Deep Learning

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Assignment 1

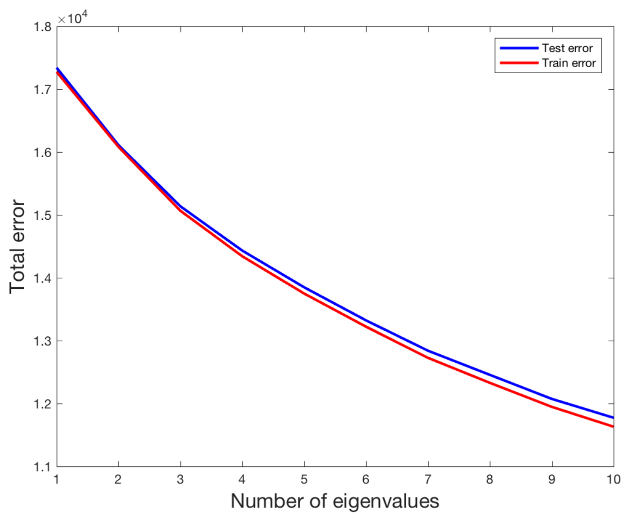
1 Data modelling: PCA and K-means

1.1 PCA

We train our PCA algorithm on the training set, and we are asked to provide a plot of the error function :

$$E(D) = \sum_{n \in Testset} \|I_n - \left(\sum_{d=1}^{D} c_{n,k} e_k + m \right) \|_2$$

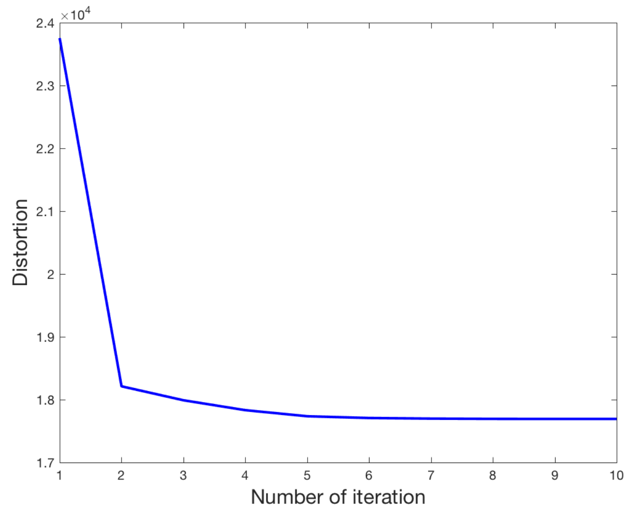
Figure 1: Quality of the learned model



1.2 K-means

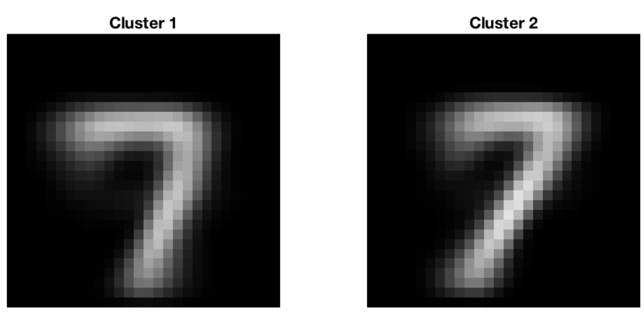
After having programmed the k-means algorithm, we evaluate the distortion criterion for k=2, and verify that it decreases as shown in the graphics below.

Figure 2: Decrease of the distortion criterion



We then perform the algorithm ten times and keep the solution that minimises the distortion. Here are the resulting digit clusters.

Figure 3: Digit clusters

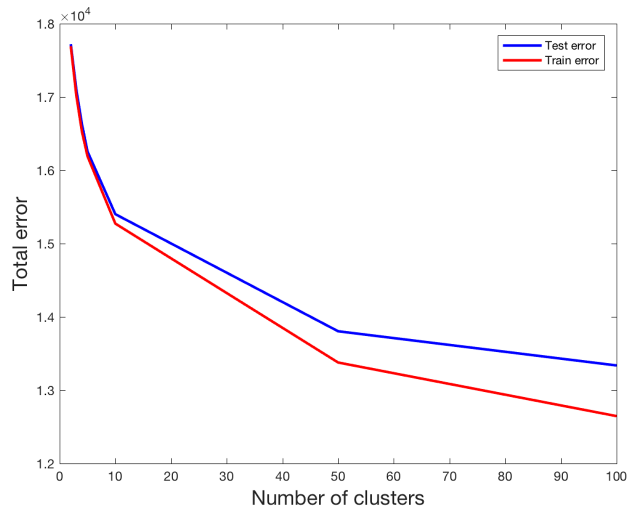


Finally, we repeat the procedure for k = 3,4,5,10,50,100 and observe that the cost decreases as k increases, as shown below.

We computed the cost of the k-means algorithm according to this formula (c_n being the centroid to which I_n was affected) :

$$E(k) = \sum_{n \in Testset} \|I_n - c_n\|_2$$

Figure 4: Decrease of the k-means distortion cost



PCA and K-means perform similarly on train set data (from 1.8×10^4 to 1.2×10^4), but the PCA generalises better on unseen data. This difference can be explained because PCA is a dimensionality reduction technique whereas k-means is a clustering algorithm. In order to mitigate this error, a good practice could be to combine both techniques.

2 Ising model: MCMC and learning

2.1 Part-1: Brute-force evaluations

In this exercise, we consider an Ising model based on a 4x4 lattice. We first consider exhaustively all $2(N^2)$ states, energies and associated probabilities. The energy is of the form:

$$E(\mathbf{x}; J) = \sum_{(i,j) \in \mathcal{E}} x_i x_j = -\frac{J}{2} \mathbf{x}^T \mathbf{A} \mathbf{x}$$

where J is a scalar and $\mathcal{E}$ is the set of edges of a rectangular grid, $\mathbf{A}$ is the 16x16 incidence matrix of the network.

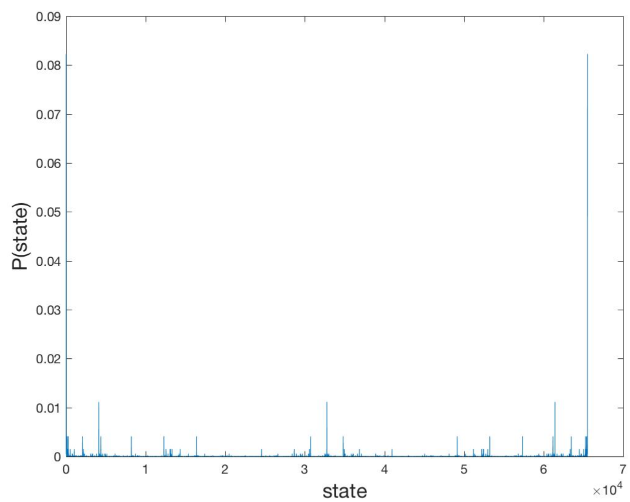
We then compute the partition function:

$$Z = \sum_{\mathbf{x}} \exp(-E(\mathbf{x}; J))$$

As shown below, the probability that the network is in any state $\mathbf{x}$ is given by:

$$P(\mathbf{x}; J) = \frac{1}{Z} \exp(-E(\mathbf{x}; J))$$

Figure 5: Probabilities of each state

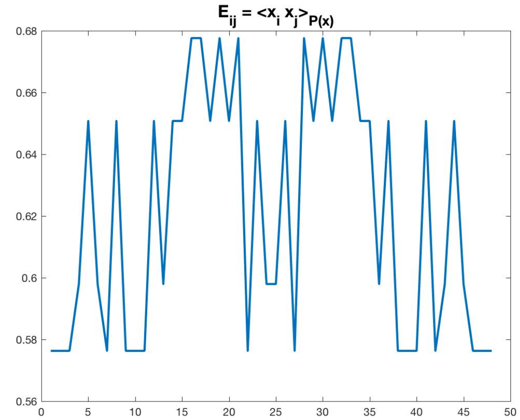


And, for all features $\phi_k(x), k = 1, ..., K$, we compute their expectations under the model:

$$E_k^{PJ} = \langle \phi_k(\mathbf{x}) \rangle_{P(\mathbf{x};J)} = \sum_{\mathbf{x}} P(\mathbf{x}; J) \phi_k(\mathbf{x})$$

In figure 6, we can see the result of the calculation of this expectation.

Figure 6: Features expectations

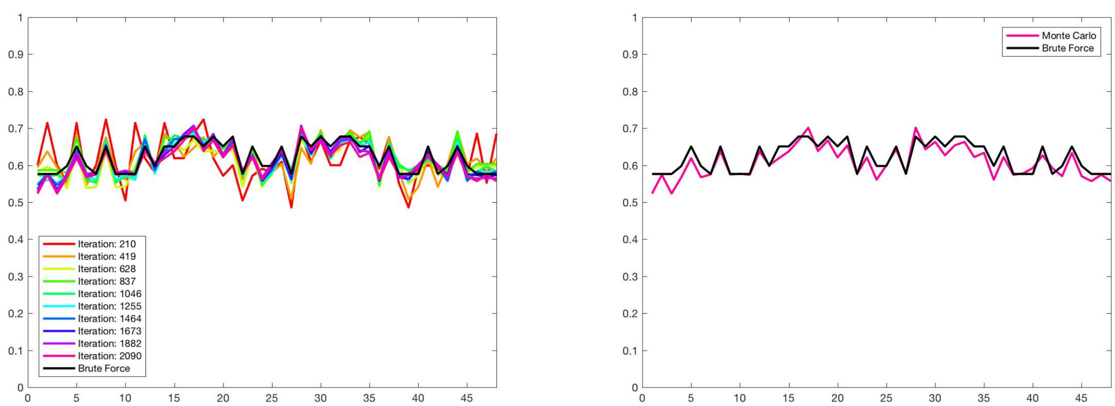


2.2 Part-2: MCMC evaluations

We use Gibbs sampling to obtain Monte Carlo estimates of the quantities computed in the previous subsection:

$$E_k^{MC}(N) = \sum_{n=1}^{N} \phi_k(\mathbf{x}_n)$$

Figure 7: Progression of Monte Carlo estimates and Comparison between Monte Carlo and Brute Force



2.3 Part-3: Parameter Estimation

In this part, we want to estimate the value of J that is most likely to be responsible for the values of E^{PJ}. In other words, we try to find the maximum likelihood estimate of J. We seek therefore to maximise the log-likelihood of J, which can be expressed:

$$log(L(x_1, ..., x_n; J)) = -NlogZ - \sum_n E(x_n; J)$$

This value is maximum for $\frac{\partial log(L(x_1,...,x_n;J))}{\partial J} = 0$. As we observe that the Ising model described is a member of the exponential family, this value is obtained for:

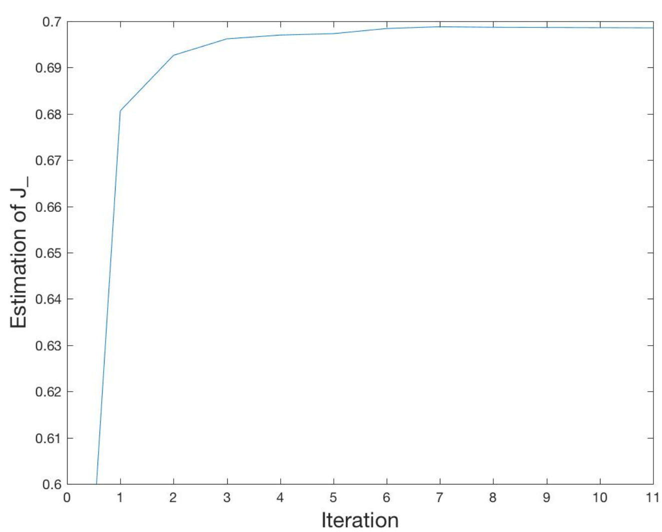
$$\langle \phi_k \rangle_{emp} = \langle \phi_k \rangle_{P(\mathbf{x};J)}$$

We use a gradient ascent technique to find a global maximum with this formula:

$$J = J - \delta(\langle \phi_k \rangle_{emp} - \langle \phi_k \rangle_{P(\mathbf{x};J)})$$

δ is the step we use to update J. After some experiments, a value of $\delta = 0.015$ is the one which yields the most precise result. We then find a value of $J = 0.6986$.

Figure 8: Estimation of J by gradient ascent



We then plot the expectations.

Figure 9: Progression of the estimation per iteration and Comparison between estimated and desired models