

CENTER FOR

ASTROPHYSICS

HARVARD & SMITHSONIAN



STROOPWAFEL: a Dutch cookie and an Adaptive Importance Sampling algorithm

Floor Broekgaarden

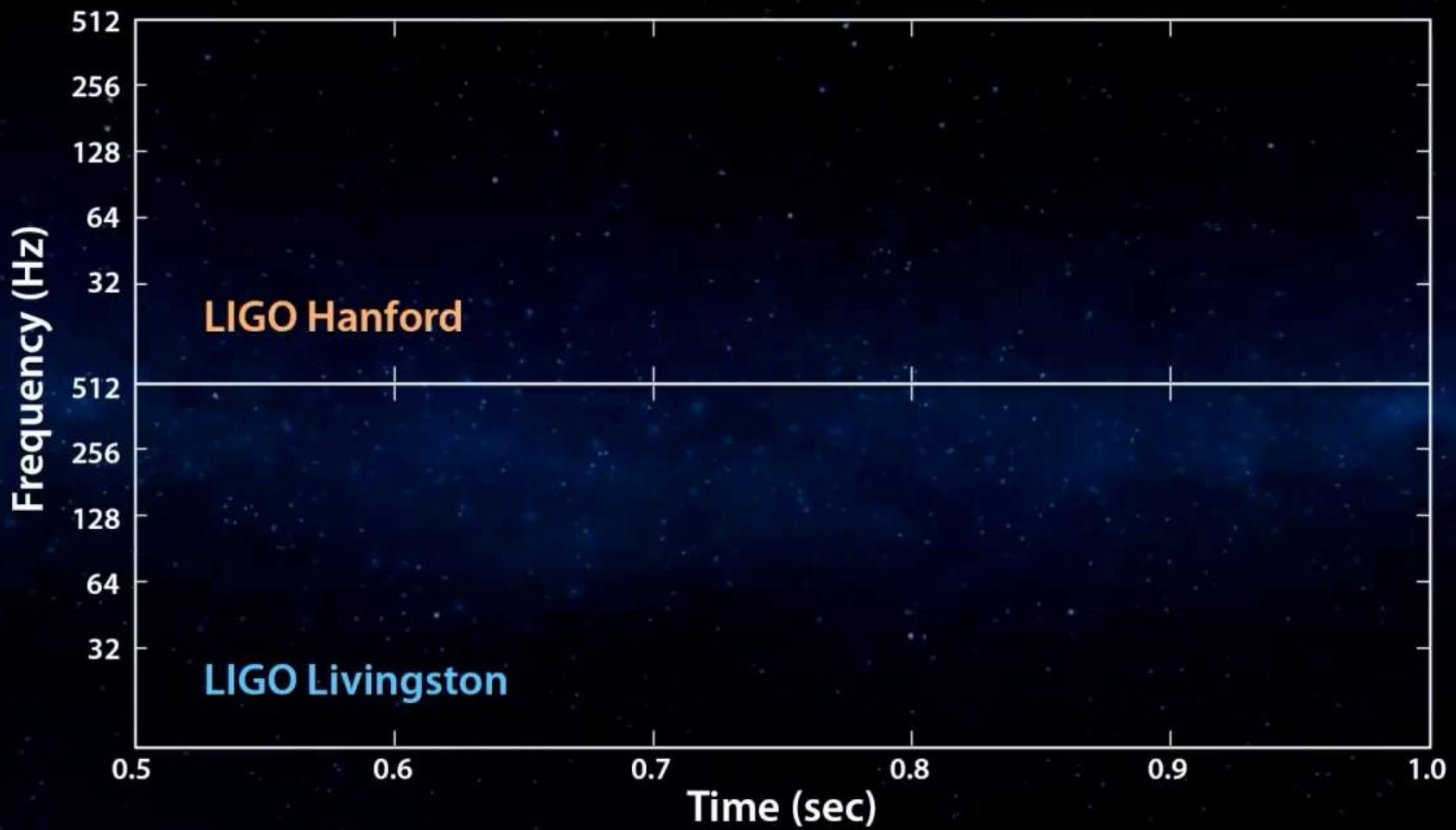
Center for Astrophysics | Harvard &
Smithsonian

In collaboration with:

Jonathan Gair, Floris Kummer, Lokesh Khandelwal, Stephen Justham, Ilya Mandel, Selma de Mink, Luyau Lin, Coen Neijssel, Alejandro Vigna-Gomez, Simon Stevenson, Tom Wagg, Lieke van Son, Michelle Wassink, Poojan Agrawal, Serena Vinciguerra, Reinhold Willcox, Jeff Riley, Mike Lau and many others...

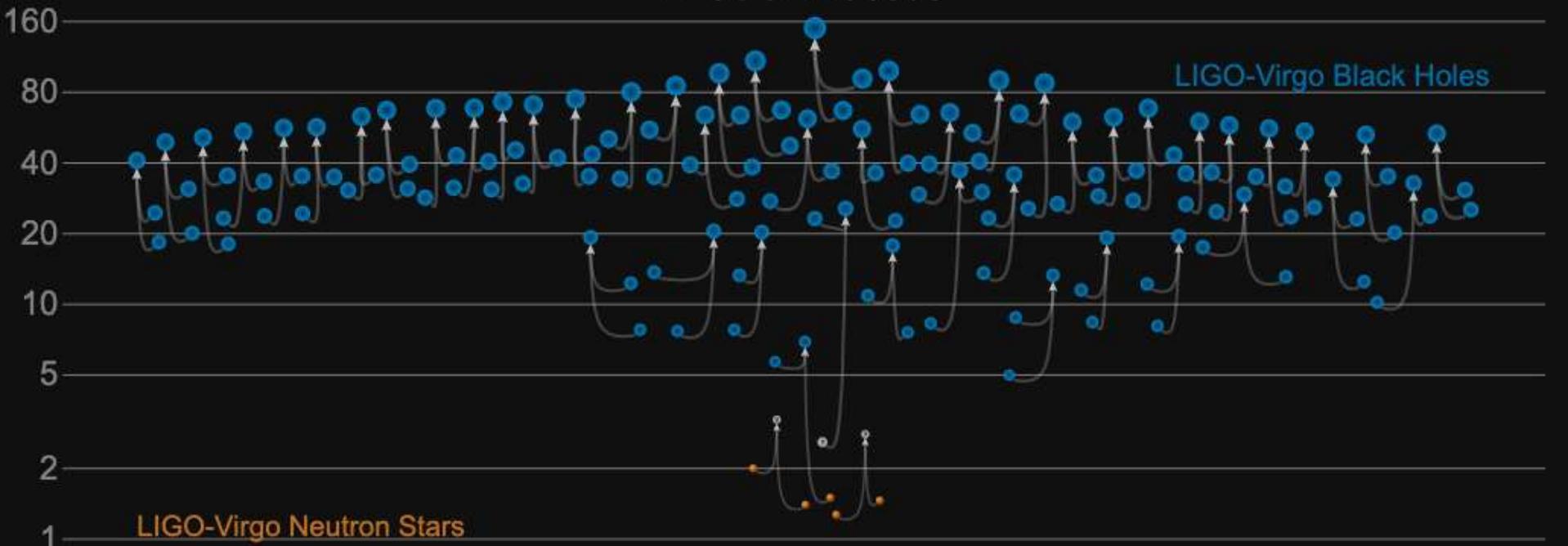
ICERM meeting | 20 November 2020

Statistical Methods for the Detection, Classification, and Inference of
Relativistic Objects



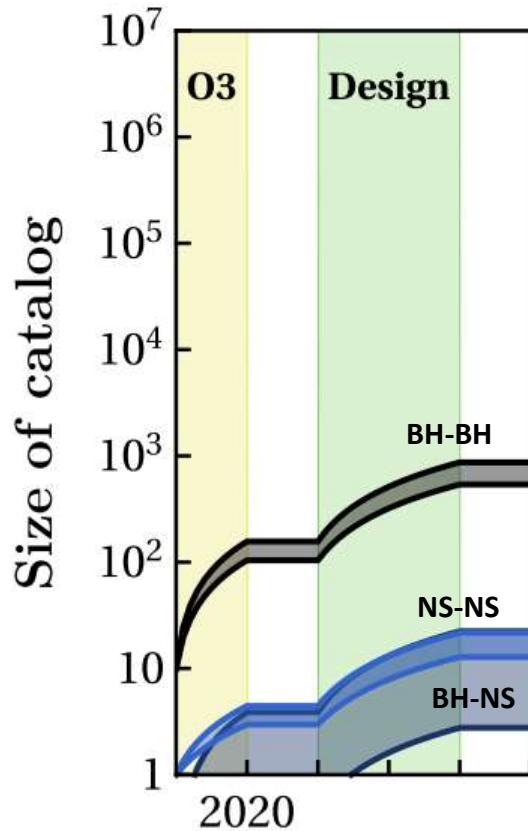
Masses in the Stellar Graveyard

in Solar Masses

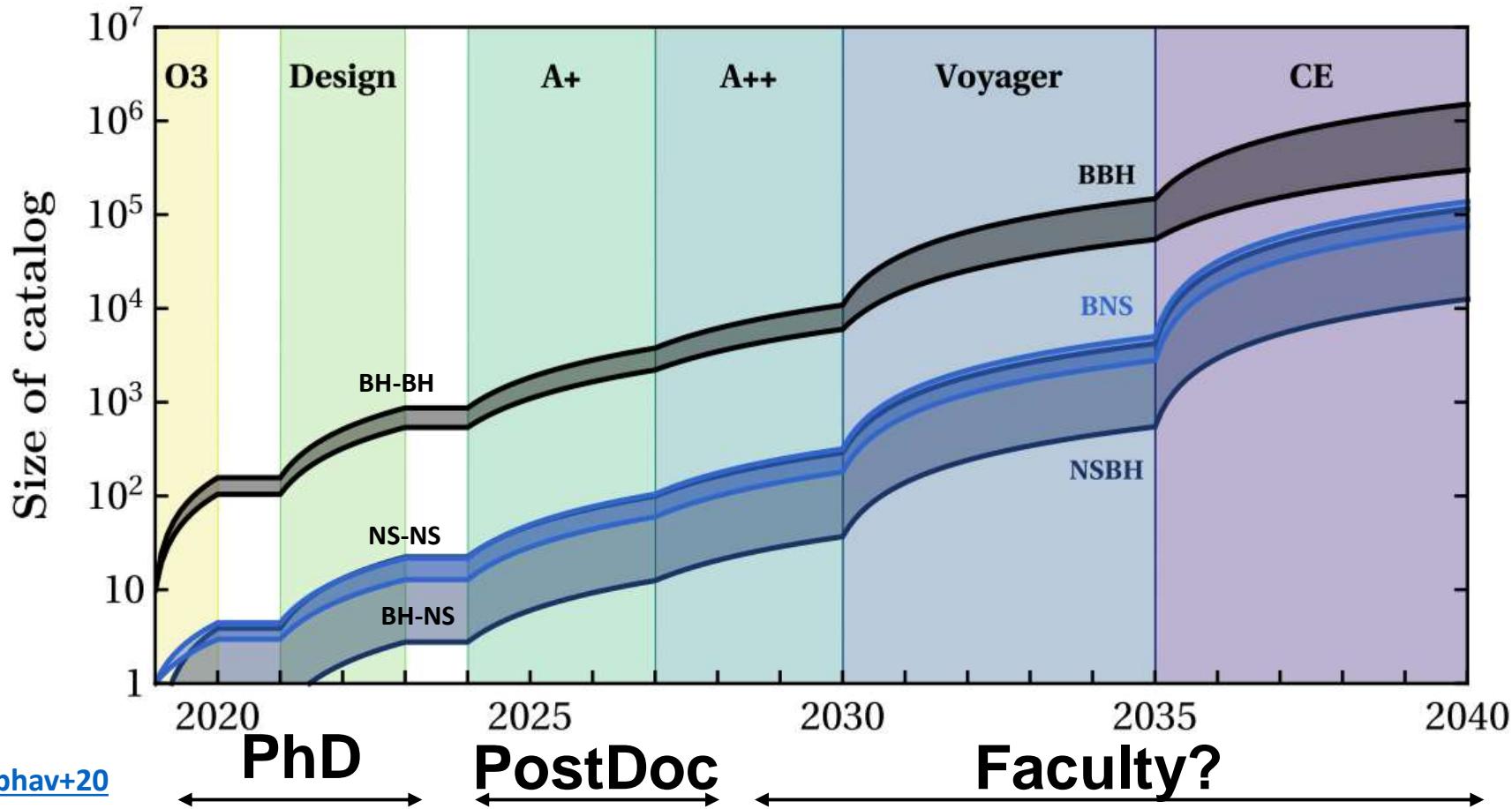


GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

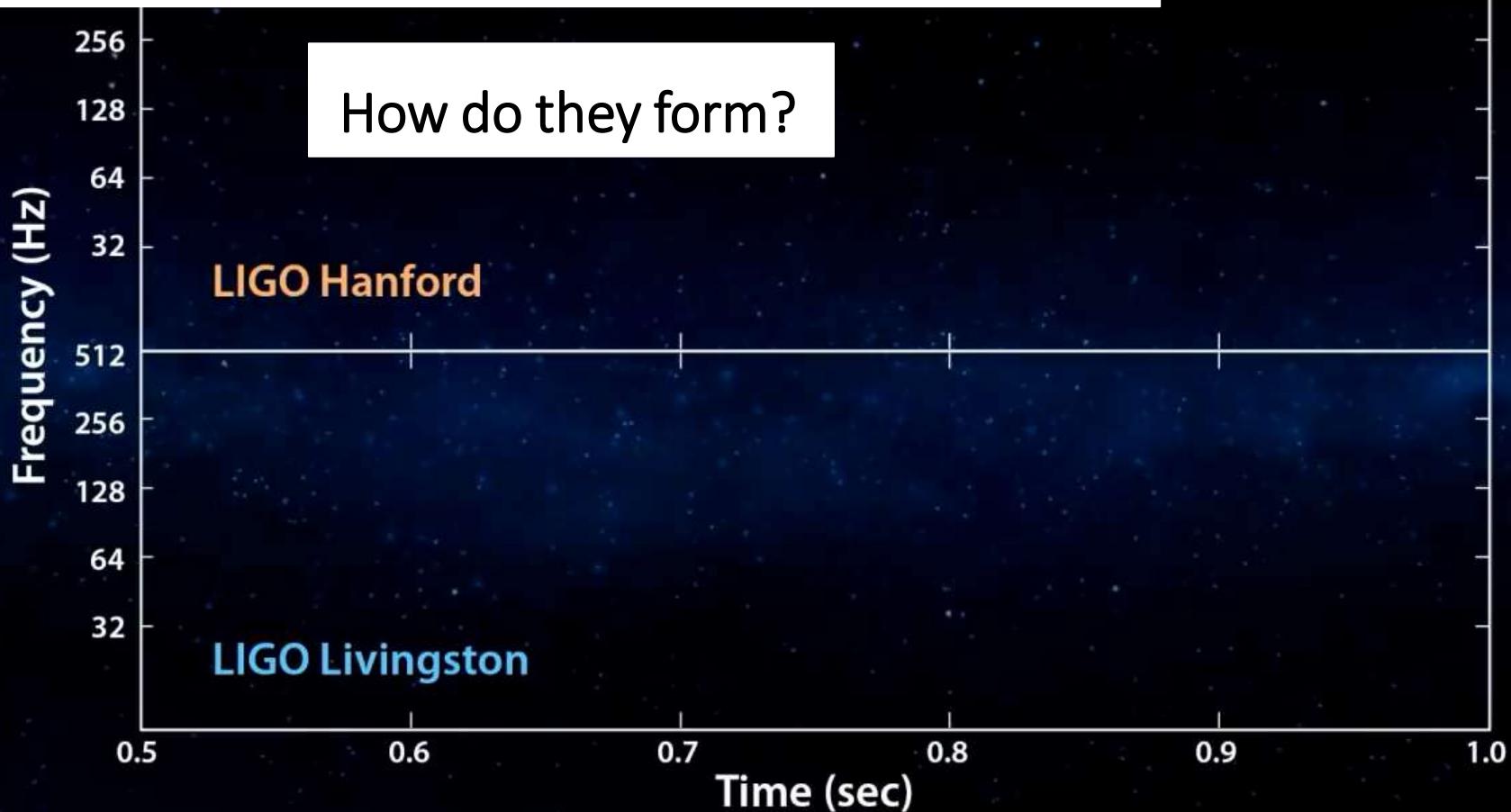
Building up a population of observations



Building up a population of observations



What can we learn from gravitational waves?



$z = 0$

How do they form?

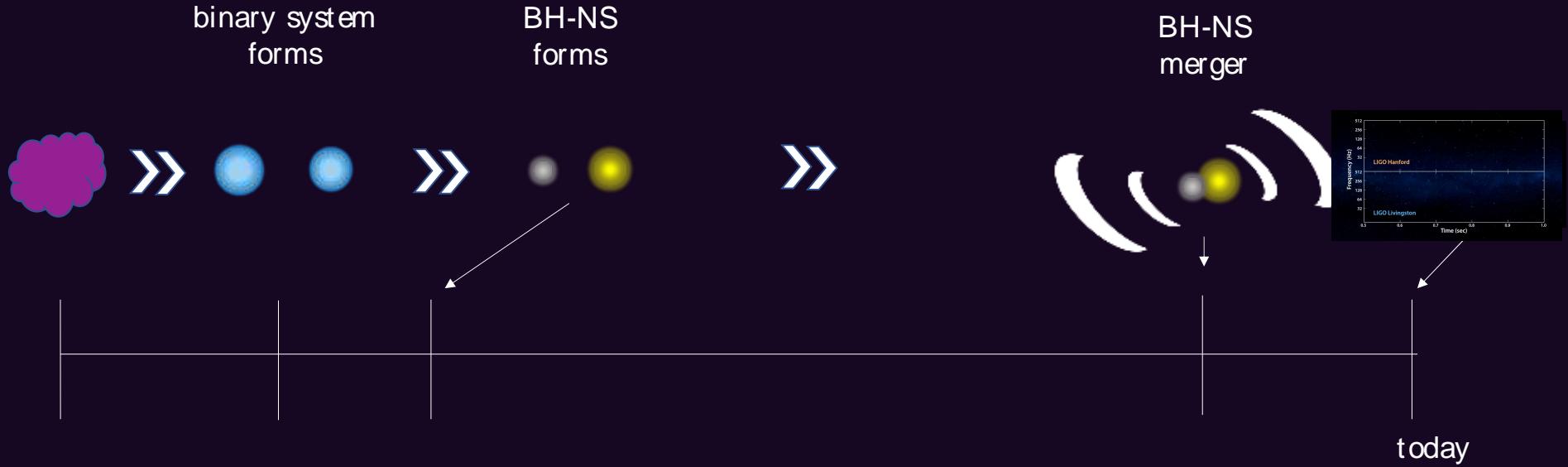


Image not to scale!

e.g. Paczynski+76, Smarr & Blandford+76

Floor Broekgaarden

Rapid binary populatio n synthesis

Stevenson+17, Barrett+18, Vigna-Gomez+18
Based on tracks from Hurley+00,02, Pols+98

Small
images:
Tauris+17



$$\mathbf{x}_i = (m_{1,i}, m_{2,i}, a_i, \dots)$$

Initial conditions: mass 1, mass 2,
separation, ...

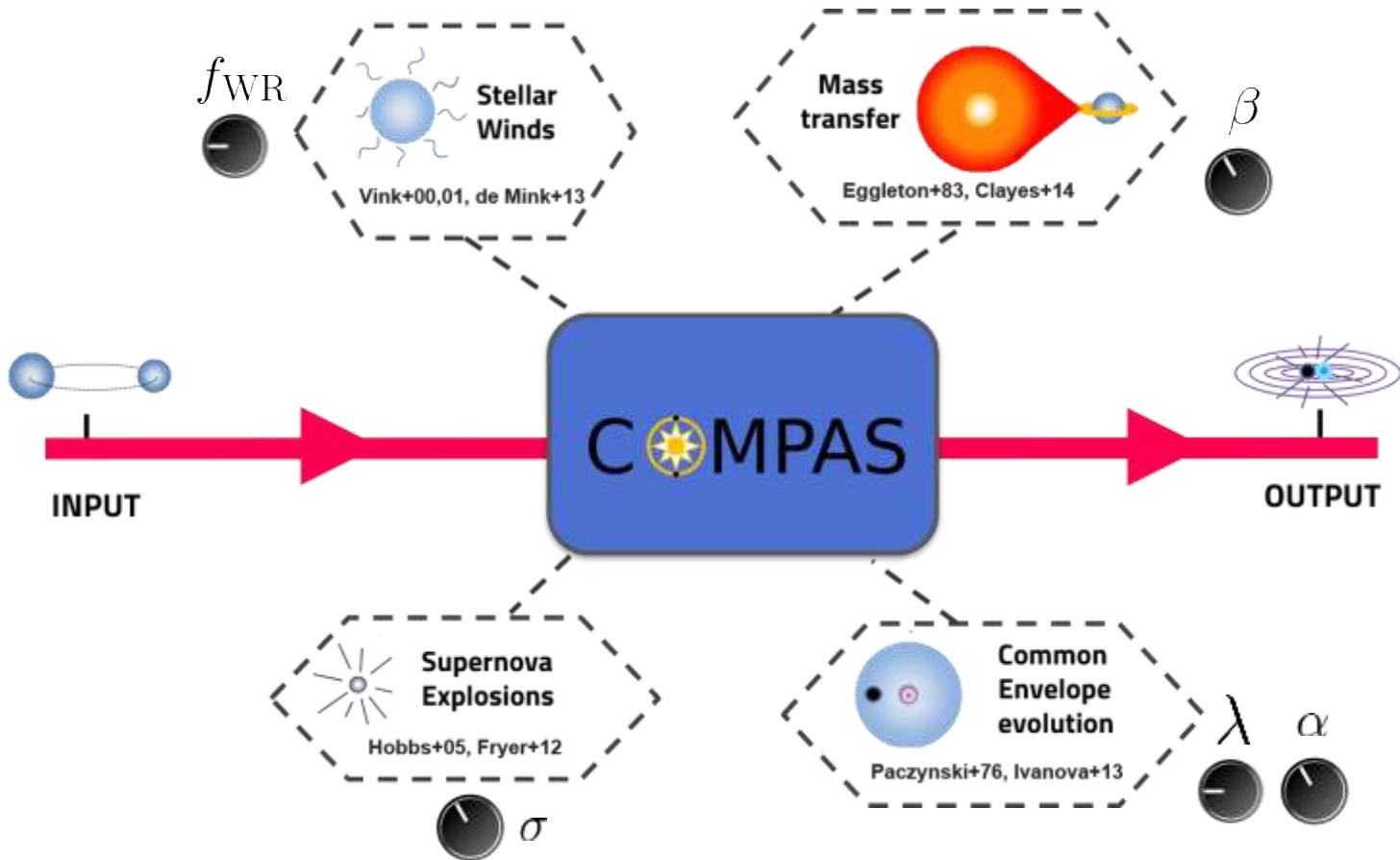
$$\mathbf{u}(\mathbf{x}_i) = (m_{1,f}, m_{2,f}, \dots)$$

final conditions: mass 1, mass 2, ...

Rapid binary populatio n synthesis

Stevenson+17, Barrett+18, Vigna-Gomez+18
Based on tracks from Hurley+00,02, Pols+98

Small
images:
Tauris+17



Why GitHub? Team Enterprise Explore Marketplace Pricing

Search Sign in Sign up

TeamCOMPAS / COMPAS

Code Issues 35 Pull requests Actions Projects Wiki Security Insights

1 dev 6 branches 87 tags Go to file Code

ilyamandel Merge pull request #460 from TeamCOMPAS/minor_enhancements_and_star_f... 9 hours ago 899 commits

github reading docker.yml 7 days ago

defaults Update pythonSubmit.py 12 hours ago

docs Merge pull request #462 from jeffriley/minor_enhancements_and_star_f... 21 hours ago

postProcessing Fixing broken comment 13 days ago

sro Update Options.cpp 9 hours ago

.gitignore removed dockerfile and hid it in my .gitignore - this should never be... 8 days ago

Dockerfile Add dockerfile and makefile for docker 5 months ago

LICENSE Create LICENSE 9 months ago

README.md Update README.md 2 months ago

About

COMPAS rapid binary population synthesis code

compas.science

Readme MIT License

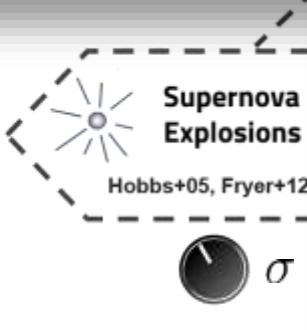
Releases 87 tags

Packages No packages published



Select Mergers: Population & Statistics

COMPAS is a rapid binary population synthesis code that is designed so that evolution prescriptions and modelable. COMPAS draws properties for a binary star system from a set evolves it from zero-age main sequence to the end of its life as two been used for inference from observations of gravitational-wave stars, X-ray binaries, and luminous red novae.



Supernova Explosions

Hobbs+05, Fryer+12

Getting started

Specifications

Post-processing

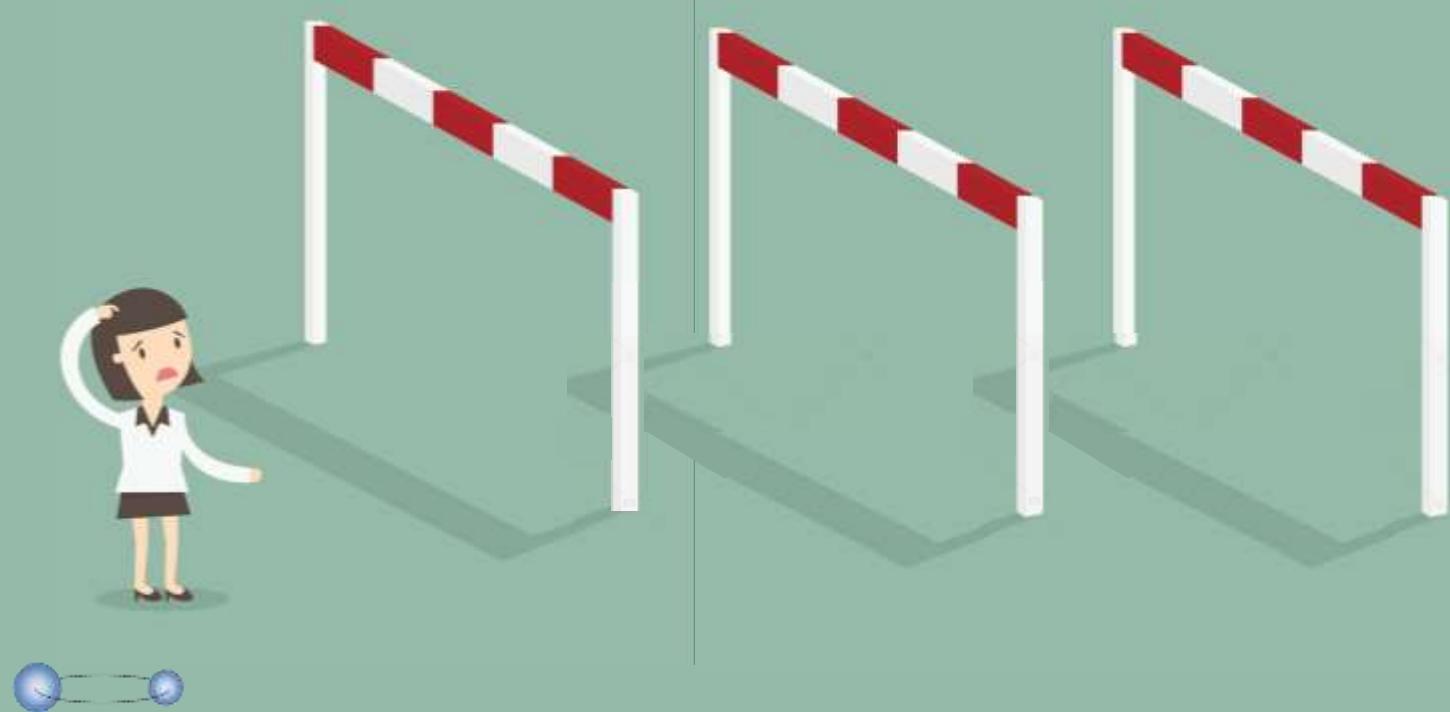
Running COMPAS inside a Docker container

Contact

Please email your queries to compas-user@googlegroups.com. You are also welcome to join the [COMPAS User Google Group](#) to engage in discussions with COMPAS users and developers.

3 computation al challenges

1 Simulating populations



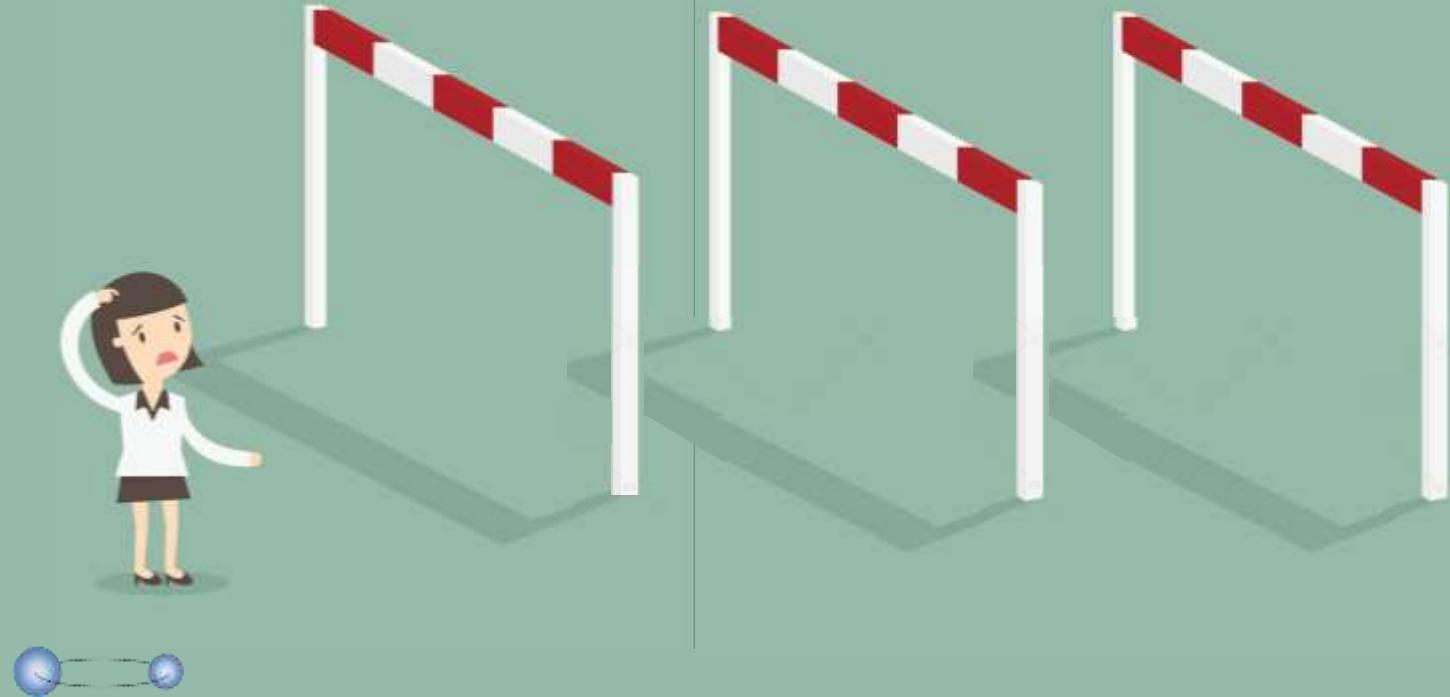
Cost:

~1 sec

3 computation al challenges

1 Simulating populations

2 Testing physics



Cost:



$\sim 1 \text{ sec}$

$\times 10^6$

3 computation al challenges

1 Simulating populations



Cost:

$\sim 1 \text{ sec}$

$\times 10^6$

2 Testing physics



3 Rare events

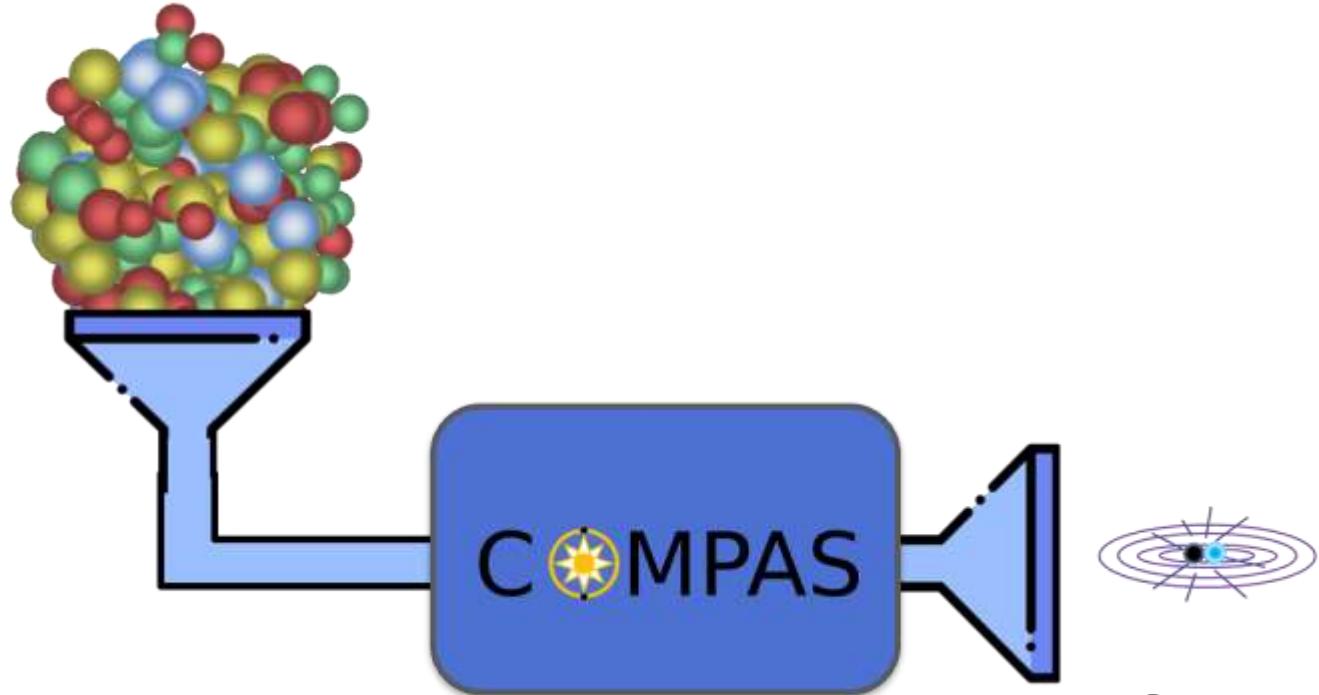


$\times 10 - 10^6$

3

Rare events:

1000 binaries



3 computation al challenges

1 Simulating populations



Cost:

$\sim 1 \text{ sec}$

$\times 10^6$

2 Testing physics



$\times 10 - 10^6$

$\times 10 - 10^3$

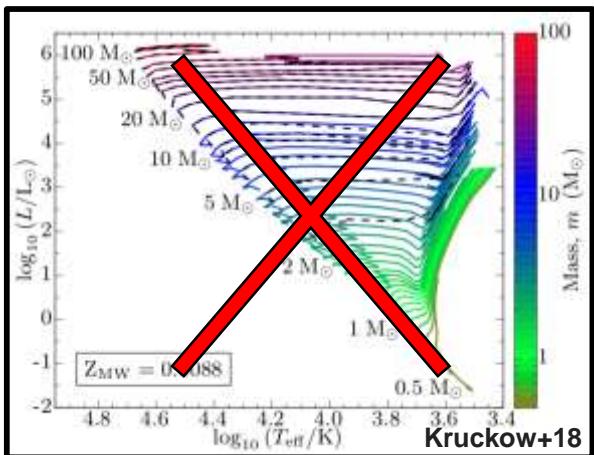
$\sim 3 - 3 \times 10^6$ years computing time

Floor Broekgaarden

3 Rare events

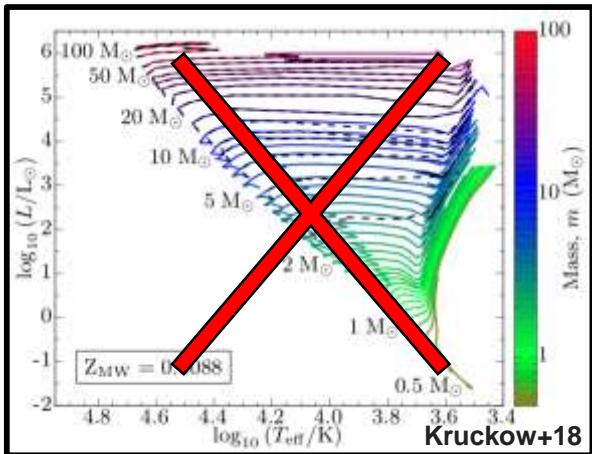


Current binary population synthesis models pay a high price...

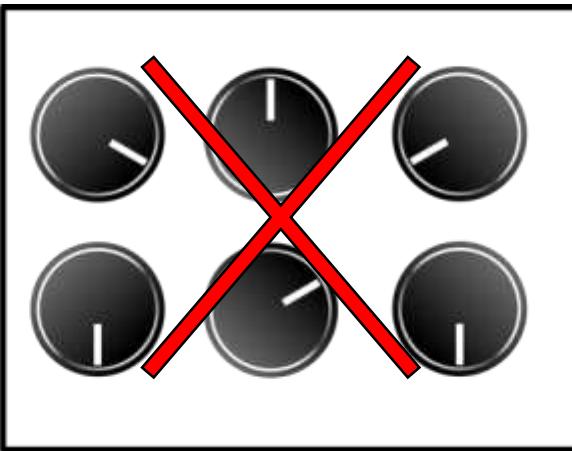


Do not include detailed
prescriptions

Current binary population synthesis models pay a high price...

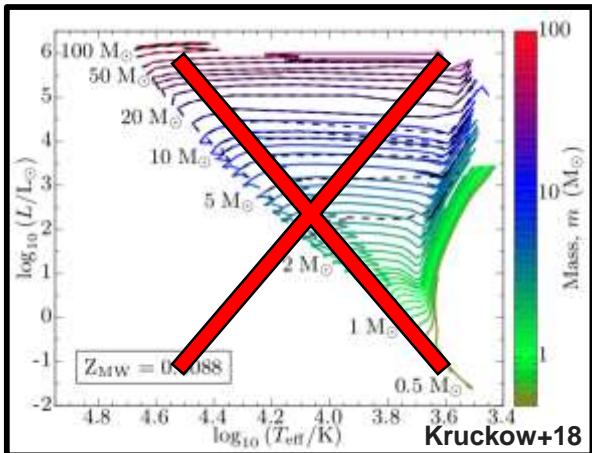


Do not include detailed prescriptions

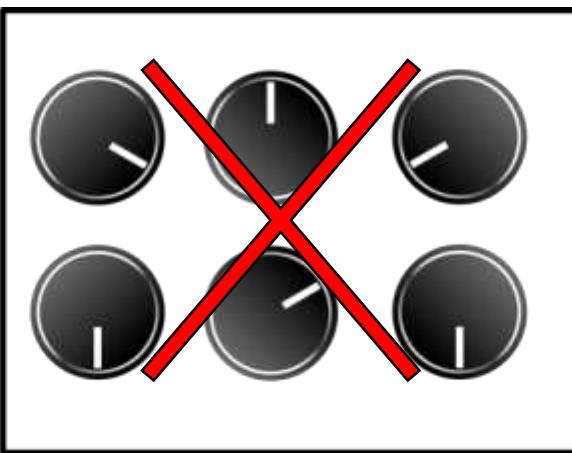


Perform only a small parameter study

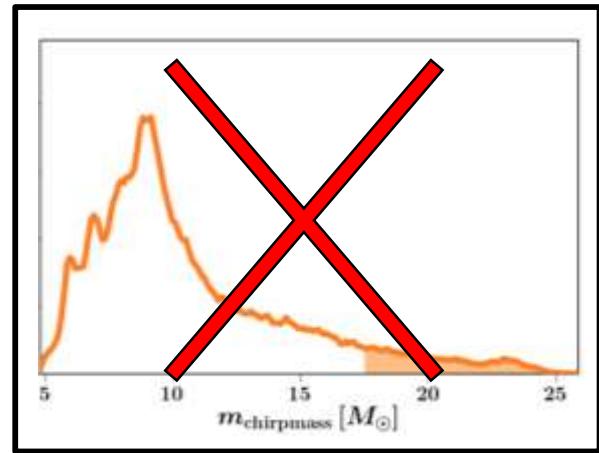
Current binary population synthesis models pay a high price...



Do not include detailed prescriptions



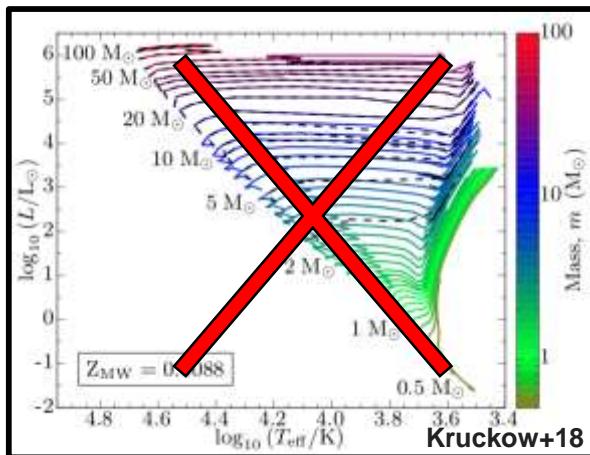
Perform only a small parameter study



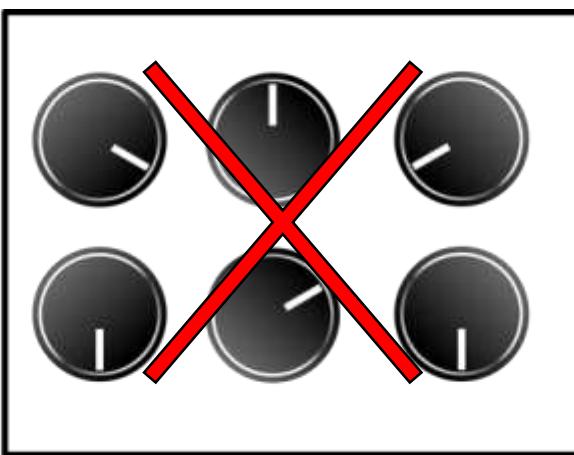
Do not explore tails of distributions



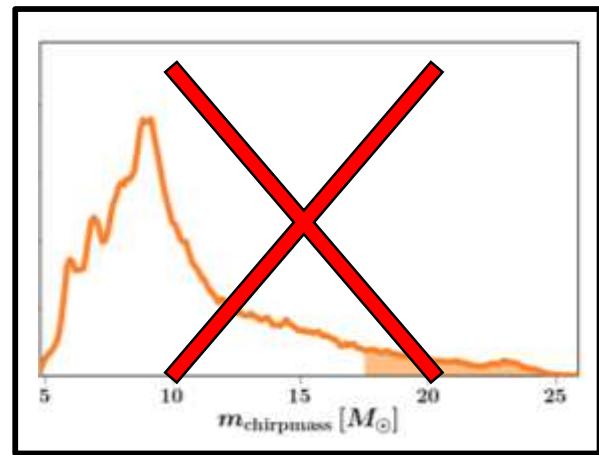
Can we improve this? Population synthesis models pay a high price...



Do not include detailed prescriptions



Perform only a small parameter study



Do not explore tails of distributions

Previous work

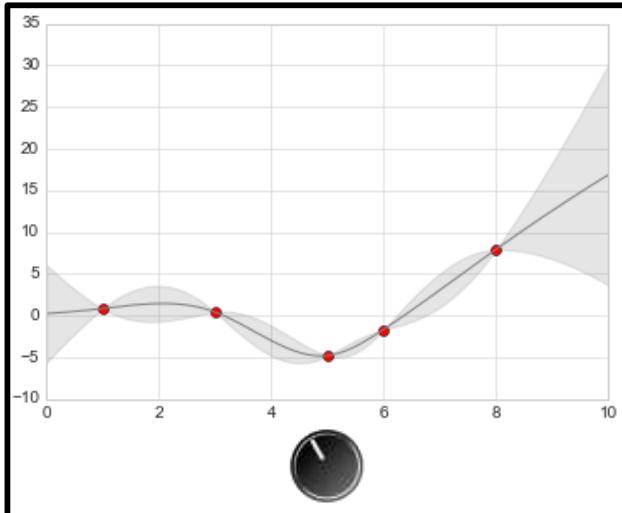
Analytical formalisms

Kolb 1993, Politano 1996, Kalogera 1996
Kalogera&Webbink 1998, Kalogera+2000



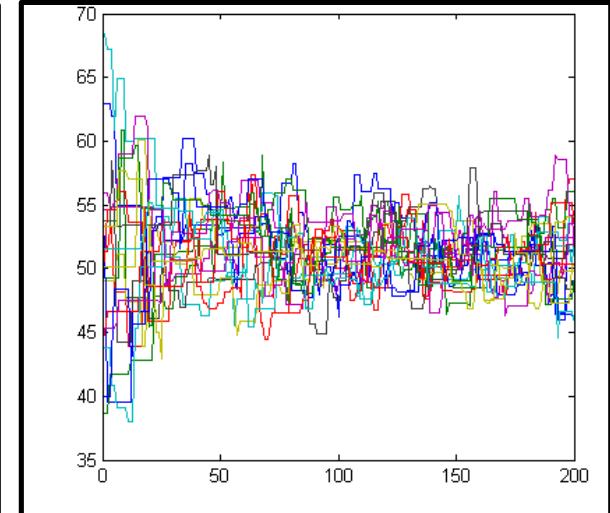
Emulators (GPR)

Barret+2017, Taylor & Gerosa 2018,
Wong & Gerosa 2019



Markov Chain Monte Carlo

Andrews+2017; (dart_board)



We present:

(drum roll)

New sampling algorithm:

Simulating The Rare Outcomes Of Populations
With AIS For Efficient Learning



Adaptive Importance Sampling (AIS)



Marin+06, Douc+07, Owen+09, Martino+15
AIS: Torrie & Valleau 1977, Hesterberg 1995,
Cappe+2004, Pennanen & Koivu 2006,
Cornuet+2012, Ortiz & Pack Kaelbling (2013), ...

New sampling algorithm:

Simulating The Rare Outcomes

With STROOPWAFFEL



Adaptive Importance Sampling

Monthly Notices
of the Royal Astronomical Society

Issues ▾ Advance articles Submit ▾ Purchase Alerts About ▾ All Monthly Notices of the RAS ▾


Volume 490, Issue 4
December 2019

Article Contents

ABSTRACT
1 INTRODUCTION
2 METHOD
3 RESULTS
4 DISCUSSION
5 SUMMARY AND CONCLUSIONS

STROOPWAFFEL: simulating rare outcomes from astrophysical populations, with application to gravitational-wave sources*

Floor S Broekgaarden, Stephen J Justham, Selma E de Mink, Jonathan Gair, Ilya Mandel, Simon Stevenson, Jim W Barrett, Alejandro Vigna-Gómez, Coenraad J Neijssel

Monthly Notices of the Royal Astronomical Society, Volume 490, Issue 4, December 2019,
Pages 5228–5248, <https://doi.org/10.1093/mnras/stz2558>

Published: 27 September 2019 Article history ▾

Cite Permissions Share ▾

ABSTRACT

Gravitational-wave observations of double compact object (DCO) mergers are providing new insights into the physics of massive stars and the evolution of binary systems. Making the most of expected near-future observations for understanding stellar physics will rely on comparisons with binary population

New sampling algorithm:

Simulating The Rare Outcomes Of Populations
With AIS For Efficient Learning



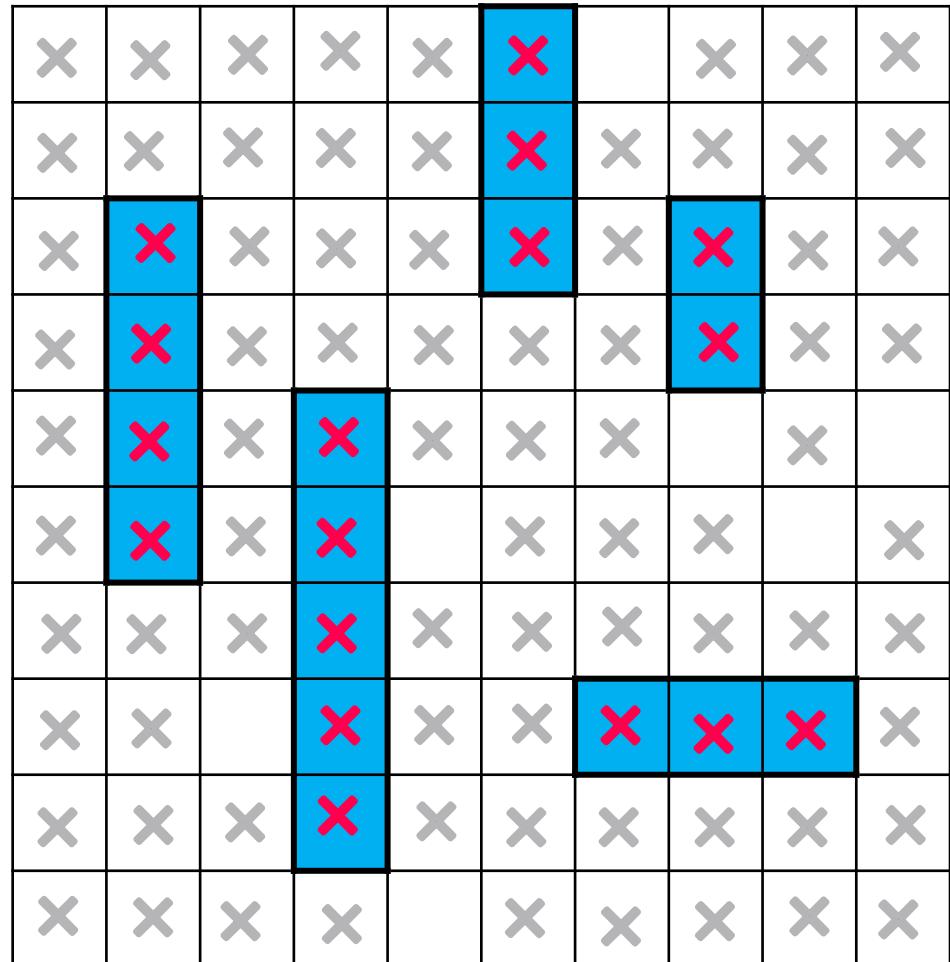
Adaptive Importance Sampling (AIS)



Marin+06, Douc+07, Owen+09, Martino+15
AIS: Torrie & Valleau 1977, Hesterberg 1995,
Cappe+2004, Pennanen & Koivu 2006,
Cornuet+2012, Ortiz & Pack Kaelbling (2013), ...

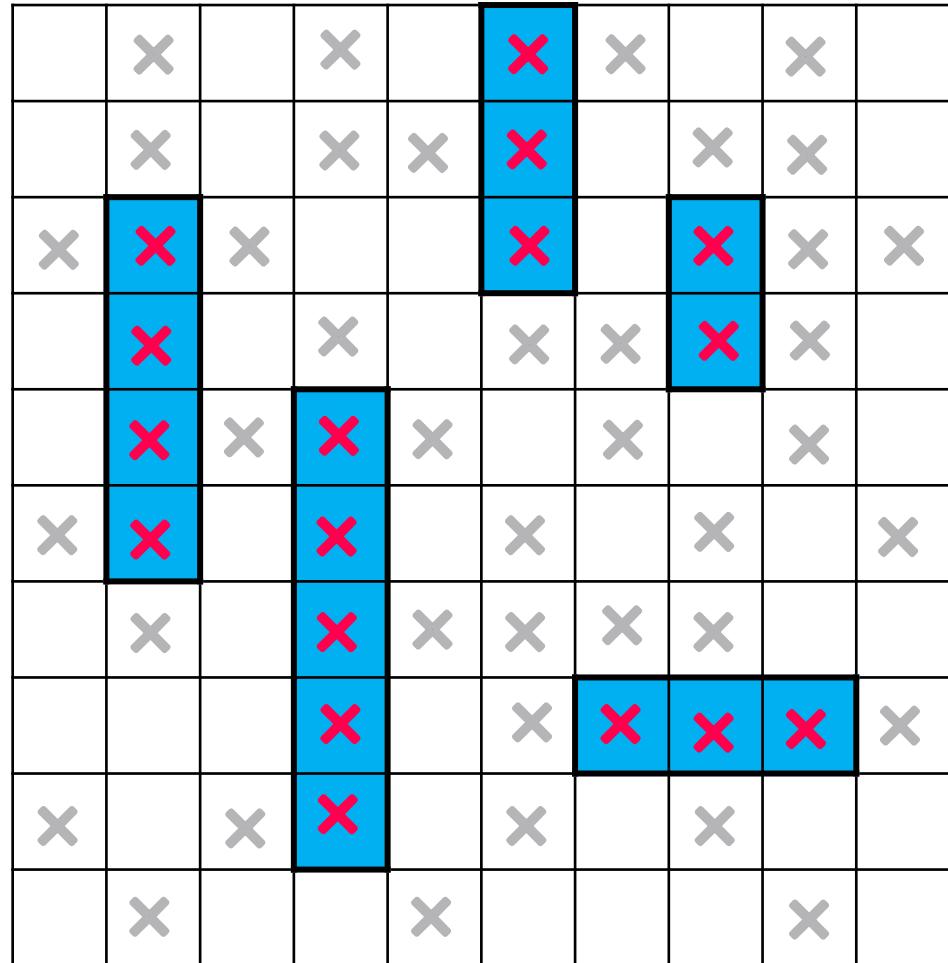
**Traditional models
use “random shooting”:**

requires
< 96 >
shots
to complete the game



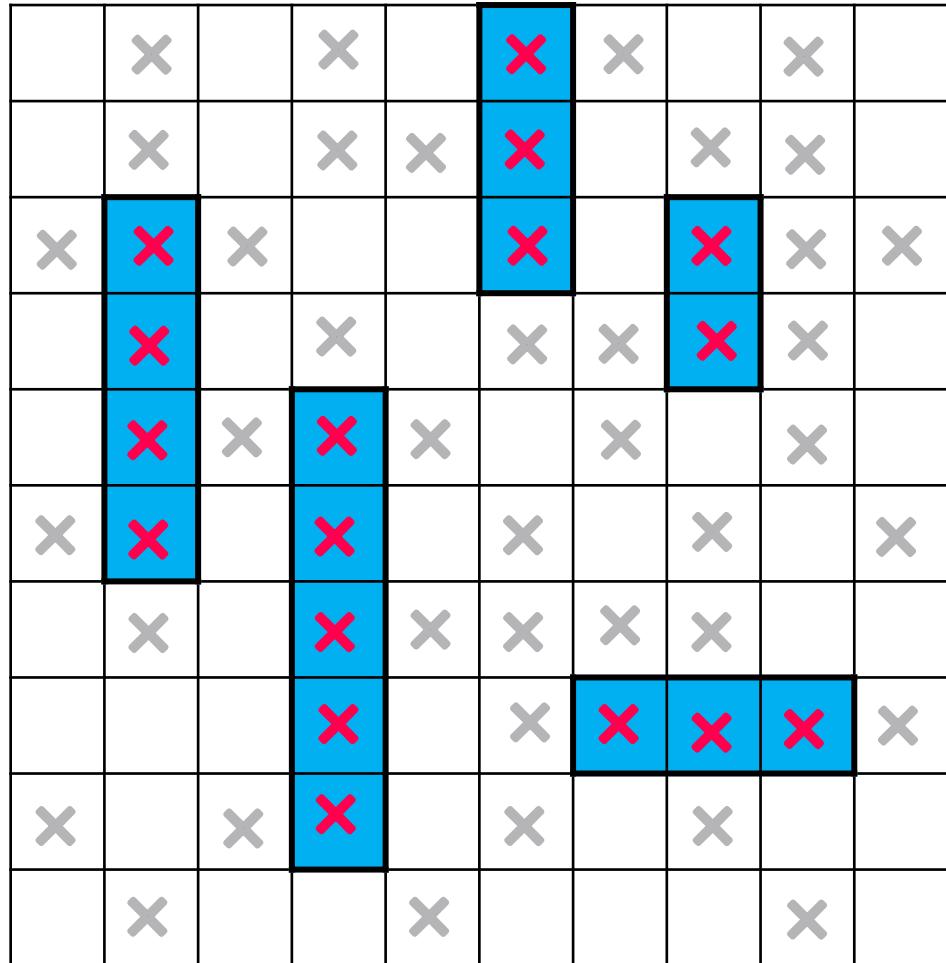
STROOPWAFEL
uses “*explore/refine*”:

requires
<? ? >
shots
to complete the game



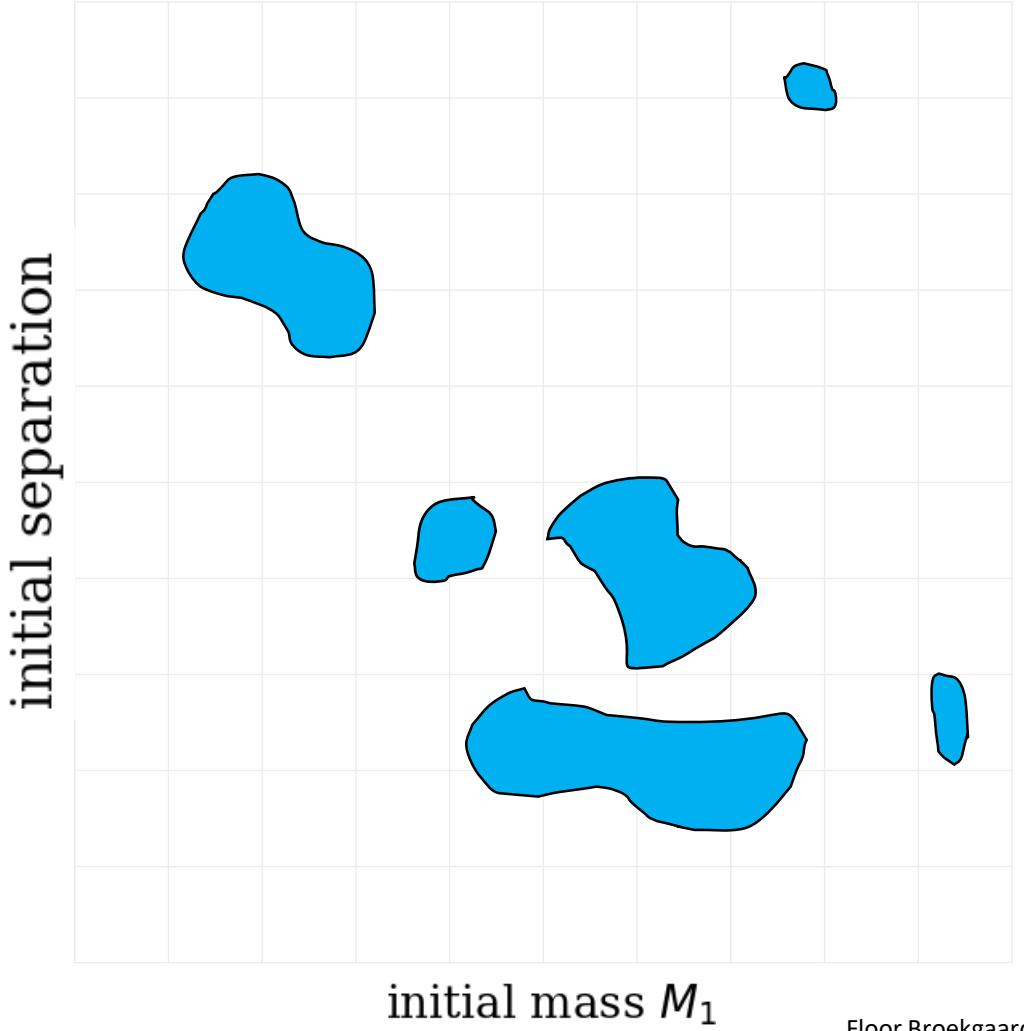
STROOPWAFEL
uses “*explore/refine*”:

requires
< 65 >
shots
to complete the game



Binary population synthesis:

- High-dimensional space,
- Unknown “islands” that form, e.g., BH-BH mergers
- unknown “rate” $\sim \frac{1}{1000}$



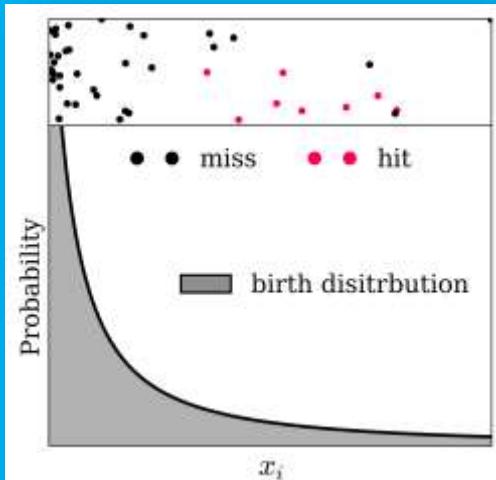
initial mass M_1

Floor Broekgaarden

prior

$$\pi(\mathbf{x}_i) = \pi(m_{1,i}, m_{2,i}, a_i, \dots)$$

1) Exploring phase



prior

$$\pi(\mathbf{x}_i) = \pi(m_{1,i}, m_{2,i}, a_i, \dots)$$

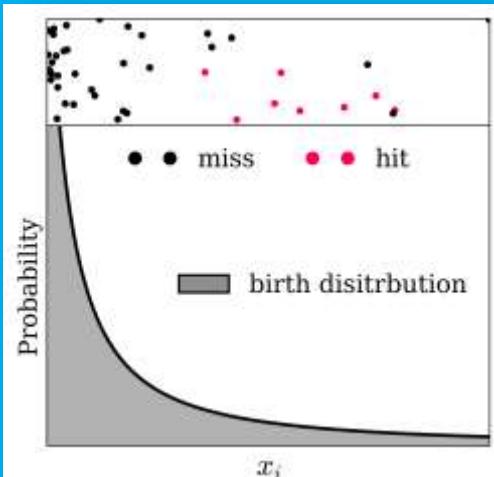
Gaussian around hits

$$q(\mathbf{x}) = \frac{1}{N_{\text{T,expl}}} \sum_{k=1}^{N_{\text{T,expl}}} q_k(\mathbf{x}; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

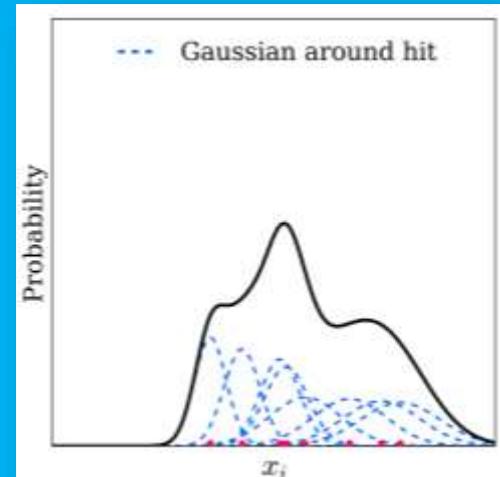
Gaussian width

$$\sigma_{j,k} = \kappa \cdot \frac{1}{\pi_j(x_k) N_{\text{expl}}^{1/d}},$$

1) Exploring phase



2) Create adapted distribution



prior

$$\pi(\mathbf{x}_i) = \pi(m_{1,i}, m_{2,i}, a_i, \dots)$$

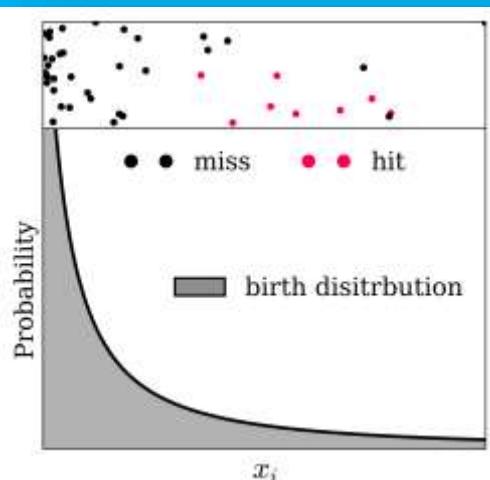
Gaussian around hits

$$q(\mathbf{x}) = \frac{1}{N_{\text{T,expl}}} \sum_{k=1}^{N_{\text{T,expl}}} q_k(\mathbf{x}; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

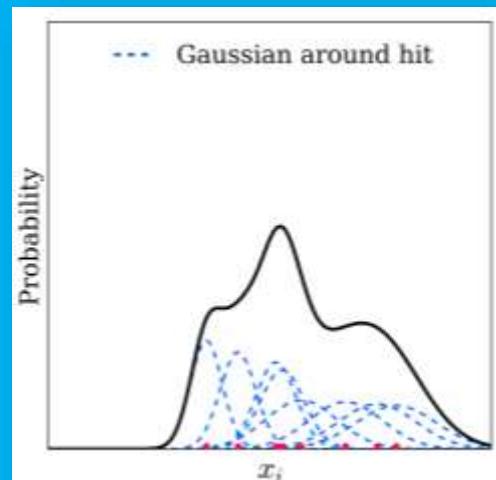
Gaussian width

$$\sigma_{j,k} = \kappa \cdot \frac{1}{\pi_j(x_k) N_{\text{expl}}^{1/d}},$$

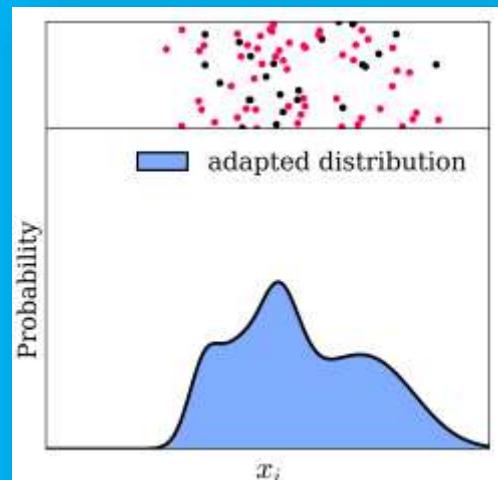
1) Exploring phase



2) Create adapted distribution

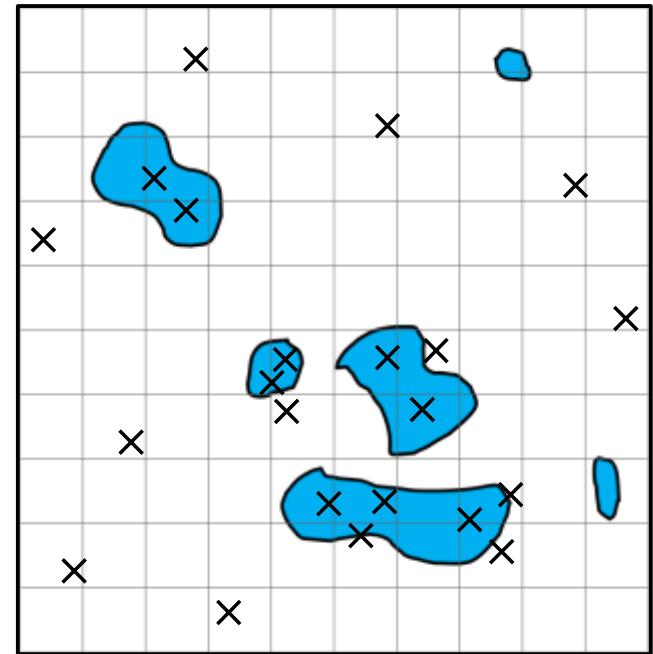
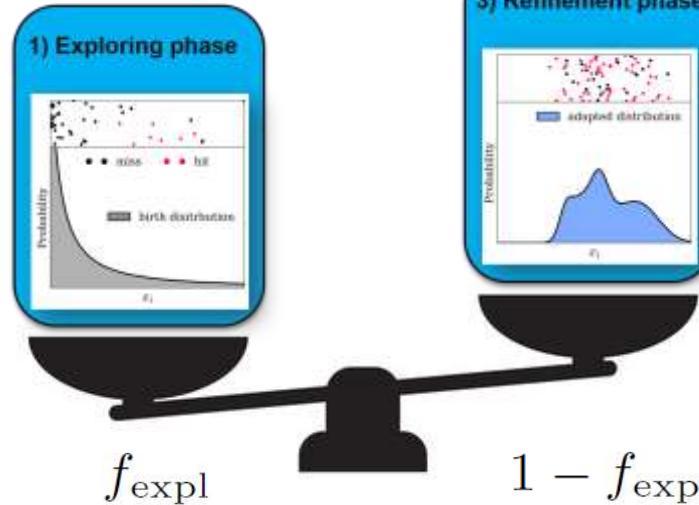


3) Refinement phase



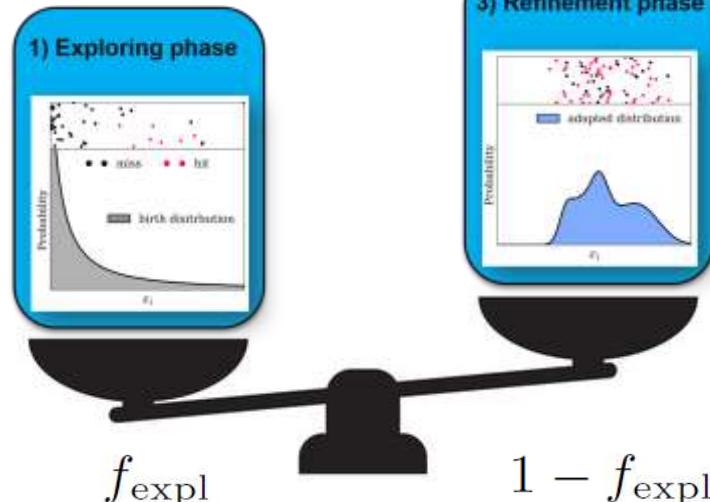
When to switch from exploring to refinement?

Getting more “hits”
Missing an “island”



When to switch from exploring to refinement?

Getting more “hits”
Missing an “island”



Alternative to e.g. Effective Sample Size (ESS) Hesterberg 1995; Liu 2008

Uncertainty from refining

$$f_{\text{expl}} = 1 - \frac{z_1(\sqrt{1-z_1} - \sqrt{z_2})}{\sqrt{1-z_1}(\sqrt{z_2(1-z_1)} + z_1)}$$

↑
uncertainty from missing an island

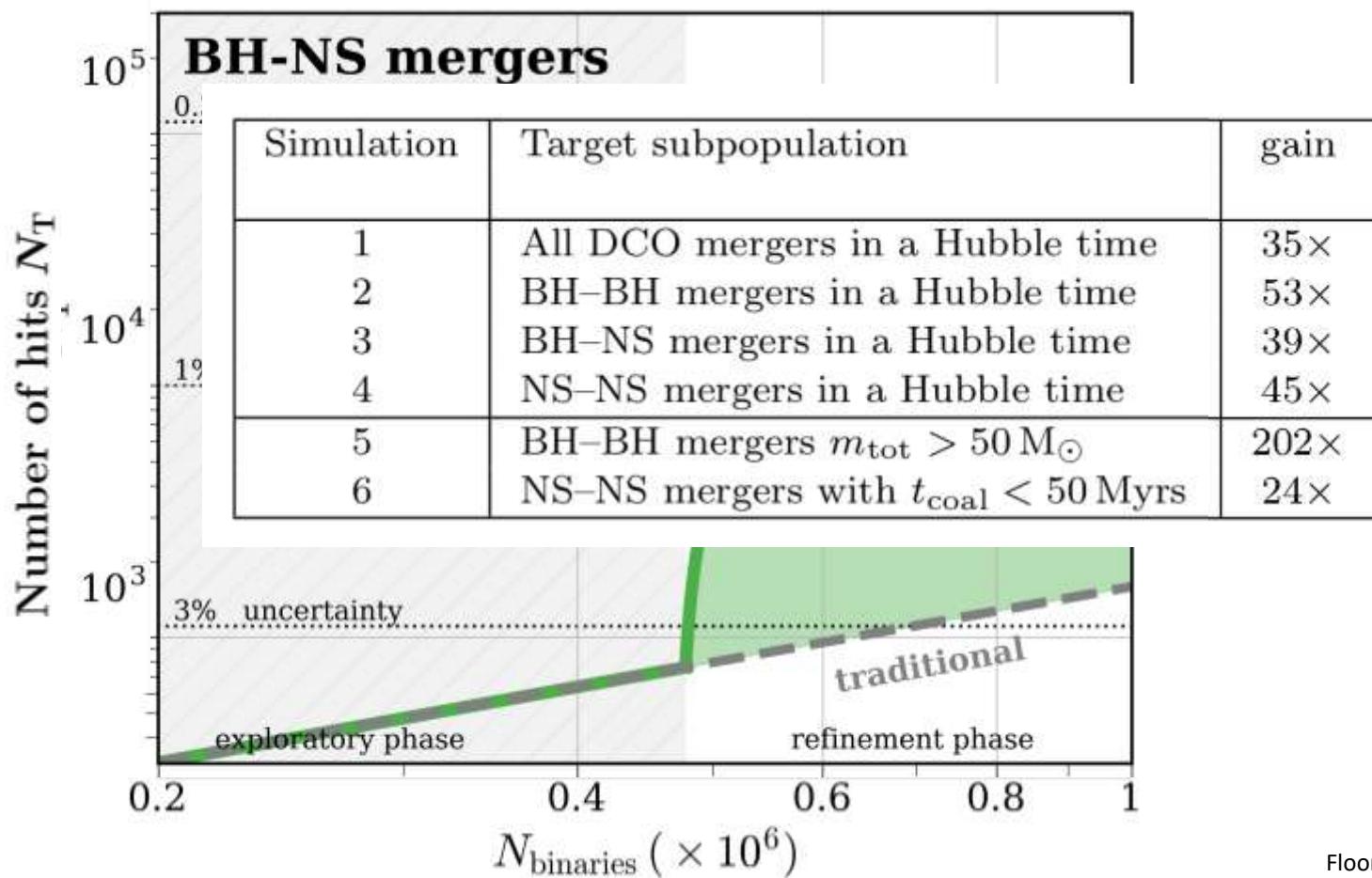
Ingredients

- 6 different target populations
- 3 parameters:
 m_1 , m_2 & separation
- traditional vs STROOPWAFEL sampling

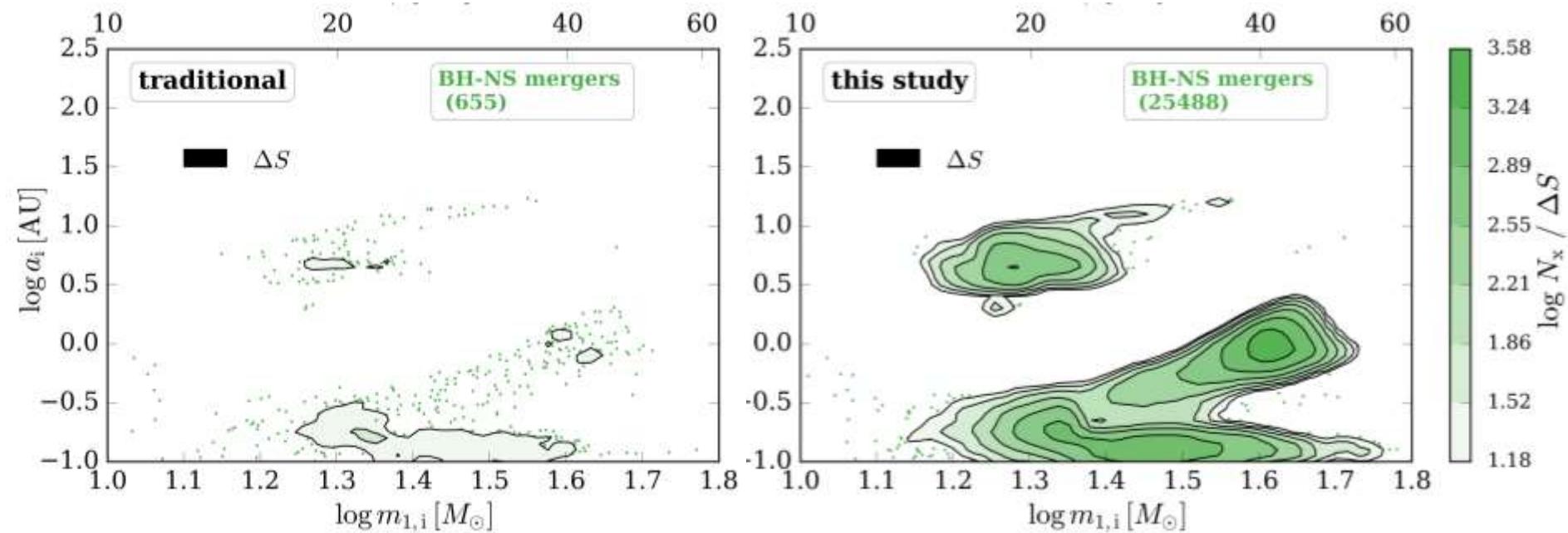
Simulation	Target subpopulation
1	All DCO mergers in a Hubble time
2	BH-BH mergers in a Hubble time
3	BH-NS mergers in a Hubble time
4	NS-NS mergers in a Hubble time
5	BH-BH mergers $m_{\text{tot}} > 50 M_{\odot}$
6	NS-NS mergers with $t_{\text{coal}} < 50 \text{ Myrs}$



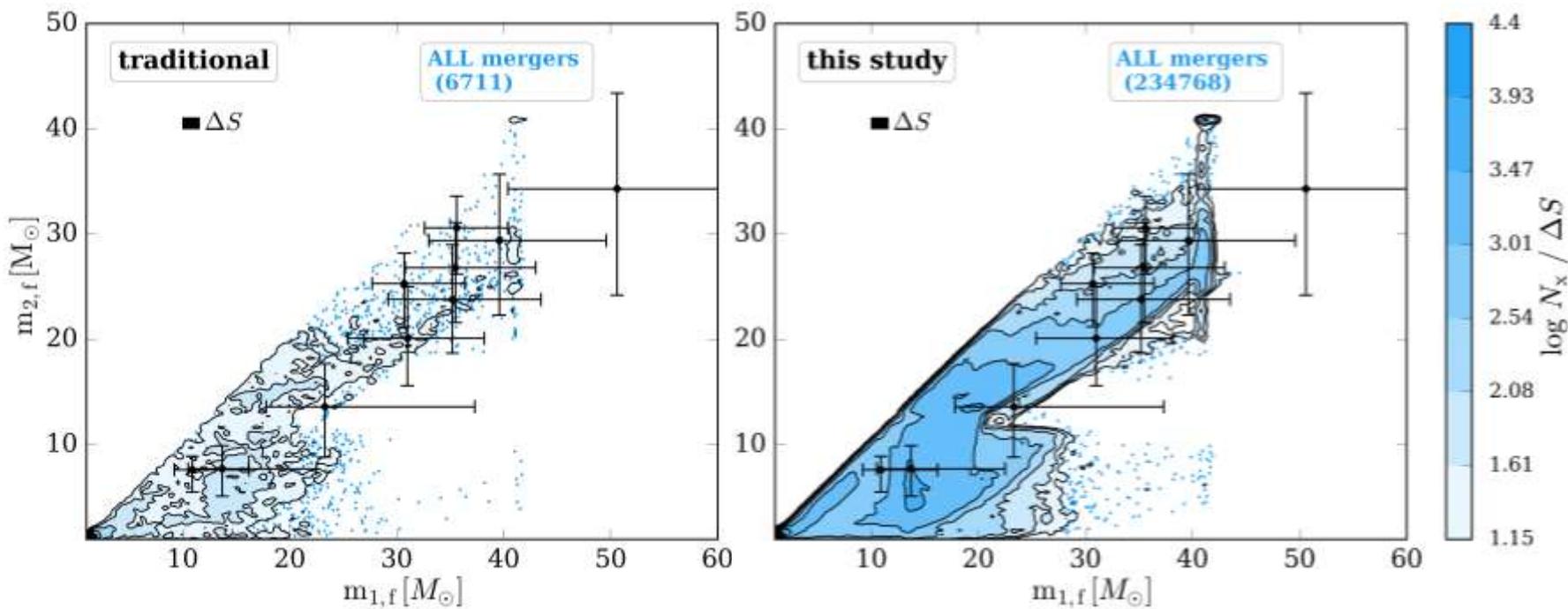
Higher efficiency



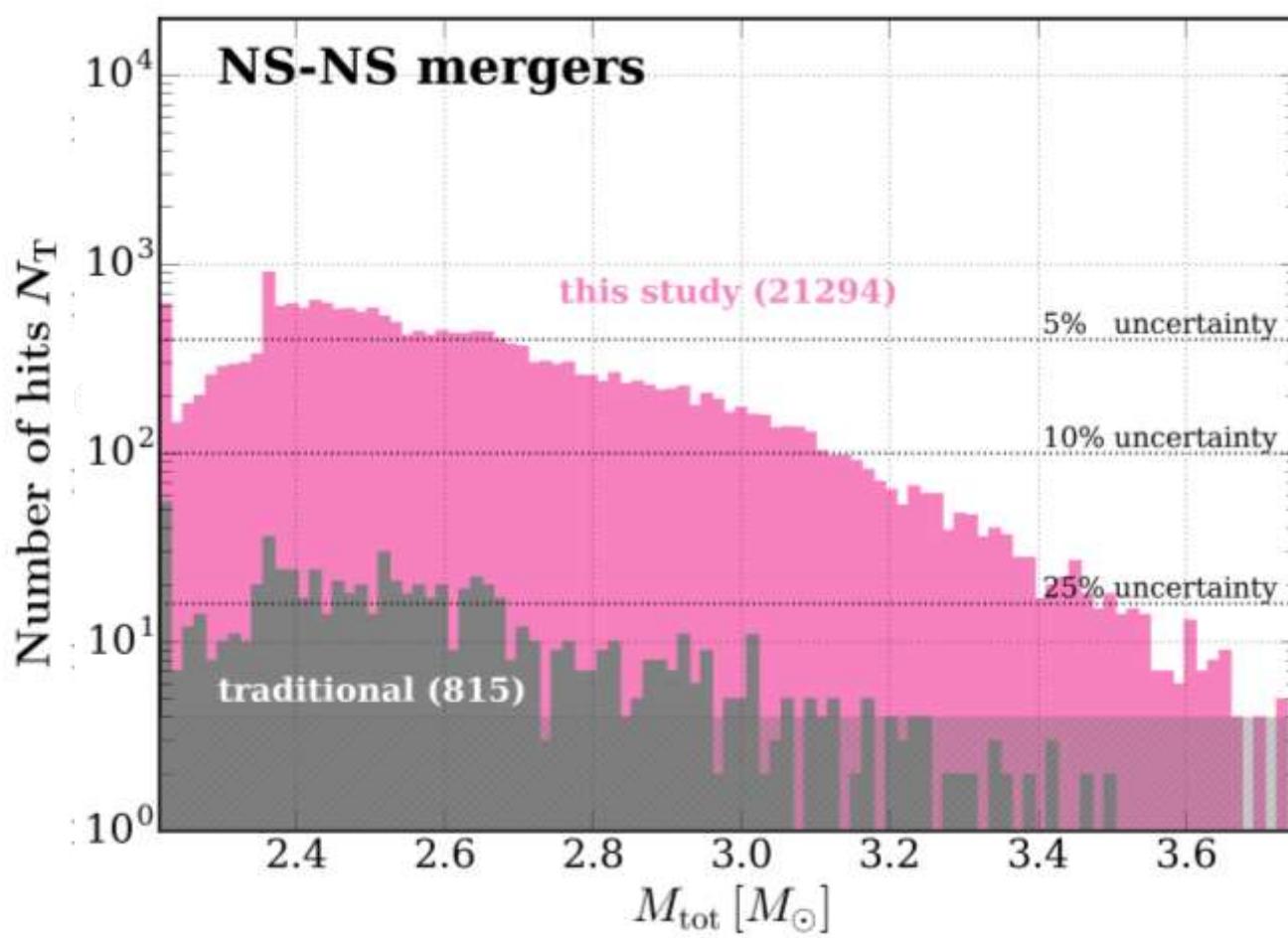
Higher resolution on input parameters



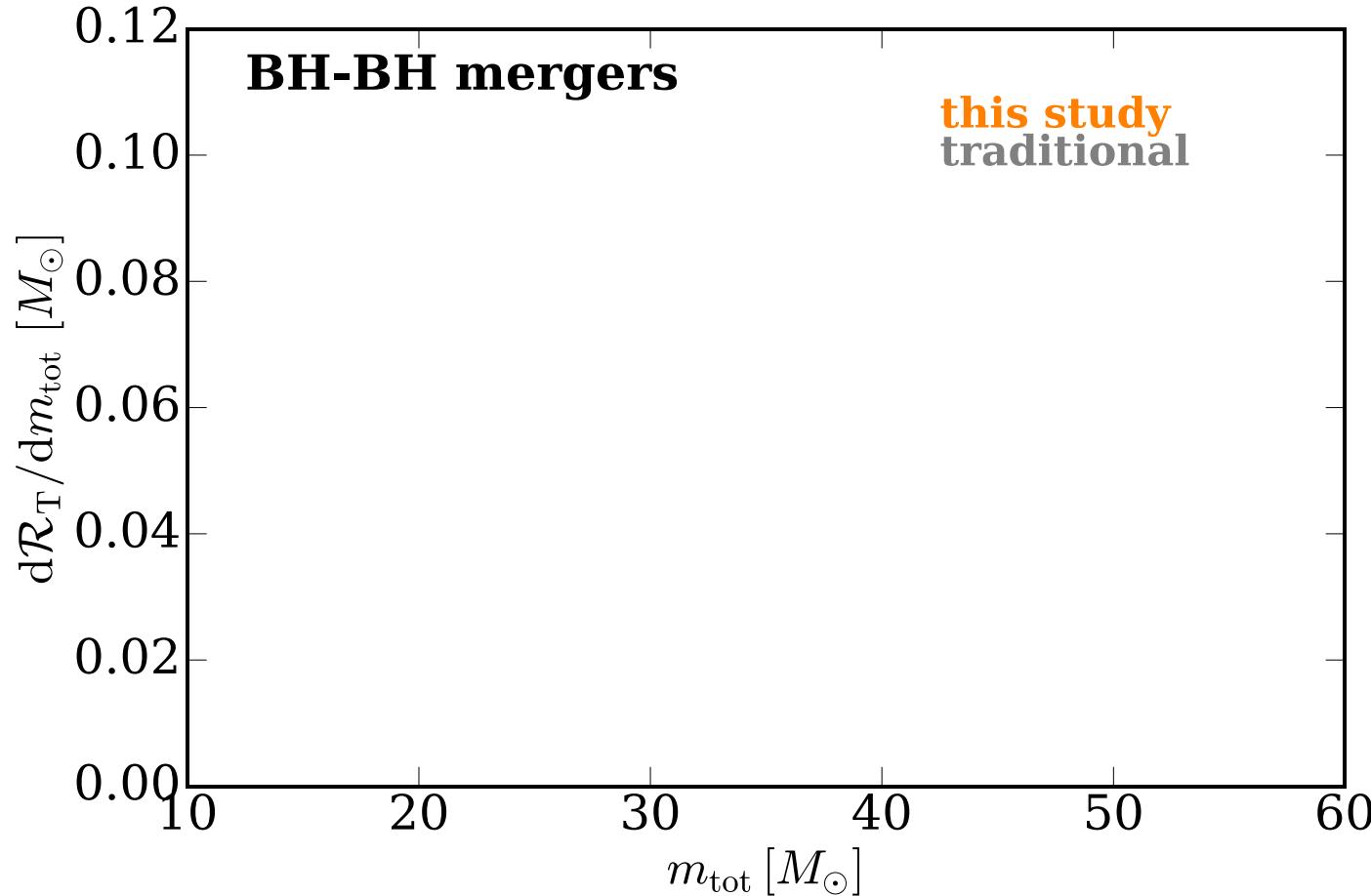
Higher resolution output



Resolve tails of distributions



Better distribution functions



STROOPWAFEL paper accepted & public release of code & data:

Monthly Notices

of the Royal Astronomical Society

Issues ▾

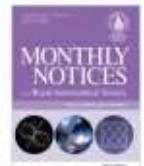
Advance articles

Submit ▾

Purchase

Alerts

About ▾



Volume 490, Issue 4
December 2019

Article Contents

ABSTRACT

1 INTRODUCTION

2 METHOD

3 RESULTS

4 DISCUSSION

5 SUMMARY AND CONCLUSIONS

STROOPWAFEL: simulating rare outcomes from astrophysical populations, with application to gravitational-wave sources[★]

Floor S Broekgaarden , Stephen J Justham, Selma E de Mink, Jonathan Gair, Ilya Mandel, Simon Stevenson, Jim W Barrett, Alejandro Vigna-Gómez, Coenraad J Neijssel

Monthly Notices of the Royal Astronomical Society, Volume 490, Issue 4, December 2019,
Pages 5228–5248, <https://doi.org/10.1093/mnras/stz2558>

Published: 27 September 2019 Article history ▾

“ Cite

Permissions

Share ▾

ABSTRACT

Gravitational-wave observations of double compact object (DCO) mergers are providing new insights into the physics of massive stars and the evolution of binary systems. Making the most of expected near-future observations for understanding stellar physics will rely on comparisons with binary population

zenodo

August 20, 2019

Data STROOPWAFEL: Simulating rare outcomes from astrophysical populations, with application to gravitational-wave sources

291 views 107 downloads See more details...

FloorBroekgaarden Update README.md 6d3744 1 minute ago 16 commits

algorithm add publicly available stroopwafel code 8 months ago

code_for_plotting_figures_paper update Fig 3 8 months ago

data added demo of data 8 months ago

fryer2012remnantBUG added explanation about fryerremnantbug in COMPAS from Paper ... 8 months ago

other add STROOPWAFEL_ings 8 minutes ago

paper added stroopwafel paper 8 months ago

presentations added astrosat.pptx 8 months ago

README.md Update README.md 1 minute ago

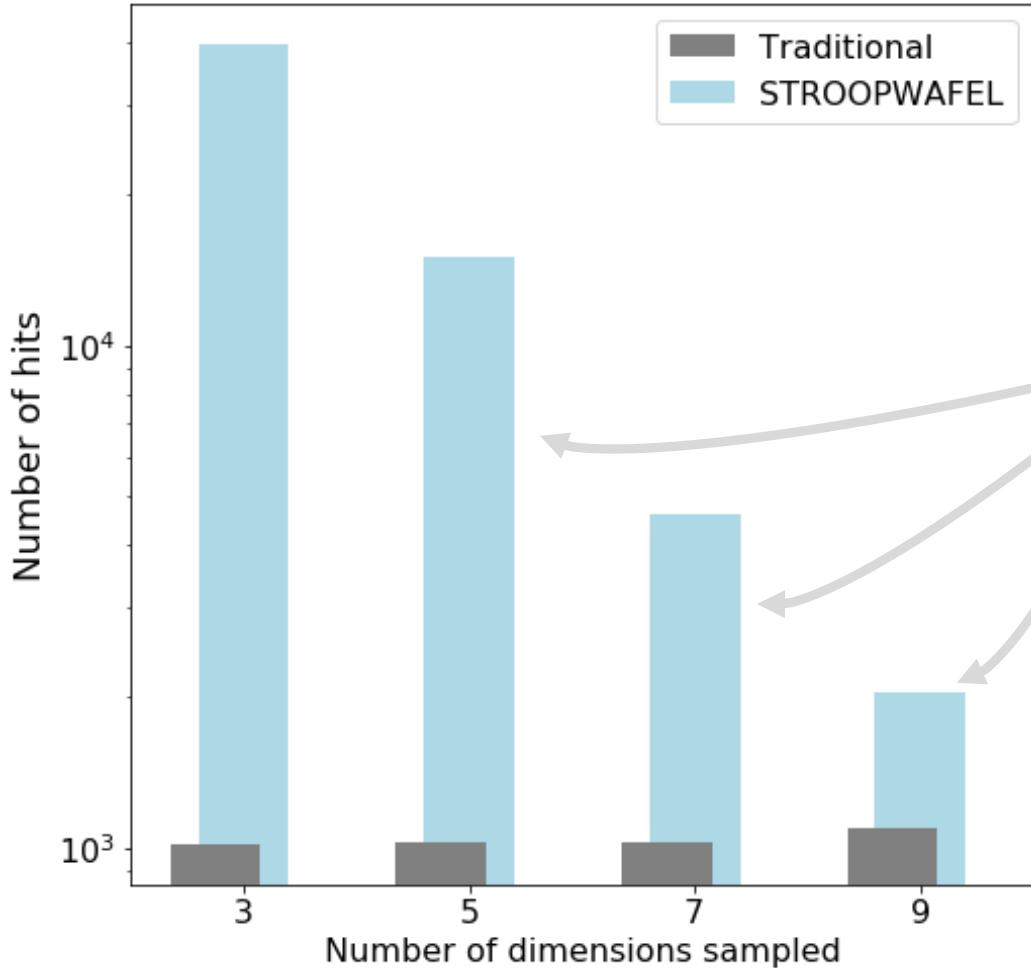
README.md

STROOPWAFEL

The STROOPWAFEL algorithm code. In addition this folder also contains the data, demos and all material that is used for the paper: STROOPWAFEL: Simulating rare outcomes from astrophysical populations, with application to gravitational-wave sources <https://academic.oup.com/mnras/article-abstract/490/4/5228/8575207> redirectedFromPDF.

A pip installable version of STROOPWAFEL by Lokesh Khedelwal can be found at <https://pypi.org/project/stroopwafel/>

$$N_{binaries} = 10^6$$



Higher dimensions?:
From Khandelwal in prep.

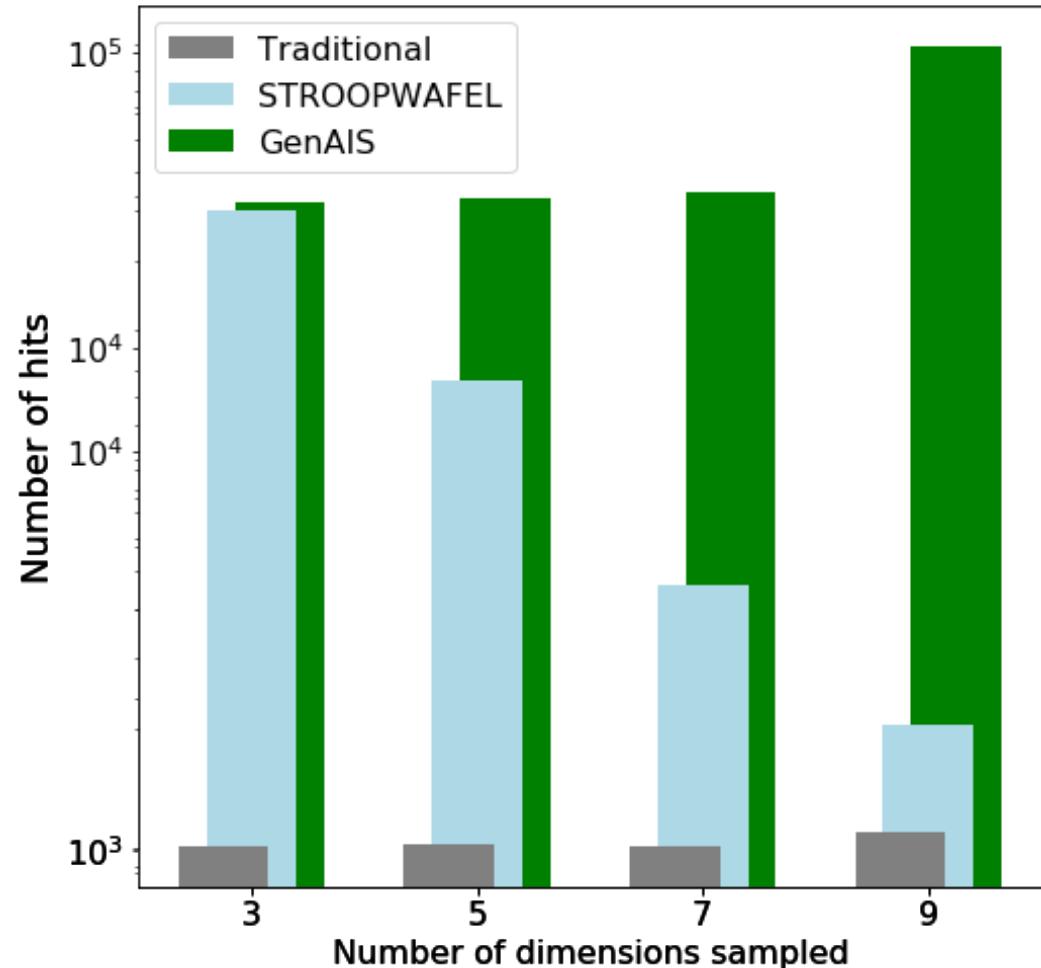


**Curse of dimensionality suffered by
STROOPWAFEL**



Lokesh Khandelwal

$$N_{binaries} = 10^6$$



Higher dimensions?:

From Khandelwal in prep.



Curse of dimensionality suffered by
STROOPWAFEL
improved using GenAIS (generational
Adaptive Importance Sampling)

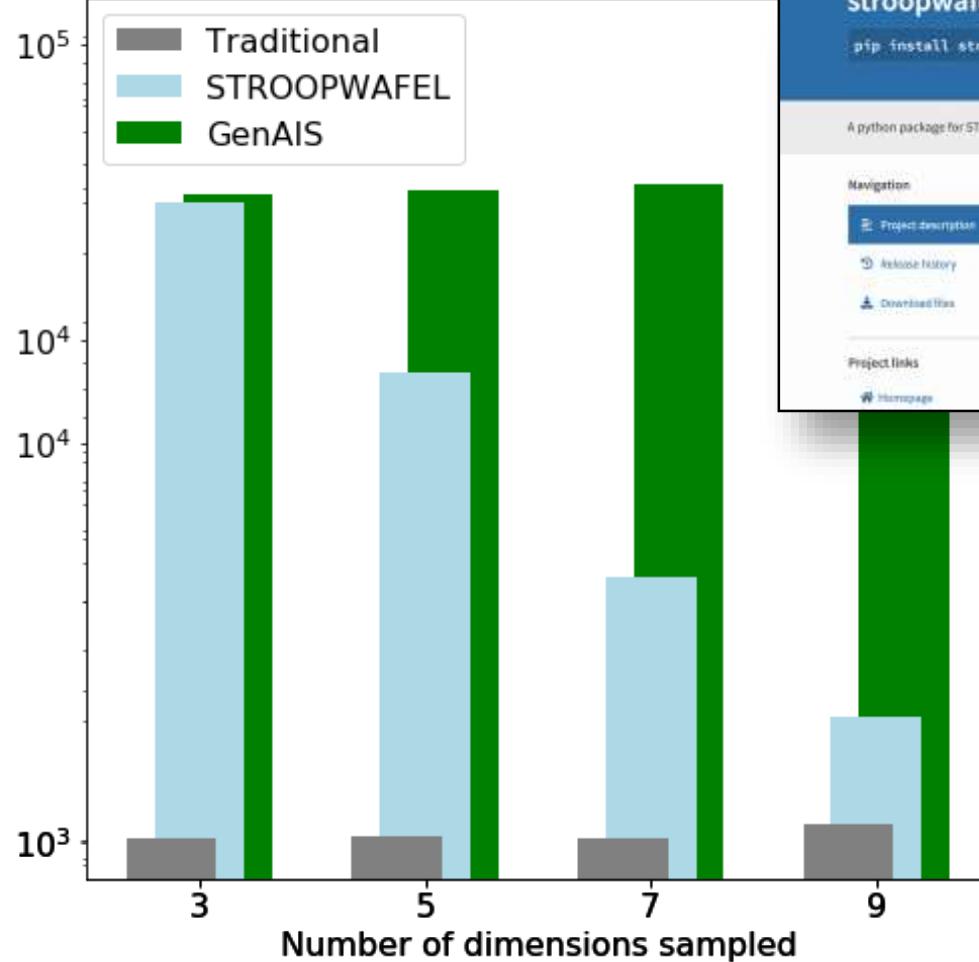
Based on Wraith+2009



Lokesh Khandelwal

$$N_{binaries} = 10^6$$

Number of hits



stroopwafel 1.0.0
pip install stroopwafel

A python package for STROOPWAFEL and GenAIS algorithms

Navigation

- Project description
- Release history
- Download files

Project description

STROOPWAFEL

Based on <https://github.com/lkhanDEL/stroopwafel>

A short documentation of stroopwafel implementation can be found here → https://docs.google.com/document/d/1Tbc_nzxEkTJgjyH0oCdq30Z77ycrG154Okhp8cGK

Project links

- Homepage

Installation

Based on Wraith+2009

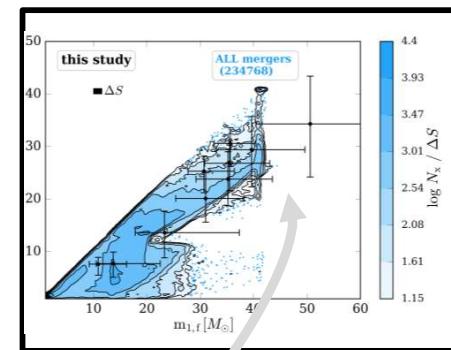
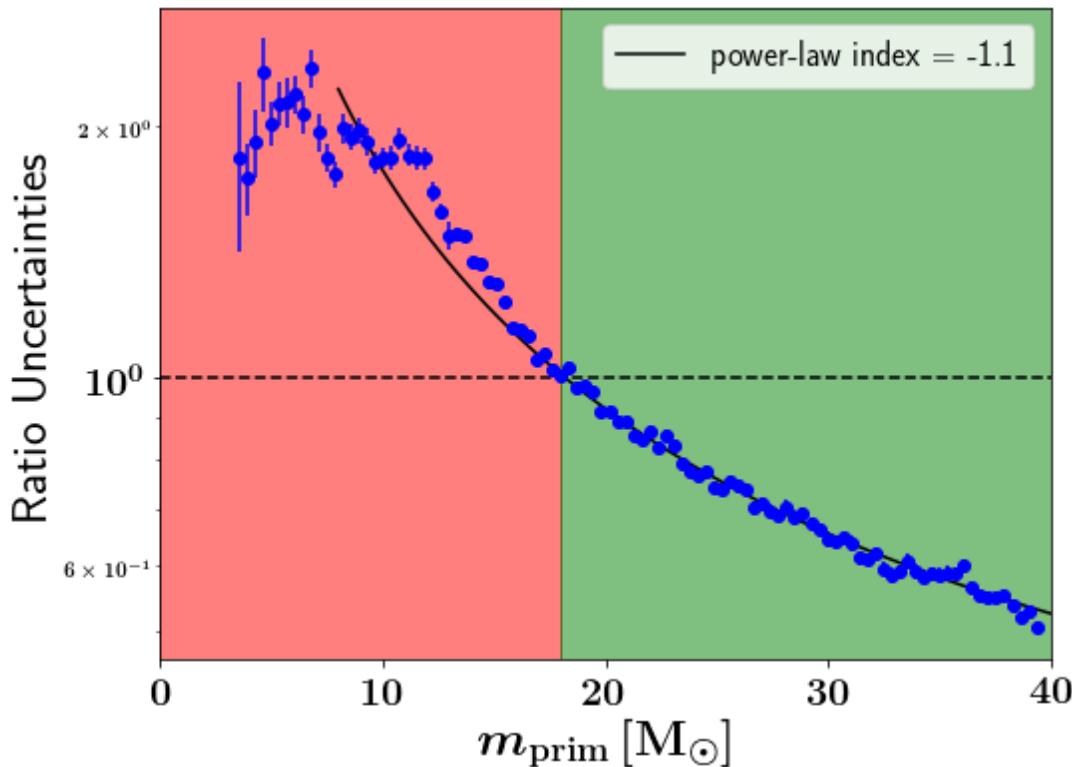


Lokesh Khandelwal

Floor Broekgaarden

Observational selection biases?

Kummer et al. in prep.



We want to improve
resolution for the
massive BH-BHs

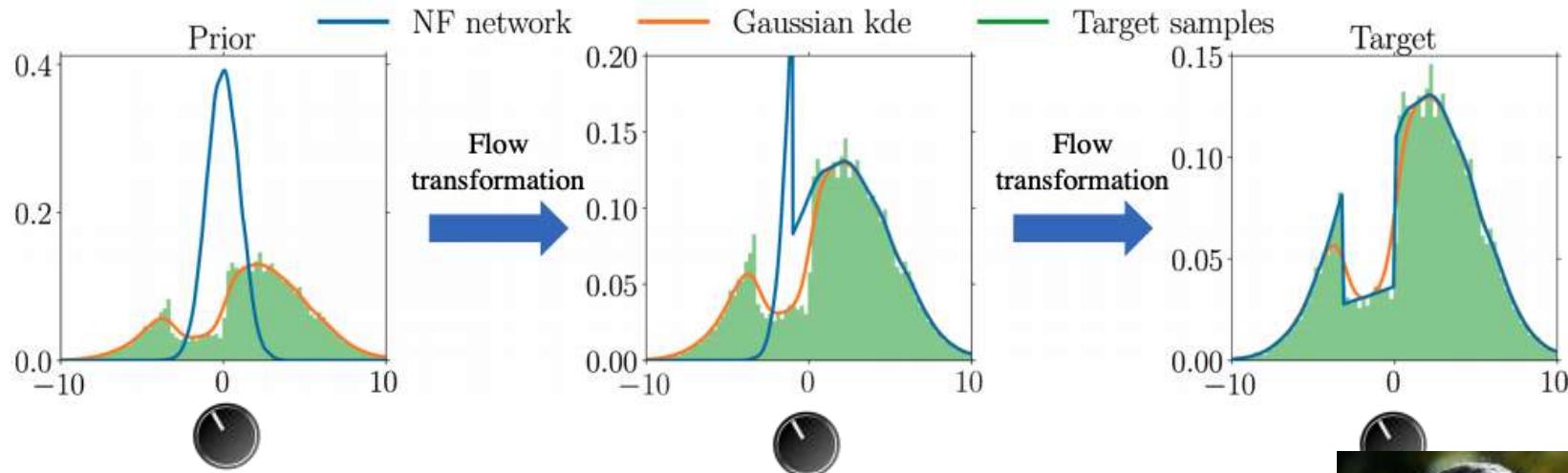


Floris Kummer



Hierarchical Bayesian modeling with normalizing flows

ArXiv: 2002.09491
and 1909.06373



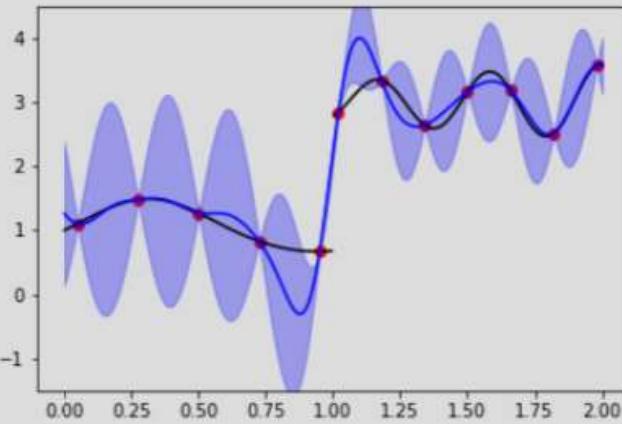
Kaze Wong
finishing PhD



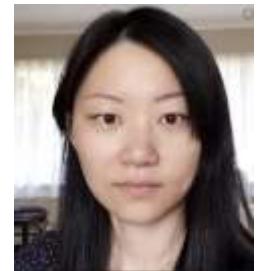
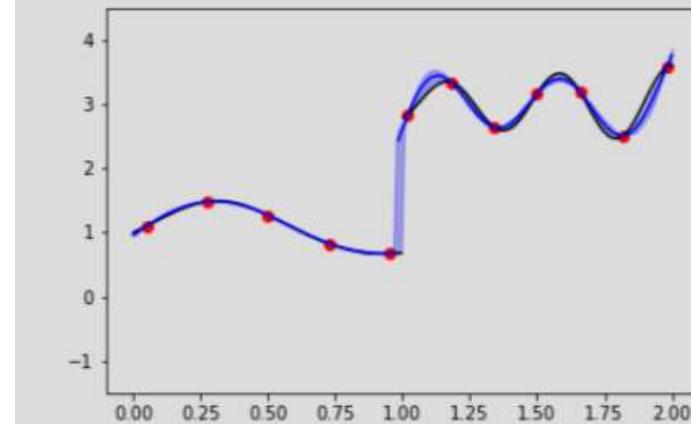
See also Kaze et al. recent work on using this for GWTC-2 in:
ArXiv: 2011.03564 and 2011.01865

Future work: Gaussian process classifiers & emulation + local response surface model

Traditional Gaussian Process Regression



Luyao Lin (in prep.)



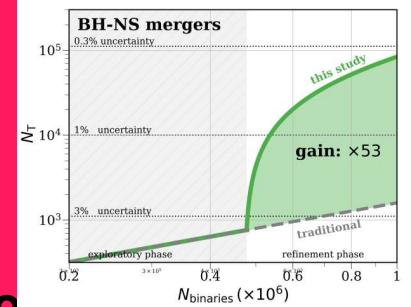
Luyao Lin



Derek Bingham

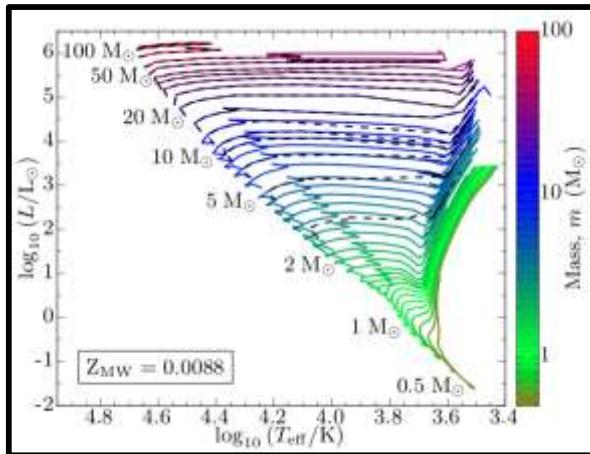
I Conclusions

- STROOPWAFEL is an sampling algorithm that obtains $\times 30 - 200$ more hits or equivalently: speed up

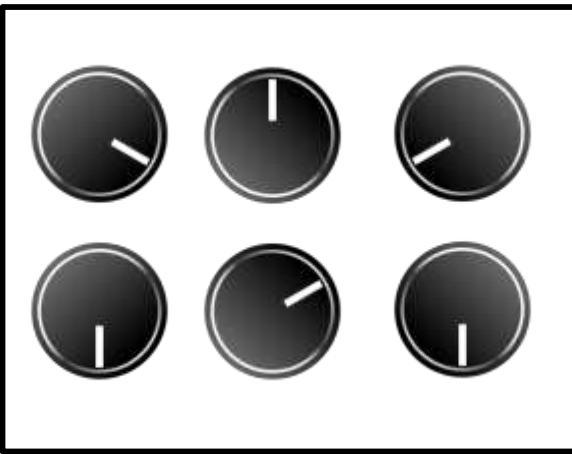


Broekgaarden et al., 2019
Data, code and all scripts available
on Zenodo/github

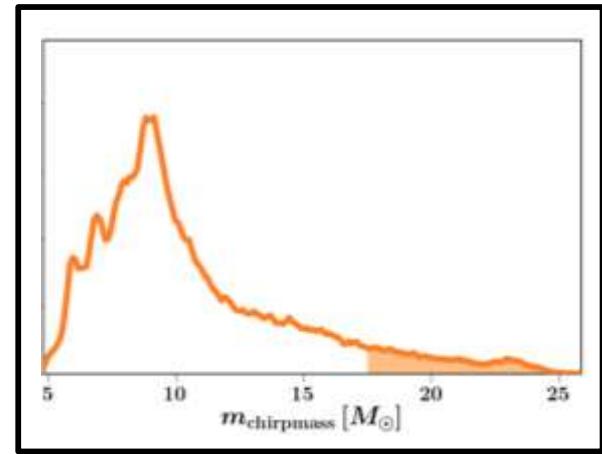
STROOPWAFEL helps next generation simulations:



Include detailed prescriptions



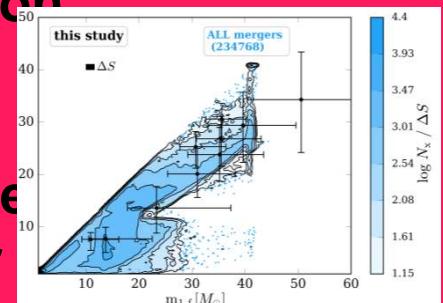
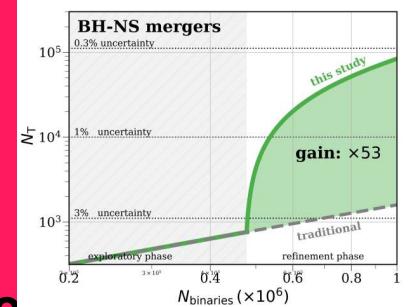
Perform larger
parameter study



Explore tails of
distributions

I Conclusions

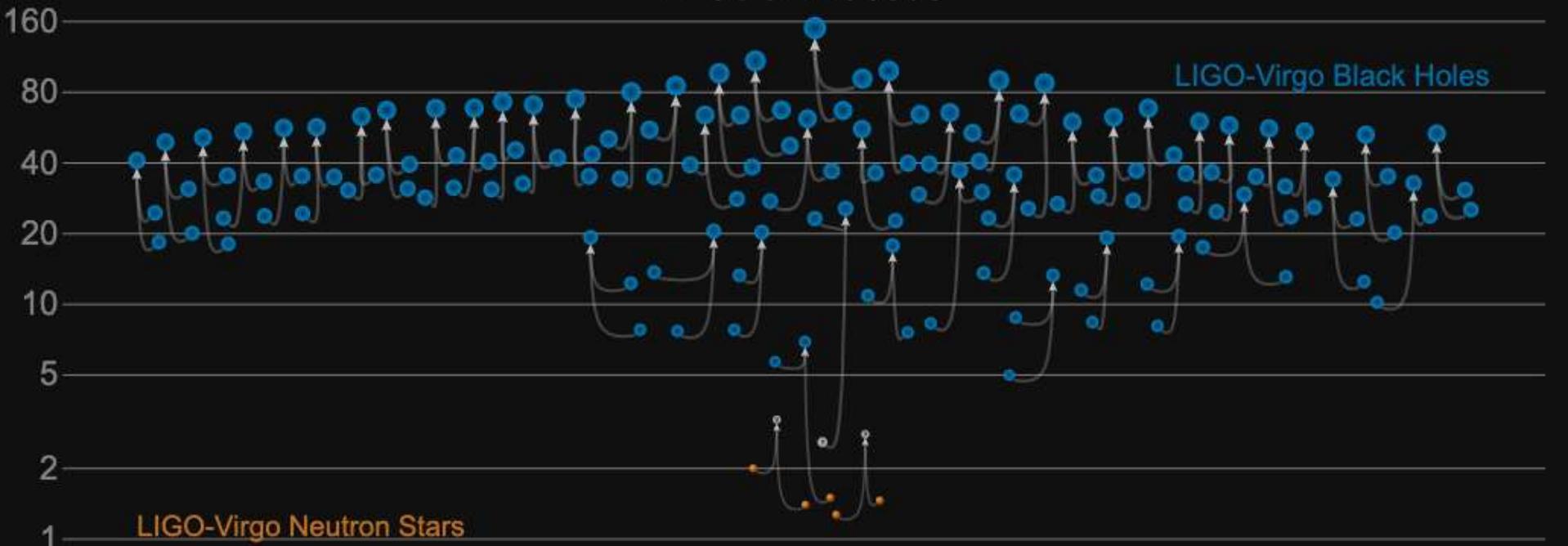
- STROOPWAFEL is an sampling algorithm that obtains $\times 30 - 200$ more hits or equivalently: speed up
- STROOPWAFEL improves resolution of population synthesis studies of GW events
- Future improvements are being actively developed + STROOPWAFEL is being implemented in other codes



Broekgaarden et al., 2019
Data, code and all scripts available
on Zenodo/github

Masses in the Stellar Graveyard

in Solar Masses



GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

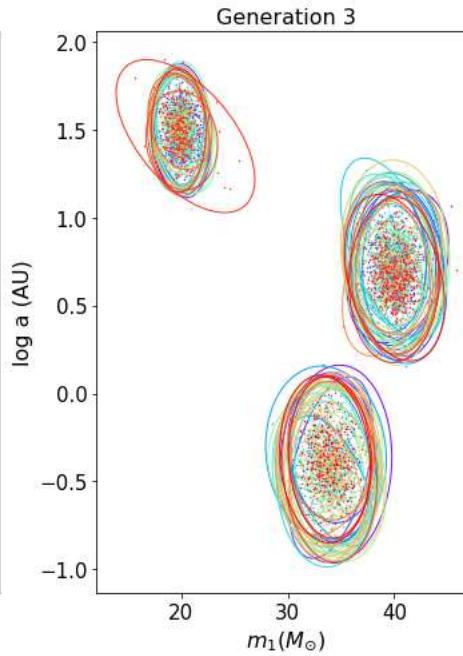
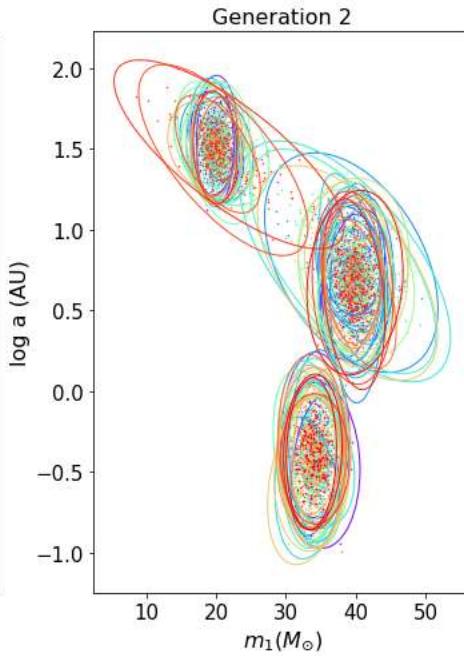
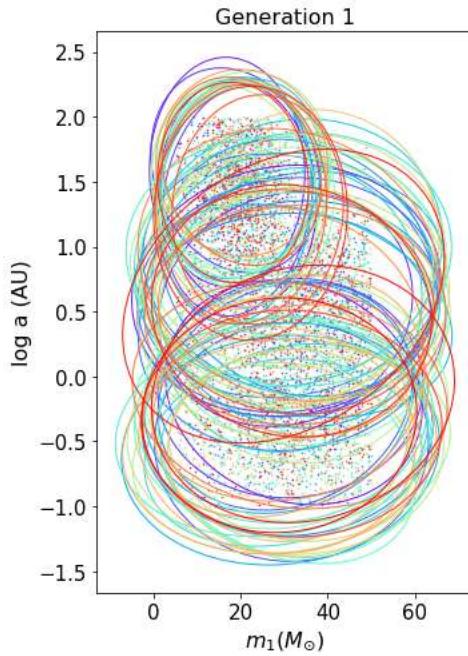


Thank you!



Back-up slides

Future Prospects: more dimensions

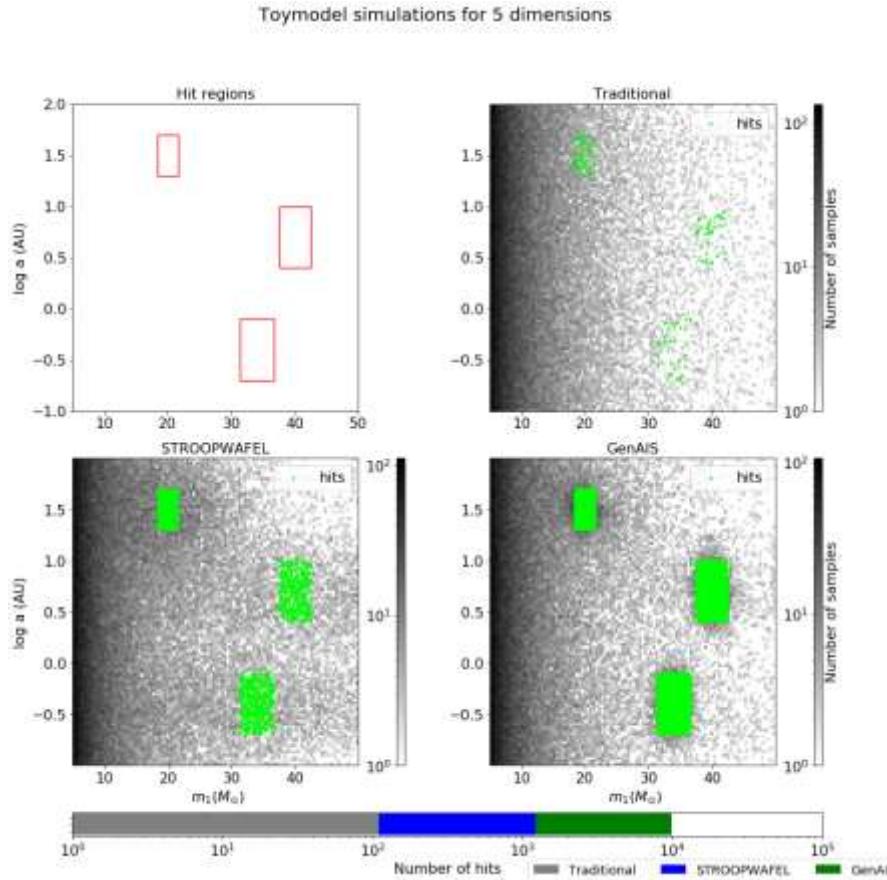


Floris Kummer



Lokesh Khandelwal

Future Prospects: more dimensions



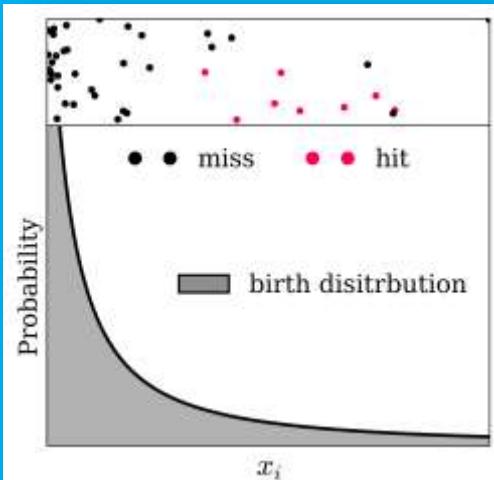
Floris Kummer



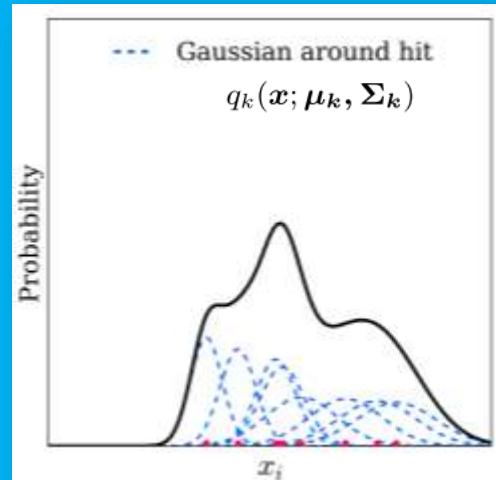
Lokesh Khandelwal

Floor Broekgaarden

1) Exploring phase

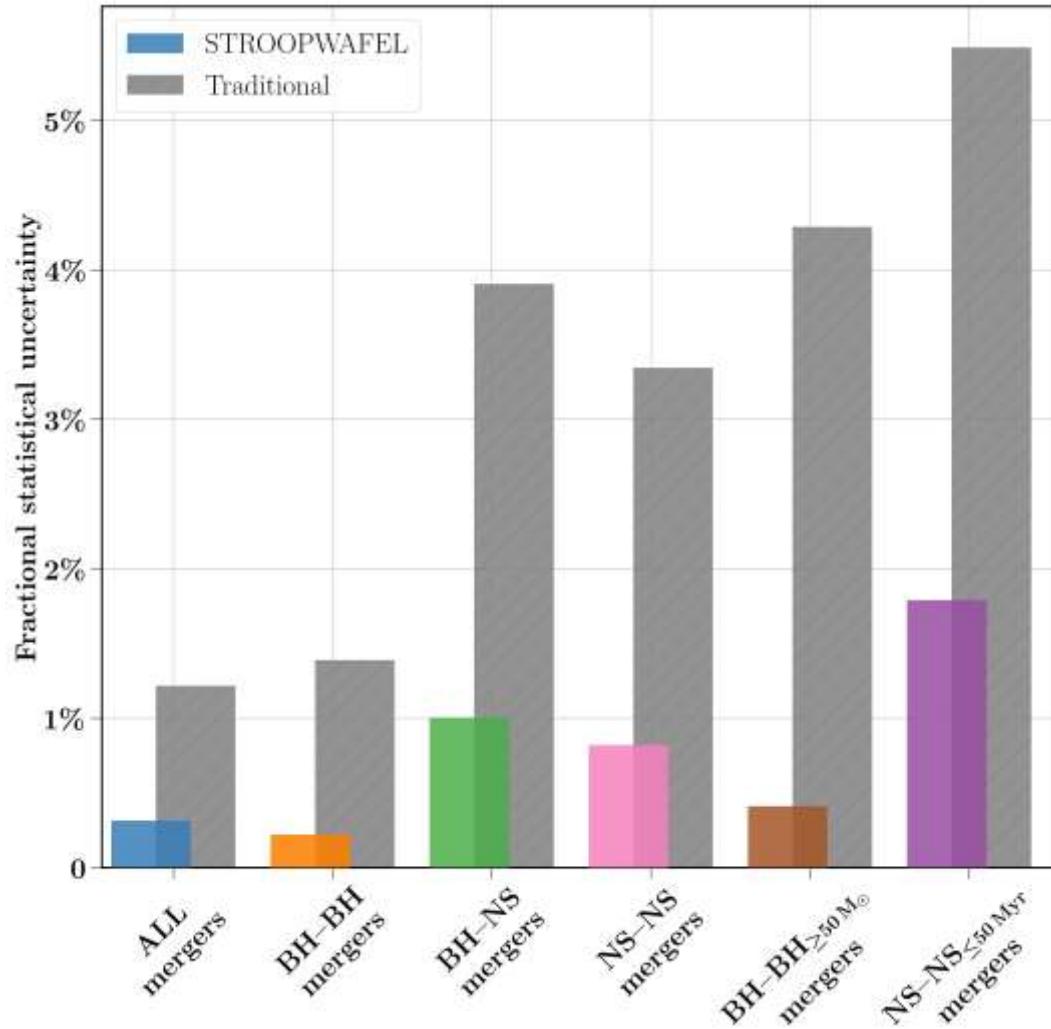


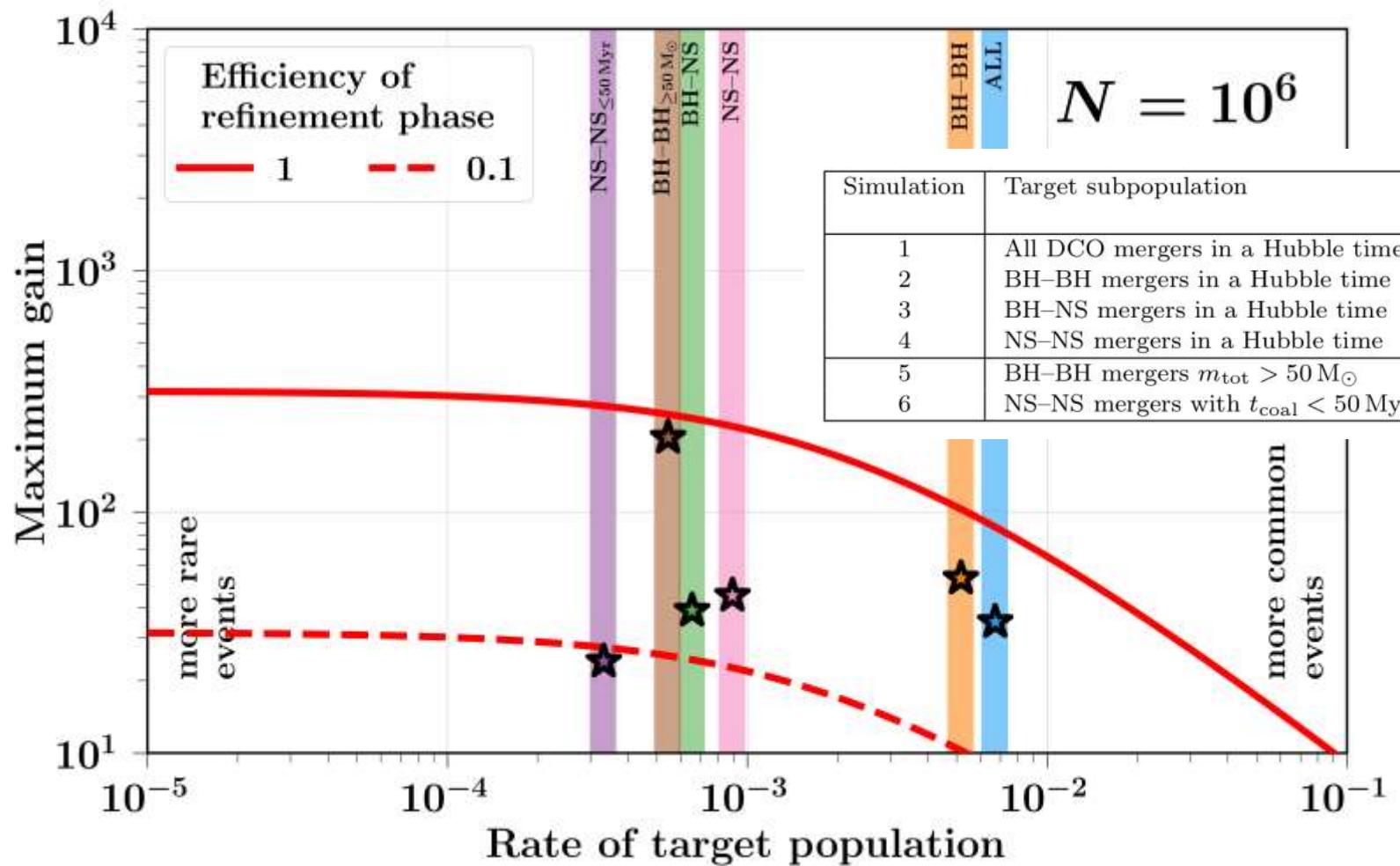
2) Create adapted distribution

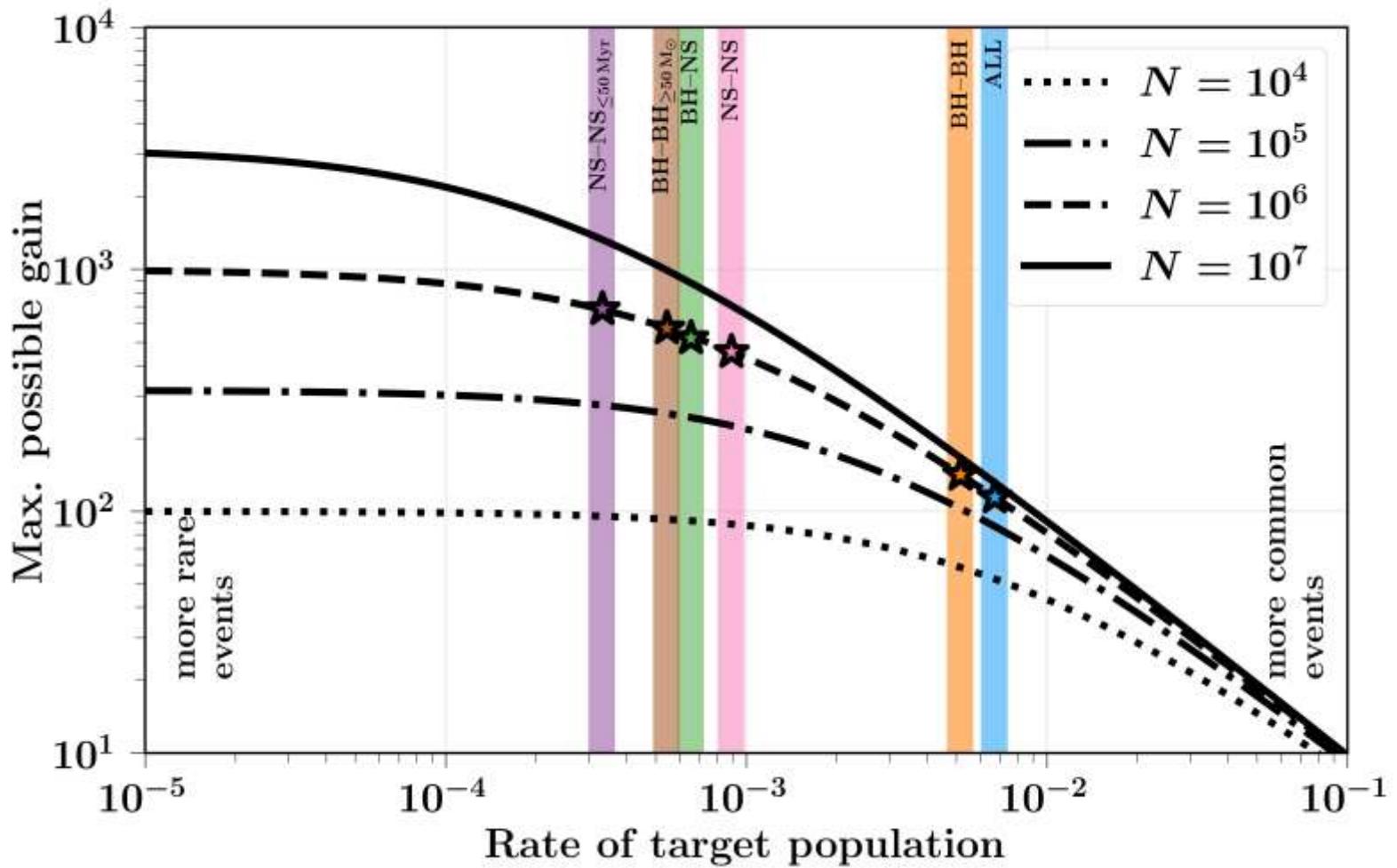


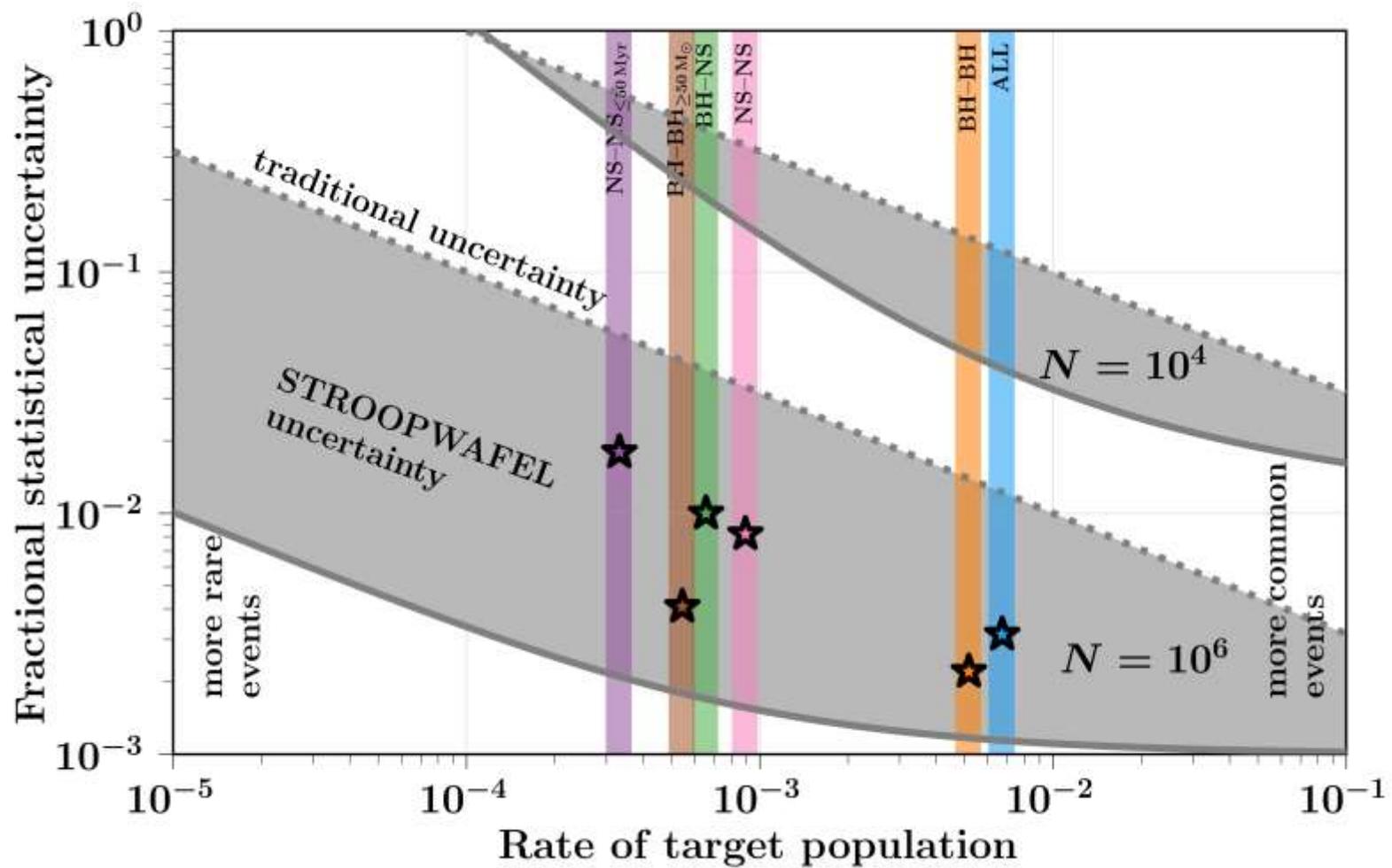
$$\Sigma_k = \begin{bmatrix} \sigma_{1,k}^2 & 0 & \dots \\ 0 & \ddots & \\ \vdots & & \sigma_{d,k}^2 \end{bmatrix},$$

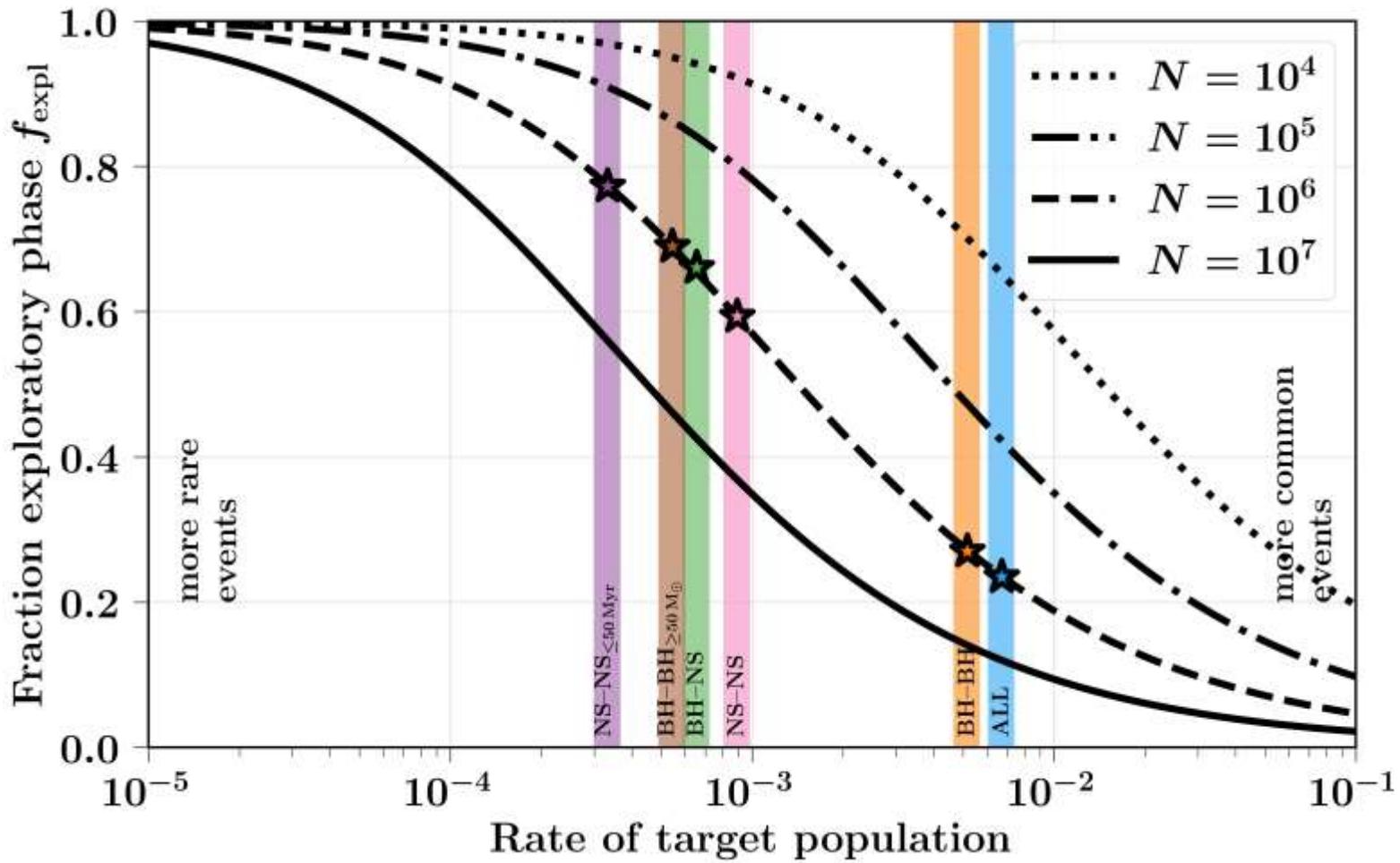
$$\sigma_{j,k} = \kappa \frac{1}{\pi_j(x_k) N_{\text{expl}}^{1/d}},$$



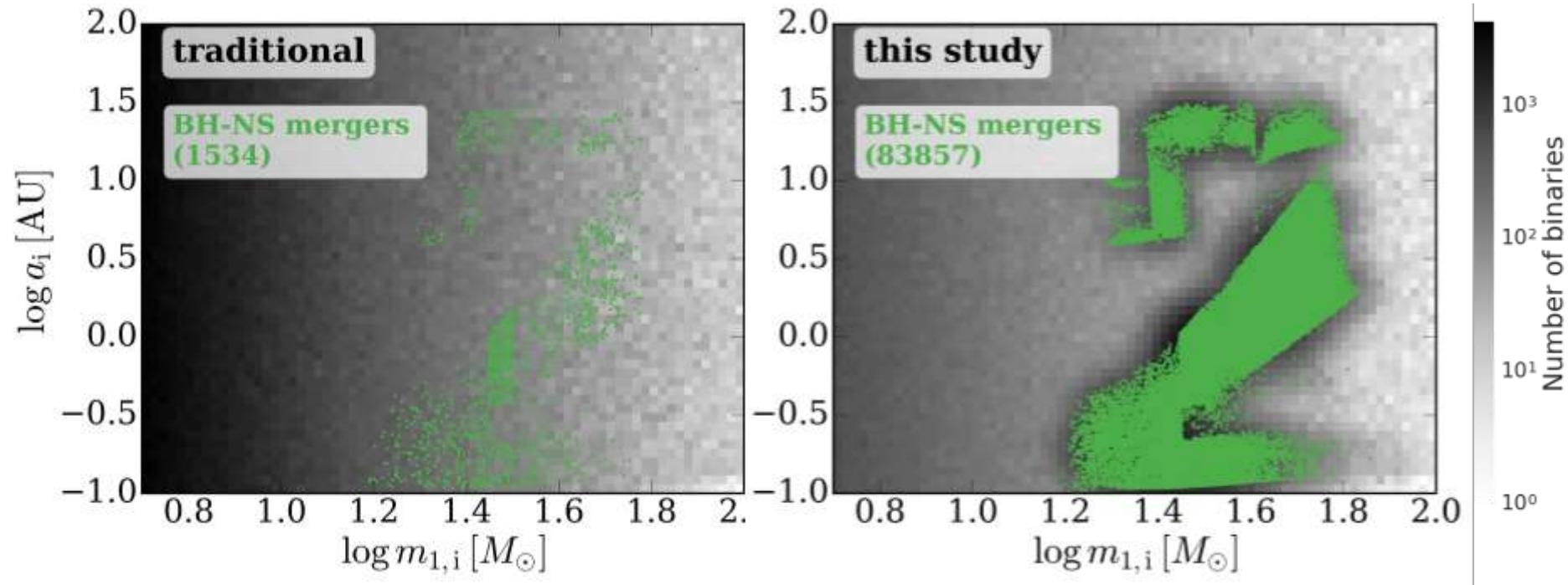




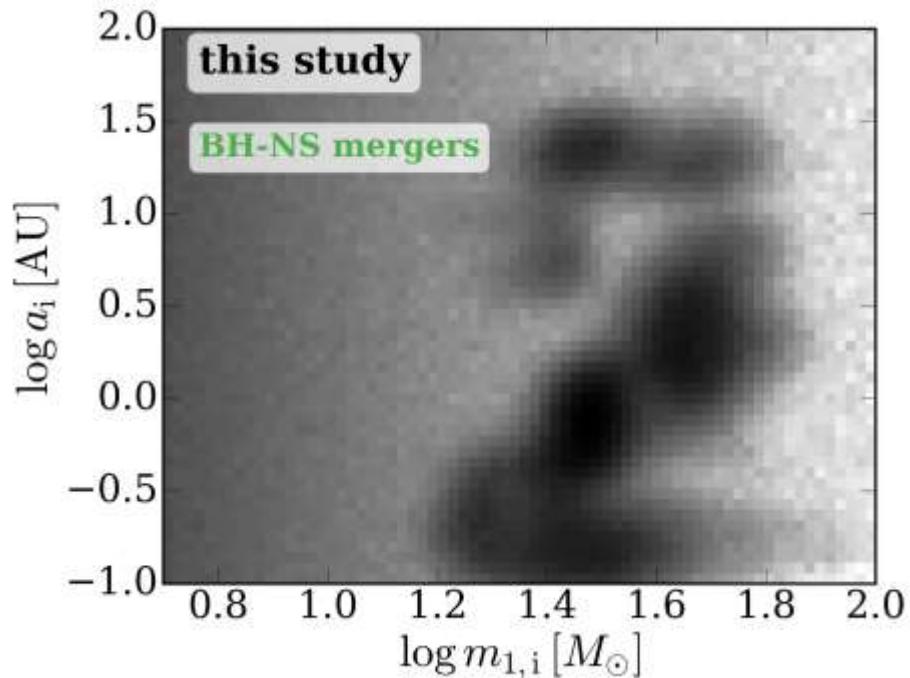
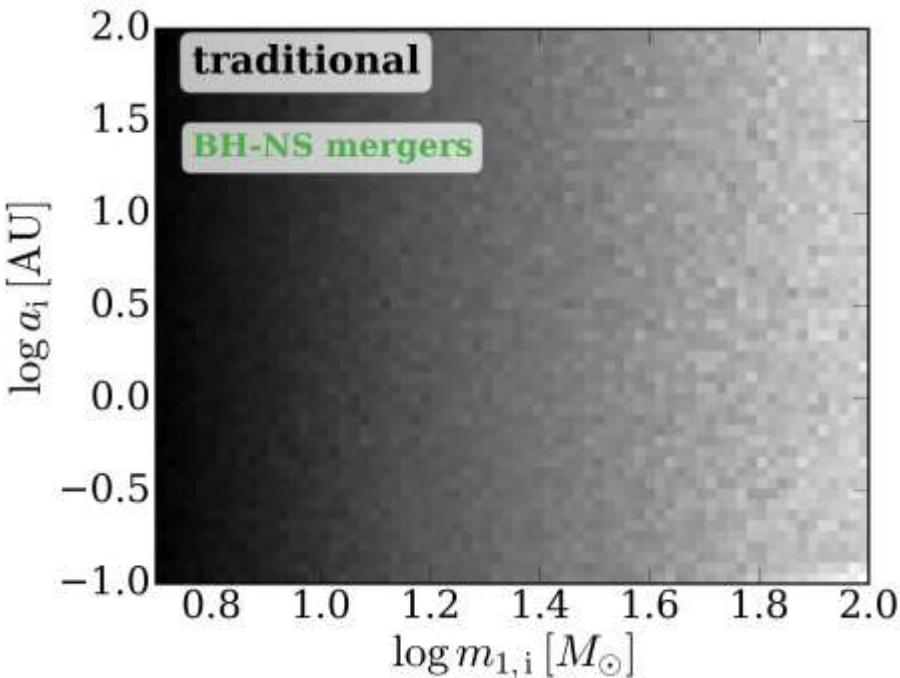


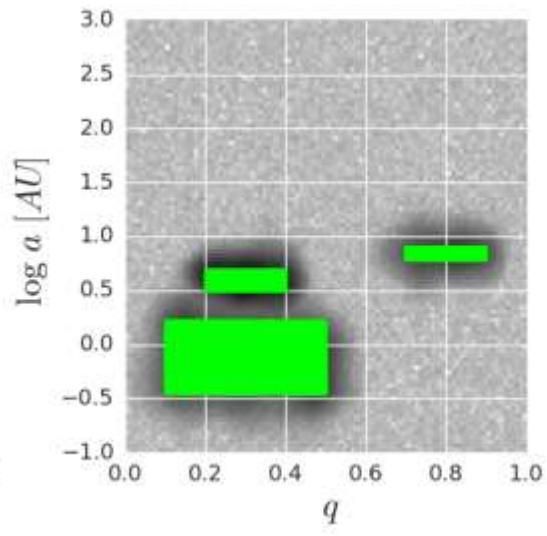
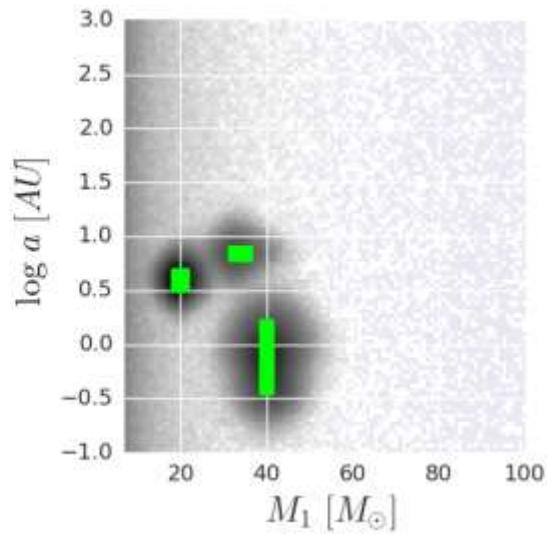
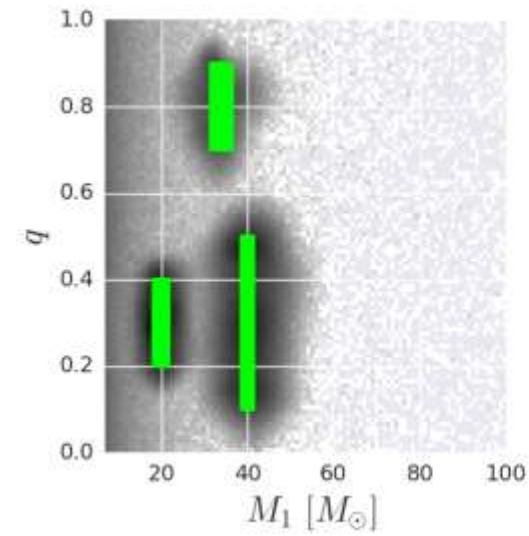


Better resolution progenitors

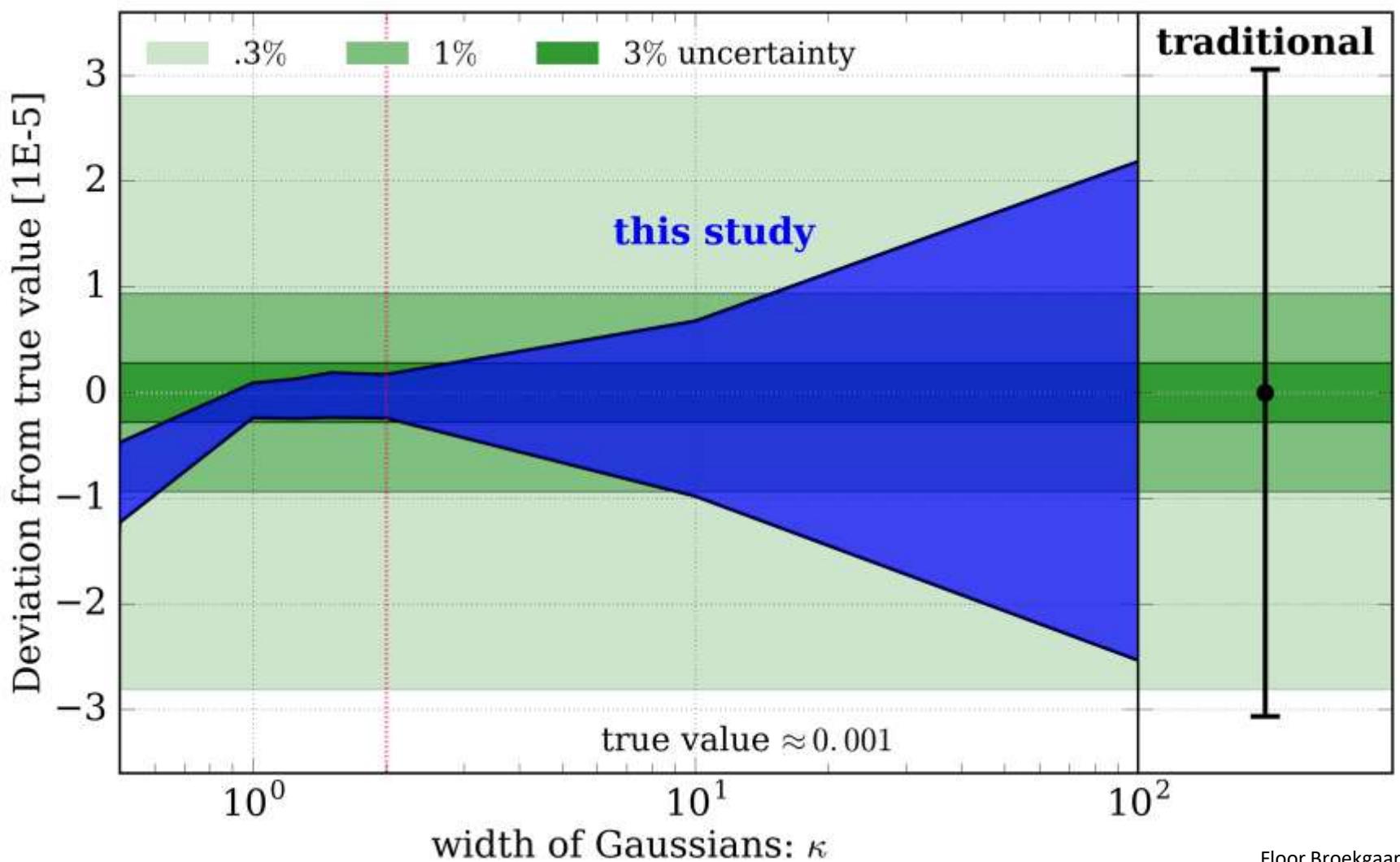


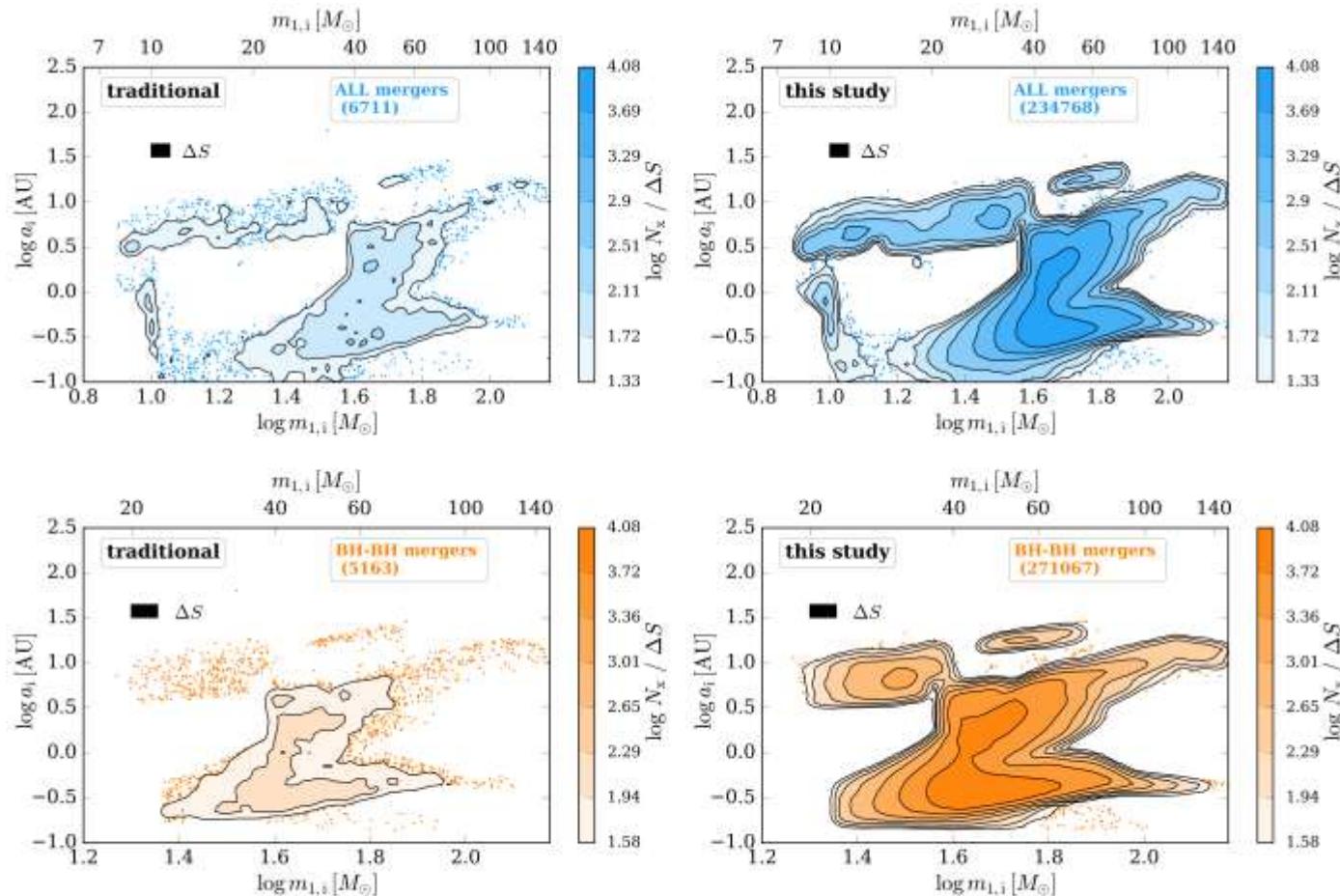
Refinement phase

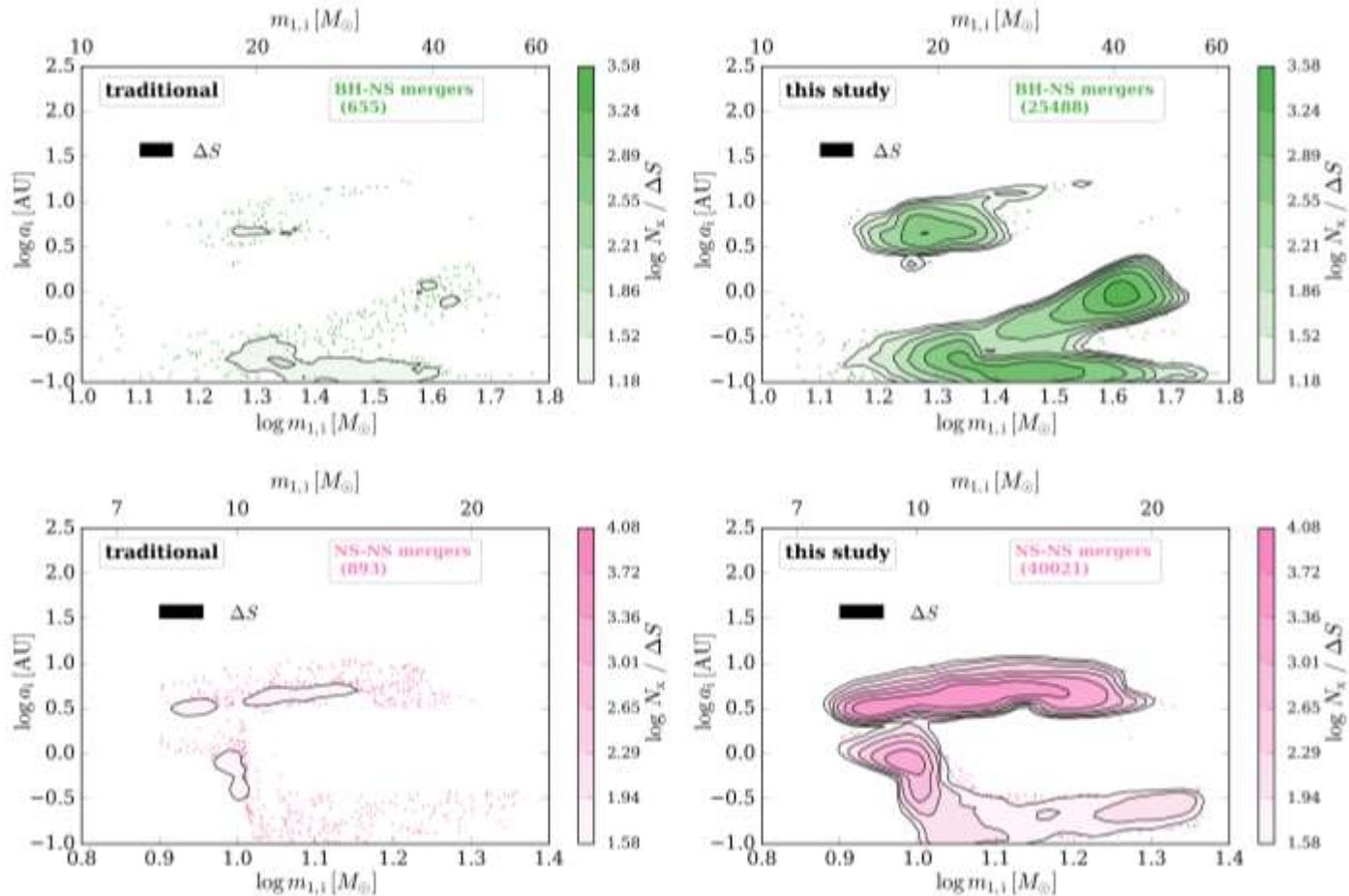


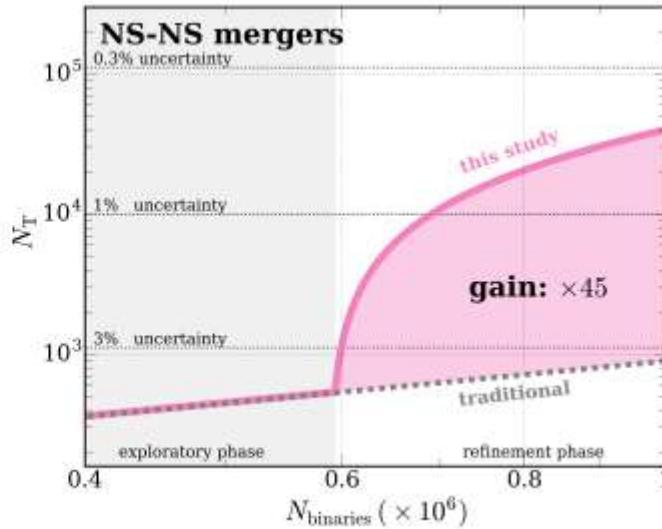
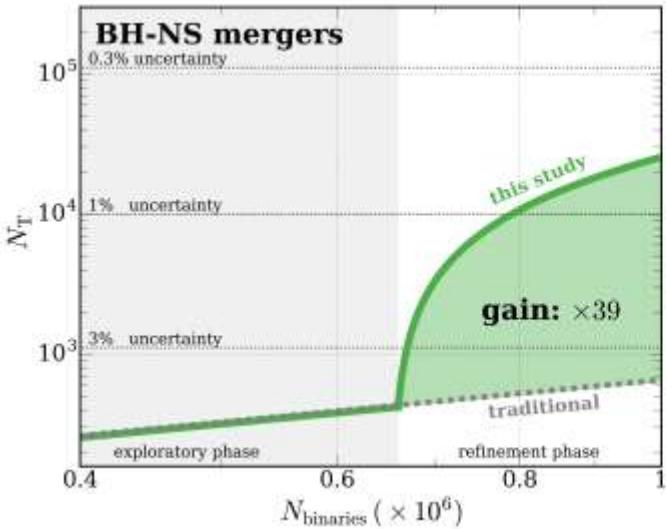
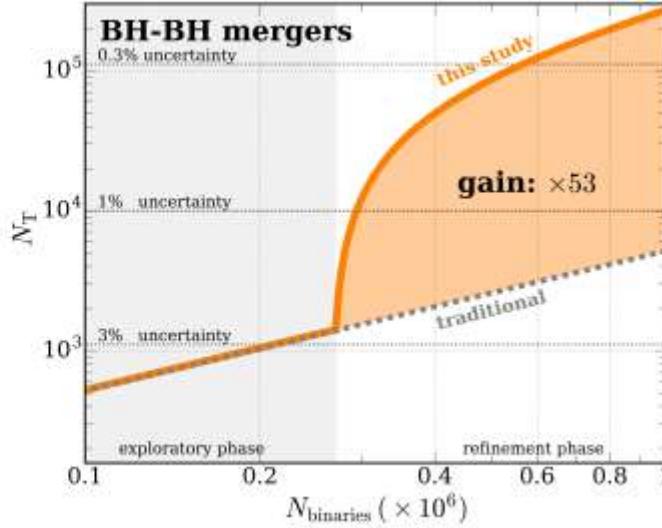
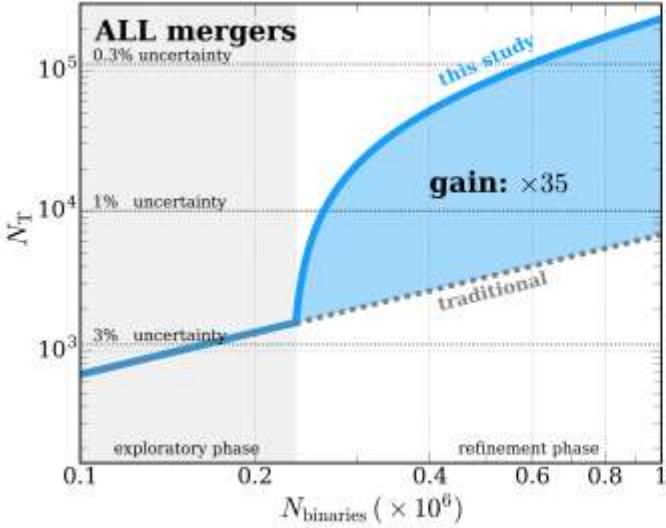


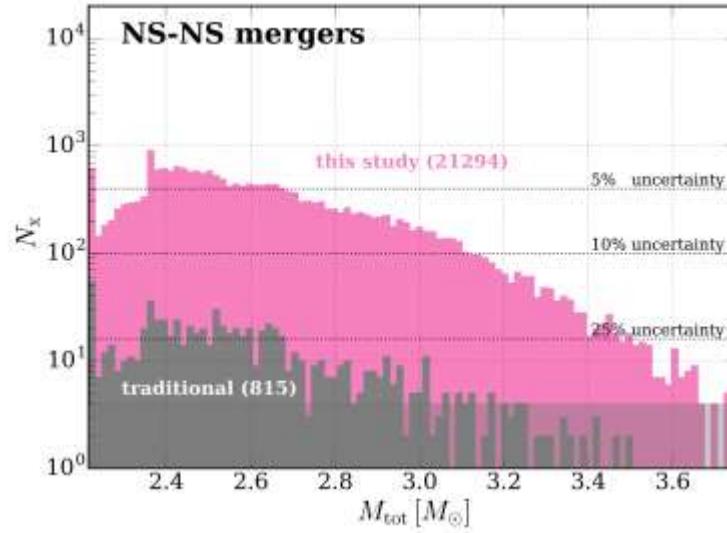
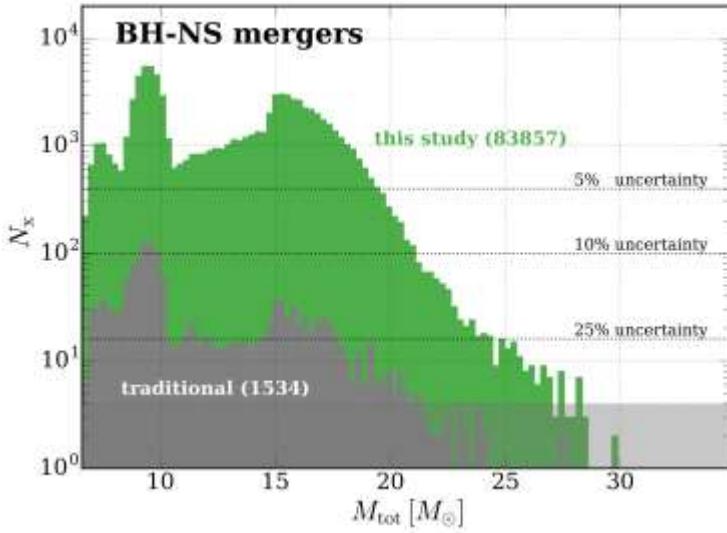
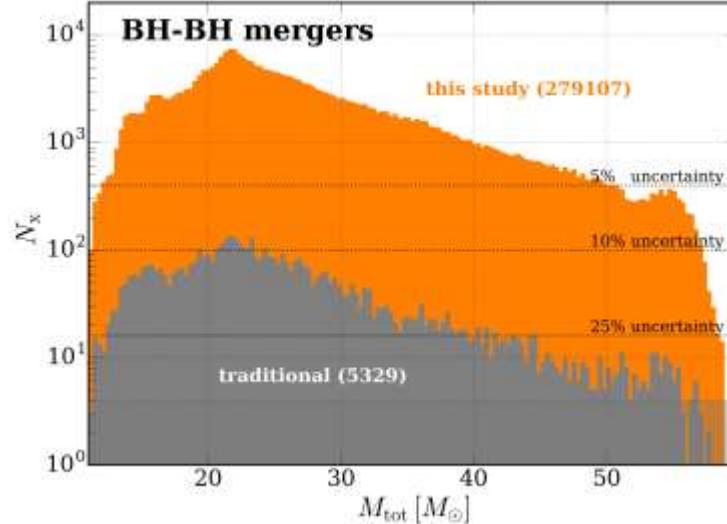
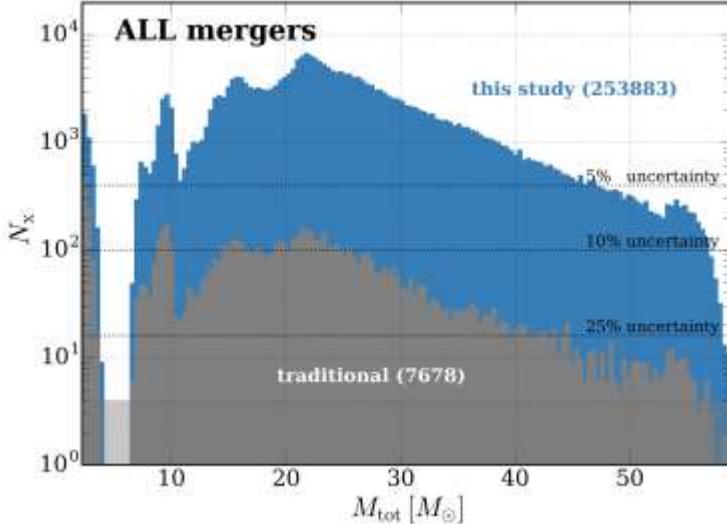
Number of binaries
 10^0
 10^1
 10^2
 10^3

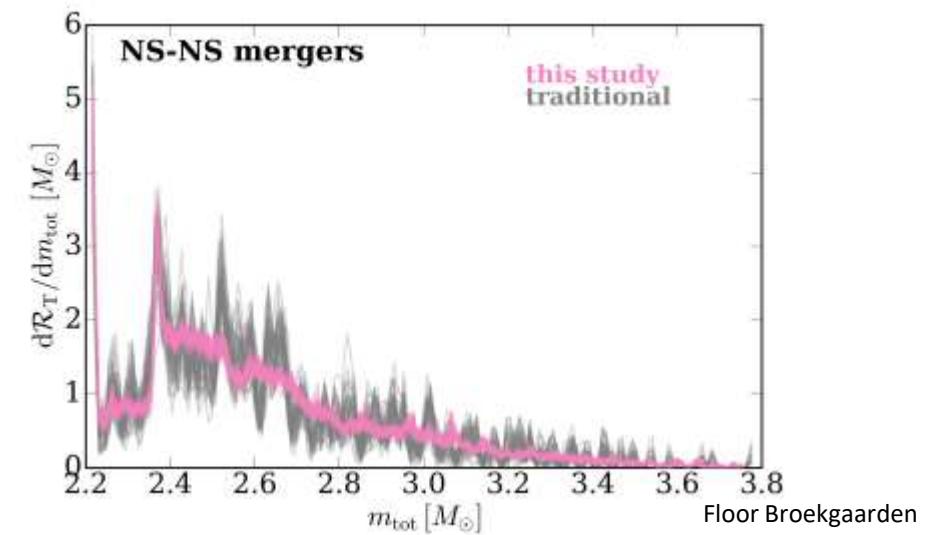
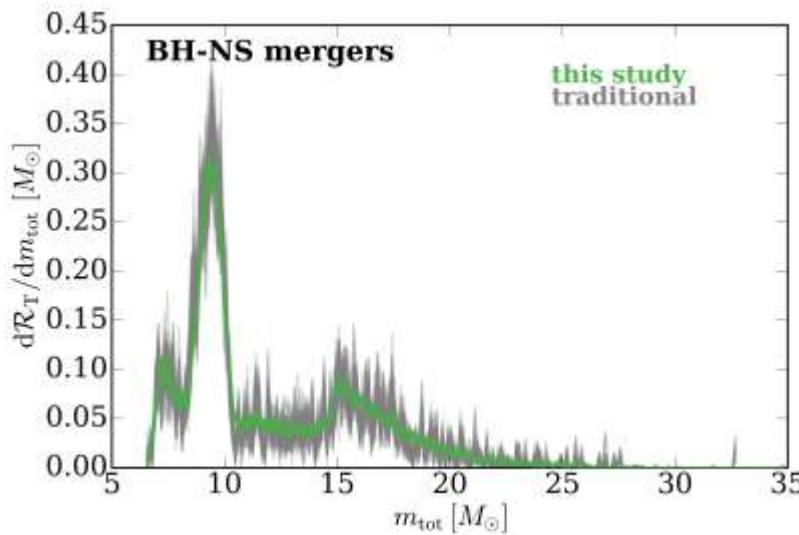
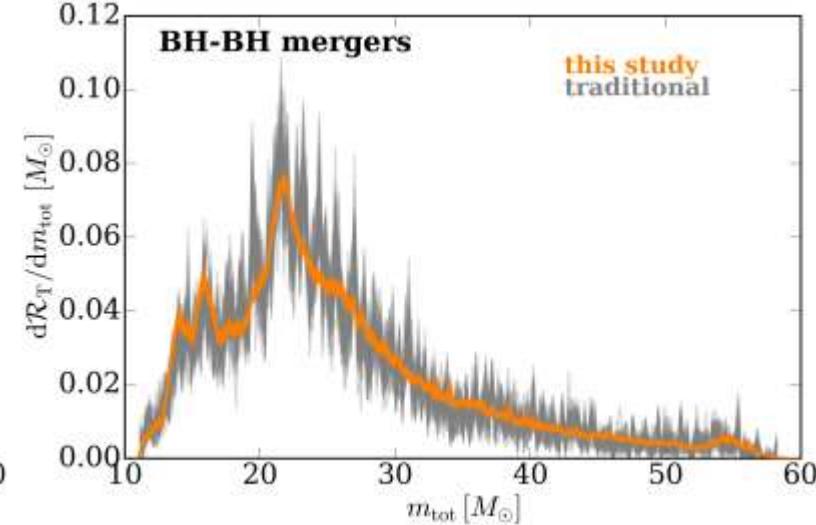
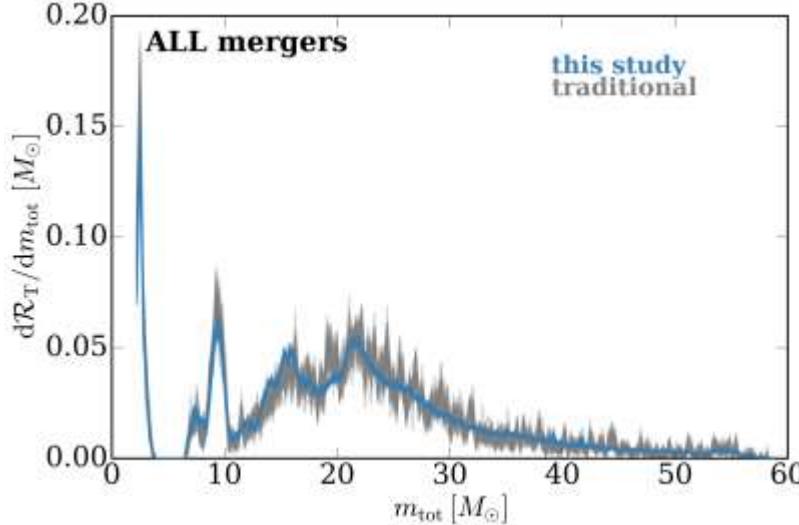






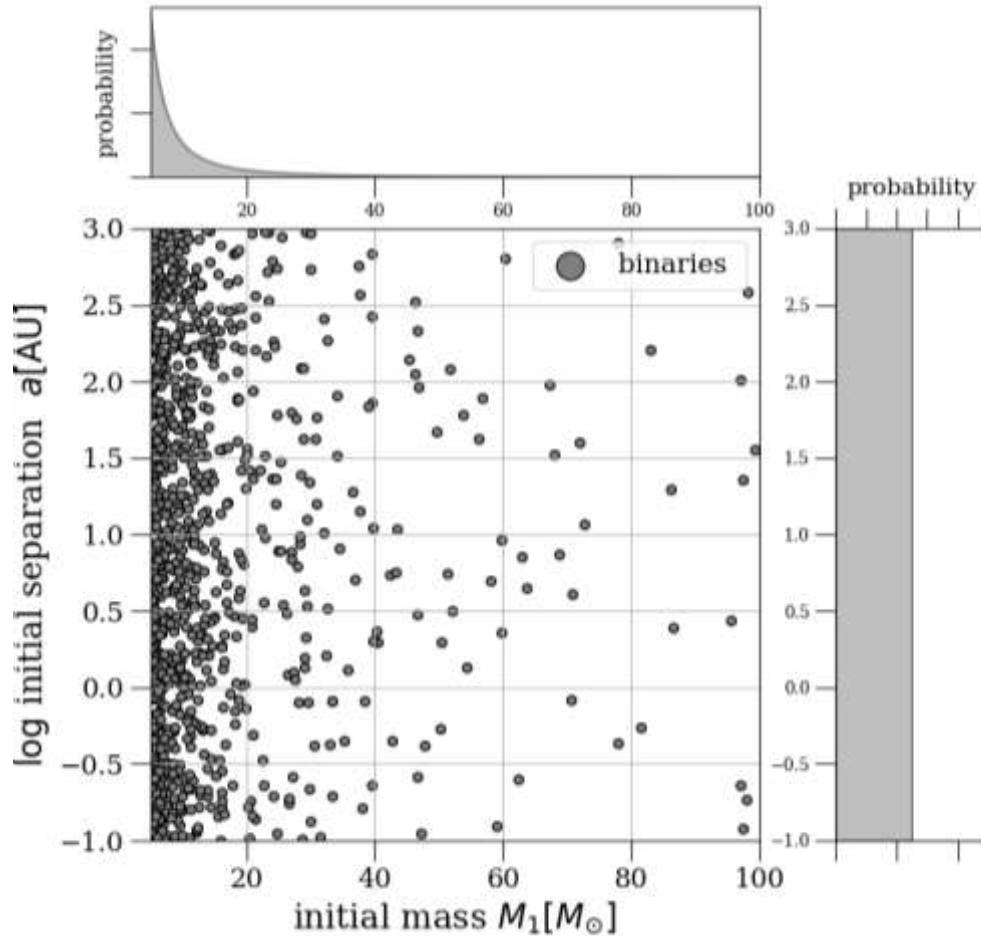




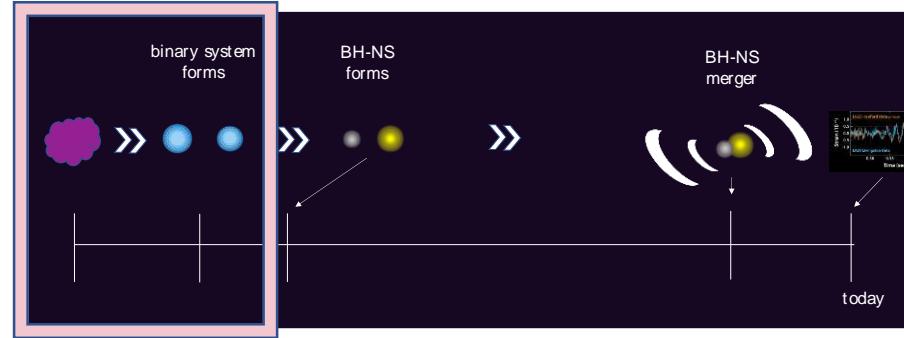
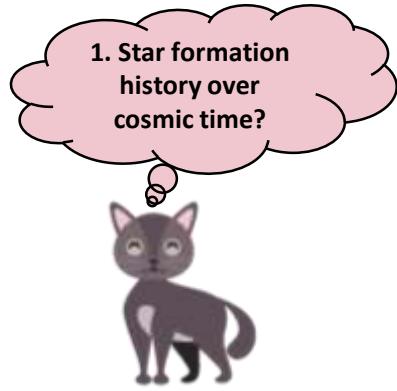


1

Populations:

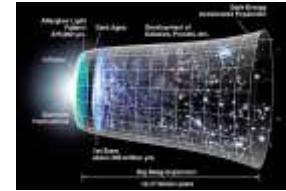


Two processes impact the predicted gravitational wave observations:



Describes “model for the Universe”:

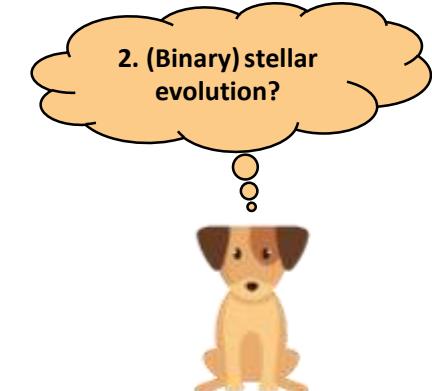
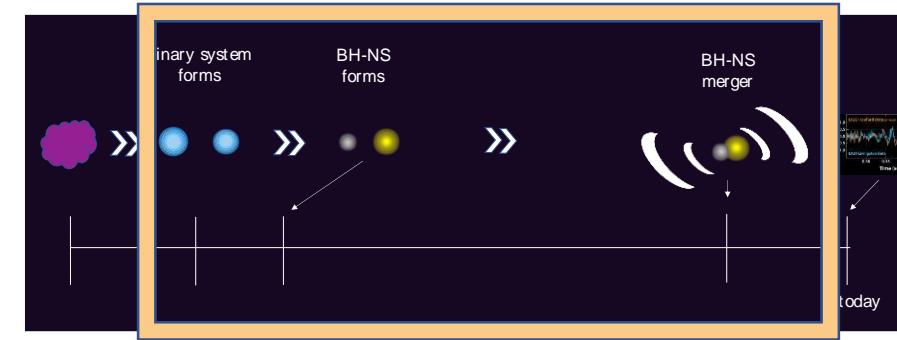
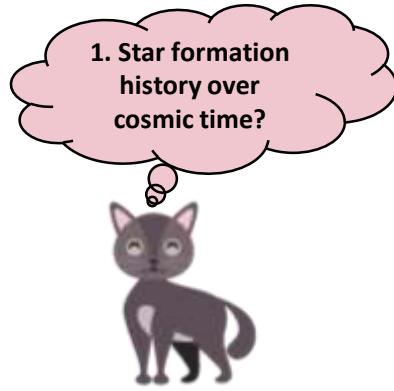
$$\text{MSSFR}(z_{\text{form}}) = \underbrace{\frac{d^2 M_{\text{SFR}}}{dt_{\text{s}} dV_c}}_{\text{Star formation rate SFR(z)}} \times \underbrace{\frac{dM_{\text{SFR}}}{dZ}}_{\text{Chemical evolution Z(z)}}$$



Star formation rate
SFR(z)

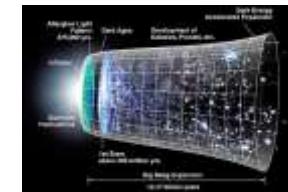
Chemical evolution **Z(z)**

Two processes impact the predicted gravitational wave observations:

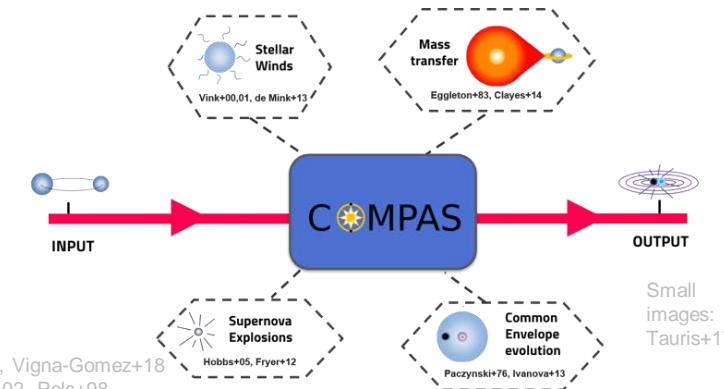


Describes “model for the Universe”:

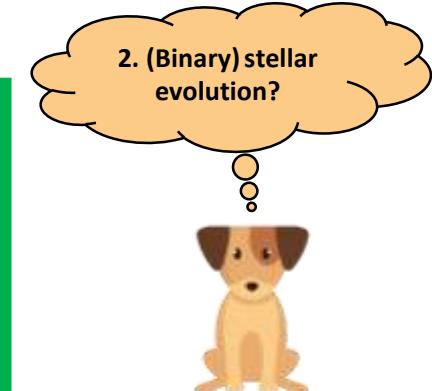
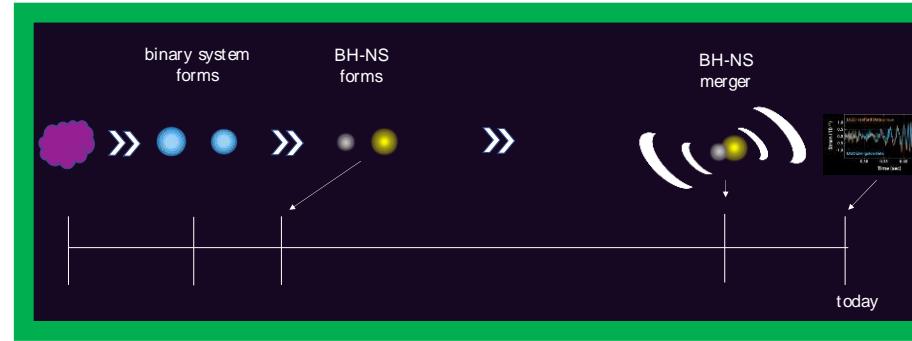
$$\text{MSSFR}(z_{\text{form}}) = \underbrace{\frac{d^2 M_{\text{SFR}}}{dt_{\text{s}} dV_c}}_{\text{Star formation rate SFR}(z)} \times \underbrace{\frac{dM_{\text{SFR}}}{dZ}}_{\text{Chemical evolution } Z(z)}$$



Describes “isolated binary evolution”:

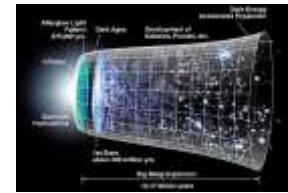


We need to understand the combined effect!



Describes “model for the Universe”:

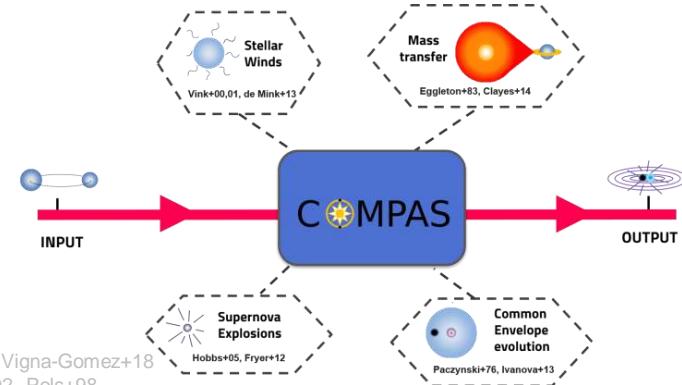
$$\text{MSSFR}(z_{\text{form}}) = \underbrace{\frac{d^2 M_{\text{SFR}}}{dt_{\text{s}} dV_c}}_{\text{Star formation rate SFR}(z)} \times \underbrace{\frac{dM_{\text{SFR}}}{dZ}}_{\text{Chemical evolution } Z(z)}$$



Star formation rate
SFR(z)

Chemical evolution $Z(z)$

Describes “isolated binary evolution”:



We need to understand the combined effect!

1. Star formation history over cosmic time?



2. (Binary) stellar evolution?



Describes

MSSFR(z_{fb})

Requires simulating many metallicities...



New algorithm that speeds up simulations

100x



Broekgaarden+19

And billions of binaries...
many TBs of data &
 $>20\,000\,000$ CPU hours...

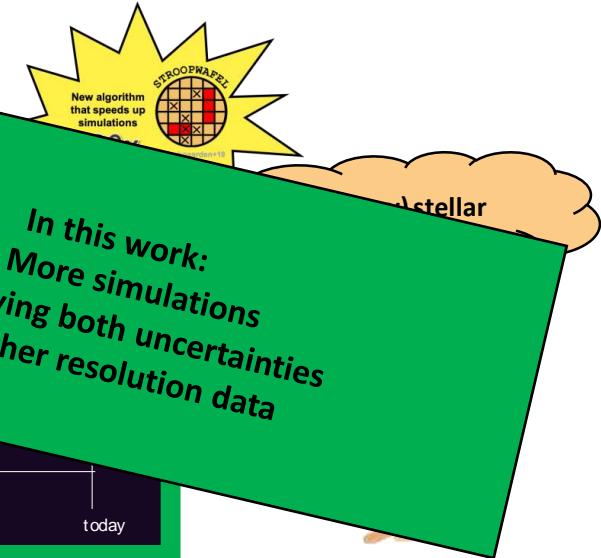


Star formation
surveys



OUTPUT

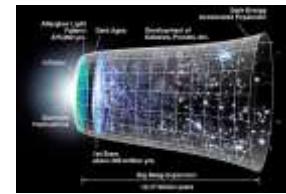
STROOPWAFEL allows to study the impact from black holes



28 models:

based on Neijssel+19

$$\text{MSSFR}(z_{\text{form}}) = \underbrace{\frac{d^2 M_{\text{SFR}}}{dt_{\text{s}} dV_c}(z_{\text{form}})}_{\text{Star formation rate SFR}(z)} \times \underbrace{\frac{dM_{\text{SFR}}}{dZ}(z_{\text{form}})}_{\text{Chemical evolution } Z(z)}$$



Star formation rate
SFR(z)

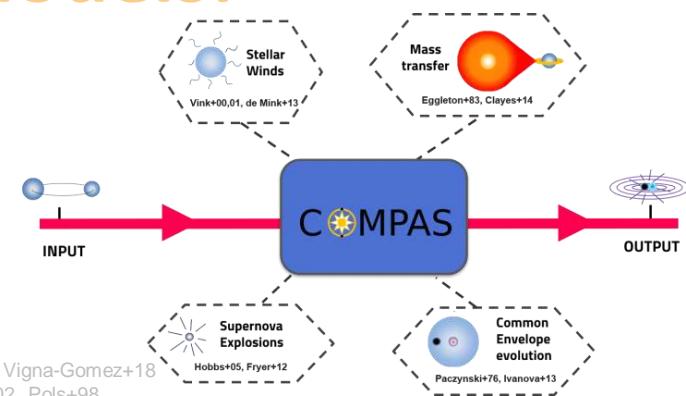
Madau & Dickinson 2014
Strolger+2004
Madau & Fragos 2017

Galaxy-Stellar-Mass-function:
Panter+2004,
Furlong (single/double Schechter)+2015

Mass-Z relation:
Langer & Norman 2006 (+offset)
Ma+2016

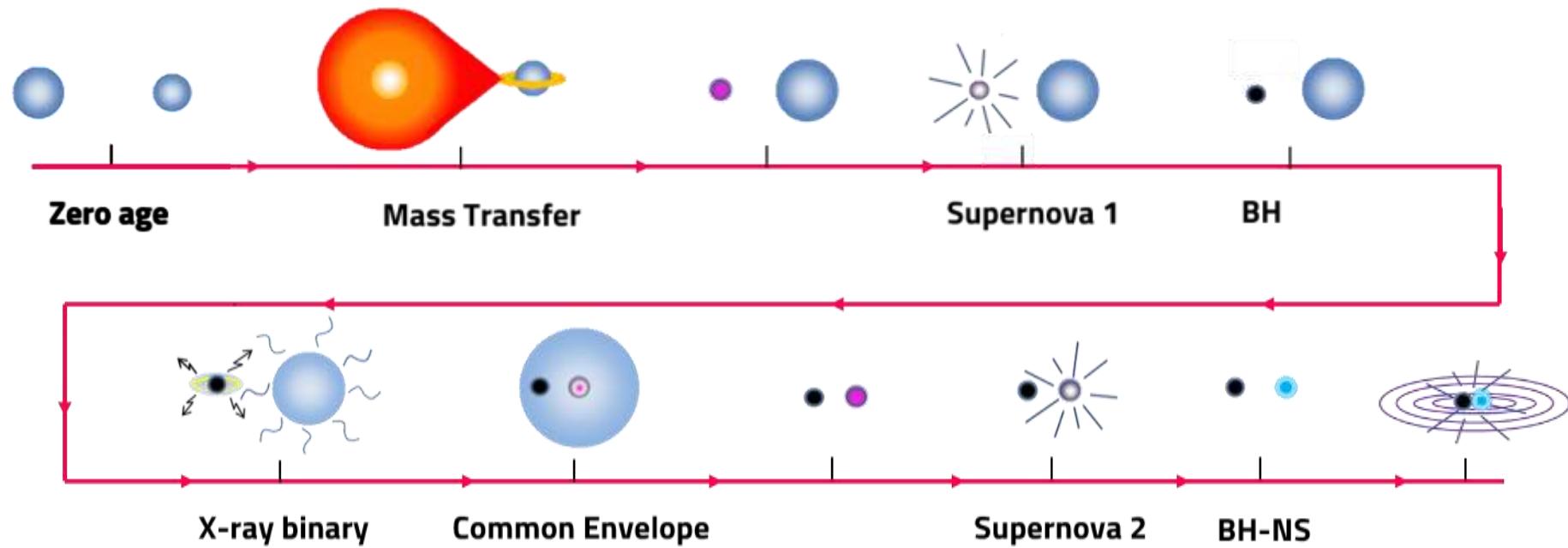
12 models:

Stevenson+15,17,19, Barrett+18, Vigna-Gomez+18
Based on tracks from Hurley+00,02, Pols+98

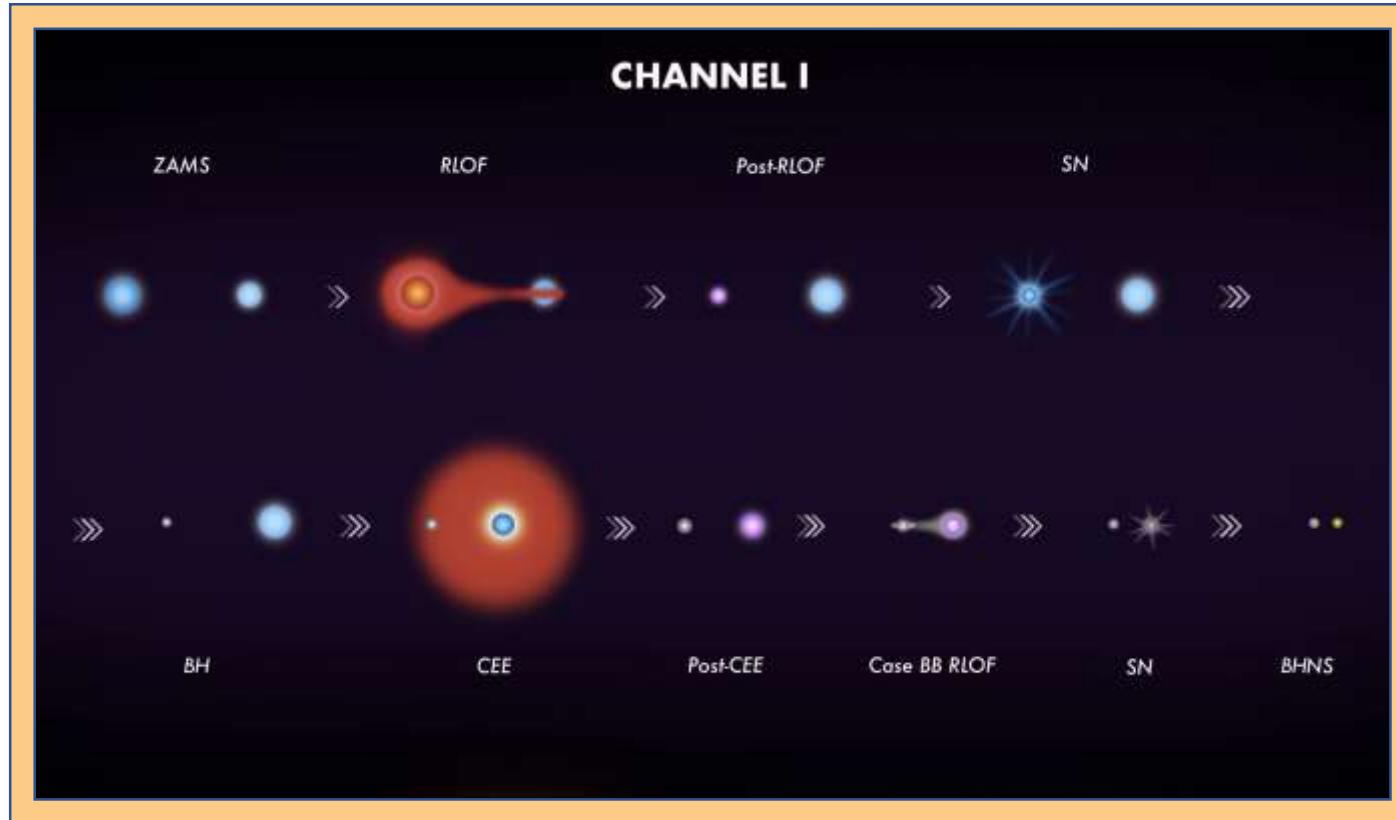


I Classic channel BH-NS merger:

e.g. Paczynski+76, Smarr & Blandford+76 | Figure based on Tauris+17



Modelling gravitational-wave sources



$z = 0$

How do they form?

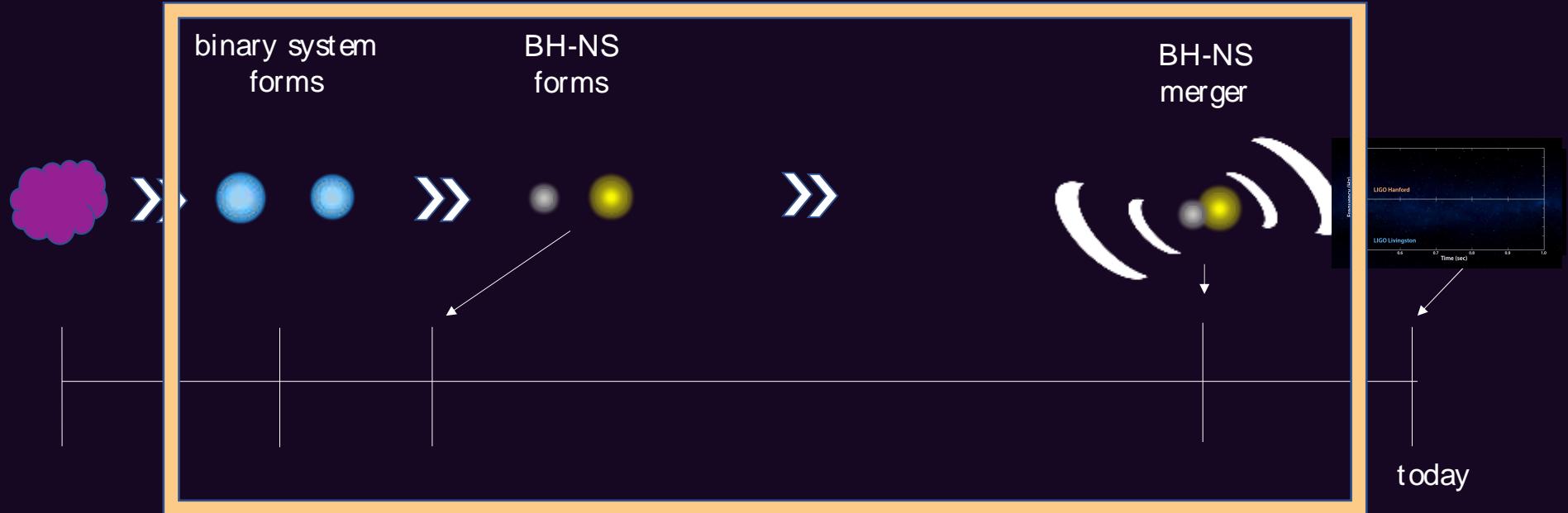


Image not to scale!

e.g. Paczynski+76, Smarr & Blandford+76

Floor Broekgaarden

Other puzzling methods



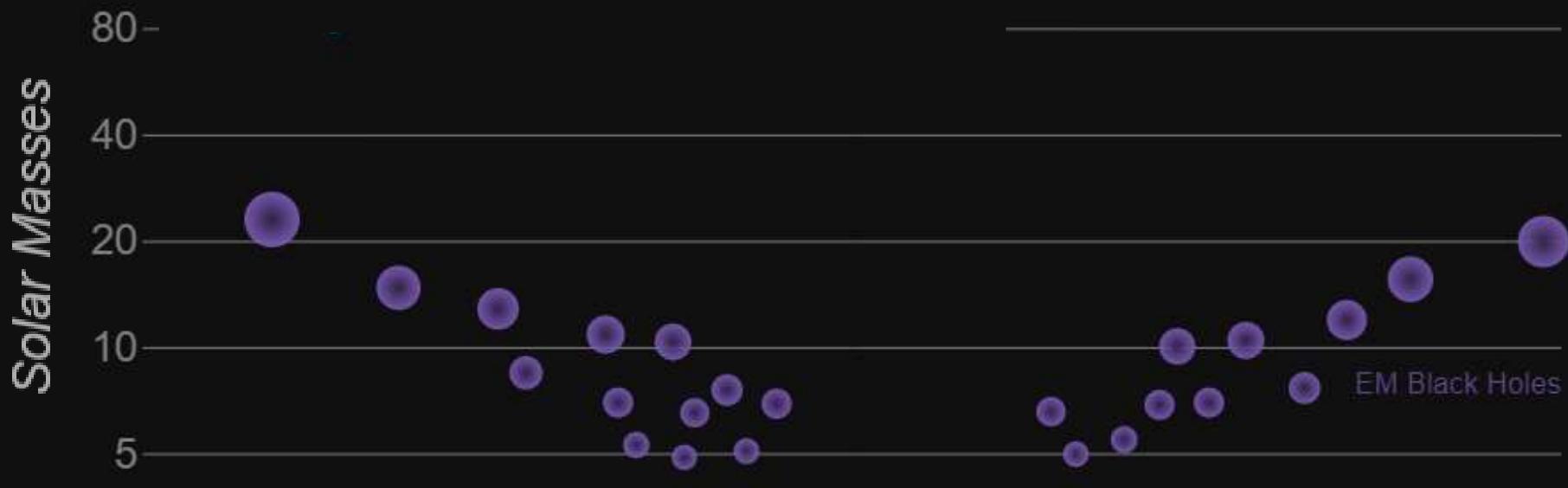
Q: How many stellar black holes are thought to exist in our Milky Way at this moment?

~100 000 000

~50 000 000

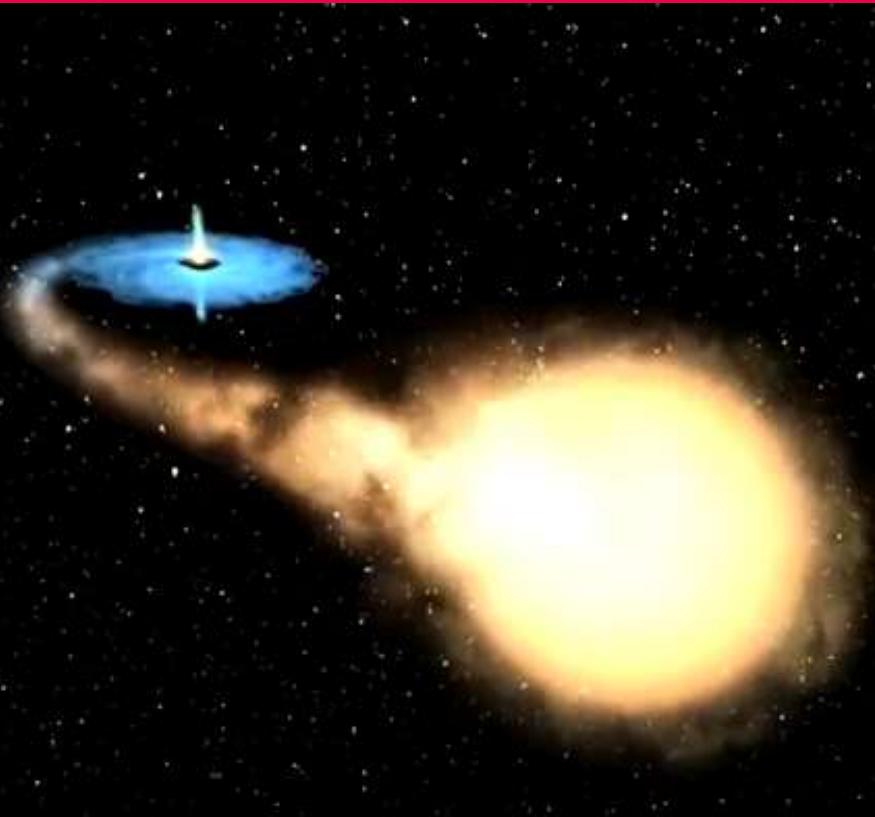


| Up to 2015:
~20 stellar BHs detected with
known masses

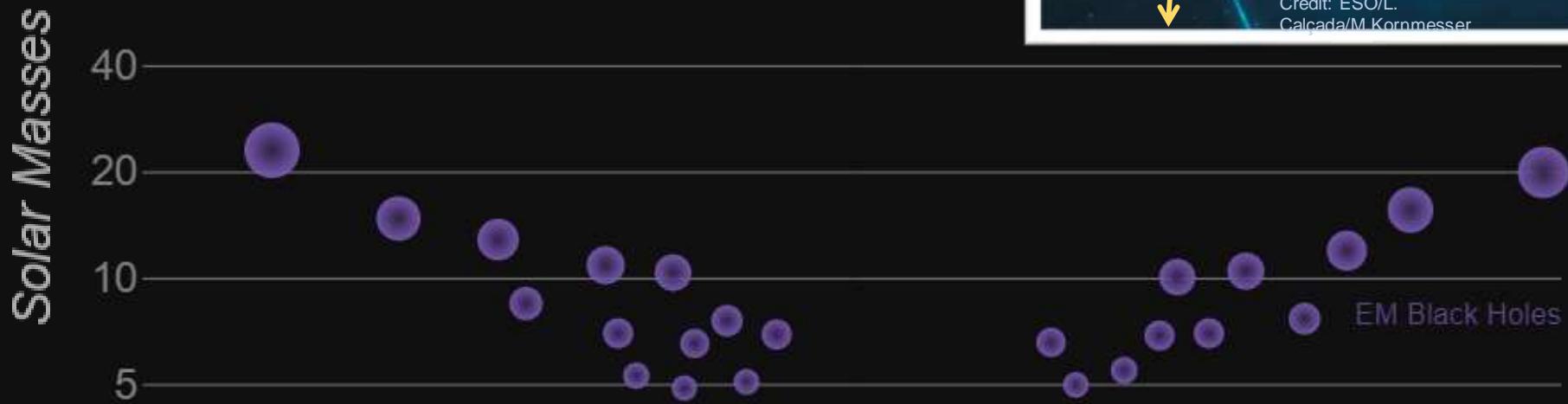
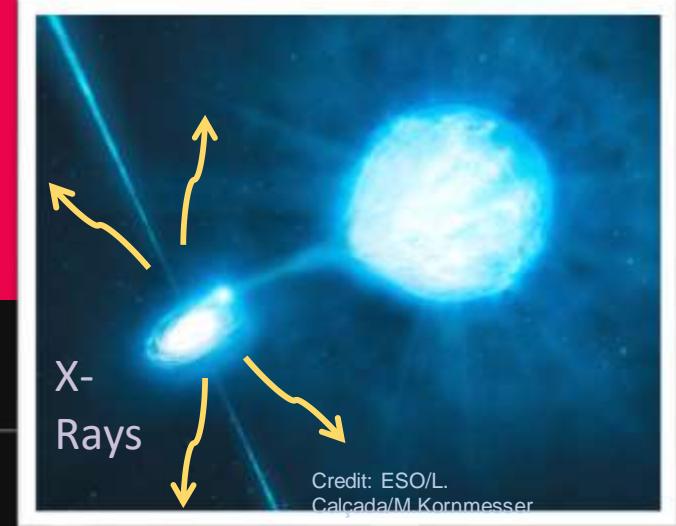


Q: The first detection of a stellar mass black hole in 1971 (through X-rays)

Bolton +72, Webster and Murdin+72



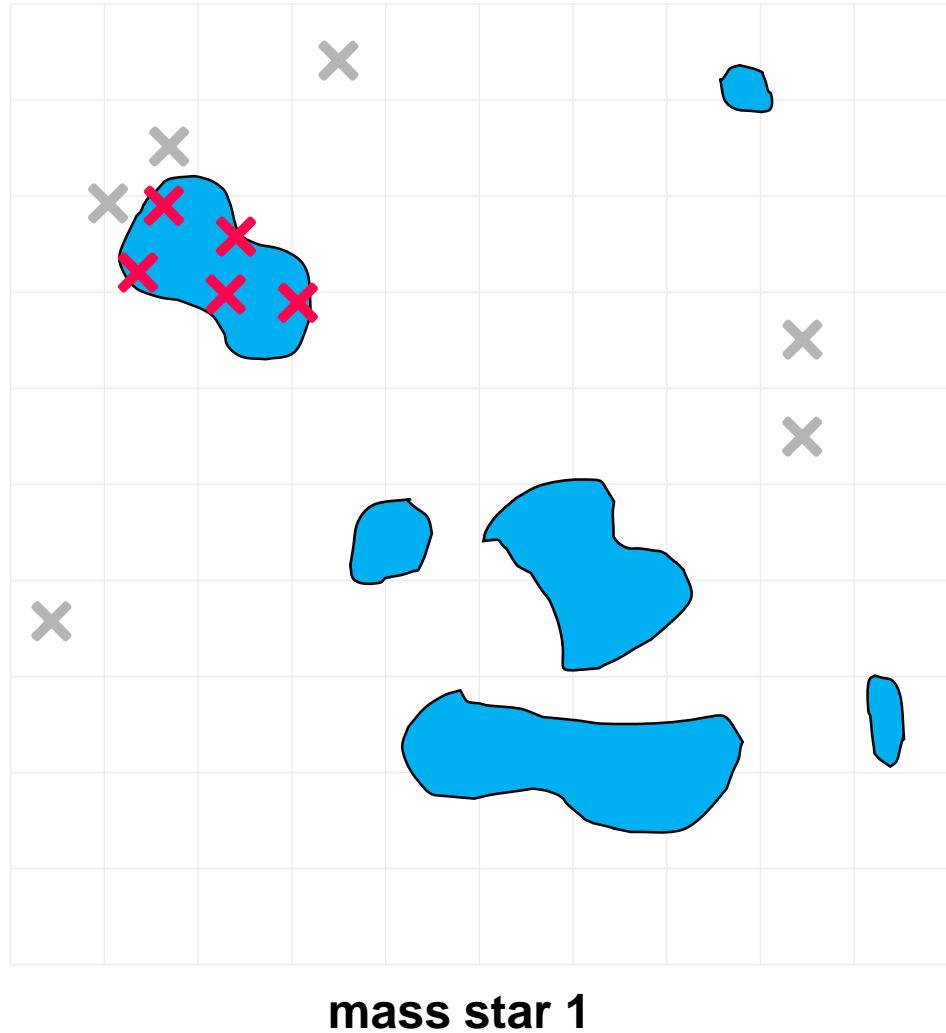
| Up to 2015:
~20 stellar BHs detected with
known masses



1. “*Adaptive learning*”



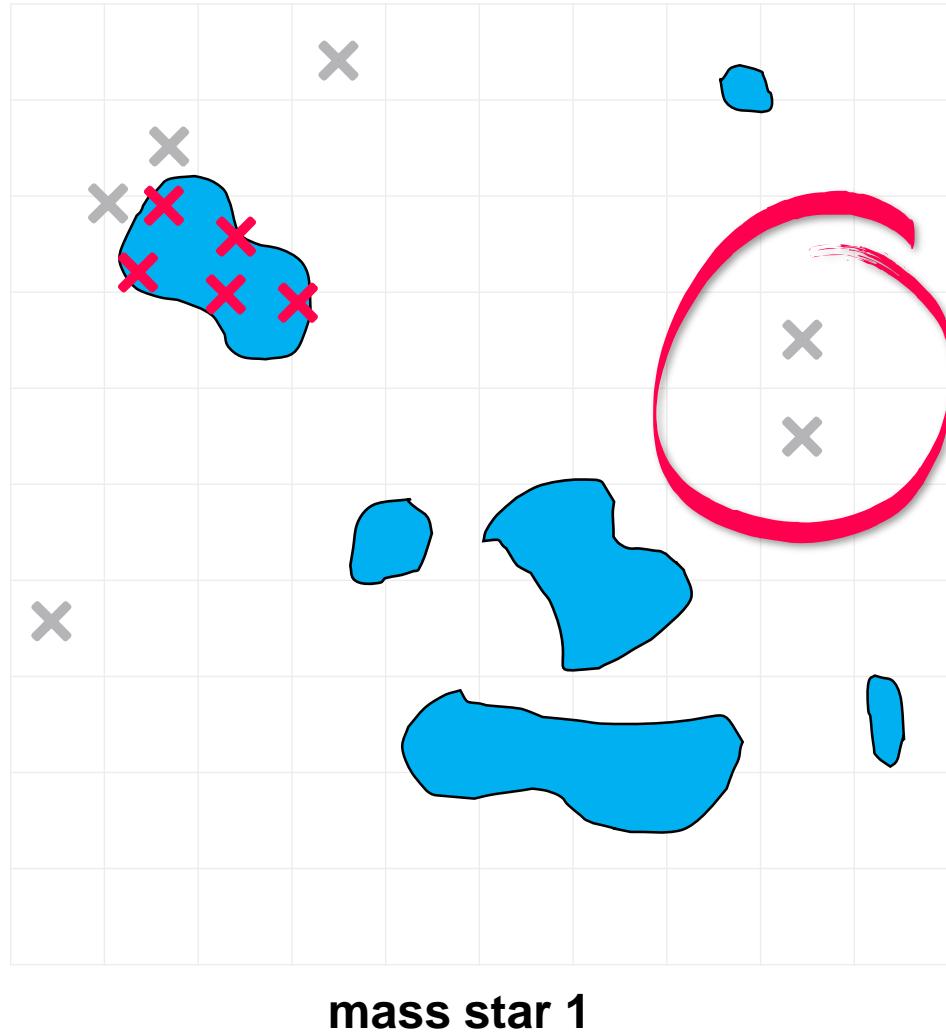
mass star 2



1. “*Adaptive learning*”



mass star 2



1. “Adaptive learning”



2. Latin hypercube (“Sudoku”) sampling

mass star 2

	9				6		
	6	3	4		2	9	
2	3		5		7		
	6	7					
	4			3		9	
					6	8	
		2		1	9	3	4
4	9			2	5	1	
	7			3	8		

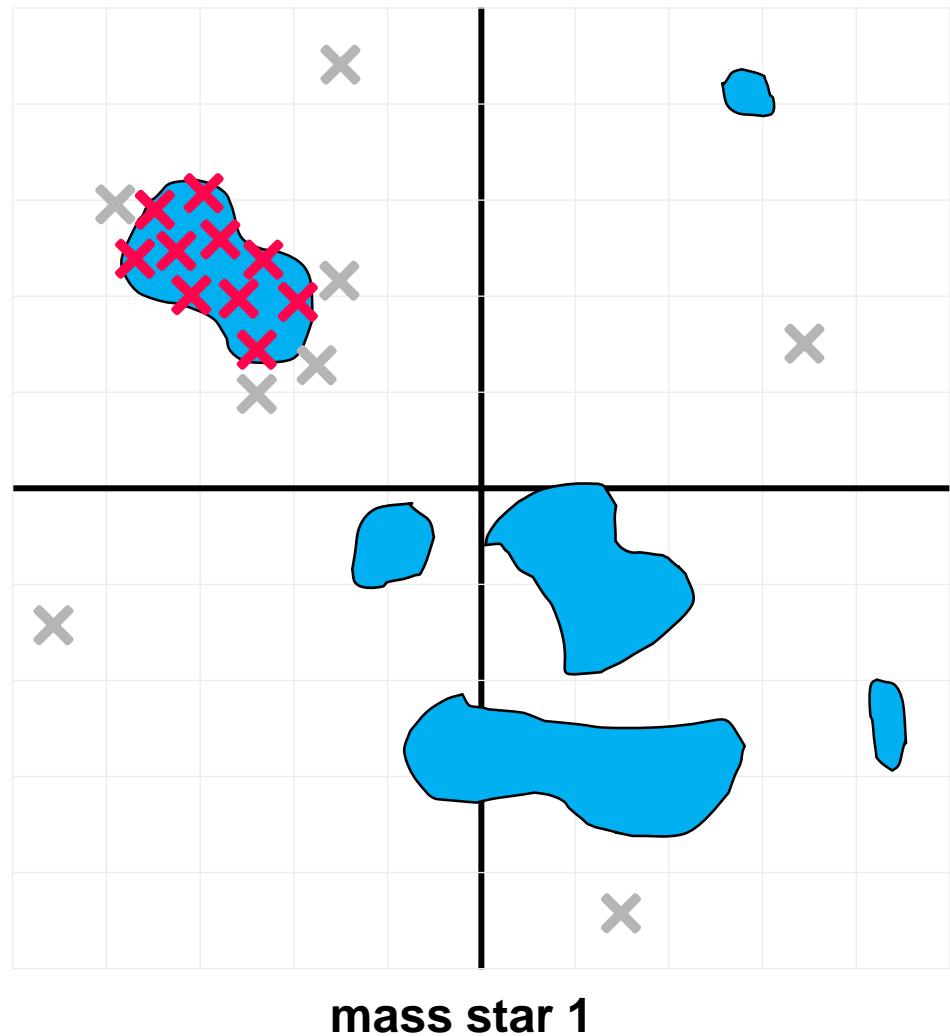
mass star 1

1. “Adaptive learning”



2. Latin hypercube (“Sudoku”) sampling

mass star 2



1. “Adaptive learning”



2. Latin hypercube (“Sudoku”) sampling

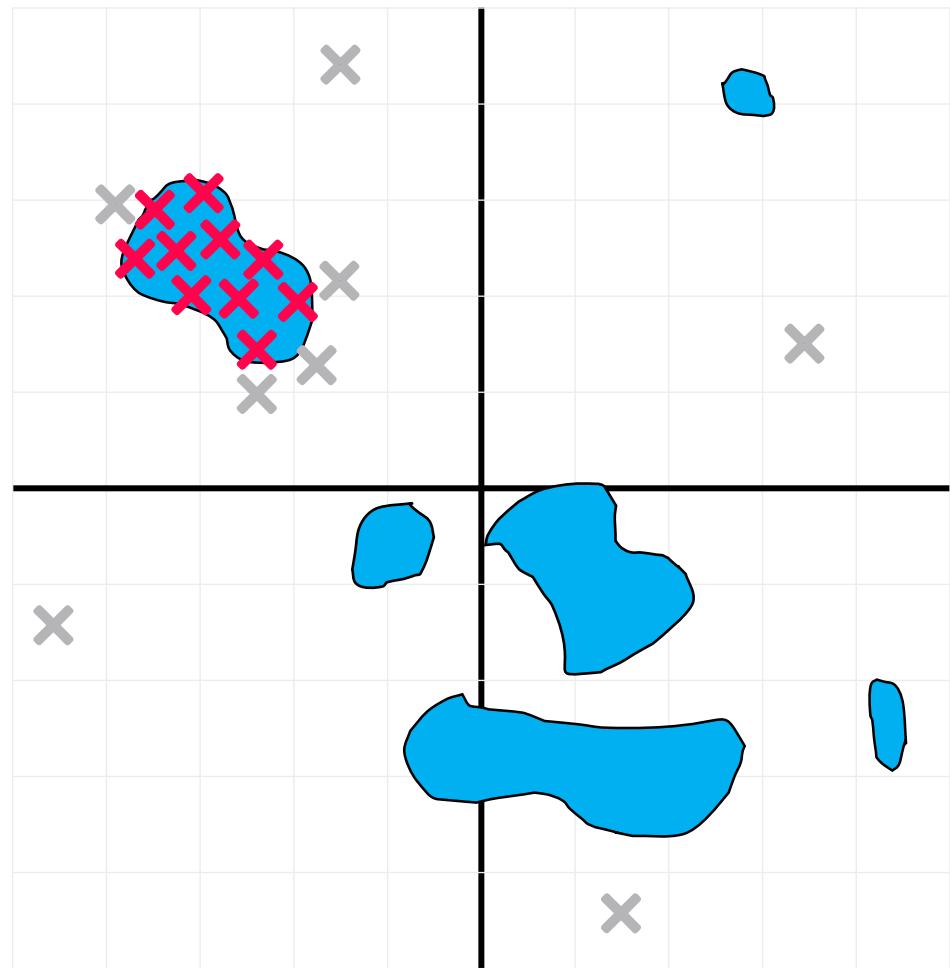
requires



to complete the game?

mass star 2

mass star 1



1. “Adaptive learning”



2. Latin hypercube (“Sudoku”) sampling

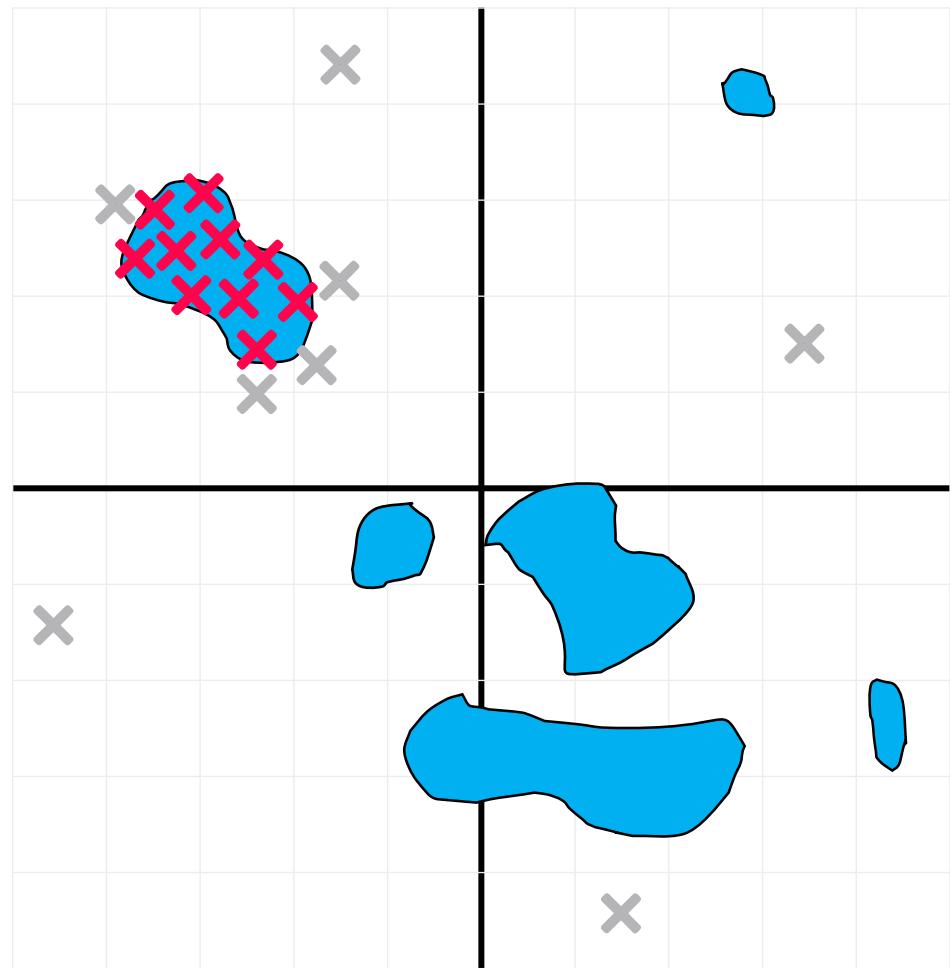
requires

< 61 >

to complete the game?

mass star 2

mass star 1

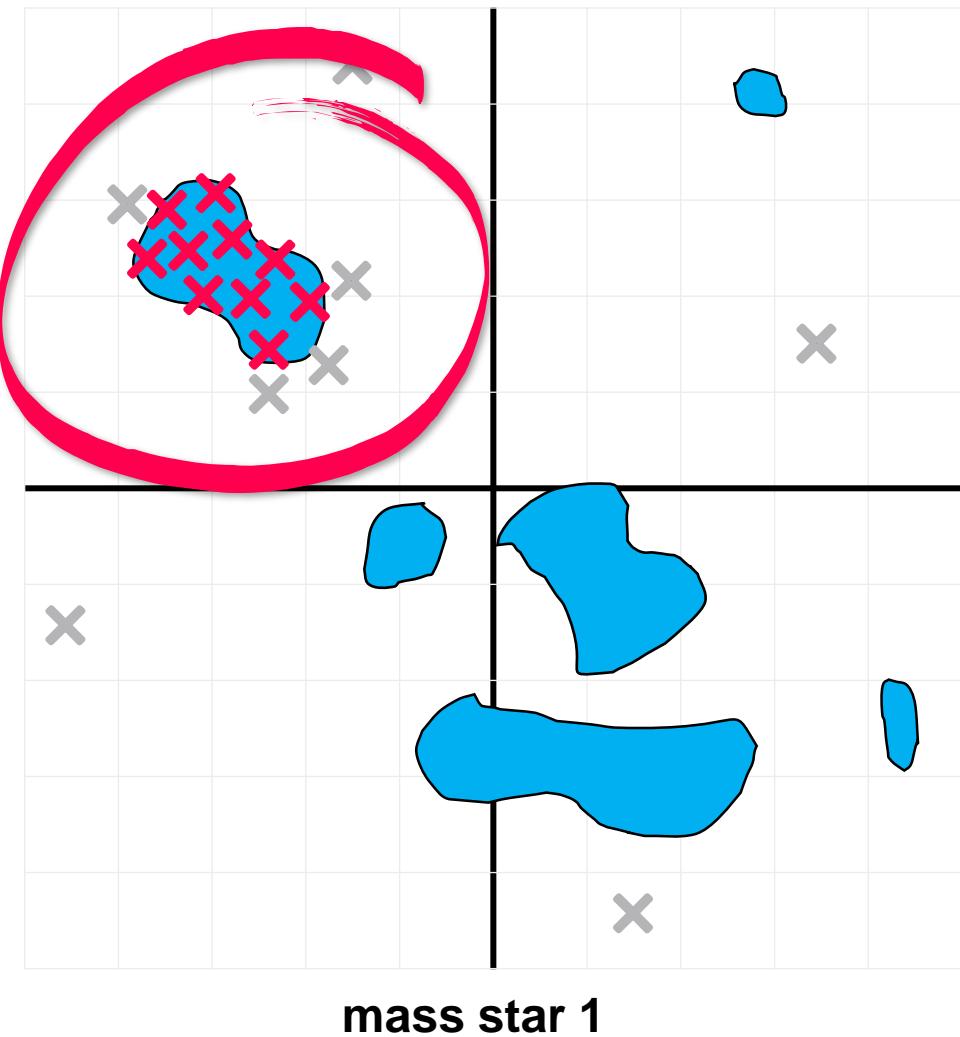


1. “Adaptive learning”



2. Latin hypercube (“Sudoku”) sampling

mass star 2



1. “Adaptive learning”

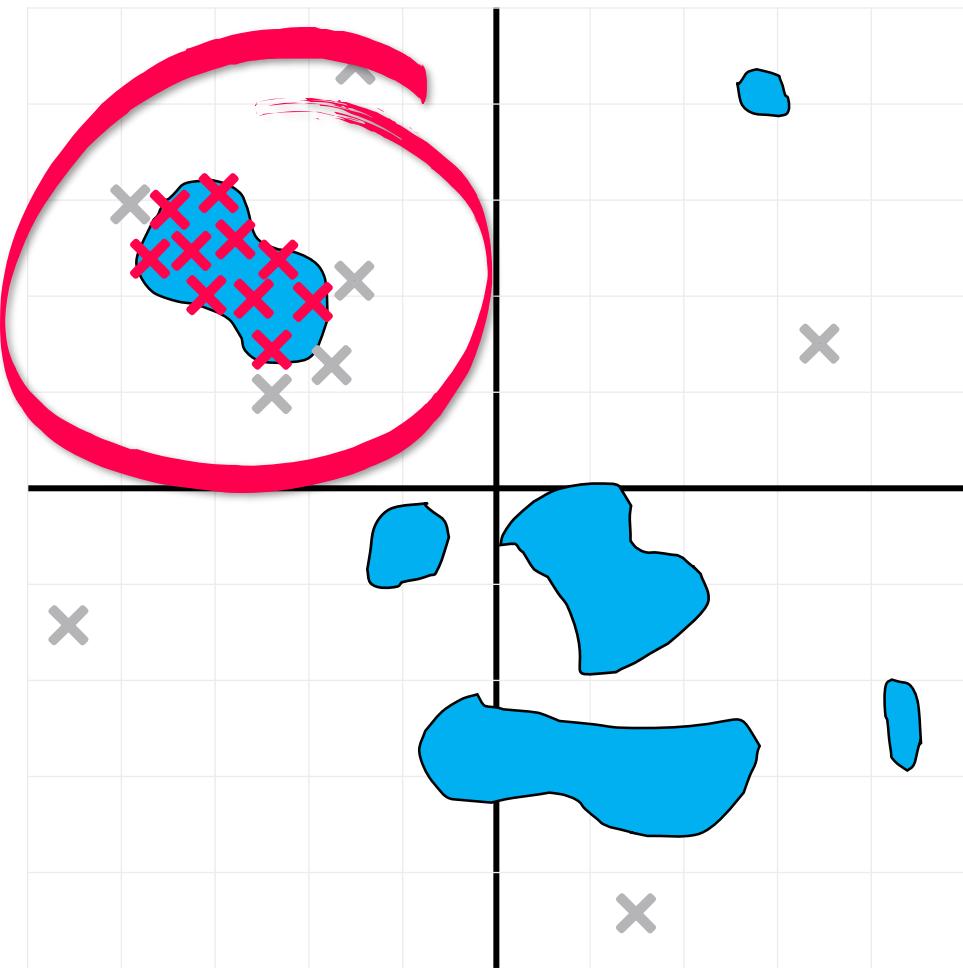


2. Latin hypercube (“Sudoku”) sampling

3. Predict outcome in between (“Emulators”)

mass star 2

mass star 1



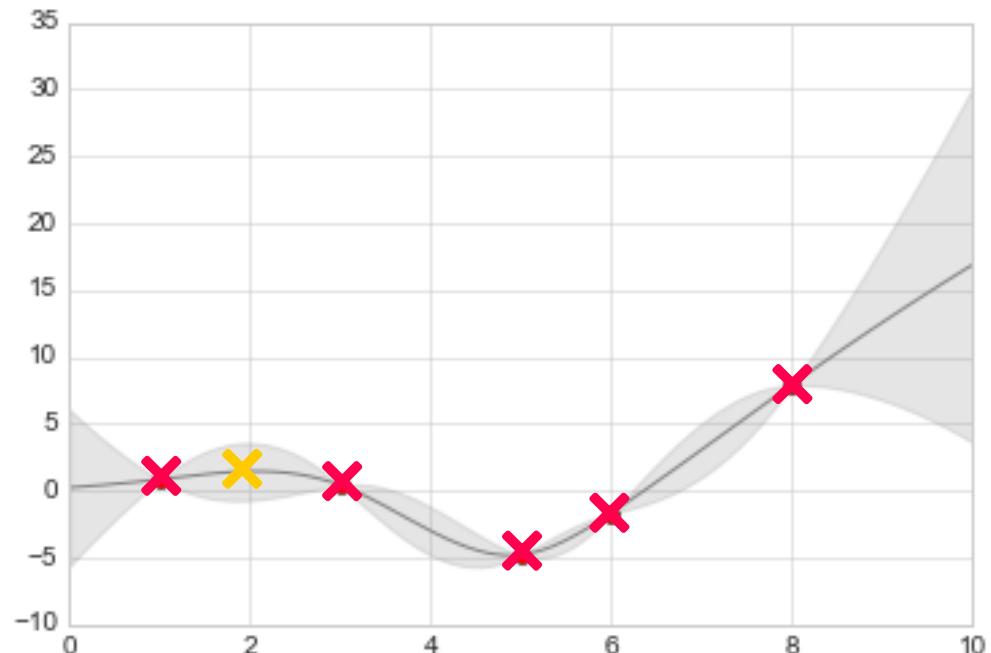
1. “Adaptive learning”



2. Latin hypercube (“Sudoku”) sampling

3. Predict outcome in between (“Emulators”)

Emulators



1. “Adaptive learning”

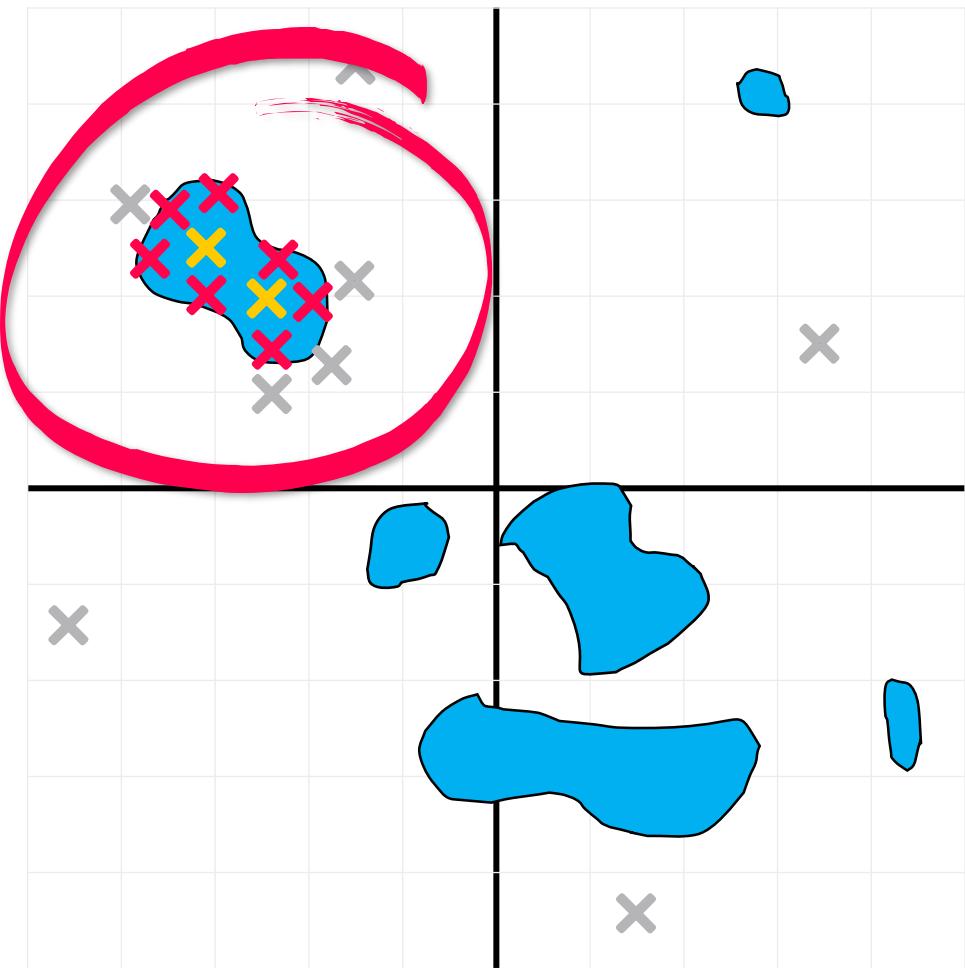


2. Latin hypercube (“Sudoku”) sampling

3. Predict outcome in between (“Emulators”)

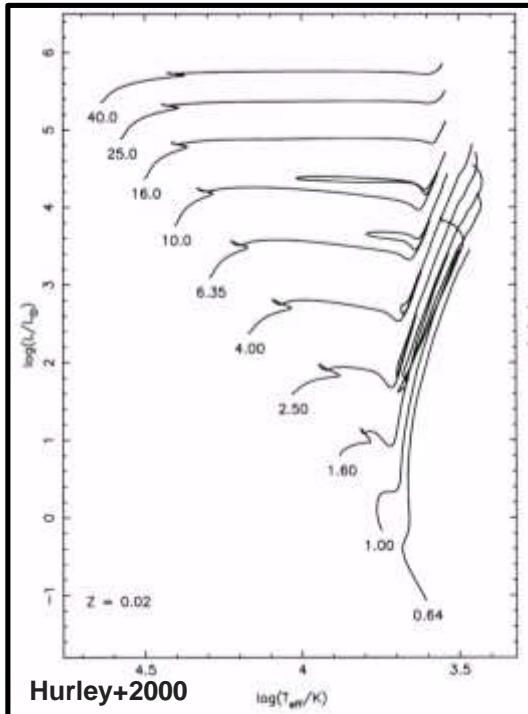
mass star 2

mass star 1



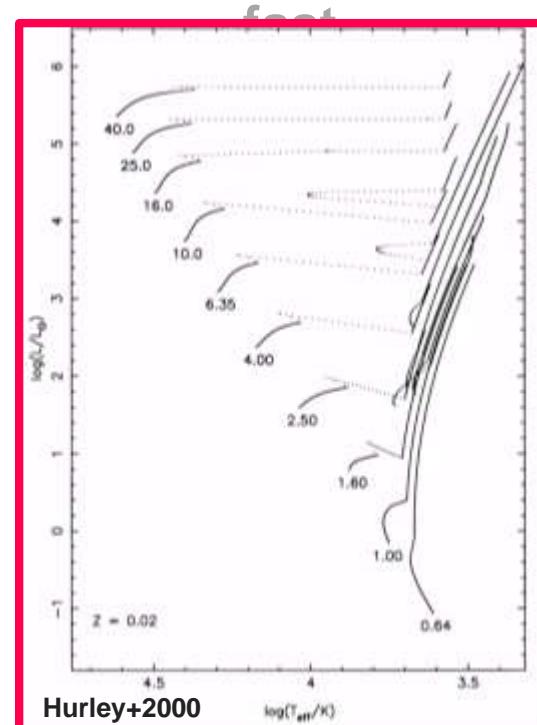
2 ways to implement (single) stellar evolution:

“Full” stellar calculations
relatively slow



e.g. MESA

Analytical fits or
interpolations



e.g. StarTrack, binary_c, COMPAS

Floor Broekgaarden