course:

Searching the Web (NDBI038)

Searching the Web and Multimedia Databases (BI-VWM)

lecture 3:

Vector model of information retrieval

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Today's lecture outline

- vector model
  - motivation
  - term weighting
  - querying
  - implementation by inverted index
- latent semantic indexing
  - motivation
  - singular-value decomposition
  - Querying
- word2vec

Vector model – motivation

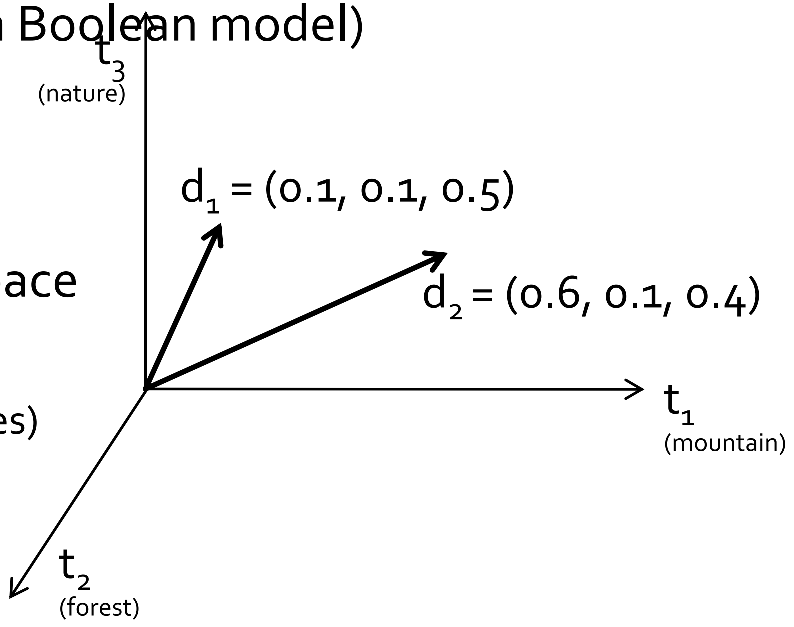
- content-based similarity search model
  - query is represented the same as document
    – by a vector in a high-dimensional space
  - the **distance** of query vector to a document vector
    ≈ **dissimilarity** of the query text and document text
    ≈ the **relevance** of the document to the query
  - cognitive model – people **search for similar** things

Vector model – motivation

- highly parameterizable
  - different term representation schemas (positioning the vectors in space)
  - different similarity (distance) measures
- ranking of documents in the query result
- support of relevance feedback

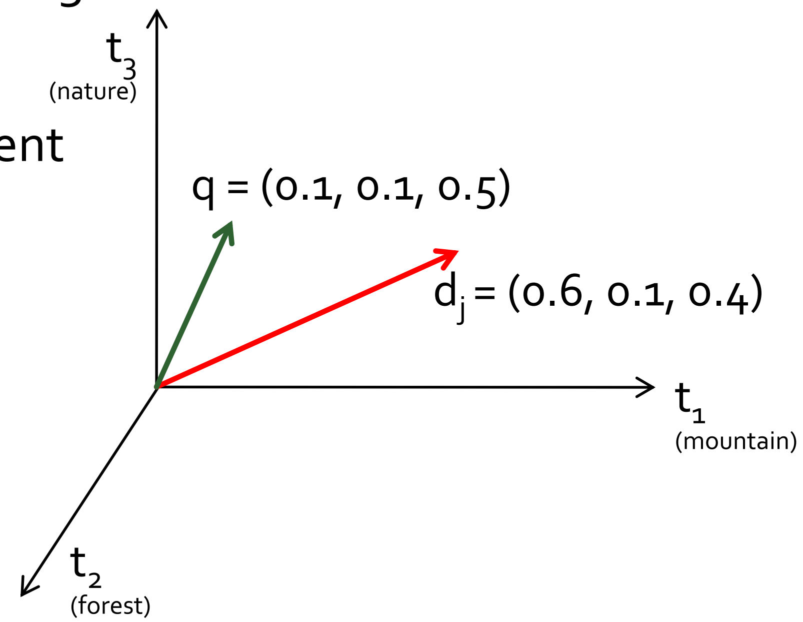
Vector model – the basics

- also known as the **bag of words** (BoW) model
- document = bag of terms
  - bag = multi-set = set allowing multiple occurrences of the same element
  - vocabulary of **m** terms (the same as in Boolean model)
  - document $\mathbf{d}_j$ modeled by a vector of dimension **m**
  - extremely high-dimensional vector space
    - note that usually **m** > 10^4 for an English collection (could be more for other languages)



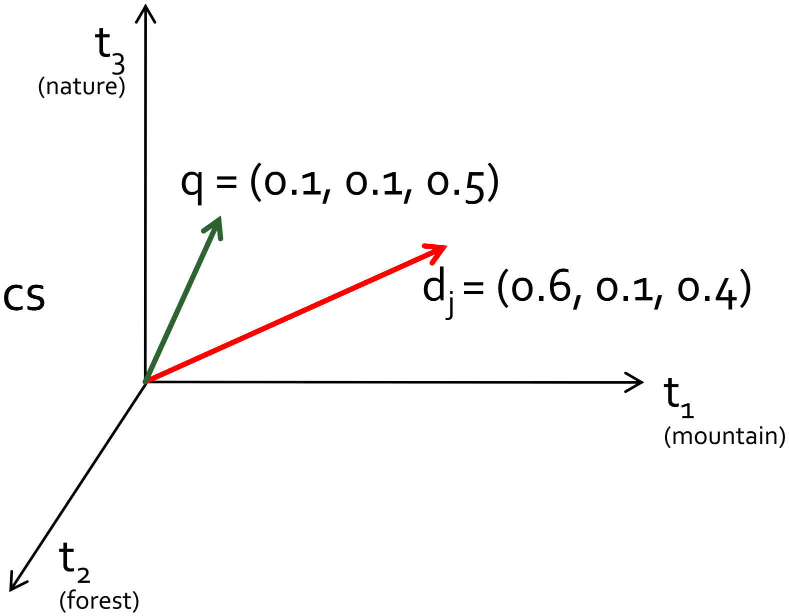
Vector model – the basics

- query = bag of terms, i.e., the same as document
  - could be specified by **a few keywords** (e.g., typing into Google) or by a **query document** (e.g., searching for plagiarism)
  - no Boolean condition or a query language
  - query modeled by a vector **q** of dimension **m**, the same as document



Vector model – the basics

- the space structure
  - each **dimension** of the space belongs to a **term** from the vocabulary
  - a coordinate value **x** in a dimension **i** means the weight **x** (importance) of term $\mathbf{t}_i$
- document/query vector
  - point (vector) in the space represents a unique combination of term weights
  - describes the basic term-based statistics of a document/query
    - a compact descriptor
    - quite well defined semantics



Vector model – the basics

- term-by-document matrix A
  - similar to the one shown in last lecture (Boolean model), i.e., stores document vectors $\mathbf{d}_j$/term vectors $\mathbf{t}_i$
  - not binary values – occurrence of a term in documents, but real values – weight $\mathbf{w}_{ij}$ of a term $\mathbf{t}_i$ in document $\mathbf{d}_j$
- zero weight = a term has no significance in the document or it doesn't occur in the document
  - usually sparse matrix (99%)

$$\mathbf{A} = \begin{array}{c c} & \begin{array}{cccc} t_1 & t_2 & \cdots & t_m \end{array} \\ \begin{array}{c} d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_n \end{array} & \left(\begin{array}{cccc} w_{11} & w_{21} & \cdots & w_{m1} \\ w_{12} & w_{22} & \cdots & w_{m2} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ w_{1n} & w_{2n} & \cdots & w_{mn} \end{array} \right) \end{array}$$

Vector model – the term weights

- how to construct the term weights?
  - manual weighting leads to inconsistency and labor
  - automatic weighting based on term frequency statistics (in each document and in the entire collection)

- what is good weighting model?
  - high weights for **important** terms, and vice versa
    - important term
      = representative, discriminative, semantically significant term
  - low weights for indiscriminative terms (e.g., stop words) and terms appearing in many documents

Vector model – the term weights

- document-scope statistics
  - more frequent terms in a document are more important

    f_{ij} = **frequency of term t_i in document d_j**

  - normalized term frequency of term t_i in document d_j

    $tf_{ij} = f_{ij} / max_i\{f_{ij}\}$, where max returns the highest frequency of term t_i over the entire collection

    - weights normalized to 0..1
    - alone not robust enough, a document concatenated with itself would obtain double weights

Vector model – the term weights

- collection-scope statistics
  - terms present in many documents are less important

    df_i = **document frequency of term t_i**

    = number of documents containing term t_i

idf_i = **inverse document frequency of term t_i,**

$= \log_2 (n / df_i)$, where n is the total number of documents

Vector model – the term weights

- *tf-idf*, popular *weighting scheme*

  $$w_{ij} = tf_{ij}\, idf_i = tf_{ij} \log_2 (n / df_i)$$

  - the *idf* component generalizes the effect of removing stop words
    - logarithm used to inhibit the effect of term frequency (*tf*)
    - stop words and other frequent terms would obtain very low *idf*, and thus the entire weight
- pros
  - experimentally, *tf-idf* proved as the best w.r.t. precision/recall
- cons
  - using *idf* requires static collection for efficient query implementation (by inverted index)

- many other ways of determining term weights proposed

Vector model – the term weights

Example:

Given a document **d** containing terms with given frequencies in **d** as

d = < mountain(3), forest(2), nature(1) >

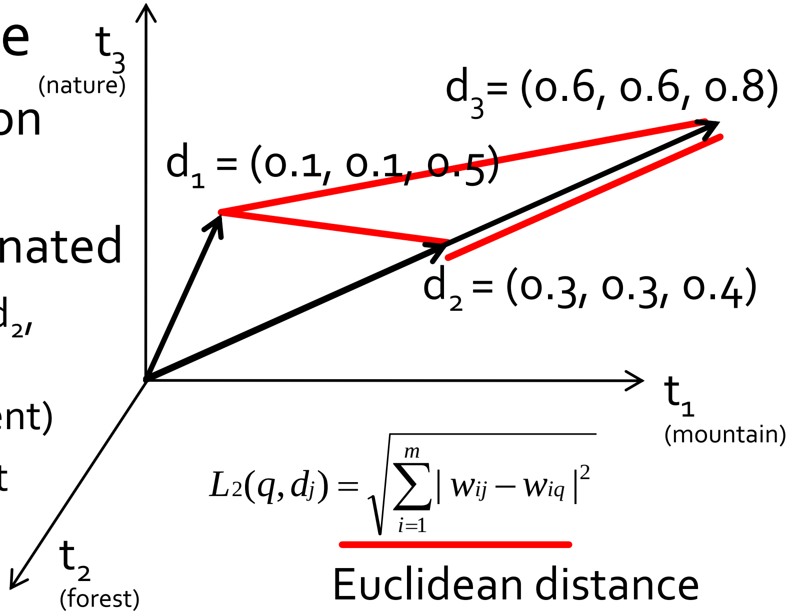
Assume 10,000 documents and document frequencies of the terms as mountain(50), forest(1300), nature(250)

Then the *tf-idf* weights of the terms in **d** are:

mountain:	$tf = 3/3$; $idf = \log_2(10000/50) = 7.6$;	**tf-idf = 7.6**
forest:	$tf = 2/3$; $idf = \log_2(10000/1300) = 2.9$;	**tf-idf = 2.0**
nature:	$tf = 1/3$; $idf = \log_2(10000/250) = 5.3$;	**tf-idf = 1.8**

Vector model – the similarity

- now we have the vector model and the weights, but what about the similarity of vectors?
- intuitivelly, one would choose the Euclidean (L_2) distance as the dissimilarity measure
- Euclidean distance not appropriate
  1. not robust w.r.t. weight multiplication e.g., consider documents d_2 and d_3, where d_3 is two copies of d_2 concatenated
     - although the text in d_3 is twice the same as d_2, the distance d_2 and d_3 is larger than d_2 and d_1 (where text in d_1 is completely different)
     - could be solved by vector normalization, but
  2. implementation problem (later)



$$L_2(q, d_j) = \sqrt{\sum_{i=1}^{m} | w_{ij} - w_{iq} |^2}$$

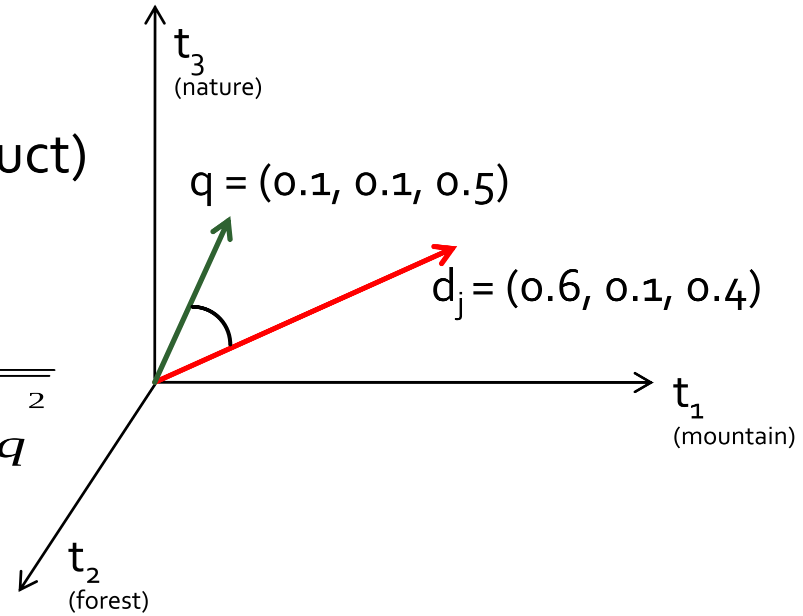
Euclidean distance between d_1, d_2, d_3

Vector model – the similarity

- robust similarity comparing the **directions (angles)**, not the **positions** (that is why we talk about vectors not points)
- inner product
  - sim($\boldsymbol{d_j}, \boldsymbol{q}$) = $\boldsymbol{d}_j \bullet \boldsymbol{q}$ = $\sum_{i=1}^{m} w_{ij} w_{iq}$
- cosine similarity (normed inner product)
  - CosSim($\boldsymbol{d_j}$, $\boldsymbol{q}$) =

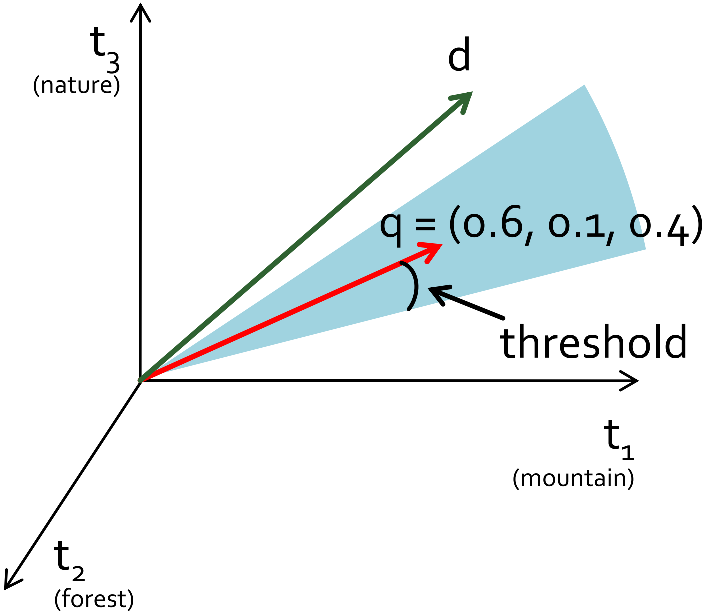
$$\frac{\vec{d}_j \cdot \vec{q}}{\left|\vec{d}_j\right| \cdot \left|\vec{q}\right|} = \frac{\sum_{i=1}^{m} (w_{ij} \cdot w_{iq})}{\sqrt{\sum_{i=1}^{m} w_{ij}^{2} \cdot \sum_{i=1}^{m} w_{iq}^{2}}}$$

  - cosine of the angle between **q** and $\mathbf{d}_j$ (high value = high similarity)
  - by arccos(CosSim) we obtain the **angle distance** (low dist.=high similarity)



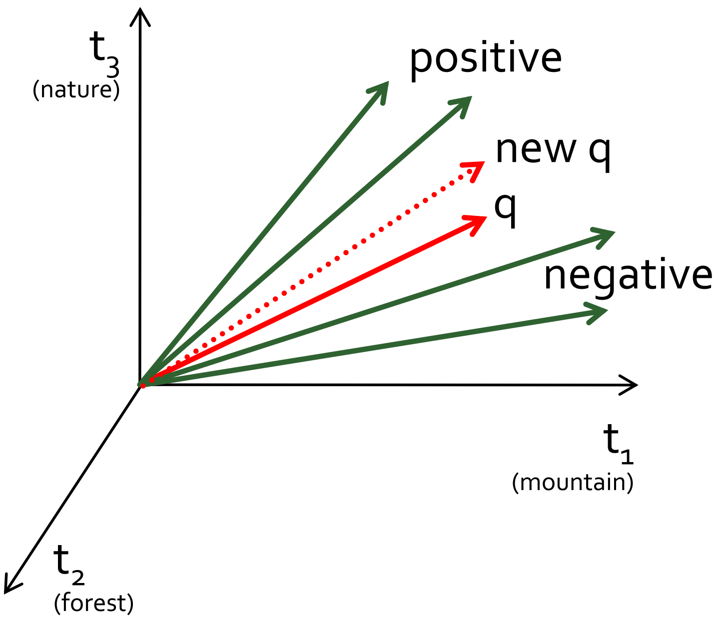
Vector model – querying

- **vector range query** used, specified by
  - a query vector **q**, that stands for the desired document
  - a **similarity threshold** that should be exceeded, when measuring the similarity of **q** and a document vector **d**
    - if CosSim(q,d) > threshold, then **d** goes to the query result
- the result of range query is a ranked set of documents, ordered by their similarity to **q**



Vector model – querying

- relevance feedback supported
  - by **shifting query vector**
  - consider a **query collection**, the query vector **q** is obtained by treating all documents in the query collection as one



- the user can
  - manually shift the query vector (adjusting the weights)
  - add relevant documents from the previous search to the query collection, obtaining a shifted query vector
  - remove irrelevant documents from query collection

Vector model – implementation

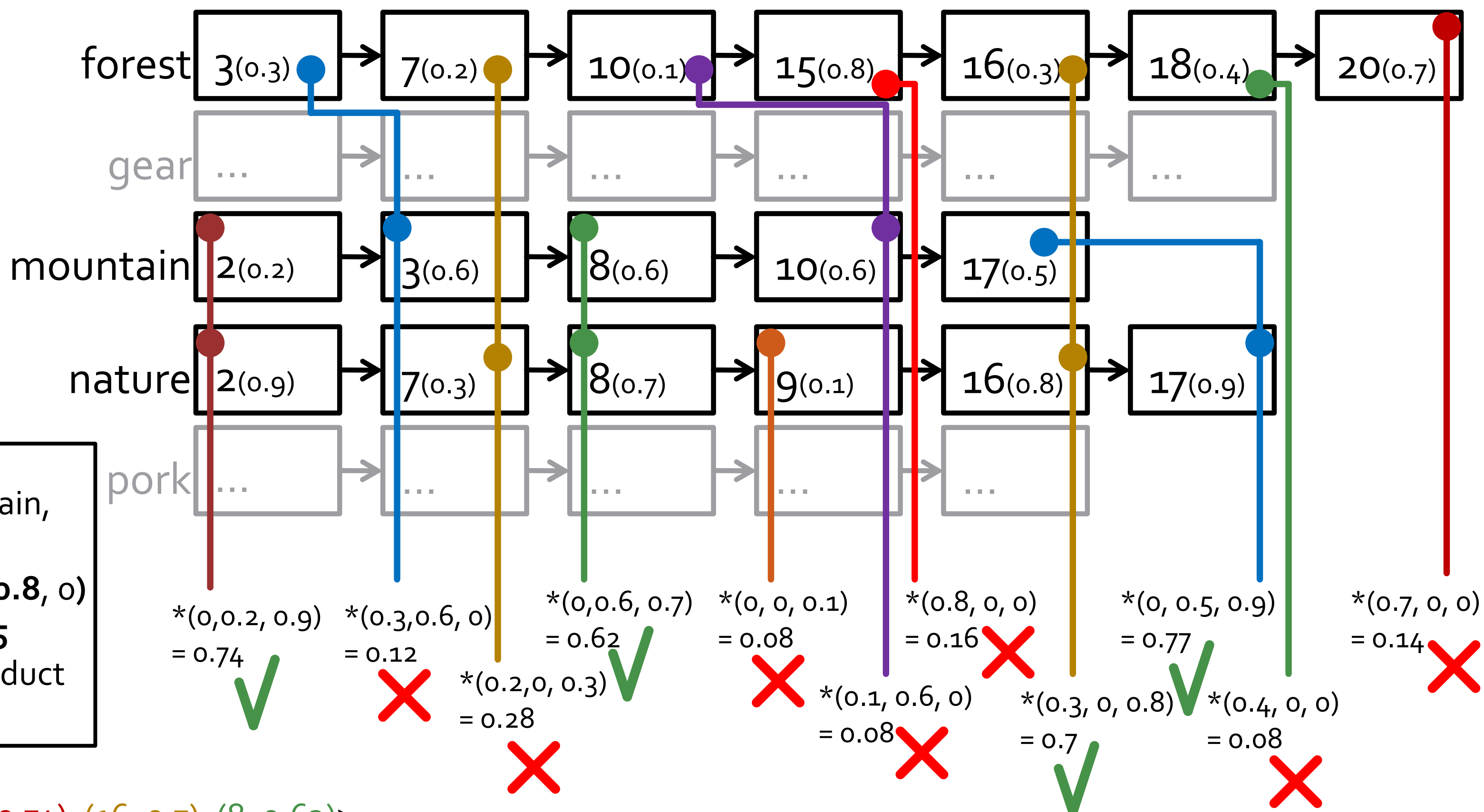
- mostly the inverted index is used
- the same data structure as for the Boolean model (previous lecture)
  - term vectors represented by inverted lists of ordered document ids the term appears in
  - additionally, the term weight is stored together with document id as DocID(term weight)

forest | 3(0.3) → 7(0.2) → 10(0.1) → 15(0.8) → 16(0.3) → 18(0.4) → 20(0.7)

- query processing similar to that of Boolean query
  - merge-sort style (aligning list cursors to the same ids)
  - but computing the cosine similarity/inner product of **q** and $\mathbf{d}_j$, not a Boolean expression

Vector model – implementation

- example of processing vector query using inverted index



Query:
"forest, mountain, nature"
q=(0.2, 0, **0.1**, **0.8**, 0)
threshold = 0.5
using inner product as similarity

Query result =
<(17, 0.77) , (2, 0.74), (16, 0.7), (8, 0.62)>

Vector model – implementation

- why inverted index?
  - useful also for Boolean queries for free
    - both vector and Boolean query evaluated by single index traversal
  - compact representation of sparse matrix (not storing zero weights)
  - term vectors indexed

- only small part of the matrix is accessed
  - zero weights (99% of the matrix) are skipped
  - only inverted lists of terms with nonzero weights in the query vector are accessed
    - query typically only a few keywords → a few lists

Vector model – implementation

- note that inverted index is only suitable when inner product or cosine similarity are used as the similarity
  - consider Euclidean distance and normed vector space* – the query would return the same as for cosine similarity, BUT
  - subtraction of coordinates, not multiplication!
    - hence, we cannot skip the inverted lists of terms with zero weights in the query, as they would contribute to the Euclidean distance

$$L_2(q, d_j) = \sqrt{\sum_{i=1}^{m} | w_{ij} - w_{iq} |^2} \qquad sim(q, d_j) = \sum_{i=1}^{m} w_{ij} w_{iq}$$

Euclidean (L2) distance vs. inner product

*The vector coordinates are divided by the vector size, so we obtain unitary vectors.

Vector model – pros and cons

- pros
  - simple and well-defined approach, geometric model
  - query-by-example (no need of a query expression)
  - provides partial matching (ranking)
  - effective model, efficient implementation
- cons
  - too simple queries, lack of the expressive power of Boolean query, syntax in the text is not considered
  - the geometerization (weighting and vector similarity) lacks of a strict semantic information
  - term independence assumed, cannot deal with linguistic phenomena like synonymy and homonymy

Latent semantic indexing (LSI) – motivation

- need to address the cons of vector model
  - vector model assumes too many independent low-level terms – but terms are not independent!
    - leads to lower precision/recall
    - e.g., two documents **"more people look for jobs"** and **"unemployment is on rise"** would not be ranked similar, while others **"George Bush fired his secretary"** and **"fires in Australian bush got strenghtened"** would be ranked as similar
  - consider higher-level concepts rather than low-level terms
    - groups of semantically similar terms (e.g., street, road, path)
    - also solves synonymy and homonymy
    - dimensionality reduction (much less concepts than terms)

The LSI model – the idea

- latent semantic indexing (LSI) = extension of the vector model
  - preprocessing of the term-by-document matrix A by the singular value decomposition (SVD)

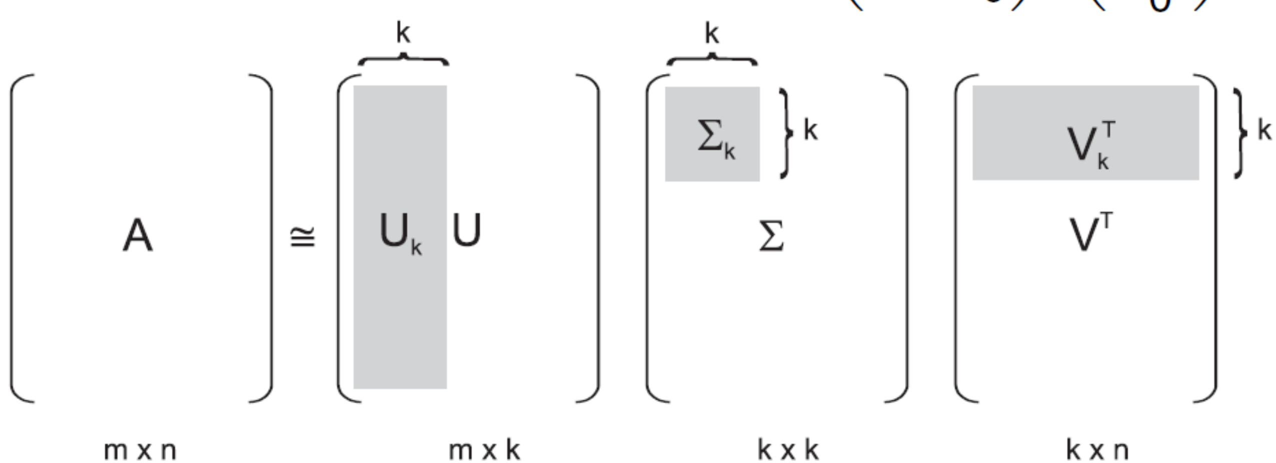
$$A = U\Sigma V^T$$

- the result is a dense *concept-by-document* matrix $\mathbf{\Sigma V^T}$
  - a document is now modeled by concepts not terms
- concept bases (vectors) in **U**
  - concept = linear combination of terms
    - statistically significant (found in the matrix A)
    - the **latent semantics** – the concept appears in multiple documents
  - concepts ordered by significance (decreasing singular values in the diagonal matrix $\mathbf{\Sigma}$) $\rightarrow$ only the first concepts in **U** are most significant

The LSI model – rank-k SVD

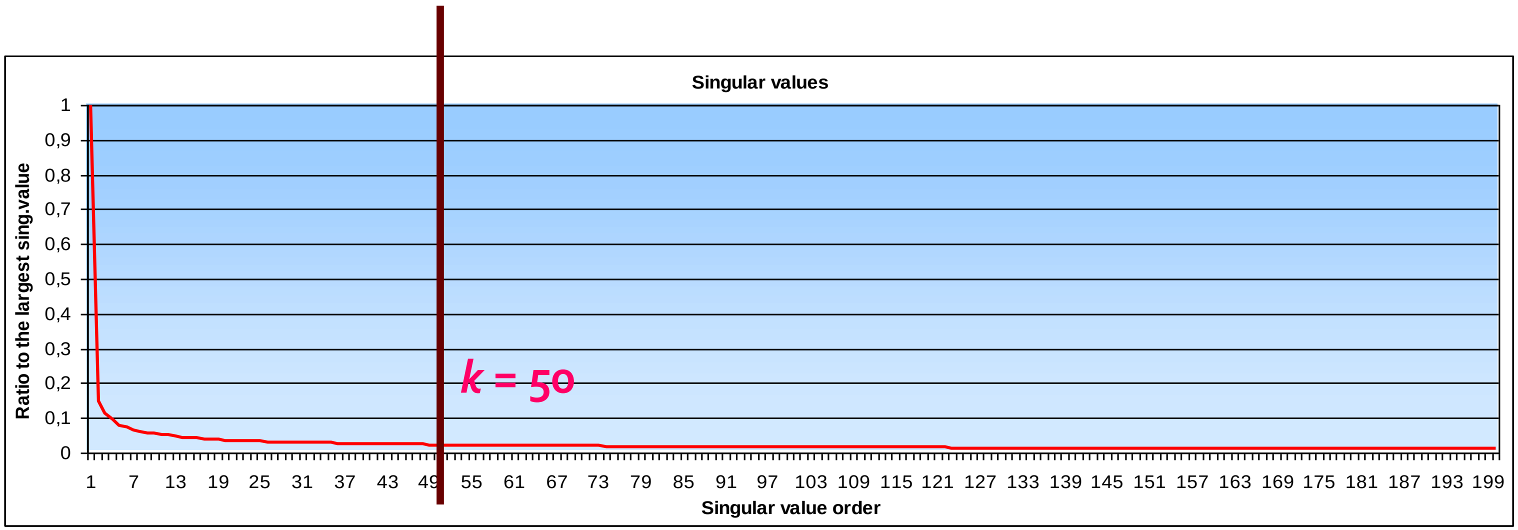
- more generally, consider SVD and a user-defined number k – the number of most significant concepts to be considered
  - we ignore the less significant concepts (beyond the k)
  - dimensionality reduction
  - more efficient algorithms of SVD decomposition

$$A = U\Sigma V^T \approx A_k = (U_k U_0) \begin{pmatrix} \Sigma_k & 0 \\ 0 & \Sigma_0 \end{pmatrix} \begin{pmatrix} V_k^T \\ V_0^T \end{pmatrix}$$



The LSI model – rank-k SVD

- usually, the singular values in Σ (significance of concepts) decrease quickly, so only several tens or hundreds is sufficient to consider
  - compare 10^1-10^3 concepts (LSI)
    to 10^4-10^5 terms (vector model)



The LSI model – the retrieval

- what is important to information retrieval
  - concept-by-document matrix, the data (document vectors)

$$D_k = \Sigma_k V_k^T$$

  - query projection
    (from the term space to the concept space)

$$q_k = U_k^T q$$

  - still cosine similarity used

The LSI model – the retrieval

- dense but low-dimensional matrix $\mathbf{D_k}$
  - dimensionality below 10^3
- also the query vector $\mathbf{q_k}$ is dense

term-by-document matrix A

document / term \	D_1	D_2	D_3	D_4	D_5
database	0	0.48	0.05	0	0.70
vector	0.23	0	0.23	0	0
index	0.43	0	0	0	0
image	0	0	0.10	0	0.54
compression	0	0	0	0	0.21
multimedia	0.12	0.52	0.62	0	0
metric	0	0	0.32	0.40	0
space	0.42	0	0	0.24	0

concept-by-document matrix D_k

document / concept \	D_1	D_2	D_3	D_4	D_5
$concept_1$	-0.21	0.48	-0.05	0.10	0.70
$concept_2$	0.23	0.20	-0.23	0.45	0
$concept_3$	-0.43	0.02	0.32	0.24	-0.06
$concept_4$	0.34	-0.01	0.10	0	0.54
$concept_5$	0.31	0.9	-0.78	0.52	0.21

The LSI model – implementation

- the inverted index not efficient data structure for LSI
  - dense concept-by-document matrix
  - dense query vector
  - processing equivalent to sequential scan of all the document vectors
- other indexing techniques needed,
  e.g., the metric access methods (later lectures)

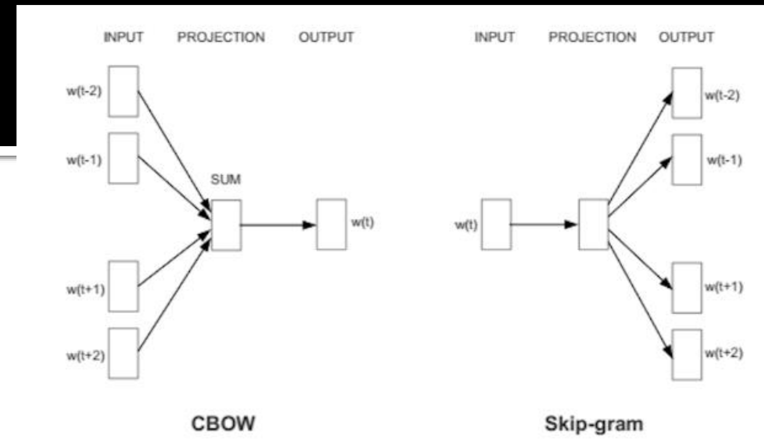
The LSI model – pros and cons

- pros
  - LSI model reveals "latent semantics" in the collection
    - the search is term-independent, it is **concept-based**
    - LSI model partially solves the problem of **synonymy** and **homonymy**
  - dimensionality reduction
    - e.g., from hundreds of thousands to several hundreds
- cons
  - concepts just statistically significant linear combinations of terms
    - not well-defined semantics (not a linguistic category)
  - dense matrix and query vector
    - inverted index not appropriate
  - expensive preprocessing of the matrix A
    - SVD algorithms have complexity $O(n^2+m^3)$

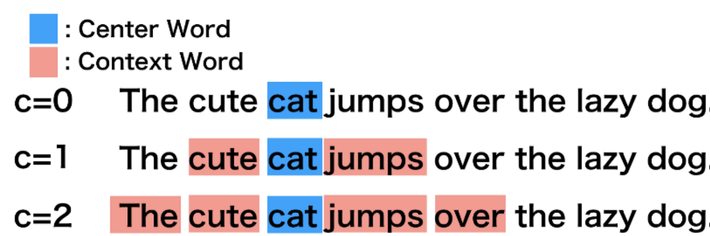
Word2vec

- distributed word representations
- machine learning approach to word embeddings
  - by Tomáš Mikolov (2013), https://github.com/tmikolov/word2vec
  - extensible to genes, code, likes/follows, playlists, social media graphs and other verbal or symbolic series
  - unsupervised, just text corpus needed for training
  - useful for NLP, search, recommendation, sentiment analysis, etc.
  - 3-layer backpropagation (gradient descent) neural network

Word2vec



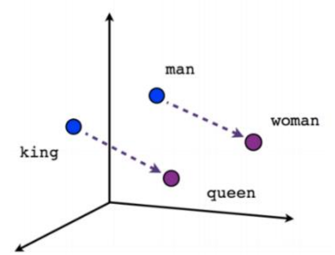
- two algorithms/network models
  - continuous bag of words (CBOW)
  - skip-grams
- N-gram context (window) for each (**center/focus**) word



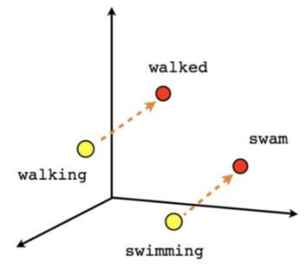
- CBOW
  - input layer: 1-of-V coding for **c** context words
    - a V-dimensional vector [0,0,0,…,1,…,0,0,0], where V is the vocabulary size
  - projection layer – continuous (target) representation of focus word
  - output layer – 1-of-V representation of focus word
- Skip-gram
  - the opposite of CBOW + order of words in context matters

Word2vec

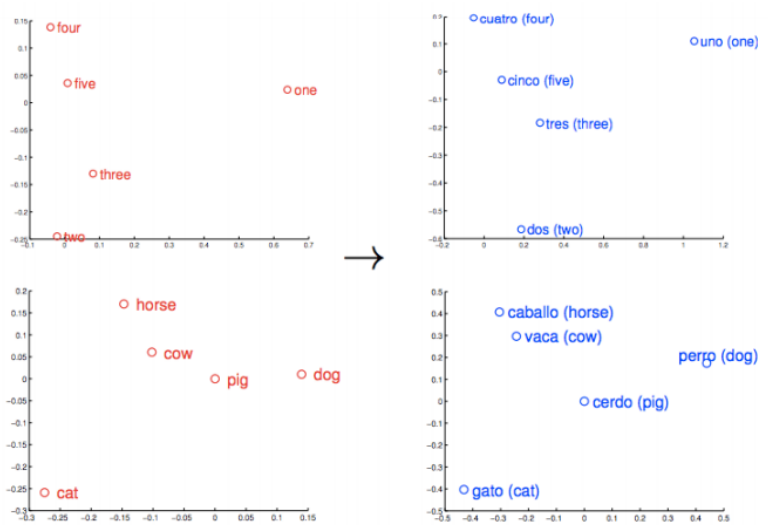
- vector arithmetic for NLP and information retrieval
  - vec("king") – vec("man") + + vec("woman") ≈ vec("queen")

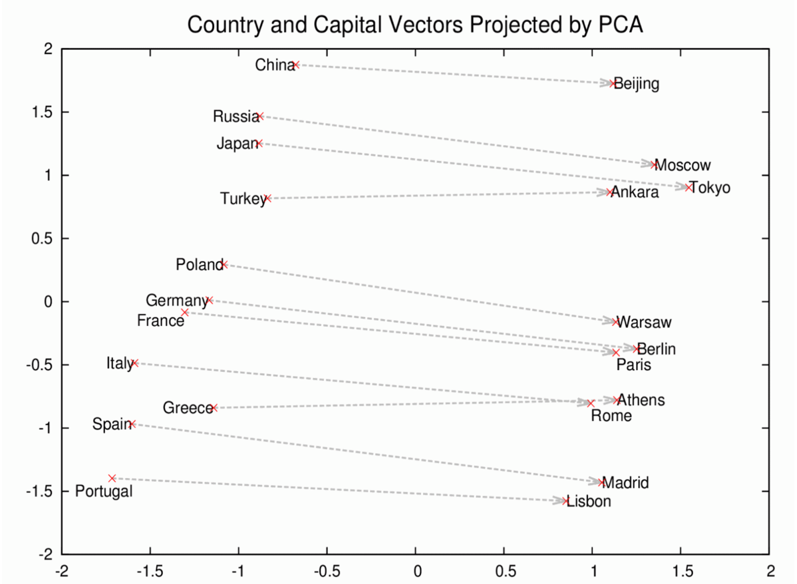


Male-Female



Verb tense





Word2vec vs. LSI (SVD)

- word representations vs. document (and word) representations
- word2vec much faster (scalable)
  - no need to build/store huge matrices, no decomposition, parallel proc.
- word2vec performs much better
  - window contexts, not entire documents
- however, SVD could be adapted to work like word2vec