Practical work 03 – 03.10.2023
Classification with Bayes - System Evaluation

Summary for the organisation :

- Submit the solutions of the practical work before the date specified in Moodle.
- **Rule 1**. Submit an archive (***.zip**!) with your Python notebooks (one per exercise), including datasets and all necessary files.
- **Rule 2**. The archive file name must contain the number of the practical work, followed by the family names of the team members by alphabetical order, for example `02_dupont_muller_smith.zip`. Put also the name of the team members in the body of the notebook (in first cell). Only one submission per team.
- **Rule 3**. We give a **fail** for submissions that do not compile (missing files are a common source of errors...). So, make sure that your whole notebooks give the expected solutions by clearing all cells and running them all before submitting.

Exercice 1 Classification system using Bayes

The objective of this exercise is to build a bayesian classification systems to predict whether a student gets admitted into a university or not based on their results on two exams [1].

You have historical data from previous applicants that you can use as a training set. For each training example n, you have the applicants scores on two exams $(x_{n,1}, x_{n,2})$ and the admissions decision y_n. Your task is to build a classification model that estimates an applicants probability of admission based on the scores from those two exams.

a. Bayes - Histograms

Implement a classifier based on Bayes using histograms to estimate the likelihoods.

a) Read the training data from file `ex1-data-train.csv`. The first two columns are x_1 and x_2. The last column holds the class label y.
b) Compute the priors of both classes $P(C_0)$ and $P(C_1)$.

1. Data source : Andrew Ng - Machine Learning class Stanford

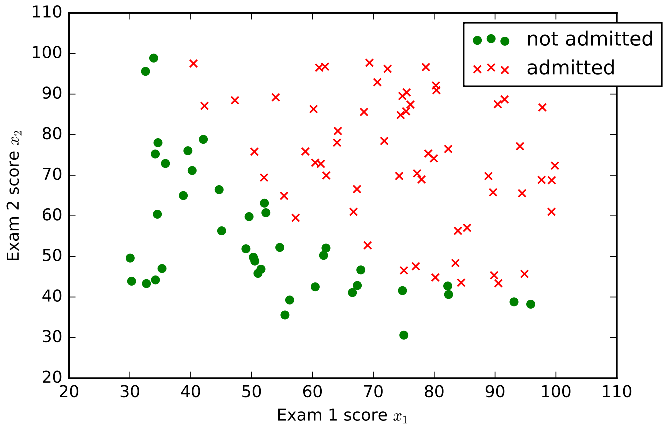

FIGURE 1 – Training data

c) Compute histograms of x_1 and x_2 for each class (total of 4 histograms). Plot these histograms. Advice : use the numpy `histogram(a,bins='auto')` function.
d) Use the histograms to compute the likelihoods $p(x_1|C_0)$, $p(x_1|C_1)$, $p(x_2|C_0)$ and $p(x_2|C_1)$. For this define a function `likelihood_hist(x,hist_values,edge_values)` that returns the likelihood of x for a given histogram (defined by its values and bin edges as returned by the numpy `histogram()` function).
e) Implement the classification decision according to Bayes rule and compute the overall accuracy of the system on the test set `ex1-data-test.csv`. :

   — using only feature x_1
   — using only feature x_2
   — using x_1 and x_2 making the naive Bayes hypothesis of feature independence, i.e. $p(X|C_k) = p(x_1|C_k) \cdot p(x_2|C_k)$

   Which system is the best ?

b. Bayes - Univariate Gaussian distribution

Do the same as in a. but this time using univariate Gaussian distribution to model the likelihoods $p(x_1|C_0)$, $p(x_1|C_1)$, $p(x_2|C_0)$ and $p(x_2|C_1)$. You may use the numpy functions `mean()` and `var()` to compute the mean μ and variance σ^2 of the distribution. To model the likelihood of both features, do the naive Bayes hypothesis of feature independence, i.e. $p(X|C_k) = p(x_1|C_k) \cdot p(x_2|C_k)$.

Exercice 2 System evaluation

Let's assume we have trained a digit classification system able to categorise images of digits from 0 to 9, as illustrated on Figure 2.

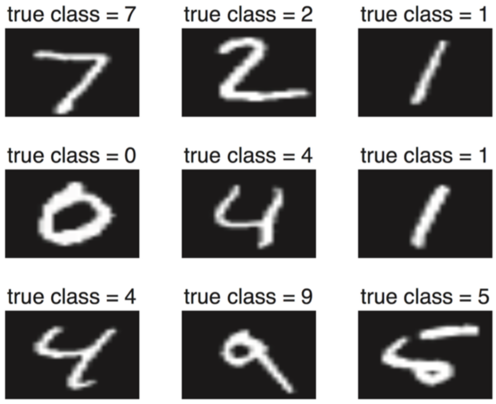

FIGURE 2 – Digit classification system

After training, the system has been run against a test set (independent of the training set) including $N_t = 10'000$ samples. The system is able to compute estimations of a posteriori probabilities $P(C_k|\mathbf{x})$ for $k = 0, 1, 2 \ldots, 9$.

In file `ex1-system-a.csv`, you find the output of a first system A with the a posteriori probabilities $P(C_k|\mathbf{x})$ in the first 10 columns and with the ground truth y in the last column.

a) Write a function to take classification decisions on such outputs according to Bayes'rule.
b) What is the overall error rate of the system ?
c) Compute and report the confusion matrix of the system.
d) What are the worst and best classes in terms of precision and recall ?
e) In file `ex1-system-b.csv` you find the output of a second system B. What is the best system between (a) and (b) in terms of error rate and F1.

Exercice 3 Review questions

a) Assuming an univariate input **x**, what is the complexity at inference time of a Bayesian classifier based on histogram computation of the likelihood ?

b) Bayesian models are said to be *generative* as they can be used to generate new samples. Taking the implementation of the exercise 1.a, explain the steps to generate new samples using the system you have put into place. **Optional**. Provide an implementation in a function `generateSample(priors, histValues, edgeValues, n)` that returns n new samples $[(x_1^{(1)}, x_2^{(1)}), (x_1^{(2)}, x_2^{(2)}), \ldots, (x_1^{(n)}, x_2^{(n)})]$ in an array.

c) What is the minimum overall accuracy of a 2-class system that is built on a training set that includes 5 times more samples in class A than in class B ?

d) Let's look back at the PW02 exercise 3 of last week. We have built a KNN classification systems for images of digits on the MNIST database.

   How would you build a Bayesian classification for the same task ? Comment on the prior probabilities and on the likelihood estimators. More specifically, what kind of likelihood estimator could we use in this case ? **Optional** : implement it and report performance !

e) Read https://theintercept.com/2019/07/26/europe-border-control-ai-lie-detector/. The described system is "*a virtual policeman designed to strengthen Europeens borders*". It can be seen as a 2-class problem, either you are a suspicious traveler or you are not. If you are declared as suspicious by the system, you are routed to a human border agent who analyses your case in a more careful way.

   i) What kind of errors can the system make ? Explain them in your own words.
   ii) Is one error more critical than the other ? Explain why.
   iii) According to the previous points, which metric would you recommend to tune your ML system ?

f) When a deep learning architecture is trained using an unbalanced training set, we usually observe a problem of bias, i.e. the system favors one class over another one. Using the Bayes equation, explain what is the origin of the problem.