VISCOSITY OF LAVA

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ABSTRACT

Palmer, by assuming that the Alika flow of Hawaii when liquid was turbulent, found that its viscosity was 15 times that of water. Becker, who also assumed turbulent motion, found that the viscosty of the 1840 flow of Hawaii was 60 times that of water. Dimensional analysis, however, proves that these flows moved with laminar motion and therefore the viscosities computed by Becker and Palmer are in error.

By using the Jeffreys formula, the viscosity of the Alika flow is found to have been about 287,000 times the value calculated by Palmer.

The specific gravity, the thickness, and the gradient of the last 6 miles of the McCartys flow are known. If it is assumed that the viscosity was similar to that of the 1887 flow of Hawaii, the velocity in this area, calculated by the Jeffreys formula, is 4.8 miles per hour. This figure, together with a consideration of the flow mechanism of the flow, indicates that it covered the last 6 miles in about 12 hours.

The volume of the last 6 miles of the flow is known, and, as the time necessary to cover this distance has been calculated, the rate of extrusion of lava can also be calculated. It is found to be approximately 178,320,000 cubic feet per hour.

INTRODUCTION

The literature on the viscosity of lava flows is meager. Becker[1] computed the viscosity of the 1840 basaltic flow of Hawaii, and found that it was 60 times as viscous as water. Palmer[2] calculated the viscosity of the Alika flow of Hawaii, also basaltic, and found it to be 15 times that of water. Becker and Palmer assumed that the flows were turbulent. However, it will be shown that both flows must have moved by laminar motion and therefore the calculations of both Becker and Palmer produce erroneous results. The viscosity of the Alika flow, assuming laminar motion, has been computed and is of a much higher order of magnitude than the result obtained by Palmer.

Considerable work has been done on the viscosities of dry melts of basalt at temperatures similar to those found in basaltic flows. The viscosities computed by Becker and Palmer are not in harmony with any of this experimental work; but it will be shown that vis-

[1] George F. Becker, "Some Queries on Rock Differentiation," *Amer. Jour. Sci.*, Vol. III (4th ser., 1897), p. 29.

[2] Harold S. Palmer, "A Study of the Viscosity of Lava," *Bull. Hawaiian Vol. Observ.*, Vol. XV (1927), pp. 1–4.

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cosities calculated on the assumption that the flows moved by laminar motion are in close agreement with this work.

CHEZY AND KUTTER FORMULAS

The Chezy and Kutter formulas, familiar to hydraulic engineers, apply to water and generally to all liquids that have turbulent flow, i.e., flow with eddies. The velocity of these flows is almost independent of the viscosity of the liquid and depends on the density of the liquid, on the slope, and on the dimensions and roughness of the channel.

The Chezy formula[3] is

$$V = c\sqrt{rs} \, ,$$

where V is the mean velocity in feet per second, c is a coefficient depending on the dimensions, slope, and roughness of the channel, r is the hydraulic radius which is defined as the ratio of the area of the cross section to the length of its wetted perimeter, and s is the slope or tangent of the angle of inclination. The coefficient, c, is usually computed from Kutter's formula:

$$c = \frac{\frac{1.811}{n} + 41.66 + \frac{0.00281}{s}}{1 + \frac{n}{\sqrt{r}}\left(41.66 + \frac{0.00281}{s}\right)} \, ,$$

in which s and r are the same as in the Chezy formula, n is a coefficient expressing the roughness of the channel surfaces, and the various numerical coefficients are those derived empirically from experiments with channels of many types.

PALMER'S CALCULATION OF VISCOSITY

Palmer[4] analyzed the Alika flow (1919) of Mauna Loa by means of the Chezy and Kutter formulas and found that a stream of water with the same cross section, flowing on the same gradient and in a channel of similar roughness, would flow 11 times as fast as did the Alika flow. He drew from this relation and from the fact that the

[3] Charles W. Harris, *Hydraulics* (New York: John Wiley & Sons, 1936), pp. 108–11.

[4] *Op. cit.*, pp. 2–4.

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gas-charged fluid lava had a specific gravity of the order of 1.4 the incorrect conclusion that the lava was 15 times as viscous as water, on the assumption that "velocity is inversely proportional to the viscosity." Such an assumption, however, is true only of liquids which move by laminar flow, in which the filaments are parallel in any unit of the liquid. It does not hold for turbulent flows. The correct inference to be drawn from Palmer's calculation is that, because the observed velocity of the lava flow was 11 times as slow as that calculated for the stream of water, the lava flow could not have been turbulent, but must have moved by laminar flow—a state of movement to which the Chezy and Kutter formulas do not apply.

APPLICATION OF DIMENSIONAL ANALYSIS

Since the application of dimensional analysis to flow problems has become accepted, it is customary to plot the resistance of open-channel flows as shown in Figure 1. The ordinate of this diagram is known as the "friction factor" and is the product $8gsd/V^2$, where g is the acceleration of gravity, s is slope, d is the depth, and V is average velocity. The abscissa is known as "Reynold's number" and is the product $4\rho Vd/u$, where V is average velocity, d is the depth, ρ is density of the liquid, and u is viscosity. Roughly, the diagram contains two branches, A and B. Branch A applies only to liquids moving by laminar flow, and curves B, B', and B'' only to liquids moving by turbulent flow. With A goes a formula such as the one given by Jeffreys.[5] For branch B several empirical formulas are known, of which the most familiar is that of Chezy.

Palmer finds, on the basis of the Chezy and Kutter formulas, that the Alika flow moved 11 times more slowly than a stream of water flowing under identical conditions. As far as the abscissa is concerned, this results merely in a horizontal shift in the diagram, which is unimportant, since curve B is almost horizontal. With the abscissa, however, the fact that V for the lava was 11 times as small as for water means that $8gsd/V^2$ is 121 times as large as for water. Such a flow would be represented in the diagram by a point 121 times as high above the origin as most points of curve B, and therefore the Alika flow could not have been turbulent but must have been lami-

[5] Harold Jeffreys, "The Flow of Water in an Inclined Channel of Rectangular Section," *Phil. Mag.*, Vol. XLIX (1925), p. 794.

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nar. However, Palmer assumed that the flow ran in a channel whose roughness coefficient (*n* in Kutter's formula) was approximately 0.014. If we assume that the channel was much rougher than this and had a roughness coefficient (*n* in Kutter's formula) of 0.037, which would seem to be much nearer the truth, then a point on a curve lying between curves B' and B'' of Figure 1 would represent a stream of water of the same dimensions as the Alika flow running

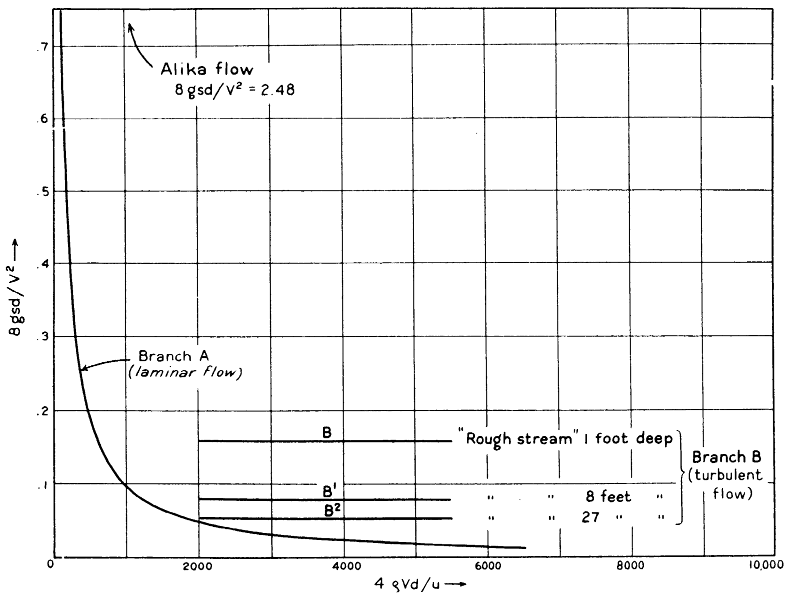

FIG. 1.—Branch A is a curve for laminar flows. If the depth, velocity, and slope of such a flow are known, the viscosity of the liquid can be obtained from this curve. Curves B, B', and B'' are for turbulent flows of various depths, having rough bottoms. If the depth, velocity, and slope of a turbulent flow are known, the viscosity of the liquid can be obtained from these curves.

under identical conditions. Calculation shows that $8gsd/V^2$ for the Alika flow is 2.48. As the value of $8gsd/V^2$ for any point on a curve lying between curves B' and B'' must have been less than 0.1, it is evident that on this assumption also the Alika flow must be represented by a point on branch A and that therefore it was not turbulent. Palmer's error lies in the fact that he applies to branch B a property which belongs only to A—namely, that velocity is inversely proportional to viscosity.

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If the computed velocity of the stream of water had been about the same as that of the lava flow, two solutions for the viscosity of the lava would have been possible. The conditions of velocity, density, slope, and depth in this case could have been satisfied by a relatively fluid liquid flowing with turbulence and by another liquid of greater viscosity flowing with laminar motion. This can be seen graphically in Figure 1. From the value of $8gsd/V^2$, calculated from the velocity, slope, and depth of the flow, a value for $4\rho Vd/u$ can be obtained from the curve for laminar flow. From this value of $4\rho Vd/u$, a value for u (viscosity of lava) may be calculated, since V, d, and ρ are all known. Similarly, a smaller value for u (viscosity) may be obtained from the curve for turbulent flow. Gravity does not succeed in making the liquid of low viscosity flow more rapidly than the more viscous liquid, because the liquid of low viscosity uses up energy in turbulence, whereas in the case of the more viscous liquid the force of gravity is not so largely expended in overcoming internal resistance. Which is the correct solution can be determined only if it is known whether the flow was turbulent or laminar.

Becker[6] calculated the viscosity of the 1840 flow of Hawaii and found that it was 60 times that of water. He made the same error as did Palmer, for he also assumed that velocity is inversely proportional to viscosity. An analysis similar to that made for the Alika flow proves that the 1840 flow was also of the laminar type.

Since, however, Becker failed to give the essential data on which his computations rest, no further analysis of the 1840 flow is profitable.

In the application of this analysis to these problems, the help of Professor J. P. DenHartog is specifically acknowledged.

VISCOSITIES OF ALIKA AND 1887 FLOWS

As the Alika flow was of the laminar type, its viscosity can be calculated by the Jeffreys formula.[7] This formula applies to cross sections in which the width is much greater than the depth; however, no large error should be involved if this formula is used with cross sections of other dimensions. The Jeffreys formula in c.g.s. units is

$$V = \frac{g \sin A \, d^2 p}{3u} ,$$

[6] *Op. cit.*, p. 29.

[7] *Op. cit.*, p. 794.

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where V is mean velocity with regard to depth, g is acceleration of gravity, A is angle of inclination, d is depth, u is ordinary coefficient of viscosity, and p is specific gravity. Jaggar[8] observed the pahoehoe stream in the main channel of the Alika flow to be moving at the rate of 11 miles per hour. He believes that the channel was from 20 to 40 feet wide and from 15 to 30 feet deep. Recent topographic mapping shows the gradient of the channel to be about 500 feet in $\frac{3}{4}$ of a mile, or about 12.6 per cent. If we assume that the depth of the Alika flow was 20 feet (Jaggar thought it to be between 15 and 30 feet), and that its specific gravity when liquid was 1.4, the vis-

TABLE 1

Substance	Temperature C.	Coefficient of Viscosity	Authority
Ice, glacier	....	12×10^{13}	Deeley
Pitch	0°0	51×10^{10}	Trouton and Andrews
Pitch	15.0	1.3×10^{10}	Trouton and Andrews
Wax, shoemaker's	8.0	4.7×10^{6}	Trouton and Andrews
Water	20.0	.01005	Bingham and Jackson
Glycerin	20.3	8.30	Schöttner
Sugar	109.0	2.8×10^{4}	Tammann
Alika flow	....	4.3×10^{4}	Calculated by the Jeffreys formula
1887 flow	....	4.77×10^{4}	Calculated by the Jeffreys formula

cosity as calculated by the Jeffreys formula is 4.3×10^4 poises. This is about 287,000 times the value calculated by Palmer for this flow.

Thanks to the careful mapping of the 1887 flow of the Kau district, Hawaii, by Stearns,[9] it is possible to estimate its viscosity. From Stearns's work it appears that the average velocity of the 1887 flow was 0.7 mile an hour, its average thickness is 7 feet 3 inches, average gradient 354 feet to the mile, average width 4,000 feet, and the specific gravity when liquid must have been approximately 1.4. Using these data and the Chezy and Kutter formulas it is found that water would flow under the same conditions 40 times as fast as did the 1887 flow. Therefore, the 1887 flow was also of the laminar type. Using the Jeffreys formula and Stearns's data, its viscosity is

[8] Palmer, *op. cit.*, p. 2.

[9] Harold T. Stearns and William O. Clark, "Geology and Water Resources of the Kau District, Hawaii," *U.S. Geol. Surv. Water-Supply Paper 616* (1930), p. 74.

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4.77×10^4 poises, or about the same as that calculated for the Alika flow, using the same method.

From the table of viscosities of familiar substances (Table 1), it will be seen that the viscosities of both the Alika and 1887 flows as calculated by the Jeffreys formula seem reasonable. They are smaller than the viscosities of glacial ice, pitch, and shoemaker's wax, but larger than those of water and glycerin. It will also be noted that the viscosities of the two flows are similar to that of sugar at 109° C. Sugar at this temperature is a somewhat pasty liquid, which nevertheless flows out of an inclined test tube.

EXPERIMENTAL WORK ON VISCOSITY

In order to measure the relative viscosity of the lava in the lake of Halemaumau and in the lower southwest rift cone, Jaggar[10] built a cylinder 27 inches long with an inside diameter of 3 inches, which was attached by a reducer to a 1-inch pipe. A flat iron cap with a central circular orifice 1.5 inches in diameter was fastened over the open end of the cylinder. After being heated over the hot lava, the cylinder and pipe were thrust into the lava to a depth of 1 meter and held there for four minutes. At the first locality it was found on withdrawal that the lava had filled approximately 83 per cent of the cylinder, while at the lower southwest rift cone only a third of the cylinder was filled. The temperature of the lava in Halemaumau pit was found to be 1,200° C. and the temperature in the lower southwest rift cone was 1,100° C. Although it is not possible with these data to make an accurate calculation of the viscosity of the lava either in the lake or in the cone, it is obvious that the viscosity was of a very high order, much greater than that calculated by Palmer and Becker and probably of the same order as that here determined for the Alika and 1887 flows. In so far as the lava in the lake and in the southwest rift cone is similar to that which formed the Alika flow, these experiments favor the estimate of viscosity obtained with the Jeffreys formula over those made by Palmer.

Volarovich[11] has measured the viscosity of dry melts of basalt at various temperatures. His results are shown in Table 2.

[10] T. A. Jaggar, "Experimental Work at Halemaumau," *Bull. Hawaiian Vol. Observ.*, Vol. IX (1921), pp. 28–29.

[11] M. P. Volarovich, D. M. Tolstoy, and L. I. Korčemkin, "A Study of the Viscosity of Molten Lavas from Mount Alaghez," *Compt. Rend.* (*Doklady*) *Acad. Sci. U.S.S.R.*, Vol. I(X), No. 8(85) (1936), p. 334.

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It will be noticed that two values for viscosity are given for 1,160° C. This is due to the fact that at 1,160° C. the basalt began to crystallize during measurement. Consequently, the viscosity at this temperature increased with progressive crystallization.

TABLE 2

BASALT

Temperature C.	Absolute Value of Viscosity—Poises
1,400°	3.53×10^2
1,340	6.50×10^2
1,320	8.20×10^2
1,280	1.34×10^3
1,220	2.96×10^3
1,160	6.82×10^3
1,160	1.36×10^4

Kani[12] also has measured the viscosity of dry melts of an olivine basalt at various temperatures. His results, which check rather closely those of Volarovich, are shown in Table 3.

TABLE 3

OLIVINE BASALT

Temperature C.	Absolute Value of Viscosity—Poises
1,400°	1.37×10^2
1,350	1.76×10^2
1,300	2.96×10^2
1,250	6.56×10^2
1,200	3.18×10^3
1,150	3.79×10^4

The temperatures of the Alika and 1887 flows were probably between 1,150° and 1,200° C. It will be noticed that the viscosities obtained by the Jeffreys formula for these flows are slightly higher than those of the dry melts at these temperatures. One should, however, expect the viscosities of the Alika and 1887 flows to be lower than those of these dry melts at similar temperatures, because lava flows when molten contain dissolved gases as well as bubbles of gas that may make up as much as 50 per cent of the volume of the flow. It is impossible to say in a quantitative way how the presence of this

[12] K. Kani, "The Measurement of the Viscosity of Basalt Glass at High Temperatures, II," *Imperial Acad. Tokyo Proc.*, Vol. X (1934), p. 82.

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gas would affect the viscosity. However, it seems unlikely that it would lower it to anything like the figures given by Becker and Palmer. The fact that the viscosities here calculated are higher than those of the dry melts, whereas because of their dissolved gases we should expect them to be lower, may be explained, first, by assuming that crystallization had already started while the flows were still in motion; second, by the fact that in the case of the 1887 flow the velocity used was the average rate at which the front of the flow advanced rather than the channel speed, which, if used in the calculation, would have given a lower figure for the viscosity of the flow; third, the temperature of the flows may have been lower than 1,150° C., the lowest temperature at which viscosities were experimentally measured; and, fourth, the 1887 flow undoubtedly had to fill in depressions in the terrane over which it flowed. This ponding delayed advance and consequently the estimate of its velocity is undoubtedly too low and therefore its calculated viscosity is too high.

SUMMARY ON VISCOSITY

In conclusion, it has been shown that the viscosities calculated by Palmer and Becker for the Alika and 1840 flows are based on the faulty assumption that the flows were turbulent. The viscosity of the Alika flow calculated by means of the Jeffreys formula, which assumes laminar flow, was found to be 4.3×10^4 poises or 287,000 times that calculated by Palmer. The viscosity of the 1887 flow was 4.77×10^4 poises. The experimental data of Jaggar, Volarovich, and Kani are consistent with these estimates but are not in harmony with those of Palmer and Becker.

APPLICATION OF THE JEFFREYS FORMULA TO THE McCARTYS FLOW

THE McCARTYS FLOW

The McCartys flow[13] in Valencia County, New Mexico, is a pahoehoe basaltic flow, more than 30 miles long, which is probably of historic age. The last 6 miles of the flow were studied in great detail during the field seasons of 1934 and 1935. The following assumptions and calculations are made with the object of obtaining an

[13] Robert L. Nichols, "Quaternary Geology of the San José Valley, New Mexico," *Geol. Soc. Amer. Proc. for 1933* (Abst.), p. 453.

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estimate of the velocity of the flow in this area and the rate of extrusion of lava from the source.

VELOCITY

The gradient of the last 6 miles of the McCartys flow is about 30 feet per mile. The roughness of the terrane over which it moved was probably similar to that of a channel having a high roughness coefficient. Unfortunately, since the flow is uneroded, it is difficult to

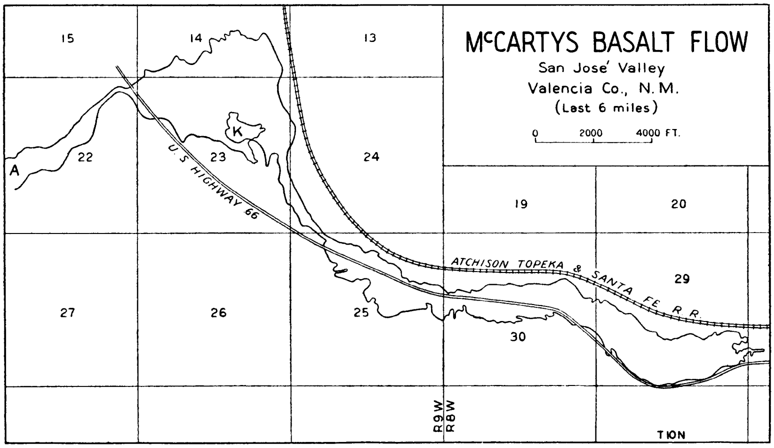

FIG. 2.—Map showing the last 6 miles of the McCartys flow, central New Mexico. The area marked *K* is a kipuka.

get accurate data on its thickness in this area. However, an approximate figure may be obtained from the depth of the larger cracks, from the depth of the collapse depressions, by a study of the edge of the flow around a kipuka, and by a comparison with the Laguna flow,[14] which is similar petrographically but eroded so that its thickness can be measured. Measurements around the kipuka shown in Figure 2 indicate that the flow in this area is about 15 feet thick; however, where the flow is narrow it is probably as much as 50 feet thick. Thirty feet is probably a good figure for the average thickness of the last 6 miles of the flow and its average width is about 1,500 feet. Using these data and the Chezy and Kutter formulas,[15] it appears that a stream of water of the same dimensions as the

[14] *Ibid.*

[15] Harris, *op. cit.*, pp. 108–11.

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McCartys flow, flowing under identical conditions, would flow with a speed of 24 miles per hour. If it is assumed that the McCartys flow moved with less than one-half this speed, and this seems reasonable, an analysis similar to that made for the 1887 and Alika flows of Hawaii proves that the McCartys flow flowed by laminar motion.

If it is assumed that the viscosity of the McCartys flow was similar to that of the 1887 flow of Hawaii as calculated above, an assumption which is probably not greatly in error as both are basaltic flows, and if it is also assumed that the specific gravity when liquid was 2.0, then by using the data given above and the Jeffreys formula,[16] which assumes laminar flow, it is found that the average velocity for the last 6 miles was 4.8 miles per hour. At this velocity the lava would have moved from point *A* on Figure 1 to the end of the flow in approximately 1.2 hours.

The foregoing analysis, however, assumes that the lava moved as a single-unit flow. This, however, is not in agreement with the field facts which suggest that the flow, at least in part, moved by the flow-unit mechanism.[17] A flow which moves by multiple units will flow more slowly than one of similar dimensions which flows as a single unit. The slower movement results, first, from the fact that there is a time interval between the extrusion of individual units, and, second, because the units themselves move more slowly than would the whole mass as a single unit, since velocity, other things being equal, is proportional to the square of the thickness. The total time required to cover the last 6 miles must, therefore, have been in excess of the minimum figure of 1.2 hours. Just how much longer it took is difficult to say. However, if we assume that the flow is composed of 3 flow units of equal thickness, a reasonable assumption, then each unit being one-third the total thickness of the flow would take 9 times as long to move the last 6 miles (approximately 10 hours) as would the flow moving as a single unit. If it is further assumed that the time interval between the extrusion of the units was 5 hours, then approximately 40 hours would be required to cover the last 6 miles. However, the field evidence indicates that only a small fraction of the last 6 miles moved by the flow-unit mechanism. In

[16] Jeffreys, *op. cit.*, p. 794.

[17] Nichols, "Flow-Units in Basalt," *Jour. Geol.*, Vol. XLIV (1936), pp. 617–30.

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view of this, 12 hours is probably a good estimate for the time required to cover this distance.

Furthermore, the rate of flow varied at different points within the last 6 miles as shown by the following analysis.

The Jeffreys formula[18] in c.g.s. units is

$$V = \frac{g \cdot \sin A \cdot d^2 p}{3u},$$

where V is mean velocity with regard to depth, g is the acceleration of gravity, A is angle of inclination, d is depth, u is the absolute viscosity, and p is specific gravity. If it is assumed that the density and viscosity of the lava remained constant while the flow was advancing over this 6 miles, and that the gradient was more or less uniform, then it can be seen from the formula that the velocity of the flow must have varied as the square of its thickness. As the lava, due to topography, does vary in thickness, its velocity could not have been uniform but must have been more rapid at one place and less rapid at another. This is, of course, in accord with observations on historic flows in the Hawaiian Islands and elsewhere.

RATE OF EXTRUSION

The area of the flow downstream from point A on Figure 2 is approximately 38,000,000 square feet. If we take 30 feet as the average depth for this part of the flow, its volume is 1,140,000,000 cubic feet. However, collapse depressions resulting from the collapse of the roofs of lava tubes are common in part of this area. The area of the lava tubes was approximately 3,500,000 square feet. If it is assumed that their average height was 20 feet, a figure which seems reasonable when compared with the depth of the depressions which now occupy their sites, and that their cross sections were square, their volume was about 70,000,000 cubic feet. This figure should be subtracted from the volume of the flow calculated above, giving 1,070,000,000 cubic feet for the corrected volume.

Assuming that all of this lava flowed through the lava tube at A in approximately 12 hours, according to the foregoing estimate, about 89,160,000 cubic feet flowed through every hour. However,

[18] *Op. cit.*, p. 794.

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as the flow may have been increasing in area upstream from A while it was advancing downstream from A, it is apparent that this is a minimum figure for the rate of extrusion from the cone which fed the flow. If we multiply 89,160,000 cubic feet by 2 for this factor, we are probably not far from the truth. If 120 pounds is taken as the average weight of each cubic foot, then approximately 10,699,000 tons of lava per hour were extruded.

Stearns and Clark[19] found that the rate of extrusion for the 1887 flow of Hawaii was 2,250,000 tons per hour. This is about one-fifth the rate of extrusion for the McCartys flow.

ACKNOWLEDGMENTS.—Professor J. P. DenHartog, as acknowledged in the text, furnished the requisite data and method for the application of dimensional analysis to the problem of the flow of lava. Helpful suggestions of a more general character have also been made by Professors Kirk Bryan and R. A. Daly of Harvard University and Dr. Richard Tousey of Tufts College. Professor Bryan also assisted the author in the preparation of the manuscript. Thanks are due to Dr. Karl T. Benedict, Mr. Robert S. Folsom, and Mr. Charles E. Stearns for field assistance.

[19] *Loc. cit.*

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