Graph Signal Processing Exercise Set 4

Graph Neural Networks

May 7, 2025

Learning Goals

By the end of this exercise, you should be able to...

- Understand graph neural networks

Feature matrices from sentences

Consider the graph with adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

To the nodes the following sentences are attached:

Node	Sentence
v_1	The horse is red.
v_2	The horse is blue.
v_3	The hen was red.
v_4	The hen was blue.

Encode this as a feature matrix: For each word create a vector containing for each node a 1 if the word is present in the attached sentence, or 0 otherwise.

GCN by hand

Use the feature matrix from the previous exercise. Your task is to do the calculations done by a graph convolutional network with 2 layers by hand. Consider a GCN with two layers, ReLU activation and weight matrices

$$W^{(0)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \ W^{(1)} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Compute the output of each layer and the final result of the forward pass of the so defined GCN.

Note that this exercise shows that GCNs can easily create over-smoothing, so one has to use more advanced techniques to avoid this (regularization, skip connections).

Graph Convolutional Networks k-hops

Explain how a graph convolutional network with 2 layers aggregates information from two hops apart, while only using 1-hop operators in each layer.