VISCOMETRY APPLIED TO THE BINGHAM SUBSTANCE ICE SLURRY

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A modified theory of Bingham substances is applied to describe the behaviour of an ice slurry. This could be a substantial contribution to answer the question, if ice slurries show Newtonian behaviour at low ice fractions. A time behaviour of the cooling fluid is observed. The final results of a new online viscometry method are outlined and applied to experimental data. Measuring the mass flow, the density and the pressure drop over a defined distance for two forcings of the system gives the input data for a mathematical transformation, which results in the rheogram. The method is also applied to data which have been determined at other institutes (e.g. CEMAGREF, Antony, France) and show very good agreement.

1. INTRODUCTION

Besides having environmental benefits an application of ice slurries can also be very economical. In 1996 a comparison between an ice slurry and a conventional brine for a concrete application was presented (see Ref. [1]). The result is that the ratio of the transported energy of cold to the energy used for pumping is a factor six to thirty higher for the ice slurry application. Such estimates are directly influenced by two physical properties, the enthalpy of the transport fluid, which preferably should be high, and the shear stress, which should be small. Only then an economic transport can be expected.

Knowledge on enthalpy of ice slurries is available in the literature (Ref. [2], [3] and [4]), and there is very good agreement between a general accepted model and measurements for the case where the ice crystals are in the thermodynamic equilibrium with the surrounding multiple component fluid. On the other hand, measurements of heat transfer coefficients and shear stresses - represented by rheograms - show large scattering. The deviations are even larger when results of different research groups are compared. This led to some basic investigations, performed at the University of Applied Sciences of Central Switzerland (UASCS), which hopefully will bring some insight, how to overcome this confusing situation.

2. ROTARY VISCOMETRY

Several methods to determine the viscosity of fluids are known, e.g. the Searle and Couette type rotary viscometry, the absolute and relative capillary viscometry, the Höppler falling sphere viscometry, etc. (see e.g. Ref. [5]). A rotary method, with two cylinders relatively accelerated to each other, has been applied to ice slurries by Egolf et al., who obtained high viscosities [1]. In the case of an application of a rotary viscometer, the friction of the accelerating cylinder should not lead to a substantial heat generation, which would lower the ice fraction by melting of ice in the gap between the cylinders. Therefore, a Danish group (DTI) applied a rotary viscometer with refrigerated cups [2]. At present their determined shear stresses yield the lowest available data. Another group at CEMAGREF used a viscometer of tubular type (Ostwald rheometer) and obtained higher values of shear stresses.

At the UASCS a large number of measurements have been performed with a rheometer - Haake Type RV 20 - by dipping the measuring device partially into an open cylindrical ice slurry tank with a volume of 0.3 m^3. In the viscometer the small inner cylinder is accelerated against its outer quiescent counterpart. The experimental data show large deviations and, therefore, we decided not to publish the entire data set. Two measurements - shown in figure 1 - are compared with results from other research groups.

An important question (posed at the First IIR Workshop on Ice Slurries in Yverdon-les-Bains) is, if ice slurries really show Bingham behaviour. Otherwise a description by power laws may be more accurate (compare with curves in figure 1).

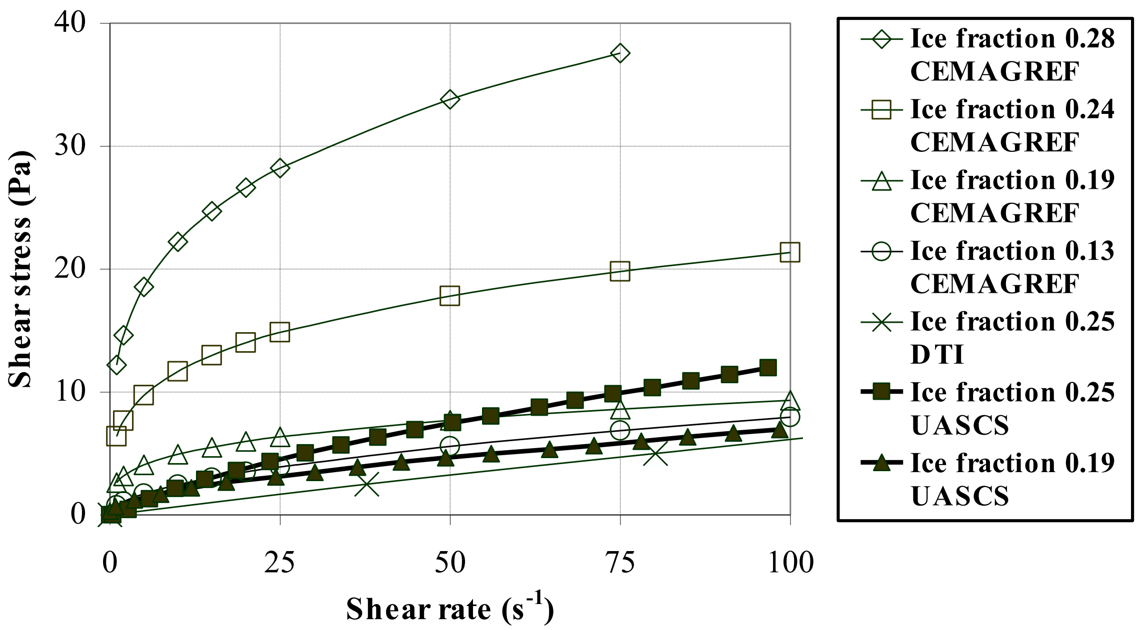

Figure 1: Rheograms measured by different research groups lead to completely different results. The Danish group obtains the lowest shear rates. The additives used were the following: 1) CEMAGREF 11 mass % ethanol, 2) DTI 10 mass % ethanol, 3) UASCS 10 mass % talin.

3. A MODIFIED BINGHAM THEORY

The rheograms in figure 1 do not show ideal Bingham behaviour. An ideal Bingham exhibits no deformation up to a finite level of stress, named critical shear stress τ_0. Only when the stress is larger than the critical value the material starts to flow. It is known that numerous suspensions - containing solid particles - show an offset in the rheogram, but otherwise nearly Newtonian behaviour. These substances are named Bingham fluids after E. C. Bingham, who in 1916 described the behaviour of paint by a model, which is now known as Bingham theory [6]. A list of numerous materials showing such behaviour was given by Bird et al. [7].

In the early beginning of ice slurry research, in oral communications, it was claimed that ice slurries are of Bingham type. On the other hand at very low shear stresses the be-haviour is close to that of a Newtonian fluid with a very large viscosity (see figure 1). Therefore, also other models, e.g. the Ostwald-de Waele model (see Ref. [8]) may be ap-plied to obtain appropriate results. But power law models describing the rheologic behaviour always show a decreasing derivative with increasing shear velocity. In our rotary viscometer measurements the derivative clearly is constant up to high values of $\dot{\gamma}$. One has to take attention, that in the case of dissipative heat generation by wall friction at the boundaries of the cylinders, ice will melt and the viscosity will decrease toward higher shear velocities, giving an erraneous support to power law descriptions. In our opinion the application of a modified Bingham model, proposed in 1987 by Papanastasiou [9], gives the best results in the entire shear rate domain. In 1992 this model was applied to plastics by Abdali and Mitsoulis [10]. Numerical simulations of a Bingham plastics past a sphere in a cylindrical tube were performed by Blackery and Mitsoulis, also taking the modified Bingham approach into consideration [11].

The rheogram of an ideal Bingham is described by (for example see Ref. [1])

$$\tau = \mu\dot{\gamma} + \tau_0 . \tag{1}$$

The modified Bingham model contains a further parameter m, which describes a constant time

$$\tau = \mu\dot{\gamma} + \left[1 - \exp\left(-m\dot{\gamma}\right)\right]\tau_0 . \tag{2}$$

The modification can be neglected if

$$m\dot{\gamma} >> 1 . \tag{3}$$

For the shear velocity in a tube the following relation is valid

$$\dot{\gamma} = - \left.\frac{du}{dr}\right|_{r=R} . \tag{4}$$

If condition (3) is valid, then it follows by taking the derivative of the Bingham profile at the tube wall, which leads to the largest possible derivative of the velocity component

$$\dot{\gamma} = -\frac{1}{2\mu}\frac{dp}{dx}\left(r_2 - r_1\right), \tag{5}$$

respectively with

$$\frac{dp}{dx} = -2\frac{\tau_0}{r_1}, \tag{6}$$

and relation (3)

$$\frac{\mu}{\tau_0}\frac{1}{\frac{r_2}{r_1} - 1} << m \,. \tag{7}$$

After m has been determined as a function of the temperature, (e.g. for a practical system) it can be checked, if results of the ideal Bingham model are sufficient or if the generalized approach must be taken into consideration.

4. TIME BEHAVIOUR

The following experiments were performed with the central part of FIFELAB, which is shown in figure 2. The ice slurry is produced with an Integral ice generator which is connected to a refrigeration machine running with R404A. On the right the experimental set-up to measure pressure drops and to apply the online viscometry, which is described in chapter 5, is shown.

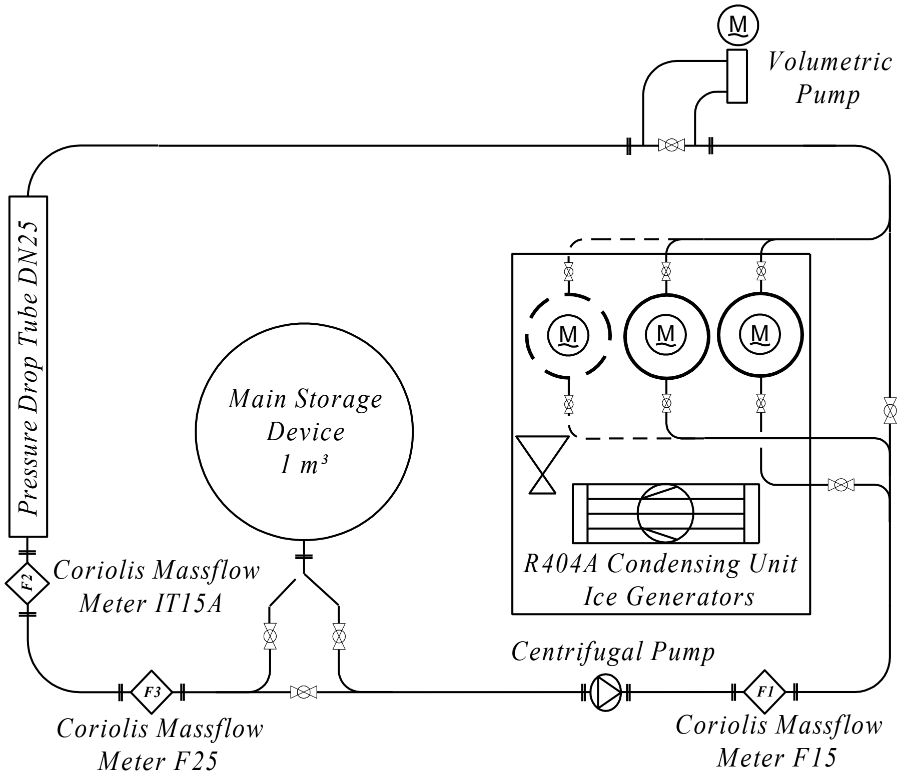

Figure 2: The experimental set-up to measure pressure drops and to apply the online viscometry.

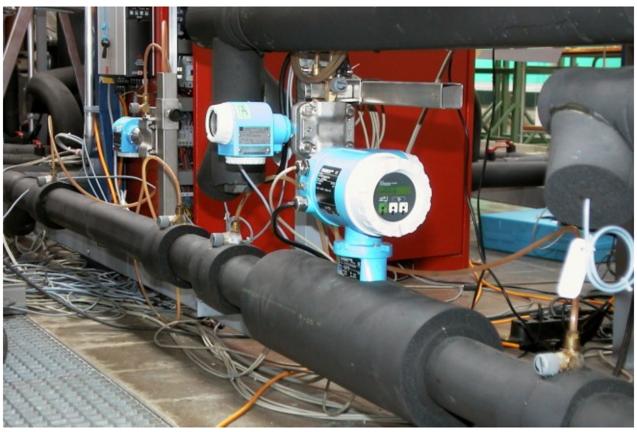

Figure 3: The well insulated test section is equipped with coriolis mass flow meters and a pressure drop measuring device to measure directly the pressure difference between two chambers which are located at a distance of exactly one meter. The inner diameter of the piping system is 27.2 mm.

Approximately one year ago a curious observation was made. In experiments with constant ice slurry temperatures (and densities) the trough formed on the surface of the ice slurry - shaped around the slab of the mixing element - smoothly altered its form and depth during the day time. A first interpretation was that the viscosity of the fluid may alter as a function of time. Also the minimal angular velocity to create a homogeneous temperature field, controlled by data acquisition of ten temperature sensors at different locations in the tank, could be reduced during these experiments.

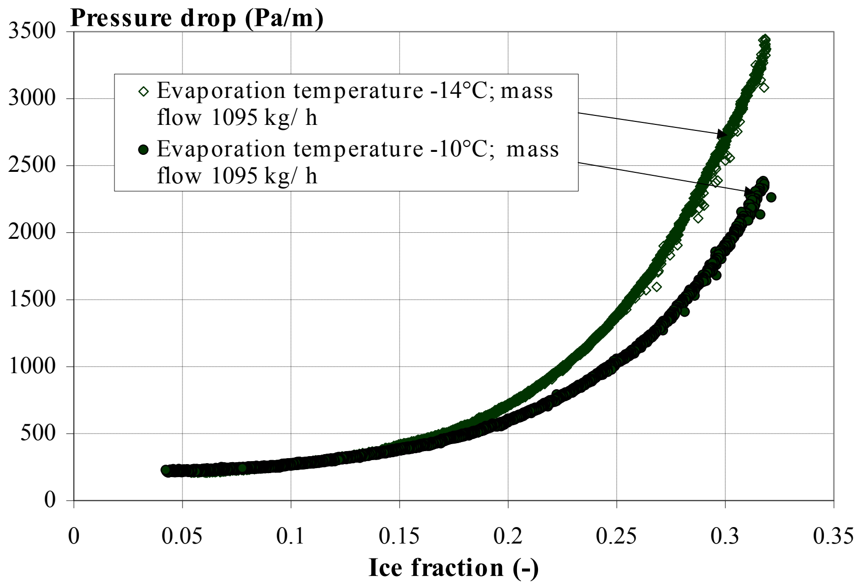

Figure 4: For two different evaporation temperatures the dependence of the pressure drop is shown as a function of the ice fraction.

For further investigations into this phenomenon the pressure drop was measured during an experiment, where the ice slurry was slightly heated by heat conduction through the insulated wall of the tube till its initial ice fraction of 32 % completely vanished. The corresponding decrease of the pressure drop as a function of the ice fraction is shown in figure 4.

The experiment has been repeated with a different evaporation temperature in the condensing unit. At high ice fractions large differences in the pressure drop - up to fourty percent - can be observed.

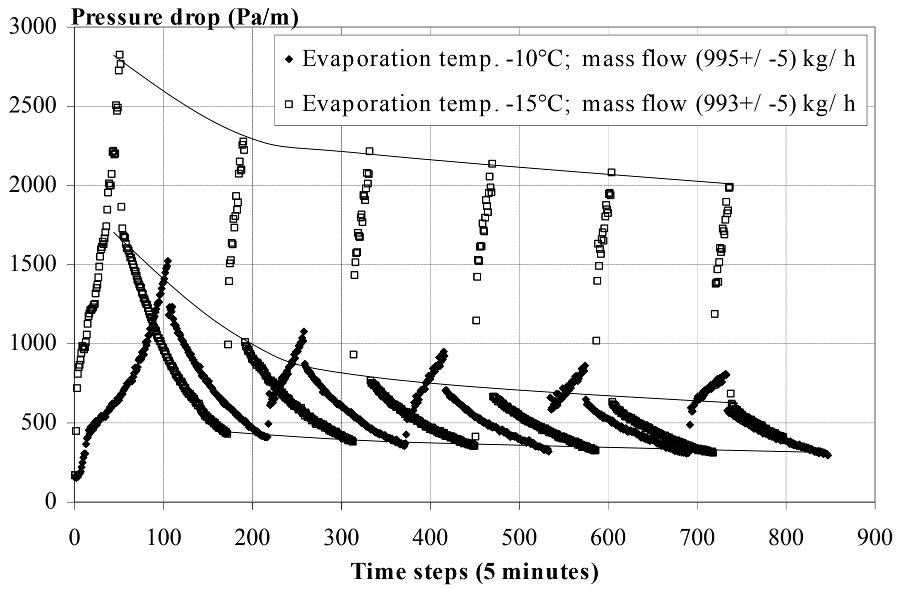

Figure 5: Periodic cooling and heating with a peak to peak amplitude of nine percent ice fraction in the main storage tank. Again a time-dependent decrease in pressure drop is observed. The heating is caused by the heat input from the environment to the ice slurry.

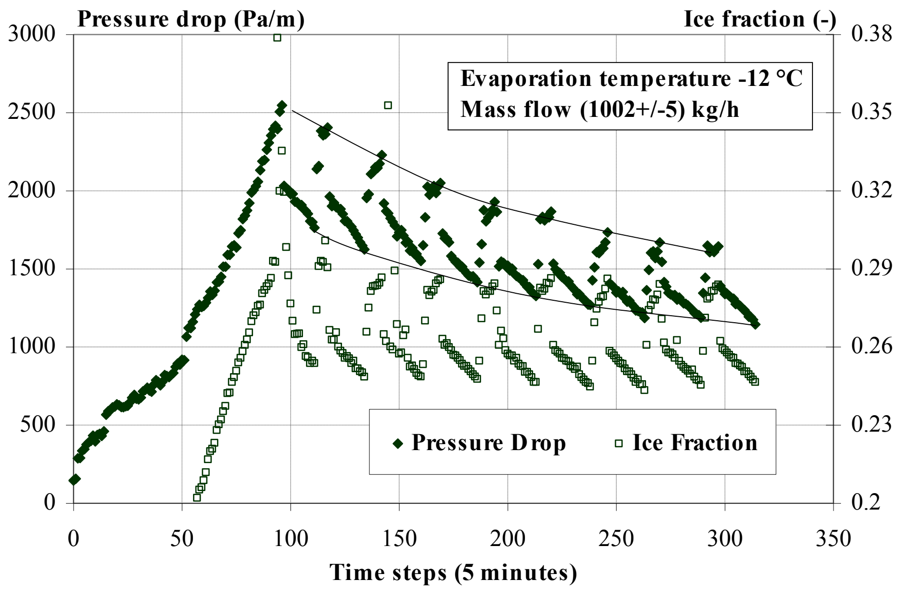

Figure 6: In this experiment the ice fraction varies only at a peak to peak amplitude of two percent in the main storage tank. The pressure drop also decreases with negligible heating and cooling.

Figure 5 shows a periodic cooling and heating process of the ice slurry with a peak-to-peak amplitude from 13 % to 22 % ice fraction. During these cycles the pressure drop was continuously measured. It can be seen that ice coming directly from the outlet of the ice generator leads to a very high pressure drop. When a storage tank is filled with ice, the mean life time of the ice particles increases and the pressure drop decreases with time. A second experiment, leading to the results in figure 6, was performed to show that the time delay from the production of the ice to its usage (here in a pressure drop experiment) is relevant for the decrease in pressure difference or shear stress. The reduction of the pressure drop is also observable when only a negligible periodic cooling and heating is applied. But in both measurements the mixing device in the main storage tank was fully running. Therefore, we assume that ice slurry of this composition clearly shows a time behaviour. The mechanisms for this must be discovered and, then they have to be thoroughly investigated. Furthermore, it can be seen that in the shear stress the influence of different evaporation temperatures vanishes in the long-time limit. The asymptotic value of the shear stress toward infinite time after ice slurry production may be of greatest importance.

The observed phenomenon may be an explanation for the different results shown in figure 1. It cannot be concluded that measurements leading to high values of shear stress are wrong. The physical properties just have been evaluated with a shorter mean life time of the ice crystals. It is recognized that the size of the used storage tank (in comparison to the refrigeration power) is a direct measure for the experimentally determined shear stresses.

5. ONLINE VISCOMETRY

When samples of ice slurries are taken from a producing system to a measuring equipment, it is very difficult to avoid heat input, which decreases ice fraction. For this reason an online viscometry method has great advantages and was developed. In the control of production processes such a method may have numerous applications.

For a Newtonian fluid the mass flow is given by the formula of Hagen Poiseuille [1]

$$\dot{m} = \frac{\pi}{8\mu}\left(-\frac{dp}{dx}\right)\rho R^4 . \tag{8}$$

This equation can be rewritten

$$\mu = \frac{\pi}{8\dot{m}}\left(-\frac{dp}{dx}\right)\rho R^4 . \tag{9}$$

Knowing the mass flow, the pressure drop, the density and the radius of the pipe the viscosity can be immediately calculated. The question which we have posed is: can this me-thod be generalized and applied to an ideal Bingham fluid (see figure 7)? The detailed theory is presented in an internal research report [12]. Here only a brief description can be outlined.

For the mass flow of a laminar Bingham flow, it is known that (see Ref. [1])

$$\dot{m} = \frac{\pi}{8\mu}\left(-\frac{dp}{dx}\right)\rho R^4\left[1-\frac{4}{3}\left(\frac{r_1}{r_2}\right)+\frac{1}{3}\left(\frac{r_1}{r_2}\right)^4\right], \qquad \frac{r_1}{r_2} = -2\tau_0\frac{1}{r_2}\frac{dx}{dp} = -2\tau_0\frac{1}{r_2}\frac{l}{\Delta p}, \qquad \Delta p < 0 . \tag{10}$$

Because the ideal Bingham is characterized by two quantities - the viscosity and the critical shear stress - it is clear that two equations must be produced to solve this problem.

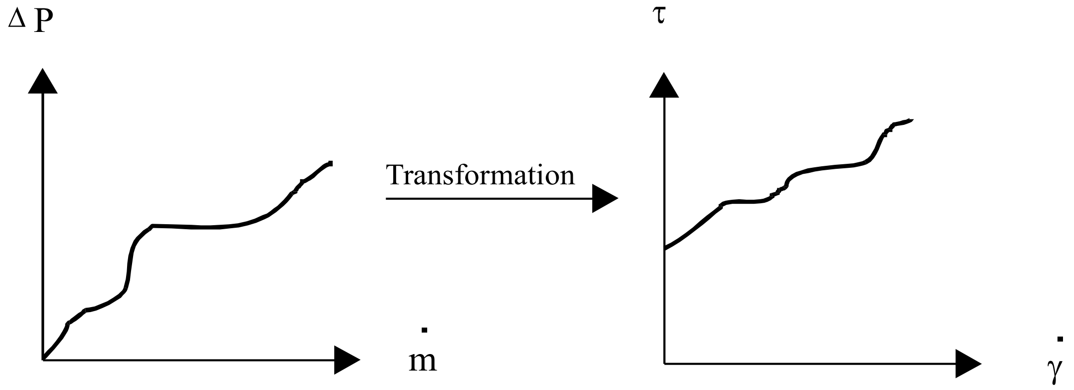

Figure 7: The mathematical transformation Γ is derived to find the rheogram from measurements of the mass flow, the pressure drop and the density.

Performing measurements with two different mass flows is the solution to this problem. Then two sets of data are obtained, a first characterized by the index *1* and a second with index 2. These define the coefficients of a system of two equations in the variables μ and τ_0

$$8l\dot{m}_1\mu + \frac{16}{3}\pi\rho_1\frac{l^4}{\Delta p_1{}^3}\tau_0{}^4 + \frac{8}{3}\pi\rho_1 R^3 l\tau_0 + \pi\rho_1\Delta p_1 R^4 = 0$$
$$8l\dot{m}_2\mu + \frac{16}{3}\pi\rho_2\frac{l^4}{\Delta p_2{}^3}\tau_0{}^4 + \frac{8}{3}\pi\rho_2 R^3 l\tau_0 + \pi\rho_2\Delta p_2 R^4 = 0. \qquad (11a,b)$$

The equation is of power four in τ_0. It can be analytically solved. The solution is given by

$$\tau_0 = \frac{1}{2}\left(\sqrt{\alpha+\beta} - \sqrt{-\alpha-\beta-\gamma}\right), \qquad (12)$$

with coefficients defined by

$$\alpha = \frac{4\cdot 2^{\frac{1}{3}}a_2}{\left(27a_1{}^2 + \sqrt{729a_1{}^4 - 6912a_2{}^3}\right)^{\frac{1}{3}}},\quad \beta = \frac{\left(27a_1{}^2 + \sqrt{729a_1{}^4 - 6912a_2{}^3}\right)^{\frac{1}{3}}}{3\cdot 2^{\frac{1}{3}}},\quad \gamma = \frac{2a_1}{\sqrt{\alpha+\beta}}, \qquad (13a\text{-}c)$$

with the abbreviations

$$a_1 = \frac{1}{2}\left(\frac{R}{l}\right)^3 \frac{\rho_1 \dot{m}_2 - \rho_2 \dot{m}_1}{\rho_1 \frac{\dot{m}_2}{\Delta p_1{}^3} - \rho_2 \frac{\dot{m}_1}{\Delta p_2{}^3}}, \qquad a_2 = \frac{3}{16}\left(\frac{R}{l}\right)^4 \frac{\Delta p_1 \dot{m}_2 - \Delta p_2 \dot{m}_1}{\frac{\dot{m}_2}{\Delta p_1{}^3} - \frac{\dot{m}_1}{\Delta p_2{}^3}} \tag{14a,b}$$

If the critical shear stress is calculated, the viscosity follows with

$$\mu = b_1 \tau_0 + b_2, \tag{15}$$

with the following coefficients

$$b_1 = \frac{\pi}{3\dot{m}_1}\rho_1 R^3 \left[\frac{\rho_1 \dot{m}_2 - \rho_2 \dot{m}_1}{\rho_1 \dot{m}_2 - \rho_2 \dot{m}_1 \left(\frac{\Delta p_1}{\Delta p_2}\right)^3} - 1\right], \quad b_2 = \frac{\pi}{8l\dot{m}_1}\rho_1 R^4 \left[\frac{\rho_1 \Delta p_1 \dot{m}_2 - \rho_2 \Delta p_2 \dot{m}_1}{\rho_1 \dot{m}_2 - \rho_2 \dot{m}_1 \left(\frac{\Delta p_1}{\Delta p_2}\right)^3} - \Delta p_1\right]. \tag{16a,b}$$

Figure 3 shows the experimental testing section to determine the mass flow, density and pressure drop. With the equations (12) to (14) the critical shear stress is calculated and with equations (15) and (16) the viscosity. The results are shown in figures 8 and 9. The theory has also been applied to mass flows, densities and pressure drops measured at other institutes. After the transformation Γ the analytical results are the dynamic viscosities and critical shear stresses. These are also added to the figures below (see Cemagref (Online viscometry) and DTI (Online viscometry)).

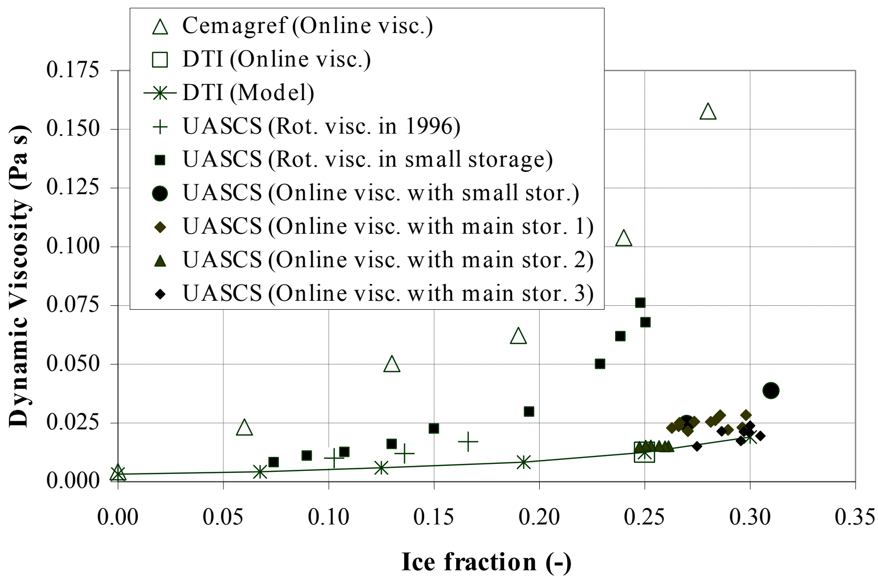

Figure 8: Eight different series of measurements of the dynamic viscosity, performed at CEMAGREF, DTI and UASCS with rotary and online viscometry. One series is a model calculation presented by Christensen et al. (see Ref. [13]).

The measurements of CEMAGREF were obtained with hardly any storage of the ice slurry. Therefore, the mean lifetime of an ice particle - between production and determination of the rheogram - was very small. That explains the high values. DTI operated with a large storage tank and obtained very small shear stresses. In our first measurements, performed in 1996 (Ref. [1]), the storage tank also was small and the determined shear stresses are of medium values. Newer measurements have been performed with a small and a large storage tank. Again a difference, depending on mean life time of the ice particles, can be observed. The shear stress data of ice slurry obtained with the large tank are as small as the DTI values and compare quite well. Comparing in figures 8 and 9 the data related to the main storage tank (series No. 3), it is observed that - with a longer duration of mixing -the critical shear stress can be further decreased. But then the dynamic viscosity increases by such a ratio, that the apparent viscosity remains approximately constant.

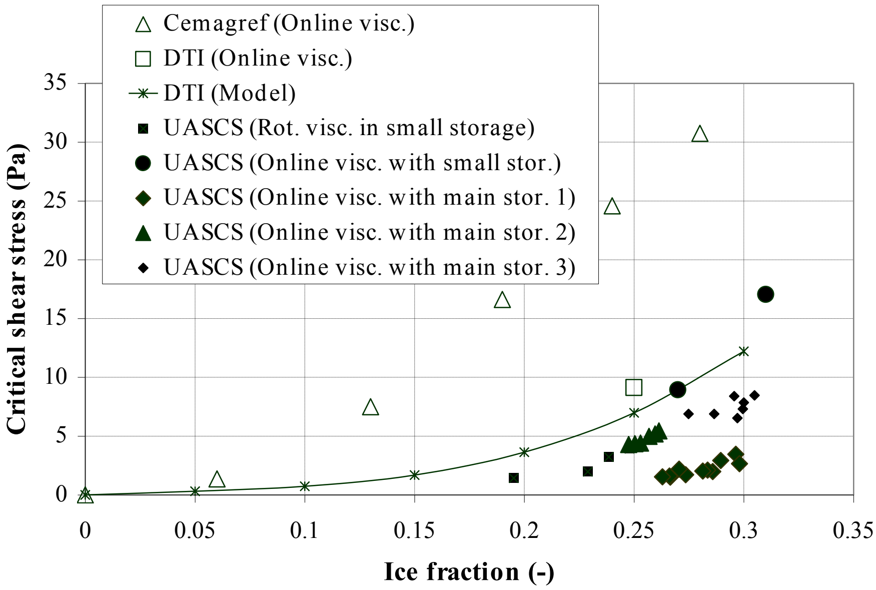

Figure 9: The corresponding critical shear stresses which are related to the dynamic viscosities - presented in figure 8 - are shown.

Ben Lakdhar [14] measured pressure drops as a function of the maximal velocity (mass flow) and also determined the rheograms for the same conditions. Therefore, in figure 7 experimental material exists for the graphics on the left and on the right. This gives the possibility to test the online viscosimetry method (the mathematical transformation Γ) presented in this chapter, or to control if the two different sets of measurements are in agreement with the Bingham theory. If the method is applied, good agreement is obtained throughout the parameter sets.

6. CONCLUSIONS AND OUTLOOK

Comparing measured viscosities and critical shear stresses and estimating the mean life time of a large ensemble of ice particles between the production and the measuring procedure gives a very consistent picture. A higher mean life time leads to a small shear stress. The insight that ice slurries show time behaviour can be a help to define further experiments, that are performed with the same time constants of the experimental apparatuses. Then comparisons should lead to better agreement and more reliable properties can be established.

Furthermore, the mechanism of the time behaviour must be detected and investigated. Polarized light and microscopic analyses, as performed by List [15] or Sari et al. [16], could be of great help. In some microscopic experiments small air bubbles were detected. But this cannot be the reason for the time behaviour, because in all our viscometry experiments the density was in agreement with the physical-properties model [4]. Another possibility: a recrystallization process of ice particles?

The shear stresses are lower than measured in first experiments. Therefore, a comparison of the transport fluid "ice slurry" with a conventional brine, as reported in Ref. [1], should be corrected. For a sufficiently mixed ice slurry even better results will be obtained. This knowledge is very important for systems design. As mixing is power consuming, but also decreases the shear stress there will be an optimal storage and mixing time, which depends on several parameters, e.g the lengths of the piping system, etc.

NOMENCLATURE

Standard

a_1, a_2, b_1, b_2	constants
l	length
m	time constant
$\dot{m}$	mass flow
p	pressure
r	radial coordinate
r_1	plug radius
r_2	pipe radius
R	pipe radius
u	velocity
x	axial coordinate

Greek

α, β, γ	constants
$\dot{\gamma}$	shear velocity
μ	viscosity
ρ	density
τ	shear stress
τ_0	critical shear stress

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