with zero mean can be calculated as the eigenvectors of the data covariance matrix $E[XX^T]$. Interpret the vector X as graph nodes of a suitable graph and then explain how the principal components are related to the frequency components of this graph.

What is the connection between the variance in the data explained by each principal component and the total variation of the frequency components?