阻抗导纳圆图和阻抗匹配

- 阻抗和导纳圆图
- λ/4阻抗变换器、信号源与负载阻抗的匹配
- 阻抗匹配和调谐
- 小反射理论和宽带阻抗变换器

小反射理论和宽带阻抗变换器

变换

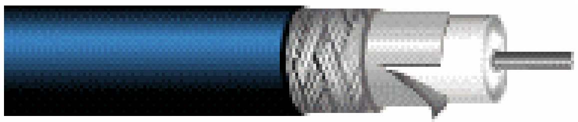
包含:

变换元件

(1) 尺寸过渡

(2) 阻抗匹配

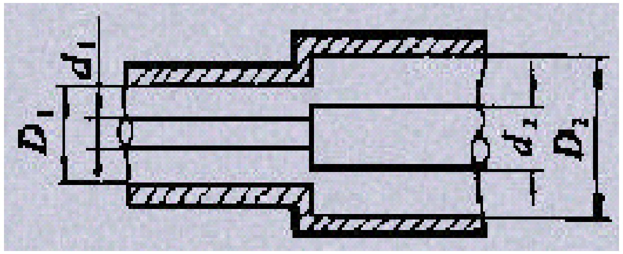
同轴波导

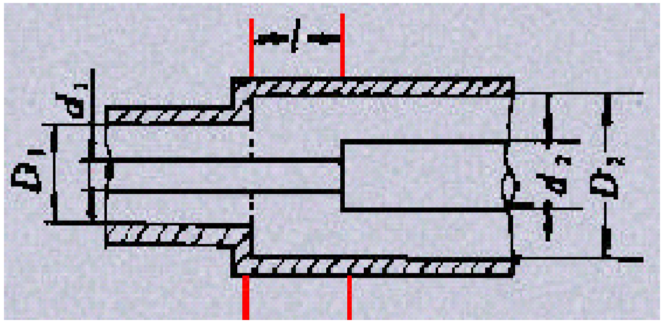


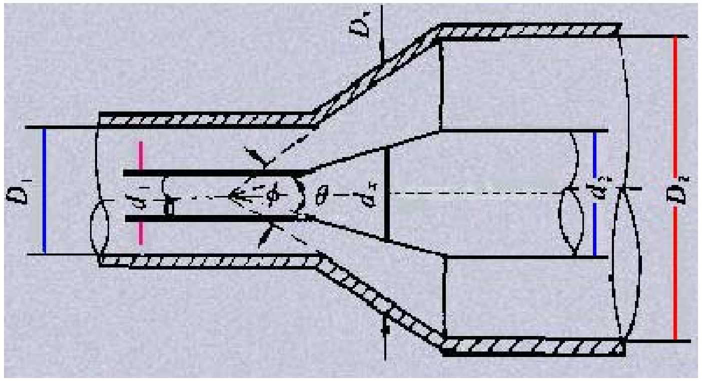
1. 尺寸过渡：

特性阻抗相同，但尺寸不同的同轴线连接

特性阻抗不同，尺寸不同的同轴线连接



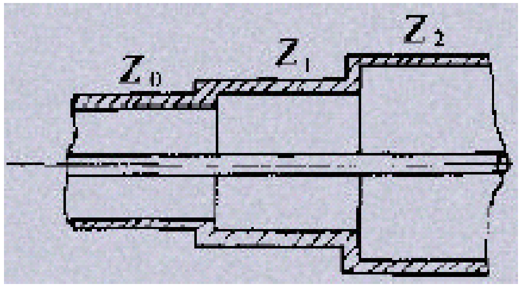


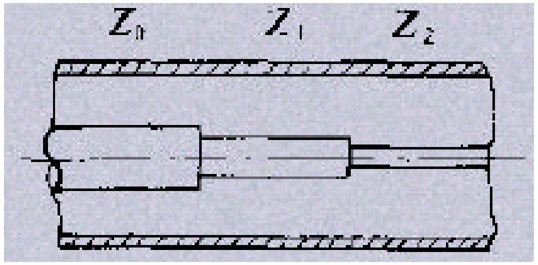


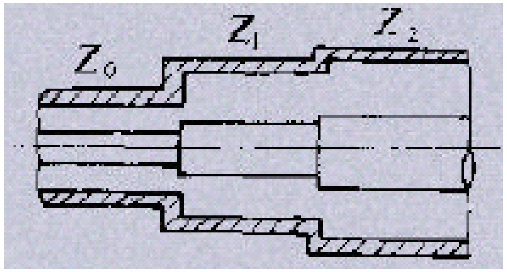
同轴波导

2. 阻抗匹配：

特性阻抗不同的同轴线连接

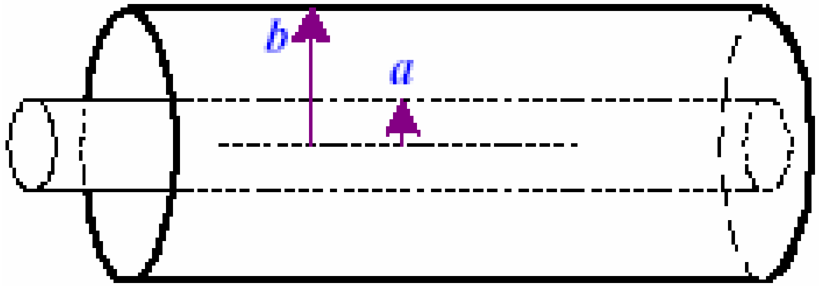






$$Z_1 = \sqrt{Z_0 \cdot Z_2}$$

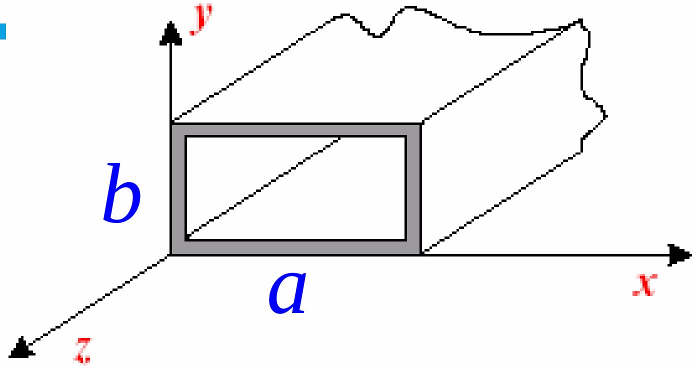
$$Z_0 = \sqrt{\frac{L_0}{C_0}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon}}\ln\left(\frac{b}{a}\right)$$

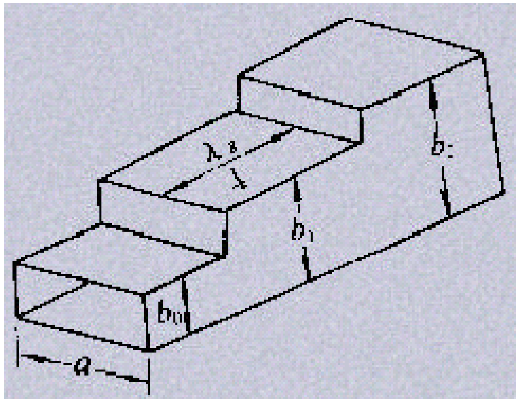


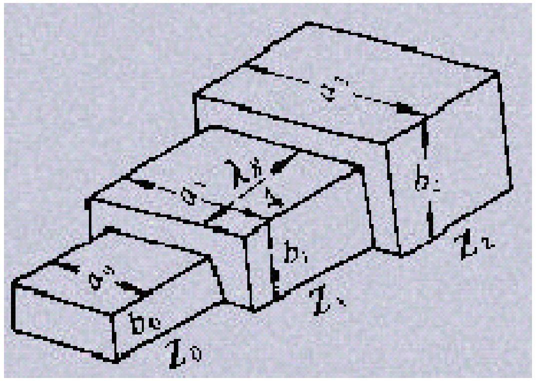
$$\boxed{Z_0 \propto \ln\left(\frac{b}{a}\right)}$$

矩形波导

$$Z_0 \propto \left(\frac{b}{a}\right) \cdot \frac{1}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}$$





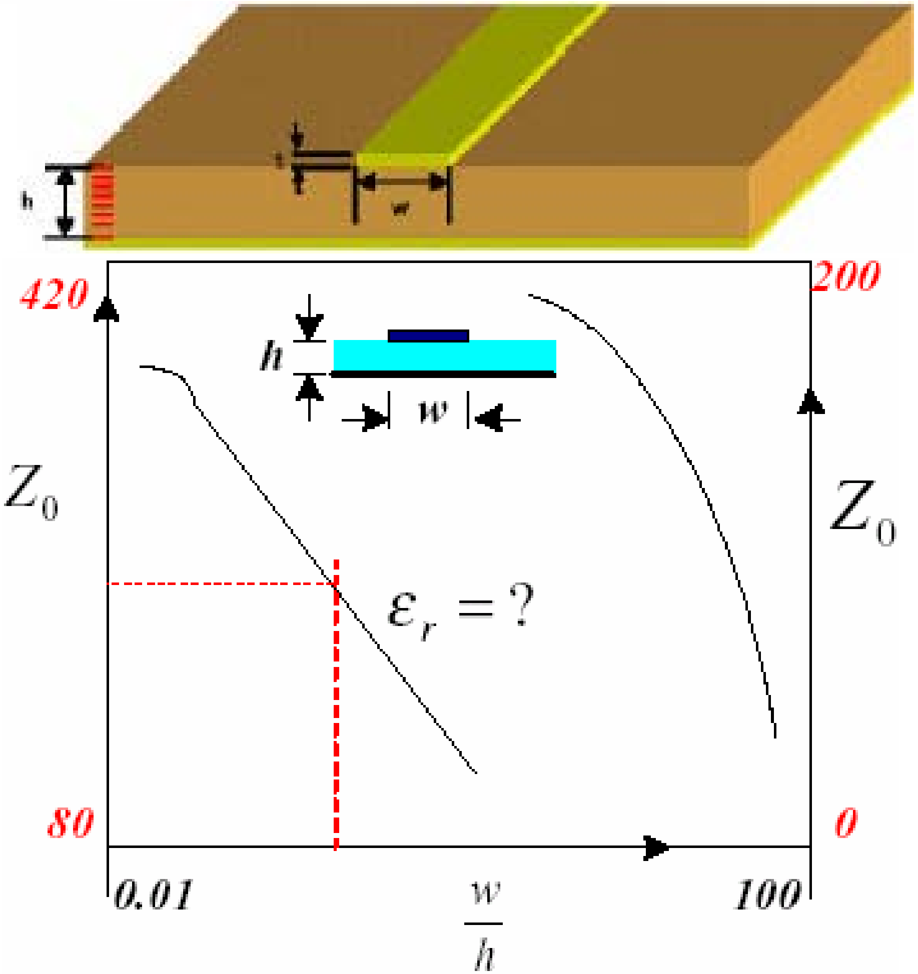


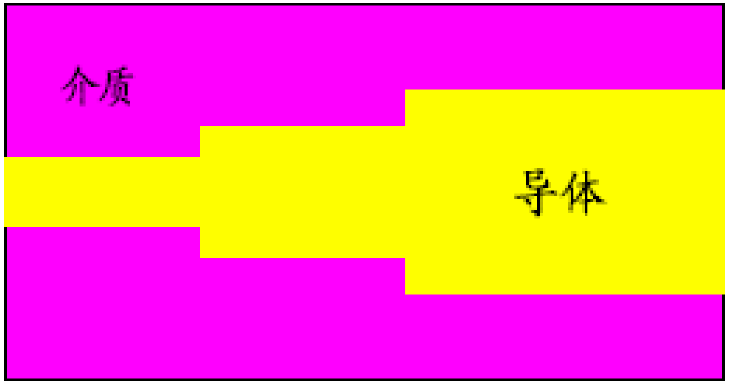
a不变情况下

$$Z_0 \propto b$$

微带线

顶视图

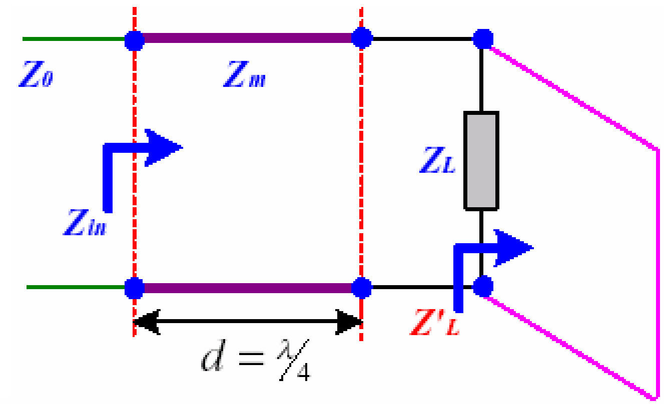


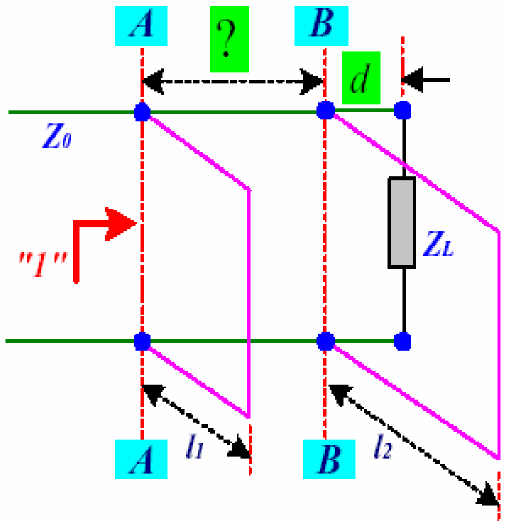


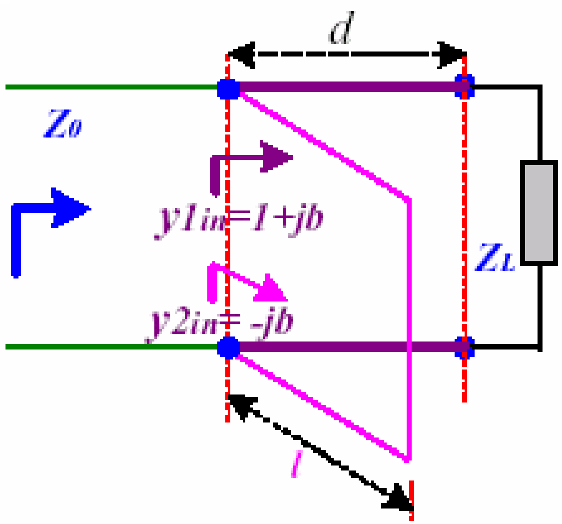
调整微带线的“宽窄”!

1.6.2 问题的提出

窄：匹配长度与信号波长(频率)的密切关系







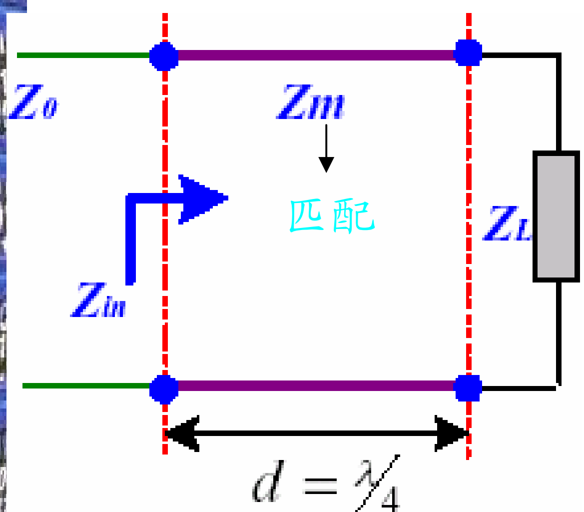
复习　串连1/4波长线实现匹配

- 复习：　$Z_0 = R_0$ 无损耗线：

$$Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0 tg(\beta l)}{Z_0 + jZ_L tg(\beta l)}$$

$$l = \frac{\lambda}{4}$$

$$Z_{in}(\frac{\lambda}{4}) = \lim_{\beta l \to \frac{\pi}{2}} (Z_m \frac{Z_L + jZ_m tg(\beta l)}{Z_m + jZ_L tg(\beta l)}) = (Z_m)^2 \left(\frac{1}{Z_L} \right)$$



$$Z_{in} = Z_0$$

串联另外一种(Zm)传输线，实现匹配

四分之一波长阻抗变换器——“阻抗倒置”

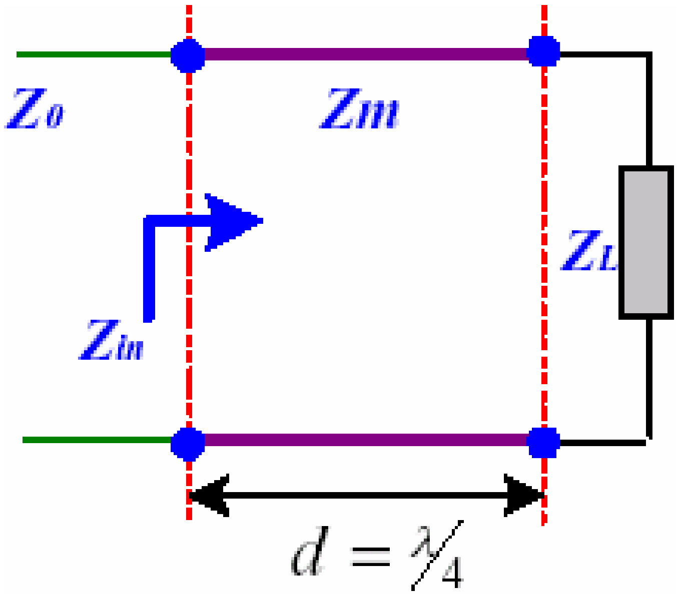
不一定有现成产品

要求 Z_L 为纯实数

串连1/4波长线进行阻抗匹配

复习

- 如果纯阻性负载 Z_L 没有虚部 $Z_L = R_L$



$$Z_{in} = (Z_m)^2\left(\frac{1}{R_L}\right) = Z_0$$

$$\therefore Z_m = \sqrt{Z_0 \cdot R_L}$$

串连一根1/4波长传输线，特征阻抗为: $Z_m = \sqrt{Z_0 \cdot R_L}$

串联的是另外一种(Zm)传输线，实现匹配

小反射理论

串连1/4波长线进行阻抗匹配：实现单频率匹配

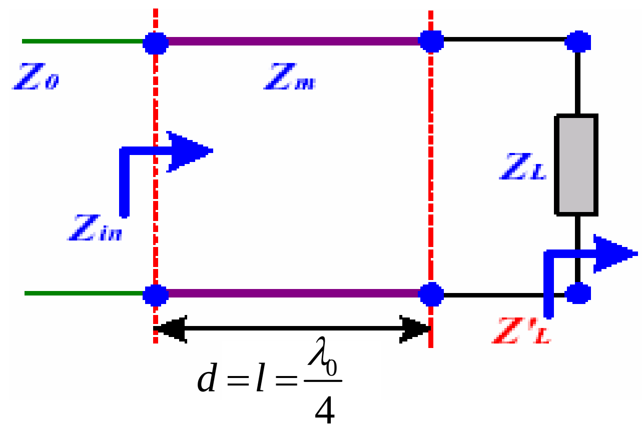
为获得更大的带宽，可采用多节阻抗变换器

先讨论多个小的不连续产生的反射引起的总反射系数

即为小反射理论

例

已知：在频率为f_0时匹配



$$Z_m = \sqrt{Z_L \cdot Z_0}$$

$$Z_{in} = Z_m \cdot \frac{Z_L + jZ_m tg\theta}{Z_m + jZ_L tg\theta}$$

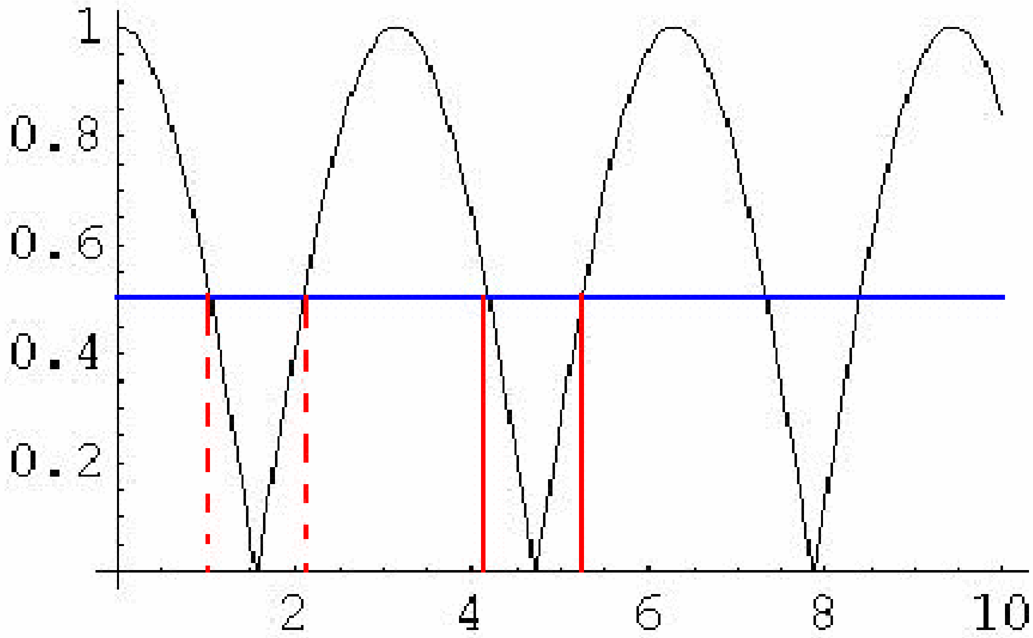
$$\theta = \beta \cdot l = \frac{2\pi}{\lambda} \frac{\lambda_0}{4} = \frac{\pi}{2} \frac{\lambda_0}{\lambda} = \frac{\pi}{2} \frac{f}{f_0}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \xrightarrow{\text{目的}} = 0$$

考察模: $$|\Gamma| = \frac{1}{\sqrt{1 + \left[\left(\frac{2\sqrt{Z_0 \cdot Z_L}}{Z_L - Z_0}\right) \cdot \frac{1}{\cos\theta}\right]^2}} \approx |\Gamma_0| \cdot |\cos\theta|$$

1.6.3 “匹配”是窄带的

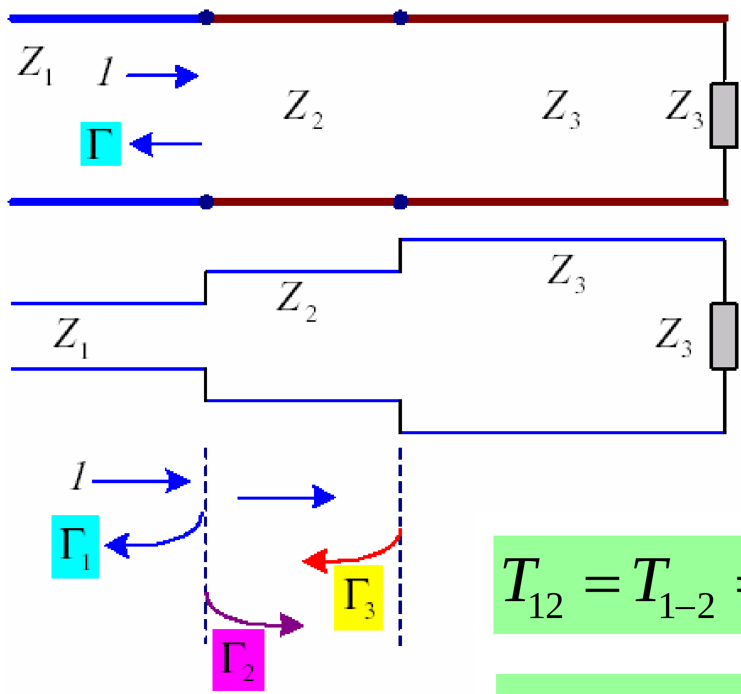
$|\cos\theta|$



$$|\Gamma| \propto |\cos\theta|$$

$$\theta = \beta \cdot l = \frac{2\pi}{\lambda}\frac{\lambda_0}{4} = \frac{\pi}{2}\frac{\lambda_0}{\lambda} = \frac{\pi}{2}\frac{f}{f_0}$$

1.6.4 连接点的多次反射现象



$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\Gamma_2 = \ldots = -\Gamma_1$$

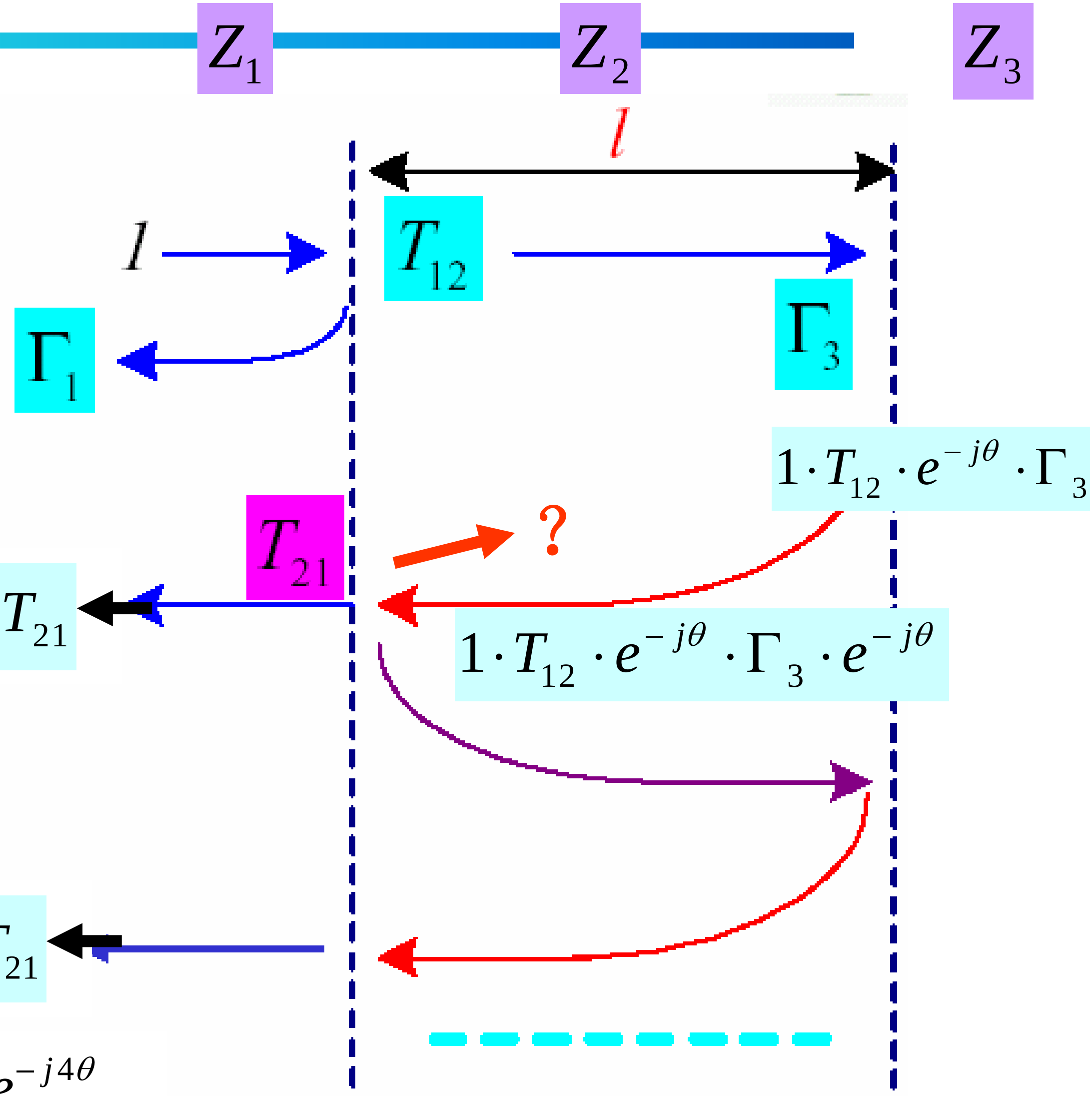
$$\Gamma_3 = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$

$$T_{12} = T_{1-2} = 1 + \Gamma_2 = 1 - \Gamma_1$$

$$T_{21} = T_{2-1} = 1 + \Gamma_1 = 1 - \Gamma_2$$

多次反射

$$\theta = \beta \cdot l = 2\pi \cdot \left(\frac{l}{\lambda}\right)$$

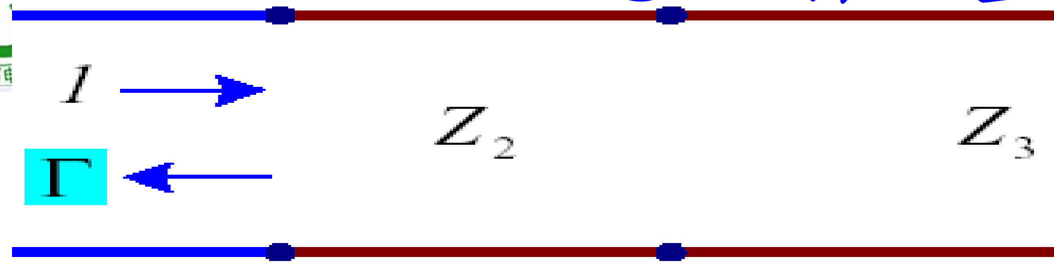


$$\left[1 \cdot T_{12} \cdot e^{-j\theta} \cdot \Gamma_3 \cdot e^{-j\theta}\right] \cdot T_{21}$$

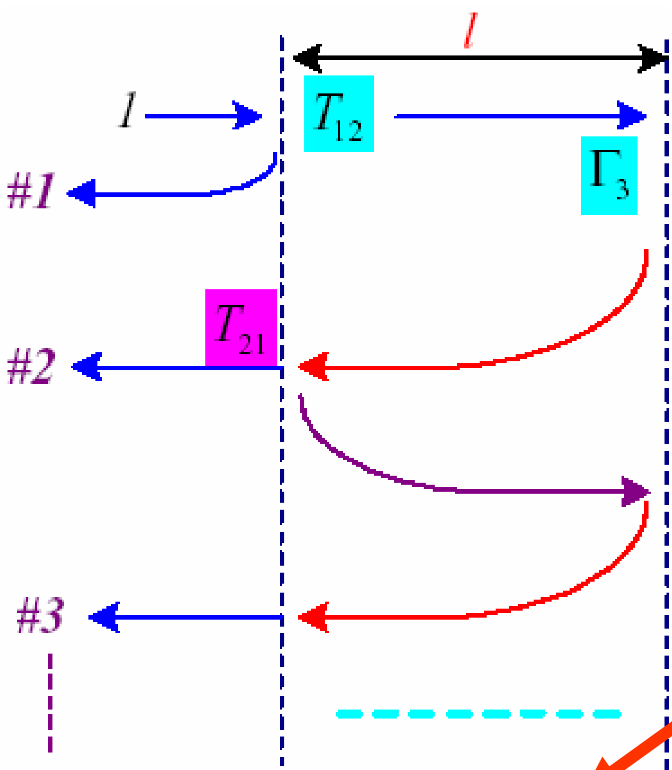
$$\left[(?) \cdot \Gamma_2 \cdot e^{-j\theta} \cdot \Gamma_3 \cdot e^{-j\theta}\right] \cdot T_{21}$$

$$= T_{12} \cdot T_{21} \cdot \Gamma_2 \cdot (\cdot\Gamma_3)^2 e^{-j4\theta}$$

总反射：叠加



总反射系数



$$\Gamma = \Gamma_1 + \left(T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-j2\theta}\right) + \dots$$

参考几何级数：

当 $|x| < 1$ 有：$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

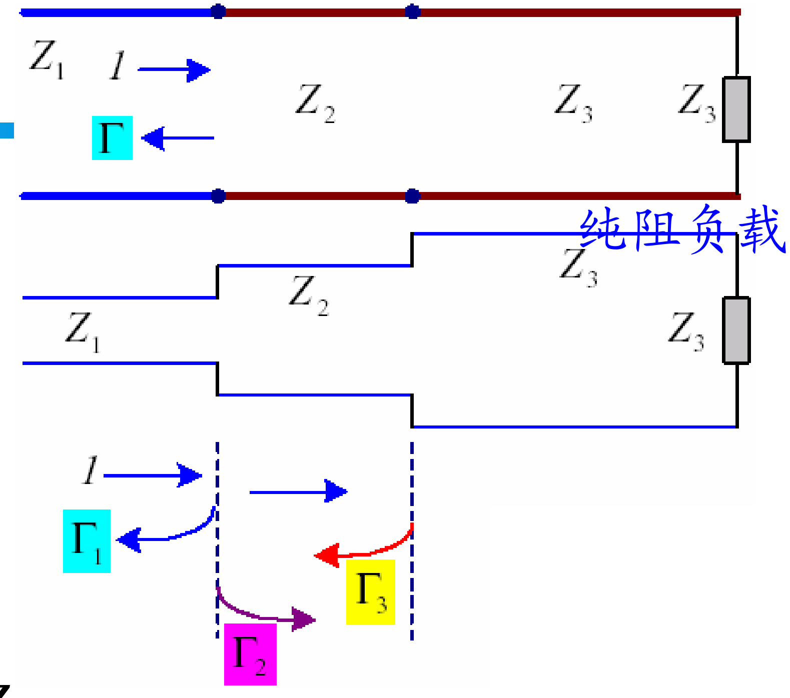
$$\Gamma = \Gamma_1 + \left(T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-j2\theta}\right) \cdot \frac{1}{1-\Gamma_2 \cdot \Gamma_3 \cdot e^{-j2\theta}}$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-j2\theta}}{1+\Gamma_1 \cdot \Gamma_3 \cdot e^{-j2\theta}} \approx \Gamma_1 + \Gamma_3 \cdot e^{-j2\theta}$$

$$T_{12} = T_{1-2} = 1+\Gamma_2 = 1-\Gamma_1$$

$$T_{21} = T_{2-1} = 1+\Gamma_1 = 1-\Gamma_2$$

例1.12 $\lambda/4$ 变换器



$$Z_1=100\Omega, Z_2=150\Omega, Z_3=225\Omega$$

利用 $\Gamma=\dfrac{\Gamma_1+\Gamma_3\cdot e^{-j2\theta}}{1+\Gamma_1\cdot\Gamma_3\cdot e^{-j2\theta}}\approx\Gamma_1+\Gamma_3\cdot e^{-j2\theta}$

考虑最坏情况下近似式带来的误差比

解： 局部反射系数为

$$\Gamma_1=\frac{Z_2-Z_1}{Z_2+Z_1}=\frac{150-100}{150+100}=0.2 \qquad \Gamma_3=\frac{Z_3-Z_2}{Z_3+Z_2}=\frac{225-150}{225+150}=0.2$$

最大差别为：$\theta=0$ 或 $\theta=180^0$ 时：

总系数的精确值

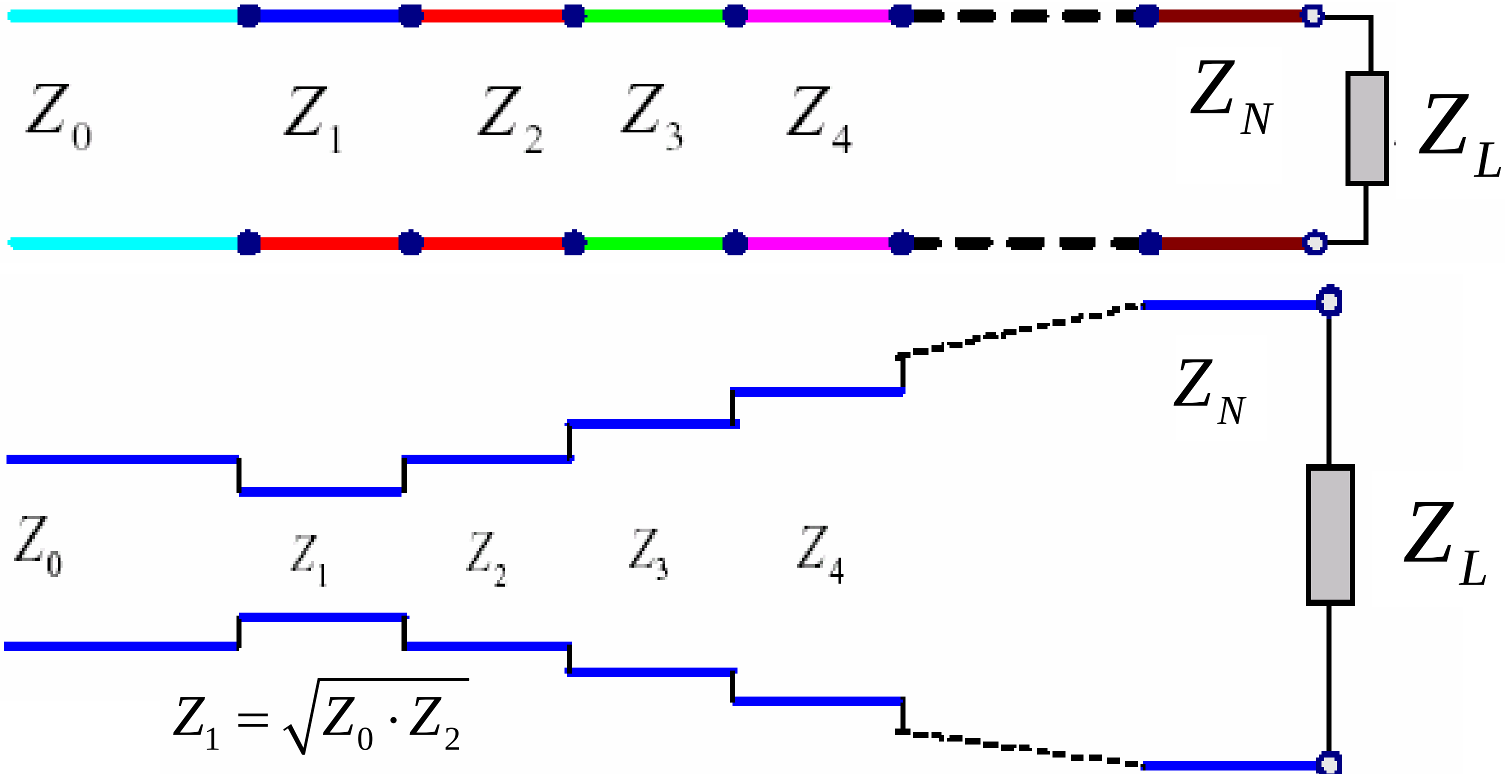
$$\Gamma=\frac{\Gamma_1+\Gamma_3}{1+\Gamma_1\cdot\Gamma_3}=\frac{0.2+0.2}{1+0.2\times0.2}=0.384$$

总系数的近似值 $\Gamma\approx\Gamma_1+\Gamma_3=0.2+0.2=0.4$

误差为：

$$\frac{0.4-0.384}{0.384}\approx0.04$$

1.6.5 多段λ/4阻抗变换



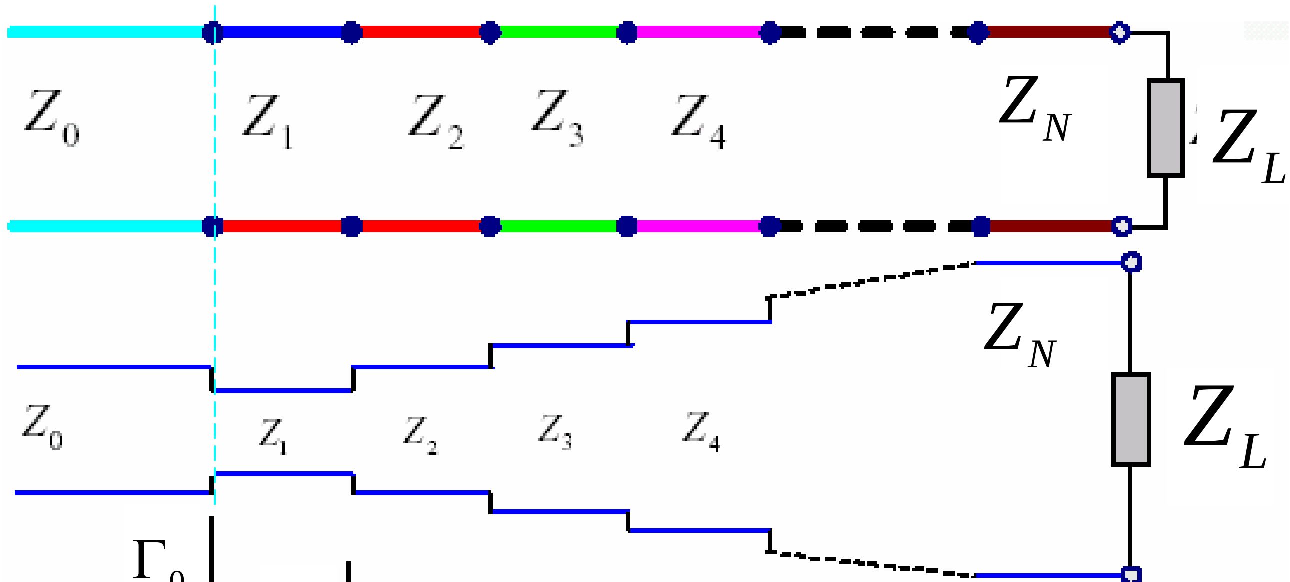
$$Z_1 = \sqrt{Z_0 \cdot Z_2}$$

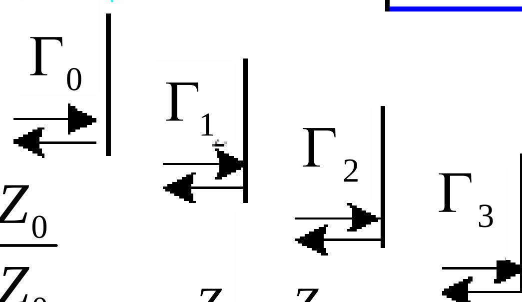
$$Z_2 = \sqrt{Z_1 \cdot Z_3}$$

$$Z_N = \sqrt{Z_{N-1} \cdot Z_L}$$

$$Z_n = \sqrt{Z_{n-1} \cdot Z_{n+1}}$$

考虑信号只经过“1次”反射





$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

设Zn单调增加（或减小）

总的反射系数

$$\Gamma=\Gamma_0+\Gamma_1\cdot e^{-j2\theta}+\Gamma_2\cdot e^{-j4\theta}+\ldots+\Gamma_i\cdot e^{-j2i\theta}+\ldots+\Gamma_N\cdot e^{-j2N\theta}$$

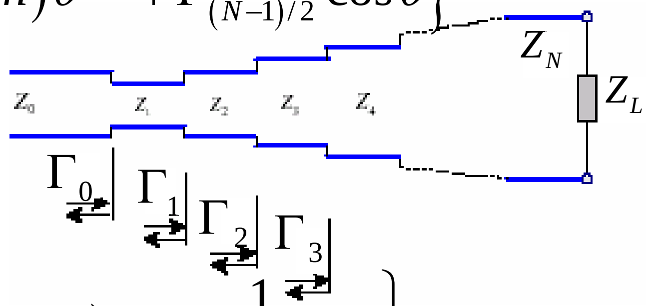
假定变换器做成对称的，即 $\Gamma_0=\Gamma_N,\Gamma_1=\Gamma_{N-1},\Gamma_2=\Gamma_{n-2},\cdots\cdots$

$$\Gamma=e^{-jN\theta}\left\{\Gamma_0\left(e^{jN\theta}+e^{-jN\theta}\right)+\Gamma_1\left(e^{j(N-2)\theta}+e^{-j(N-2)\theta}\right)+\cdots\right\}$$

$$\theta=\beta\cdot l=\frac{\pi}{2}\cdot l$$

N为奇数时最后一项为 $\Gamma_{(N-1)/2}\left(e^{j\theta}+e^{-j\theta}\right)$

$$\Gamma=2e^{-jN\theta}\left\{\Gamma_0\cos N\theta+\Gamma_1\cos(N-2)\theta+\cdots+\Gamma_n\cos(N-2n)\theta\cdots+\Gamma_{(N-1)/2}\cos\theta\right\}$$

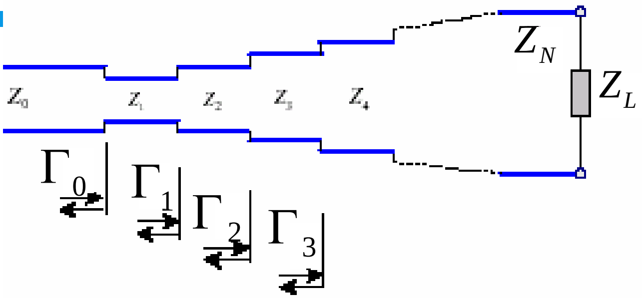


N为偶数时最后一项为 $\Gamma_{N/2}$

$$\Gamma=2e^{-jN\theta}\left\{\Gamma_0\cos N\theta+\Gamma_1\cos(n-2)\theta+\cdots+\Gamma_n\cos(N-2n)\theta\cdots+\frac{1}{2}\Gamma_{N/2}\right\}$$

总的反射系数

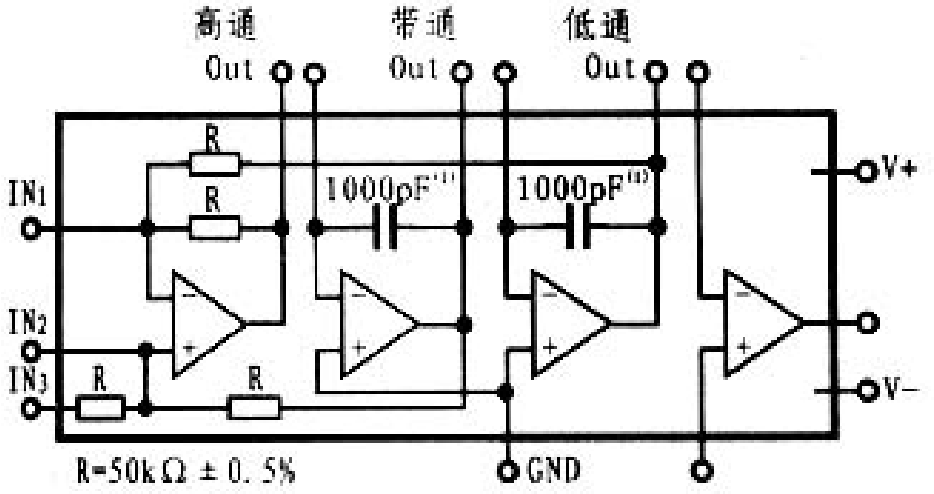
任何函数都可利用傅立叶级数展开
可把反射系数设计成我们期望的形式



最平特性多节阻抗变换器

等波纹特性多节阻抗变换器

贝塞耳滤波器
巴特沃斯滤波器
梳状滤波器
椭圆函数滤波器



通用有源滤波器UAF42的内部结构框图如图所示

1.6.6 最平特性多节阻抗变换器

出发点：使各连接点反射在输入处叠加的总反射系数

反射的频率特性为最平坦特性

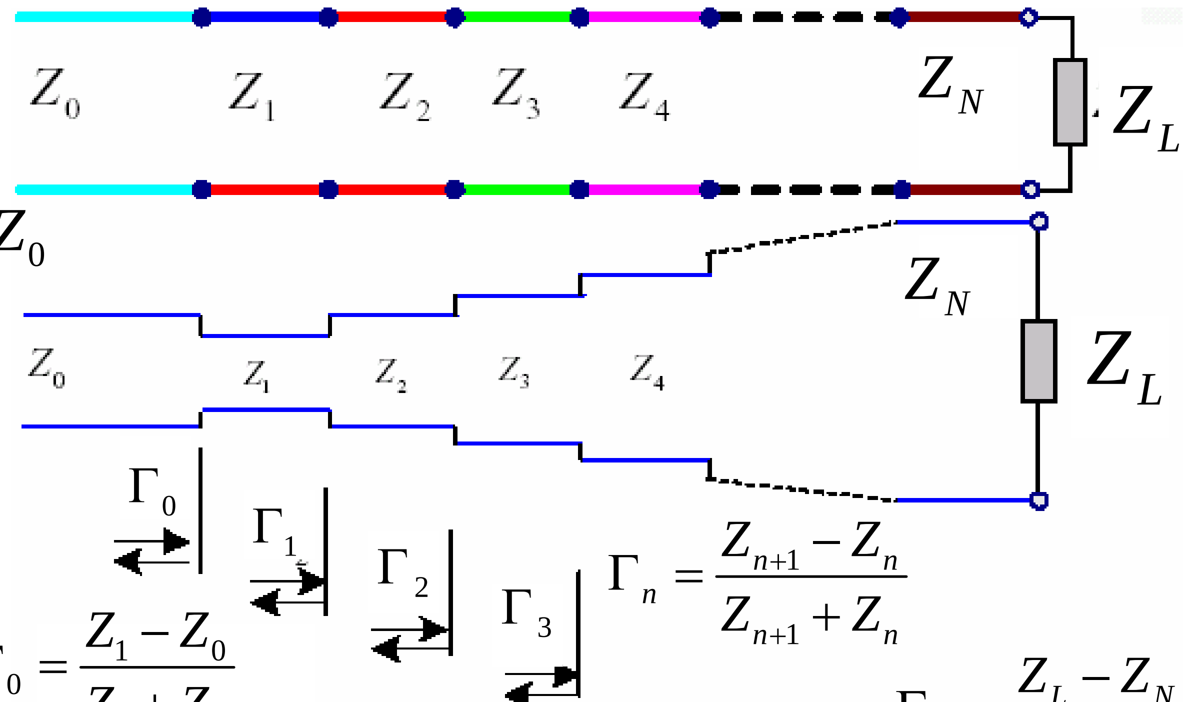
如图，各节特性阻抗满足

$$Z_L > Z_N, \cdots, Z_2 > Z_1, Z_1 > Z_0$$

考虑最平坦通带特性为

$$\Gamma = A \cdot \left(1 + e^{-j2\theta}\right)^N$$

如何求常数A?



$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

1.6.6 最平特性多节阻抗变换器

考虑最平坦通带特性为 $\Gamma = A\cdot\left(1+e^{-j2\theta}\right)^N$

如何求常数A?

$\theta = 0$ 时总反射系数为:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

由

$$\Gamma = A\cdot 2^N$$

所以 $A = 2^{-N}\dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$\Gamma = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0}\cdot\left(1+e^{-j2\theta}\right)^N$$

利用二项式 $(1+x)^N = C_0^N + C_1^N x^1 + C_2^N x^2 + \cdots + C_N^N x^N = \sum_{n=0}^{N} C_n^N x^n$

其中二项式系数 $C_n^N = \dfrac{N(N-1)(N-2)\cdots[N-(n-1)]}{n!} = \dfrac{N!}{(N-n)!\,n!}$

展开 $\Gamma = 2^{-N}\dfrac{Z_L - Z_0}{Z_L + Z_0}\cdot\sum_{n=0}^{N} C_n^N e^{-j2n\theta}$

即为最平坦通带特性总反射系数

1.6.6 最平特性多节阻抗变换器

$$\Gamma=\Gamma_0+\Gamma_1\cdot e^{-j2\theta}+\Gamma_2\cdot e^{-j4\theta}+\ldots+\Gamma_i\cdot e^{-j2i\theta}+\ldots+\Gamma_N\cdot e^{-j2N\theta}$$ （1）式

为得到最平坦特性，必须使（1）式 各对应系数相等

与最平坦通带特性总反射系数 $\Gamma=2^{-N}\dfrac{Z_L-Z_0}{Z_L+Z_0}\cdot\sum_{n=0}^{N}C_n^N e^{-j2n\theta}$

因此有: $$\Gamma_n=2^{-N}\frac{Z_L-Z_0}{Z_L+Z_0}\cdot C_n^N$$

而 $C_n^N=C_{N-n}^N$

所以有: $$\Gamma_{N-n}=\Gamma_n=2^{-N}\frac{Z_L-Z_0}{Z_L+Z_0}\cdot C_n^N$$

如果知道: Z_L,Z_0,N

就可以由

$$\Gamma_0=\frac{Z_1-Z_0}{Z_1+Z_0}\qquad \Gamma_1=\frac{Z_2-Z_1}{Z_2+Z_1}$$

$$\Gamma_n=\frac{Z_{n+1}-Z_n}{Z_{n+1}+Z_n}\qquad \Gamma_N=\frac{Z_L-Z_N}{Z_L+Z_N}$$

计算出各节的特性阻抗

并设计最平特性多节阻抗变换器

1.6.6 最平特性多节阻抗变换器

为简化进一步近似 利用

$$\ln\frac{Z_{n+1}}{Z_n}=2\frac{Z_{n+1}-Z_n}{Z_{n+1}+Z_n}+\frac{2}{3}\left(\frac{Z_{n+1}-Z_n}{Z_{n+1}+Z_n}\right)^3+\cdots$$

其中 $\frac{Z_{n+1}-Z_n}{Z_{n+1}+Z_n}$ 为第n+1个连接处的反射系数

反射系数很小时，只取第一项，所以 $\ln\frac{Z_{n+1}}{Z_n}\approx 2\frac{Z_{n+1}-Z_n}{Z_{n+1}+Z_n}=2\Gamma_n$

所以 $\ln\frac{Z_{n+1}}{Z_n}=2\Gamma_n=2^{-N}\cdot C_n^N\, 2\frac{Z_L-Z_0}{Z_L+Z_0}$

$$\Gamma_n=2^{-N}\frac{Z_L-Z_0}{Z_L+Z_0}\cdot C_n^N$$

由 同理有 代入

$$\ln\frac{Z_L}{Z_0}\approx 2\frac{Z_L-Z_0}{Z_L+Z_0}$$

有

$$\ln\frac{Z_{n+1}}{Z_n}=2^{-N}C_n^N\ln\frac{Z_L}{Z_0}$$

最平特性多节阻抗变换器近似设计公式

1.6.6 最平特性多节阻抗变换器

例 设计一最平特性多节阻抗变换器，取N=2，被匹配的阻抗为Z_L, Z_0

求两节四分之一波长段特性阻抗

思考严格解?

对比近似解

解 由 $C_n^N = \dfrac{N!}{(N-n)!n!}$

代入

所以当N=2时 $C_0^2 = C_2^2 = 1, C_1^2 = 2$

$$\ln\frac{Z_{n+1}}{Z_n} = 2^{-N} C_n^N \ln\frac{Z_L}{Z_0}$$

所以

n=0时 $\ln\dfrac{Z_1}{Z_0} = \dfrac{1}{4}\ln\dfrac{Z_L}{Z_0}$ $\quad \dfrac{Z_1}{Z_0} = \left(\dfrac{Z_L}{Z_0}\right)^{\frac{1}{4}}$ $\quad Z_1 = Z_L^{\frac{1}{4}} Z_0^{\frac{3}{4}}$

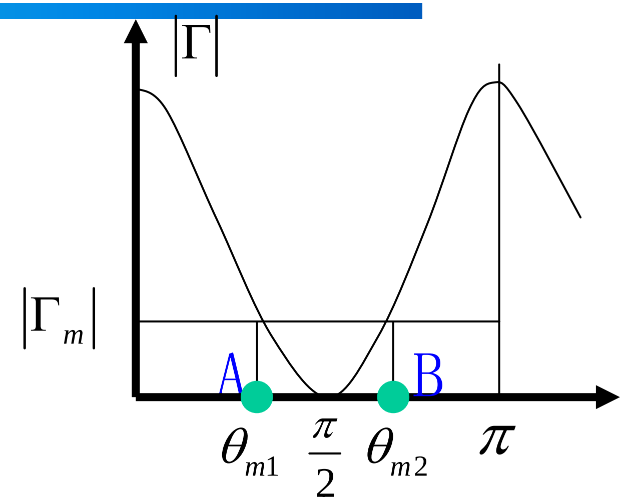
即为第一节四分之一波长段特性阻抗

n=1时 $\ln\dfrac{Z_2}{Z_1} = \dfrac{1}{2}\ln\dfrac{Z_L}{Z_0}$ $\quad \dfrac{Z_2}{Z_1} = \left(\dfrac{Z_L}{Z_0}\right)^{\frac{1}{2}}$ $\quad Z_2 = Z_L^{\frac{3}{4}} Z_0^{\frac{1}{4}}$

即为第二节四分之一波长段特性阻抗

带宽

1.6.6 最平特性多节阻抗变换器



如图 $|\Gamma_m|$ 是设计要求的最大反射系数

$\theta_{m2}-\theta_{m1}$ 是设计要求带宽

由 $$\Gamma = 2^{-N}\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\left(1+e^{-j2\theta}\right)^N$$

对A点有：

$$|\Gamma_m| = \left|2^{-N}\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\left(1+e^{-j2\theta_m}\right)^N\right| = \left|\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\left(\frac{1+e^{-j2\theta_m}}{2}\right)^N\right|$$

$$= \left|\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\left(e^{-j\theta_m}\frac{e^{j\theta_m}+e^{-j\theta_m}}{2}\right)^N\right| = \left|e^{-jN\theta_m}\right|\left|\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\left(\frac{e^{j\theta_m}+e^{-j\theta_m}}{2}\right)^N\right|$$

由 $$\ln\frac{Z_L}{Z_0}\approx 2\frac{Z_L-Z_0}{Z_L+Z_0}$$

$$= \left|\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\cos^N\theta_m\right| \approx \frac{1}{2}\left|\ln\frac{Z_L}{Z_0}\cdot\cos^N\theta_m\right|$$

所以

$$\theta_m = \arccos\left|\frac{2\Gamma_m}{\ln\frac{Z_L}{Z_0}}\right|^{\frac{1}{N}} < \frac{\pi}{2}$$

所以对应A点 θ_{m1}

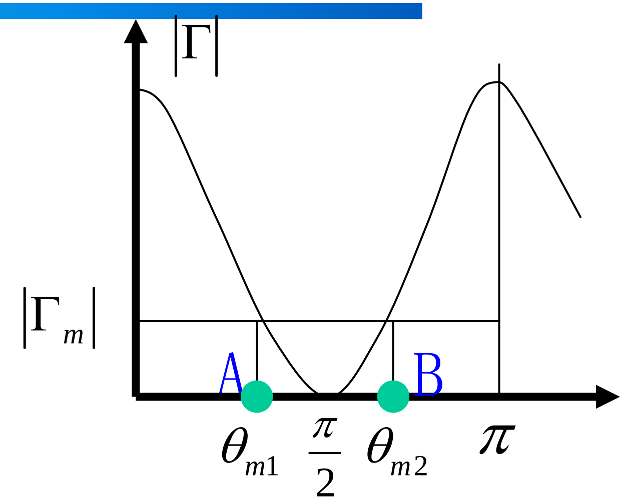
带宽

1.6.6 最平特性多节阻抗变换器

如图 $|\Gamma_m|$ 是设计要求的最大反射系数

$\theta_{m2}-\theta_{m1}$ 是设计要求带宽

$$\theta_{m1}=\arccos\left|\frac{2\Gamma_m}{\ln\frac{Z_L}{Z_0}}\right|^{\frac{1}{N}}$$



定义相对带宽

$$W=\frac{\theta_{m2}-\theta_{m1}}{\theta_0}$$

由于 θ_{m2},θ_{m1} 相对于 $\theta_0=\frac{\pi}{2}$ 对称，所以

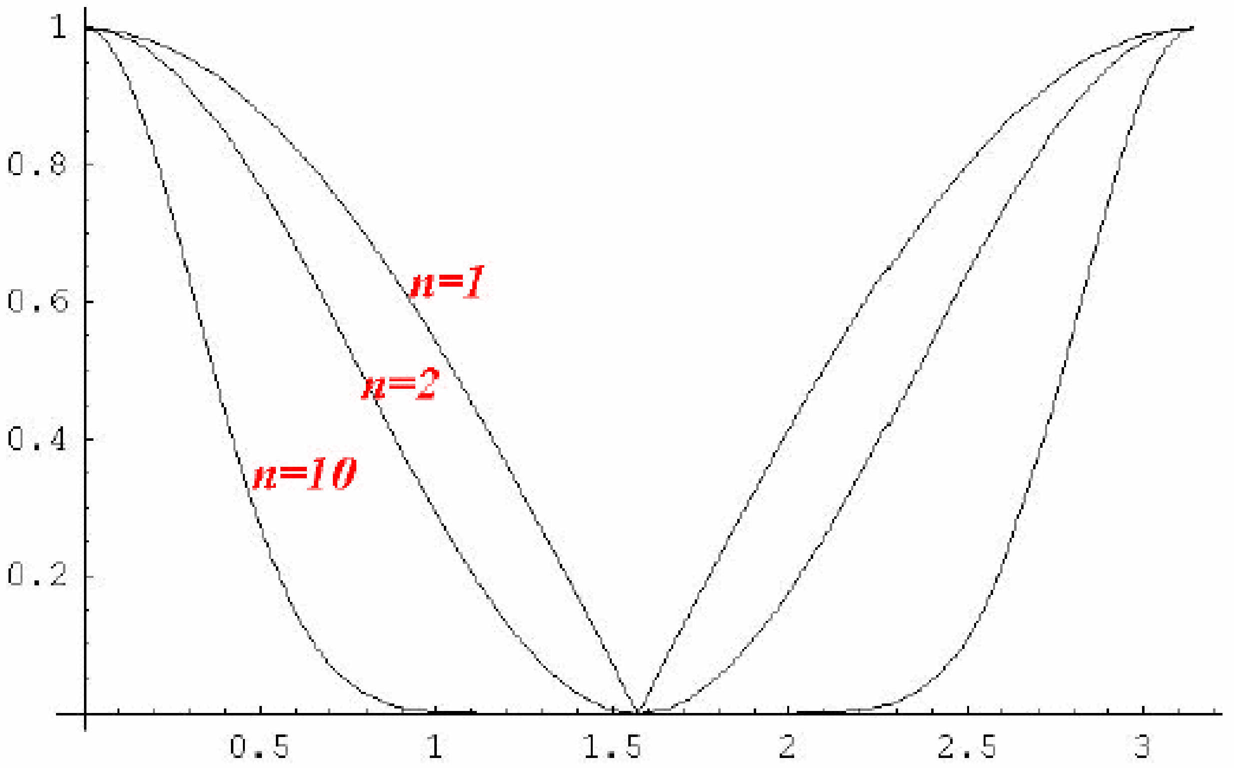
$$\theta_{m2}=\pi-\theta_{m1} \quad \theta_{m2}+\theta_{m1}=\pi \quad \theta_0=\frac{\theta_{m1}+\theta_{m2}}{2}=\frac{\pi}{2}$$

所以相对带宽为

$$W=\frac{\theta_{m2}-\theta_{m1}}{\theta_0}=\frac{\theta_{m2}-\theta_{m1}}{\frac{\pi}{2}}=2-\frac{4}{\pi}\theta_{m1}=2-\frac{4}{\pi}\arccos\left|\frac{2\Gamma_m}{\ln\frac{Z_L}{Z_0}}\right|^{\frac{1}{N}}$$

图示

$$\left|\Gamma_m\right| \approx \frac{1}{2}\left|\ln\frac{Z_L}{Z_0}\cdot\cos^N\theta_m\right|$$



$\left|\cos^n\theta\right|$

(1) 段数

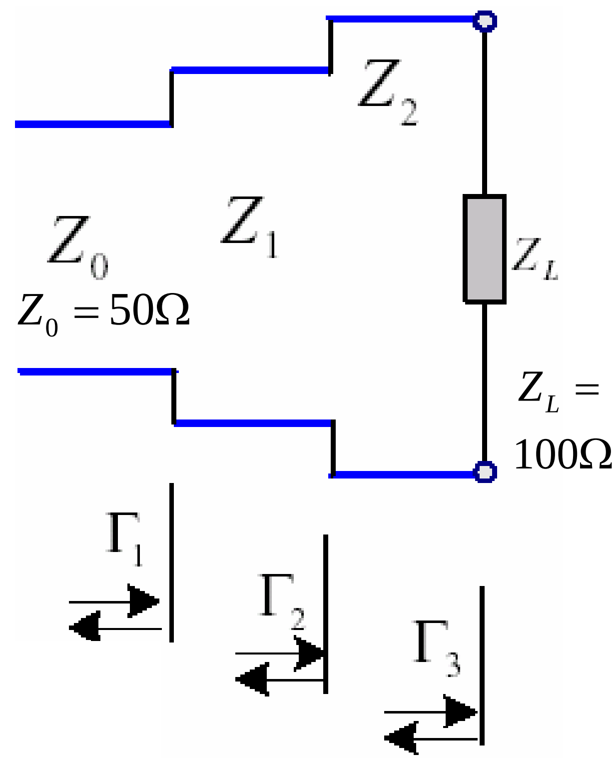
(2) 带宽

例题: $\Rightarrow \lambda \Rightarrow \dfrac{\lambda}{4}$

设计二段最平坦匹配线，使得 100 Ohm负载在频率为10GHz时匹配到50 Ohm填充空气的传输线上。求当反射系数 Γ_m =0.1时的带宽

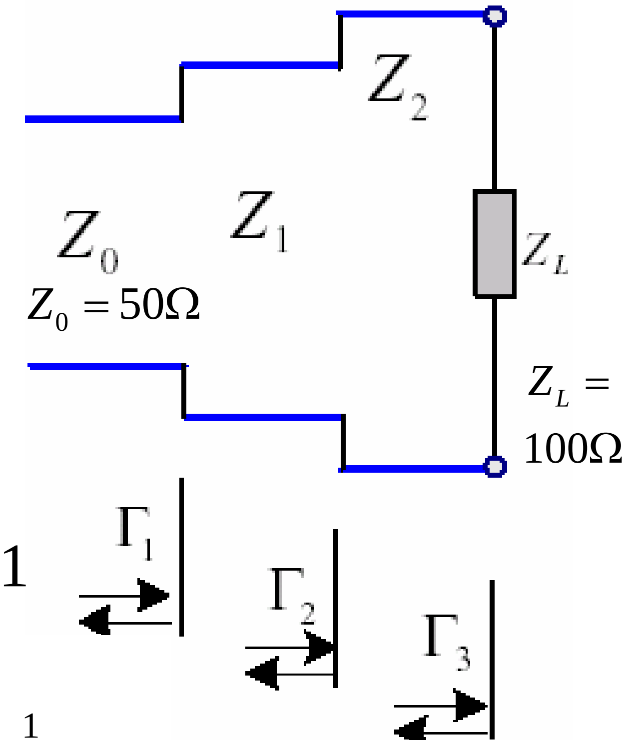
$\Gamma_{\max}$

n	a_1	a_2	a_3	a_4	a_5	$\cdots$	2^n
$n=1$	1	1					2
$n=2$	1	2	1				4
$n=3$	1	3	3	1			8
$n=4$	1	4	6	4	1		16
…							



$$\Rightarrow \lambda \Rightarrow \frac{\lambda}{4}$$

设计二段最平坦匹配线，使得 100 Ohm负载在频率为10GHz时匹配到50 Ohm填充空气的传输线上。求当反射系数 Γ_m =0.1时的带宽



由前面例题有:

$$Z_1 = Z_L^{\frac{1}{4}} Z_0^{\frac{3}{4}} \qquad Z_2 = Z_L^{\frac{3}{4}} Z_0^{\frac{1}{4}}$$

所以:

$$Z_1 = 100^{\frac{1}{4}} 50^{\frac{3}{4}} = \left(100 \times 50^3\right)^{\frac{1}{4}} = 50 \times 2^{\frac{1}{4}} = 59.5$$

$$Z_2 = 100^{\frac{3}{4}} 50^{\frac{1}{4}} = \left(50 \times 100^3\right)^{\frac{1}{4}} = 100 \times \left(\frac{1}{2}\right)^{\frac{1}{4}} = 84.1$$

由相对带宽

$$W = 2 - \frac{4}{\pi} \arccos \left| \frac{2\Gamma_m}{\ln \frac{Z_L}{Z_0}} \right|^{\frac{1}{N}}$$

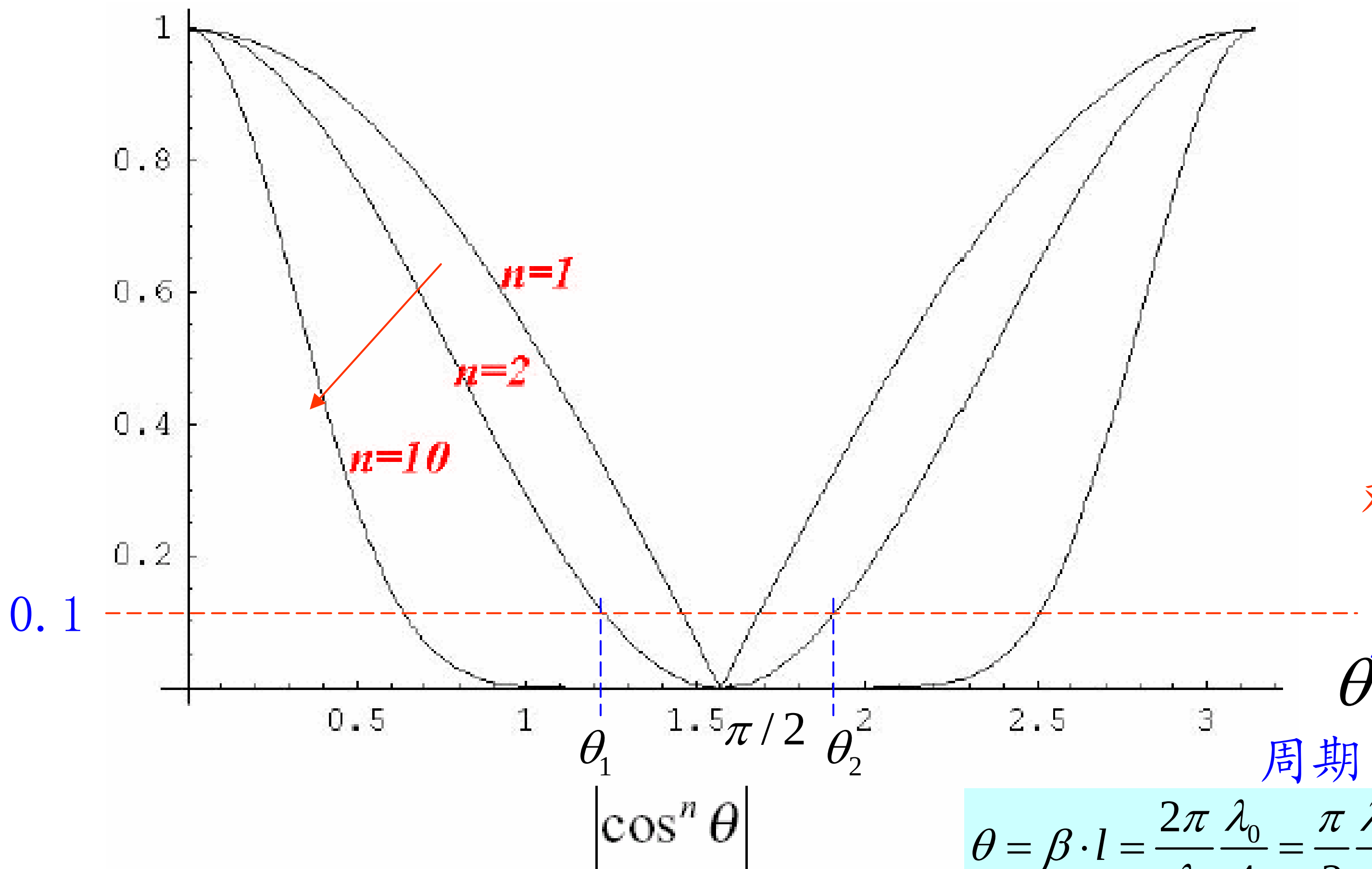
所以

$$= 2 - \frac{4}{\pi} \arccos \left| \frac{0.2}{\ln 2} \right|^{\frac{1}{2}} = 0.72\,?$$

图示

$$|\Gamma| = \ldots = 2^n \cdot |\Gamma_1| \cdot |\cos^n \theta|$$

相对带宽 $\dfrac{\theta_2 - \theta_1}{\dfrac{\pi}{2}}$



对应频率

周期 π

$$\theta = \beta \cdot l = \frac{2\pi}{\lambda}\frac{\lambda_0}{4} = \frac{\pi}{2}\frac{\lambda_0}{\lambda} = \frac{\pi}{2}\frac{f}{f_0}$$

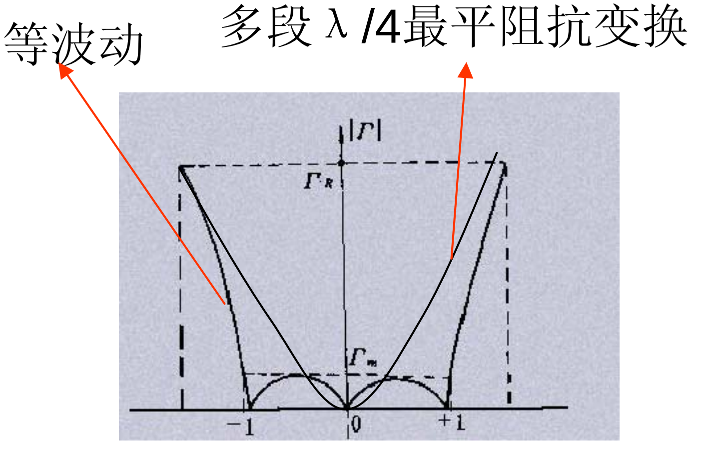
可换算成频率

1.6.7 等波纹特性多节阻抗变换器

阻抗变换的出发点：切比雪夫多项式

反射系数模随Θ变化按切比雪夫多项式变化

即 $|\Gamma|$ 是θ的切比雪夫函数



该阻抗变换的特点：

(1) 工作频带内等波动

(2) 给定最大允许反射系数值（相同节数）的条件下，带宽最大

设计 $\Gamma(\theta)$ 符合切比雪夫多项式

(1) 切比雪夫多项式

(2) 将反射系数同切比雪夫多项式关联

(3) 定义域变换

切比雪夫多项式(Chebyshev Polynomial)

定义： N阶切比雪夫多项式

$$T_N(x)=\begin{cases}\cos(N\cdot arc\cos x) & |x|\le 1\\ \cosh(N\cdot\cosh^{-1}x) & |x|>1\end{cases}$$

$$\cosh(x)=\frac{e^{x}+e^{-x}}{2}$$

递推公式：

$$T_{n+1}(x)=2x\cdot T_n(x)-T_{n-1}(x)\ (n>1)$$

$$T_0(x)=1$$

$$T_1(x)=x$$

$$T_2(x)=2x\cdot T_1(x)-T_0(x)=2x^2-1$$

$$T_3(x)=2x\cdot T_2(x)-T_1(x)=4x^3-3x$$

$$T_4(x)=8x^4-8x^2+1$$

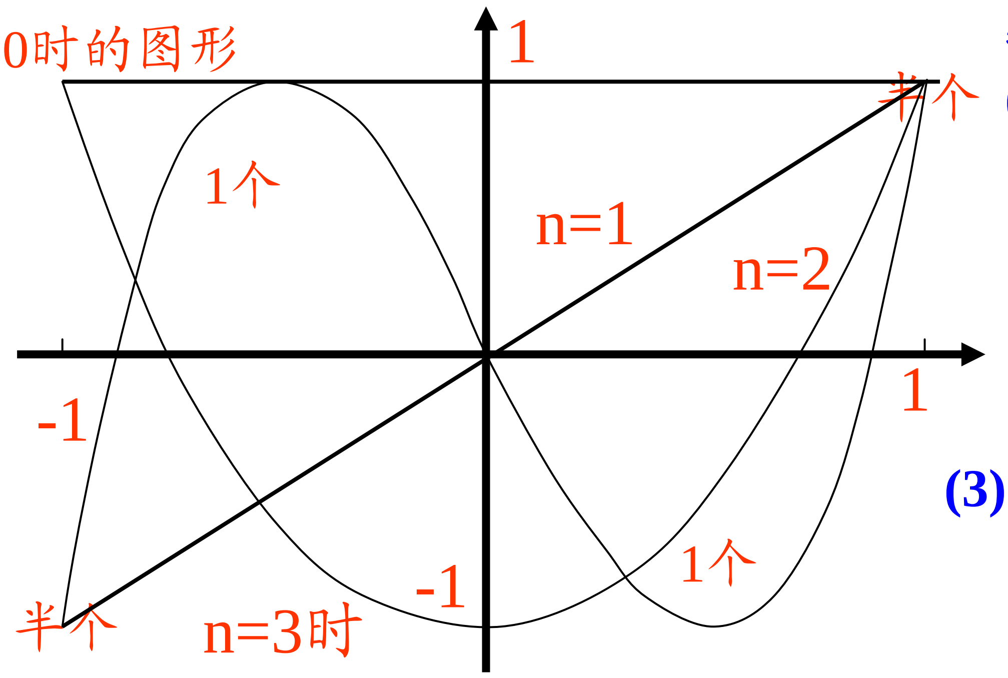
示意图(-1<x<+1)

$$T_n(x)=\cos(n\cdot \arccos x) \quad |x|\le 1$$

$$T_0(x)=1 \quad T_1(x)=x$$

$$T_2(x)=2x^2-1$$

$$T_{n+1}(x)=2x\cdot T_n(x)-T_{n-1}(x) \quad (n>1)$$



特点:

(1)定义域为 $|x|\le 1$ | $|x|>1$

定义域为	$\lvert x\rvert\le 1$	$\lvert x\rvert>1$
值域	$\lvert T_n(x)\rvert\le 1$	$\lvert T_n(x)\rvert\ge 1$

(2) 过定点(+1,+1)，即

$$T_n(1)=1 \quad T_n(-1)=(-1)^n$$

(3) $T_n(x)=0$ 即0点都在区域(-1,1)

零点个数为阻抗变换器节数n

n个波

(4) 对称性(轴/中心对称)

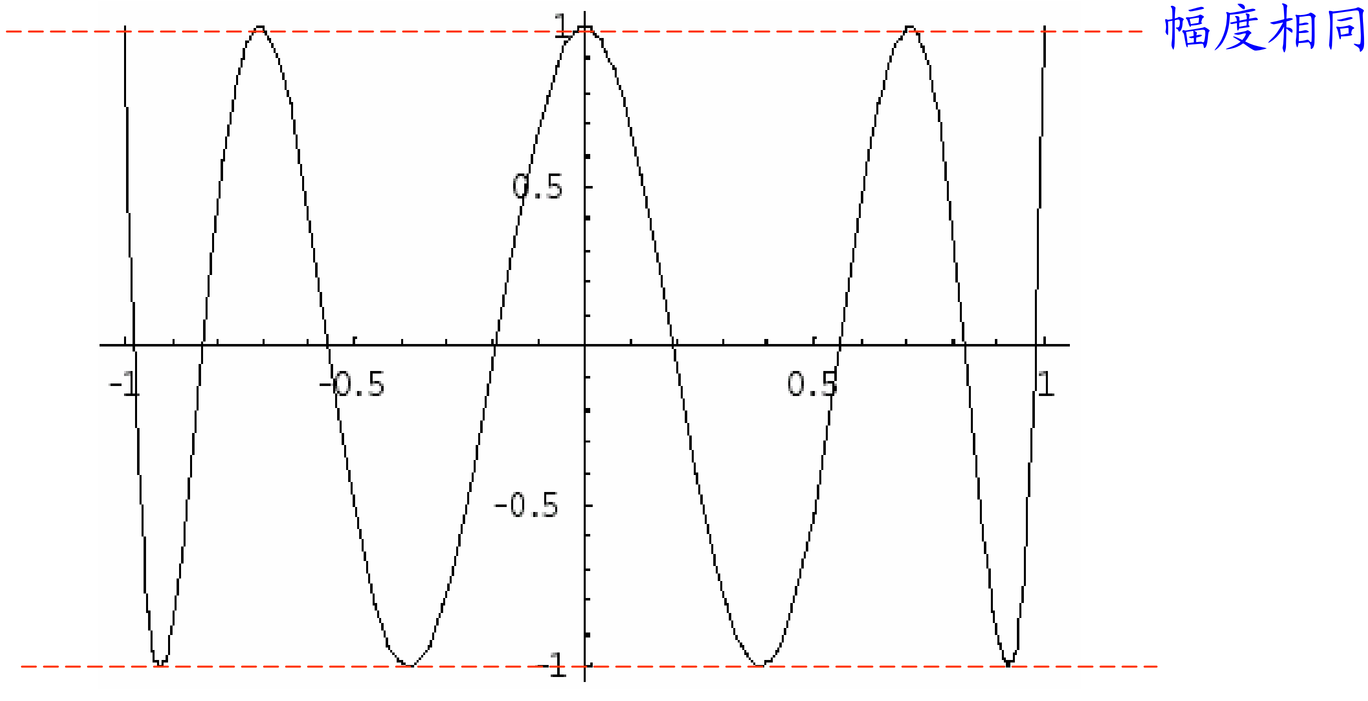
例题：画出n=8时定义域[-1, +1]的契比雪夫函数

$$T_N(x)=\begin{cases}\cos(N\cdot arc\cos x)\\ \cosh(N\cdot\cosh^{-1}x)\end{cases}$$

$$T_{n+1}(x)=2x\cdot T_n(x)-T_{n-1}(x)$$

$$T_3(x)=\ldots=4x^3-3x$$

$$T_4(x)=8x^4-8x^2+1$$



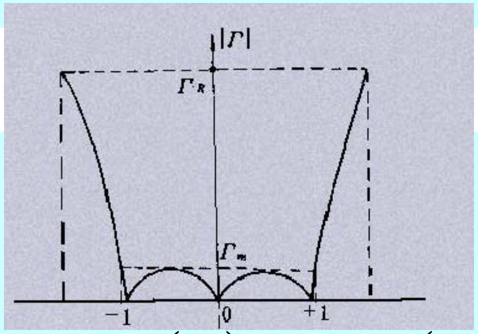
$$\cosh(x)=\frac{e^x+e^{-x}}{2}$$

多节变换器总反射系数:

$$\cosh(x)=\frac{e^{x}+e^{-x}}{2}$$

$$\Gamma=\Gamma_0+\Gamma_1\cdot e^{-j2\theta}+\Gamma_2\cdot e^{-j4\theta}+\ldots+\Gamma_i\cdot e^{-j2i\theta}+\ldots+\Gamma_N\cdot e^{-j2N\theta}$$ （1）式

在用切比雪夫多项式时



$$T_N(x)=\begin{cases}\cos(N\cdot arc\cos x) & |x|\le 1\\ \cosh\left(N\cdot\cosh^{-1}x\right) & |x|>1\end{cases}$$

$$T_0(x)=1$$
$$T_1(x)=x$$
$$T_{n+1}(x)=2x\cdot T_n(x)-T_{n-1}(x)$$

由切比雪夫多项式曲线可看出 $|x|\le 1$ 时，反射系数有等波纹性质

所以取 $x=\dfrac{\cos\theta}{\cos\theta_m}$ 代入 所以有

$$T_N\left(\frac{\cos\theta}{\cos\theta_m}\right)=\cos\left[N\left(\arccos\frac{\cos\theta}{\cos\theta_m}\right)\right]$$

也可以写成

$$T_N(\sec\theta_m\cdot\cos\theta)=\cos\left[N\arccos(\sec\theta_m\cdot\cos\theta)\right]$$

多节变换器总反射系数:

$$\Gamma=\Gamma_0+\Gamma_1\cdot e^{-j2\theta}+\Gamma_2\cdot e^{-j4\theta}+\ldots+\Gamma_i\cdot e^{-j2i\theta}+\ldots+\Gamma_N\cdot e^{-j2N\theta}$$ （1）式

对切比雪夫变换器 反射系数应按切比雪夫多项式变化

假定变换器做成对称的，即 $\Gamma_0=\Gamma_N,\Gamma_1=\Gamma_{N-1},\Gamma_2=\Gamma_{n-2},\cdots\cdots$

$$\Gamma=e^{-jN\theta}\left\{\Gamma_0\left(e^{jN\theta}+e^{-jN\theta}\right)+\Gamma_1\left(e^{j(N-2)\theta}+e^{-j(N-2)\theta}\right)+\cdots\right\}$$
$$=2e^{-jN\theta}\left\{\Gamma_0\cos N\theta+\Gamma_1\cos(N-2)\theta+\cdots+\Gamma_n\cos(N-2n)\theta+\cdots\right\}$$

N为奇数时最后一项为 $\Gamma_{(N-1)/2}\cos\theta$ N为偶数时最后一项为 $\frac{1}{2}\Gamma_{N/2}$

根据上式，反射系数写成 $$\Gamma=Ae^{-jN\theta}T_N\left(\frac{\cos\theta}{\cos\theta_m}\right)$$

$$=2e^{-jN\theta}\left\{\Gamma_0\cos N\theta+\Gamma_1\cos(N-2)\theta+\cdots+\Gamma_n\cos(N-2n)\theta+\cdots\right\}$$

切比雪夫阻抗变换器

考虑切比雪夫通带特性为 $\Gamma = Ae^{-jN\theta}T_N\left(\dfrac{\cos\theta}{\cos\theta_m}\right)$

如何求常数A?

$$= 2e^{-jN\theta}\left\{\Gamma_0\cos N\theta + \Gamma_1\cos(N-2)\theta + \cdots + \Gamma_n\cos(N-2n)\theta + \cdots\right\}$$

$\theta = 0$ 时总反射系数为： 由

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma = A\cdot T_N(\sec\theta_m)$$

所以 $A = \dfrac{Z_L - Z_0}{Z_L + Z_0}\dfrac{1}{T_N(\sec\theta_m)}$

$$\Gamma = e^{-jN\theta}\frac{Z_L - Z_0}{Z_L + Z_0}\cdot\frac{T_N(\sec\theta_m\cos\theta)}{T_N(\sec\theta_m)}$$

通带内最大反射系数为 $|\Gamma_m|$ 对应 $\theta = \theta_m$ $\quad \sec\theta_m\cdot\cos\theta_m = 1$

$$T_N(\sec\theta_m\cdot\cos\theta_m) = 1$$

所以

$$\boxed{|\Gamma_m|} = \left|e^{-jN\theta_m}\right|\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\cdot\left|\frac{T_N(\sec\theta_m\cos\theta)}{T_N(\sec\theta_m)}\right| \boxed{= \frac{Z_L - Z_0}{Z_L + Z_0}\cdot\frac{1}{T_N(\sec\theta_m)}}$$

代入 总反射系数

设计变换器时，通常给定 Z_L, Z_0, Γ_m, N 可以通过 求 θ_m

$$\boxed{\Gamma = e^{-jN\theta}|\Gamma_m|T_N(\sec\theta_m\cos\theta)}$$

$$\boxed{= 2e^{-jN\theta}\left\{\Gamma_0\cos N\theta + \Gamma_1\cos(N-2)\theta + \cdots + \Gamma_n\cos(N-2n)\theta + \cdots\right\}}$$

把 展开即可求反射系数 $\Gamma_0, \Gamma_1, \cdots\Gamma_n, \cdots$

例1.14 已知

$Z_L=100\Omega, Z_0=50\Omega$ 最大允许反射系数为: $|\Gamma_m|=0.05$ 节数N=2

用切比雪夫阻抗变换器设计匹配，求两节变换器的特性阻抗 Z_1, Z_2

解 由 $\Gamma_0=\dfrac{Z_1-Z_0}{Z_1+Z_0}$ $\Gamma_1=\dfrac{Z_2-Z_1}{Z_2+Z_1}$ $\Rightarrow Z_1=\dfrac{1-\Gamma_0}{1+\Gamma_0}Z_0$ $Z_2=\dfrac{1-\Gamma_1}{1+\Gamma_1}Z_1$

所以先求 Γ_0,Γ_1 把N=2代入 $\boxed{\Gamma=e^{-jN\theta}|\Gamma_m|T_N(\sec\theta_m\cos\theta)}$

$$\boxed{=2e^{-jN\theta}\{\Gamma_0\cos N\theta+\Gamma_1\cos(N-2)\theta+\cdots+\Gamma_n\cos(N-2n)\theta+\cdots\}}$$

所以 $|\Gamma_m|T_2(\sec\theta_m\cdot\cos\theta)=2\Gamma_0\cos2\theta+\Gamma_1$ 因为N为偶数

考虑 $\boxed{T_2(\sec\theta_m\cdot\cos\theta)=\sec^2\theta_m(1+\cos2\theta)-1}$

所以

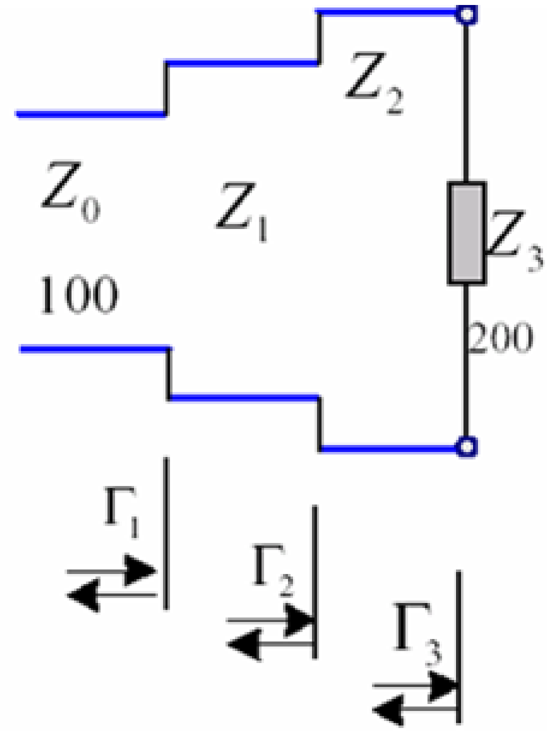
$$\Gamma_0=\frac{1}{2}|\Gamma_m|\sec^2\theta_m \qquad \Gamma_1=|\Gamma_m|(\sec^2\theta_m-1)$$

由 $\boxed{|\Gamma_m|=\dfrac{Z_L-Z_0}{Z_L+Z_0}\cdot\dfrac{1}{T_N(\sec\theta_m)}}$ 所以 $T_2(\sec\theta_m)=\dfrac{Z_L-Z_0}{Z_L+Z_0}\cdot\dfrac{1}{|\Gamma_m|}=6.67$

而 $T_2(\sec\theta_m)=2\sec^2\theta_m-1 \Rightarrow \sec\theta_m=1.96 \quad \Rightarrow \Gamma_0,\Gamma_1 \Rightarrow Z_1,Z_2$

例题:

设计二段等波动匹配线，使得 100 Ohm传输线匹配到200 Ohm传输线上。最大反射系数为0.05



解:

$$n = 2 \quad |\Gamma|_{\max} = 0.05 \Rightarrow \Gamma_m = 0.05$$

$$\Gamma_m \cdot T_2(\sec\theta_m) = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200 - 100}{200 + 100} = 0.33$$

$$0.05 \cdot \left(2\sec^2\theta_m - 1\right) = 0.33$$

$$\Rightarrow \theta_m = 59.1^\circ$$

$$T_2(\sec\theta_m) = 2x^2 - 1 = 2\sec^2\theta_m - 1$$

超越方程求解：近似

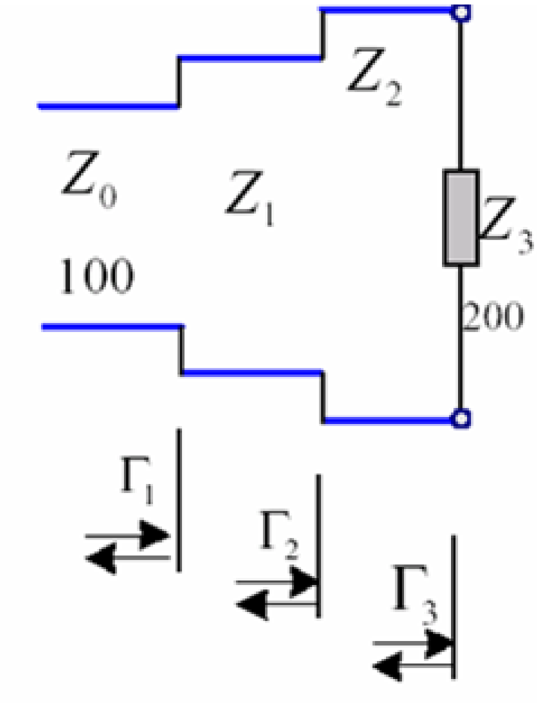
（一）书上

（二）：（简单）

$$0.05\cdot\left(2\sec^2\theta_m-1\right)=0.33$$

$$\sec^2\theta_m=3.8$$

$$\boxed{\begin{aligned}\Gamma_0&=\frac{1}{2}\left|\Gamma_m\right|\sec^2\theta_m\\ \Gamma_1&=\left|\Gamma_m\right|\left(\sec^2\theta_m-1\right)\end{aligned}}$$



$$\begin{cases}\Gamma_0=\dfrac{Z_1-Z_0}{Z_1+Z_0}\\ \Gamma_1=\dfrac{Z_2-Z_1}{Z_2+Z_1}\end{cases}\Rightarrow\begin{cases}Z_1=\dfrac{1-\Gamma_0}{1+\Gamma_0}Z_0\\ Z_2=\dfrac{1-\Gamma_1}{1+\Gamma_1}Z_1\end{cases}$$

小结:

与二项式匹配变换对比, 契比雪夫多项式匹配通带宽更加优化