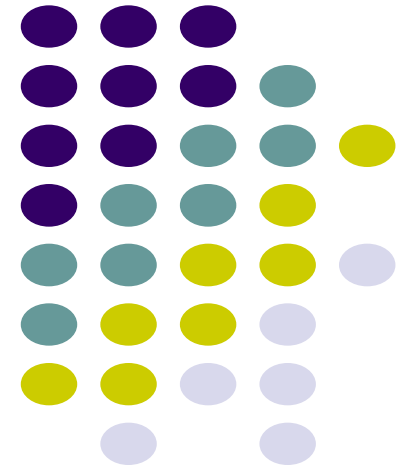
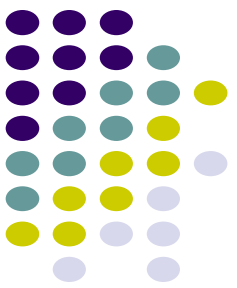


# Fuzzy D'Hondt's Algorithm for On-line Recommendations Aggregation

Ladislav Peška and Štěpán Balcar

Department of Software Engineering,  
Charles University, Prague,  
Czech Republic



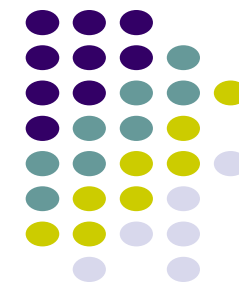


# Task, Motivation

- **Fair (proportional) representation of multiple recommending algorithms based on their on-line performance**
    - Aggregate multiple lists of recommended items  $R_i$  corresponding to the recommending strategy  $rs_i$  into a single list of items  $R$  based on the current performance of  $rs_i$
  - **Why?**
    - Multiple recommending strategies may be both **relevant** (up to some extent) and **highly diverse**
    - Multiple strategies can be used to model **various user's interests**, which should be present in recommendations [Steck 2018]
- ⇒ Fair aggregation of multiple base recommenders may provide relevant, yet highly diverse recommendations or cover multiple users interests

[Steck 2018] Harald Steck, Calibrated recommendations. RecSys 2018

# Motivating Example: Assembling Police Photo Lineups



Recommend candidates similar to the suspect, so the lineup is unbiased

- Two strategies: CB attribute-based & visual-based (DCNN)

Results:

- Visual RS is better *(58% vs 37% of selections, not so good for rare ethnics)*
- However, very small candidates intersection *(1.8% for top-20 candidates)*

⇒ CB strategy was relevant up to some extent

⇒ CB proposed some relevant candidates which Visual could not

⇒ *Both strategies should be present in final recommendations*

⇒ *Simple weighted aggregation would over-sample visual strategy*

⇒ *Thompson sampling MAB [Broden 2018] would probably over-sample best method as well*

*What about something that is tailored to preserve fair distribution?*

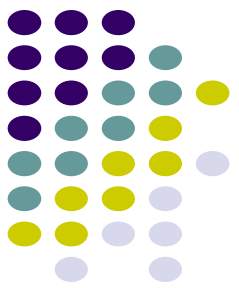


Biased lineup, [floridainnocence.org](http://floridainnocence.org)



(Fairly) unbiased lineup

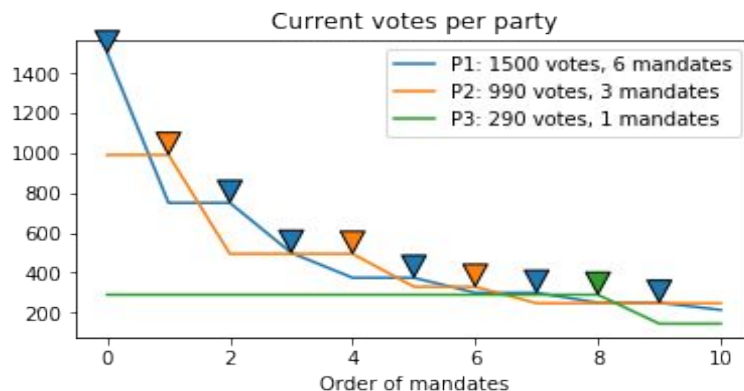
[Broden 2018] Bjorn Broden et al. Ensemble recommendations via thompson sampling: An experimental study within e-commerce. IUI 2018



# D'Hondts Election Algorithm

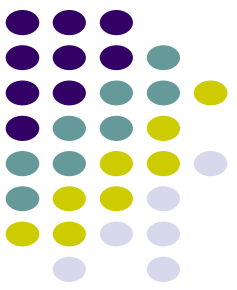
What about something that is tailored to preserve fairness?

- Range of algorithms designed to convert votes to mandates in public elections
    - D'Hondt's election algorithm [D'Hondt 1882]
      - One of the widest-used method in proportional election systems (e.g., 23 countries from Europe, including Denmark)
      - Core principle:
        - Party with the most current votes  $v_{curr}$  wins the next candidate, the best candidate of the winning party is selected
        - Upon a selection of a candidate ( $k$ -th from this party), the volume of party's current votes is reduced  $v_{curr} = \frac{v_{orig}}{k+1}$
- ⇒ Algorithm outputs an ordered list of „current best“ candidates
- ⇒ Volume of per-party votes corresponds with the volume of per-party candidates
- ⇒ Minimize the level of over-representation of the most over-represented party



[D'Hondt 1882] Victor D'Hondt, Systeme pratique et raisonne de representation proportionnelle. C.Muquardt, 1882

Peska & Balcar: Fuzzy D'Hondt's Algorithm for Online RS  
Aggregation



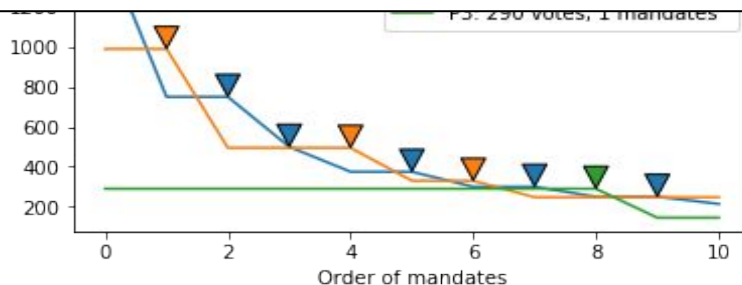
# D'Hondts Election Algorithm

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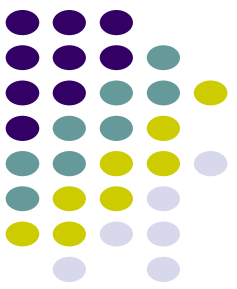
**Can be considered as parallel hybridization algorithm**



[D'Hondt 1882] Victor D'Hondt, Systeme pratique et raisonne de representation proportionnelle. C.Muquardt, 1882

Peska & Balcar: Fuzzy D'Hondt's Algorithm for Online RS  
Aggregation





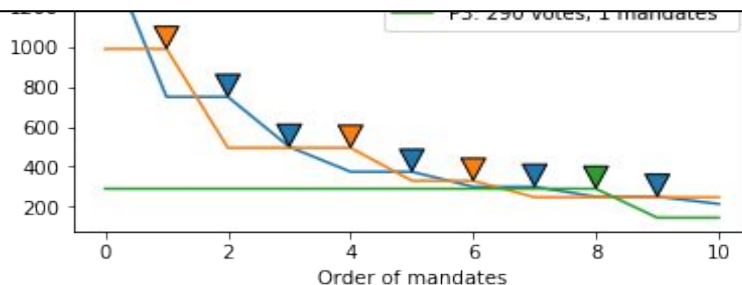
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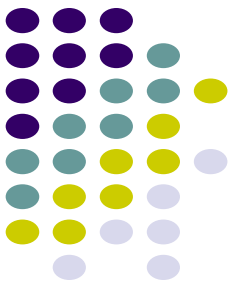


## Are RS aggregations and Elections the same problem?



[D'Hondt 1882] Victor D'Hondt, Systeme pratique et raisonne de representation proportionnelle. C.Muquardt, 1882

Peska & Balcar: Fuzzy D'Hondt's Algorithm for Online RS  
Aggregation



# D'Hondts Election Algorithm - Limitations

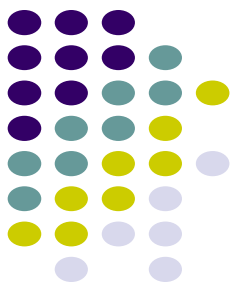
- Politics:
  - *Can a person join multiple parties simultaneously?*
- Recommender systems:
  - *Can be an item recommended by multiple RS strategies?*

**NOPE**



**Sure, why not**

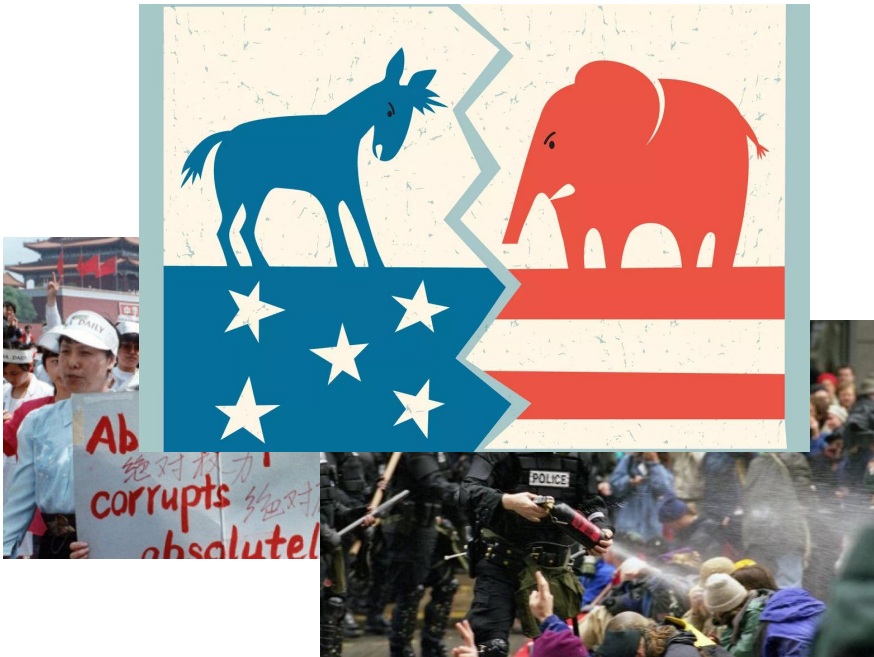
- *What about some bonus for items recommended by multiple RS?*



# D'Hondts Election Algorithm - Limitations

- Politics:
  - Can a person join multiple parties simultaneously?*
- Recommender systems:
  - Can be an item recommended by multiple RS strategies?*

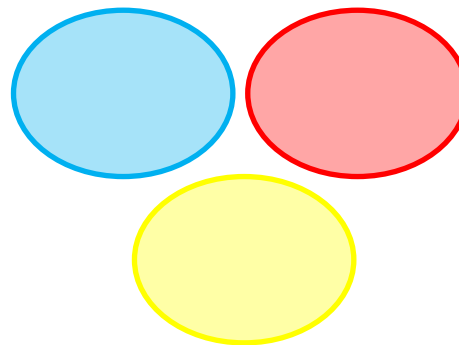
**NOPE**



**Sure, why not**

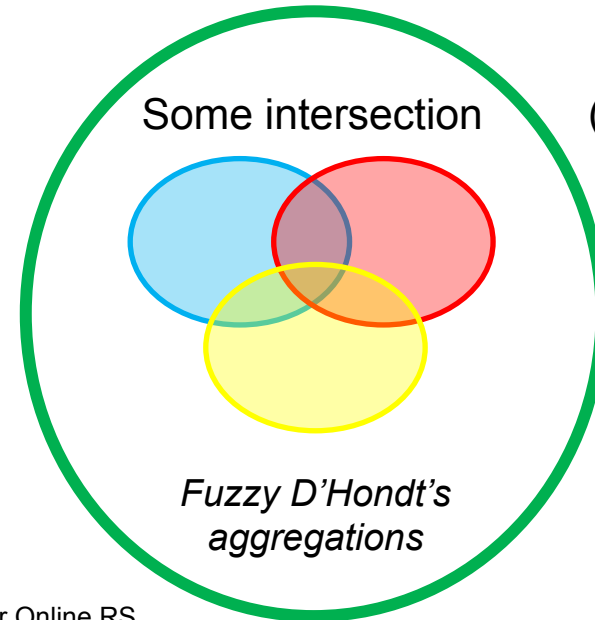
- But how large is the intersection?*

(Almost) no intersection

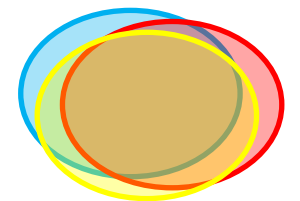


*D'Hondt's is fine*

Some intersection

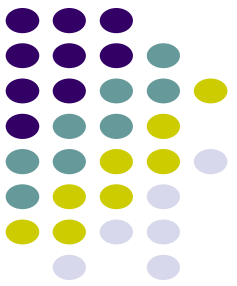


(Almost) full intersection



*Weighted aggregations should be fine*





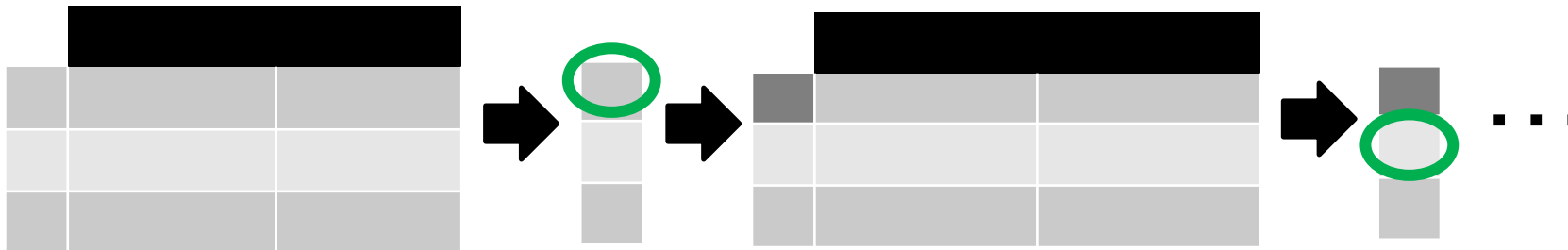
# Fuzzy D'Hondt's RS Aggregations

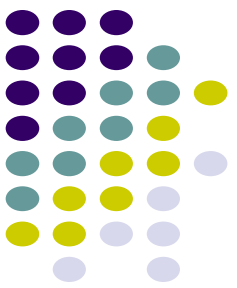
• *Parties = RS strategies, Candidates = Items*

- **Item can be recommended by multiple RS with varying relevance score**  
(unit vector normalization per RS)
- Selecting best candidate w.r.t. current per-party votes and party-candidate relevance score

$$c_{best} = \underset{c_i \in C}{\operatorname{argmax}} \left( \sum_{p_i \in P} v_{curr,i} \times r_{i,j} \right)$$

- Reduce per-party votes based on the party-candidate relevance  $k_i += r_{i,best}, v_{curr,i} = \frac{v_{orig,i}}{k_i+1}$
- Iterate until the desired volume of candidates is selected





# Fuzzy D'Hondt's Algorithm

• *Parties = RS strategies, Candidates = Items*

- **Item can be recommended by multiple RS with varying relevance score**

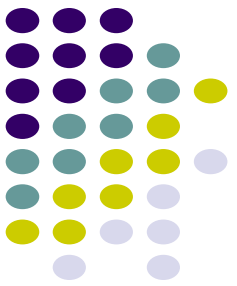
*(unit vector normalization per RS)*

- Selecting best candidate w.r.t. current per-party votes and party-candidate relevance score

$$c_{best} = \operatorname{argmax}_{c_i \in C} \left( \sum_{p_i \in P} v_{curr,i} \times r_{i,j} \right)$$

- Reduce per-party votes based on the party-candidate relevance  $k_i += r_{i,best}$ ,  $v_{curr,i} = \frac{v_{orig,i}}{k_i+1}$
- Iterate until the desired volume of candidates is selected

***But who will assign the votes?***



# Fuzzy D'Hondt's Algorithm

• *Parties = RS strategies, Candidates = Items*

- **Item can be recommended by multiple RS with varying relevance score**

*(unit vector normalization per RS)*

- Selecting best candidate w.r.t. current relevance score

$c_i$

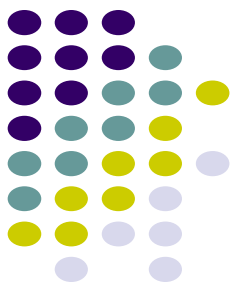
- Reduce per-party votes based on

- Iterate until the desired volume of candidates

This is beyond the hybridization problem...  
Anyway, we changed the technique recently

$$v_{i,i} = \frac{v_{orig,i}}{k_i+1}$$

*Also, where is the „on-line“ from the title?*



# Iterative Votes Assignments

• *Idea: More votes for methods recommending successful candidates and vice versa.*

- Optimizing criteria: maximize the difference of votes for good and bad candidates

$$\max \left( \sum_{\forall c_j \in \mathcal{F}^+} \left( \sum_{\forall p_i \in \mathcal{P}} v_i * r_{i,j} \right) - \lambda_{neg} \sum_{\forall c_j \in \mathcal{F}^-} \left( \sum_{\forall p_i \in \mathcal{P}} v_i * r_{i,j} \right) \right)$$

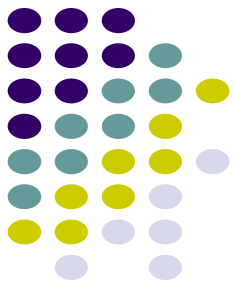
- Single SGD step upon registering each positive or negative feedback event

- Positive: clicks on recommended items
- Negative: ignoring items (*weak feedback, low  $\lambda_{neg}$* )
- *Afterwards, normalize  $\sum v_i = 1$*

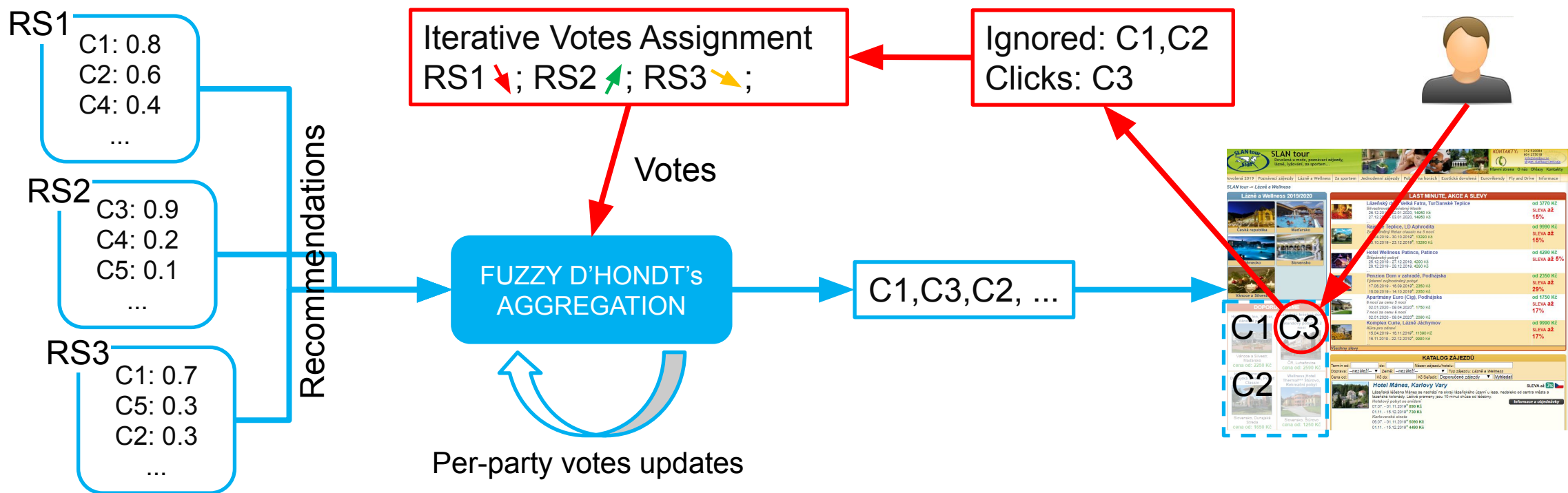
$$v_i = v_i + \eta_{pos} * \left( r_{i,j} - \sum_{\forall k \neq i} r_{k,j} \right)$$

$$v_i = v_i - \eta_{neg} * \left( r_{i,j} - \sum_{\forall k \neq i} r_{k,j} \right)$$

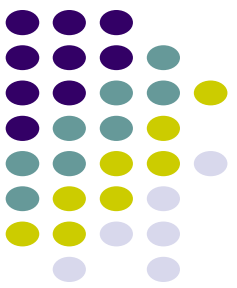
- *Adapting on performance changes over time*
- *Can be pre-set based on previous performance*
- *Can be fine-tuned for individual (long-term) users*



# Overview of the Fuzzy D'Hondt's Aggregations





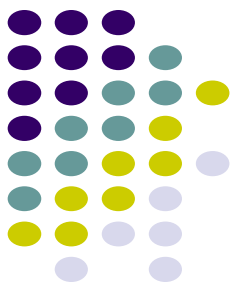


# Evaluation Procedure

- Travel agency website, procedure based on our previous work<sup>[Peska 2018]</sup>
- Several RS: doc2vec (tours description), CB attributes similarity and CF word2vec
  - Recommendations for short sections of user's history
  - Selecting from previously best-performing algorithms: 4 base RS
- On-line A/B testing (*one RS strategy per user*), July 2019
  - Two best-performing individual RS (Cosine CB, word2vec)
  - Fuzzy D'Hondt's Aggregations (4 base RS)
  - BEER(TS, SB): multiarmed bandits with Thompson sampling<sup>[Broden 2018]</sup> (4 base RS)
- Click through rate (CTR), visits after recommendation (VRR)
  - Aggregated per recommendation and per user

<sup>[Broden 2018]</sup> Bjorn Broden et al. Ensemble recommendations via thompson sampling: An experimental study within e-commerce. IUI 2018

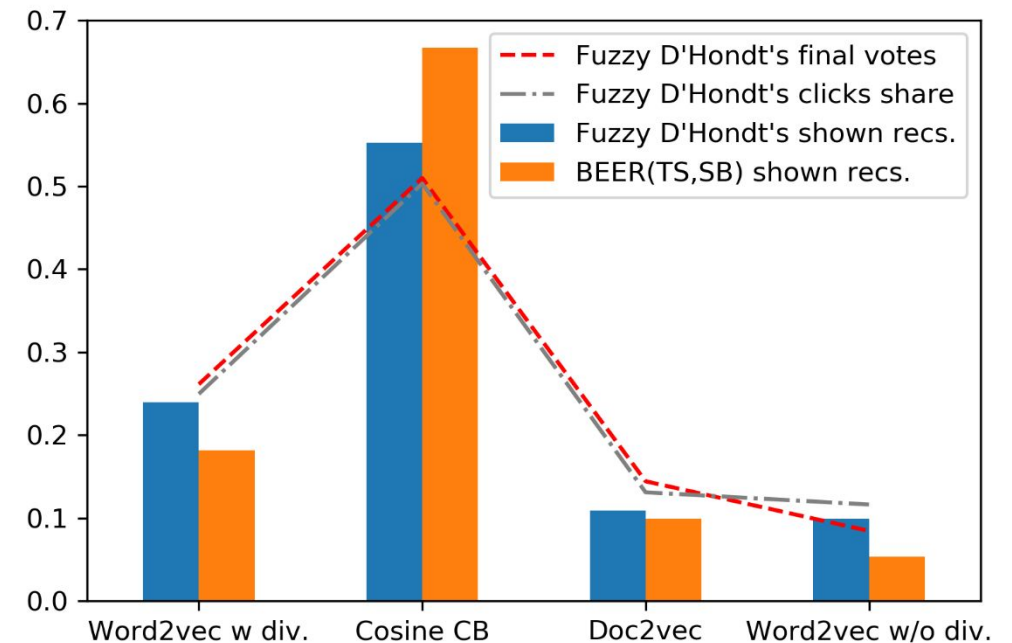
<sup>[Peska 2018]</sup> Peska & Vojtas, Off-line vs. On-line Evaluation of Recommender Systems in Small E-commerce. REVEAL (RecSys) 2018

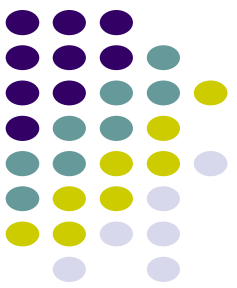


# Results

- In total 2 299 users, 78 356 recommended objects, 7 663 #VRR, 576 #CTR
- Fuzzy D'Hondt's achieved best scores in VRR and VRR per user
- The representation of individual base RS in results seems fair considering their final votes and click-through shares
- *Fuzzy D'Hondt's recommendations are rather static -> lower CTR compared to Bandits?*

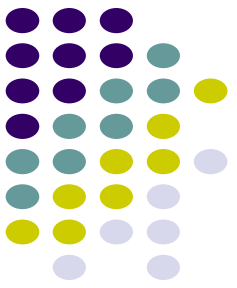
	CTR	VRR	CTR_U	VRR_U
<b>Cosine CB</b>	<b>.0086</b>	0.089	0.262	2.707
<b>Word2vec</b>	.0071	0.104	0.220	3.250
<b>BEER(TS,SB)</b>	.0077	0.078	<b>0.282</b>	2.861
<b>Fuzzy D'Hondt's</b>	.0063	<b>0.118</b>	0.239	<b>4.504</b>





# Conclusions & Outlook

- Fuzzy extension of D'Hondt's election algorithm with iterative votes assignment can be used to aggregate multiple RS strategies w.r.t. the principle of proportionality
- Only the basic algorithm variant explored
  - Better definition (learning?) of positive-negative feedback threshold
  - More refined feedback definition (e.g., actions after click, negative only if visible) - depends on domain
  - Stochastic candidates selection
  - Penalize candidates relevance through time
- More extensive evaluation needed
  - Additional domains & baselines
  - Analyze domain parameters (RS diversity etc.)



# Thank you!

## Questions, comments?